DOCUMENTO CEDE 2007-21
ISSN 1657-7191 (Edición Electrónica)
SEPTIEMBRE DE 2007

CEDE

HUMAN CAPITAL FORMATION, INEQUALITY, AND COMPETITION FOR JOBS*

DANIEL MEJÍA†
MARC ST-PIERRE‡

Abstract

This paper develops a model where heterogeneous agents compete for the best available jobs. Firms, operating with different technologies, rank job candidates in the human capital dimension and hire the best available candidate due to complementarities between the worker's human capital and technologies used in the production process. As a result, individuals care about their relative ranking in the distribution of human capital because this determines the firm they will be matched with and therefore the wage they will receive in equilibrium. The paper rationalizes a different channel through which peer effects and human capital externalities might work: competition between individuals for the best available jobs (or prizes associated with the relative position of individuals). We show that more inequality in the distribution of endowments negatively affects aggregate efficiency in human capital formation as it weakens competition for jobs between individuals. However, we find that the opposite is true for wage inequality, namely, more wage inequality encourages competition and, as a result, agents exert more effort and accumulate more human capital in equilibrium.

Keywords: Human Capital, Inequality, Competition, Relative Ranking.

JEL Classification: J24, J31, O15, D33.

---

* The authors wish to thank seminar participants at Brown, Universidad de los Andes, and Universidad del Rosario as well as Luis Eduardo Arango, Oded Galor, Peter Howitt, Glenn Loury, Darío Maldonado, Malhar Nabar, Carlos Esteban Posada, and Felipe Valencia for many helpful comments and suggestions. Lydia Boroughs kindly helped us with the numerical methods employed in the simulations of the model. The usual disclaimer applies.
† Corresponding author. Department of Economics, Universidad de los Andes, Bogotá, Colombia. Email: dmejia@uniandes.edu.co
‡ Brown University and University of the South. E-mail: Mastpier@sewanee.edu

CAPITAL HUMANO, DESIGULADAD Y COMPETENCIA POR PUESTOS DE TRABAJO

Resumen

Este artículo desarrolla un modelo en donde agentes heterogéneos compiten por los mejores puestos de trabajo disponibles en el mercado laboral. Las firmas, que operan con diferentes tecnologías, hacen un ranking de los individuos en el mercado laboral (en la dimensión del capital humano) y contratan al mejor candidato disponible debido a complementariedades en la producción entre el capital humano de los trabajadores y las tecnologías utilizadas en la producción. Como resultado de esto, los individuos se preocupan por su posición relativa en el ranking ya que ésta determina la firma con la que terminarán emparejados en equilibrio y, por lo tanto, el salario que recibirán. Este artículo racionaliza un canal diferente mediante el cual los efectos de compañeros de grupo (peer effects) pueden funcionar: competencia entre los individuos por los mejores puestos de trabajo (o premios asociados con la posición relativa de los individuos en el ranking). El artículo muestra que la mayor desigualdad en las dotaciones iniciales necesarias para la acumulación de capital humano afecta de manera negativa la eficiencia agregada en la formación de capital humano ya que desincentiva la competencia entre los individuos. Sin embargo, la mayor desigualdad en los salarios (retornos a la educación) afecta de manera positiva la eficiencia agregada en la formación de capital humano ya que incentiva a los individuos a ejercer mayor esfuerzo y a acumular mas capital humano como consecuencia de la mayor competencia por los mejores puestos de trabajo.

Palabras clave: Capital Humano, Desigualdad, Competencia, Ranking Relativo.

Clasificación JEL: J24, J31, O15, D33.

1. Introduction

This paper develops a model of human capital accumulation and competition for jobs where there are strategic interactions between heterogeneous agents that compete for the best available jobs. We argue that higher inequality in the distribution of t he endowments necessary to accumulate human capital negatively affects aggregate efficiency in human capital formation. This effect is beyond the standard Jensen's inequality channel because inequality also affects individuals' incentives to accumulate human capital when they confront competition from their close peers for the best available jobs. Intuitively, as the mass of close competitors for any given job position increases, the incentives to differentiate from each other, by exerting higher effort and accumulating more human capital, also increases. However, more wage inequality (i.e. more inequality in the returns to human capital accumulation) has the opposite effect, fostering competition for job positions between individuals, and, by doing so, increasing aggregate efficiency in human capital formation. In equilibrium, individuals' optimal choices depend on both, the distribution of endowments that are complementary to time and effort invested in the accumulation of human capital (the distribution of opportunities), and on the distribution of wages (the distribution of returns to human capital accumulation). As we will show, changes in the degree of inequality in each of these distributions have opposite effects on individuals' choices and on aggregate efficiency in human capital formation.

One of the main working assumptions in the model we develop in this paper is that there are heterogeneous firms that operate with different technologies. Our objective is not to explain why this happens in equilibrium[1] but, rather, how wage dispersion across firms (due to the firm's use of different technologies in the production process) affects individual's incentives to accumulate human capital and, therefore, aggregate efficiency in human capital formation and in production.

On the demand side of the labor market, we will assume that firms, operating with different technologies, rank individuals in the human capital dimension and hire the best available candidate due to complementarities between the worker's human capital and technologies used in the production process. On the supply side, in choosing the optimal level of investment in the accumulation of human capital, individuals take two effects into

[1] That is, why different firms operate with different technologies. The reader is refered to Caselli (1999) and Acemoglu et al. (2001) for possible explanations.

account when evaluating the marginal benefit from exerting one extra unit of effort in the accumulation of human capital. The first effect is the usual direct marginal increase in income that results from a marginal increase in human capital (as in a standard model *à-la*-Becker (1964)). The second effect comes from the marginal change in the relative position of the individual that results when she invests one extra unit of time and effort in the accumulation of human capital, which, in turn, determines her relative position in the human capital distribution and, thus, the wage she will receive in equilibrium. As a result of this last effect, there is a so-called "rat-race" where individuals try to out-compete other individuals for the best available jobs. Although more effort is exerted in equilibrium when individuals compete with each other for the best available job positions than in a standard model where they do not compete[2], this so-called 'excessive competition' increases aggregate efficiency in human capital formation (and production) because the pricing of human capital in the labor market fully compensates individuals for their extra investment. That is, we will assume that there is a perfectly competitive labor market where individuals' human capital is remunerated according to its marginal product. In fact, when making the optimal decision on the amount of investment in human capital, individuals trade-off the disutility from exerting more effort in the competition for jobs for the greater utility they obtain from being able to match with firms that operate with better technologies and, hence, pay higher wages. If labor markets were not fully competitive and, for instance, wages were determined by Nash bargaining between firms and employees, then the excessive competition would not be fully efficient.[3]

Our model assumes that individuals' concerns for relative ranking are instrumental. That is, individuals care about their relative position in the distribution of human capital not because they derive utility from relative ranking *per se*, but because their relative position determines the wage they will receive in equilibrium. On the other hand, we will assume that firms care about the relative ranking (in the distribution of human capital) of the individual they hire because the technologies they use in production are complementary to the worker's human capital.[4]

The literature on how inequality affects human capital formation has focused mostly on

[2] Because, for instance, they take their rank in the distribution of human capitalas as given.

[3] See Moen (1999).

[4] We don't explicitly model the process by which firms choose the worker they hire, but, instead, assume that due to complementarities in production, all firms would like to hire the best available candidate in the labor market. The individual that ranks first in the distribution of human capital will accept the offer from the firm operating with the most advance technology (because it pays the highest wage). Then, the best available candidate for the second firm is the individual ranking second in the distribution, and so on and so forth.

the role of credit market imperfections, wherein relatively poor individuals face financial constraints to pay for the costs associated with human capital accumulation, as they cannot use future earnings as collateral for the loans necessary to cover these costs. Furthermore, if there are decreasing returns to human capital accumulation, it is precisely these individuals (the poor) who have the largest returns to resource investments in education. As a result, a redistribution of resources from rich to poor individual would increase aggregate efficiency in the accumulation of human capital because of the reallocation of resources towards more profitable investments. This theoretical idea has been extensively developed in the literature since the work of Galor and Zeira (1993) and Banerjee and Newman (1993). Other developments have been proposed by De Gregorio (1996) and Bénabou (1996, 2000).[5] Empirical evidence has been found in favor of the hypothesis that inequality affects human capital accumulation in the presence of credit constraints (see Flug et al., 1998 and De Gregorio, 1996). In a recent paper, Mejía and St-Pierre (2007) show that inequality in the endowments that are complementary to effort in the schooling process (inequality of opportunities) affects aggregate efficiency in the accumulation of human capital without relying on credit market imperfections. The argument in that paper is that there are crucial complementary factors to the schooling process that are non-purchasable when the time for making investment decisions in education comes (i.e. parental schooling level, pre and post natal care, etc.). Because there are decreasing returns to time investment in human capital accumulation, and time investment in education is complementary to these factors, more inequality negatively affects aggregate human capital. Other papers in the literature have also explored political economy channels through which inequality affects human capital formation and economic growth. In particular, Glomm and Ravikumar (1992) and Ferreira (2001) emphasize the choice of public versus private schooling made through a political process as a key determinant of how inequality affects human capital formation (see Glomm and Ravikumar, 1992, and Ferreira, 2001).

The main contribution of this paper is to provide a rationale for a new, perhaps complementary, channel through which the inequalities of endowments and returns affect the incentives for human capital accumulation. An important difference with the existing literature is that the model we propose in this paper includes strategic interactions between individuals. That is, an individual's return from the accumulation of human capital depends not only on his own choices and on the production technologies, but also on the entire distribution of endowments and returns. In other words, we argue that in deciding the optimal investment in human capital formation, there are strategic interactions be-

[5] See Aghion et al. (1999) for a throrough review of the literature.

tween individuals. In this respect our model is also related to existing works on human capital externalities, and to the literature on peer effects in education. While most of the empirical literature on peer effects has focused on the effect of average education of peers on different measures of educational attainment of each student in a given class (that is, on linear-in-means peer effects), two recent papers find that, in fact, the structure of peer effects is highly non-linear. That is, students benefit differently from the inclusion of a new student in the class depending on their relative position in the class and the relative position of the entering student. In particular, students benefit significantly more from the inclusion in their class of new students that are similar to them (see Hoxby and Weingarth, 2007, and Ding and Lehrer, 2006), just as our model would predict. Human capital externalities have also been modeled in the literature as an average mean effect, that is, it is average human capital in the economy that affects each individual's marginal productivity in production (Lucas, 1988). In the existing literature on peer effects and human capital externalities individuals benefit from being close to more educated students or colleagues because of close collaboration and spillovers in the classroom or in the workplace. Our paper departs from the existing literature in two important aspects. First, individuals are affected differently from an entering student in their cohort depending on their relative position and the relative position of the entering student. In particular, an individual is affected more by the choices made by those individuals close to her in the distribution than by the choices of individuals who are very different (as was shown empirically by Hoxby and Weingarth, 2007, and Ding and Lehrer, 2006). And, second, we argue that individuals are affected by other individuals not because of close collaboration and spillover effects in the classroom or the workplace but because they are competing with each other for the best available jobs. While our model does not rule out important effects due to collaboration and cooperation, we do propose another potentially important way through which peer effects or human capital externalities might work: competition for the best available jobs (or other relevant prizes associated with relative position). Thus, our paper has important implications (predictions) for the empirical literature on peer effects and human capital externalities. Namely, we argue that in measuring human capital externalities or peer effects one should not only account for the mean human capital in the population but, also, for higher moments of the distribution of education. In particular, human capital externalities (due to competition) should be larger in societies with less inequality of opportunity and, also, in those parts of the distribution of endowments with a greater mass of individuals. Also, the model predicts that peer effects and human capital externalities associated with competition should be larger in environments where the prizes associated with the relative position in the final dimension (grades, achievements, etc.) are more differentiated.

In addition to this introduction, the paper contains four sections. Section 2 discusses how concerns for relative position have been introduced in the economic literature and presents a short review of related contributions. In Section 3 we present the simple version of the model with two individuals and two firms. In section 4 we develop the general model. Section 5 concludes.

2. Concerns for relative ranking in the economics literature

Since the seminal work of Thorstein Veblen (1899), *A Theory of the Leisure Class*, several economists have argued that concerns for status (or the relative position in some relevant dimension(s)) have important economic consequences.[6] A central discussion in the literature that deals with concerns for relative ranking has to do with how we should understand such concerns, that is, whether they are direct or instrumental. While in the former case people have concerns for status because they obtain utility from having high status in its own sake, in the latter people care about status because status directly affects the goods and services that individuals ultimately consume (Postlewaite, 1998). While the strongest argument for incorporating direct concerns for relative position in the utility function is an evolutionary one,[7] the case for not incorporating direct concerns for status in the utility function is that economic models that incorporate them typically allow for very diverse behavior, there are almost no restrictions on equilibrium behavior and, as a result, the models lose predictive power.[8] In other words, differences in individual's preferences over status may directly account for differences in equilibrium choices.

Most of the contributions that have emphasized the importance of concerns for relative ranking have focus on conspicuous consumption. The idea is the following: because wealth is unobservable, the consumption of conspicuous goods serves as a signal of non observable ability. Furthermore, if there are complementary interactions between individuals (for instance, at the household level between men and women, or at the workplace between employees and employers) conspicuous consumption might be welfare enhancing, even when

[6] The reader is refered to Bastani (2007) for a thorough review of the literature on concerns for relative ranking.

[7] As Postlewaite (1998) explains, the desire to ascend to the top of the social hierarchy may have had selection value over the course of human evolution (and thus may be hardwire in humans) as high-ranked members usually enjoy access to better mates, more food, etc. which increases their survival probability and that of their offspring.

[8] See Postlewaite (1998).

the costs of conspicuous consumption[9] are taken into account, as they allow for a better (more efficient) matching (among others, see Cole et al., 1992 and 1995, Bagwell and Bernheim, 1996, and Rege, 2000). While concerns for status might generate excessive competition, this does not mean that excessive competition is inefficient (as has been argued by Frank, 1999 and others). In fact, when status can be purchased in a competitive market, the cost of acquiring status is simply a transfer payment that adds to the seller's wealth. For instance, Becker and Murphy (2000, ch. 4) show that competition for mates is fully efficient if the value that someone brings to the marriage is fully priced. In the same book, Becker, Murphy and Werning take Frank's (1999) example of wearing high heels and argue that "the demand for high heels is efficient, even when such shoes cause foot and back damage, if the marriage, or other, markets that match men and women compensates women fully for the utility gain to their husbands or other companions from their wearing high heels. This behavior is efficient even when it lowers the relative attractiveness of other women, including women who also wear high heels." (see Becker and Murphy, 2000, ch. 8). In fact, when women decide to wear high heels they trade-off the cost of wearing high heels for the utility gain they obtain from getting better husbands. Thus, wearing high heels can be understood as an equilibrium outcome of a game where women compete with each other for the best available partners.

Only a few contributions in the economics literature on human capital and labor markets have incorporated concerns for relative ranking. In particular, Moen (1999) studies the incentives to invest in human capital in a model with labor market frictions and unemployment. In his model, an unemployed worker's chances of getting a job depends on his human capital relative to that of other unemployed workers because firms prefer to hire the most productive applicant due to rent sharing between them and the workers. Relative ranking affects the job finding rate and, as a result, there is a rat-race between unemployed individuals competing for job positions. Because wages are assumed to be determined by rent sharing between firms and workers (that is, the gains from education will not fully accrue to the workers in the form of higher wages) excessive competition might lead to inefficient overinvestment in human capital.

The most related contribution to this paper is a recent paper by Hopkins and Kornienko (2006). They study the effects of inequality in a tournament model where individuals compete for different rewards. Individuals, given their resources, make a simultaneous investment and output decision and then each individual is rewarded according to her

[9] Conspicuous consumption (or "Veblen effects") exists when consumers are willing to pay a higher price for a functionally equivalent good (see Bagwell and Bernheim, 1996).

relative position. The authors also emphasize the differential effect of inequality of resources and of inequality of rewards on individual equilibrium choices. While our main focus is on the relationship between inequalities of opportunities and wages and aggregate efficiency, theirs is on how changes in inequality of resources and rewards affect welfare for different segments of the population. In particular, they find that more inequality of resources lowers utility for agents in the middle and upper parts of the distribution, whereas an increase in the inequality of resources leads to lower utility for the relatively poor agents in society.[10]

3. A simple illustration: The 2 agents - 2 firms model.

This section presents a simple model with two firms and two agents that captures some of the main results that will be presented in the next section of the paper.

3.1. Firms

Let us assume that there are two firms, l and h, that produce a single homogeneous good, q_j, using a production function that combines technology and human capital as follows:

$$q_j = a_j * h_j, \tag{1}$$

where: $a_j > 0$ is the technology used by firm $j = \{l, h\}$. Assume, without loss of generality, that $a_h > a_l$. h_j is the human capital of the individual hired by firm j. Furthermore, we assume that each firm hires only one individual.[11]

Firms pay their workers their marginal product per unit of human capital employed in production. That is, firm l pays the worker it hires $w_l = a_l$ per unit of human capital and firm h pays the worker it hires $w_h = a_h$ per unit of human capital employed in the production process.

In this framework job positions differ in their payments because different firms operate with different technologies. The assumption that the production technology is linear in human capital greatly simplifies the analysis and also allows us to isolate the standard effect of inequality in the distribution of human capital on aggregate production efficiency

[10] Galí and Fernandez (1999) also develop a tournament model of competition for places at college but their main interest was to compare the efficiency of two different mechanisms in allocating rewards: markets vs. tournaments.

[11] One can also think about one firm that has two available job positions, each operating with a different technology.

that works through Jensen's inequality (see Mejía and St-Pierre, 2007).[12],[13] That is, if the amount of output produced is a concave function of human capital then a more unequal distribution of this factor of production across individuals would reduce aggregate efficiency in production. The linear production technology also implies that all wage dispersion in the model is explained by the dispersion of technologies across firms.[14]

Because technologies are complementary to human capital in the production process, the firm operating with the most advanced technology would like to hire the individual with the highest human capital available in the labor market.[15] That is, we assume that firms rank individuals in the human capital dimension and that they make job offers to the individual with the highest human capital available in the job market. We will also assume that there is a perfectly assortative matching and that there are no search costs.

3.2. Individuals

There are two individuals with endowments of the complementary factors to the schooling process equal to θ_p and θ_r. θ_i, with $i = \{p, r\}$, can be thought of as a measure of opportunities for human capital accumulation, where opportunities are a combination of all factors that complement individual's effort in the educational process, such as parental education, school and teacher quality, etc. (see Mejía and St-Pierre, 2007). Without loss of generality we will assume that $\theta_r \geq \theta_p$. That is, individual r (the rich individual) has a larger (or equal) endowment of the complementary factors to the schooling process than individual p (the poor individual).

Individuals accumulate human capital combining effort and the complementary factors to the schooling process according to the following human capital production function:

$$h = h(e, \theta), \tag{2}$$

[12] This assumption also implies that the distribution of wages is independent of the distribution of human capital in the economy, which greatly simplifies the analysis and allows us to isolate changes in the distribution of returns to human capital accumulation from changes in the distribution of endowments.

[13] The analysis that follows would go through with any production function where human capital and technology are complements.

[14] This implication is in line with Caselli's (1999) model and with the empirical evidence found in Faggio et al. (2007) in the sense that the diffusion of heterogeneous technologies accross firms in the UK has increased both the spread of productivity and of wages.

[15] Or, alternatively, in the case of one firm with two job positions that operate with different technologies, the firm would prefer to match the individual with the highest human capital to the job position with the advanced technology.

where e stands for effort and θ for the endowment of the complementary factors.

Assumption $A1$: $h(\cdot,\cdot)$ is differentiable, $h_e(\cdot,\cdot) > 0$, $h_\theta(\cdot,\cdot) > 0$, $h_{ee}(\cdot,\cdot) < 0$, $h_{\theta\theta}(\cdot,\cdot) < 0$, $h_{e\theta}(\cdot,\cdot) > 0$, and $\lim_{e\to 0} h_e(e,\cdot) = \infty$.

According to $A1$, human capital is an increasing and strictly concave function of both effort and the complementary factors, and the marginal effect of effort on the accumulation of human capital is increasing in the complementary factors. In other words, effort is complementary to the endowment of the complementary factors in the production of human capital. Also, effort is strictly necessary for the accumulation of human capital.

Each individual i maximizes a utility function that depends positively on consumption and negatively on effort. Furthermore we assume that the utility function is separable in the two arguments.[16] Each individual's problem is:

$$\underset{\{e\}}{Max} \quad U(c,e) = u(c) - v(e) \tag{3}$$

Assumption $A2$: $u(c)$ and $v(\cdot)$ are differentiable, $u'(\cdot) > 0$, $u''(\cdot) \leq 0$, $v'(\cdot) > 0$, $v''(\cdot) > 0$ and $\lim_{e\to+\infty} v'(e) = +\infty$.

Consumption equals income which, in turn, is equal to the expected wage per unit of human capital times the stock of human capital accumulated that the individual brings to the labor market. That is, consumption equals the expected wage times the amount of human capital that an individual brings to the market, $E(w) * h$.

Before going to the job market both individuals accumulate human capital and they know that the two firms will rank them in the human capital dimension and will hire the individual with the highest human capital available in the market (due to complementarities between the worker's human capital and technologies[17]). As a result , individual i's perceived probability of being hired by the firm that operates with the advanced technology (that is, the firm that pays the high wage) is a function of her human capital, h_i, and the human capital of individual j, h_j. Individual i's expected wage is given by:

$$E(w_i) = p(h_i, h_j) * w_h + (1 - p(h_i, h_j)) * w_l, \tag{4}$$

where $p(h_i, h_j)$ is the probability, as perceived by individual i, of being hired by the firm that pays the high wage.

[16] This is perfectly equivalent to a situation where consumption and leisure are the only arguments in the utility function and where leisure time is sacrificed when time and effort are devoted to the accumulation of human capital.

[17] See footnote 4.

Assumption $A3$: $p_{h_i} > 0$, $p_{h_j} < 0$, $p_{h_i h_i} < 0$, and $\left|p_{h_i h_j}\right| < \varepsilon$, where ε is an arbitrarily small number.

$A3$ says that the probability of being hired by the advanced technology firm for individual i increases as her human capital increases and decreases with the human capital of the other individual. Furthermore, this probability is strictly concave on h_i.

Assuming, again, without loss of generality, that $u(c) = c$, individual i takes individual j's effort as given and chooses her own effort to maximize her utility.[18] Individual i's problem is:

$$\max_{\{e_i\}} E(w_i)h(e_i, \theta_i) - v(e_i). \tag{5}$$

The first order condition of individual i's problem is:[19]

$$\frac{\partial p(h_i, h_j)}{\partial h_i} \frac{\partial h_i(\hat{e}_i, \theta_i)}{\partial e_i}(w_h - w_l)h(\hat{e}_i, \theta_i)) + E(w_i)h_{e_i}(\hat{e}_i, \theta_i) - v'(\hat{e}_i) = 0. \tag{6}$$

The second and third term on the left hand side of equation 6 are the standard terms in models of human capital accumulation *à-la*-Becker - Ben-Porath: the direct marginal benefit and cost from exerting one extra unit of effort in the accumulation of human capital. The first term captures how an extra unit of time and effort allocated to the accumulation of human capital affects the probability of being hired by the firm that pays the high wage (that is, the firm operating with the advanced technology). In other words, when firms pay different wages because, for instance, they operate with different technologies, individuals have an extra incentive to invest time and effort in the accumulation of human capital to increase the probability of being hired for the best available job.[20]

3.3. Labor market equilibrium and comparative statics results

A Nash equilibrium of the game of competition for jobs is a pair of strategies $\{e_r, e_p\}$ that satisfy the first order conditions for both agents, r and p, in equation 6. These two first

[18] In other words we assume that both agents make human capital investment decisions simultaneously. That is, they play a Nash-Cournot game of competition for the best available jobs.

[19] Assumptions A1 through A3 guarantee that the maximization problem in equation 5 has a unique and interior solution and that the first order condition in equation 6 is sufficient.

[20] Standard models of human capital accumulation do not incorporate this effect because they implicitly assume that all available jobs operate with the same technology. As a result, there is no incentive for competition between applicants as the wage rate per unit of human capital is the same in all available jobs.

order conditions describe the reaction function (the choice of effort) of each agent to every possible choice of effort by the other agent.

Before proceeding it is worth specifying a benchmark case where individuals do not take into account the effect of effort on the probability of being hired by the firm operating with the high technology (the first term in equation 6). In the benchmark case individuals either take as given the probability of being hired by the firm operating with the advanced technology, or, alternatively, take as given the expected wage. In other words, in the benchmark case individuals take their rank in the distribution of human capital as given and cannot affect it by exerting more effort in the accumulation of human capital. The important point of setting up this benchmark case is that individuals are not able to affect the probability of being hired by the advanced technology firm by exerting more effort. In the benchmark case the first order condition is:

$$E(w_i)h_{e_i}(e_i^*, \theta_i) - v'(e_i^*) = 0, \tag{7}$$

where $E(w_i)$ is taken as given by individual i, and, when comparing the benchmark case with the case where individuals compete with each other for the best available job in Proposition 1 below we will assume that $E(w_i)$ is the same that would result if the two individuals had engaged in a contest, although they cannot affect this probability in the former case.

Proposition 1: Effort and hence human capital accumulation are higher when individuals compete for job positions than in the benchmark case where there is no competition.

$Proof$: If $p_{e_i} = \frac{\partial p(h_i,h_j)}{\partial h_i}\frac{\partial h_i(e_i,\theta_i)}{\partial e_i} > 0$, that is, if the probability of individual i being hired by the firm operating with the advanced technology increases as her human capital increases (i.e. as his effort increases), then $p_{e_i}(w_h - w_l)h(\hat{e}_i, \theta_i)) > 0$ and, using equation 6, $(p_h w_h + (1 - p_h)w_l)h_{e_i}(\hat{e}_i, \theta_i) - v'(\hat{e}_i) < 0$. However, in the benchmark case, $(p_h w_h + (1 - p_h)w_l)h_{e_i}(e_i^*, \theta_i) - v'(e_i^*) = 0$ and so it must be that $\hat{e}_i > e_i^*$ if the function $(p(h_i, h_j) * w_h + (1 - p(h_i, h_j)))h(e_i, \theta_i) - v(e_i)$ is strictly concave in e_i, as it is by assumptions $A1$ through $A3$■

Intuitively, when there is competition between individuals for the best available job positions they will exert more effort because they have an extra incentive to accumulate human capital beyond the standard marginal benefit (the second term in equation 6). This extra incentive is the marginal increase in the probability of being hired by the firm operating with the advanced technology that results form an extra unit of time and effort allocated to the accumulation of human capital.

Proposition 2: Higher inequality in the distribution of the complementary factors decreases average human capital in the economy. The decrease in average human capital as inequality increases is larger when individuals compete for jobs than in the benchmark case where they take their rank in the human capital dimension as given.

Proof : Define the average endowment of the complementary factors as $\overline{\theta}$, and let $\theta_r = \overline{\theta} + \delta$, and $\theta_p = \overline{\theta} - \delta$. The parameter δ captures inequality in distribution of the complementary factors. With this definition, the larger is δ, the larger is inequality in the distribution of the complementary factors. From $A3$ and the assumption that $h_{e\theta} > 0$ from $A1$, $\left|\frac{\partial p(h_p, h_r)}{\partial \delta}\right| > \frac{\partial p(h_r, h_p)}{\partial \delta}$, where $\frac{\partial p(h_p, h_r)}{\partial \delta} < 0$■

Because the probability of being hired by the advanced technology firm is a strictly concave function of effort and the endowment of the complementary factors and effort are complements in the accumulation of human capital, a higher degree of inequality in the distribution of endowments reduces aggregate effort invested in the accumulation of human capital. That is, when inequality increases, the probability perceived by the poor individual of being hired by the firm that operates with the advanced technology decreases more than the same probability perceived by the relatively rich individual increases.[21] This result follows directly from Jensen's inequality after noticing that $p_{e_i e_i} < 0$ (from $A1$ and $A3$).

Proposition 3: As the difference between wages in the two available job positions $(w_h - w_l)$ increases, average human capital in the economy increases. That is, a larger difference in wages (i.e. the technologies employed by the two firms) increases the incentives to exert more effort and accumulate more human capital.

Proof : The results follows directly from the first order condition (equation 6) by noticing that the larger is $w_h - w_l$, the larger is the return from exerting effort that is associated with the increase in the probability of being hired by the advanced technology firm (first term of equation 6)■

Notice that inequality of endowments and inequality of returns (wages) affect differently the incentives to compete for the best available job positions. While more inequality of endowments disencourages competition between individuals for the best available jobs, more wage inequality does the opposite.[22]

[21] In equilibrium, because $\theta_r > \theta_p$ and $h_{e\theta} > 0$, the rich individual has more human capital than the poor individual.

[22] These results are in line with those obtained by Hopkins and Kornienko (2006).

In order to have some sense of the magnitude of the effect of inequality in the endowments of the complementary factors, and of inequality in returns, on effort and human capital accumulation, Figure 1 (a) and (b) present the results obtained from the calibration of the model presented above.[23] Effort and hence human capital accumulation are higher when individuals compete for job positions than in the benchmark case (Proposition 1).[24] This is seen in Figure 1a by comparing the two lines for any given level of endowment inequality. Note also from this figure that as inequality increases average human capital in the economy decreases, but in the case of competition for jobs it decreases faster (Proposition 2). Figure 1b shows how average human capital changes as the difference between wages in the two available job positions increases for a given level of endowment inequality. As the wage difference becomes larger, individuals have a higher incentive to compete for the high paying job position and thus exert more effort and accumulate more human capital (Proposition 3).

[INSERT FIGURE 1 HERE]

4. The General Model

4.1. Firms

Suppose that there is a continuum of firms indexed by j that produce a homogeneous good according to the following production function:

$$q_j = a_j * h_j,$$

where, as in equation 1, $a_j > 0$ is the technology used by firm j and h_j is the human capital of the individual hired by firm j. Assume that each firm hires only one individual. Furthermore, assume that $a_j \sim H(a)$.[25] There is perfect competition in the labor market

[23] We use the following functional forms for the calibration of the model: $h(e_i, \theta_i) = Ae_i^{\alpha}\theta_i^{1-\alpha}$, with $0 < \alpha < 1$, $p(h_i, h_j) = \frac{h_i}{h_i+h_j}$, and $v(e_i) = \frac{e_i^2}{2}$. Note that these functional forms satisfy assumptions $A1$ through $A3$. We set $A = 1$ and $\alpha = 3/4$. The results presented in Figure 1 (a) and (b) are robust to large variations of these parameters.

[24] For the benchmark case we take the probability of being hired by the advanced technology firm to be the probability that would obtain if the two agents had engaged in a contest for the high paying position. Note that in this case individiduals take as given the probability that results in equilibrium but cannot affect it by exerting more effort.

[25] We assume that the CDF $H(.)$ is strictly increasing and continous.

so firms remunerate human capital according to its marginal product. That is, the wage rate paid by firm j is equal to a_j. Wages, therefore, are distributed according to $H(a)$.

4.2. Individuals

There is a continuum of individuals indexed by i. As in the two agents - two firms model, each individual has a given endowment of the factors that complement time and effort in the educational process, θ_i. The endowment of the complementary factors is distributed in the population according to $G(\theta)$, with support in $[a, b]$. Human capital is accumulated (produced) using individual's effort and the complementary factors, according to $h(e, \theta)$ (same as in equation 2). The human capital production function satisfies $A1$ above.

Individuals derive utility from consumption and disutility from effort according to:

$$U(c, e) = u(c) - v(e). \tag{8}$$

The utility function in equation 8 satisfies $A2$.

4.3. Matching between firms and workers in the labor market

Following the approach of Hopkins and Kornienko (2004), if we let $F(h)$ be the distribution of human capital across individuals, individual i's ranking in the distribution of human capital will be given by:

$$\gamma F(h(e, \theta) + (1 - \gamma)F^{-}(h(e, \theta)), \tag{9}$$

where $F^{-}(h) = \lim_{\acute{h} \to h^{-}} F(\acute{h})$ is the mass of individuals with human capital strictly less than h,[26] and $\gamma \in [0, 1)$ is a parameter that captures the decrease in the payoff from "ties".[27]

[26] A simpler definiton of rank would be just having $F(h)$ (as in Frank, 1985). The problem with this definition is that if all agents accumulate the same level of human capital, $\widehat{h}$, then, because $F(\widehat{h}) = 1$, all agents would have the highest ranking, and since there is a continumm of individuals, each having zero weight, an individual that increases her investment in human capital just above $\widehat{h}$ would see no increase in her ranking (see Hopkins and Kornienko, 2004).

[27] If all agents were to choose a level of human capital equal to $\widehat{h}$, then they would have ranking γ whereas if one individual chooses a level of human capital slightly greater than $\widehat{h}$ her ranking would be 1 $(> \gamma)$ (see Hopkins and Kornienko, 2004).

We will assume that, in hiring workers, firms rank individuals according to their human capital and hire the best available job candidate in the market due to complementarities in production between the worker's human capital and technologies. Thus, the firm with the most advanced technology would like to hire the individual with the highest human capital available in the market (the individual that ranks first in the distribution of human capital), the firm ranked second would like to hire the individual with the highest human capital available in the market (the individual who ranks second in the distribution of human capital), and so on and so forth. That is, there is a perfectly assortative matching between firms and individuals in the labor market.

Recalling that $H(a)$ denotes the distribution of technologies across firms and that the wage rate is equal to the marginal product of human capital, $w_j = a_j$, then individual i's ranking in the distribution of human capital coincides exactly with her ranking in the distribution of wages in the economy. That is:

$$\gamma F(h(e,\theta) + (1-\gamma)F^{-}(h(e,\theta)) \ = H(w_i) \qquad \Rightarrow \tag{10}$$

$$R\left[\gamma F(h(e,\theta) + (1-\gamma)F^{-}(h(e,\theta))\right] = w_i, \tag{11}$$

where w_i is the wage rate per unit of human capital that individual i receives and $R = H^{-1}$ is the inverse function (the quantile function) of the CDF of a.[28]

4.4. Individuals' optimization problem

Individuals take as given other individuals' effort and choose their own effort, e, to maximize $U(c,e)$.[29] In the following, we assume that $u(c) = c$ without loss of generality[30]. As usual, the objective of an agent with endowment $\theta \in [a,b]$ is to solve the following problem:

$$\max_{e\,\in\,[e_a,+\infty]} R\left[\gamma F(h(e,\theta) + (1-\gamma)F^{-}(h(e,\theta))\right] h(e,\theta) - v(e), \tag{12}$$

where $R\left[\gamma F(h(e,\theta) + (1-\gamma)F^{-}(h(e,\theta))\right] h(e,\theta) \ \ (= w * h)$ is the level of income (and consumption) that an agent with an endowment θ will attain.

[28] Recall that we have assumed before that the CDF $H(a)$ is strictly increasing and continous, so it has an inverse (quantile) function.

[29] That is, individuals play a simultaneous move game of competition for jobs.

[30] As long as the utility is monotonically increasing in c, the presence of (strict) concavity would only strengthen our results.

Assumption A4:

(i) $v(\cdot)$ is differentiable, $v'(\cdot) > 0$, $v''(\cdot) > 0$ and $\lim_{e \to +\infty} v'(e) = +\infty$ (from $A2$),

(ii) $h(\cdot, \cdot)$ is differentiable, $h_{e\theta}(\cdot, \cdot) > 0$, $h_e(\cdot, \cdot) > 0$,$h_\theta(\cdot, \cdot) > 0$, $h_{ee}(\cdot, \cdot) < 0$, $h_{\theta\theta}(\cdot, \cdot) < 0$ and $\lim_{e \to e_a} h_e(e, \theta) = +\infty$ for all $\theta \in (a, b]$,

(iii) $R(\cdot)$ is differentiable, $R'(\cdot) \geq 0$ and $R''(\cdot) \leq 0$.

4.5. Labor market equilibrium

A symmetric Nash equilibrium solution is a mapping $e : [a, b] \to [e_a, +\infty$ that assigns a choice of effort $e(\theta)$ for any possible endowment level θ, where $e(\theta)$ is chosen to solve the problem in equation 12. Notice that the assumption A4 is sufficient to ensure that the mapping $e(\cdot)$ is a function[31]. In the following, let $h^{eq}(\theta) \equiv h(e(\theta), \theta)$ be the equilibrium human capital mapping. The next results provide a characterization of the solution mapping $e(\cdot)$.

Proposition 4: If the solution $e(\cdot)$ exists then:
(i) $h^{eq}(\cdot)$ is strictly increasing
(ii) $e(\cdot)$ is continuous
(iii) $e(\cdot)$ is differentiable.

Proof: see the Appendix.

Proposition 5: Under $A4$, a solution function (symmetric Nash equilibrium of the game of competition for jobs) $e(\cdot)$ exists, is unique, and is characterized by the following differential equation with the initial condition $e(a) = e_a$.

$$e'(\theta) = -[R'(G(\theta))g(\theta)\frac{h(e, \theta)}{R(G(\theta))h_e(e, \theta) - v'(e)} + \frac{h_\theta(e, \theta))}{h_e(e, \theta)}] \tag{13}$$

Proof: see the Appendix.

4.6. Inequality of endowments, inequality of wages, and human capital accumulation.

As argued in the simple model presented in section 3, inequality of endowments (opportunities) and inequality of wages (returns) affect the incentives to invest time and effort

[31] Indeed, the objective function in 12 is strictly quasi-concave by A4 and, hence, the solution to the maximization problem is always unique if it exists.

in the accumulation of human capital in a different way. More inequality of opportunities disencourages competition for the best available jobs and, as a result, individuals exert less effort and accumulate less human capital in equilibrium. However, more inequality of wages, by increasing the incentives to compete for the best available job positions, induces higher competition between agents and, hence, aggregate (and average) human capital accumulation is higher.

In order to evaluate these predictions in the general model we will use functional forms for the distribution of wages (technologies) in the economy and for the distribution of endowments across individuals. As we will explain below the functional forms that we will use for the two distributions will allow us to solve explicitly for the equilibrium level of effort for each agent in the Nash equilibrium of the game of competition for jobs. Importantly, these functional forms will also allow us to implement increases in inequality without affecting the mean of each distribution. In other words, we will study how effort and human capital accumulation respond as we implement a mean-preserving spread in the distribution of endowments, and, separately, a mean-preserving spread in the distribution of wages.

Assumption $A5$ (wage distribution): Let wages be distributed across firms (or job positions) according to the following CDF:

$$H(w;\kappa)=\begin{cases} 0 & for \quad w<0 \\ Kw^{\kappa} & for \quad w\in\left[0,\frac{1+\kappa}{\kappa}\right] \\ 1 & for \quad w>\dfrac{1+\kappa}{\kappa} \end{cases}, \tag{14}$$

where $K=\left(\dfrac{\kappa}{1+\kappa}\right)^{\kappa}$ and with $\kappa\in(0,1]$.

The Appendix describes in detail some of the main characteristics of the wage distribution function defined in equation 14. However, a few points are worth mentioning about this particular distribution: first, the mean wage is always equal to 1 ($E(w)=1 \quad \forall\kappa$). Second, the parameter κ is an inverse measure of wage inequality. That is, as κ increases wage inequality decreases. In particular, as κ increases, two commonly used measures of inequality respond in the expected way. That is, the ratio of the median to the mean wage increases (i.e. there is less wage inequality), and the wage Gini coefficient decreases (which, again, means that there is less wage inequality).[32]

Assumption $A6$ (distribution of endowments): Let endowments be distributed across individuals according to the following CDF:

[32] See the appendix for the full derivation of these results.

$$G(\theta;\phi) = \begin{cases} 0 & for \quad \theta < 0 \\ \Phi\theta^{\phi} & for \quad \theta \in \left[0, \frac{1+\phi}{\phi}\right] \\ 1 & for \quad \theta > \dfrac{1+\phi}{\phi} \end{cases}, \tag{15}$$

where $\Phi = \left(\dfrac{\phi}{1+\phi}\right)^{\phi}$ and with $\phi \in (0, 1]$.

As in the case for the wage distribution, in this case the mean endowment is always equal to 1, and, for the endowment distribution described in equation 15, the parameter ϕ is an inverse measure of inequality in the distribution of endowments. That is, as ϕ increases endowments are more equally distributed across agents, the median to the mean endowment increases, and the Gini coefficient for the distribution of endowments decreases.[33]

Assumption $A7$ (human capital production function and cost function): Let us assume that the human capital production function takes the following functional form:[34]

$$h(e, \theta) = Ae^{\alpha}\theta^{1-\alpha}, \quad \text{with} \quad \alpha \in (0, 1). \tag{16}$$

Also, we will assume that the function $v(e)$ takes the following functional form:[35]

$$v(e) = e^{\mu}, \quad \text{with} \quad \mu > 1. \tag{17}$$

4.6.1. Equilibrium effort and comparative static results

Recall from equation 10 that individual i's ranking in the distribution of wages coincides with her ranking in the distribution of human capital in the population. From equation 10 we are able to obtain the wage rate that each individual receives by taking the inverse of the distribution of wages (technologies) in the economy (see equation 11). Using the distribution functions 14 and 15, the function $R(G(\theta)) \equiv H^{-1}(G(\theta))$ that appears in equation 13 is given by:

$$R(G(\theta)) = \left[\frac{G(\theta)}{K}\right]^{\frac{1}{\kappa}} = \left[\frac{\Phi}{K}\right]^{1/\kappa} \theta^{\frac{\phi}{\kappa}} \tag{18}$$

Using the distribution functions for wages (technologies) and for endowments in equations 14 and 15 respectively, equation 16, 17, and 18, the solution to the differential equation

[33] These results follow exactly in the same way as for the wage distribution (see the appendix for details).

[34] This functional form staisfies all conditions in Assumption $A1$.

[35] This functional form satisfies the conditions in Assumption $A2$.

that describes the Nash equilibrium of the game of competition for jobs is:[36]

$$\widehat{e} = \left[\left(\frac{\Phi}{K} \right)^{\frac{1}{\kappa}} \left(\frac{\alpha A}{\mu} \right) \frac{\mu + \mu \frac{\phi}{\kappa} - \mu\alpha}{\mu + \alpha \frac{\phi}{\kappa} - \mu\alpha} \right]^{\frac{1}{\mu-\alpha}} \theta^{\frac{1+\frac{\phi}{\kappa}-\alpha}{\mu-\alpha}}. \tag{19}$$

Proposition 6: Equilibrium effort and, hence, human capital accumulation are higher for all agents when inequality of endowments is lower. That is, there is a negative relationship between inequality of opportunities (as measured by inequality in the distribution of the complementary factors of the schooling process) and aggregate efficiency in human capital formation.

Proof: It follows directly by noticing that all terms in equation 19 depend positively on the parameter ϕ,[37] which, as explained after equation 15, is a direct measure of equality in the distribution of endowments■

Because human capital is an increasing function of both effort (e) and the endowment of the complementary factors (θ), a higher value of the parameter ϕ (more equality of opportunities) implies a higher level of human capital accumulation for all individuals. Thus, the model predicts a negative relationship between inequality of opportunities and aggregate efficiency in human capital formation.

Intuitively, as in the simple model with 2 agents - 2 firms, more equality in the distribution of endowments generates more competition between agents for the best available job positions, and, as a result, in the Nash equilibrium of the game all agents end up exerting more effort and accumulating more human capital. In choosing the optimal amount of time and effort invested in the accumulation of human capital individuals trede-off the benefits from an extra unit of time invested in the accumulation of human capital (which, as explained throughout the paper, includes the marginal change in the relative position in the distribution of human capital and the corresponding higher wage) with the disutility cost of exerting more effort.

Proposition 7: Equilibrium effort and, hence, human capital accumulation are higher for all agents when inequality in the distribution of wages (rewards) in the economy is larger.

[36] See the appendix for the full derivation of this result.

[37] The third term inside the parenthesis of equation 19 depends positively on ϕ because, by assumption, $\mu > 1$, $\alpha < 1$, so $\mu > \alpha$.

Proof: Again, it follows directly by noticing that all terms in equation 19 depend negatively on the parameter κ,[38] which, as explained after equation 14, is a direct measure of equality in the distribution of wages in the economy$\blacksquare$

More wage inequality generates incentives to compete for the best available job positions and, as a result, individuals exert more effort in equilibrium and accumulate more human capital.

Despite the fact that closed form solutions to the Nash equilibrium of the game of competitions for jobs cannot be found for many other distributions (as for the case presented above) one can, in principle, solve the model numerically with other distributions in order to check whether the main predictions of the model still hold.

In order to evaluate the main qualitative results of the model with a different distribution we solve equation 13 numerically using the same functional forms that we used in the 2 agents - 2 firms model (see footnote 23) and assume, without loss of generality, that $e_a = 1$, that is, the agent with the lowest endowment of the complementary factors exerts a level of effort equal to 1.[39] In the first set of calibrations we will fix the degree of inequality in the distribution of wages by assuming that $H(a) \sim U(0, 1)$.That is, we assume that technologies (and therefore wages) are distributed according to a standard uniform. Also, we assume that $G(\theta) \sim U(1 - \varepsilon, 2 + \varepsilon)$ and, in doing the mean preserving spread in the distribution of endowments, we will increase the parameter ε.

Figures 2 (a) and (b) present the results of the numerical solutions of the general model for two different values of the relative importance of effort in the accumulation of human capital, α (see footnote 23) . Each figure shows the result of the simulation of a mean preserving spread in the distribution of endowments (an increase in the parameter ε in the distribution of endowments $G(\theta)$). As can be seen in these figures more inequality in the distribution of endowments (higher ε) is associated with lower aggregate efficiency in human capital formation (as measured by average human capital in the population). In other words, the simulations of the equilibrium of the model using the uniform distribution also suggest that there is no trade-off between equality of opportunity and aggregate efficiency in human capital formation, just as in the case with 2 agents and 2 firms, and the case presented above where we were able to find a closed form solution of the equilibrium of the

[38] Again, note that $\mu > \alpha$ so the third term inside the parenthesis in equation 19 depends negatively on κ.

[39] We use a 4(5) imbedded pair Runge-Kutta Scheme called the Dormand-Prince 4(5) (explicit) scheme to solve numerically the differential equation 13 (see Ascher and Petzold, 1998, ch. 4). We thank Lydia Boroughs for kindly helping us with this methodology.

game.

[INSERT FIGURE 2 HERE].

Figures 3 (a) and (b) present the results of a similar exercise but this time we fix the degree of inequality in the distribution of endowments and do a mean preserving spread in the distribution of wages. In both cases (when α, the relative importance of effort in the accumulation of human capital, is 3/4 and 1/2) a higher degree of inequality in the distribution of wages is associated with higher aggregate efficiency in human capital formation. This result, again, confirms the result obtained in the simple and in the general models presented above, namely, that more wage inequality (more inequality of returns) fosters competition for the best available job positions between individuals and, thus, induces individuals to exert more effort and accumulate more human capital in equilibrium.

[INSERT FIGURE 3 HERE].

5. Concluding remarks

This paper develops a model where heterogeneous agents compete for the best available job positions. One of the main working assumptions is that different firms operate with different technologies and, as a result, have open job positions with different remuneration. Because technologies are complementary to human capital in the production process, the firm operating with the most advanced technology is matched with the individual with the highest human capital in the job market, the second firm with the second individual in the distribution of human capital and so on and so forth. As a result of this, when individuals are choosing the optimal investment in human capital formation not only do they take into account the benefit of a marginal increase in their human capital and the marginal cost of one extra unit of investment, but, also, how that extra unit of time and effort invested in the accumulation of human capital affects their relative position in the distribution, which, in turn, affects the firm they will be matched with and, as a result, the wage they will receive in equilibrium.

We propose a new channel through which inequality affects aggregate efficiency in human capital formation. In particular we find that a more equal distribution of the endowments that are complementary to time and effort in the educational process increases aggregate efficiency in human capital formation. However, more inequality in the returns to human capital accumulation, by increasing the incentives to compete for the best available job positions, increases average human capital formation in the economy.

The paper proposes a different, perhaps complementary, explanation for the existence of peer effects and/or human capital externalities. While the explanation so far advanced in the literature for the existence of peer effects and human capital externalities is based on close collaboration, cooperation, and spillovers between individuals in the classroom or in the workplace, our explanation is based on a different story: individuals compete with each other for the best available job positions (or for differentiated prizes associated with relative ranking in the human capital dimension). The proposed model rationalizes a mechanism for non-linear peer effects. In particular we argue that in estimating peer effects or human capital externalities one should not only take into account the first moment of the distribution of opportunities or human capital (as most of the empirical literature on peer effects has done) but also should account for higher moments of the distribution. Also, according to the intuition developed in the model about how peer effects and human capital externalities operate through competition, one should expect larger peer effects in environment where prizes associated with the relative position in a final dimension (grades, achievement, etc.) are more differentiated.

6. Appendix

Proof of Proposition 4.

STEP 1: we show that $h^{eq}(\cdot)$ is non decreasing.

This proof is adapted from Lemma A1 of Hopkins and Kornienko (2004). It should be clear that $e(\theta) \geq e^*(\theta)$ for all $\theta \in [a, b]$ since it is a dominated strategy to play a level of effort below the benchmark level. If $e(\theta) = e^*(\theta)$ for some $\theta \in [a, b]$ then the result follows immediately as $h(e^*(\theta), \theta)$ is increasing in θ. Let us consider the case where $e(\theta) > e^*(\theta)$.

Notice that for any other choice $\tilde{e} \in (e^*(\theta), e(\theta))$, we have:

$$R\left[\gamma F(h(e,\theta)) + (1-\gamma)F^{-}(h(e,\theta))\right] h(e,\theta) - v(e) \geq$$
$$R\left[\gamma F(h(\tilde{e},\theta)) + (1-\gamma)F^{-}(h(\tilde{e},\theta))\right] h(\tilde{e},\theta) - v(\tilde{e})$$
(AP1)

We now show the following inequality:

$$\frac{\partial h(e,\theta)}{\partial \theta} R\left[\gamma F(h(e,\theta)) + (1-\gamma)F^{-}(h(e,\theta))\right] >$$
$$\frac{\partial h(\tilde{e},\theta)}{\partial \theta} R\left[\gamma F(h(\tilde{e},\theta)) + (1-\gamma)F^{-}(h(\tilde{e},\theta))\right]$$
(AP2)

Notice that we can write the LHS of the above inequality as follows:

$$\frac{\partial h(e,\theta)}{\partial \theta} R\left[\gamma F(h(\tilde{e},\theta)) + (1-\gamma)F^{-}(h(\tilde{e},\theta))\right] + \frac{\partial h(e,\theta)}{\partial \theta}\left(R\left[\gamma F(h(e,\theta)) + (1-\gamma)F^{-}(h(e,\theta))\right] - \right.$$
$$\left. R\left[\gamma F(h(\tilde{e},\theta)) + (1-\gamma)F^{-}(h(\tilde{e},\theta))\right]\right)$$
(AP3)

Now, the first term in the LHS of (AP3) is at least as large as the RHS of (AP2) since we assumed that $h_{e\theta} > 0$ in Assumption A4. Notice also that $R\left[\gamma F(h(e^*(\theta),\theta)) + (1-\gamma)F^{-}(h(e^*(\theta),\theta))\right] h(e,\theta) - v(e)$ is decreasing in e as long as $e > e^*(\theta)$.

Therefore, we have that:

$$R\left[\gamma F(h(e,\theta)) + (1-\gamma)F^{-}(h(e,\theta))\right] >$$
$$R\left[\gamma F(h(\tilde{e},\theta)) + (1-\gamma)F^{-}(h(\tilde{e},\theta))\right]$$

That is, it must be that an increase in effort implies an increase in rank because, otherwise, an individual could increase his utility by decreasing his effort level below $e(\theta)$. Since $h_\theta > 0$ then the inequality (AP2) is verified. Therefore, the marginal return on effort following an increase in the level of endowment is larger for richly endowed individuals. In turn, the optimal choice of effort necessarily increases as well.

STEP 2 $h^{eq}(\cdot)$ is strictly increasing

By contradiction, suppose that it is not. Then there exists $\theta_0 < \theta_1$ with $\bar{h} = h^{eq}(\theta_0) = h^{eq}(\theta_1)$. Because $h^{eq}(\cdot)$ is non-decreasing, $h^{eq}(\theta) = \bar{h}$ for all $\theta \in [\theta_0, \theta_1]$. That is, there is a mass point in the distribution of human capital at $\bar{h}$ and therefore $F(\bar{h}) > F^-(\bar{h})$. It follows that $R(\gamma F(\bar{h}) + (1-\gamma)F^-(\bar{h})) < R(F(\bar{h})) \leq R(\gamma F(\bar{h}+\epsilon) + (1-\gamma)F^-(\bar{h}+\epsilon))$ for all $\epsilon > 0$.

Notice that $e(\theta_1) < d$. If this was not true then we would have $h^{eq}(\theta_0) \equiv h(e(\theta_0), \theta_0) \leq h(d, \theta_0) < h(d, \theta_1) = \bar{h}$, a contradiction. Since $h(\cdot, \theta)$ and $v(\cdot)$ are both continuous in e, then for any $d > \delta > 0$, we have $\lim_{\delta \to 0} v(e(\theta_1) + \delta) = v(e(\theta_1))$ while $\lim_{\delta \to 0} h(e(\theta_1) + \delta, \theta_1) = \bar{h}$. From the preceding paragraph, however, $R(\gamma F(h(e(\theta_1) + \delta, \theta_1)) + (1-\gamma)F^-(h(e(\theta_1) + \delta, \theta_1))) > R(\gamma F(\bar{h}) + (1-\gamma)F^-(\bar{h}))$ for any $\delta > 0$.

Therefore, there exists a small enough $\bar{\delta} > 0$ such that for any $\delta < \bar{\delta}$,

$$R(\gamma F(h(e(\theta_1)+\delta,\theta_1)) + (1-\gamma)F^-(h(e(\theta_1)+\delta,\theta_1)))h(e(\theta_1)+\delta,\theta_1) - v(e(\theta_1)+\delta) >$$
$$R(\gamma F(\bar{h}) + (1-\gamma)F^-(\bar{h}))\bar{h} - v(e(\theta_1))$$

Thus, an individual with an endowment θ_1 could increase her utility by choosing a slightly higher level of effort $e(\theta_1) + \delta$, which leads to a contradiction.

STEP 3 We show that $e(\cdot)$ is continuous.

By contradiction, suppose not, so there is a jump in the equilibrium solution at some endowment level of the complementary factors $\hat{\theta} \in [a, b]$ so that $\lim_{\theta \to \hat{\theta}} e(\theta) = \hat{e} \neq e(\hat{\theta})$. Notice that $h^{eq}(\cdot)$ being strictly increasing implies the continuity of $R(\gamma F(h^{eq}(\cdot) + (1-\gamma)F^-(h^{eq}(\cdot)))$, that is

$$\lim_{\theta \to \hat{\theta}} R(\gamma F(h^{eq}(\theta) + (1-\gamma)F^-(h^{eq}(\theta)))$$
$$= R(\gamma F(h^{eq}(\hat{\theta}) + (1-\gamma)F^-(h^{eq}(\hat{\theta}))) = R(\gamma F(h(\hat{e},\hat{\theta}) + (1-\gamma)F^-(h(\hat{e},\hat{\theta})))$$

Since, $v(\cdot)$, $h(\cdot,\cdot)$ and $R(\gamma F(h^{eq}(\cdot) + (1-\gamma)F^-(h^{eq}(\cdot)))$ are continuous at $(\hat{e}, \hat{\theta})$ then a standard argument applies.

STEP 4 We show that $e(\cdot)$ is differentiable on $[a,b]$. From the previous steps, notice that $\gamma F(h(e,\theta)+(1-\gamma)F^{-}(h(e,\theta))=F(h(e,\theta))$ for all $\theta\in[a,b]$. That is, there are no mass points.

Let $\hat{\theta}=\theta+\delta$ for some δ. We have,

$$R(F(h(e(\theta),\theta))h(e(\theta),\theta)-v(e(\theta))\geq R(F(h(e(\hat{\theta}),\theta))h(e(\hat{\theta}),\theta)-v(e(\hat{\theta}))$$

Similarly, we have,

$$R(F(h(e(\hat{\theta}),\hat{\theta}))h(e(\hat{\theta}),\hat{\theta})-v(e(\hat{\theta}))\geq R(F(h(e(\theta),\hat{\theta})))h(e(\theta),\hat{\theta})-v(e(\theta))$$

By the Mean Value theorem we have,

$$R(F(h(e(\hat{\theta}),\theta))h(e(\hat{\theta}),\theta)=R(F(h(e(\theta),\theta))h(e(\theta),\theta))+$$
$(R^{'}(F(h(e_1,\theta))f(h(e_1,\theta)h_e(e_1,\theta))h(e_1,\theta)+R(F(h(e_1,\theta)))h_e(e_1,\theta)))(e(\hat{\theta})-e(\theta))$ for some $e_1\in[0,d]$

so that,

$$(v(e(\hat{\theta}))-v(e(\theta)))-(R^{'}(F(h(e_1,\theta))f(h(e_1,\theta)h_e(e_1,\theta))h(e_1,\theta)$$
$$+R(F(h(e_1,\theta)))h_e(e_1,\theta)))(e(\hat{\theta})-e(\theta))\geq 0$$

Similarly, using again the mean value theorem yields

$$(v(e(\hat{\theta}))-v(e(\theta)))-$$
$$(R^{'}(F(h(e_2,\theta))f(h(e_2,\theta)h_e(e_2,\theta))h(e_2,\theta)+R(F(h(e_2,\theta)))h_e(e_2,\theta)))(e(\hat{\theta})-e(\theta))\leq 0$$
for some $e_2\in[0,d]$

Combining these two last inequalities, we obtain:

$$\frac{v(e(\hat{\theta}))-v(e(\theta))}{(R^{'}(F(h(e_1,\theta))f(h(e_1,\theta)h_e(e_1,\theta))h(e_1,\theta)+R(F(h(e_1,\theta)))h_e(e_1,\theta))\delta}\leq\frac{e(\hat{\theta})-e(\theta)}{\delta}\leq$$
$$\frac{v(e(\hat{\theta}))-v(e(\theta))}{(R^{'}(F(h(e_2,\theta))f(h(e_2,\theta)h_e(e_2,\theta))h(e_2,\theta)+R(F(h(e_2,\theta)))h_e(e_2,\theta)))\delta}$$

By continuity, both the RHS and LHS of the expression converges to the same limit at δ approaches 0 ensuring that $\lim_{\delta\to 0}\frac{e(\hat{\theta})-e(\theta)}{\delta}$ exists. By definition, this establishes that $e(\cdot)$ is differentiable at θ.

END OF PROOF.

Proof of Proposition 5.

STEP 1

From the previous step and since the functions $R(\cdot)$, $F(\cdot)$, $h(\cdot,\theta)$ and $v(\cdot))$ are continuously differentiable[40] then the problem (12) can be characterized by the following (sufficient) first order condition[41]: for any $\theta \in (a, b]$:

$$R^{'}(F(h^{eq}(\theta)))f(h^{eq}(\theta))h_e(e,\theta)h(e,\theta) + R(F(h^{eq}(\theta)))h_e(e,\theta) - v^{'}(e) = 0 \qquad (20)$$

Alternatively, (20) can be rewritten as follows using the fact that $G(\cdot) \equiv F(h^{eq}(\cdot))$ and $g(\cdot) \equiv f(h^{eq}(\cdot))$.

$$R^{'}(G(\theta))g(\theta)\frac{d(h^{eq})^{-1}(h^{eq}(\theta))}{dh}h_e(e,\theta)h(e,\theta) + R(G(\theta))h_e(e,\theta) - v^{'}(e) = 0 \qquad (21)$$

STEP 3 The above first order condition can be rewritten as the following differential equation using the fact that $1 = \frac{d(h^{eq})^{-1}(h^{eq}(\theta))}{dh}(h_e(e,\theta)e^{'}(\theta) + h_\theta(e,\theta))$ and given the denominator does not vanish in the RHS of the following expression:

$$e'(\theta) = -[R'(G(\theta))g(\theta)\frac{h(e,\theta)}{R(G(\theta))h_e(e,\theta) - v'(e)} + \frac{h_\theta(e,\theta)}{h_e(e,\theta)}] \qquad (22)$$

STEP 4 In equilibrium, the ranking of an individual with endowment $\theta = a$ is $G(a) = 0$ and so her utility in equilibrium is: $-v(e(a))$. Clearly $e(a) = e_a$ dominates any other strategy for that player. We now show that equilibrium solution is continuous at $\theta = a$.

In equilibrium, an individual with endowment $\theta = a$ must not be able to increase her utility by increasing her effort and increasing her utility by achieving a higher rank. This implies that in particular that:

$\lim_{\theta \to a^+} R(F(h(e(\theta), a)))h(e(\theta), a) - v(e(\theta)) \geq -v(e_a)$

Because the LHS of the above equation converges to: $R(F(h(e(\theta),\theta)))h(e(\theta),\theta) - v(e(\theta))$ by continuity and that, necessarily, $R(F(h(e(\theta),\theta)))h(e(\theta),\theta) - v(e(\theta)) \geq -v(e_a)$ we obtain the following:

$\lim_{\theta \to a^+} R(F(h(e(\theta),\theta)))h(e(\theta),\theta) - v(e(\theta)) = -v(e_a)$

[40] The fact that $F(\cdot)$ is differentiable follows from the identity $G(\cdot) \equiv F(h^{eq}(\cdot))$ in which both $h^{eq}(\cdot)$ and $G(\cdot)$ are differentiable.

[41] The reader may check that the equilibrium level of effort is interior for all individuals but the poorest.

Lastly, observe that $\lim_{\theta \to a^+} R(F(h(e(\theta), \theta))) = 0$ by continuity. Therefore, we are left with the following equation:

$\lim_{\theta \to a^+} -v(e(\theta)) = -v(e_a)$

Since $v(\cdot)$ is strictly increasing and continuous, this implies $\lim_{\theta \to a^+} e(\theta) = e_a$. In other words, the equilibrium solution is continuous at $\theta = a$.

STEP 5

The differential equation 13 with initial condition $e(a) = e_a$ has a unique (continuous and differentiable) solution by virtue of the fundamental theorem of differential equations. That is, a unique equilibrium solution exists.

END OF PROOF.

Characteristics of the wage distribution function (equation 14).

Some of the characteristics of the wage cumulative distribution function, $H(w;\kappa)$, described in equation 14 are:

(i) *Mean*: $E(w) = 1 \quad \forall \kappa$

(ii) *Median*: $F(w_m) = \dfrac{1}{2} \;\Rightarrow\; w_m = \dfrac{1+\kappa}{\kappa 2^{1/\kappa}}$

(iii) As $\kappa \to 1$, the distribution function in equation 14 approaches the Uniform distribution.

(iv) Define the first measure of inequality in the distribution of w as: $\Omega_w = \dfrac{median}{mean}$. That is:

$$\Omega_w = \frac{1+\kappa}{\kappa 2^{\frac{1}{\kappa}}}. \tag{23}$$

A higher value of Ω_w corresponds to a lower degree of inequality of wages (because the median of the distribution is closer to the mean). Note also that:

$$\frac{\partial \Omega_w}{\partial \kappa} > 0 \quad \text{for} \quad \kappa \in (0,1].$$

Therefore, both κ and Ω_w are measures of inequality in the distribution of wages in the economy. that is, as κ and Ω_w increase, wage inequality decreases.

(v) Define the Gini coefficient of the distribution wages as:[42]

[42] See Lambert (2001), chapter 2.

$$Gini_w = 2 \int_0^{\frac{1+\kappa}{\kappa}} wH(w;\kappa)h(w;\kappa)dw - 1 \tag{24}$$

Solving the previous equation using the distribution given in equation 14 we have:

$$Gini_w = \frac{2(1+\kappa)}{2\kappa+1} - 1$$

Using the last equation, note that: $\dfrac{\partial Gini_w}{\partial \kappa} < 0$. As the parameter that captures the degree of inequality in the distribution wages in the economy increases, the wage Gini coefficient, which is a measure of inequality of wages, decreases.

Derivation of equation 19 (solution to the differential equation 13).

Using equations 14, 15,16, 17, and 18, the differential equation 13 can be written as:

$$e'(\theta) = -\left[\frac{\phi}{\kappa}\left(\frac{\Phi}{K}\right)^{\frac{1}{\kappa}} \theta^{\frac{\phi}{\kappa}-1} \frac{A\left(\frac{e}{\theta}\right)^{\alpha}}{\alpha A\left(\frac{\Phi}{K}\right)^{\frac{1}{\kappa}} \theta^{\frac{\phi}{\kappa}-1}\left(\frac{e}{\theta}\right)^{\alpha-1} - \frac{\mu e^{\mu-1}}{\theta}} + \frac{1-\alpha}{\alpha}\left(\frac{e}{\theta}\right)\right] \tag{25}$$

Let´s assume that the solution to the differential equation in equation 25 is of the form $\widehat{e}(\theta) = \Lambda\theta^{\lambda}$, where Λ and λ are given constant terms that depend on the parameters of the model. If this is the case, then $\widehat{e}'(\theta) = \Lambda\dfrac{\widehat{e}}{\theta}$.Plugging this into the LHS of equation 25 and after some algebraic manipulation yields:

$$\widehat{e} = \left[\left(\frac{\Phi}{K}\right)^{\frac{1}{\kappa}}\left(\frac{\alpha A}{\mu}\right)\frac{\mu + \mu\frac{\phi}{\kappa} - \mu\alpha}{\mu + \alpha\frac{\phi}{\kappa} - \mu\alpha}\right]^{\frac{1}{\mu-\alpha}} \theta^{\frac{1+\frac{\phi}{\kappa}-\alpha}{\mu-\alpha}}, \tag{26}$$

which confirms that the solution to the differential equation 19 is of the form $\widehat{e}(\theta) = \Lambda\theta^{\lambda}$, where:

$$\Lambda = \left[\left(\frac{\Phi}{K}\right)^{\frac{1}{\kappa}}\left(\frac{\alpha A}{\mu}\right)\frac{\mu + \mu\frac{\phi}{\kappa} - \mu\alpha}{\mu + \alpha\frac{\phi}{\kappa} - \mu\alpha}\right]^{\frac{1}{\mu-\alpha}},$$

and,

$$\lambda = \frac{1 + \frac{\phi}{\kappa} - \alpha}{\mu - \alpha}.$$

References

Acemoglu, D., Aghion, P., and Violante, G., 2001. Deunionization, technical change and inequality. Carnegie-Rochester Conference Series of Public Policy, 55 (1), 229-264.

Aghion P., Caroli E., and Garcia-Peñalosa, C., 1999. Inequality and economic growth: The perspective of the new growth theories. Journal of Economic Literature 37 (4), 1615-1660.

Ascher, U., and L. Petzold, 1998. Computer Methods for Ordinary Differential Equations and Differential-Algebraic Equations. Society for Industrial and Applied Mathematics.

Bagwell, L. and Bernheim, D., 1996. Veblen Effects in a Theory of Conspicuous Consumption. The American Economic Review 86 (3), 349-373.

Banerjee, A. and Newman A., 1993. Occupational choice and the process of development. Journal of Political Economy 101 (2), 274-298.

Bastani, S., 2007. Towards a Theory of Relative Preferences. Uppsala University, Mimeo.

Becker, G., 1964. Human Capital. Chicago: University of Chicago Press.

Becker, G. and Murphy, K., 2000. Social Economics: Market Behavior in a Social Environment. Cambridge, MA: Harvard University Press.

Ben-Porath, Y., 1967. The Production of Human Capital and the Life Cycle of Earnings. Journal of Political Economy 75 (4), 352-365.

Bénabou, R., 1996. Equity and efficiency in human capital investment: The Local Connection. Review of Economic Studies 63 (2), 237-264.

Bénabou, R., 2000. Meritocracy, redistribution, and the size of the pie, in: Arrow, K., Bowles, S. and Durlauf S. (Eds.), Meritocracy and Inequality, Princeton University Press, pp. 317-339.

Benabou, R., 2000. Unequal societies: Income distribution and the social contract. American Economic Review 90 (1), 96-129.

Caselli, F., 1999. Technological revolutions. American Economic Review 89, 78-102.

Cole, H., Mailath, G. and Postlewaite, A., 1992. Social Norms, Saving Behavior, and Growth. The Journal Political Economy 100 (6), 1092-1125.

Cole, H., Mailath, G. and Postlewaite, A., 1995. Incorporating Concern for Relative Wealth Into Economic Models. Federal Reserve Bank of Minneapolis Quaterly Review 19 (3), 12-21

Cole, H., Mailath, G. and Postlewaite, A., 1995. Response to “Aristocratic Equilibria”. The Journal Political Economy 103 (2), 439-443.

De Gregorio, J., 1996. Borrowing constraints, human capital accumulation, and growth. Journal of Monetary Economics 37 (1), 49-71.

Ding, W., and Lehrer, S., 2006. Do Peer Effects Affect Student Achievement in China's Secondary Schools?. NBER working paper # 12305.

Faggio, G., Salvanes, K., Reenen, J., 2007. The evolution of inequality in productivity and wages: Panel data evidence. NBER working paper # 13351.

Fernández, R. and Galí, J., 1999. To each according to...? Markets, tournaments and the matching problem with borrowing constraints. Review of Economic Studies 66, 799-824.

Ferreira, F., 2001. Education for the masses? The interaction between wealth, educational and political inequalities. Economics of Transition 9 (2), 533-552.

Flug, K., Spilimbergo A., and Wachteinheim, E., 1998. Investment in education: do economic volatility and credit constraints matter? Journal of Development Economics 55 (2), 465-481.Frank, 1999

Frank, R., 1999. Luxury Fever: Why Money Fails to Satisfy in an Era of Excess. Princeton University Press.

Galor, O. and Zeira J., 1993. Income distribution and macroeconomics. Review of Economic Studies 60 (1), 35-52.

Glomm, G. and B. Ravikumar, 1992. Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality. Journal of Political Economy 100 (4), 818-834.

Hopkins, E. and Kornienko, T., 2004. Running to Keep in the Same Place: Consumer Choice as a Game of Status. American Economic Review 94 (4), 1085-1107.

Hopkins, E. and Kornienko, T., 2006. Which Inequality? The Inequality of Resources Versus the Inequality of Rewards. Mimeo.

Hoxby, C., and Weingarth, C., 2007. School Reassignment and the structure of Peer Effects. Working paper, Harvard.

Lucas, R., 1988. On the mechanics of economic development. Journal of Monetary Economics 22 (1), 3-42.

Mejía, D. and St-Pierre, M., 2007. Unequal Opportunities and Human Capital Formation. Journal of Development Economics, in press.

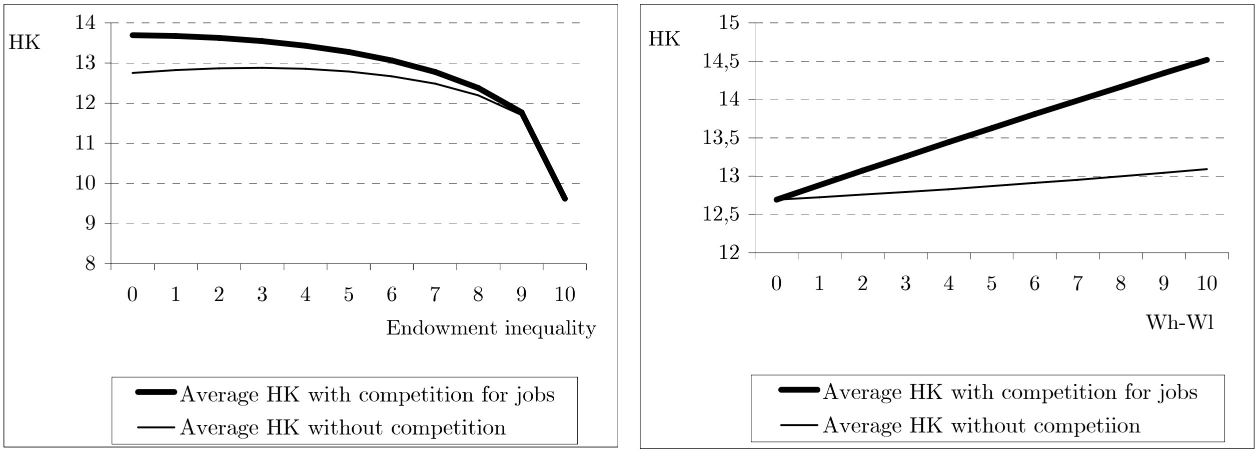
Moen, E., 1999. Education, Ranking, and Competition for Jobs. Journal of Labor Economics 17 (4), 694-723.

Postlewaite, A., 1998. The Social Basis of Interdependent Preferences. Economic European Review 42, 779-800.

Rege, M., 2000. Why Do People Care about Social Status?. Journal of Economic

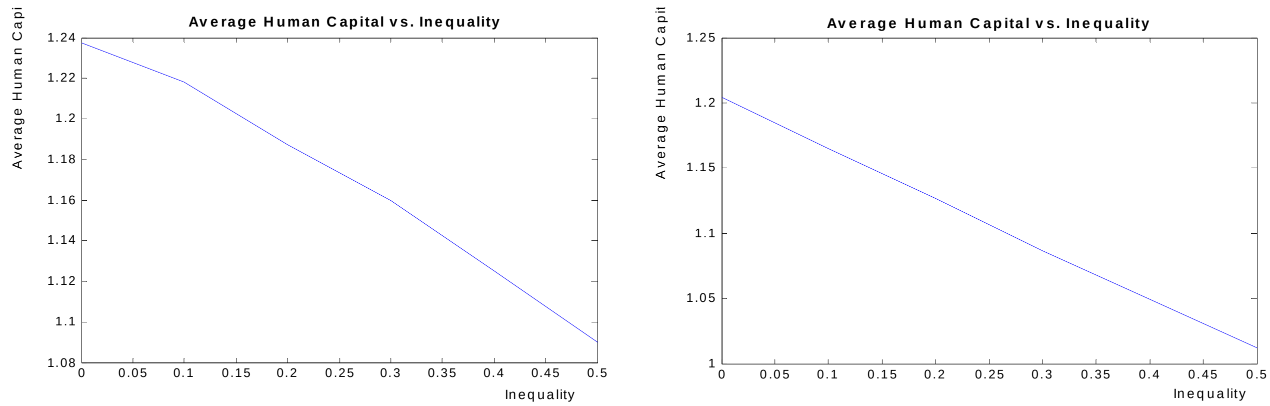
Behavior and Organization.

Veblen, T., 1899. The Theory of Leisure Class: An Economic Study of Institutions. New York: Macmillan



(a) (b)

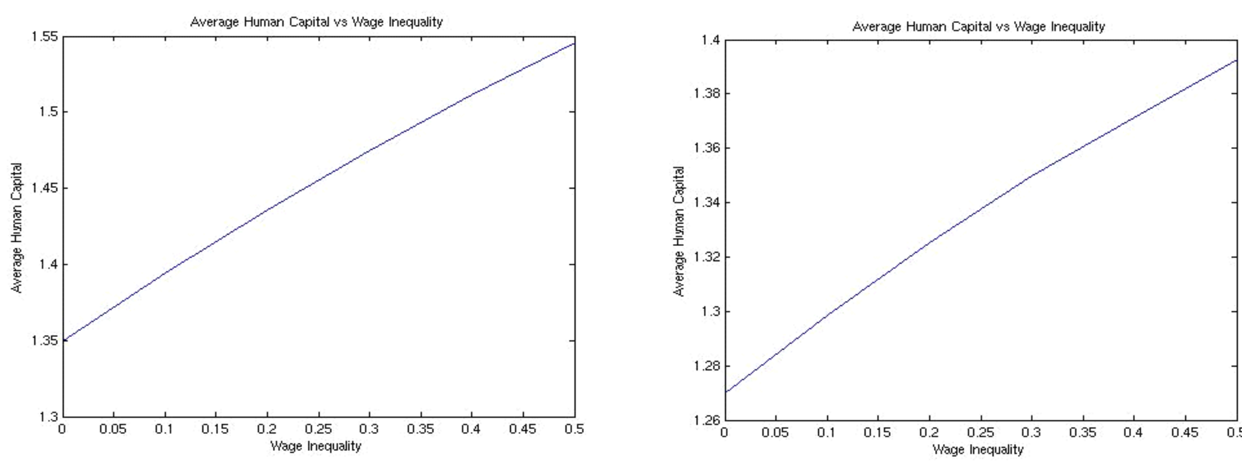
Figure 1: Calibration results for the simple model.



(a)

(b)

Figure 2: Aggregate efficiency in human capital formation vs. endowment inequality (inequality of opportunities). In panel (a) $\alpha = 3/4$ and in panel (b) $\alpha = 1/2$.



(a)

(b)

Figure 3: Aggregate efficiency in human capital formation vs. wage inequality (inequality of returns). In panel (a) $\alpha = 3/4$ and in panel (b) $\alpha = 1/2$.