Assortative Matching and the Education Gap

Ximena Peña*
Georgetown University

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Abstract

This paper attempts to explain the decrease and reversal of the education gap between males and females. The education decisions of heterogeneous agents are modelled as an assignment game with endogenous types. In the first stage agents choose their education level and in the second they participate in the labor and marriage markets. Competition among potential matches ensures that the efficient education levels can always be sustained in equilibrium, but there may be additional equilibria with inefficient investments. Asymmetries intrinsic to the modelled markets (relative abundance of females and the observed evolution of the wage differential) generate dissimilar education decisions from sets of agents with identical cost distributions. The model theoretically reproduces the behavior of the observed education gap.

Key Words: Assortative matching, heterogeneity, efficiency.

JEL Classification Numbers: C78, D13, D61.

1 Introduction

In countries like the U.S., Colombia and Brazil, women have closed the education gap in college education and even surpassed men in college attainment, reversing the historical attainment advantage enjoyed by the latter. This paper studies the shaping forces behind the decrease and reversal of the education gap.

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xp@georgetown.edu, http://www12.georgetown.edu/students/xp

Education has returns in the two main markets young adults participate in: labor and marriage market. In the labor market education enhances productivity and thus generates higher returns via the wage rate. In the marriage market, if spousal attributes are complementary, people marry with a like education partner and thus education is a vehicle to match with a better type. This paper highlights the importance of the search for a spouse in the education decisions of both sides of the market[1].

The education decisions are modelled as an assignment game with endogenous types that can be described in two stages. The first stage is noncooperative: agents observe their education costs and simultaneously choose education investments. The returns to players are affected by the (equilibrium) investment choices of other agents. In the cooperative second stage agents match, produce "household good" and work. Assuming transferable utility, spouses bargain over the fraction of the household surplus each appropriates. Higher investments in education generate an improved set of potential matches and a (weakly) higher share of the surplus. The result of the model is a set of matched agents, a split of the surplus and a distribution of education across agents.

The decrease and eventual reversal of the education gap is explained by introducing asymmetries intrinsic to the modelled markets that affect the marginal benefit of education. Namely, the observed evolution of the gender (marginal) wage gap in the labor market and the relative abundance of females in the marriage market. Hence, the model generates asymmetric education decisions from sets of agents with identical cost distributions.

The classical paper in the marriage market literature belongs to Becker (1973). The author proves that complementarity of the inputs is a sufficient condition for matching to be positive assortative (PAM) in an frictionless environment.

Several models endogenize investment levels in a matching environment. Cole, Mailath and Postlewaite (CMP 2001a, 2001b) solve the hold-up problem by endogenizing investment specificity and introducing competition among agents for complementary investments. The model predicts the existence of multiple equilibria, and the ex-ante efficient levels of investment can always be attained given the optimal bargaining rule. Makowski and Ostroy (1995) consider a finite population model in which individuals choose occupations. Equilibria are efficient when an individual's benefit from an occupational choice coincides with the social contribution of that choice and there are no complementarities. The lack of complementarities rules out the coordination-failure inefficiency present in CMP. Pe-

[1] To keep the model tractable, I assume away the effect of education and marriage decisions on social outcomes such as fertility.

ters and Siow (2002) develop a model where parents invest in their children's wealth and spousal wealth is a public good in marriage. The authors find that when the marriage market is large, the hedonic return to investment internalizes the external benefits, and the competitive equilibrium is efficient.

This paper is closest to CMP (2001a), albeit in a different spirit, for the case of continuous best response functions. The contribution of the paper is to introduce asymmetries natural to the modelled markets, not exploited in previous papers, to reproduce the behavior of education attainment.

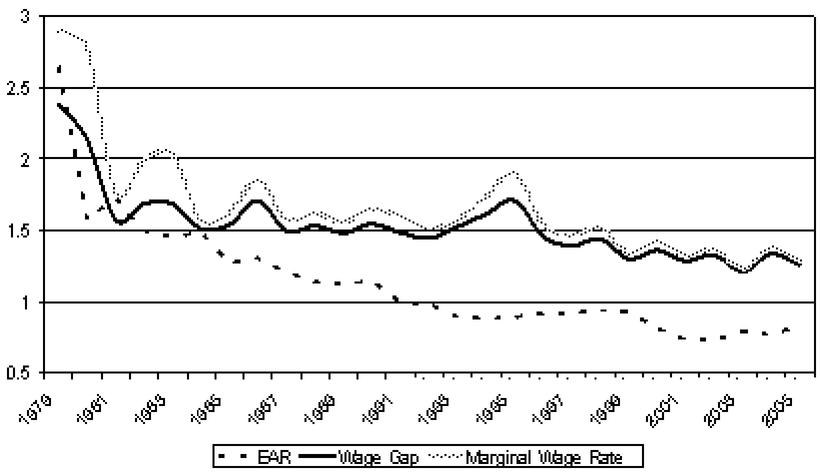
Other authors have attempted to explain some aspect of the education attainment ratio. Ríos-Rull and Sánchez (2002) focused on the puzzlingly high ratio of male to female college graduates in the U.S. in the late 1970's. They model parental investment decisions in children's education given potential gender differences in the costs of education and different sharing rules of household earning across parents. Echevarría and Merlo (1999) use a two-sex overlapping generations model where men and women of each generation bargain over consumption, number of children and investment in the education of their children, conditional on gender. The contribution of the present paper to this side of the literature is two-fold. On one hand, there are no papers to our knowledge that explain the decrease and reversal of the education gap. On the other, papers studying asymmetries in education choices have used the household decision problem while we model the problem from the individual's perspective. Since young adults *own* the college education decision so long as education costs are appropriately modelled, this seems a better approach.

The rest of the paper is organized as follows. The next section presents some stylized facts and a motivating example is worked out in Section 3. Section 4 develops the model, characterizes the equilibria, its efficiency and comparative statics results. Section 5 concludes. We appendicize all technical proofs and provide intuitive arguments in the main text.

2 Stylized Facts

Women have been consistently investing more in education than men, to the extent that they have reversed the historical schooling attainment advantage enjoyed by the latter. Until the late 1970's the ratio of college attainment of men to women was around 1.6 in the U.S. (Ríos-Rull and Sánchez, 2002). The education ratio experienced a dramatic decline in the recent years: 55 percent of U.S. college students today are women. This

Figure 1. Education Attainment Ratio, Wage Gap and Marginal Wage Gap (Male/Female, College+,25-40 years of age)



trend is not specific to developed countries since a similar situation is observed in several Latin American countries, including Brazil and Colombia. Figure (1) shows the education attainment ratio for individuals between 25 and 40 years of age with at least a college degree in Colombia[2]. The dashed line shows the ratio of males to females in the group, which consistently declined from 2.1 in the late 1970's to 0.79 in 2004. Staring in the early 90's women have surpassed men in their education attainment.

Despite a decreasing trend since the 1950's gender wage differentials persist in all industrialized nations, where gender wage differential is denifed as the ratio between average male wage over average female wage. In the U.S. it has decreased from 1.66 in the 1970's to 1.33 during the 1990s (Blau and Khan, 1992). A similar behavior is observed in developing countries. The thick solid line in Figure 1 shows the decrease in the wage gap in Colombia for agents with education of college or more: from over 2 in the early 80's to under 1.3 in 2005. Males are still paid higher level wages than females at any education level. Define the marginal wage rate as the difference in mean wage between agents with less than college and those with completed college or more, which measures the additional earnings associated to completing college education. The grey line in Figure 1 is the marginal wage

[2] Colombian calculations based on the September shift of the National Household Survey (NHS), significant for the 7 main cities.

2003				
Male\Female	Less than High School	Less than College	College+	Total
Less than High School	38%	11%	1%	49%
Less than College	10%	24%	4%	39%
College+	1%	4%	7%	12%
Total	49%	39%	12%	100%

Table 1: Assortative Matching (Colombia, 25-40 years of age)

gap, i.e. the ratio between marginal male and female wages. Marginal incentives move very closely to the observed change in the wage gap. Not only do women face lower wage *levels*, but also lower *marginal* wages. Labor market incentives suggest that men should be more educated. This makes the relative female over-education even more surprising.

Where are female incentives coming from? This paper includes the marriage market to capture two main effects: assortative matching across education levels and the effect of the relative scarcity of men. People do not marry randomly. Fernández, Guner and Knowles (2001) use household surveys from 34 countries to calculate the degree of correlation of spouses' education (marital sorting). They find that the average Pearson correlation between spousal education for the sample is 0.610, with a standard deviation of 0.106. Table (1) describes the education attainment of married females by the education attainment of their spouses in Colombia in 2003. Assortative mating within education classes is large since we observe that a the majority of matches fall along the main diagonal; the correlation coefficient between spousal education is 0.63.

Females face tougher competition than males in the marriage market. Factors affecting genders in a (potentially) different way such as death rates, imprisonment rates, immigration patterns or sexual orientation generate a relative scarcity of males in the marriage market. To measure the relative abundance of females we calculate the ratio between the fraction of matched males over fraction of matched females. Colombia data shows that for people between 20 and 65 years of age, this ratio was on average 1.15 for the 1979-2005 period, with a standard deviation of 0.02 and for agents between 25 and 40 years of age the figure is 1.07 with the same standard deviation. That is, 15% and 7% more males are matched than females, respectively. This generates increased competition and additional incentives for women to educate.

Summarizing, there is a puzzling decrease and reversal of the post-secondary education attainment ratio. Decreasing wage differentials and marginal wage gap suggest that still today men have more incentives to educate. Hence, the marriage market is introduced to study what the effects assortative matching and the relative scarcity of men have on the education decisions of agents.

3 Motivating Example

Let us motivate the model by an example. Assume a large population on each side of the market: males and females. This implies that a change of an attribute by a single agent on either side does not affect the division of matches unaffected by the attribute change. Also, an agent who increases his or her education level can match with other agents in the economy. Let's solve a parametrized Social Planner's problem for the marriage marlet in isolation, which is always supportable as an equilibrium to illustrate the matching and education investment decisions.

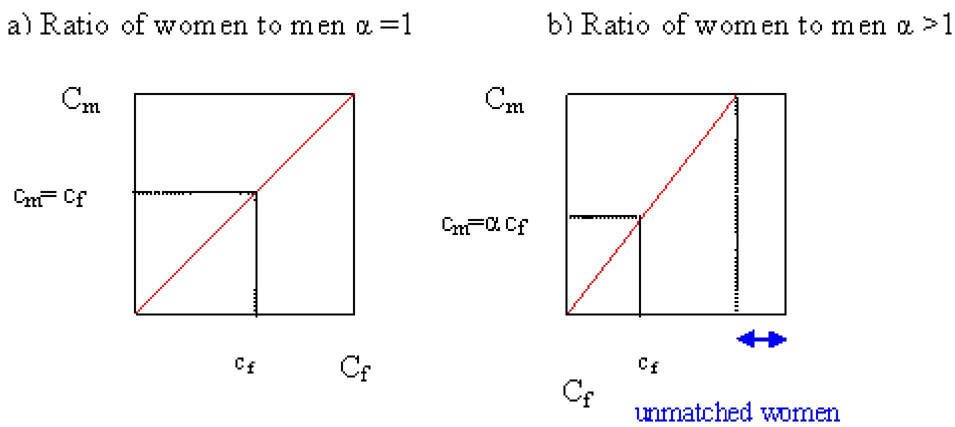
Females and males are indexed by their education costs c_f and c_m, which are uniformly distributed on $[\underline{c}, \overline{c}]$. Let $d_f(\cdot)$ be the education choice function of a female with cost c_f, mapping it into an education level x; $d_m(\cdot)$ is the education choice function of a c_m male, mapping costs into education level y. The cost for a female of acquiring education x is $c_f x$ and the cost for a male of acquiring education y is $c_m y$. The household production generated by an (x, y) couple is given by, $\pi(x, y) = x^{1/3}y^{1/3}$. Note that $\pi_x, \pi_y, \pi_{xy} > 0$. Aggregate surplus is maximized by the association of likes: i.e. lowest cost female matching with the lowest cost male and so on. This type of positive assortative matching (PAM) is a consequence of the complementarity of the couple's education levels ($\pi_{xy} > 0$).

What is the effect of a having more females than males? All potential matches will be made, starting with the lowest cost females, until there are no more available males; some high cost females will be left unmatched. The relative abundance of females is measured as the ratio of matched males over matched females: $\alpha \equiv \frac{\overline{c}-\underline{c}}{\widetilde{c}_f-\underline{c}}$ with $\widetilde{c}_f$ being the cost of the "last" matched female. An increase in α is equivalent to an increase of the ratio of females to males.

A matching is a rule associating a female to her mate. Given uniform cost functions, the matching is a straight line:

$$c_m = \mu^*(c_f) = \alpha c_f \quad \text{with} \quad \alpha \equiv \frac{\overline{c} - \underline{c}}{\widetilde{c}_f - \underline{c}}$$

Figure 2. Matching given Uniform Cost Distributions



Efficiency requires that for each c_f and c_m, education choices must solve:

$$\underset{x,y}{Max}\ \ x^{1/3}y^{1/3} - c_f x - c_m y \ \ \text{st.} \ \ c_m = \alpha c_f \tag{b}$$

The second order condition is satisfied and we have an interior maximum:

$$d_f\left(c_f\right) = \frac{1}{27\alpha c_f^3} \quad \text{and} \quad d_m\left(c_m\right) = \frac{1}{27\alpha^2 c_f^3}$$

The ratio of education attainment of men to women is $\frac{d_m(c_m)}{d_f\left(c_f\right)} = \frac{1}{\alpha}$. If $\alpha = 1$, then $\frac{d_m\left(c_f\right)}{d_f\left(c_f\right)} = 1$ and both sides of the market are equally educated. However, when $\alpha > 1$ for a given c_f, $\frac{d_m\left(c_f\right)}{d_f\left(c_f\right)} < 1$ and females are more educated than their respective matches, even for the lowest-cost pair. Conversely, if $\alpha < 1$, $\frac{d_m\left(c_f\right)}{d_f\left(c_f\right)} > 1$ and males are more educated. This simple example illustrates the subtle point thatin the marriage market an asymmetric gender composition of the population generates, through the matching, different levels of education investments between males and females.

4 Model

The education decisions are modelled as an assignment game with endogenous types. In the first stage agent's education investments are determined while in the second agents match and split the surplus.

Let $C = \langle C_f, C_m \rangle$ be the distributions of education costs that admit a density across females and males, respectively. At $t = 0$ agents sample a constant cost of education $c_i \in [0,1]$, $i = m, f$, which captures their natural ability and budget constraint. The set of individuals I is divided into disjoint classes: a set m of males and f of females on $[0,1]$. Let $d = \langle d_f, d_m \rangle$ be education choice functions; d_i maps education cost c_i into desired education level: $d_i : [0,1] \longrightarrow [0,1]$. Let x and y be the chosen levels of education for women and men, respectively. After educating, agents participate in the labor and marriage markets where they match, produce and bargain over the surplus.

In the second stage education choices have already been made and each agent is identified by their publicly observable education level. In the labor market agents face wage schedules $w = \langle w_f(x), w_m(y) \rangle$, such that w is C^2, increasing and concave[3]. Let us assume that in equilibrium all workers are employed[4].

A married couple, female x and male y, generates divisible output $\pi(x,y)$, where the surplus function π is C^2, symmetric, strictly increasing in x and y, and strictly supermodular (SPM) $\frac{\partial^2 \pi(x,y)}{\partial x \partial y} > 0$. We normalize: $\pi(x,0) = \pi(0,y) = \pi(x,\varnothing) = \pi(\varnothing,y) = 0 \;\; \forall x,y$, where $\varnothing$ means no match. Home production captures the quantity and quality of children, and the enjoyment of each other's company.

Marriage is individually rational. The total surplus of an (x,y) couple is given by the home production and labor market outcomes of spouses: $s\,(x,y) = \pi(x,y) + w_m(y) + w_f(x)$. Let π and w_f , w_m be such that s is strictly concave in x, y.

4.1 Second Stage

Being a two stage game, it is solved by backward induction. In stage 2, for given education levels, we have an assignment game: the allocation of the scarce resource, highly educated agents, to maximize total social surplus. An outcome is a set of matched pairs and a split of the surplus.

Let $v_f\,(x) \geq 0$ be the return to a type x woman and $v_m\,(y) \geq 0$ the return to a y man. Individuals choose the partner's education to maximize their share of household surplus

[3] Concavity of w_i is only needed to ensure that a single agent's problem is well-behaved.

[4] A wage gap implies that males receive wages that are weakly higher than females by education level: $w_m(y) \geq w_f(x) \; \forall \; x = y$. This assumption is not necessary to obtain the results in the model. What will prove important is a point-wise condition between the derivatives, i.e. $w'_m(y)$ vs. $w'_f(x) \; \forall \; x = y$. This will be discussed in Proposition 4.

while balancing two opposing effects: the joint surplus increases but so does the share of surplus appropriated by the spouse. Therefore, agents' payoffs $v = \langle v_f(x), v_m(y) \rangle$, are the upper envelope of the shares generated by the potential partners.

$$v_f(x) = \underset{y \in [0,1]}{Max} \{s(x,y) - v_m(y)\} \quad \text{and} \quad v_m(y) = \underset{x \in [0,1]}{Max} \{s(x,y) - v_f(x)\} \tag{1}$$

Even though marriage has to be mutually acceptable, in equation (1) agents maximize over all possible partners. This follows from the partner's share being endogenous[5]. The split of surplus captures what agents are willing to offer for different spouses. Since education levels are complementary, the own marginal product increases with spousal education and a low type male will always be outbid by a high type one for a female with high education level. Thus, competition generates mutually acceptable matches and an association of likes (Proposition 1).

A *matching* μ is a function $\mu : [0,1] \to [0,1] \cup \{\emptyset\}$, that associates to an x woman the education level of her mate $y = \mu(x)$ where μ is one-to-one on $\mu^{-1}(M)$ and $\emptyset$ is interpreted as no match. A couple who are not matched under μ, but who prefer each other to their assignments, can *block* the matching since by rematching and sharing the resulting surplus, they are strictly better off. In a *stable matching* there are no blocking pais.

A *stable bargaining outcome* is feasible[6], individually rational and satisfies $\forall x, y$:

$$\begin{aligned} v_f(x) + v_m(y) &= s(x,y) && \text{for matched couples} \\ v_f(x) + v_m(y) &\geq s(x,y) && \text{for unmatched couples} \end{aligned} \tag{2}$$

In a stable bargaining outcome, within a gender, agents with the same attributes receive equal (gross) payoffs: "equal treatment" in CMP. As a result, there are no blocking pairs.

Recall that education decisions are given in this stage and assume that the education choice functions d are strictly monotone (decreasing) and hence invertible[7]. A matching μ is *feasible* if:

$$d_f^{-1}(x) = d_m^{-1}(\mu(x)) \qquad \forall x \in [0,1] \tag{3}$$

[5]To see that the problem is well defined in this rendition, let us restate the it by first combining the two expressions in (1) and changing index: $v_f(x) = max_y\{s(x,y) + min_t\{-s(t,y) + v_f(t)\}\}$ and rewriting $v_f(x) = max_y min_t\{s(x,y) - s(t,y) + v_f(t)\}$.

[6]For the case of discontinuous attribute choices, see CMP's (2001a) definition of feasibility.

[7]This is shown in Proposition 3.

An equilibrium in the second stage, taking the education decisions as given, is a matching and a split of the surplus.

Definition 1 *Given d and C that admit a density a* ***matching equilibrium*** *is a pair (μ, v) such that:*

1. *Individuals maximize their share (Condition 1).*
2. *The bargaining outcome is stable (Condition 2).*
3. *The matching μ is stable and feasible (Condition 3).*

Studying the the Social Planner's problem (SP) is interesting since in this stage the first and second welfare theorems obtain (Proposition 0). Given d, the SP chooses the *efficient matching* μ to maximize social welfare. Let $G = \langle G_m, G_f \rangle$ be the education cumulative distributions of the matched males and females, respectively, induced by the education choice functions d.

$$S(d) = \underset{\mu}{Max} \int_0^1 s(x, \mu(x)) G_f(x) \quad s.t. \;\; d_f^{-1}(x) = d_m^{-1}(\mu(x)) \;\; \forall x \in [0,1] \tag{4}$$

Given SPM s, the matching is characterized by PAM. If the mass of females exceeds that of males, provided that all possible pairs are matched, low education women are left unmatched[8].

Gretsky, Ostroy and Zame (1992) -GOZ- prove that the SP's problem can be decentralized and hence equilibrium matchings are efficient. They also show that the associated matching pattern is equivalent to the existence of a stable bargaining outcome.

Proposition 0 *(GOZ, 1992) Given d, the Social Planner's problem can be decentralized, that is, the first and second welfare theorems obtain.*

Let us characterize the matching equilibrium. Given Proposition 0, as long as the education choice functions are monotone and continuously differentiable, there is an increasing, continuously differentiable and unique matching.

[8] This implication of the frictionless model does not fit well what we observe in the data: single women have different levels of education, and many of them are highly educated. If frictions were introduced in the marriage market, we would obtain a statistical version of this result, i.e. highly educated women are more likely to marry.

Proposition 1 *Given d, C that admit a density, and SPM s:*

1. *μ is increasing.*
2. *If d is strictly monotone then the matching is unique: $\mu(x) = d_m\left(d_f^{-1}(x)\right) \forall x \in [0,1]$. If d is C^1, so is μ.*

Existence of a stable split of the surplus between spouses is immediate from Proposition 0. Proposition 2 characterizes the stable bargaining outcome, and shows that it induces efficient education levels since agents internalize the returns to their investments.

Proposition 2 *For given d strictly monotone and C^1, C that admit a density and SPM s, we can show that:*

1. *For any stable bargaining outcome, v is strictly increasing and C^1:*

$$v_f'(x) = s_1(x, \mu(x)) \forall x \in [0,1] \quad \text{and} \quad v_m'(\mu(x)) = s_2(x, \mu(x)) \forall y \in [0,1] \tag{5}$$

2. *v is strictly concave if the following condition holds*[9]*: $-\frac{s_{11}(x,\mu(x))}{s_{12}(x,\mu(x))} > \frac{d_f^{-1\prime}(x)}{d_m^{-1\prime}(\mu(x))}$ $\forall x$.*

4.2 First Stage

Let us now turn to the determination of d. Recall that agents sample an education cost from distributions C that admit a density and choose the education level to maximize the (net) payoff:

$$V_f(c_f) = \underset{x \in [0,1]}{Max} \{v_f(x) - c_f x\} \text{ and } V_m(c_m) = \underset{y \in [0,1]}{Max} \{v_m(y) - c_m y\} \tag{6}$$

From Proposition 2 v is strictly increasing, strictly concave and differentiable and there is a unique solution to the agent's problem.

Proposition 3 *Given cost distributions C that admit a density*:

[9] Note that if s is convex in x, y, then v is convex.

1. $d = \langle d_f, d_m \rangle$ *are continuous, differentiable and strictly decreasing:*

$$d_f(c_f) = s_1^{-1}\left(c_f, \mu\left(c_f\right)\right) \quad \forall c_f \in [0,1] \quad \text{and } d_m(c_m) = s_2^{-1}\left(c_f, \mu\left(c_f\right)\right) \quad \forall c_m \in [0,1] \tag{7}$$

2. $V = \langle V_f, V_m \rangle$ *display the standard properties of value functions: continuity and strict concavity.*

As long as C admits a density, results stated in Proposition 3 ensure that the required conditions for Proposition 2 are met. On one hand, given that d is strictly monotone and C^1, v is C^2. In addition, we can restate the condition for strict concavity[10] of v_f as: $-s_{12}\left(x, \mu\left(x\right)\right) < s_{22}\left(x, \mu\left(x\right)\right) \forall x$. For the payoff function to be strictly concave, in absolute value, the cross-partial derivative needs to be greater than the own-second derivative. The intuition behind this is that given an increase in the spousal education level, the indrease in the marginal household surplus due to the complementarity of education levels needs to outweight the decreasing surplus growth rate.

An equilibrium in the game are education choice functions and a split of surplus such that a single player's education decision is a best response given the other players' choices and education distributions, in turn, are consistent with both stable payoffs and matching.

Definition 2 *Given C that admit a density, an* ***education equilibrium*** *(EE) is an array (d, v) such that:*

1. *For each c_f in the support of C_f:*

$$d_f(c_f) = \arg\max_{x \in [0,1]} \left\{ v_f\left(x\right) - c_f x \right\}$$

and for each c_m in the support of C_m[11]*:*

$$d_m(c_m) = \arg\max_{y \in [0,1]} \left\{ v_m\left(y\right) - c_m y \right\} \tag{8}$$

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[10] Recall that from Proposition 2 v_f is strictly concave if $-\frac{s_{11}(x,\mu(x))}{s_{12}(x,\mu(x))} > \frac{d_f^{-1\prime}(x)}{d_m^{-1\prime}(\mu(x))}$. Now, using expression (7) it can be restated as $-\frac{s_{11}(x,\mu(x))}{s_{12}(x,\mu(x))} > \frac{s_{11}(x,\mu(x))}{s_{22}(x,\mu(x))}$. Simplifying, as long as $-s_{12}\left(x, \mu\left(x\right)\right) < s_{22}\left(x, \mu\left(x\right)\right)$ $\forall x$, v_f is strictly concave.

[11] An alternative way to express this condition which evidences the equivalence with Nash Equilibirum is: $\forall c_i \in [0,1]$ and $\forall \widetilde{d}_i : V_i\left(d_i\left(c_i\right)\right) \geq V_i\left(\widetilde{d}_i\left(c_i\right)\right)$, $i = m, f$. Note that an education equilibrium is a Subgame Perfect Nash Equilibrium, since it survives backward induction.

2. (μ, v) *is a* ***matching equilibrium*** *given the* μ *induced by* d *and* C.

The efficient education decisions can be characterized using the SP problem, since the second welfare theorem holds for the whole game (Proposition 4). For given C that admit a density, efficiency requires that the matching $\mu^*(C)$ and education choice functions d^* must maximize net social welfare $S(C)$. Again, given SPM of S, the efficient matching implies PAM[12].

$$\begin{aligned} S(C) &= \underset{d^*,\mu^*}{Max} \int_0^1 [s(x, \mu(x)) - c_f x - {d_m^*}^{-1}(\mu(x))\, \mu(x)] dC_f \\ s.t.\ \mu(x) &= d_m^* \left(d_f^{*-1}(x) \right) \quad \forall x \end{aligned} \tag{9}$$

CMP solve the hold-up problem by introducing competition among agents. However, inefficient equilibria may exist as well due to a coordination failure: the absence in the other side of the market of agents with attributes that would induce the efficient education levels. We get a similar result in the context of the model.

Proposition 4 *Given* C *that admit a density*:

1. *There exists a unique solution to the Social Planner's problem* (d^*, μ^*).

2. *An efficient education equilibrium* (d, v) *exists.*

3. *If* $s_i(1,1) > 1,\ \ i = 1, 2$, *there may be an inefficient over-investment equilibria where all agents choose the maximum level of education.*

There is a unique solution to the SP's problem and the Theorem of the Maximum under convexity assumptions characterizes the solutions. The first order conditions for

[12] Kremer and Maskin (1996), assuming a discrete number of agents, find that given an asymmetric surplus function cross-matching around the median is more efficient than PAM. As the set of agents tend to infinity the *measure* of agents cross-matching tends to zero. Therefore, despite asymmetries in the marginal incentives, PAM is always efficient in this setting.

both the Social Planner's and the individual's problem are shown to be equivalent[13]; the SP's solutions are a subset of the decentralized equilibria and hence the set of decentralized equilibria is non-empty.

We now turn to comparative statics results. We introduce asymmetries that are natural to the modelled markets, decreasing marginal wage gap and relative abundance of females in the marriage market, and study their effect on the marginal return of education. The interaction between the two asymmetries theoretically replicates the decrease and reversal of the education gap.

First, let's formalize the way in which the marginal wage gap affects the education decisions of agents. The stylized facts suggest that the marginal wage for males is higher than for females. In the model, the marginal wage affects the marginal return to an education level. Therefore, given higher marginal payoff, a man would choose higher education levels than a woman of the same education cost (Lemma 1). The decreasing tendency implies that the relative incentives tended to equalize over time. However, men have higher marginal wages for the whole period and therefore more incentives to educate as compared to women.

Lemma 1 (*Marginal Incentives*) *For given $C_m = C_f$ that admit a density and an equal proportion of men and women, marginal labor incentives determine the education gap: if $w'_m(c_f) \gtreqless w'_f(c_f)$ then $\frac{d_m(c_f)}{d_f(c_f)} \gtreqless 1$.*

In the absence of other asymmetries labor market incentives, captured through a higher marginal wage for males, translates directly into an education gap favoring men.

A more challenging exercise has to do with changes in the relative abundance of agents and the effect on payoffs. The motivating example shows in a simple setting that as the ratio of females to males increases, so do incentives for females to educate through the matching.

When thinking the problem in terms of the decentralized solution, an initial intuition suggests that an increase in the relative abundance of women translates into a set of matched females described by a truncated female education distribution G_f . Given PAM, the highest education woman matches with the top education man, and so on, until there are no more men to match with. The truncation would happen at the point where the mass of matched males equals the mass of females. However, the actual effect is more

[13] Existence of an equilibrium can be established directly using a fixed point argument.

complex than the previous statement suggests. This is because education decisions, and hence education distributions, are equilibrium objects and thus changes in the relative abundance of females affect them: education decisions d determine the matching μ. Once matched, agents bargaing over the split of surplus v which in turn determines the education decisions d.

Let us state a some definitions and a simple result that will help distentagle the feedback effects described above and simplify the comparative statics.

The relative abundance of women is captured through a proportional increase in the number of females: there are $\alpha \in \mathbb{R}$ females per male for each education cost, with $\alpha \geq 1$. This definition is convenient because the education cost distributions remain equal between genders. Thus, results regarding asymmetric education decisions are not driven by differences in cost distributions.

The motivating example is easily solved because by stating the SP's problem we define a matching in education costs that avoids the feedback effect between education decisions and matching in education levels. We now do a similar thing.

Since all equilibrium matchings are efficient (Proposition 0), we can simplify the mutual causality between the matching and education decisions by defining a matching in education costs: μ^* is increasing and one-to-one and associates the education cost of her mate c_m to a c_f woman[14]. Both μ^* and μ describe the same assignment, they both associate the same "types" together, but the former describes the set of matched pairs in terms of the education costs while the latter does it using education levels. By using μ^* the matching is pinned down in terms of primitives of the model and we can identify the effect of changes in α on the education decisions, side-tracking the feedbacks between matching and education decisions.

The example shows that given uniform cost functions μ^* is a straight line. This observation is more general: μ^* is linear whenever the distribution of education costs is the same across men and women. Therefore, if females are twice as abundant -there are two females per male for each education cost-, μ^* will remain a straight line but the slope will double. This follows from the fact that cost distributions are equal together with the definition of changes in the relative abundance of women. Lemma 2 formalizes this statement.

Lemma 2 (*Affine Matching*) *For given $C_f = C_m$ that admit densities, if there are α females per male for each education cost, then $\mu^*(c_f) = \alpha c_f$ in the interval $\left[0, \frac{1}{\alpha}\right]$ and no*

[14]Recall that the SP chooses μ^* in the first stage.

match otherwise.

In the absence of asymmetries and given identical education cost distributions, education decisions are the same for both sides of the market. This implies that men and women are equally educated. Lemmas 1 and 2 lay the ground to formalize the effects of both labor and marriage market incentives. Introducing each asymmetry in isolation yields a clear cut result. On one hand, a marginal wage gap favoring men generates more educated males because it translates directly into higher relative marginal payoffs. On the other, having relatively scarse men yileds more educated women. The intuition behind this result is as follows: an increase in α is equivalent to having women distributed on a space that is more *dense*. This generates competition for a given female from other women with attributes that are *closer* to hers, generating increased competition. Tougher competition, which is captured through changes in the matching and hence through the allocated spouse, translates into higher education levels for women.

Proposition 5 *For given education cost distributions, $C_m = C_f$, that admit a density, let (d, v) be an EE.*

1. *In the absence of asymmetries ($w_m' = w_f'$ and $\alpha = 1$), $\frac{d_m(\mu^*(c_f))}{d_f(c_f)} = 1$ and there is no education gap.*

2. *If males face higher marginal wages for all education levels, ($w_m' > w_f'$ and $\alpha = 1$), there exists an EE $(\overline{d}, \overline{v})$ such that for all education levels $\frac{\overline{d}_m(\mu^*(c_f))}{\overline{d}_f(c_f)} > 1$ and males are more educated.*

3. *If there is a proportional increase in the number of females per male ($w_m' = w_f'$ and $\alpha > 1$), there exists an EE $(\overline{d}, \overline{v})$ such that $\frac{\overline{d}_m(\mu^*(c_f))}{\overline{d}_f(c_f)} < 1$ and females are more educated.*

The two asymmetries work in opposite directions: a marginal wage gap favoring men imply they should be more educated while the relative abundance of women provides additional incentives for females to educate as compared to males. It is clear that neither asymmetry in isolation is capable of reproducing the decrease and reversal of the education gap. Therefore, the interaction of labor and marriage market incentives is required to explain the proposed stylized fact, which is a direct consequence of the previous Proposition.

Corollary *Balancing out the labor and marriage market incentives the model theoretically reproduces the closing and reversal of the education gap: if* $w_m'(\alpha c_f) \gtrless w_f'(c_f)$ *then* $\frac{d_m(\alpha c_f)}{d_f(c_f)} \gtrless 1$.

The Corollary states the conditions under which either effect dominates: labor vs. marriage market incentives. This coincides with the story portrayed in the stylized facts section. The relative abundance of females has been stable throuhgout the time period, generating a latent incentive for females to purchase higher education levels than their male counterparts. However, labor market incentives captured through the marginal wage gap, were so high that they outbalanced the marriage market incentives and generated more educated males. As the marginal wage gap decreased over time, incentives for males receeded and marriage market incentives outweighted the labor market ones. Hence, during the final part of the period females had higher incentives and chose to educate more than males.

5 Conclusion and Extensions

The interaction between the two natural asymmetries from the labor and marriage markets can theoretically replicate the decrease and reversal of the education gap in this two-stage game. Being a one-shot game, dynamics are not modelled and features such as match dissolution cannot be addressed.

An interesting extension is to introduce frictions in the labor and marriage markets. In doing so, the wage differential will be endogenized rather than assumed since the continuation value in the labor market depends on the marriage market prospects, which are better for males than females given their relative scarcity. In addition, by introducing frictions, the model will no longer predict unmatched low-education females when the ratio of females to males increases, but rather we would have unmatched females of all education levels. The model with frictions could be taken to the data and tested, to determine whether it could reproduce real world data on participation rates, equilibrium unemployment, as well as matching the education distribution of unmatched agents in the marriage market. Also, it would shed light on how important are the effects of asymmetries in the demographic trends, namely the relative abundance of femlaes, on the education decisions of agents. The empirical difficulty of separately identifying the effect of changes in education costs and benefits will remain.

Omitted Proofs

Proposition 1

Proof.

1. Becker (1973) proved that SPM s is sufficient to obtain PAM in a frictionless setting: $\mu(x)$ is increasing.

2. In a feasible matching $\mu(x)$ the mass of males equals the mass of females at every education level $d_f^{-1}(x) = d_m^{-1}(\mu(x)) \quad \forall x$. Let d_f and d_m be strictly monotone (decreasing) and hence invertible. There exists a unique matching:

$$\mu(x) = d_m\left(d_f^{-1}(x)\right) \ \forall x \in [0,1] \tag{10}$$

   If d is C^1, so is μ: $\mu'(x) = \frac{d_f^{-1\prime}(x)}{d_m^{-1\prime}\left(d_m\left(d_f^{-1}(x)\right)\right)} > 0 \ \forall x \in [0,1]$.

■

Proposition 2

Proof.

1. Combining expression (2) for matched couples with the feasible matching we get: $v_f(x) + v_m(\mu(x)) = s(x, \mu(x))$. Differentiate this expression wrt $\mu(x)$ to get $v_m'(\mu(x)) = s_2(x, \mu(x)) > 0 \ \forall y \in [0,1]$. Now differentiate it wrt x: $v_f'(x) = s_1(x, \mu(x)) + [s_2(x, \mu(x)) - v_m'(\mu(x))]\mu'(x)$. Substituting the previous result we get $v_f'(x) = s_1(x, \mu(x)) > 0 \ \forall x \in [0,1]$.

2. Taking a derivative of $v_m'(\mu(x))$ wrt $\mu(x)$ from (5) we get: $v_m''(\mu(x)) = s_{22}(x, \mu(x)) < 0$. v_m is strictly concave and C^2.

   Now, taking a derivative of $v_f'(x)$ wrt x from (5) we have $v_f''(x) = s_{11}(x, \mu(x)) + s_{12}(x, \mu(x))\mu'(x)$. v_f is strictly concave if $v_f'' < 0$ which is equivalent to $-\frac{s_{11}(x,\mu(x))}{s_{12}(x,\mu(x))} > \mu'(x)$. Substituting $\mu'(x) = \frac{d_f^{-1\prime}(x)}{d_m^{-1\prime}(\mu(x))}$the expression becomes $-\frac{s_{11}(x,\mu(x))}{s_{12}(x,\mu(x))} > \frac{d_f^{-1\prime}(x)}{d_m^{-1\prime}(\mu(x))}$ $\forall x$. v_f has a continuous second derivative, and it is C^2 as long as d is C^1.

■

Proposition 3

Proof.

1. To show continuity of d, apply the Theorem of the Maximum under convexity assumptions (Sundaram, 1999).

   Taking first order condition of (6) we have $v_i'(d_i(c_i)) = c_i$ which implies: $d_i(c_i) = v_i'^{-1}(c_i)$. v_i is strictly concave and C^2. Then $d_i(c_i)$ is strictly decreasing ($v_i'' < 0$ and the inverse of a decreasing function is decreasing) and C^1 with derivative $d_i'(c_i) = v_i''^{-1}(c_i)$, $i = m, f$. Given condition (5), this can be stated in terms of the exogenous function s as $d_f'(c_f) = s_{11}^{-1}(c_f, \mu(c_f))$, and similarly for men.

2. Applying the Theorem of the Maximum under Convexity assumptions, $V = \langle V_f, V_m \rangle$ is continuous and strictly concave.

■

Proposition 4

Proof.

1. Recall the Planner's problem (9). Given that s is continuous and strictly concave (both properties are preserved under integration) and that the constraint set is convex, the FOCs are necessary and sufficient for a global optima: there is a unique solution to the Social Planner's problem (μ^*, d^*). Moreover, since the conditions for the Theorem of The Maximum under concavity (Sundaram, 1996) are met: the objective function is continuous and strictly concave and the constraint set is compact and continuous, (μ^*, d^*) are continuous functions. In addition, $S(C)$ is continuous and strictly concave.

2. From part 1 a solution to the SP's problem always exists. To show that an efficient

   equilibrium always exists, we will show that the FOCs of the SP's and decentralized problems are equivalent.

From Part 1 we know the FOCs are both necessary and sufficient for the Planner's problem. For the decentralized solution let us focus on the female education decision, given the constraints she faces in terms of payoffs and matching

$$\underset{x\in[0,1]}{Max}\ \{v_f(x) - c_f x\}$$
$$st\ \ v_f(x) = s(x,\mu(x)) - v_m(\mu(x))\ \ \forall x$$
$$\mu(x) = d_m\left(d_f^{-1}(x)\right)\ \ \forall x$$

Note that by Proposition 2 the objective function is strictly concave and continuous. The constraint set is compact, convex and continuous (Propositions 1 and 2). Therefore, the first order approach is both necessary and sufficient.

Substitute the definition of feasible bargaining into the objective function and let λ_x be the multiplier associated to a feasible matching (3) for each education level.

$$\underset{x\in[0,1]}{Max}\ \{s(x,\mu(x)) - v_m(\mu(x)) - c_f x\}\quad st\ \ d_f^{-1}(x) = d_m^{-1}(\mu(x))\ \ \forall x$$

Taking FOC wrt x :

$$s_1 + s_2\mu' - v_m'\mu' - c_f + \lambda_x\left[d_f^{-1\prime}(x) - d_m^{-1\prime}(\mu(x))\,\mu'\right] = 0 \tag{11}$$

Let us solve the Planner's problem (9) couple by couple. Then, we could just aggregate over agents (integrate). Optimality implies that for the Planner's solution the following holds:

$$s_1 + s_2\mu' - c_f - d_m^{-1\prime}(\mu(x))\,\mu(x)\,\mu' - d_m^{-1}(\mu(x))\,\mu' + \gamma_x\left[d_f^{-1\prime}(x) - d_m^{-1\prime}(\mu(x))\,\mu'\right] = 0 \tag{12}$$

The term in brackets, common to both expressions, holds with equality and drops out. Given that $d_m^{-1}(\mu(x))$ is the constant education cost of the mate of x, it's derivative is zero. From (5) we know that $v_m' = s_2(x,\mu(x))$ and so the FOCs are equivalent. It must be the case that $d = d^*$ and individual maximization yields the same education choices as the the Social Planner's. By Proposition 0 μ is an equilibrium matching iff it is efficient: the centralized and decentralized matchings coincide. This implies an efficient equilibrium always exists. ∎

3.

Proof. Let us derive conditions under which the trivial over-education equilibrium can be sustained: G are degenerate at 1 and all agents get the maximum education. For an agent to get the maximum education, from (6) the following condition must hold:

$$v_f'(x) > c_f \ \ \forall c_f \quad \text{and} \quad v_m'(y) > c_m \ \forall c_m$$

Therefore, rewriting this expression using equation (5) we get $s_1\left(x, \mu\left(x\right)\right) > c_f$ and $s_2\left(x, \mu\left(x\right)\right) > c_m$. For all agents to fully educate, given $c_i \in [0,1], i = m, f$, we need the following conditions to hold

$$s_i\left(1,1\right) > 1 \quad i = 1,2$$

The benefits of getting full education need to surpass the costs for every possible education cost, in particular, the highest one. Hence, for some parameter configurations, there are multiple equilibria[15].

Note that the previous condition, given the education distribution of agents, is equivalent to the SP's condition for optimality: in some cases it might be an efficient equilibrium for all the agents to fully educate. However, all other individuals fully educating might be the result of lack of coordination, rather than of efficient decision-making. Hence, when a single agent chooses given the that al other agents get full education he finds it profitable to get maximum education but had the SP made the decisions, the resulting education distribution would not necessarily be degenerate at full education, and the condition would not necessarily hold.

In addition, note that as long as the previous condition holds for every agent, over-education is an equilibrium, regardless of the relative abundance of females, that is, regardless of whether the deviating agents is matched on not since $s_i\left(1,1\right)$ is defined both for married and unmatched agents. ■

Lemma 1 (Marginal Incentives)

[15] An agent's education choice is the solution to equation (6). For an agents to choose not to educate the following condition needs to hold: $v_f'(x) < c_f \ \ \forall c_f$ and $v_m'(y) < c_m \ \forall c_m$.

Therefore, for the outcome "no education" for every agent in the economy to be an equilibrium, this condition needs to hold for every agent, including those with the lowest possible education cost. Since costs are normalized $c_i \in [0,1]$, it should be the case that $v_i'(\cdot) < 0$. This is a contradiction since by Proposition 2 v_i is strictly increasing, $i = m, f$.

Proof. Agents choose an education level to maximize their share of surplus (condition 6). Given symmetric π, for a pair such that $c_f = c_m$, $w'_m(c_f) \gtrless w'_f(c_f)$ implies that $s_2(c_f, c_f) \gtrless s_1(c_f, c_f)$ and by equation (5) we can write it as $\frac{v'^{-1}_m(c_f)}{v'^{-1}_f(c_f)} \gtrless 1$. Rewriting the education gap, using condition (7) we get $\frac{d_m(c_f)}{d_f(c_f)} = \frac{v'^{-1}_m(c_f)}{v'^{-1}_f(c_f)}$. It follows that marginal labor incentives determine the education gap: if $w'_m(c_f) \gtrless w'_f(c_f)$ then $\frac{d_m(c_f)}{d_f(c_f)} \gtrless 1$ ■

Lemma 2 (Affine Matching)

Proof. Recall that men and women can be described by two ordered disjoint sets m and f on $[0,1]$. Given $C_f = C_m$, define an education cost function assigning an education cost to each indexed agent in a set, for example: $c : i \to C_i$, $i = m, f$. Let P be a probability function on $[0,1]$ such that for every subinterval of education costs $[a,b]$, $P(\bigcup_{a<i<b} c^{-1}(i))$ is the probability of the set of men (women) with an education cost between a and b. Now, with education costs c', c'', $\mu^*(c') = c''$ iff the probability of men with education cost less than or equal to c'' equals that of the set of females with education costs less than on equal to c'. This implies $P(\bigcup_{i<c'} c^{-1}(i)) = P(\bigcup_{i<c''} c^{-1}(i))$, and by the property of the P function these probabilities are, respectively: c' and c'', so $c' = c''$, hence $\mu^*(c') = c'$.

Now, if there are α females per male for each education cost, then the probability of the set of women with an education cost in $[a,b]$ is $\alpha P(\bigcup_{a<i<b} c^{-1}(i))$. Following the previous argument, $\mu^*(c') = \alpha c'$.Note that women with education costs in $\left[\frac{1}{\alpha}, 1\right]$ will not be matched, since all available men are already matched with lower cost women. Therefore, $\mu^*(c_f) = \alpha c_f$ in the interval $\left[0, \frac{1}{\alpha}\right]$ and no match otherwise. ■

Proposition 5

Proof.

1. Since $\alpha = 1$, by Lemma 2 $\mu^*(c_f) = c_f$. If for all education levels and $w'_m = w'_f$, following a similar argument as in Lemma 1, by this implies that $\frac{\bar{d}_m(c_f)}{\bar{d}_f(c_f)} = 1$: men and women are equally educated.

2. Since $\alpha = 1$, by Lemma 2 $\mu^*(c_f) = c_f$. If for all education levels $w'_m(c_f) > w'_f(c_f)$, by Lemma 1 $\frac{\bar{d}_m(c_f)}{\bar{d}_f(c_f)} > 1$ and men are more educated.

3. Since μ^* and μ are equivalent, let us rewrite the education gap by combining the individual's education choice functions (7), the matching in costs and results from Lemma 2: $\frac{d_m((\alpha c_f))}{d_f(c_f)}$. By Proposition 3 d_m is decreasing. Thus, if $\alpha > 1$ then $\frac{d_m((\alpha c_f))}{d_f(c_f)} < 1$ and women are more educated.

■

Corollary

Proof. For given $\overline{\alpha} > 1$, if $w_m'^{-1}(\overline{\alpha} c_f) > w_f'^{-1}(c_f)$ following a similar logic to Lemma 1 the labor market incentives dominate and males are more educated: $\frac{\overline{d}_m(\overline{\alpha} c_f)}{\overline{d}_f(c_f)} > 1$. As the marginal wage gap decreases and the condition reverses, $w_m'^{-1}(\overline{\alpha} c_f) < w_f'^{-1}(c_f)$, the marriage market incentive dominates and women are more educated: $\frac{\overline{d}_m(\overline{\alpha} c_f)}{\overline{d}_f(c_f)} < 1$. ■

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