Endogenous employment and incomplete markets

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Abstract

In this paper we explore the role of effort and human capital as mechanisms to alleviate the idiosyncratic risk faced by individuals in the presence of incomplete markets. We construct a dynamic stochastic general equilibrium model where effort and human capital determine the probability of being employed the next period, for both currently employed and unemployed agents. In other words, we endogeneize the Markov chains that summarize the transition between states. While effort is a flow variable that has to be exerted every period, human capital is a stock variable that can be accumulated and also produces monetary returns. While maintaining previous results obtained in this literature such as a lower risk-free interest rate and partial insurance with a riskless asset, we also found that individuals will diversify between market and non-market mechanisms to reduce risk. As a result, in the long run the median individual will hold a negative credit balance, which better approximates the real wealth distribution when compared with previous studies. The model also sheds light on understanding long run unemployment from a supply side perspective.

Keywords Employment, Incomplete markets, Heterogeneity, Endogenous Markov chains
JEL codes D91, E21, E24, E25, J22

1 Introduction

Idiosyncratic shocks and consumption smoothing has been largely studied in the literature. Models of incomplete markets and heterogenous agents have been used to explain the risk premium (Huggett, 1993), the benefits of insurance (Hansen and İmrohoroğlu, 1992), optimal fiscal policy (Heathcote, 2005), and the distribution of income (Aiyagari (1994); Heckman, Lochner, and Taber (1998); Krusell and Smith (1998)), among others. The common characteristic

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of these models is that they use mechanisms affecting the budget constraint to smooth consumption. These mechanisms are usually identified with assets holdings (or credit balances), capital, or savings.

The purpose of this paper is to explore different mechanisms used by individuals to deal with their idiosyncratic risk that are not related to the budget constraint. We develop two models based on the framework proposed by Huggett (1993) and Aiyagari (1994), where effort and human capital are variables determining the transition dynamics between states. In the first model, effort is modelled as a flow variable that has to be chosen every period to maintain a positive probability of being employed. In the second model, human capital is a stock built through time that depreciates and can be accumulated. It increases the probability of employment while it also increases future productivity and thus future income.

In the first model we take the approach followed by the literature on unemployment insurance where an individual must exert effort to increase the probability of being employed next period (Hopenhayn and Nicolini (1997);Wang and Williamson (1996)). This can be seen as search effort when the individual is unemployed, or effort in the job when the agent is employed. We assume the level of effort required in the latter case is more effective that the one when the agent is unemployed. This assumption matches with empirical data that has been studied in search models and emphasize the role of the depreciation of human capital during unemployment (Addison and Portugal (1989); Neal (1995)).

Our approach allows to understand the outside-of-the-market behavior of households when taking labor supply decisions, and its relationship with market strategies to bear idiosyncratic risk. Therefore, the unobservable effort exerted by an individual could be approximated by the level of asset holdings, assuming they are observable. This analysis becomes an important tool when analyzing unemployment insurance when private effort affects future transitions in a dynamic stochastic general equilibrium framework.[1]

The role of the asset holdings in our model is similar to the one played in previous literature. When the individual is employed she accumulates assets, while she decreases her holdings when unemployed. Therefore, it keeps track of the employment history the individuals have had.

As it is usual in this literature, we fix a lower bound for the asset holdings to prevent situations where individuals get indebted forever. This lower bound is used to model a financial friction usually found in reality, and is calibrated accordingly. An upper bound arises naturally from the optimal decisions and the fact that the interest rate is lower than the rate implied by the intertemporal discount factor. This discourages individuals from accumulate forever their asset holdings.

The ability of the assets to smooth consumption loses importance when they are close to the lower limit. At that point effort plays a major role by increas-

[1] Cole and Kocherlakota (2001) have also studied efficient allocations when asset holdings and income are private information. Our framework could be also used there by modifying accordingly their model.

ing the likelihood of being employed next period. In fact, we found a negative relationship between effort and asset holdings that becomes less important as asset holdings increase. As a consequence of this optimal policy functions, consumption gets sufficiently smooth and both types of individuals enjoy similar levels.

Our model also sheds light on understanding unemployment in the long run. In the stationary distribution most of the individuals will hold a small negative credit balance, while few of them will be at the extremes. This means that most of the individuals combine both channels to smooth consumption rather than relying in one of them, which implies that individuals will not exert huge efforts to become or remain employed.[2] Therefore, a natural rate of unemployment arises. More importantly, the resulting stationary distribution of wealth is much closer to the real one than the wealth distributions obtained by previous studies. Another important feature of this framework is that it allows the individual to alleviate risk without increasing the risk free rate. This is also a consequence of the diversification between the market and the non-market mechanisms to alleviate risk.

In our second model we explored the role of human capital in the transition probabilities while maintaining the usual approach of human capital as a mechanism to increase earnings. Previous empirical work has also pointed out the effect of human capital on employment transitions. For example, Card and Sullivan (1988) estimate the effect of training on the probability of employment for the 1976 cohort of adult male participants in the Comprehensive Employment and Training Act (CETA). They found that the effect is positive, even for people who is already employed. Gritz (1993) also found that participation of women in private training programs increases both the frequency and duration of employment spells. Ham and LaLonde (1996) evaluated the National Supported Work Demonstration and only found a significative positive effect on the employment spells.

But evidence also suggests that the positive effect of training is decreasing. For instance, Bonnal, Fougère, and Sérandon (1997) evaluated French on-the-job training programs during the 1980's and found that these are principally beneficial for less educated young workers. We incorporate all these results by allowing human capital to affect the transition probabilities in a similar fashion as effort by assuming that the probabilities are increasing and concave in human capital.

We found again a negative relation between assets and human capital, suggesting again that individuals diversify between the two mechanisms. Moreover, we found that investment in human capital is counter cyclical: in bad times people tend to study more. Previous studies have documented this counter cyclicality via an increasing on labor productivity (see for example DeJong and Ingram (2001)). However, we show how these models underestimate human capital re-

[2]This is an interesting result that is related with diversification, which in turn is a consequence of the convexity properties of the sets. In our model it can be traced to the concavity of the probability transition to the employed state, as well as the concavity of the utility function.

turns when transitions are assumed exogenous. Endogeneizing the probabilities of employment accounts for an additional benefit of human capital and captures the non-market returns of such mechanism, which has been found significant as previously pointed out.

The organization of the paper is as follows. The next section describes the model where individuals choose effort as a strategy to improve the probability of being employed the next period. The third section defines the equilibrium in this scenario. We then describe the calibration used in the model, while section 5 devotes attention to the computation. In section 6 we show the results and its implications. Section 7 explores the role of human capital as a variable that can be accumulated to obtain higher income and increase the probability of being employed. The last section concludes.

2 The Model with Effort

Consider an exchange economy with a continuum of agents with total mass equal to one who face idiosyncratic risk. There are two commodities: one perishable consumption good and asset holdings. We only let the individual use credit balances in their budget constraint to focus on the role of effort on smoothing consumption. Each agent receives an stochastic endowment s_t at the beginning of each period. Assume the endowment can take two possible values $s_L < s_H$, which are usually associated with unemployed/employed status, respectively.

Effort is made in order to increase the probability of having a good endowment (state) next period. This probability is defined as $\Pr(s_{t+1} = s_H | s_t) = P(e_t; s_t)$, which is an increasing concave function of the effort and it depends on the current state; in particular, let $P(e_t; s_H) > P(e_t; s_L)$. This last condition implies that effort to remain employed is more effective than the effort to become employed when previously unemployed.

Let effort belong to $[0,1]$ and assume that $P(0; s) = 0$ and $P(1; s) = 1$ for any s, that is exerting no effort implies a null probability of being employed whereas exerting the maximum effort implies certainty on being employed next period. This assumption implies that there is an e^* such that for any $e < e^*$ we have $P_e(e; s_H) > P_e(e; s_L)$ and for any $e' > e^*$ we have $P_e(e'; s_H) < P_e(e'; s_L)$. In other words, the single crossing property applies to the marginal probability.

Assume each agent maximizes her expected additive separable utility function over her infinite lifetime conditional on the information available at the beginning of the period

$$\begin{aligned} U &= E_t\left[\sum_{t=0}^{\infty} \beta^t \left(u(c_t) + g(1-e_t)\right) | I_{t-1}\right] \\ &= \sum_{t=0}^{\infty} \beta^t \left[P(e_{t-1}; s_{t-1})\left(u(c_t) + g(1-e_t)\right) + \left(1 - P(e_{t-1}; s_{t-1})\right)\left(u(c_t) + g(1-e_t)\right)\right] \end{aligned}$$

where $c_t \in R^+$ is the consumption in period t, e_t is the effort made in period t, $\beta \in (0,1)$ is the discount factor, I_t is the set of information available at the beginning of time t, and $u(\cdot)$ and $g(\cdot)$ are strictly concave functions satisfying Inada conditions.

Each agent is able to smooth her consumption by holding a single asset. This asset entitles the individual to receive one unit of future consumption for each unit of asset whose price is $q > 0$. The amount of claims held must remain above the limit $a_{\min}$, a restriction that represent the financial friction faced by individual in addition to the incompleteness of the markets. Therefore, the budget constraint faced by an individual who holds a claims, has a current endowment s, and chooses consumption c and future claims a', is given by

$$c + qa' \leq s + a \tag{1}$$

The agent's problem can be represented in recursive formulation as

$$v(a, s; q) = \max_{c,e,a'} \{u(c) + g(1-e) + \beta [P(e; s) v(a', s_H) + (1 - P(e; s)) v(a', s_L)]\} \tag{2}$$

subject to (1), $c \geq 0$, $e \in [0, 1]$, and $a' \geq a_{\min}$.

This problem is well defined since $v(a, s; q)$ will inherit the concavity properties of $u(\cdot)$ and $P(\cdot)$, while also satisfying discounting and monotonicity (see Stokey and Lucas with Prescott (1989)). Therefore, the first order conditions are necessary and sufficient, and the optimal decision rules $c(a, s; q)$, $e(a, s; q)$, and $a'(a, s; q)$ are given by

$$\begin{aligned}
-g_e(1-e) &\geq \beta P_e(e; s) [v(a', s_H; q) - v(a', s_L; q)], \\
\text{with equality if } e &\in (0, 1) \\
u_c(c)\, q &\geq \beta \left[P(e; s) \frac{\partial v(a', s_H; q)}{\partial a'} + (1 - P(e; s)) \frac{\partial v(a', s_L; q)}{\partial a'} \right], \\
\text{with equality if } a' &> a_{\min} \\
c + qa' &\leq s + a
\end{aligned}$$

The first condition shows the tradeoff between the marginal disutility and the expected marginal benefits of exerting an effort. This condition is similar to the one obtained in the optimal unemployment insurance literature.

The second condition is very familiar to the literature that uses asset markets. This condition can be recast as

$$u_c(c_t) \geq \frac{\beta}{q} E_t [u_c(c_{t+1})], \text{ with equality if } a' \geq a_{\min}$$

The limiting behavior of consumption can be characterized by applying the theory of martingales. Let $Z_t = \left(\frac{\beta}{q}\right)^t u_c(c_t) \geq 0$. Therefore, $E_t [Z_{t+1} - Z_t | I_t] = \left(\frac{\beta}{q}\right)^t E_t \left[\frac{\beta}{q} u_c(c_{t+1}) - u_c(c_t) | I_t\right] \leq 0$, where I_t is the information set at time t, including e_t. The previous expectation implies that Z_t is a supermartingale. Since Z_t is nonnegative, we can apply the supermartingale convergence theorem. This theorem states that Z_t must converge almost surely to a nonnegative random variable ((Williams, 1991)).

If $\beta > q$ then Z_t must converge to zero to avoid its divergence. But then this implies that c_t must diverge to infinity. This is obtained by letting the asset holdings go to infinity since the incentives to save are greater than the ones to get more debt. This explosive solution can not be an equilibrium. A similar behavior is obtained if $\beta = q$, see Chamberlain and Wilson (2000) for an exposition. On the other hand, if $\beta < q$, then Z_t will converge to a nondegenerate nonnegative random variable. This implies that consumption and asset holdings will remain finite, a necessary condition to achieve an equilibrium.

From this set of first order conditions we can obtain the optimal decision rules of consumption, effort and next period amount of assets. It is important to note that optimal decision rules will depend on their state vector (a, s) and on the price of claims. This price will be determined in equilibrium according to a market clearing condition that we describe in the next section.

3 Equilibrium

The equilibrium in an exchange economy is usually defined as policy rules and prices that clear the markets given some aggregate states. However, the market clearing condition is always changing in this dynamic economy given that the distribution of individuals is always moving. Therefore, a definition of a stationary equilibrium is more appropriate in this context. In this definition we focus on market clearing when the distribution of wealth λ is invariant and plays the role of the aggregate variable that depends on the price q.

The law of motion of this state vector distribution is described by

$$\begin{aligned}
\lambda_{t+1}(a', s'; q) &= \Pr(a_{t+1} = a', s_{t+1} = s') \\
&= \int_{a_t} \sum_{s_t} \Pr(a_{t+1} = a', s_{t+1} = s', a_t = a, s_t = s)\, da_t \\
&= \int_{a_t} \sum_{s_t} \Pr(a_{t+1} = a' | s_{t+1} = s', a_t = a, s_t = s) \\
&\quad \cdot \Pr(s_{t+1} = s' | a_t = a, s_t = s) \cdot \Pr(a_t = a, s_t = s)\, da_t \\
&= \int_{a_t} \sum_{s_t} \Pr(a_{t+1} = a' | a_t = a, s_t = s) \\
&\quad \cdot \Pr(s_{t+1} = s' | a_t = a, s_t = s) \cdot \Pr(a_t = a, s_t = s)\, da_t \\
&= \int_{a_t} \sum_{s_t} \lambda_t(a, s; q) \cdot P(e_t; s_t) \cdot I(a', s, a)\, da_t
\end{aligned}$$

where $I(a', s, a)$ is an indicator function that takes the value of 1 if $a' = f(a, z)$ and 0 otherwise, where $f(a, z)$ is the optimal decision rule for a'. Hence

$$\lambda_{t+1}(a', z'; q) = \int_{\{a_t : a'(a,s;q)\}} \sum_{s_t} \lambda_t(a, z; q) \cdot P(e_t; s_t)\, da_t$$

A stationary distribution is thus defined as a distribution $\lambda(a, z; q)$ such that $T\lambda(a, z; q) = \int_{\{a_t : a'(a,s;q)\}} \sum_{s_t} \lambda(a, z; q) \cdot P(e_t; s_t)\, da_t = \lambda(a, z; q)$. The existence and uniqueness of the invariant distribution is established in Hopenhayn and Prescott (1992). Therefore, starting from any initial distribution, a sufficient number of iterations will converge to the invariant one. Moreover, since $a'(a, s; q)$ is bounded, the sequence of averaged assets will also converge.

Definition 1 *A stationary equilibrium is defined by policy rules $c(a, s; q)$, $e(a, s; q)$, and $a'(a, s; q)$; a value function $v(a, s; q)$; a price q; and a stationary distribution $\lambda(a, z; q)$, such that*

- *The policy and value functions solve the agent's problem (2)*
- *Markets clear:*
  1. $\int_a \sum_{s=1,2} c(a, s; q)\, \lambda(a, s; q)\, da = \int_a \sum_{s=1,2} s\lambda(a, s; q)\, da$
  2. $\int_a \sum_{s=1,2} a'(a, s; q)\, \lambda(a, s; q)\, da = 0$
- *The stationary distribution $\lambda(a, s; q)$ is induced by the policy functions and the endogenous Markov chains generated by $P(e(a, s; q); s)$.*

The first condition states the optimality of the decisions. The second one defines market clearing for assets, which means that the average holdings in the population must be zero. By Walras Law, if the market of loans is cleared, then the market of the consumption good is also cleared by making average consumption equal to the average endowment. The third condition requires that the distribution of assets remains the same over time. For that we need them to remain finite, this is assured by the lower bound and the fact that $\beta < q$. It also plays an important role that $P(e; s_H) > P(e; s_L)$.

4 Calibration

We calibrate the model according to the previous literature on heterogenous agents, mainly Huggett (1993), and unemployment insurance (Hopenhayn and Nicolini, 1997). We first assume the utility function takes the form

$$u(c) + g(1 - e) = \frac{c^{1-\sigma}}{1 - \sigma} + \frac{(1 - e)^{1-\upsilon}}{1 - \upsilon}$$

This is the standard utility function used in this type of problems. According to Mehra and Prescott (1985), estimates of the risk aversion coefficient σ are around 1.5. On the other hand, Hopenhayn and Nicolini (1997) let $\upsilon = 0$, which implies an infinitely elastic effort supply. Results with $\upsilon = 0$ show that the agent would exert enough effort to virtually assure employment next period, thus loosing important aspects of the tradeoff. Since we require effort to be bounded, we let $e \in [0, 1]$ by assuming a sufficiently low risk aversion coefficient

$\upsilon = 0.5$. The parameter υ has to be low enough in order to obtain significant variation in the effort levels across individuals with different level of assets.[3]

The rest of the parameters are calculated according to periods of 8.5 weeks approximately, that is 6 periods per year. Huggett (1993) chose this length to match the average duration of unemployment spells of 17 weeks (Bureau of Labor Statistics), which is a underestimation of the current average duration of 21.6, but it fits the 5 year trend. For this the endowments were calibrated to $s_H = 1$ and $s_L = 0.1$, where the last number assumes that individual has access to social programs when he is unemployed. Finally $\beta = 0.99322$ to match an annual discount rate of 0.96. He also specified an exogenous Markov process where $\Pr(s_{t+1} = s_H | s_t = s_H) = 0.925$ and $\Pr(s_{t+1} = s_H | s_t = s_L) = 0.5$.

This calibration replicates a coefficient of variation for the annual earnings of 20%, which is close enough to the actual data. It also generates an annual average endowment of 5.3; therefore, we set $a_{\min} = -5$ to simulate the financial friction. This bound generates in equilibrium an annual interest rate between 2.3% and 3.4% in Huggett's calculations and is close to the natural borrowing limit of $-\frac{s_L}{r}$ described by Aiyagari (1994).

In order to obtain similar quantitative results, we calibrate our endogenous Markov chain to find similar probabilities. We model the probability of having a high state tomorrow as a cdf of a beta distribution with parameters $(1, \mu_s)$, where $\mu_L = 1$ and $\mu_H = 5$. The first parameter implies that in a state of unemployment the probability follows a cdf of an uniform distribution. This assumption plays the role of a normalization since the effort exerted is the same probability of being employed. This parameterization satisfies our initial assumptions of first order stochastic dominance and the ones described by Hopenhayn and Nicolini (1997) to characterize the optimal unemployment insurance. Moreover, as shown in the next section, the optimal probabilities in equilibrium will wander around Huggett's calibration.

5 Computation

To find the optimal policy rules we first set a candidate for q, say q_1, belonging to a plausible interval of equilibrium prices. We then use value function iteration to obtain the optimal policy rules. Since all the desired properties of the value function are satisfied, convergence is achieved independently of the initial guess for the value function. To compute the solution we discretize the choices of a and e, while obtaining consumption from the budget constraint. The grid must be fine enough to achieve smooth policy functions.

While $e \in [0, 1]$ by definition of the problem, we have to find a natural upper bound for a. This upper bound exists since in the optimum the choice of future assets for an employed agent starts above the 45^0 line (when current assets are negative), and then crosses this line for some positive level of current

[3]We also consider the case where $\upsilon = 1$. Although the qualitative results remain, the quantitative effects decrease importantly. This is so because the higher υ, the closer we get to the case of exogenous Markov transitions.

holdings, say $a_{\max}$. On the other hand, an unemployed agent will always reduce her holdings to maintain her consumption. See Fig. 2 in the appendix for an example of an optimal policy rule for asset holdings.

This shape of the optimal policy implies that $a_{\max}$ plays the role of a fixed point when an agent is always employed. Moreover, it also plays the role of an upper bound since once the agent receives a bad shock she will decrease her assets. Hence, an agent with any initial wealth will converge to the interval $[a_{\min}, a_{\max}]$, and remain there forever. This upper bound can only be computed by experimentation and thus the upper bound of the grid is set large enough to include the fixed point.

After obtaining the optimal decision rules we compute the stationary distribution. To obtain it we simulate an economy of 100000 agents and iterate for 200 periods.[4] The initial distribution of states and assets will not matter for the convergence. We first fix a set of i.i.d shocks with a uniform distribution between 0 and 1 for each individual and each period. We then interpolate the optimal decision using the optimal policy rules and the current asset holdings and state. We then compare the i.i.d shock with the probability associated with the optimal effort and the current state. If the shock is smaller then the next period's state is employed, otherwise the agent will be unemployed.

After the stationary distribution is computed we calculate the excess demand for assets given the initial price q_1. Then we follow Huggett's process of bisection: if the excess demand is positive we increase the price q, if it is negative we decrease it. This algorithm follows the conjecture that the excess demand of assets is negatively correlated with its price. Although this is hard to prove in general, this is the case in the interval we examined, and it has been also true in related papers that follow the same methodology (see for example Huggett (1993) and Aiyagari (1994)). The process continues until excess demand is approximately 0 and the difference of the updated price is less than 0.001.

6 Results

Fig. 1 shows the concavity of the value function that permits the contraction to find the fixed point. It also shows how utilities diverge when asset holdings are close to the lower limit, a result that is intuitive after examining the policy rules. The optimal asset policy is shown in Fig. 2 and it follows the behavior described in the previous section. It shows how individuals with low states will decrease their holdings until the lower limit, while individuals with good shocks accumulate holdings until they reach the upper bound. This is a characteristic of the models in this branch of the literature.

In our model we also explore a different non-monetary mechanism used by individuals to alleviate risk. Individuals use effort to increase their probability of being employed next period, especially when their level of assets is approaching its lower limit. The optimal effort is decreasing on the asset holdings and is

[4] We also chose a longer horizon without obtaining significant differences.

greater for unemployed individuals since by assumption is harder to change their status. In fact, unemployed individuals almost exert the maximum effort to increase the chances of being employed when asset holdings are too low.

The probabilities associated with the exerted effort are shown in Fig. 3. These wander around the probabilities calibrated by Huggett (1993), providing a good approximation of the steady state. They also show how the individual increases them when asset holdings are close to the lower bound. As a consequence of this optimal strategy for risk bearing, consumption has very little variation across different types of individuals, except for unemployed agents whose asset holdings are close to the lower limit. Fig. 4 depicts the optimal consumption.

Fig. 5 shows the excess demand of holdings, which depends negatively on the price. The price of assets that clears the market is 0.9951, which is equivalent to an annual interest rate of 2.99% and is between the bounds obtained by Huggett (1993).

The distribution of wealth in the stationary distribution differs from the one found by Huggett (1993) shown in his Fig. 2. Fig. 6 shows how it is concentrated in a slightly negative amount of assets for both employed and unemployed individuals. This suggests that in the long run individuals are not afraid of becoming indebted since they have another non-market mechanism to smooth consumption. It is also remarkable how equal is the consumption across different types of agents, which is a consequence of the smoothing process. At the end, the incomplete markets partial insurance is successfully complemented by the effort.

This interaction between assets and effort also offers an explanation of long run unemployment from the supply side. Each individual prefers to diversify between both mechanisms to smooth consumption. Therefore, most of the individuals will hold a slightly negative amount of assets in the stationary equilibrium and will not exert the maximum effort to affect tomorrow's state. These optimal decisions generate unemployment. In fact, we found that the endogenous unemployment rate in the stationary distribution is 13.28%, which is close to the unemployment rate of 13.04% implied by the stationary distribution of the exogenous Markov matrix calibrated by Huggett (1993).

7 An Extension with Human Capital

(Work in progress)

Now consider a similar environment but this time individuals invest in human capital to increase the probability of being employed next period. We abstract from effort in this model to focus on the effect of human capital. The probability of being employed tomorrow $P(h_{t+1}|s_t)$ is now a function of the stock of human capital tomorrow h_{t+1} and still a function of the current state of employment s_t, where $P(h_{t+1}|s_H) > P(h_{t+1}|s_L)$ and concave as before. We let $h \in [0, \infty)$ and assume again the single crossing property on the marginal probability.

We also allow for the possibility of accumulation and depreciation of human capital to capture the investments on it. Moreover, we also allow for a positive effect of human capital on earnings by letting income in a given period be $h \cdot s$.[5] The budget constraint faced by an individual who hold a claims, has a current state s, has current human capital h, and chooses consumption c, future claims a' and future human capital h' is now expressed as

$$c_t + h' - (1-\delta) h + qa' = hs + a \tag{3}$$

Agents maximize their expected utility, which we can express in recursive form as

$$v(a,s,h;q) = \max_{c,a',h'} \{u(c) + \beta [P(h';s) v(a', s_H, h'; q) + (1 - P(h';s)) v(a', s_L, h'; q)]\}$$

subject to the budget constraint (3) and $a' \geq a_{\min}$

Under our assumptions the value function is concave and thus first order conditions are necessary and sufficient. The optimal decision rules $c(a,s,h;q)$, $a'(a,s,h;q)$, and $h;(a,s,h;q)$ are given by:

$$\begin{aligned}
u_c(c) &= \beta \left\{ \begin{array}{c} P_h(h';s)[v(a',s_H,h';q) - v(a',s_L,h';q)] \\ +\mathbb{E}_{s'}[u_c(c')(s'+1-\delta)|h',s] \end{array} \right\}, \\
u_c(c)q &\geq \beta \mathbb{E}[u_c(c')|h',s], \\
\text{with equality if } a' &> a_{\min} \\
c + h' + qa' &\leq h(s+1-\delta) + a
\end{aligned}$$

The last two conditions are interpreted similarly as the analogous conditions in the model with effort. The first condition equalizes the marginal cost of human capital in terms of consumption, and the expected marginal benefits. The expected marginal benefits are a combination of the benefits found before for effort and the ones obtained in the usual literature that considers the effect of human capital as another asset. This implies that the usual theoretical literature importantly underestimates the return of human capital by not considering its effect on the transition probabilities, which has been previously identified in the empirical literature, and is not captured by market returns. Moreover, this equation makes the countercyclicality stronger than the one found before.

8 Concluding remarks

We have studied a model of heterogenous agents who face idiosyncratic risk and smooth their consumption using incomplete markets and by exerting an effort that affects the transition distribution to the next state. We have found that effort is an important mechanism to smooth consumption and alleviate the risk without increasing the risk-free interest rate. In the long run individuals will diversify between the two mechanisms. This result arises because individuals can

[5] This endogeneization of the income would also allow us to endogeneize the borrowing limit to an estimate of $-\frac{hs_L}{r}$. However we abstract from this issue in this first stage.

also rely on effort to affect tomorrow's probability distribution: a classical result of diversification to decrease risk but this time with a non-market mechanism.

As a consequence, the median (employed and unemployed) individual holds a small negative credit balance and exert a medium amount of effort. This result contrasts with the ones previously obtained where the median individual holds a positive credit balance. Therefore, our framework replicates much better the real distribution of wealth.

Our model also helps to understand why there is a natural unemployment rate in the long run. In the stationary equilibrium agents will prefer to rely on assets to smooth consumption rather than exerting enough effort to reduce uncertainty in the next period. This optimal strategy generates long run unemployment.

This model could be used as a benchmark to evaluate the effect of unemployment insurance on wealth and consumption smoothness. Moreover, it suggest how asset holdings could be used as a proxy to unobservable effort. Therefore, the optimal contract could exploit this relationship to deal with private information.

We also explored a model of human capital that merges our framework with the traditional approach that human capital increases productivity and hence earnings. The model assumes that human capital increases the probability of being employed as previous empirical studies have found, and also generates monetary returns. We also found a diversification between human capital and assets, and we expect that in the long run equilibrium the same diversification result will arise. Moreover, the model replicates the countercyclicality of investment in human capital considering also the non-market benefits.

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A Figures

Figure 1: **Value function**

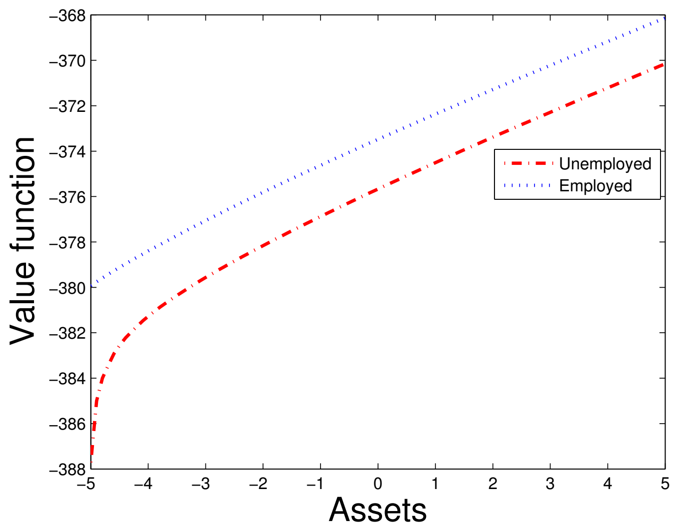

Figure 2: **Optimal policy rule for assets**

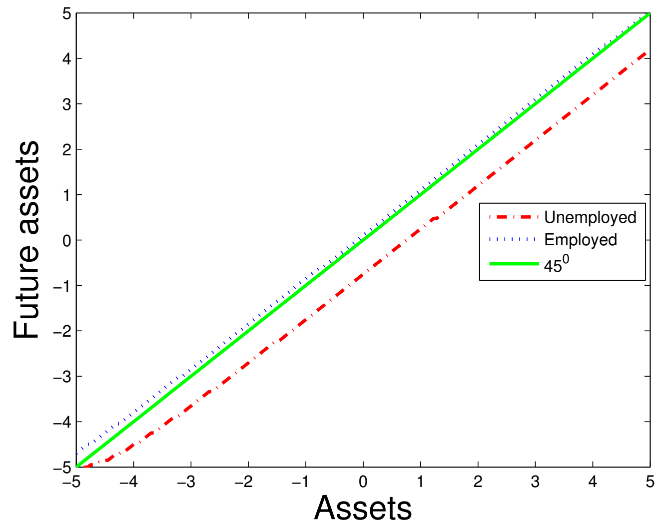

Figure 3: **Probabilities associated with optimal effort**

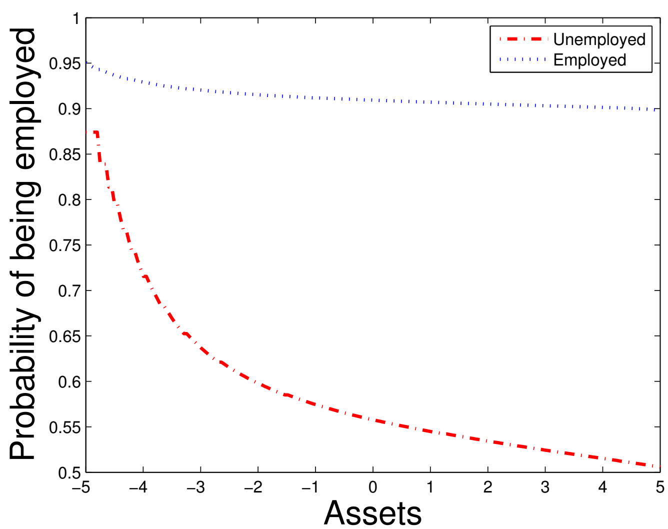

Figure 4: **Optimal policy for consumption**

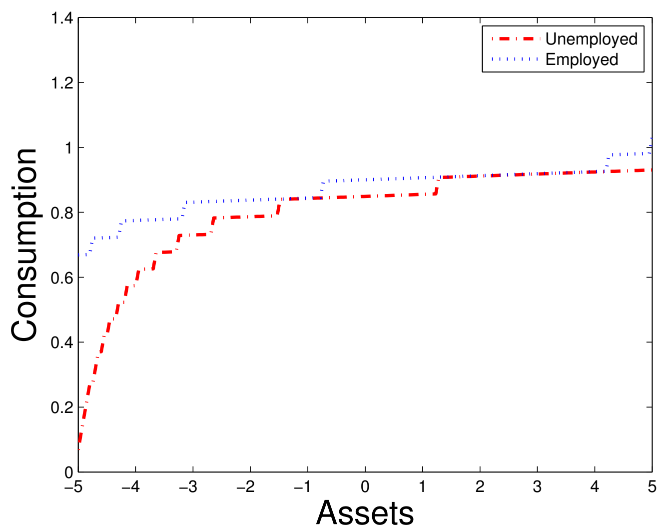

Figure 5: **Excess Demand for assets**

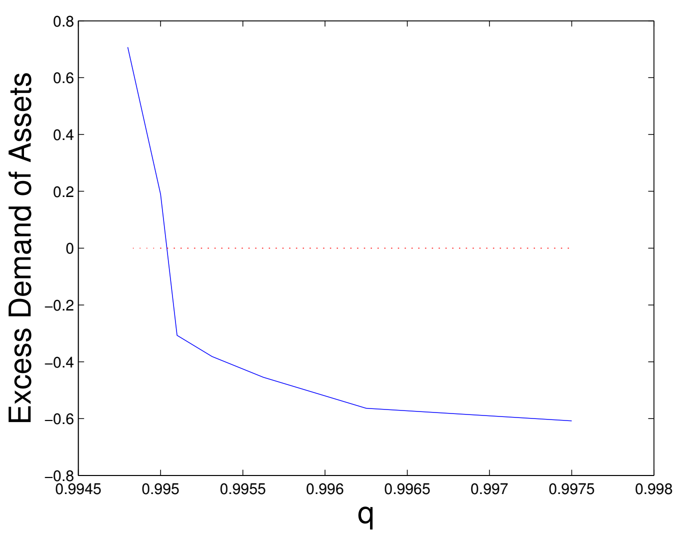

Figure 6: **Conditional stationary distribution of assets**