FOREIGN AND DOMESTIC SHOCKS: AN APPROACH TO THEIR RELATIVE IMPORTANCE AND THE CASE OF COLOMBIA

Master thesis submitted to the Department of Economics at Universidad de Antioquia.

By

Wilman Arturo Gómez Muñoz♦

Thesis advisor

Sergio Ivan Restrepo Ochoa♣

INTRODUCTION AND OVERVIEW

It is known that the Colombian economy as well as the most of the Latin American economies experienced a period of relative economic stability until the end of seventies. However, since the early eighties, a common fact in these economies but Colombia, was a debt repudiation process, economic recession and a backward movement in their economic growth and development processes. This gives raise to what is known as Latin America's lost decade.

The Colombian economy was one of the most stable in the region. It is true that the effects of the crisis were felt. However, there was not debt repudiation in Colombia. Instead, this economy continued to have credibility in the international capital market and its growth rates although less that those of previous decades were still acceptable. The growth rates of net flows of resources were even higher that the Latin America's ones (Tables 1 and 2).

Table 1

GDP Growth (%)		
	Colombia	Latin America
1985-1989	4,36	2,30
1990-1994	4,23	2,94
1995-1999	1,41	2,56
2000-2002	1,88	1,23

Source: CEPAL, DANE, Banco de la República.
Note: Latin America groups 20 countries.

♦ Master student, Department of Economics.
♣ Professor-researcher, Department of Economics and Department of Mathematics, Faculty of Economics. Universidad de Antioquia.

At the end of the nineties, there was a new crisis, but this time, the situations were very different, and Colombia, being in past times one of the most outstanding economies in the region, suffered a lack of confidence in international capital market, as well as other countries in the region, at the time that increasing debt spread, GDP growth rates deceleration and high interest were observed. These facts conveyed the 1999 recession. The deepest suffered in Colombia since at least 1940. Nonetheless, accordingly to Table 1 The Colombian economic growth decelerated after 1995 and was lower than that of Latin America as a whole, while it was substantially higher during 1985-1994.

Table 2

Aggregated resources net flow growth (%)		
	Colombia	Latin America
1985-1989	30,77	-10,84
1990-1994	125,05	59,81
1995-1999	24,93	15,09

Source: World Bank.
Note: Latin America groups 20 countries.

Therefore, it is worth to ask: i) what were the most important changes that the Colombian economy suffered from one decade to the other? ii) It would be possible that the change of the Colombian situation was explained through the nature of the shocks suffered by those countries? iii) Could these phenomena be explained from the basis of the structural and institutional changes of the Colombian economy such as the economic reforms it experienced since the beginning of nineties? iv) Was there a change in the international perception of Latin American economies confidence and on the Colombian economy particularly?

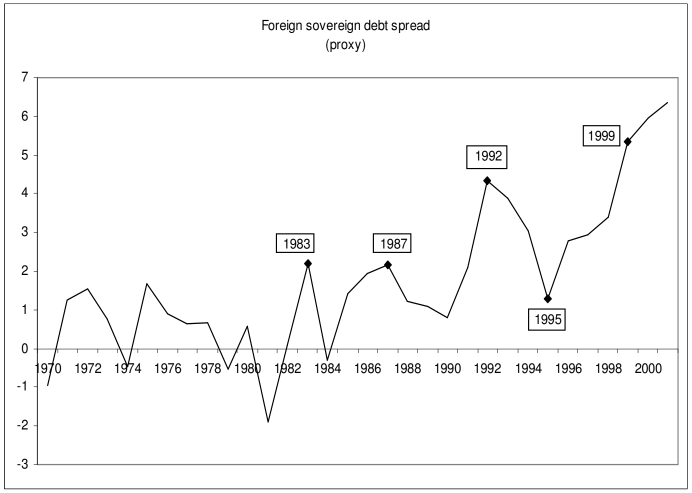
The aim to write this thesis is mostly related with questions i) and iv), questions ii) and iii) are at the moment out of the scope of this study. In a world economy such as the current one, where economies trade goods and services and capital flows every where it is pertinent to ask whether the ancient debate about business cycles could become more likely to occur in such an environment. To be more precise, the questions are: can technology shocks spill over the world through channels such as trade? Is it possible that confidence on small economies could change in such a way that increasing debt spreads caused higher costs of capital accumulation financing or even though higher costs of recovering from a depression?

Graphics 1 and 2 show a important fact for the Colombia economy: [1] foreign debt spreads for nineties was greater (and increasing) than those for eighties (the Lost Decade of Latin America). There are three remarkable points in Graphic 1: 1992 consolidation of economic opening process, 1995 a year after the peak of 1994 boom, and 1999 the deepest depression in the Colombian economic history in post was period. In Graphic 2 it is clear (but not so clear for the eighties) that foreign debt-GDP ratio increases are consistent with debt spread increases. Furthermore, graphics 3 and 4 show the increasing gap existing between Colombian real interest rate and USA interest rate for the eighties, but mostly for the nineties.

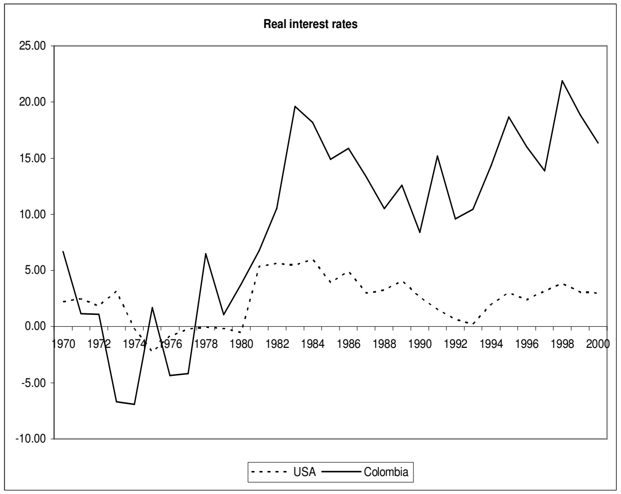
Knowing these facts, the objective of this work is to build a two country model emphasizing the role of the small open economy as the one that not only take prices, but also the one that fronts capital mobility restrictions through increasing foreign debt spreads. Chapter one of this thesis makes a detailed revision of six seminal papers on real business cycles (RBC) and international real business cycles (IRBC). In Chapter two, a two country dynamical model is built and transitional dynamics and calibration issues are shown. In chapter three stability conditions are derived and a simulation experiment is performed. Chapter four presents some conclusions and schedule later exercises to be developed with the model. Appendix one shows transition equation building for foreign assets, appendix two has a proof for the dynamics of the endogenous structural change of the world economy, and finally, appendix three explain in detail the calibration process.

[1] Source for graphics 1 to 4 are Greco (2002), and International Financial Statistics published by International Monetary Fund.

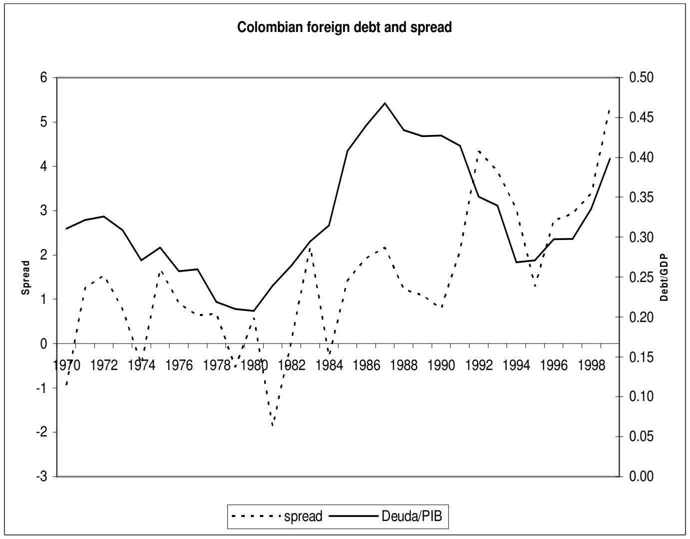
Graphic 1



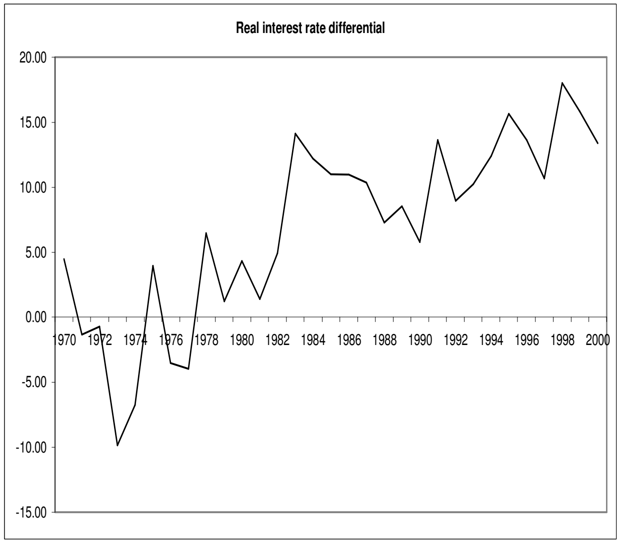
Graphic 3



Graphic 2



Graphic 4



CHAPTER I

INTERNATIONAL REAL BUSINESS CYCLES, A BRIEF SURVEY

The purpose of this article is to analyze the main articles written on the Real International Business Cycles (RIBC) tradition. This paper is the first section of the master's thesis to be submitted to the Department of Economics at THE Universidad de Antioquia.

In the first place, I will survey what is called first generation models in Real Business Cycles (RBC), which are those papers written for closed economies, the ones that started what is known as the Real Business Cycles theory. Secondly, I will review the second generation papers which are those of small open economies. Finally I will survey the third generation models which are in the line of research of international real business cycles (IRBC). These last models are born in the tradition of the multi country models.

We can cite a first pioneering work on real business cycle literature. The first one is: "Time to build and aggregate fluctuations" by Fynn E. Kydland and Edward C. Prescott (1982). They developed a general equilibrium model and fitted it for quarterly data for the US economy in the postwar period. They found a relative good fitness between the model's co-movements and those of the economy.

This is considered a seminal paper because it integrates growth models and business cycles theory in a multiperiod model which considers consumption and labor choices, and time to build modeling which states that several time periods are needed to incorporate finished capital goods as a part of new productive capital.

Three elements distinguish this work: a) non-time separable utility function which allows greater intertemporal substitution of leisure, b) the exogenous stochastic components of the model are shocks to technology and imperfect indicators of productivity. The two technological shocks differ in persistence and c) the parameters are calibrated using steady state considerations.

The model has a rule of accumulation of capital which includes the new projects of investment and the inventories investment. Therefore, the production function uses an aggregate of capital and inventories and labor. Thus, the production function is a Cobb-Douglas-CES function.

The preferences are a kind of non-time separable function in leisure. This is because time allocated to non-market activities has changes along different periods according to the change in their marginal productivity. This implies that utility function is a Constant Relative Risk Aversion function in total "consumption" which is a composite one of consumption and leisure. The argument for using such a function is that it fits well to cycle behavior of labor along business cycle. The technological shock is the sum of both a permanent and transitory component.

The equilibrium of the model exploits the result that in absence of externalities, competitive equilibria are Pareto-optima. "With homogeneous agents, the relevant pareto optimum is the one that maximizes the welfare of the stand-in consumer subject to the technology constraints and the information structure".

Quadratic approximations are made in the neighborhood of the steady state. Equilibrium decision rules for the resulting approximate economy are computed. These rules are linear, so in equilibrium the approximate economy is generated by a system of stochastic difference equations for which covariances are easily determined.

The main results are that the model is consistent with the large (percentage) variability in investment and low variability in consumption and their high correlations with the data though its magnitude is somewhat smaller. The model displays more variability in worked hours than in productivity, but not much more than that in data. It was also found, for the US economy and the model, consumption and non-inventory investment move contemporaneously with output and have serial correlation properties similar to output. Inventories and capital stocks for the model lag output also match well with data. Some of the inventory stocks cross-serial correlations with output deviate significantly, however, from those for US economy. The only variable whose lead-lag relationship does not match with data is productivity. For the US economy it is a leading indicator while there is no lead or lag in the model. This was not unexpected in view of our discussion above with regard to productivity. Thus, even though the overall fit of the model is very good, it is not surprising, given the level of abstraction, that there are elements of the fine structure of the dynamics that it does not capture

The second seminal work is written by John B. Long, Jr. and Charles I. Plosser: “Real Business Cycles”. (1983). The basic assumptions of the model are constant preferences: consumption goods are demanded in positive quantities and those goods are normal for every price level. This implies that the consumer “spreads” any unanticipated increase in wealth over time and commodities. Moreover, commodities are used as inputs for the production of other goods, causing that cross sector comovements are reproduced as will be shown later.

Production is supposed feasible and efficient with constant returns to scale, smooth substitutability of inputs and strictly diminishing marginal productivity of any given input in any given level of employment.

In a formal way, the model has an infinitely lived agent with an initial endowment, production possibilities and preferences. Plans of consumption and productions are chosen accordingly to a set of prices consistent with the equilibrium of the economy. Once the agent have done his consumption and productions choices, current technological shocks only affect future consumption and production plans. As all of the commodities are perishable, depreciation rate is 100% and the produced commodities are used to consumption or production. Therefore, the problem of the representative agent in this economy is:

$$\max \sum_{t=0}^{\infty} \beta^t u(C_t, Z_t), 0 < \beta < 1 \quad (1)$$

$$S.T. \quad Y_{t+1} = F(L_t, X_t; \lambda_{t+1}) \tag{2}$$

Y_{t+1}: is an Nx1 vector whose *ith* element, $Y_{i,t+1}$ is the total stock of commodity i available at time t+1.

$F(\cdot,\cdot;\cdot)$: is Nx1 vector-valued function concave and linearly homogeneous with respect to L_t and X_t.

L_t: Vector of labor inputs allocated at time t. In the case of no joint production, $L_{i,t}$ is the hours allocated at time t to the production of commodity i.

X_t: a matrix of commodity inputs allocated at time t. In the case of no joint production, X_t is NxN and X_{ijt} is the quantity of commodity j allocated in time t to production of commodity i.

λ_{t+1} is a random vector observable only in t+1 whereas λ_t can be currently observable. This vector is a time homogeneous Markov process and not necessarily must be stationary.

There are some special cases for the technology:

a. There is not a joint production
b. There is not technological change ($\{\lambda_t\}$ are independent and identically distributed)
c. Given L_t and Z_t the elements of Y_{t+1} are independently distributed.

These possibilities imply that "the regularities of the business cycles are not imposed to the model economy by the nature of the production functions or exogenous shocks" (p.45).

The resource constraints are

$$H = Z_t + \sum_i^N L_{it}, t = 0,1,2...... \text{ (for labor/leisure choices)} \tag{3}$$

$$Y_{it} = C + \sum_i^N X_{ijt}, \quad j = 1,2......N, \quad t = 0,1,2..... \text{(for commodity allocation)} \tag{4}$$

Relative prices are given by marginal rates of substitution evaluated in the quantities chosen for the consumption plan.

Thus we have that the current welfare function $V(S_t)$ defined as

$$V(S_t) = \max E\left[\sum_{s=t}^{\infty} \beta^{s-t} u(C_s, Z_s) \mid S_s\right]$$

and subject to equations (2),(3) and (4) this is equivalent to

$$V(S_t) = \max E\left\{\sum_{s=t}^{\infty} \beta^{s-t} u[C(S_s), Z(S_s)] \mid S_s\right\}.$$

Given $S_0 = (Y_0, \lambda_0)$ the initial state of the economy, the competitive equilibrium of the economy has a recursive evolution in a multivariate stochastic process.

As an example, a hypothetical economy is built on special assumptions about preferences and production functions. More precisely, preferences are modeled with logarithmic intra and intertemporal separable utility function, and the production function for each sector y of the Cobb-Douglas form. For the references function, coefficients are assumed greater or equal to cero; when the coefficient is equal to cero for any commodity, it is assumed that it is only used as an input in the production of any other commodity. However, the coefficient for quantities of leisure is assumed to be always greater than cero. For the case of the production function, the participation of commodity and labor inputs sum up to one; and it also has a technology λ_t parameter which is assumed to be stochastic.

With this set up, the maximization problem of the agent is posited as the following dynamic programming problem:

$$V(S_t) = \max\{u(C_t, Z_t) + \beta E[V(S_{t+1}) | S_t]\}$$

This last expression is a Bellman equation that must be solved, in the most of the cases, by “hunt and peck” or by numerical methods. Fortunately, the special forms supposed for the preferences and production functions give intelligible analytical forms which in spite of being a particular case, can shed light on what would happen in a more general case for preferences and production.

From these formulae a couple of natural results are drawn: i) the quantity of a commodity devoted to production or consumption is an increasing function of its productivity in the case of an input, or the consumption value for the case of a commodity used for consumption. ii) The quantities of a commodity or labor allocated to any productive use or consumption are all increasing functions of the total quantity available of it.

The second result is the most important because it is the essence of the propagation mechanism of the shocks toward both the future and other productive sectors, resulting the persistence behavior of the time series in the model.

Additionally, these rules decisions have two important properties:[2] i) commodities or time allocations do not depend on the contemporaneous quantities of other commodities. ii) given the value of Y_t, none of the allocations in time t depends on λ_t.

It is special in this case that labor/leisure allocation does not depend on Y_t and λ_t. When Y_t increases it causes an increase in marginal product of labor and thus an increase in the output of other commodities which lowers the prices of the commodities in t+1. Then, there are two opposite forces which compensate each other. However this is not the general case.

From the point of view of input and consumption substitutability a conjecture is drawn:

> *(...) if producers substitute between inputs (as relative prices change) less readily than consumers substitute between commodities and leisure and/or between present and future consumption, then equilibrium labor employment at time t will be positively associated with commodity stocks at time t. similarly, if consumer demand for claims to future consumption is more Elastic than in our example, input employment (including labor) at t will be positively associated with the conditional mean of* λ_{t+1}, $E(\lambda_{t+1}|\lambda_t)$. (p. 50).

[2] This only holds for the specific example presented here.

Prices and wage rates are utility denominated and behave naturally with respect to production and preferences and production parameters. For the interest rate, if we suppose a riskless claim on future consumption of a unit of some commodity the optimality condition for intertemporal choice of consumptions is:

$$\frac{\beta E\left[\frac{\partial(V(S_{t+1})\,|\,S_t)}{\partial Y_{N,t+1}}\right]}{\left[\frac{\partial V(S_t)}{\partial Y_{N,t}}\right]} = \beta E\left[\left(\frac{Y_{N,t}}{Y_{N,t+1}}\right)\middle| S_t\right] = \frac{1}{1+r_{N,t}}$$

Being "$r_{N,t}$ *the one period (short term) commodity N rate of interest at time t*" (p. 51). If it is supposed that $Y_{N,t+1}$ given S_t is lognormal, they find that $r_{N,t} = \rho + \mu - \sigma^2$. This is, the interest rate of a riskless asset on a commodity is the subjective discount rate plus the expected growth rate in availability of commodity μ minus a measure of the uncertainty about that growth rate.

Finally, dynamics of the economy is analyzed by performing two simulations: a one time shock to see the response of the quantities of production in the economy; and a sequence of shocks for each time in order to compare serial correlations, comovements, and statistical properties of the hypothetical economy and the real US economy.

First, it is shown that time evolution of outputs can be expressed as a VAR(1) and therefore it could be transformed into a Moving Average (MA) representation. After that, response functions were calculated showing the following features: i) the largest response usually occurs in the sector where the shock is given. ii) response in the other sector generally occurs gradually and it takes two or three periods to happen. iii) given the matrix which collects the coefficients of the VAR representation of the model, shocks to agriculture, mining and construction have only minor impacts on the other sector outputs as well as aggregate output which reveals that there are minor sectors of the productive process. On the other hand, the higher responses of all other sectors to shocks in manufacturing, transportation and trade, and services, highlight the central role of commodities with many productive uses in the propagation mechanism of the model.

Finally, the economy was simulated to get the time paths of all the variables. The exercise showed that the hypothetical economy still has correlations and serial properties similar to the US economy ones, although the first one patterns are smoother than those of the second one, and therefore less volatile. This result is attached to the fact that the model is developed to capture persistence and propagation in time instead of volatility properties.

Finally, a pair of research topics are left open: i) the assessment of the extend to which simple real-business cycle models of this type can account for the covariance structure of observed quantities and relative prices; and ii) searching for technical progress that enables researchers to obtain explicit solutions to models with alternative specifications of preferences and production.

A second group of works is that of small open economies whish is led by "Real Business Cycles in a Small Open Economy" (1991) by Enrique G. Mendoza. In this article the author states that most of the countries exhibit well defined empirical regularities in international indicators as well as in domestic ones, as documented by Backus and Kehoe (1989). This evidence basically says that national savings and domestic investment are positively correlated, and that current account and balance of payments are countercyclical. The first finding has caused a very long debate because it implies imperfect capital mobility; this result is robust to several modifications. However, theoretical work including elements as uncertainty, and shocks to technology and population growth in infinitely lived agents as well as in overlapping generations models, shows that there is no clear capital mobility degree.

The second finding referred above, although it is also a well documented behavior by Backus and Kehoe (1989) is not unambiguously predicted by intertemporal equilibrium approach; there is not a clear negative correlation pattern between current account and balance of payments. This is because such relation depends on how dominant is either income or substitution effect. Furthermore empirical studies did not support that kind of relation.

Given these facts, the question is: can a real-business-cycle model account for the positive savings-investment correlations and the countercyclical behavior of balance of payments?

The question is answered by using a real business cycle model for an open economy which is calibrated and simulated. A new element of this kind of model is that it has a foreign assets market which finances trade imbalances and is very important to explain investment and saving dynamics. One of the findings derived from the simulations with the model is that positive correlation between investment and saving holds for perfect capital mobility. This result is also found for a two country model with persistence of productivity disturbances (p. 799). Other investigations have shown that in an open economy when individuals can access the foreign markets, investment-saving decisions are separable, because external financing allows the coverage of the difference between them.

In the model, it is shown that for a small open economy, capital accumulation exaggerates the variability of investment and underestimates its correlation with savings. These phenomena can be avoided by introducing moderate capital-adjustment costs, thus adopting the view that financial capital is more mobile than physical capital (p.799).

The output of this economy is a composite commodity internationally tradable:

$$G(K_t, L_t, K_{t+1}) = \exp(e_t) K_t^{\alpha} L_t^{1-\alpha} + \left(\frac{\phi}{2}\right)(K_{t+1} - K_t)^2$$

L: labor.

K: capital

e_t: an exogenous disturbance, that follows a stochastic process, and because GDP is tradable, it incorporates the influence of fluctuations in terms of trade (p. 800).

With this formulation, non-traded goods are avoided, focusing the analysis on international prices fluctuations and the wealth effect induced by them.

The physical capital evolves accordingly to

$$K_{t+1} = (1-\delta)K_t + I_t,\ 0 \leq \delta \leq 1$$

I_t : Gross investment.

In the competitive international capital market, the individuals can access to an asset A_t which pays or charges $r*$ real interest rate, and it is exchanged with the rest of the world.

The time evolution for this asset is:

$$A_{t+1} = TB_t + A_t\left[1 + r^* \exp(n_t)\right]$$

TB_t : Trade balance.
n_t : Random disturbance affecting the world's real interest rate.

This kind of formulation for assets does not allow for international risk sharing. However, it is possible holding foreign debt as a mean of financing domestic investment in physical capital.

The aggregate resource constraint states that:

$$C_t + I_t + TB_t \leq \exp(e_t)K_t^{\alpha} L_t^{1-\alpha} + \left(\frac{\phi}{2}\right)\left(K_{t+1} - K_t\right)^2$$

With this formulation the model produces "fluctuations in the relative price of investment and consumption goods, which is given by the marginal rate of technical substitution between I_t and C_t: $q_t = 1 + \phi(I_t - \delta K_t)$" (p. 800).

In this economy there are infinitely lived agents who maximize their intertemporal "Stationary Cardinal Utility" by allocating optimally C_t and L_t:

$$E\left[\sum_{t=0}^{\infty}\left\{u(C_t - G(L_t))\exp\left(-\sum_{\tau}^{t-1}\nu(C_\tau - G(L_\tau))\right)\right\}\right]$$

and the instantaneous utility and time preference functions are:

$$u(C_t - G(L_t)) = \frac{\left[C_t - \frac{L_t^{\omega}}{\omega}\right]^{1-\gamma} - 1}{1-\gamma}, \ \omega, \gamma > 1$$

$$\nu(C_t - G(L_t)) = \beta \ln\left(1 + C_t - \frac{L_t^{\omega}}{\omega}\right), \ \beta > 0$$

These functions satisfy:

$u(\bullet) < 0, \quad u'(\bullet) > 0, \quad u'(0) = 0$

$\ln(-u(\bullet))$ convex

$\nu(\bullet) > 0, \quad \nu'(\bullet) > 0, \quad \nu''(\bullet) < 0$

$u'(\bullet)\exp(\nu(\bullet))$ nonincreasing

"This structure of preference features an endogenous rate of time preference, $\exp[\nu(\bullet)]$, that increases with the level of past consumption." This formulation implies that when current consumption changes, the subjective rate of substitution also chances.

Conditions numbered, as has been shown, "produce a unique invariant limiting distribution of the state variables, is suitable for dynamic programming, and ensures that consumption in every period is a normal good (p. 801)."

The dynamic programming problem and the solution technique is as follows: given the values of the state variables K_t, A_t and λ_t, the optimal intertemporal decisions induce the choice of K_t, A_t, C_t and L_t, being λ_t the realization of the state of nature with respect to the pair of disturbances (e_t, η_t).

“Optimal choices must comply with the usual nonnegativity restrictions on K_t, C_t and L_t and must also be consistent with intertemporal solvency.” This is a condition which “precludes the individuals from running Ponzi-type schemes:” $A_t \geq \Delta$, being Δ a negative constant.

The stochastic equilibrium of the economy is obtained by solving:

$$V(k_t, A_t, \lambda_t^s) = \max\left\{ \frac{\left(C_t - \frac{\hat{L}_t^\omega}{\omega}\right)^{1-\gamma} - 1}{(1-\gamma)} + \exp\left[-\beta \ln\left(1 + C - \frac{\hat{L}_t^\omega}{\omega}\right)\right] \mathrm{x} \left[\sum_{r=1}^{4} \pi_{s,r} V\left(k_{t+1}, A_{t+1}, \lambda_{t+1}^r\right)\right] \right\}$$

control variables for this problem are: K_{t+1}, A_{t+1}, C_t.

Optimization is subject to:

$$C = \exp(e_t) K_t^\alpha \hat{L}_t^{1-\alpha} - \left(\frac{\phi}{2}\right)\left(K_{t+1} - K_t\right)^2 - K_{t+1} + K_t(1-\delta) + \left[1 + r^* \exp(\eta_t)\right] A_t - A_{t+1}$$

$$\hat{L}_t = \arg\max_{L_t} \left\{ \exp(e_t) K_t^\alpha L_t^{1-\alpha} - \frac{L_t^\omega}{\omega} \right\}$$

$$K_t \geq 0,\ A_t \geq \Delta,\ C_t \geq 0,\ L_t \geq 0$$

Assuming Markov chains for the disturbances, the stochastic structure of the model is simplified:

$$\lambda_t \in \Lambda = \left\{ \left(e^1, \eta^1\right), \left(e^1, \eta^2\right), \left(e^2, \eta^1\right), \left(e^2, \eta^2\right) \right\}$$

This is, for each period, there are four possible states to occur. And there is a probability that $\lambda_t^s = (e_t^x, \eta_t^i)$ moves to state $\lambda_{t+1}^r = (e_{t+1}^x, \eta_{t+1}^i)$, $x, i = 1,2$ and $s, r = 1,4$, $\pi_{s,r}$ which has a “simple persistence” rule:

$\pi_{s,r} = (1-\theta)\Pi_r + \theta p_{s,r}$

θ: parameter gorverning the persistence of e and η

Π: long run probabilities of state λ^r and $p_{s,r} = 1$ if $s = r$ and 0 otherwise.

Transition probabilities must satisfy $0 \le \pi_{s,r} \le 1$ and $\sum_{r=1}^{4} \pi_{s,r} = 1$

Using symmetry conditions:

$$\Pi(e^1, \eta^1) = \Pi(e^2, \eta^2) = \Pi$$
$$\Pi(e^1, \eta^2) = \Pi(e^2, \eta^1) = 0.5 - \Pi$$
$$e^1 = -e^2 = e$$
$$\eta^1 = -\eta^2 = \eta$$

These restrictions facilitate the numerical analysis and short the size of the parameter set and allows relating them explicitly to the statistical moments of the shocks. The asymptotic standard deviations of the shocks are $\sigma_e = e,\quad \sigma_\eta = \eta$, and $\rho_e = \rho_\eta = \rho = 4\Pi - 1$ is the common first order serial correlation and $\rho_{e,\eta} = \theta$ the contemporaneous correlation coefficient. Thus, by setting values for e, η, Π and θ the stochastic process are well defined.

The numerical solution is done by using the method by Dimitri Bertsekas (1976), introduced to dynamic macroeconomics by Sargent (1980), and used by Greenwood et al. (1988). First, grids for K and A are chosen, getting a state space for the economy defined in a set of $K \text{ x } A \text{ x } \Delta$ of dimension $N \text{ x } M \text{ x } 4$. The election of the grids is aimed to capture "the ergodic set for the joint stationary distribution of K, A and λ, and are refined until the covariances among the state variables converge (p. 803)."

Secondly, the problem of the Bellman equation is solved by using an algorithm of successive approximations. From this solution, decision rules are derived for A_{t+1}, K_{t+1}

given combinations of A_t, K_t and λ_t, and jointly with $\pi_{s,r}$, $\forall s, r = 1,4$ give the probability of moving to other values of A, K, λ in one period.

P is a (4MNx4MN) matrix of transition probabilities used "to calculate limiting probabilities of each triple in the state space (p. 803)." Entering $X^1 = X^0 P$ where X^0 is an "intial-guess" vector, there will be convergence to a fixed point X^* "which is a unique invariant joint limiting distribution of K, A and λ. "this distribution produces what amounts to population moments of variability, comovement, and persistence of all variables in the model (p. 803)."

Using this model and a calibration for $\sigma_e, \sigma_\eta, \rho, \rho_{e,\eta}, \alpha, r^*, \gamma, \delta, \omega, \beta, \phi$ the author tries to reflect the Canadian economy. Thus, a benchmark model is calibrated for the hypothetical economy and its moments are compared to the ones of the Canadian economy.

The model mimics the absence of comovements between GDP and foreign interest rate payments or the trade balance-output ratio. However, the benchmark model has serious problems: exaggerates variability of investment; underestimate its first order autocorrelations; underestimate its correlation with GDP; the benchmark model exhibits a very large positive comovement between C and GDP; a negative persistence in TB/GDP vs. a positive one for the empirical economy; generates for L a stochastic process that shows perfect correlation with GDP. "This last result is an unavoidable feature of the model that is implied by the structure of the preferences and technology described in section I (p. 807)."

The model also shows a very low correlation between saving and investment which is implied by the "low degree of serial correlation of the shocks used to calibrate the model" (p.807). In spite of this, the model supports the argument that "the intensity of the comovement between S and I in economies with perfect capital mobility depends on the degree of persistence of the underlying technological disturbances (p. 807)."

Another exercise shows that a reduction in γ affects the behavior of foreign assets, "because less-risk-averse individuals attain optimal consumption without resorting as often to the insurance that these risk-free assets provide" (p. 807). So, the degree of correlation between consumption and GDP seems to be not depending on risk aversion, the model is not good enough to explain the movements of investment and other macroeconomic variables when $\gamma = 1.001$. This is explained by investment-saving separability and no costs of investment adjustment: "investment is set to equalize the expected marginal returns, in utility terms, of domestic capital and foreign assets" while "savings, in contrast, are determined by equating the expected intertemporal marginal rate of substitution with the resk-free real rate of return on assets" (p. 807). In small open economies "consumption smoothing" operates through the current account, thus, there is not intertemporal consumption substitution.

In a perfect foresight model the optimal investment decision occurs when marginal returns on capital and foreign assets equal each other on every period:

$$\exp(e_t)F_K(K_{t+1}, \hat{L}_{t+1}) - \delta = r^*$$

One of the ways to correct the benchmark model was to extend it to include random disturbances affecting world's real interest rate which induce three effects: a wealth effect, an itertemporal consumption substitution and redistribution between K and A. The results of moderate shocks to interest rate show that they have "minimal effects on the equilibrium stochastic process of the model". (p. 807).

A second extension was the inclusion of positive costs of adjustment, which allows lowering investment reaction by preventing sudden capital adjustments. The idea behind this is to introduce frictions in investment "without introducing imperfections" in capital markets or "imposing controls on international financial flows". Costs of adjustment are introduced by using a convex quadratic function.

With disturbances mores serially correlated and slightly larger than those of the benchmark model, the experiment can replicate the variability of investment with little costs of adjustment. Thus, agents do not change their physical capital holding quickly. A more important result is that including costs of adjustment, the model can reproduce more accurately most of the facts documented by the author, and generates "procyclical behavior in consumption, savings, investment, employment, and productivity, and produces stochastic process for the trade balance-output ratio and foreign interest payments that are almost uncorrelated with GDP, (…) the model also duplicates some of the first-order autocorrelation coefficients. Additionally, costs of adjustment allow variation of relative price of investment and consumption.

In addition, introduction of costs of adjustment causes an increase in the persistence of shocks and allow a higher correlation between S and K which is also consistent with perfect financial capital mobility. This extension also replicates variability of TB/GDP and the correlation between TB and GDP, suggesting that the current account approach can be consistent with the facts of the business cycle.

A comparison with closed-economy models simulated by other authors show that disturbances to small open economies need to be less persistent to reproduce empirical facts. However, small open economies can not mimic the results for the behavior of labor productivity. Furthermore, small open economies "exaggerate comovements between consumption and GDP" with or without costs adjustments and regardless the value of the risk aversion.

Finally, some topics of research are suggested: addition of nontradables, the use of the model to explain business cycles as induced by terms of trade fluctuations in developing countries intensively exporting primary goods and importing capital goods. Additionally, the model could be extended to study policy effects such as capital controls and tariffs.

A second seminal paper about small open economies is by By David K. Backus, Patrick J. Kehoe, and Finn E. Kydland (1994): "Dynamics of the trade balance and the terms of trade:

The J-curve?". For a sample of developed countries, the authors find that trade balance is uniformly countercyclical and negatively correlated with current and future movements in the terms of trade, but positively correlated with past movements (p.84).

The model used is an extension of other studies. Additionally, their hypothetical economy has capital, labor, two imperfect substitutable goods and fluctuations from technological shocks and government purchases. With "plausible" parameters the hypothetical economy reproduces the behavior documented by the authors.

The data for the terms of trade are the price of imports-price of exports ratio, measured using implicit price deflactors. Trade (nx) is defined as the net exports-GNP or GDP ratio, measured at constant prices, prices and income are taken in logs. Theoretical and empirical values of variables are detrended with HP filter.

The hypothetical economy is a stochastic growth model for two countries and has a large number of agents. The proposed model is a two country model version of that of Kydland and Prescott (1982) for a closed economy. Each economy produces a different good with its own technology, and labor is internationally immobile. Fluctuations are produced by tech shocks and government purchases.

The problem of the representative agent in country i is max the expected value of the intertemporal utility function:

$$u_i = E_0\left[\sum_{t=0}^{\infty} \beta^t u(c_{it}, 1-n_{it})\right]$$

$$u(c, 1-n) = \frac{\left[c^{\mu}(1-n)^{1-\mu}\right]^{\gamma}}{\gamma}$$

Where c is consumption and n is hours worked in country i.
Each economy specializes in production of a good: a for country 1 and b for country 2; using capital and labor within a neoclassical production function. Thus the resource constraint is:

$a_{1t} + a_{2t} = y_{1t} = z_{1t} F(k_{1t}, n_{2t})$

$b_{1t} + b_{2t} = y_{2t} = z_{2t} F(k_{2t}, n_{2t})$

Being $F(k_t, n_t) = k^{\theta} n^{1-\theta}$

$Z_t = (z_{1t}, z_{2t})$ is a vector of stochastic shocks of productivity.

Consumption, investment and government purchases are composites of domestic and foreign goods:

$c_{1t} + x_{1t} + g_{1t} = G(a_{1t}, b_{1t})$

$c_{2t} + x_{2t} + g_{2t} = G(a_{2t}, b_{2t})$

Being $G(a_{2t}, b_{2t}) = \left[\omega_1 a^{-\rho} + \omega_2 b^{-\rho}\right]^{-1/\rho}$, $\rho \geq -1$, and is a degree one homogenous function whose elasticity of substitution is $\sigma = 1/(1+\rho)$. g_{it} is a stochastic component.

Capital formation has a time-to-build scheme: "a unit increase in the capital stock J quarters from now involves purchases of 1/J units of final good for J consecutive quarters". (p. 90).

$k_{i,t+1} = (1-\delta) k_{i,t} + s_{i,t-J+1}$, δ is the capital rate of depreciation.

$$x_{i,t} = J^{-1} \sum_{j=0}^{J-1} s_{i,t-J}$$

The tech shocks have an autoregressive structure:

$Z_{t+1} = AZ_t + \varepsilon_{t+1}^{z}$, ε_t^z distributed normally and independently over time with variance V_z. The elements of the diagonal y A show the correlation between shocks.

The government shocks are modeled as:

$g_{t+1} = Bg_t + \varepsilon_{t+1}^{g}$, ε^g distributed normally with variance V_g.

As the function G is homogenous of degree one, in equilibrium we have: $c_{1t} + x_{1t} + g_{1t} = q_{1t}a_{1t} + q_{2t}b_{1t}$ being q_{1t}, q_{2t} prices of the two goods in period t in units of the composite good and with the resource constraint is:

$$y = \frac{\left(c_{1t} + x_{1t} + g_{1t}\right)}{q_{1t}} + \left(a_{2t} - p_t b_{1t}\right)$$

being $p_t = \frac{q_{2t}}{q_{1t}} = \frac{\{\partial G(a_{1t}, b_{1t}) / \partial b_{1t}\}}{\{\partial G(a_{1t}, b_{1t}) / \partial a_{1t}\}}$ the terms of trade evaluated at the equilibrium quantities, and $nx = \frac{\left(a_{2t} - p_t b_{1t}\right)}{y_{1t}}$ the trade-balance/GDP ratio.

"Given values for the model's parameters, we compute an equilibrium by solving numerically a quadratic approximation to a social planner's problem that weights equally the utility of consumers of the two countries" (p. 91).

The authors report a "benchmark economy" using a calibration for the model. nx and terms of trade are highly correlated in the theoretical economy and its value is in the range exhibited for the sample of countries, and the terms of trade autocorrelation is very close to the sample mean. nx is countercyclical for the benchmark economy and is within the sample range. There is strong negative correlation between trade balance and terms of trade which is also observable in data, and terms of trade and output are strongly positively correlated while in data this fact is not so obvious.

Because the cross-correlation between nx and terms of trade has a S-shape, the model reproduces one of the features of data. By performing a sensibility analysis moving the values of the elasticity of substitution, they find that it only affects the contemporaneous correlation between terms of trade and nx, but the S-curve still remains. A second shock exercise is performed for the government purchases (and tech shocks) with similar results of that for the benchmark economy. Thus, it is concluded that the introduction of government shocks does not change neither the S-curve nor the countercyclical behavior of nx.

Another interesting finding is that data dynamics shows more persistency than theory and the cross-correlation function changing 1-2 quarters faster in the theory. Such a behavior was found changing the values for σ.

Because such a behavior could be due to the dynamics of capital accumulation, a time to build experiment with $J = 2$ was developed. As a result, the cross-correlation pattern is not very different from that of the benchmark economy. A second experiment was made including a lag in the trade process. Therefore, goods exported to other country only can be used in a posterior date, having $G(a_{1,t}, b_{1,t-1})$ and $G(b_{2,t}, a_{2,t-1})$ which was labeled "one period delivery lag time to ship" (p.96). It was found that the time to ship has influence on the timing of the relationship between the trade balance and the terms of trade. So, time to build and time to ship are extensions of the benchmark economy.

Besides, two extreme experiments are made:

a) No capital formation: as a result of this experiment, the S-curve disappeared, allowing concluding, that capital formation is essential for the dynamics of trade balance and terms of trade in the benchmark economy. The reason for this is straightforward: the consumption smoothing, and no capital formation means that when a positive technological shock moves the economy upwards the output increase will be greater than the consumption increase and investment is zero or almost zero, giving rise to a trade balance surplus.
b) A government purchases as single impulse to the economy: in this experiment the S-curve disappears one more time. The reason for this is that when economy has a positive government shock there is not an investment boom. A very important conclusion here is that there is a "hazard of predicting comovements between the terms of trade and the trade balance without specifying the shock that gives rise to these movements" (p.98).

In spite of the relative goodness of the model, it has a couple of "anomalies": first, the standard deviation of the terms of trade is 0.48% in the benchmark economy, while for the

US economy is 2.92%, which is, accordingly to the authors, explained by measurement errors. Second, the discrepancies in the magnitude and character of fluctuations: “standard deviations of consumption and investment, and the correlations of output and consumption across countries” (p.99). For the hypothetical economy when $J=1$ standard deviation of investment is 31.47 times the one of GDP, however, when it is supposed that $\omega_1=\omega_2$ investment/GDP variability ratio is 30.32, but in the benchmark economy that figure was 3.48. In conclusion, this is not considered an anomaly. There is another discrepancy between data and theory: in data consumption correlation between countries is smaller than that of output while in the theoretical economy happens the opposite.

Finally is to be noted that despite of the “anomalies”, the S-curve remains. The paper adds to the literature “a consideration of the short run dynamics of trade and prices (p.101).” It is necessary to develop more work in order to solve those discrepancies.

Finally, the third group is that of multicountry models. The first to be cited is International Real Business cycles, by By David K. Backus, Patrick J. Kehoe, and Finn E. Kydland (1992). In the article, business cycle theory for closed economies is extended to try to answer whether such a model can account for prominent international as well as domestic comovements of macroeconomic variables, including correlations between countries for main aggregates and the trade balance. Two very important facts are tracked with this model. One is the empirical correlation between GDP of countries, and the other one is the theoretical result for consumption between countries which states that under complete financial markets, consumptions should be correlated because of the international risk sharing. In this way, the extensions are made to the model by Kydland and Prescott (1982) for a closed economy.

The model supposes production of a single good in order to focus on the role of financial market in the risk sharing and the determination of production and intertemporal decisions. Two additional assumptions are made: agents access to international capital markets and technological shocks of countries are different and correlated each other which allows for shocks transmission across the borders of the economies. A small transportation cost is included to try to replicate some comovements and with the purpose of reducing somewhat the high variability of the investment in the hypothetical model.

Properties of the international business cycles are studied for developed economies countries in the post war period, focusing on movements of quarterly time series detrended with H-P filter and cross-correlations of variables of such countries. For the US economy they found that consumption is as volatile as a half the output volatility, investment volatility is three times that of output, hours worked are slightly less volatile than output, and the trade balance is countercyclical contemporaneously. In the international environment, product fluctuations of other countries are positively correlated with those of US economy and so are consumption correlations. Additionally, the correlation is higher between outputs than between consumption.

In order to avoid theoretical and empirical problems measuring saving, as for example, the market value of assets, the saving in the model is defined as output minus consumption and government purchases, which is a definition able to capture the separability of saving and investment in open economies. It was found that correlation between investment and saving varies across countries and is high and positive for some economies. Trade balance is measured as the ratio of net exports to output and its variability and its standard deviation varies over time and across countries, and is countercyclical contemporaneously for all economies.

In the hypothetical economy there are two countries with identical consumers and a production technology. Only one good is produced, and tech and preferences are the same, with equal values for each economy. Labor is not internationally mobile and each country has different tech shocks. In each economy the consumer maximizes the expected utility function:

$$E_0 \sum_{t=0}^{\infty} \beta \; u(c_t^i, l_t^i) \,, \; i = h, f$$

Being $u(c,l) = (c^{\mu} l^{1-\mu})^{\gamma} / \gamma$, $0 < \mu < 1$, $\gamma < 1$

c_t^i: Consumption in t of country i.

l_t^i: Distributed lag on leisure of the each agent of country i.

$l_t = 1 - \alpha n_t - (1-\alpha)\eta\, a_t$ and $a_{t+1} = (1-\eta)a_t + n_t$, $a_t = \sum_{j=0}^{\infty}(1-\eta)^{j-1} n_{t-j}$ $0 < \eta \leq 1$ and $0 < \alpha \leq 1$, being n_t time allocated to work. We have that if $\alpha = 1$ and current utility depends only on current leisure, otherwise, current utility depends on non market activities.

Product is made with capital, labor and stocks of inventories, k_t, n_t, z_t:

$y_t^i = F(\lambda_t^i, k_t^i, n_t^i, z_t^i) = \left[(\lambda_t, k_t^{\theta}, n_t^{1-\theta})^{-\nu} + \sigma z^{-\nu}\right]^{-1/\nu}$, which is a Constant Elasticity Transformation function (CET) with $0 < \theta < 1$, $\nu > -1$, $\sigma > 0$.

To close the model, the world output which is the sum of all economies products is allocated to consumption, fixed investment and inventories accumulation in all countries, this is:

$$\sum_i (c_t^i + x_t^i + z_{t+1}^i - z_t^i) = \sum_i F(\lambda_t^i, k_t^i, n_t^i, z_t^i).$$

Net exports are gives as $nx_t^i = y_t^i - (c_t^i + x_t^i + z_{t+1}^i - z_t^i)$

A time to build structure as proposed by Kydland and Prescott (1982) is incorporated:

$$k_{t+1}^i = (1-\delta)k_t^i + s_{1t}^i,\ s_{j,t+1}^i = s_{t+1,j}^i\ ,\ \forall j = 1,.....J-1$$

δ: Depreciation rate

$s_{j,t}^i$: Investment projects in country i that are j periods from completion.

Let ϕ_j be $\forall j = 1,.....J$ the fraction of total value of investment project in the j period before completion $\phi_j = 1/J$.

$$x_t^i = \sum_{j=1}^{J} \phi_j s_{jt}^i$$

Technological shocks are modeled as an autoregressive process:

$$\lambda_{t+1} = A\lambda_t + \varepsilon_{t+1}$$

Where $\lambda_t = (\lambda_t^h, \lambda_t^f)$, A matrix of coefficients, and $\varepsilon_t = (\varepsilon_t^h, \varepsilon_t^f)$, where ε_t is a serially independent, multivariate, normal random variables with contemporaneous variance matrix V. Shocks are stochastically correlated through the off diagonal elements of A and V, and the off diagonal elements of A are spillover effects. It is supposed that λ_t is known in t when the agents make their decisions.

Equilibrium in this economy is characterized for an equivalence between competitive equilibrium and Pareto optima (p…). So, the optimum can be obtained as the solution to a planning problem:

$$\Psi E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i, l_t^i) + (1-\Psi) E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^f, l_t^f)$$

Subject to previous restrictions. There will be a set of equilibriums for a set of Ψ values, where the authors computed the competitive equilibrium for $\Psi = 0.5$. The problem is approximated in the neighborhood of the steady state after of substituting the nonlinear restriction for the model closure in the objective function. A quadratic approximation is made, and then, this new objective function is optimized subject to the other restrictions.

To calibrate the model, the authors use the parameter values of kydland and Prescott (1982). Tech parameters are estimated from international data using Solow residual method. Taking advantage of the symmetry of the economies, and that consequently one is a reflection of the other the world economy calibration is only a double replication of a closed economy (p.757).

In the steady state situation, all variables are constant and the interest rate is $r = (1-\beta)/\beta$, fixed investment equals depreciation and inventory investment is equal to zero. Then, $c + \delta k = y$ is the resource constraint in steady state and the rental price of inventories is r.

"The value of resources used to produce one unit of capital in terms of the same-date consumption good is $q = \sum_{j=1}^{J} \phi_j (1+r)^{j-1}$. The rental price of capital is therefore $q(r+\delta)$. A profit-maximizing firm's first order condition for inventories, capital and labor imply:"

$$r = \sigma \left(\frac{y}{z} \right)^{1+\nu}$$

$$q(r+\delta) = \theta \left(\frac{y}{k} \right) \left(1 - \sigma \left(\frac{y}{z} \right)^{\nu} \right)$$

$$w = (1-\theta) \left(\frac{y}{n} \right) \left(1 - \sigma \left(\frac{y}{z} \right)^{\nu} \right)$$

Being w equilibrium wages in consumption units, obtained from the consumer's first order condition: $U_l / U_c = w$. Thus, they have:

$$\frac{(1-\mu)c(\alpha r + \eta)}{r+\eta} = \mu w (1-\eta)$$

Information from national accounts and microeconomic observations were used to calibrate the model. US sharing of labor income is chosen to estimate $1-\theta$ consistently with long run "constancy" of labor income to GDP ratio, which is implied by Cobb-Douglas production function. The value for ν is not observable from data calibration, so, as Kydland and Prescott (1982) do, they use a value of 3. However, the author says that this parameter does not have influence in the international environment. The only parameter left to calibration is time to build, and following Kydland and Prescott (1982) one more time, they set $J = 4$.

For the US economy, 30% of the non-sleeping[3] time is devoted to market activities. Therefore, when $\alpha = 1$, $\mu = 0.34$ is the value used in the Cobb-Douglas preference function. The risk aversion parameter is set to -1. It is important the non additive separability for the replication of the imperfect correlation of consumption across countries. Parameters of A matrix are estimated from Solow residuals for the US and other developed countries economies by using ordinary least squares. Although estimated parameters of A show asymmetry and non zero spillover effects, some of the numerical exercises are performed imposing symmetry.

With these parameters in hand, 50 simulations of size 100 each were made, and the properties of the benchmark economy were calculated from the simulated data. One of the most important findings is that standard deviation of the benchmark economy is 91% of that of the empirical economy. The variability of consumption in the benchmark economy is very similar to that of the empirical economy. However, the theoretical economy exhibits investment variability three times the one of the empirical economy. The trade balance variability is almost seven times that for the US economy and is even greater than the figure of other countries and is almost uncorrelated with output and negative contemporaneously. Saving/investment correlation is positive but not high enough to be similar to the one of the empirical economy. In the model economy, foreign and domestic output are negatively correlated, while in the data correlation is positive for twelve countries, consumption across countries are higher than in the data. Finally, consumption cross correlation is 0.88 while for output it is -0.18.

When the domestic economy receives a positive shock which is associated with increases in domestic investment, consumption and output, because the increase in investment is the largest, a current account deficit is generated. The spillover effect generates an increase in technology in the foreign country and a fall in investment because resources are moved to the place where production is more profitable; however, consumption in the foreign country raises a little. Thus, for the hypothetical world, consumption across economies has positive correlation but investment and output correlations are negative. These facts show some

[3] Non-sleeping time is the one devoted to activities different than sleeping.

differences between theoretical and empirical economy: “In the model, investment and net exports are more variable whereas consumption is more highly correlated across countries and output is less highly correlated (p. 765).”

Several experiments were made in order to check whether changes in parameters could affect such behaviors. The first one was an asymmetric spillover effect using estimates for A for US data and European aggregates. The main results are a fall in investment/output correlation from 0.27 to -0.08 and a fall in saving/investment correlation from 0.28 to -0.04 implying that correlations of saving and investment are sensitive to changes in perturbation parameters, and that investment and net exports are more variable than in data and consumption is still highly correlated across countries even more than output.

Secondly, the risk aversion parameter was increased, to check how this and non separability of consumption and leisure could affect the consumption correlation between countries. The result was a very little fall of volatility of investment and net exports in relation to output, with output across countries still being negatively correlated, and consumption correlation exceeding that of output. In the third place, a distributed lag structure was added to the model to make leisure durable, which increased the variability of output and investment by raising the intertemporal substitution of leisure, producing a higher variability of marginal product of capital. However, this change had no effects on cross country correlations. Finally, when time to build was set to $J = 1$, standard deviation of product raises to 2.24, and investment standard deviation relative to output was ten times that of output. Thus, time to build has a stronger effect in open economies.

The authors argue that differences between the hypothetical economy and the empirical one are due to the fact that agents can move resources across countries by buying state contingent claims. Thus, when agents insure themselves against negative shocks, they will not care about what state the economy is in at the moment they are making decisions, so shocks to production will not affect consumption. Supposing a transportation cost the authors introduce frictions in the model economy. To avoid a V-shaped function which would difficult the quadratic approximation, they choose a quadratic function to model these costs: $G(nx) = \tau\, n\, x^2$, being $\tau > 0$ a parameter. Therefore, resource constraint must be rewritten as:

$$\sum_i (c_t^i + x_t^i + z_{t+1}^i - z_t^i) = \sum_i F(\lambda_t^i, k_t^i, n_t^i, z_t^i) - G(nx_t^i)\,.$$

Marginal cost of transportation is $2\tau\, nx$ in each country because of the symmetry. Inclusion of this kind of cost reduces net exports/output variability and also reduces investment/output variability, however, with no substantial effect on cross country correlations of consumption an output.

As an extreme experiment, the economies are closed, allowing just connection through the spillover effect of technology shocks. This exercise reduced output variability a little bit more than in the case with costs of transformation, or in other words autarky has similar result to those of trading frictions. With no trade, consumption correlation across countries is higher than correlation of output. Then, because no assets are traded, discrepancies can not be explained by imperfect capital movements.

In order to understand the discrepancies between the data and the hypothetical economy jointly with the international business cycles from the perspective of neoclassical real business cycles more theoretical work is needed. Such advances in the case of open economies might consider shocks different from those of technology, international relative prices movements and their correlation with trade balance. All of these imply further extensions of the model presented, and keep open the study of models for high frequency fluctuations.

And the last one is by Christian Zimmermann (1995): International Real Business Cycles among Heterogeneous countries. This author begins by stating that most of the business cycles models for open economies assume symmetric countries despite they show differences in their aggregates variability with small economies showing higher variability and being more influenced by foreign fluctuations. Thus, the model is supposed to have a small open economy, a big neighbor and a distant and large third country (rest of the world).

Data of the countries studied show that investment volatility is higher than that of output and consumption as employment is less volatile than product, while imports and exports are the most volatile (p. 3). The more procyclical variable is consumption and trade balance is countercyclical and shows a J-curve.

The model economy has three countries; each one populated by representative agents who maximize utility functions and access the same production function. The infinite lived agents can not move across countries, but there is free trade of goods. The size of countries is given by the number of representative agents. Each agent chooses consumption and time labor to maximize:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_{it},1-n_{it})\right] \quad (1)$$

Being $u(c_{it},1-n_{it})=\frac{\left[c_{it}^{\mu}(1-n_{it})^{1-\mu}\right]^{\gamma}}{\gamma}$

c_{it} : consumption in country i at time t

n_{it} : share of time devoted to market activities in country i at time t

$0<\beta<1$, $0<\mu<1$, and $\gamma<1$

If $\gamma=0$ $u(.)=\mu\log(c_{it})+(1-\mu)\log(1-n_{it})$

Each country produces a good y_{it} using capital and labor:

$y_{iy}=z_{it}F(k_{it},n_{it})$ is a Cobb-Douglas production function, and z_{it} is a stochastic technology parameter, $F(k_{it},n_{it})=k_{it}^{\theta}n_{it}^{1-\theta}$, $0<\theta<1$.

As good mobility is allowed, a share of the product is exported:

y_{ijt} : quantity per capita of good produced in country i and used by agent of country j in country j.

α_i : the number of households in country i.

The product of a country is used domestically y_{iit} ; abroad y_{ijt} in country j and y_{ikt} in country k. Thus, total product of the country is given by:

$$\alpha_i y_{it} = \alpha_i y_{iit} + \alpha_j y_{ijt} + \alpha_k y_{ikt}, \; i \neq j \neq k .$$

Demands from different origins are modeled as an Armington aggregator. This aggregate demand is used to be consumed or invested.

$$c_{it} + x_{it} = G(y_{iit}, y_{jit}, y_{kit}) \quad (4)$$

$$G(y_{iit}, y_{jit}, y_{kit}) = \left(\omega_{ii} y_{iit}^{-\rho} + \omega_{ji} y_{jit}^{-\rho} + \omega_{ki} y_{kit}^{-\rho}\right)^{-1/\rho} \quad (5)$$

$\omega_{ii}, \omega_{ji}, \omega_{ki} \geq 0$; $\rho \geq -1$; and $\sigma = \dfrac{1}{1+\rho}$ is the elasticity of substitution.

Capital evolution is the typical one:

$$k_{i,t+1} = (1-\delta)k_{i,t} + x_{it} \;, \; 0 < \delta < 1 \quad (6)$$

being δ the depreciation rate

Technological shocks are specific for each country and have a Markov process:

$$z_{t+1} = \bar{z} + A(L) z_t + B(L)\varepsilon_{t+1}$$

z_t Shocks Vector.

$$\varepsilon_{t+1} = \begin{bmatrix} \varepsilon_{1,t+1} \\ \varepsilon_{2,t+1} \\ \varepsilon_{3,t+1} \end{bmatrix}$$

$$A(L) = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$ Spillover and persistence effects

$$B(L) = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$ Contemporaneous correlations coefficients of ε_{t+1}

$$\varepsilon_{t+1} \neg N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ v_{31} & v_{32} & v_{33} \end{pmatrix} \right)$$

The model is designed to have several types of heterogeneities:

- Countries have different population size.
- There are some asymmetries in preferences between country i and country j. In order to maintain zero trade balance (in the steady state), asymmetries are implied in the third country.
- Different persistency in the technological shocks.
- Differences in the spillover effects.
- Spillover effects depend on the relationships between countries.
- Technology innovations can be correlated in a different way between countries.
- Variance of innovations differs between countries reflecting their different structure and nature.

For the steady state calibration, competitive equilibrium is defined as that which having prices of goods and factors which clear markets and that produce zero trade balance for all countries. Prices and goods sets, and production and utility functions properties (convexity and concavity respectively) guaranty that equilibrium is unique. For the steady state, the problem of these economies firms is:

$$\max_{(k,n)} \quad z_i k_i^{\theta} n_i^{1-\theta} + w_i n_i - (r+\delta)k_i \tag{8}$$

w_i : real wage.

From de first order condition for firm $k_i = \dfrac{\theta y_i}{r+\delta} \Rightarrow x_i = \dfrac{\delta\theta y_i}{r+\delta}$, $w_i = (1-\theta)\dfrac{y_i}{n_i}$

For the consumer we have:

$$\max_{(c,n)} \quad \frac{[c_i^{\mu}(1-n_i)^{1-\mu}]^{\gamma}}{\gamma}$$

s.t. $w_i n_i + (r+\delta)k_i = c_i + \delta k_i$, which implies:

$$w_i = \frac{c_i}{1-n}\frac{1-\mu}{\mu} \Rightarrow c_i = (1-\theta)y_i \frac{1-n_i}{n_i}\frac{\mu}{1-\mu}$$

Using the household budget constraint:

$$n = \frac{(1-\theta)\dfrac{\mu}{1-\mu}}{1+(1-\theta)\dfrac{\mu}{1-\mu} - \dfrac{\delta\theta}{r+\delta}}$$

Terms of trade are derived using the Armington aggregator into the consumer problem and optimizing for the three origin demands:

$$p_{ijt} = \frac{\dfrac{\partial G(.)}{\partial y_{jit}}}{\dfrac{\partial G(.)}{\partial y_{iit}}} = \frac{w_{jit}}{w_{iit}}\left(\frac{y_{iit}}{y_{jit}}\right)^{\rho+1}$$

So, the aggregated terms of trade for each country are:

$$p_{it} = \frac{y_{jit} p_{ijt} + y_{kit} p_{ikt}}{y_{jit} + y_{kit}} \tag{10}$$

If terms of trade are set equal to one in the steady state, and trade balance is set equal to zero:

$$y_{iit} = \frac{y_{it}}{1 + \left(\frac{w_{ji}}{w_{ii}}\right)^{\frac{1}{1+\rho}} + \left(\frac{w_{ki}}{w_{ii}}\right)^{\frac{1}{1+\rho}}}$$

$$y_{jit} = \frac{y_{it}}{1 + \left(\frac{w_{ii}}{w_{ji}}\right)^{\frac{1}{1+\rho}} + \left(\frac{w_{ki}}{w_{ji}}\right)^{\frac{1}{1+\rho}}}$$

$$y_{kit} = \frac{y_{it}}{1 + \left(\frac{w_{ii}}{w_{ki}}\right)^{\frac{1}{1+\rho}} + \left(\frac{w_{ji}}{w_{ki}}\right)^{\frac{1}{1+\rho}}}$$

Trade balance is: $nx_{it} = y_{ijt} + y_{ikt} - p_{it}(y_{jit} + y_{kit})$

Calibration process is identical to the one of Kydland and Prescott (1982), and for the open economy parameters calibration, this one is compared to that of Backus *et al.* (1994). For quarterly data, parameters were calibrated to reproduce (or mimic) “microeconomic studies” and long term values.

$r = \frac{1}{\beta} - 1 = 0.01$ for real interest rate.

$\delta = 0.025$

$\theta = (r + \theta)\frac{k}{y}$

$n = 0.3$

$\frac{c}{y} = 0.75$

$\gamma = -1$ an arbitrary value

To calibrate $w_{ij}'s$ steady state values of origin demands are calculated by averaging import shares from data, and then it was imposed the "aggregated sums up to the level of output", so:

$$w_{ij} = \left(\frac{y_{ji}}{y_i} \right)^{\rho+1}, \ i, j \in \{1,2,3\}$$

$\sigma = \dfrac{1}{1+\rho} = 1{,}5$ as in Backus *et al.* (1994)

In a first time, $B(L)$ is a identity matrix and the spillovers of technology are calculated from Solow residuals using production, capital, labor and their respective share parameters. Values for $A(L)$ matrix are estimated from a VAR model and V matrix is the estimated covariance matrix of the residuals. It was found that technology shocks have more volatility in the small countries and technological cross correlations of countries are small for distant economies. $AR(1)$ assumption for technology process is consistent with data.

From the spillover effects was found a negative sign between Switzerland and ROW1, which can be explained by the fact that a positive shock in an economy will cause a competitive disadvantage in the partner. When the sample was shortened to include more countries, different spillover effects were obtained, being less significant the domestic ones than the foreign ones, which can be explained by the fact that trade increased faster than output in those countries.

Equilibrium decision rules were calibrated using the method proposed by Kydland and Prescott (1982) and computational method is that of Hansen and Prescott (1995). Decisions are made on labor, investment and imports in an infinite horizon context, and are computed numerically by using dynamic programming techniques. Second welfare theorem is used to posit the problem in the central planer's point of view. Thus, the central planer's problem in the steady state is:[4]

$$\max_{\{n_{it},c_{it},k_{i,t+1}\}} \sum_{t=0}^{\infty}\sum_{i}^{3}\beta^t\alpha_i \frac{\left[c_{it}^{\mu}(1-n_{it})^{1-\mu}\right]^{\gamma}}{\gamma}$$

s.t. $c_{it}+k_{i,t+1}+(1-\delta)k_{i,t}=z_{it}k_{it}^{\theta}n_{it}^{1-\theta}$

"It is easy to show that, predictably, the steady state equilibrium for this problem is identical to competitive equilibrium for the representative agents previously shown, if α_i corresponds to the size of the countries". (p. 11).

The global utility is:

$$\sum_{t=0}^{\infty}\beta\sum_{i=1}^{3}\alpha_i u\left\{G\left[z_{it}F(k_{it},n_{it})-\frac{\alpha_j}{\alpha_i}y_{ijt}-\frac{\alpha_k}{\alpha_i}y_{ikt},y_{jit},y_{kit}\right]-x_{it},1-n_{it}\right\} \qquad (11)$$

s.t. $k_{i,t+1}=(1-\delta)k_{i,t}+x_{i,t}$, $\forall i\in\{1,2,3\}$ (5)

$z_{t+1}=\bar{z}+A(L)z_t+B(L)\varepsilon_{t+1}$ (6)

with $j,k\in\{1,2,3\}$, $j\neq i, k\neq i$

A second order Taylor series is needed to approximate in the neighborhood of the steady state for $\{z_{it},k_{it},n_{it},y_{it},y_{it},x_{it};\forall i | i\neq j,k\}$.

The linearized iterative solution to the problem, decisions for $n_{i,t}$, $y_{ij,t}$ and $x_{ij,t}$ as functions of k_{jt}, z_{jt}, for $i,j\in\{1,2,3\}$, are simulated 200 times with a size of 100; data were detrended by using H-P filter, correlations and standard deviations are computed, average of those values are calculated.

[4] It should be remembered that this is possible because there is no distortion given by *ad-valorem* taxes.

For the experimental economies, the model successes in explaining common facts of the countries: higher volatility of investment than that of output, and output more volatile than consumption and employment. However, volatility of imports and exports are understated. Besides, aggregates are procyclical and trade balance is countercyclical. Trade balance follows a J-curve but terms of trade are acyclical. The model also reproduces differences between countries: for small country aggregates are more volatile, but for the case of Canada this is not the case, and persistence is lower for small countries, consumption is more procyclical for large countries, and cross-correlations are higher between neighbors.

One of the failures is that volatility of imports varies too much across countries but not so the consumption variability, and the cross-correlations of consumption are higher than those of output. “This is a well known problem with international real business cycle models” (p. 12) and is not corrected with size differences or other asymmetries. Low volatility of terms of trade causes imports and exports to be too stable which is very surprising because consumption smoothing is not observed in the data.

Sensibility analysis was made by slightly modifying parameter values of the model to check how this could affect the dynamics and the theoretical statistics. When trend linkages are removed the only transmition channel left is technology innovations. Thus, consumption risk sharing is not allowed; as a consequence, cross-correlations decreases. Therefore, the business cycle variabilities are mostly coming from technological shocks correlation, which means that trade is not very important. Secondly, spillover effects are eliminated and trade is allowed. A positive technological shock produces a longer response in investment than consumption and the few cross-correlations between consumptions are caused by risk sharing through trade. When spillover effects are allowed it is found that large countries do not exhibit J-curve and “trade tends to amplify the variation of output”. (p. 13).

A time to build structure á la Kydland-Prescott (1982) is introduced with $j = 2$ quarters and the capital law formation as:

$$k_{i,t+1} = (1-\delta)k_{i,t} + s_{j,it}$$

$s_{j,it}$ is investment started in $t-1$ that becomes to be effective in $t+1$, and $s_{ji,l+1} = s_{j+1,il}$, $j=1$. Then, total investment is:

$$x_{i,t} = \frac{1}{2}\sum_{j=1}^{2} s_{ijt}$$

This causes a smoother behavior of the aggregated variables, but terms of trade an consumption have more volatility than in the benchmark economy, and correlation are smaller, and a lower J-curve.

What happens if trade takes longer than one period? This question is answered introducing a time to ship scheme. The resulting behavior is a smoother consumption than in the benchmark economy, more volatile terms of trade which causes consumption behavior to be more expensive. The resulting fact is a delayed J-curve. Further more, costs of transportation are included. As a result, consumption and output patterns are not greatly modified.

Additional experiments were performed for different elasticity of substitution values. When σ is higher, trade balance, output and terms of trade correlations are higher, but not imports. J-curve reaches positive values faster than in the benchmark economy.

Counterfactual analyses were made to check what parameters could better explain typical behaviors of small economies. This was made inserting large economy parameters into the small economy structure. When this is done and small economy has the same standard deviation of nearest large country output volatility gets smaller than that of consumption. There are also changes in terms of trade volatility (decreasing effect) and there is not a great change in J-curve. Later, by supposing equal spillover effects and setting Eigen values to the benchmark values, consumption, exports and terms of trade volatilities are higher, and J-curves are steeper.

A last experiment is performed by imposing equality in import shares of closer countries. The result is a lower volatility of trade balance in small countries and J-curve less steeper, and a lower volatility of imports and exports.

In the conclusions, the author point out that the simulated world economies fit reasonably well some properties of economies in last two decades, although focusing only on technology perturbations, the models seems very simple to approach such a complexity as the one of real world. Under other specifications such as time to ship and costs of transportation, the model shows no trade effects on business cycles. However, size effects could explain some variability of empirical and theoretical economy results.

CHAPTER II

A TWO COUNTRY MODEL WITH A SPECIAL TYPE OF ASYMMETRY

Several models have been developed to capture aggregated variables behavior in the open economies, and to explain the comovement between aggregates across countries. These set up models include time to build, time to ship, investment costs of adjustment, different spillover effects of shocks, and differences in production, preferences and population parameters has been also modeled.

However, there is an asymmetry not included yet, which is very special (and normal) for small developing open economies, such as the Latin-American ones: imperfect capital mobility and the possibility of sudden stops. Gómez and Posada (2004) modeled a small open economy as the Colombian economy, including physical capital accumulation and foreign debt (foreign assets), in an imperfect capital mobility environment. This is captured as an interest rate spread as an increasing function of foreign debt. However, interest rate spreads are not only determined by the size of debt but also by international market perception about the performance of the economy.

The economy to be modeled here, differently to those by Zimmerman (1995) for instance, will include the kind of asymmetry noted above. A big economy will front an internationally competitive risk free interest rate (given by its capital marginal productivity), while the small country will front a higher domestic interest rate which is the world interest rate plus a margin that depends on the ratio foreign debt-GDP,[5] and a stochastic process which captures perception of risk in the international capital market. Thus, interest rate spread can also be correlated with technological perturbations to the production functions of the countries. It is interesting to build a two country model because is the way to capture how the dynamics of both countries affects each other, and specially to know how the asymmetries imposed makes one of the countries more sensible and volatile to perturbations. Thus whether there are or not phenomena such like long run

[5] This is somewhat similar to Gómez y Posada (2004).

structural changes could be answered by developing a model as the proposed one. A set up model in such a spirit is built in Baxter (1995).

1. The small and dependant economy

1.1 The firms

The small economy is inhabited by representative families, firms, and the government. Firms competitively demand labor and capital, and face adjustment costs of investment. When these economies trade every one has a relative price: terms of trade, which is the export prices-import prices ratio. Supposing that for each economy the numerary is the consumption, and its price is one, then terms of trade is $\frac{Px}{Pm}$, and the price of imported goods in terms of comparable units will be the inverse of terms of trade, it is, the terms of trade for the big economy $\frac{Pm}{Px}$.

Accordingly, the problem for the firms is maximizing the expected present value of the future profits:[6]

$$\max E\left[\sum_{j=0}^{\infty}\prod_{i=0}^{j}\lambda_{h,t+i}G_{h,t+j}\right] \quad (1)$$

where: $G_{h,t} = Y_{h,t} - s_{h,t}L_{h,t}Z_{h,t} - C_h(In_{h,t}) - I_{h,t}$ are the profits[7]

With subjection to $Y_{h,t} = A_{h,t}K_{h,t}^{\alpha_h}(Z_{h,t}L_{h,t})^{1-\alpha_h}$ which is a Cobb-Douglas technology very usual in these kind of set ups.

[6] Notice an important fact: in traditional models, it is assumed that the interest rate is constant and hence the discount factor is a constant quantity each period of time. However, this model has non constant interest rate and so; the discount factor for each period is a multiplication for all the periodic discount factors.

[7] Gómez and Posada (2004) investment of the firm has the form of imported goods. In a different way, in this new model firms invest (valuating in terms of the numerary), and part of this investment is domestic and foreign saving financed, and the same for the consumption of the families.

$C_h(In_{h,t}) = \frac{\gamma_h}{2}\frac{(K_{h,t} - K_{h,t-1})^2}{K_{h,t-1}}$: Adjustment costs of investment (net investment).

$I_{h,t} = K_{h,t+1} - (1-\delta_h)K_{h,t}$: Gross investment.

$\lambda_{h,t} = \frac{1}{1+r_{h,t}}$: Discount factor.

$r_{h,t}$: Interest rate.

Being

$K_{h,t}$: Aggregated physical capital

$Z_{h,t}L_{h,t}$: Effective labor.

$Z_t = Z_0 g_{hZ}^t$ is time evolution of efficiency of labor, $L_t = L_0 g_{hL}^t$ is time evolution of population (total labor force). $g_{hZ} = (1+\frac{\dot{Z}}{Z})$ is labor efficiency gross growth rate, $g_{hL} = (1+\frac{\dot{L}}{L})$ is population gross growth rate. Thus, $g_{hZL} = g_{hZ}g_{hL}$ is effective labor gross growth rate.

Time-to-build will not be modeled here.[8] However, it should be noted that cost of adjustment of investment has a similar effect of that the time-to-build one, because these costs make a slower response of capital than it could be when adjustment is costless. This prevents the model of having an over-reaction when the economy is eventually shocked.[9]

[8] This is because the model will be calibrated for annually data.
[9] See Gómez y Posada (2004) for a discussion about this.

In this economy physical capital financing has two sources: domestic savings and foreign debt if the economy was a net debtor, and on the opposite, if it was a net creditor, it would be financing capital accumulation in the rest of the world.

Given an initial stock of capital, the firm has to solve for the following stochastic dynamic programming:

$$V_h(K_h) = \max_{K_{t+1}} \left\{ G_h(\bullet) + E\left[\lambda_{h,t+1} V_h(K_{h,t+1})\right] \right\}$$

$$V_h(K) = \max_{K_{h,t+1}} \left\{ \left[Y_{h,t} - s_{h,t} L_{h,t} Z_{h,t} - \frac{\gamma_h}{2} \frac{(K_{h,t+1} - K_{h,t})^2}{K_{h,t}} - K_{h,t+1} + (1-\delta_h) K_{h,t} \right] + E\left[\lambda_{h,t+1} V_h(K_{h,t+1})\right] \right\}$$

The first order condition for this problem is:

$$V_h'(K_{h,t}) = \left[\frac{-\gamma_h \left(K_{h,t+1} - K_{h,t}\right)}{K_{h,t}} - 1 \right] + E\left[\lambda_{h,t+1} V_h'(K_{h,t+1})\right] = 0$$

Where we have:

$$\left[1 + \frac{\gamma_h \left(K_{h,t+1} - K_{h,t}\right)}{K_{h,t}} \right] = E\left[\lambda_{h,t+1} V_h'(K_{h,t+1})\right]$$

By using the envelope theorem:

$$\frac{\partial V_h(K_{h,t})}{\partial K_{h,t}} = PmagK_{h,t} + \frac{\gamma_h In_{h,t}}{K_{h,t}} + \frac{\gamma_h}{2} \left[\frac{In_{h,t}}{K_{h,t}} \right]^2 + (1-\delta_h)$$

Being $PmagK_{h,t}$ marginal product of capital.

Actualizing up to $t+1$ and taking expectations:

$$E\left[V_h'(K_{h,t+1})\right]=E\left[PmagK_{h,t+1}+\frac{\gamma_h In_{h,t+1}}{K_{h,t+1}}+\frac{\gamma_h}{2}\left[\frac{In_{h,t+1}}{K_{h,t+1}}\right]^2+(1-\delta_h)\right]$$

Being $In_{h,t}=K_{h,t+1}-K_{h,t}$ net investment.

Besides,

$$E\left[\lambda_{h,t}V_h'(K_{h,t+1})\right]=E\left[\lambda_{h,t+1}\right]E\left[V_h'(K_{h,t+1})\right]+\operatorname{cov}\left[\lambda_{h,t+1},V_h'(K_{h,t+1})\right]$$

Therefore, the final first order condition for the firm's problem looks like:

$$\left[1+\frac{\gamma_h\left(K_{h,t+1}-K_{h,t}\right)}{K_{h,t}}\right]=E\left[\lambda_{h,t+1}V_h'(K_{h,t+1})\right]=E\left[\lambda_{h,t+1}\right]E\left[V_h'(K_{h,t+1})\right]+\operatorname{cov}\left[\lambda_{h,t+1},V_h'(K_{h,t+1})\right]$$

This expression takes into account discount factor and value function for the firm in $t+1$.

The previous equation can also be written as:

$$\left[1+\frac{\gamma_h\left(K_{h,t+1}-K_{h,t}\right)}{K_{h,t}}\right]=E\left[\lambda_{h,t+1}\right]E\left[PmagK_{h,t+1}+\frac{\gamma_h In_{h,t+1}}{K_{h,t+1}}+\frac{\gamma_h}{2}\left[\frac{In_{h,t+1}}{K_{h,t+1}}\right]^2+(1-\delta_h)\right]$$
$$+\operatorname{cov}\left[\lambda_{h,t+1},V_h'(K_{h,t+1})\right]$$

The result implies that in order to get an optimum firms have to invest up to marginal cost of capital adjustment in a unit is equal to marginal profit generated in the future by the new capital present valued at the market discount rate. Notice that future profit of capital adjustment is given by the saving of future costs of adjusting capital in the appropriated quantity in the present, plus the remaining capital once depreciation is reposed.

1.2. The families

The economy is inhabited by a constant quantity H of identical families each one composed of representative agents, which accumulate wealth as physical capital and foreign assets, and offer labor inelesticaly.[10] The families' objective is to maximize the intertemporal utility function with subjection to an intertemporal budget constraint which incorporates income from physical capital, foreign assets and wages. [11]

$$\max E\left[\sum_{t=0}^{\infty} \beta_h^{\ t} \frac{u_h(\tilde{C}_{h,t}) L_{h,t}}{H_h}\right]$$

Being $\tilde{C}_{h,t}$ *per* capita consumption in t, and $\beta_h = \dfrac{1}{1+\rho_h}$ subjective discount factor, and ρ_h subjective discount rate. $u_h(\tilde{C}_{h,t})$ is an additive separable utility function, which satisfies $u' > 0$, and $u'' < 0$. Furthermore, this utility function captures risk aversion. Thus, this utility function has the form:

$$u_h(\tilde{C}_{h,t}) = \frac{\tilde{C}_{h,t}^{1-\theta_h}}{1-\theta_h}$$

To guaranty convergence and finiteness of the utility function it must be expressed in terms of effective labor:

---

[10] The objective of this thesis is not to explain labor fluctuations, and that is the reason for making this assumption about labor.

[11] In the case of small economies like Colombia, the foreign asset stock is negative, that is, it is a net debtor.

$\widetilde{C}_{h,t} = \frac{C_{h,t}}{L_{h,t}}$, being $C_{h,t}$ the aggregated consumption of the economy. Then, it is possible to write: $\frac{\widetilde{C}_{h,t}}{Z_{h,t}} = \frac{C_{h,t}}{L_{h,t}Z_{h,t}} = c_{h,t}$ which is the consumption per unit of effective labor. Therefore, $\widetilde{C}_{h,t} = Z_{h,t}c_{h,t}$.

So, the utility function becomes:

$$u_h(\widetilde{C}_{h,t}) = u_h(Z_{h,t}c_{h,t}) = \frac{(Z_{h,t}c_{h,t})^{1-\theta_h}}{1-\theta_h}$$

The problem of the representative family is:

$$\max E\left[\sum_{t=0}^{\infty} \beta_h^{\ t} \frac{(Z_{h,t}c_{h,t})^{1-\theta_h}}{1-\theta_h} \frac{L_{h,t}}{H_h}\right]$$

A representative agent accumulates wealth which is allocated to physical capital and foreign assets. The time evolution of that wealth is the intertemporal budget constraint of the agent:

$$w_{h,t} = b_{h,t} + q_{h,t}k_{h,t}$$

Being $q_{h,t}$ price of capital or Tobin's Q, $b_{h,t}$ the stock of foreign assets in terms of effective labor, and $k_{h,t}$ is the stock of physical capital in units of effective labor.

Time evolution of wealth is:

$$W_{h,t+1} = (1 + r_{h,t})W_{h,t} + s_{h,t}L_{h,t}Z_{h,t} - C_{h,t} - \Psi_{h,t}$$

Being $\Psi_{h,t}$ government expenditures, which are supposed equal to tax revenues. After reexpressing in terms of effective labor, wealth evolution is:

$$g_{hZL} w_{h,t+1} = (1 + r_{h,t}) w_{h,t} + s_{h,t} - c_{h,t} - \psi_{h,t}$$

Given the budget constraint and the utility function, the families must solve for the following stochastic dynamic problem:

$$J_h(c_h) = \max_{c_{h,t+1}, w_{h,t+1}} \left\{ \left[u_h(Z_{h,t} c_{h,t}) \frac{L_{h,t}}{H_h} \right] + \beta_h E\left[J_h(c_{h,t+1}) \right] \right\} \qquad \text{(A)}$$

Replacing the budget constraint into the previous Bellman equation, we have:

$$J_h(w_h) = \max_{w_{h,t+1}} \left\{ \left[u_h(Z_{h,t} c_{h,t}) \right] + \beta_t E\left[J_h(w_{h,t+1}) \right] \right\}$$

Here it is not explicitly shown the quantity of consumption imported from abroad, but it is possible to distinguish between imported and domestic quantities of consumption by introducing an imperfect substitution scheme by means of a CES function:

$$c_{h,t} = \left[\varphi_h \; {}_h c_{ht}^{\sigma_h} + (1 - \varphi_h) \, {}_f c_{ht}^{\sigma_h} \right]^{\frac{1}{\sigma_h}}$$

Being, ${}_h c_{ht}$, consumption of domestically produced goods in terms of effective labor and ${}_f c_{ht}$ consumption of imported goods in terms of effective labor.[12] As a consequence, the budget constraint is slightly modified:

[12] Accordingly to this, it is to be supposed that there is a homogenous product and that the decision of importing part of it for consumption obeys to the fact that there exists separability between consumption and investment decisions. This is the reason for a non one-to-one relationship between domestic saving and investment.

$$g_{ZL} w_{h,t+1} = (1+r_t^h) w_{h,t} + s_{h,t} -_h c_{h,t} -_f c_{h,t} - \psi_{h,t}$$

For this reason, an alternative Bellman equation for the problem is:

$$J_h(c_{h,t}, w) = \max_{w_{h,t+1}} \left\{ \left[u_h(c_{h,t}, w_{h,t}) \frac{L_{h,t}}{H_h} \right] + \beta E\left[J(c_{t+1}, w_{t+1}) \right] \right\} \qquad \text{(B)}$$

It is not very difficult to solve for this problem. Families will determine optimal consumption level and then they will choose the optimal level for domestic and imported consumption respectively, by solving:

$$\text{minimize total consumption value} =_h c_{h,t} + \tau^{-1} {}_f c_{h,t}$$

With subjection to $c_{h,t} = \left[\varphi_h \; {}_h c_{h,t}^{\sigma_h} + (1-\varphi_h) {}_f c_{h,t}^{\sigma_h} \right]^{\frac{1}{\sigma_h}}$

The solution for this problem is the standard one, it is, and the optimum is the point where the slopes these two expressions equal each other.[13]

Thus the things, first order condition for problem (A) is:

$$-u_h'(c_{h,t}) g_{LZ} + \beta_h E\left[J_h'(c_{h,t+1}) \right] = 0$$
$$u_h'(c_t) g_{LZ} = \beta_h E\left[J_h'(c_{h,t+1}) \right]$$

By using the envelope theorem:

[13] Because terms of trade are supposed constant, is not very realistic since for every time, the solution for the problem is the same, that is, the share for each item of consumption is always the same. In a more real version (to be developed in later papers) variable terms of trade will make variable these shares of consumptions.

$$\frac{\partial J_h(w_{h,t})}{\partial w_{h,t}} = \left(1 + r_t^h\right) u_h'(c_{h,t}) = J_h'(c_{h,t})$$

Updating to $t+1$ and taking expectations we have:

$$E\left[J_h'(c_t)\right] = E\left[(1 + r_{t+1}^h) u_h'(c_{h,t+1})\right]$$

Replacing in the first order condition we have:

$$u_h'(c_{h,t}) g_{ZL} = \beta_h E\left[(1 + r_{t+1}^h) u_h'(c_{h,t+1})\right]$$

$$u_h'(c_{h,t}) g_{ZL} = \beta_h E\left[(1 + r_{t+1}^h)\right] E\left[u_h'(c_{t+1})\right] + \beta_h \operatorname{cov}\left[(1 + r_{t+1}^h), u_h'(c_{h,t+1})\right]$$

This equation has the standard interpretation: a decrease in the utility caused in t by an extra saving unit must be compensated, in $t+1$, with the extra utility from consumption of the extra saving unit plus its returns, discounted with the subjective discount factor.

Up to here the small (and dependant) open economy is partially characterized, and its dynamic behavior is given by the transition equations for families' wealth, capital of the firms and first order conditions for families and firms.

1.3. The interest rate spread and disturbances

A special quality of the small and dependant economies is the long term imperfect capital mobility caused for institutional arrangements or for being highly-risky perceived into international capital market. Institutional problem will be not modeled here. However, highly-risk problem will be modeled.

Let us suppose that the economy is to pay a spread on the world interest rate. This margin will be a decreasing function of the foreign asset relative to GDP,[14] and for a random disturbance which reflects rest of world perception about the economy. So, even though the economy has a low foreign asset-GDP ratio or null, the economy could be affected by international uncertainty. Regarding this subject, Rowland and Torres (2004) using a panel data sample for 29 emerging issuer debt countries find significant positive effect of debt/GDP ratio, debt/exports ratio, and debt service/GDP on debt spreads, while negative from GDP growth rate, exports/GDP ratio and foreign reserves/GDP ratio.

According to this, small economy interest rate is:

$$r_t^h = r_t^f - r_1\left(\frac{B_{h,t}}{Y_{h,t}}\right) + \varepsilon_{rt}$$

Being r_t^f world interest rate, $\frac{B_{h,t}}{Y_{h,t}} = \frac{b_{h,t}}{y_{h,t}}$ foreign asset-GDP ratio. ε_{rt} is a stochastic shock with cero mean and persistency. $\varepsilon_{rt} \to (0, \sigma_\varepsilon)$, $corr(\varepsilon_{rt}\varepsilon_{rs}) \neq 0, \forall s \neq t$. Technology shocks of the economies affect their technological coefficient in the next way:

$$A_{ht} = \mu_{Ah} + \rho_{hh} A_{h,t-1} + \rho_{fh} A_{f,t-1} + \varepsilon_{At}^h$$

$$A_{ft} = \mu_{Af} + \rho_{hf} A_{h,t-1} + \rho_{ff} A_{f,t-1} + \varepsilon_{At}^f$$

There is a relationship between technology shocks and small economy interest rate:

$$\varepsilon_t = \Gamma \varepsilon_{t-1}$$

Being

[14] This is a kind of risk modeling in front of the possibility of debt repudiation or not payment capacity.

$$\Gamma = \begin{bmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ 0 & \rho_{22} & \rho_{23} \\ 0 & 0 & \rho_{33} \end{bmatrix}$$

y

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{rt} \\ \varepsilon_{At}^{h} \\ \varepsilon_{At}^{f} \end{bmatrix}$$

Correlations structure captures the possibility of shock persistence and that is the reason why diagonal elements of Γ are non negative entries. Besides, a positive technology shock in the small economy can generate a higher productive capacity and capital accumulation, and hence, a higher payment capacity. These facts will generate a good perception of the economy in the rest of the world and a higher level of confidence, therefore a lower spread will be charged on the small economy. Because of this, $\rho_{12} \leq 0$. In the same way a positive technology shock in the big economy by itself does neither cause a significative effect on the perception about the small economy nor generates an immediate fall in the small interest rate. Thus it is expected that $\rho_{13} = 0$.

Furthermore, a big economy that influences the results and decisions of the small economy can through imports and exports generate spillover effects. [15]

Technology movement equations subsystem is:

$$\begin{bmatrix} A_{ht} \\ A_{ft} \end{bmatrix} = \begin{bmatrix} \mu_{Ah} \\ \mu_{Af} \end{bmatrix} + \begin{bmatrix} \rho_{hh} & \rho_{fh} \\ \rho_{hf} & \rho_{ff} \end{bmatrix} \begin{bmatrix} A_{h,t-1} \\ A_{f,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{At}^{h} \\ \varepsilon_{At}^{f} \end{bmatrix}$$

The full system for technology and interest rate is:

[15] It is not necessary to suppose the big economy is technology creation leader.

$$\begin{bmatrix} r_t^h \\ A_{ht} \\ A_{ft} \end{bmatrix} = \begin{bmatrix} r_t^f \\ \mu_{Ah} \\ \mu_{Af} \end{bmatrix} + \begin{bmatrix} -r_1 & 0 & 0 \\ 0 & \rho_{hh} & \rho_{fh} \\ 0 & \rho_{hf} & \rho_{ff} \end{bmatrix} \begin{bmatrix} (b_t/y_t) \\ A_{h,t-1} \\ A_{f,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{rt} \\ \varepsilon_{At}^h \\ \varepsilon_{At}^f \end{bmatrix}$$

Including the correlations structure for random shocks we have:

$$\begin{bmatrix} r_t^h \\ A_{ht} \\ A_{ft} \end{bmatrix} = \begin{bmatrix} r_t^f \\ \mu_{Ah} \\ \mu_{Af} \end{bmatrix} + \begin{bmatrix} -r_1 & 0 & 0 \\ 0 & \rho_{hh} & \rho_{fh} \\ 0 & \rho_{hf} & \rho_{ff} \end{bmatrix} \begin{bmatrix} (b_t/y_t) \\ A_{h,t-1} \\ A_{f,t-1} \end{bmatrix} + \begin{bmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ 0 & \rho_{22} & \rho_{23} \\ 0 & 0 & \rho_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{rt-1} \\ \varepsilon_{At-1}^h \\ \varepsilon_{At-1}^f \end{bmatrix}$$

2. The big economy

2.1. The firms

For the case of the firms the structure and the problem is the same of that of the firms in the small economy:

$$\max E\left[\sum_{j=0}^{\infty}\prod_{i=0}^{j}\lambda_{f,t+i}G_{f,t+j}\right]$$

Being:

$$G_{f,t} = Y_{f,t} - s_{f,t}L_{f,t}Z_{f,t} - C_f(I_{f,t}) - I_{f,t}$$

With subjection to:

$$Y_{f,t} = A_{f,t}K_{f,t}^{\alpha_f}(Z_{f,t}L_{f,t})^{1-\alpha_f} \quad : \quad \text{The production function}$$

$$C_f(I_{f,t}) = \frac{\gamma_f}{2}\frac{(K_{f,t} - K_{f,t-1})^2}{K_{f,t-1}}$$: Adjustment investment costs

$$I_{f,t} = K_{f,t+1} - (1-\delta)K_{f,t}$$: Gross investment

$$\lambda_{f,t} = \frac{1}{1 + r_{f,t}}$$: Market discount factor

$r_{f,t}$: Interest rate

Being

$K_{f,t}$: Aggregated capital

$Z_{f,t}L_{f,t}$: Effective labor

Physical capital financing problem is the same as in the small economy

The stochastic dynamic problem for the firms is:

$$V_f(K_f) = \max_{K_{f,t+1}} \{\Pi_f(\bullet) + E[\lambda_{f,t+1}V(K_{f,t+1})]\}$$

$$V_f(K_f) = \max_{K_{f,t+1}} \left\{ \left[Y_{f,t} - s_{f,t}L_{f,t}Z_{f,t} - \frac{\gamma_f}{2}\frac{(K_{f,t+1} - K_{f,t})^2}{K_{f,t}} - K_{f,t+1} + (1-\delta)K_{f,t} \right] + E[\lambda_{f,t+1}V_f(K_{f,t+1})] \right\}$$

First order condition is:

$$V_f'(K_{f,t}) = \left[\frac{-\gamma_f(K_{f,t+1} - K_{f,t})}{K_{f,t}} - 1 \right] + E[\lambda_{f,t+1}V_f'(K_{f,t+1})] = 0$$

This means:

$$\left[1 + \frac{\gamma_f(K_{f,t+1} - K_{f,t})}{K_{f,t}} \right] = E[\lambda_{f,t+1}V_f'(K_{f,t+1})]$$

Using envelope theorem:

$$\frac{\partial V_f(K_{f,t})}{\partial K_{f,t}} = PmagK_{f,t} + \frac{\gamma_f In_{f,t}}{K_{f,t}} + \frac{\gamma_f}{2}\left[\frac{In_{f,t}}{K_{f,t}}\right]^2 + (1-\delta)$$

Being $PmagK_{f,t}$ marginal product of capital

Updating up to $t+1$ and taking expectations:

$$E\left[V_f'(K_{f,t+1})\right] = E\left[PmagK_{f,t+1} + \frac{\gamma_f In_{f,t+1}}{K_{f,t+1}} + \frac{\gamma_f}{2}\left[\frac{In_{f,t}}{K_{f,t+1}}\right]^2 + (1-\delta)\right]$$

Being $In_{f,t} = K_{f,t+1} - K_{f,t}$ net investment.

Besides, it is known that:

$$E\left[\lambda_{f,t} V'(K_{f,t+1})\right] = E\left[\lambda_{f,t+1}\right] E\left[V_f'(K_{f,t+1})\right] + \operatorname{cov}\left[\lambda_{f,t+1}, V_f'(K_{f,t+1})\right]$$

The final form of the first order condition for the firms is:

$$\left[1 + \frac{\gamma_f\left(K_{f,t+1} - K_{f,t}\right)}{K_{f,t}}\right] = E\left[\lambda_{f,t+1} V_f'(K_{f,t+1})\right] = E\left[\lambda_{f,t+1}\right] E\left[V_f'(K_{f,t+1})\right] + \operatorname{cov}\left[\lambda_{f,t+1}, V_f'(K_{f,t+1})\right]$$

It can be written as:

$$\left[1 + \frac{\gamma_f\left(K_{f,t+1} - K_{f,t}\right)}{K_{f,t}}\right] = E\left[\lambda_{f,t+1}\right] E\left[PmagK_{f,t+1} + \frac{\gamma_f In_{f,t+1}}{K_{f,t+1}} + \frac{\gamma_f}{2}\left[\frac{In_{f,t+1}}{K_{f,t+1}}\right]^2 + (1-\delta)\right] + \operatorname{cov}\left[\lambda_{f,t+1}, V_f'(K_{f,t+1})\right]$$

By modeling two different interest rates we have an interesting result: firms in the small economy are more vulnerable when there is a shock, while firms in the big economy are not directly vulnerable to any kind of perturbation. Then, it is expected that when there is a shock on the interest rate spread, firms in the big economy are less affected, and as a consequence their product will be less volatile, whereas the case of the small economies will be the opposite one: the model will be able to reproduce for small economies, wider and longer cycles than those of the big economies, and higher costs of recovering for the small economies.

2.2. The families

As the families in the small economy, these families are a constant quantity H composed by representative agents who have a similar intertemporal utility function, intertemporal budget constraint, and a similar problem for choosing saving, physical capital foreign assets and labor. So the problem of these families is:

$$\max E\left[\sum_{t=0}^{\infty}\beta_f^t \frac{u_f(\tilde{C}_{f,t})L_{f,t}}{H_f}\right]$$

Being $\tilde{C}_{f,t}$ per capita consumption in t, and $\beta_f = \dfrac{1}{1+\rho_f}$ subjective discount factor, and ρ_f subjective discount rate. $u_f(\tilde{C}_{f,t})$ additive separable utility function, which satisfies $u'_f > 0$ and $u''_f < 0$. Furthermore, this utility function captures risk aversion. Thus, this utility function has the form:

$$u_f(\tilde{C}_{f,t}) = \frac{\tilde{C}_{f,t}^{1-\theta_f}}{1-\theta_f}$$

Reexpresing in terms of effective labor the problem takes the form:

$$\max E\left[\sum_{t=0}^{\infty}\beta_f^t \frac{\left(Z_{f,t}c_{f,t}\right)^{1-\theta_f}}{1-\theta_f}\frac{L_{f,t}}{H}\right]$$

As for the representative agent in the small economy wealth is allocated to physical capital and foreign assets

$$w_{f,t} = b_{f,t} + q_{f,t}k_{f,t}$$

Being $q_{f,t}$ price of capital or Tobin's Q, $b_{f,t}$ the stock of foreign assets in terms of effective labor, and $k_{f,t}$ is the stock of physical capital in units of effective labor. At this point it is necessary to outline that the time evolution of wealth for these agents is different to that of the small economy. This is because in the big economy, agents accumulates capital which is paid at a "risk free" interest rate, while resources lent to the rest of the world (the small economy) are paid a risk free interest rate plus the margin discussed above as a function of foreign debt-GDP ratio, and the random disturbance. Therefore, time evolution for wealth of agents in the big economy is:

$$W_{f,t+1} = (1+r_{h,t})B_t + (1+r_{f,t})q_{f,t}K_{f,t} + s_{f,t}L_{f,t}Z_{f,t} - C_{f,t} - \Psi_{f,t}$$

$$W_{f,t+1} = (1+r_{f,t})(B_{f,t} + q_{f,t}K_{f,t}) + r_1\left(B_t / Y_t\right)B_t + \varepsilon_{r,t}B_t + s_{f,t}L_{f,t}Z_{f,t} - C_{f,t} - \Psi_{f,t}$$

Being $\Psi_{f,t}$ government expenditures, supposed equal to tax revenues.

Differently from the small economy, in this economy there is not equivalence between choosing foreign assets and wealth because in this economy this to stocks are not paid the same interest rate. Therefore, in this economy, the families have to explicitly decide what quantity of foreign asset will borrow (will lend) in the world resources market. In other words, this economy will have to decide about $B_{f,t+1}$. Re-expressing in terms of effective labor:

$$g_{fZL}(b_{f,t+1}+q_{t+1}k_{t+1})=(1+r_{f,t})(b_{f,t}+q_{f,t}k_{f,t})+r_1(b_{h,t}/y_{h,t})b_{f,t}+\varepsilon_{r,t}b_{f,t}+s_{f,t}-c_{f,t}-\psi_{f,t}$$

Thus, the stochastic dynamic problem for the families in the big economy is:

$$J_f(c,b)=\max_{c_{f,t+1},b_{f,t+1}}\{[u_f(b_{f,t},c_{f,t})]+\beta_f E[J_f(c_{f,t+1},b_{f,t+1})]\}$$

Solving for $c_{f,t}$ in the budget constraint, we have that:

$$c_{f,t}=(1+r_{f,t})(b_{f,t}+q_{f,t}k_{f,t})-g_{fZL}(b_{f,t+1}+q_{t+1}k_{t+1})+r_1(b_{h,t}/y_{h,t})b_{f,t}+\varepsilon_{r,t}b_{f,t}+s_{f,t}-\psi_{f,t}$$

$$J_f(b_{f,t})=\max_{b_{f,t+1}}\{[u_f(b_{f,t})]+\beta_f E[J_f(b_{f,t+1})]\}$$

The problem for choosing imported and domestically quantities of consumption has the similar form as the one for the small economy:

$$\text{minimize total consumption value} = \tau^{-1}{}_h c_{ft} + {}_f c_{ft}$$

With subjection to $c_{f,t}=\left[\varphi_f\, {}_h c_{ft}^{\sigma_f}+(1-\varphi_f)\,{}_f c_{ft}^{\sigma_f}\right]^{\frac{1}{\sigma_f}}$

The first order condition for the stochastic dynamic problem is:

$$-u_f'(c_{f,t})g_{LZ}+\beta_f E[J_f'(c_{f,t+1})]=0$$
$$u_f'(c_{f,t})g_{LZ}=\beta_f E[J_f'(c_{f,t+1})]$$

Using the envelope theorem:

$$\frac{\partial J_f(b_{f,t})}{\partial b_{f,t}} = \left(1 + r_{f,t} - r_1\left(b_t / y_t\right) + \varepsilon_{rt}\right) u_f'(c_{f,t}) = J_f'(c_{f,t})$$

Updating up to $t+1$ and taking expectations:

$$E\left[J_f'(c_{f,t+1})\right] = E\left[(1 + r_{f,t+1} - r_1\left(b_{h,t+1} / y_{h,t+1}\right) + \varepsilon_{rt+1}) u_f'(c_{f,t+1})\right]$$

The first order condition takes the form:

$$u_f'(c_{f,t}) g_{ZL} = \beta_f E\left[(1 + r_{f,t+1} - r_1\left(b_{h,t+1} / y_{h,t+1}\right) + \varepsilon_{rt+1}) u'(c_{f,t+1})\right]$$

$$u_f'(c_{f,t}) g_{ZL} = \beta_f E\left[(1 + r_{h,t+1})\right] E\left[u_f'(c_{f,t+1})\right] + \beta_f \operatorname{cov}\left[(1 + r_{h,t+1}), u'(c_{f,t+1})\right]$$

This result has the same interpretation as the first order condition for the families' problem in the small economy.

By incorporating adjustment costs of capital it is clear that foreign assets and physical capital are no longer perfect substitutes and modeling differential interest rates for the big economy makes this substitutability even weaker. [16]

3. Model closure and equilibrium characterization

Let us resume the main equations for this world economy:

The small economy

$$\left[1 + \frac{\gamma\left(K_{t+1} - K_{t+1}\right)}{K_t}\right] = E[\lambda_t] E\left[PmagK_{t+1} + \frac{\gamma In_{t+1}}{K_{t+1}} + \frac{\gamma}{2}\left[\frac{In_{t+1}}{K_{t+1}}\right]^2 + (1-\delta)\right] + \operatorname{cov}\left[\lambda_t, V'(K_{t+1})\right]$$

[16] See for instance, Barro and Sala-i-Martin (1995), Obstfeld and Rogoff (1999), Gómez and Posada (2004),

$$I_t = K_{t+1} - (1-\delta)K_t$$

$$g_{hZL} w_{t+1} = (1 + r_{h,t})w_t + s_t - c_t - \psi_t$$

$$u_h'(c_{h,t}) g_{hZL} = \beta_h E[(1 + r_{h,t+1})] E[u_h'(c_{t+1})] + \beta_h \operatorname{cov}[(1 + r_{h,t+1}), u_h'(c_{h,t+1})]$$

The big economy

$$\left[1 + \frac{\gamma_f (K_{f,t+1} - K_{f,t+1})}{K_{f,t}}\right] = E[\lambda_{f,t}] E\left[PmagK_{f,t+1} + \frac{\gamma_f In_{f,t+1}}{K_{f,t+1}} + \frac{\gamma_f}{2}\left[\frac{In_{f,t+1}}{K_{f,t+1}}\right]^2 + (1-\delta)\right] + \operatorname{cov}[\lambda_{f,t}, V_f'(K_{f,t+1})]$$

$$I_{f,t} = K_{f,t+1} - (1-\delta)K_{f,t}$$

$$g_{fZL}(b_{f,t+1} + q_{t+1}k_{t+1}) = (1 + r_{f,t})(b_{f,t} + q_{f,t}k_{f,t}) + r_1(b_t / y_t)b_t + \varepsilon_{r,t} b_t + s_{f,t} - c_{f,t} - \psi_{f,t}$$

$$u_f'(c_{f,t}) g_{fZL} = \beta_f E[(1 + r_{h,t+1})] E[u_f'(c_{f,t+1})] + \beta_f \operatorname{cov}[(1 + r_{h,t+1}), u'(c_{f,t+1})]$$

The full model has four state variables, capital and foreign asset for each country, two prices (two Tobin's Q) and two flow variables, total consumption for each country. Taking this into account, dynamic stability of the model requires at least four stable roots.

Equilibrium in the world economy is characterized by the sequence of values $\{k_{h,t}, b_{h,t}, c_{h,t}, q_{h,t}, k_{f,t}, b_{f,t}, c_{f,t}, q_{f,t}\}$, that satisfies the dynamic equations system shown above.

From the first order condition for firms in both small and big economies, forward solving for $In_{h,t}$ and $In_{f,t}$:

$$\frac{\gamma_h In_{h,t}}{K_{h,t}} = \sum_{j=0}^{T} \prod_{i=0}^{j} E[\lambda_{h,t+i+1}] E\left\{Pmgk_{h,t+j+1} + \frac{\gamma_h}{2}\left(\frac{In_{h,t+j+1}}{k_{h,t+j+1}}\right)^2 - \delta_h\right\} + \sum_{j=0}^{T} \prod_{i=0}^{j} E[\lambda_{h,t+i+1}] \operatorname{cov}\{\lambda_{h,t+j+2}, V_h'(k_{h,t+j+2})\}$$

$$+ \prod_{i=1}^{T} E[\lambda_{h,t+i}] \left\{E\left[\frac{\gamma_h In_{h,t+T+1}}{k_{h,t+T+1}}\right] + 1\right\} + \operatorname{cov}\{\lambda_{h,t+1}, V_h'(k_{h,t+1})\} - 1$$

And excluding bubbles in the price of capital, and given that $0 < \lambda_{h,t} < 1$, third term in the right hand of this equation tends to zero when $T \to \infty$, and hence it is possible to say that net investment in small economy is:

$$In_{h,t} = \left[\begin{aligned} & \sum_{j=0}^{T} \prod_{i=0}^{j} E[\lambda_{h,t+i+1}] E\left\{ Pmgk_{h,t+j+1} + \frac{\gamma_h}{2}\left(\frac{In_{h,t+j+1}}{k_{h,t+j+1}}\right)^2 - \delta_h \right\} + \sum_{j=0}^{T} \prod_{i=0}^{j} E[\lambda_{h,t+i+1}] \operatorname{cov}\{\lambda_{h,t+j+2}, V_h'(k_{h,t+j+2})\} \\ & + \operatorname{cov}\{\lambda_{h,t+1}, V_h'(k_{h,t+1})\} - 1 \end{aligned} \right] \frac{K_{h,t}}{\gamma_h}$$

$$In_{h,t} = (q_{h,t} - 1)\frac{K_{h,t}}{\gamma_h}$$

$$\begin{aligned} q_{h,t} = & \sum_{j=0}^{T} \prod_{i=0}^{j} E[\lambda_{h,t+i+1}] E\left\{ Pmgk_{h,t+j+1} + \frac{\gamma_h}{2}\left(\frac{In_{h,t+j+1}}{k_{h,t+j+1}}\right)^2 - \delta_h \right\} + \sum_{j=0}^{T} \prod_{i=0}^{j} E[\lambda_{h,t+i+1}] \operatorname{cov}\{\lambda_{j,t+j+2}, V_h'(k_{h,t+j+2})\} \\ & + \operatorname{cov}\{\lambda_{h,t+1}, V_h'(k_{h,t+1})\} \end{aligned}$$

Price of capital for $t+1$ is given by:

$$E[\lambda_{h,t+1}] q_{h,t+1} = \left[\begin{aligned} & \sum_{j=1}^{T} \prod_{i=1}^{j} E[\lambda_{h,t+i+1}] E\left\{ Pmgk_{h,t+j+1} + \frac{\gamma_h}{2}\left(\frac{In_{h,t+j+1}}{k_{h,t+j+1}}\right)^2 - \delta \right\} + \sum_{j=1}^{T} \prod_{i=1}^{j} E[\lambda_{h,t+i+1}] \operatorname{cov}\{\lambda_{h,t+j+2}, V_h'(k_{h,t+j+2})\} \\ & + \operatorname{cov}\{\lambda_{h,t+2}, V_h'(k_{h,t+2})\} \end{aligned} \right]$$

Therefore, the transition equation for Tobin's Q is:

$$q_{h,t+1} = \frac{q_{h,t}}{E[\lambda_{h,t+1}]} - E\left\{ Pmgk_{h,t+1} + \frac{\gamma_h}{2}\left(\frac{In_{h,t+1}}{k_{h,t+1}}\right)^2 - \delta_h \right\} - \operatorname{cov}\{\lambda_{h,t+1}, V_h'(k_{h,t+1})\} + \operatorname{cov}\{\lambda_{h,t+2}, V_h'(k_{h,t+2})\}$$

Then the whole set of transition equations for the small economy has the form:

$$g_{hZL} k_{h,t+1} = (q_{h,t} - 1)\frac{k_{h,t}}{\gamma} + k_{h,t}$$

$$q_{h,t+1} = \frac{q_{h,t}}{E[\lambda_{h,t+1}]} - E\left\{ Pmgk_{h,t+1} + \frac{\gamma}{2}\left(\frac{In_{h,t+1}}{k_{h,t+1}}\right)^2 - \delta_h \right\} - \text{cov}\{\lambda_{h,t+1}, V'(k_{h,t+1})\} + \text{cov}\{\lambda_{h,t+2}, V'(k_{h,t+2})\}$$

$$g_{ZL}^{f}(b_{f,t+1} + q_{t+1}k_{t+1}) = (1 + r_t^f)(b_{f,t} + q_{f,t}k_{f,t}) + r_1(b_t / y_t)b_t + \varepsilon_{r,t}b_t + s_{f,t} - c_{f,t} - \psi_{f,t}$$

$$u_f'(c_{f,t})g_{ZL} = \beta_f E[(1 + r_{t+1}^h)]E[u_f'(c_{f,t+1})] + \beta_f \text{ cov}[(1 + r_{t+1}^h), u'(c_{f,t+1})]$$

Similarly for the big economy:

$$g_{fZL}k_{f,t+1} = (q_{f,t} - 1)\frac{k_{f,t}}{\gamma} + k_{f,t}$$

$$q_{f,t+1} = \frac{q_{f,t}}{E[\lambda_{f,t+1}]} - E\left\{ Pmgk_{f,t+1} + \frac{\gamma}{2}\left(\frac{In_{f,t+1}}{k_{f,t+1}}\right)^2 - \delta_f \right\} - \text{cov}\{\lambda_{f,t+1}, V'(k_{f,t+1})\} + \text{cov}\{\lambda_{f,t+2}, V'(k_{f,t+2})\}$$

$$g_{ZL}^{f}(b_{f,t+1} + q_{t+1}k_{t+1}) = (1 + r_t^f)(b_{f,t} + q_{f,t}k_{f,t}) + r_1(b_t / y_t)b_t + \varepsilon_{r,t}b_t + s_{f,t} - c_{f,t} - \psi_{f,t}$$

$$u_f'(c_{f,t})g_{ZL} = \beta_f E[(1 + r_{t+1}^h)]E[u_f'(c_{f,t+1})] + \beta_f \text{ cov}[(1 + r_{t+1}^h), u'(c_{f,t+1})]$$

In order to impose closure to the model, we have to recall that time evolution of foreign asset is:

$$g_{h,ZL}b_{h,t+1} = (1 + r_{h,t})b_{h,t} + ca_{h,t}$$

Being $ca_{h,t}$ small economy current account which is defined as:[17]

$$ca_{h,t} = y_{h,t} - {}_h c_{h,t} - \tau_h c_{f,t} + \tau_f c_{h,t} - \psi_{h,t} - i_{h,t} - \frac{C(I_{h,t})}{Z_{h,t}L_{h,t}} - \delta_h k_{h,t}$$

$$ca_{h,t} = y_{h,t} - {}_h c_{h,t} - \tau_h c_{f,t} + \tau_f c_{h,t} - \psi_{h,t} - \frac{(q_{h,t} - 1)k_{h,t}}{\gamma} - \frac{(q_{h,t} - 1)^2 k_{h,t}}{2\gamma} - \delta_h k_{h,t}$$

[17] Appendix 1 shows briefly how this equation takes this effective labor form.

This implies that foreign asset for the small economy evolves as:

$$g_{h,ZL}b_{h,t+1} = (1+r_{h,t})b_{h,t} + y_{h,t} -_{h} c_{h,t} - \tau\ _{h}c_{f,t} + \tau\ _{f}c_{h,t} - \psi_{h,t} - \frac{(q_{h,t}-1)k_{h,t}}{\gamma} - \frac{(q_{h,t}-1)^2 k_{h,t}}{2\gamma} - \delta_h k_{h,t}$$

Similarly, for the big economy we have:

$$g_{f,ZL}b_{f,t+1} = (1+r_{h,t})b_{f,t} + y_{f,t} -_{f} c_{f,t} - \tau^{-1}\ _{h}c_{f,t} + \tau^{-1}\ _{f}c_{h,t} - \psi_{f,t} - \frac{(q_{f,t}-1)k_{f,t}}{\gamma} - \frac{(q_{f,t}-1)^2 k_{h,t}}{2\gamma} - \delta_f k_{f,t}$$

Each economy valuates its macroeconomic variables in terms of its relative prices. Therefore, to impose closure, the second thing we have to keep in mind is that debt must be measured in the same units. According to this, in the equilibrium it must be satisfied that:

$$ca_{h,t} = -\tau ca_{f,t}$$

$$y_{h,t} -_{h} c_{h,t} - \tau\ _{h}c_{f,t} + \tau\ _{f}c_{h,t} - \psi_{h,t} - \frac{(q_{h,t}-1)k_{h,t}}{\gamma} - \frac{(q_{h,t}-1)^2 k_{h,t}}{2\gamma} - \delta_h k_{h,t} =$$
$$-\tau\left[y_{f,t} -_{f} c_{f,t} - \tau^{-1}\ _{h}c_{f,t} + \tau^{-1}\ _{f}c_{h,t} - \psi_{f,t} - \frac{(q_{f,t}-1)k_{f,t}}{\gamma} - \frac{(q_{f,t}-1)^2 k_{f,t}}{2\gamma} - \delta_f k_{f,t} \right]$$

This equivalent to:

$$y_{h,t} - c_{h,t} + \tau\ _{h}c_{f,t} - \psi_{h,t} - \frac{(q_{h,t}-1)k_{h,t}}{\gamma} - \frac{(q_{h,t}-1)^2 k_{h,t}}{2\gamma} - \delta_h k_{h,t} =$$
$$-\tau y_{f,t} + \tau c_{f,t} -_{f} c_{h,t} + \tau\psi_{f,t} + \frac{\tau(q_{f,t}-1)k_{f,t}}{\gamma} + \frac{\tau(q_{f,t}-1)^2 k_{f,t}}{2\gamma} + \tau\delta_f k_{f,t}$$

Equilibrium must guaranty equality between small economy chosen debt (or positive foreign assets) and big economy chosen foreign assets (or debt), measured in the same value units this is:

$$b_{h,t} = -\tau b_{f,t}$$

It must also be satisfied:

$$r_{h,t} = r_{f,t} + r_s\left(\frac{b_{h,t}}{y_{h,t}}\right) + \varepsilon_{r,t} = r_{f,t} + r_s\left(\frac{-\tau b_{f,t}}{y_{f,t}}\right) + \varepsilon_{r,t}$$

This implies that:

$$b_{h,t+1} = \frac{(1+r_{h,t})b_{h,t} + ca_{h,t}}{g_{h,ZL}} = -\tau b_{f,t+1} = \frac{\tau(1+r_{h,t})b_{f,t} + \tau ca_{h,t}}{g_{f,ZL}}$$

The consequence of this is a foreign asset redundant equation, that is, in the equilibrium, foreign assets in a country is a perfect linear combination of the foreign asset in the other one. Therefore a single equation is necessary to capture the evolution of foreign assets. Thus, the dynamic system becomes: [18]

$$g_{hZL}k_{h,t+1} = (q_{h,t} - 1)\frac{k_{h,t}}{\gamma} + k_{h,t}$$

$$q_{h,t+1} = \frac{q_{h,t}}{E[\lambda_{h,t+1}]} - E\left\{Pmgk_{h,t+1} + \frac{\gamma}{2}\left(\frac{In_{h,t+1}}{k_{h,t+1}}\right)^2 - \delta_h\right\} - \text{cov}\{\lambda_{h,t+1}, V'(k_{h,t+1})\} + \text{cov}\{\lambda_{h,t+2}, V'(k_{h,t+2})\}$$

$$b_{h,t+1} = \frac{1}{g_{hZL}}\left[(1+r_{h,t})b_{h,t} + y_{h,t} - c_{h,t} + \tau_h c_{f,t} - \psi_t - \frac{(q_{h,t}-1)k_{h,t}}{\gamma} - \frac{(q_{h,t}-1)^2 k_{h,t}}{2\gamma} - \delta_h k_{h,t}\right]$$

$$u_h'(c_{h,t})g_{ZL} = \beta_h E\left[(1+r_{t+1}^h)\right]E\left[u_h'(c_{t+1})\right] + \beta_h \text{cov}\left[(1+r_{t+1}^h), u_h'(c_{h,t+1})\right]$$

$$g_{fZL}k_{f,t+1} = (q_{f,t} - 1)\frac{k_{f,t}}{\gamma} + k_{f,t}$$

$$q_{f,t+1} = \frac{q_{f,t}}{E[\lambda_{f,t+1}]} - E\left\{Pmgk_{f,t+1} + \frac{\gamma}{2}\left(\frac{In_{f,t+1}}{k_{f,t+1}}\right)^2 - \delta_f\right\} - \text{cov}\{\lambda_{f,t+1}, V'(k_{f,t+1})\} + \text{cov}\{\lambda_{f,t+2}, V'(k_{f,t+2})\}$$

$$u_f'(c_{f,t})g_{ZL} = \beta_f E\left[(1+r_{t+1}^h)\right]E\left[u_f'(c_{f,t+1})\right] + \beta_f \text{cov}\left[(1+r_{t+1}^h), u'(c_{f,t+1})\right]$$

[18] Otherwise, no elimination of such a redundant equation would cause the system to be not soluble.

It is expected then that the system has at least three stable roots as proposed by Blanchard y Kahn (1980).

4. Steady state and calibration

As usual in this subject literature, steady state will be imposed to calibrate de model. For the small economy, calibrated parameters by Gómez y Posada (2004) are to be used, and differently from them, terms of trade for small country (Colombia for the case) will differ from 1 which is a common value when economies are supposed to be very similar.

In steady state, besides meeting of first order and transition equations, it must be met that all effective labor variables remain constant, that is, it should happen that:

$$b_{h,t+1} = b_{h,t} = b_h$$
$$y_{h,t+1} = y_{h,t} = y_h$$
$$c_{h,t+1} = c_{h,t} = c_h$$
$$k_{h,t+1} = k_{h,t} = k_h$$
$$q_{h,t+1} = q_{h,t} = q_h$$

in the small economy and

$$b_{f,t+1} = b_{f,t} = b_f$$
$$y_{f,t+1} = y_{f,t} = y_f$$
$$c_{f,t+1} = c_{f,t} = c_f$$
$$k_{f,t+1} = k_{f,t} = k_f$$
$$q_{f,t+1} = q_{f,t} = q_f$$

in the big economy simultaneously.

Evaluating in the steady state we will have:

$$g_{hZL}k_h = (q_h - 1)\frac{k_h}{\gamma} + k_h$$

$$q_h = q_h(1+r_h) - \left\{ Pmgk_h + \frac{\gamma}{2}\left(\frac{In_h}{k_h}\right)^2 - \delta_h \right\}$$

$$b_h = \frac{1}{g_{hZL}}\left[(1+r_h)b_h + y_h - c_h + \tau_h c_f - \psi_h - \frac{(q_h-1)k_h}{\gamma} - \frac{(q_h-1)^2 k_h}{2\gamma} - \delta_h k_h \right]$$

$$u_h'(c_h)g_{ZL} = \beta_h[(1+r_h)][u_h'(c_h)]$$

$$g_{fZL}k_f = (q_f - 1)\frac{k_f}{\gamma} + k_f$$

$$q_f = q_f(1+r_f) - \left\{ Pmgk_f + \frac{\gamma}{2}\left(\frac{In_f}{k_f}\right)^2 - \delta_f \right\}$$

$$b_f = \frac{1}{g_{f,ZL}}\left[(1+r_h)b_f + y_f - c_f + \tau^{-1}{}_f c_h - \psi_f - \frac{(q_f-1)k_f}{\gamma} - \frac{(q_f-1)^2 k_f}{2\gamma} - \delta_f k_f \right]$$

$$u_f'(c_f)g_{ZL} = \beta_f[(1+r_h)][u_f'(c_f)]$$

For the big economy (the US in this case) parameters calibrated by Backus, Kehoe y Kydland (1992) will be used (see Table 1). We suppose here that the rest of the world remains unmodified, so we do not need to impose net exports equal cero. The US economy is supposed to be big economy in this calibration because it is the largest Colombia's trade partner.[19] It is important to note however, that such parameters are those of more general technology and preferences functions than the ones presented here: if $\mu = 1$ we will have CRRA preferences with ineslasticaly labor supply, and there it will be not relevant time evolution parameter of labor supply. Moreover, if $\sigma = 0$ we will have a traditional Cobb-

[19] Regarding this, Avella and Ferguson (2004) found not empirical business cycles correlation across Colombia-USA economies, but this result is questionable. USA economy explains almost 50% of Colombian exports.

Douglas production function. Remaining parameters will be re-calibrated where necessary. Tables 2 and 3 present parameter values and steady state variables for the world economy respectively.[20]

Table 1

Preferences: $\beta = 0.99$ **,** $\mu = 0.34$ **,** $\gamma = -1.0$ **,** $\alpha = 1$
Technology: $\theta = 0.36$ **,** $\nu = 3$ **,** $\sigma = 0.01$ **,** $\delta =$ **0.025,** $J = 4$
Technology shocks: $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & {}_{22} \end{bmatrix} = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix}$ $\operatorname{var} \varepsilon^h = \operatorname{var} \varepsilon^f = 0.00852^2$ **,** $corr(\varepsilon^h, \varepsilon^f) = 0.258$

Source: reproduced from Backus, Kehoe y Kydland (1992).

Table 2

Parameters	**Economy**	
	Small	**Large**
α	0.420	0.58
γ	2.000	2
θ	4.000	4.028512
δ	0.027	0.1
ρ	0.046	0.041020356
g_Z	0.015	0.016
g_L	0.022	0.01181465
r_f	0.040	0.04
r_1	-0.191	0
ψ	0.150	0.1
τ	0.665	1.503759398

[20] See appendix 3 for details about the calibration process.

Table 3

Steady state		
	Small economy	Large economy
Total wealth	6.0343	31.5864
Foreign assets	-0.7958	1.1967
Interest rate	0.1098	0.0400
k/y	2.9230	4.1002
f'(k)	0.1437	0.1415
f''(k)	-0.0131	-0.0021
f(k)	2.1743	7.0187
k	6.3555	28.7779
Tobin's Q	1.0747	1.0560
Net investment	0.2373	0.8059
Gross investment	0.4096	3.6837
Investment cost	0.0089	0.0226
Consumption	1.3720	2.7084
Debt service	-0.0874	0.1314
in/y	0.1091	0.1148
b/y	-0.3660	0.1705
re/y	-0.0402	0.0187
Gov. Expenditure	0.3261	0.7019
c/y	0.6310	0.3859
Net exports	0.0576	-0.0978
Foreign assets transition	0.0000	0.0000

CHAPTER III

MODEL DYNAMICS AND SIMULATION

Up to this phase of the work it is necessary to know the dynamic properties of the model before some kind of simulation is performed. Here we have to suppose that the world economy does not suffer any random shock, it is, the system is in a deterministic environment. Since the model dynamic simultaneous equations system is nonlinear, it is necessary to linearize it in order to get a numerical path solution and thus perform simulations. Using a first order Taylor approximation in the neighborhood of the steady state, and expressing it in terms of deviations from the steady state, we have that the linearized system is:

$$\widetilde{k}_{h,t+1} = \frac{1}{g_{hZL}}\left[(q_h - 1)\frac{1}{\gamma_h} + 1\right]\widetilde{k}_{h,t} + \left(\frac{k_h}{g_{hZL}\gamma_h}\right)\widetilde{q}_{h,t}$$

$$\widetilde{q}_{h,t+1} = -\left[q_h r_1 \alpha_h b_h k_h^{-\alpha_h - 1} + f_h''(k_h)\right]\widetilde{k}_{h,t+1} + \left[(1 + r_h)\right]\widetilde{q}_{h,t} + q_h r_1 k^{-\alpha_h}\widetilde{b}_{h,t+1} - \frac{1}{\gamma_h}(q_h - 1)\widetilde{q}_{h,t+1}$$

$$\widetilde{b}_{h,t+1} = \frac{1}{g_{hZL}}\left[f_h'(k_h) - 0.2 f_h'(k_h) - \frac{(q_h - 1)}{\gamma_h} - \frac{(q_h - 1)^2}{2\gamma_h} - \delta_h - \alpha_h r_1 b_h k_h^{-\alpha_h - 1}\right]\widetilde{k}_{h,t}$$
$$- \frac{1}{g_{hZL}}\left[\frac{k_h}{\gamma_h} + \frac{(q-1)k_h}{\gamma_h}\right]\widetilde{q}_{h,t} + \frac{1}{g_{hZL}}\left[(1 + r_h) + r_1 b_h k_h^{-\alpha_h}\right]\widetilde{b}_{h,t} - \frac{1}{g_{hZL}}\widetilde{c}_{h,t}$$

$$\widetilde{c}_{h,t+1} = \left[\frac{\beta^*(1 + r_h)}{g_{hZL}}\right]^{\frac{1}{\theta_h}}\widetilde{c}_{h,t} - \frac{c_h}{\theta_h}\left[\frac{\beta_h^*(1 + r_h)}{g_{hZL}}\right]^{\frac{1}{\theta_h} - 1}\frac{\alpha_h \beta_h^* r_1 b_h k_h^{-\alpha_h - 1}}{g_{hZL}}\widetilde{k}_{h,t+1}$$
$$+ \frac{c_h}{\theta_h}\left[\frac{\beta_h^*(1 + r_h)}{g_{hZL}}\right]^{\frac{1}{\theta_h} - 1}\frac{\beta_h^* r_1 k_h^{-\alpha_h}}{g_{hZL}}\widetilde{b}_{h,t+1}$$

$$\widetilde{k}_{f,t+1} = \frac{1}{g_{fZL}}\left[(q_f - 1)\frac{1}{\gamma_f} + 1\right]\widetilde{k}_{f,t} + \left(\frac{k_f}{g_{hZL}\gamma_f}\right)\widetilde{q}_{f,t}$$

$$\tilde{q}_{f,t+1} = -f_f''(k_f)\tilde{k}_{f,t+1} + \left[(1+r_f)\right]\tilde{q}_{f,t} - \frac{1}{\gamma_f}(q_f - 1)\tilde{q}_{f,t+1}$$

$$\tilde{c}_{f,t+1} = \left[\frac{\beta_f^*(1+r_h)}{g_{fZL}}\right]^{\frac{1}{\theta_f}}\tilde{c}_{f,t} - \frac{c_f}{\theta_f}\left[\frac{\beta_f^*(1+r_h)}{g_{fZL}}\right]^{\frac{1}{\theta_f}-1}\frac{\alpha_h\beta_f^* r_1 b_h k_h^{-\alpha_h-1}}{g_{fZL}}\tilde{k}_{h,t+1}$$
$$+\frac{c_f}{\theta_f}\left[\frac{\beta_f^*(1+r_f)}{g_{fZL}}\right]^{\frac{1}{\theta}-1}\frac{\beta_f^* r_1 k_h^{-\alpha_k}}{g_{fZL}}\tilde{b}_{h,t+1}$$

Putting this as matrix entries the system can be written as:

$$
\begin{bmatrix} \vec{c}_{h,t+1} \\ \tilde{c}_{f,t+1} \\ \tilde{q}_{h,t+1} \\ \tilde{q}_{f,t+1} \\ \tilde{k}_{h,t+1} \\ \tilde{k}_{f,t+1} \\ \tilde{b}_{h,t+1} \end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & 0 & -\frac{c_h}{\theta_h}\left[\frac{\beta_h^*(1+r_h)}{g_{hZL}}\right]^{\frac{1}{\theta_h}-1}\frac{\alpha_h\beta_h^* r_1 b_h k_h^{-\alpha_h-1}}{g_{hZL}} & 0 & \frac{c_h}{\theta_h}\left[\frac{\beta_h^*(1+r_h)}{g_{hZL}}\right]^{\frac{1}{\theta_h}-1}\frac{\beta_h^* r_1 k_h^{-\alpha_h}}{g_{hZL}} \\
0 & 0 & 0 & 0 & -\frac{c_f}{\theta_f}\left[\frac{\beta_f^*(1+r_h)}{g_{fZL}}\right]^{\frac{1}{\theta_f}-1}\frac{\alpha_h\beta_f^* r_1 b_h k_h^{-\alpha_h-1}}{g_{fZL}} & 0 & \frac{c_f}{\theta_f}\left[\frac{\beta_f^*(1+r_f)}{g_{fZL}}\right]^{\frac{1}{\theta}-1}\frac{\beta_f^* r_1 k_h^{-\alpha_k}}{g_{fZL}} \\
0 & 0 & -\frac{1}{\gamma_h}(q_h-1) & 0 & -\left[q_h r_1 \alpha_h b_h k_h^{-\alpha_h-1}+f_h''(k_h)\right] & 0 & q_h r_1 k^{-\alpha_h} \\
0 & 0 & 0 & -\frac{1}{\gamma_f}(q_f-1) & 0 & -f_f''(k_f) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} \vec{c}_{h,t+1} \\ \tilde{c}_{f,t+1} \\ \tilde{q}_{h,t+1} \\ \tilde{q}_{f,t+1} \\ \tilde{k}_{h,t+1} \\ \tilde{k}_{f,t+1} \\ \tilde{b}_{h,t+1} \end{bmatrix}
$$

$$
+\begin{bmatrix}
\left[\frac{\beta_h^*(1+r_h)}{g_{hZL}}\right]^{\frac{1}{\theta_h}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \left[\frac{\beta_f^*(1+r_h)}{g_{fZL}}\right]^{\frac{1}{\theta_f}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \left[(1+r_h)-\frac{1}{\gamma_h}(q_h-1)\right] & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \left[(1+r_f)-\frac{1}{\gamma_f}(q_f-1)\right] & 0 & 0 & 0 \\
0 & 0 & \left(\frac{k_h}{g_{hZL}\gamma_h}\right) & 0 & \frac{1}{g_{hZL}}\left[(q_h-1)\frac{1}{\gamma_h}+1\right] & 0 & 0 \\
0 & 0 & 0 & \left(\frac{k_f}{g_{hZL}\gamma_f}\right) & 0 & \frac{1}{g_{fZL}}\left[(q_f-1)\frac{1}{\gamma_f}+1\right] & 0 \\
-\frac{1}{g_{hZL}} & 0 & -\frac{1}{g_{hZL}}\left[\frac{k_h}{\gamma_h}+\frac{(q-1)k_h}{\gamma_h}\right] & 0 & \Delta_1 & 0 & \frac{1}{g_{hZL}}\left[(1+r_h)+r_1 b_h k_h^{-\alpha_h}\right]
\end{bmatrix}
\begin{bmatrix} \vec{c}_{h,t} \\ \tilde{c}_{f,t} \\ \tilde{q}_{h,t} \\ \tilde{q}_{f,t} \\ \tilde{k}_{h,t} \\ \tilde{k}_{f,t} \\ \tilde{b}_{h,t} \end{bmatrix}
$$

$$\eta_{1,1} = -\frac{c_h}{\theta_h}\left[\frac{\beta_h^*(1+r_h)}{g_{hZL}}\right]^{\frac{1}{\theta_h}-1} \frac{\alpha_h \beta_h^* r_1 b_h k_h^{-\alpha_h - 1}}{g_{hZL}}$$

$$\eta_{1,2} = \frac{c_h}{\theta_h}\left[\frac{\beta_h^*(1+r_h)}{g_{hZL}}\right]^{\frac{1}{\theta_h}-1} \frac{\beta_h^* r_1 k_h^{-\alpha_h}}{g_{hZL}}$$

$$\eta_{1,3} = -\frac{c_f}{\theta_f}\left[\frac{\beta_f^*(1+r_h)}{g_{fZL}}\right]^{\frac{1}{\theta_f}-1} \frac{\alpha_h \beta_f^* r_1 b_h k_h^{-\alpha_h - 1}}{g_{fZL}}$$

$$\eta_{1,4} = \frac{c_f}{\theta_f}\left[\frac{\beta_f^*(1+r_f)}{g_{fZL}}\right]^{\frac{1}{\theta}-1} \frac{\beta_f^* r_1 k_h^{-\alpha_k}}{g_{fZL}}$$

$$\eta_{1,5} = -\left[q_h r_1 \alpha_h b_h k_h^{-\alpha_h - 1} + f_h''(k_h)\right]$$

$$\eta_{1,6} = q_h r_1 k^{-\alpha_h}$$

$$\eta_{1,7} = -f_f''(k_f)$$

$$\eta_{1,8} = -\frac{1}{\gamma_h}(q_h - 1)$$

$$\eta_{1,9} = -\frac{1}{\gamma_f}(q_f - 1)$$

$$\eta_{2,1} = \left[\frac{\beta_h^*(1+r_h)}{g_{hZL}}\right]^{\frac{1}{\theta_h}}$$

$$\eta_{2,2} = \left[\frac{\beta_f^*(1+r_h)}{g_{fZL}}\right]^{\frac{1}{\theta_f}}$$

$$\eta_{2,3} = \left[(1+r_h)\right]$$

$$\eta_{2,4} = \left[(1+r_f)\right]$$

$$\eta_{2,5} = \left(\frac{k_h}{g_{hZL}\gamma_h}\right)$$

$$\eta_{2,6} = \frac{1}{g_{hZL}}\left[(q_h - 1)\frac{1}{\gamma_h} + 1\right]$$

$$\eta_{2,7} = \left(\frac{k_f}{g_{hZL}\gamma_f}\right)$$

$$\eta_{2,8} = \frac{1}{g_{fZL}}\left[(q_f - 1)\frac{1}{\gamma_f} + 1\right]$$

$$\eta_{2,9} = -\frac{1}{g_{hZL}}\left[\frac{k_h}{\gamma_h} + \frac{(q-1)k_h}{\gamma_h}\right]$$

$$\eta_{2,10} = \frac{1}{g_{hZL}}\left[f_h'(k_h) - 0.2 f_h'(k_h) - \frac{(q_h - 1)}{\gamma_h} - \frac{(q_h - 1)^2}{2\gamma_h} - \delta_h - \alpha_h r_1 b_h k_h^{-\alpha_h - 1}\right] = \Delta$$

$$\eta_{2,11} = \frac{1}{g_{hZL}}\left[(1 + r_h) + r_1 b_h k_h^{-\alpha_h}\right]$$

$$\eta_{2,12} = -\frac{1}{g_{hZL}}$$

$$\begin{bmatrix} \tilde{c}_{h,t+1} \\ \tilde{c}_{f,t+1} \\ \tilde{q}_{h,t+1} \\ \tilde{q}_{f,t+1} \\ \tilde{k}_{h,t+1} \\ \tilde{k}_{f,t+1} \\ \tilde{b}_{h,t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & \eta_{1,1} & 0 & \eta_{1,2} \\ 0 & 0 & 0 & 0 & \eta_{1,3} & 0 & \eta_{1,4} \\ 0 & 0 & \eta_{1,8} & 0 & \eta_{1,5} & 0 & \eta_{1,6} \\ 0 & 0 & 0 & \eta_{1,9} & 0 & \eta_{1,7} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{c}_{h,t+1} \\ \tilde{c}_{f,t+1} \\ \tilde{q}_{h,t+1} \\ \tilde{q}_{f,t+1} \\ \tilde{k}_{h,t+1} \\ \tilde{k}_{f,t+1} \\ \tilde{b}_{h,t+1} \end{bmatrix} + \begin{bmatrix} \eta_{2,1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \eta_{2,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \eta_{2,3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \eta_{2,4} & 0 & 0 & 0 \\ 0 & 0 & \eta_{2,5} & 0 & \eta_{2,6} & 0 & 0 \\ 0 & 0 & 0 & \eta_{2,7} & 0 & \eta_{2,8} & 0 \\ \eta_{2,12} & 0 & \eta_{2,9} & 0 & \eta_{2,10} & 0 & \eta_{2,11} \end{bmatrix} \begin{bmatrix} \tilde{c}_{h,t} \\ \tilde{c}_{f,t} \\ \tilde{q}_{h,t} \\ \tilde{q}_{f,t} \\ \tilde{k}_{h,t} \\ \tilde{k}_{f,t} \\ \tilde{b}_{h,t} \end{bmatrix}$$

In a more compact form

$$\tilde{X}_{t+1} = \Gamma_1 \tilde{X}_{t+1} + \Gamma_2 \tilde{X}_t$$

$$\tilde{X}_{t+1} - \Gamma_1 \tilde{X}_{t+1} = \Gamma_2 \tilde{X}_t$$

$$[I - \Gamma_1]\tilde{X}_{t+1} = \Gamma_2 \tilde{X}_t$$

$$\tilde{X}_{t+1} = [I - \Gamma_1]^{-1}\Gamma_2 \tilde{X}_t$$

The matrix $[I - \Gamma_1]^{-1}$ does exist because $[I - \Gamma_1]$ is an upper triangular matrix, and thus the model becomes:

$\tilde{X}_{t+1} = \Omega \tilde{X}_t$.

Being $\Omega = [I - \Gamma_1]^{-1} \Gamma_2$.

In Appendix 2 it is shown that the dynamic system has at least one characteristic unit root meaning that as we have expressed the time evolution of the variables in terms of deviations from the steady state level, when the world economy is moved away from that state, there will be a steady state change, a structural change, and a convergence process to the new long run values of variables. This result and what was said in chapter II mean that to the model is locally stable, it is need that Eigen values are such that $\lambda_{1,2,3} > 1$, $\lambda_{4,5,6} < 1$ and $\lambda_7 = 1$.

The result shown above is very appealing and strong: once unstable paths are eliminated, the change in the steady state values of the variables in the world economy and their respective adjust process, the change can be such that one of the economies can become poorer while the other one is getting richer, depending on the type or origin of the shock.

Because Eigen values do exist, matrix Ω can be diagonalized as $\Omega = P \Lambda Q$, being P Eigen vectors matrix, Λ diagonal matrix containing Eigen values and $Q = P^{-1}$.

Then time path of deviations from steady state is given by $\tilde{X}_{t+1} = P \Lambda Q \tilde{X}_t$ and for an initial vector of deviations it can be found a particular solution path:

$$\tilde{X}_t = P \Lambda^t Q \tilde{X}_0$$

$$\Lambda^t = \begin{bmatrix} \lambda_1^t & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2^t & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3^t & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_4^t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_5^t & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_6^t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda_7^t \end{bmatrix}$$

Table 4 shows numeric Eigen values corresponding to parameters and calibration shown in tables 2 and 3 respectively. As expected, the model has at least one unit root (neutrally stable), tree stable roots and tree unstable roots.

Table 4

Root	Value
λ_1	1.8899751
λ_2	1.1887240
λ_3	1.0871118
λ_4	1.0000000
λ_5	0.96631133
λ_6	0.85105503
λ_7	0.61271370

The form of the system once the unstable roots are eliminated is:

$$\begin{bmatrix} \vec{c}_{h,t+1} \\ \widetilde{c}_{f,t+1} \\ \widetilde{q}_{h,t+1} \\ \widetilde{q}_{f,t+1} \\ \widetilde{k}_{h,t+1} \\ \widetilde{k}_{f,t+1} \\ \widetilde{b}_{h,t+1} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} & s_{17} \\ s_{21} & s_{22} & s_{23} & s_{24} & s_{25} & s_{26} & s_{27} \\ s_{31} & s_{32} & s_{33} & s_{34} & s_{35} & s_{36} & s_{37} \\ s_{41} & s_{42} & s_{43} & s_{45} & s_{45} & s_{46} & s_{47} \\ s_{51} & s_{52} & s_{53} & s_{54} & s_{55} & s_{56} & s_{57} \\ s_{61} & s_{62} & s_{63} & s_{64} & s_{65} & s_{66} & s_{67} \\ s_{71} & s_{72} & s_{73} & s_{74} & s_{75} & s_{76} & s_{77} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_4^t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_5^t & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_6^t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda_7^t \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \\ n_7 \end{bmatrix}$$

Being

$$\begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \\ n_7 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} & s_{17} \\ s_{21} & s_{22} & s_{23} & s_{24} & s_{25} & s_{26} & s_{27} \\ s_{31} & s_{32} & s_{33} & s_{34} & s_{35} & s_{36} & s_{37} \\ s_{41} & s_{42} & s_{43} & s_{45} & s_{45} & s_{46} & s_{47} \\ s_{51} & s_{52} & s_{53} & s_{54} & s_{55} & s_{56} & s_{57} \\ s_{61} & s_{62} & s_{63} & s_{64} & s_{65} & s_{66} & s_{67} \\ s_{71} & s_{72} & s_{73} & s_{74} & s_{75} & s_{76} & s_{77} \end{bmatrix}^{-1} \begin{bmatrix} \vec{c}_{h,0} \\ \widetilde{c}_{f,0} \\ \widetilde{q}_{h,0} \\ \widetilde{q}_{f,0} \\ \widetilde{k}_{h,0} \\ \widetilde{k}_{f,0} \\ \widetilde{b}_{h,0} \end{bmatrix}$$

Time deviations from the steady state for the world economy variables are:

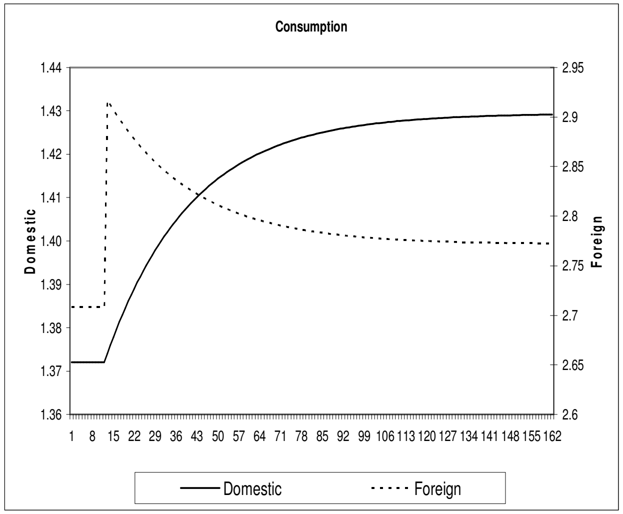
$$\begin{bmatrix} \vec{c}_{h,t+1} \\ \widetilde{c}_{f,t+1} \\ \widetilde{q}_{h,t+1} \\ \widetilde{q}_{f,t+1} \\ \widetilde{k}_{h,t+1} \\ \widetilde{k}_{f,t+1} \\ \widetilde{b}_{h,t+1} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} & s_{17} \\ s_{21} & s_{22} & s_{23} & s_{24} & s_{25} & s_{26} & s_{27} \\ s_{31} & s_{32} & s_{33} & s_{34} & s_{35} & s_{36} & s_{37} \\ s_{41} & s_{42} & s_{43} & s_{45} & s_{45} & s_{46} & s_{47} \\ s_{51} & s_{52} & s_{53} & s_{54} & s_{55} & s_{56} & s_{57} \\ s_{61} & s_{62} & s_{63} & s_{64} & s_{65} & s_{66} & s_{67} \\ s_{71} & s_{72} & s_{73} & s_{74} & s_{75} & s_{76} & s_{77} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \lambda_4^t n_4 \\ \lambda_5^t n_5 \\ \lambda_6^t n_6 \\ \lambda_7^t n_7 \end{bmatrix}$$

In this fashion we are left seven equations and four unknowns to be solved for: $\eta_4, \eta_5, \eta_6, \eta_7$.

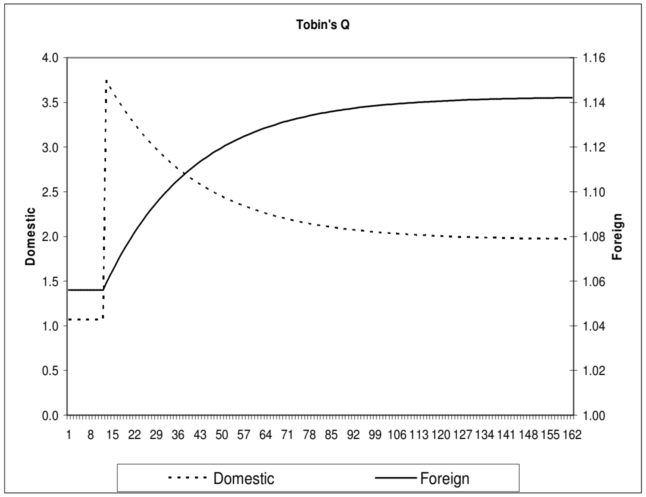
A first simulation exercise was to move the world economy from its steady state accordingly to the facts observed in 1994 in the Colombian economy: an increase in Colombian foreign asset, an increase in private consumption and in physical capital and a private consumption increase. This was the last year a high per capita GDP growth rate was observed in Colombia. Graphics 4 to 8 show increases suffered by private consumption, physical capital and foreign assets for the simulated small economy. When convergence process is finished, the economy has a structural change and also capital stocks did so although the change is almost imperceptible.

The result from this boom in consumption and investment is a higher foreign debt in the small economy (a lower foreign asset value) and the consequent higher interest rates for the small economy what means an important burden of foreign debt service, this cause a decrease in private consumption growth rate. Increases in private consumption and physical capital explain net exports result (Graphic 9) and, as Graphic 7 shows capital in big economy decreases because families in that economy allocate resources to accumulate (positive) foreign assets, lending to the families and firms in the small economy. As it was expected, simulated time paths for these economies exhibit long run structural change. What is important and remarkable from this result, is the fact that the change in steady state values do not correspond to shifts in parameter values, but the “natural” answer, an endogenous one, to the initial alteration in consumption and accumulation decisions. Alteration simulated in this case was a deviation from steady state of capital and foreign assets in Colombia, supposing that they were in steady state and those growth rates observed in 1994 give the value of the gap values in that moment.

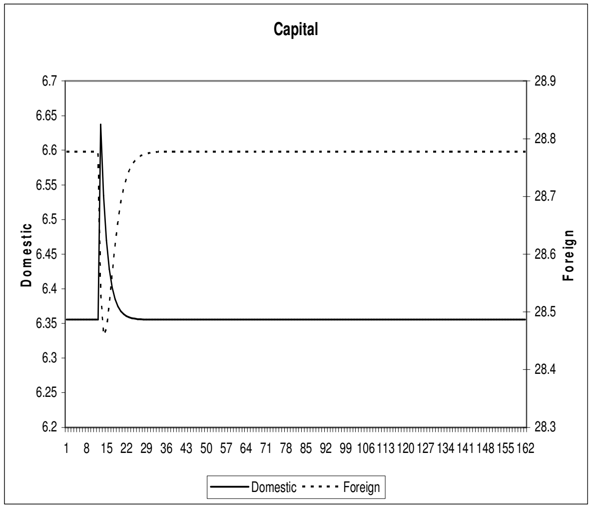
Graphic 5



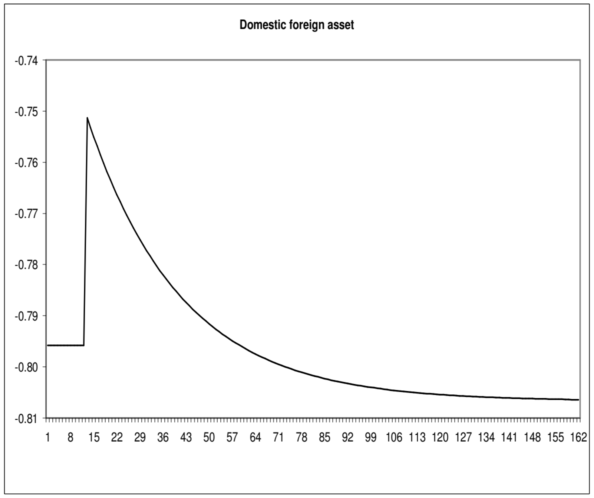
Graphic 6



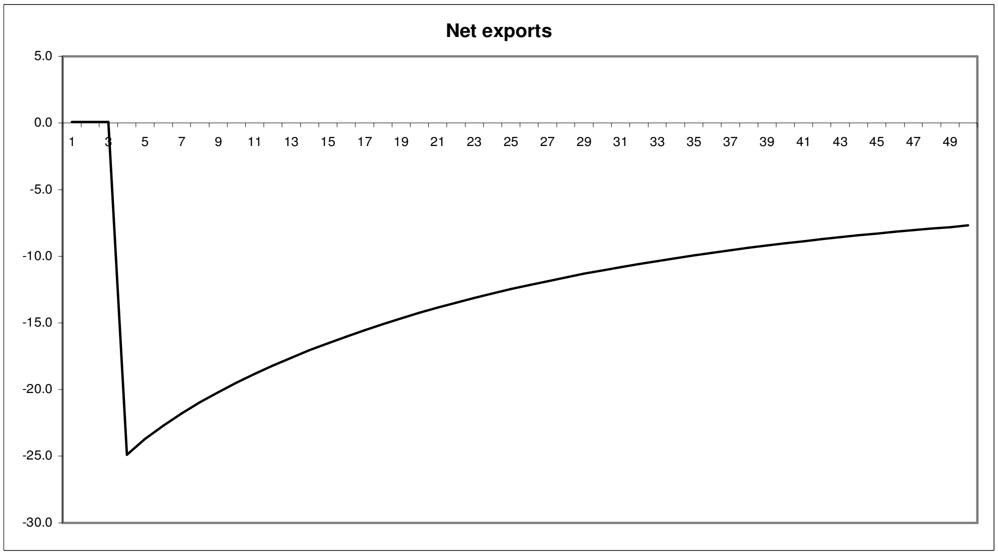
Graphic 7



Graphic 8



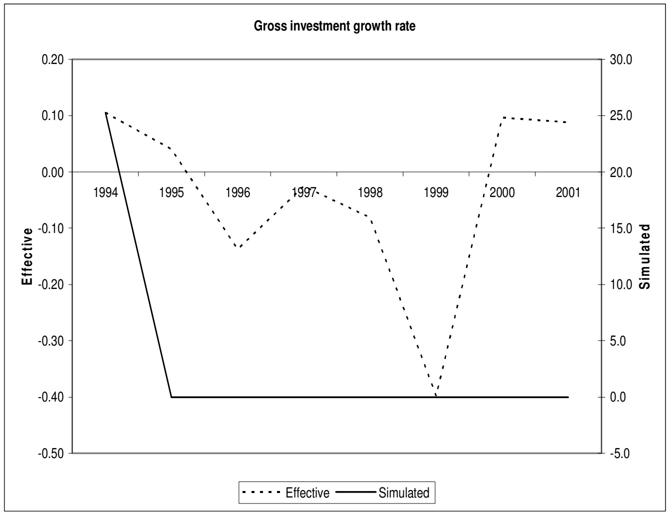
Graphic 9



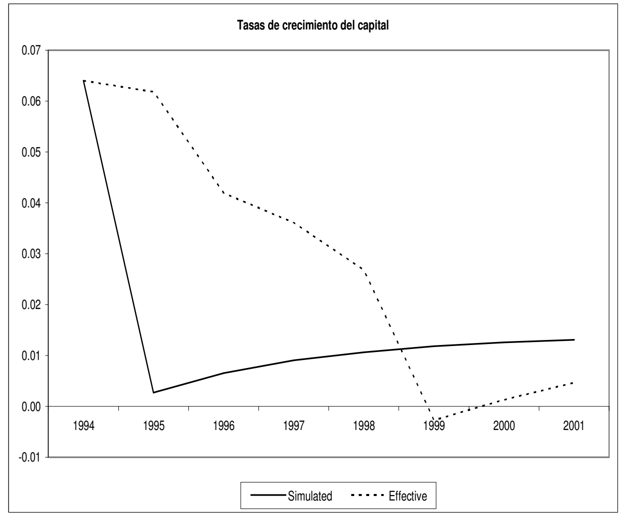
Graphics 10 to 14 show growth rates for the main small economy variables predicted by the model and the effective ones.[21] For consumption and output, results are similar to those of Gómez and Posada (2004). The model economy shows growth rates along the adjustment path, while it appears that the real economy fluctuates around them, however a new shock not foreseen in the model occurred in 1999 that made so severe the adjustment that highest depression in Colombian economic history became. Investment growth predicted was huge compared to the effective one, as we said before, explained by the excessive response of Tobin's Q. Capital growth rate fits not very well, at least during four years after the shock, it seems that after fifth year the real economy could converge behave as the model economy. Finally, for foreign assets growth rate, graphic 14 shows that the real economy has a growth rate higher than that of the predicted by the model. Although the model does not fit very well the empirical economy values, it is at least capable to show the direction of the movements and the medium-long term tendencies.

[21] This kind of comparison is also made by Gómez and Posada (2004).

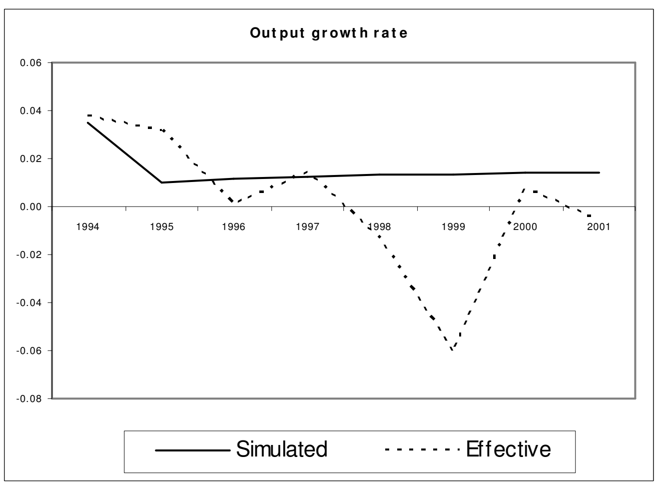
Graphic 10



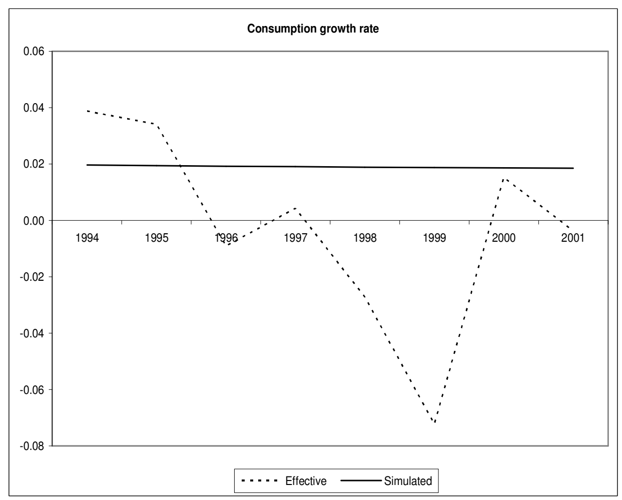
Graphic 12



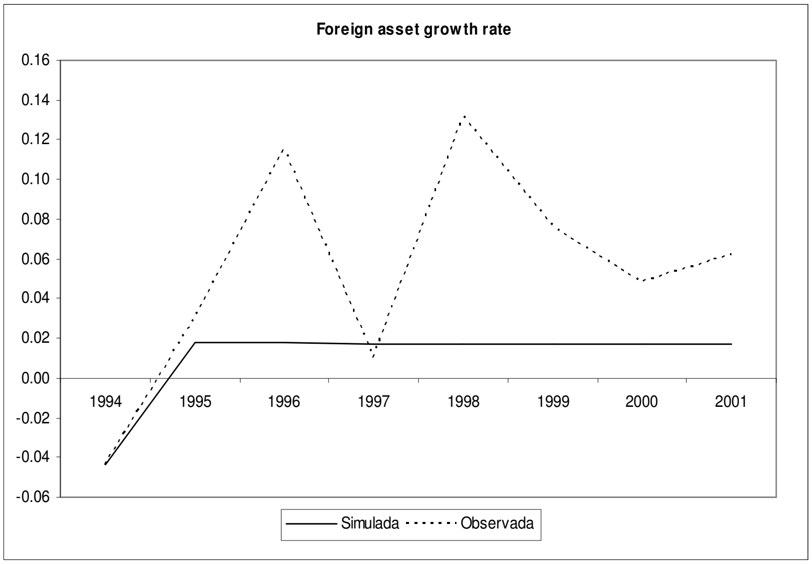
Graphic 11



Graphic 13



Graphic 14

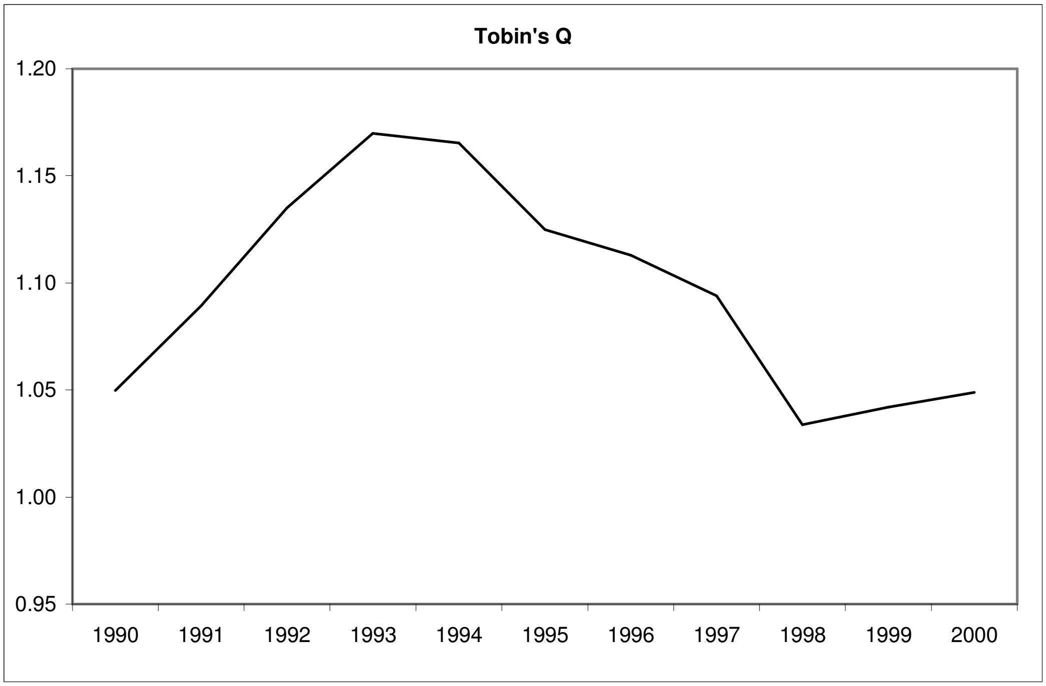


What is surprising is the Tobin's Q reaction for all economies and mostly for the Colombian economy. This excessive reaction in the price of capital is the explanation of such an increase in investment growth rates and the consequent net exports result. Graphic 15 shows the evolution of Tobin's Q, calculated from transition equation for physical capital in the small economy. It is very clear that the model over-reacts to the deviation of capital and foreign assets from their steady state. As a matter of fact, while simulated small economy Tobin's Q deviation from steady state is 2.67 (248.4% of the long run value), what is really observed for the empirical economy is a deviation of 0.091(8.46% of the long run value)

A second simulation was made,[22] but this time, including the deviation suffered by private consumption, that is, there were three variables initially deviated from the steady state: consumption, capital and foreign assets. The result was almost the same as in the first simulation; the only new thing is that consumption growth rate in the model economy fits a little better, at least for the first year, the empirical growth rate (see Graphics 16 to 20).

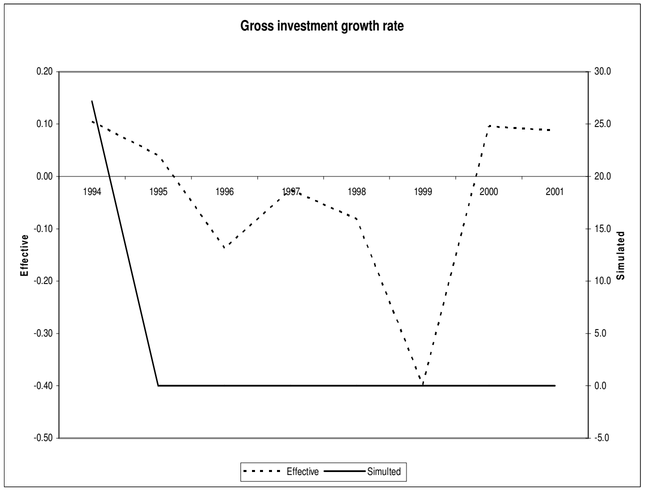
[22] Graphics not reported.

Graphic 15

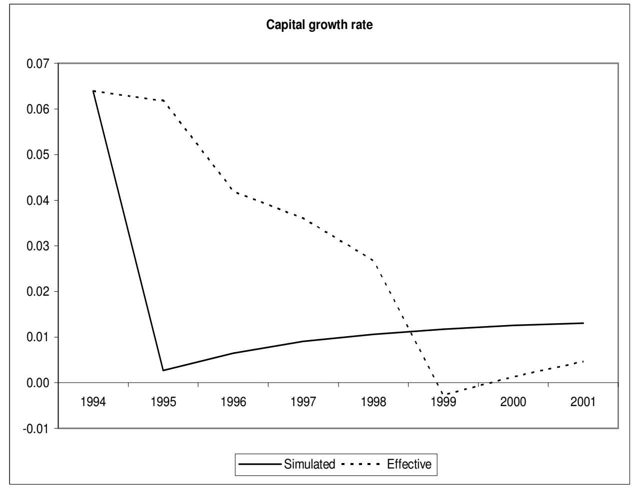


Tobin's Q
1.20
1.15
1.10
1.05
1.00
0.95
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000

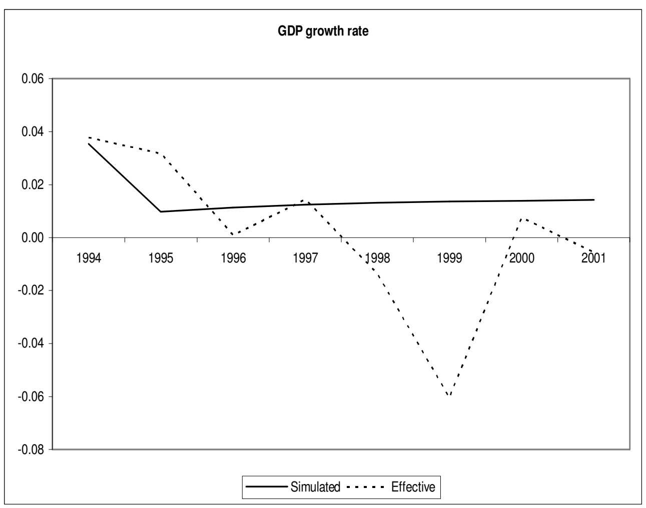
Graphic 16



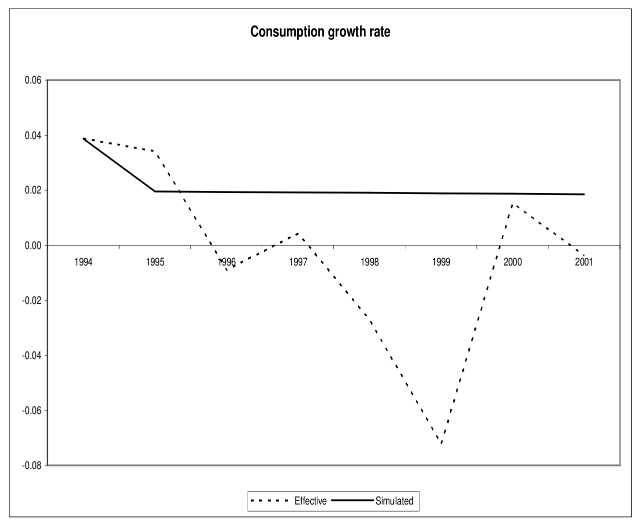
Graphic 18



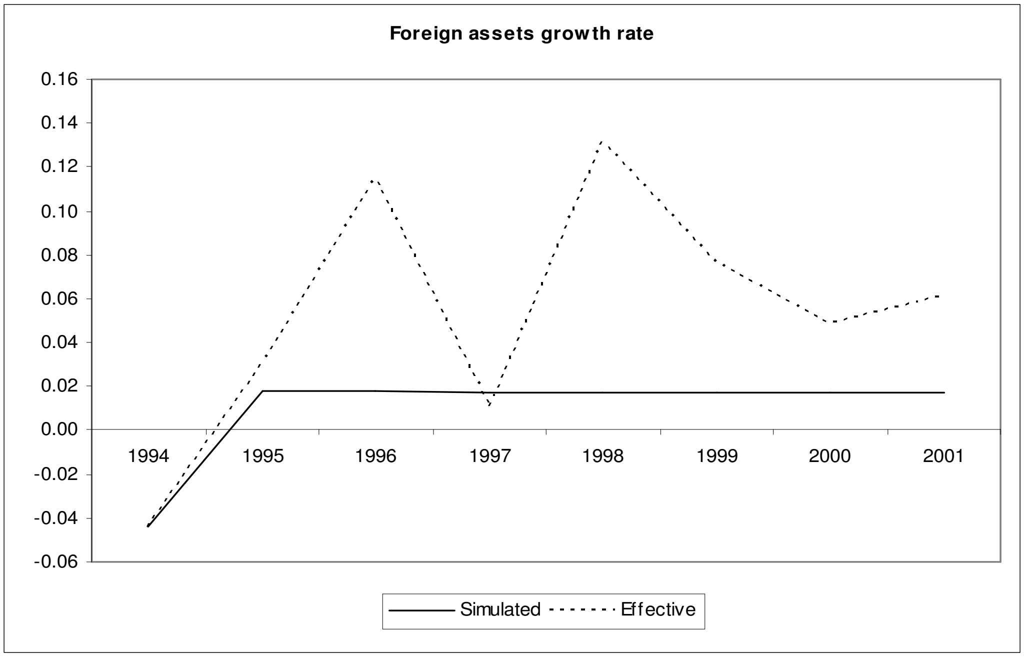
Graphic 17



Graphic 19



Graphic 20


Foreign assets growth rate
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
1994
1995
1996
1997
1998
1999
2000
2001
Simulated
Effective

CHAPTER IV

CONCLUSIONS

The theme for this thesis was inspired in empirical observations about several facts observed for Latin American countries and especially for Colombia from the early eighties up to the end of the twentieth century. Those facts are: long run decreasing real GDP growth rates, increases in domestic interest rates, increasing foreign debt spreads, increasing differential interest rates across countries, mostly between small and open economies, and deterioration of trade balance.

In this thesis, a two country model was built to capture a special type of asymmetry between a small economy and a big economy: imperfect capital mobility induced spread on the world interest rate. This margin is a function of two elements: the foreign asset relative to GDP, and random disturbances which reflects rest of world perception about the economy. Thus, the big economy is characterized for financing investment decisions restricted to the world risk free interest rate, while saving and foreign assets decision are affected by interest rate payable by the small economy, if big economy is net creditor. However, all decisions in the small economy are affected by inclusive spread interest rate, which is higher than the risk free interest rate, if small economy is net debtor.

It is analytically proved that the linearized version of the theoretical dynamic world economy has at least one unit root. This means that the model is able to reproduce a structural change as an endogenous answer to perturbations on main stock and control variables as well as to perturbations to home and foreign technology parameter, and to confidence in the international capital market.

A simulation exercise performed for the facts observed in 1994 for the Colombian economy shows that the model fits some of the behaviors the main variables exhibited. The model is able to predict medium term dynamics but it is not able to fit the whole short run very well. However, a virtue of the model is the possibility to reproduce an increase in long run levels.

In the simulated case, there were increases in the long run consumption levels in both economies, a decrease in long run value of foreign assets of small economy (became more indebted) and changes in physical capital stocks, although this change was almost imperceptible. Net exports did also experience such a structural chance. It is expected therefore, that an increase in foreign debt could replicate an opposite scenario as the observed and the simulated one, inducing the small economy to poorer having to pay for increasing debt by reducing consumption and capital long run levels. Other regularity the model can reproduce is the explosive response of investment, although in this case it was excessive mostly explained by the huge reaction of capital prices as commented above.

Four simulation exercises are left to be done and will be developed for later papers: shocks to terms of trade, shocks to spread throughout the perturbation term, and technology shocks.

A policy recommendation can be derived from these findings: great effort must be done to reduce uncertainty about the Colombian economy (to dampen the perturbation term effect), and to reduce public and private foreign indebtedness, thus, interest rate burden will be smaller and so costs of recovering.

References

Avella, Mauricio. And Fergusson, Leopoldo. (2004). El ciclo económico, enfoques e ilustraciones. Los ciclos económicos de Estados Unidos y Colombia.

Backus, David K., Patrick J. Kehoe, and Finn E. Kydland (1992). *International Real Business cycles***.** Journal of Political Economy. Vol. 100, no 4.

Backus, David K., Patrick J. Kehoe, and Finn E. Kydland (1994). *Dynamics of the trade balance and the terms of trade: The J-curve?* The American Economic Review. March. Vol. 84 No.1.

Barro, Robert. Xavier Sala-i-Martin. (1995). *Economic growth.* McGraw-Hill.

Basu, Susanto. Taylor, Alan M. (1999). Business cycles in international historical perspective. NBER wp 7090.

Baxter, Marianne (1995). International trade and business cycles. NBER wp 5025.

Blanchard, O. Kahn, C. (1980). The solution of linear difference models under rational expectarions. *Econometrica vol 48, No. 5.*

Berk,-Jan-Marc. (1997). Flows as a Channel for the Transmission of Business Cycles. Banca-Nazionale-del-Lavoro-Quarterly-Review; 50(201), June 1997, pages 187-212.

Cardia, Emanuela. (1991). The dynamics of a small open economy in response to monetary, fiscal, and productivity shocks. *Journal of Monetary Economics 28.*

Chyi,-Yih-Luan. (1998) Business Cycles between Developed and Developing Economies, Review-of-International-Economics; 6(1), February 1998, pages 90-104.

Cooley, T. Prescott, E. (1995). Frontiers of business cycles research. Cooley, T. Editor. Princeton University Press.

Correia, I. Neves, J. Revelo, S. (1995). Business cycles in a small open economy. European Economic Review, No. 39 p. 1089-1113.

Elliott,-Graham; Fatas,-Antonio. (1995). International Business Cycles and the Dynamics of the Current Account. University of California San Diego; INSEAD. Centre for Economic Policy Research, Discussion Paper: 1280, November 1995, pages 36.

Fabrizio,-Stefania; Lopez,-J.-Humberto (1996). Domestic, Foreign or Common Shocks? International Monetary Fund Working Paper: WP/96/107, September 1996, pages 14.

Gómez, Wilman y Posada, Carlos E. (2004). Un “choque” del activo externo neto y el ciclo económico colombiano 1994-2001. Borradores de economía No. 285, Banco de la República. Forthcoming in Lecturas de Economía No. 62.

Greco (2002) . El crecimiento económico colombiano en el siglo XX. Banco de la República-Fondo de Cultura Económica.

Hamann, F. Riascos, A. (1998). Ciclos económicos en una economía pequeña y abierta: una aplicación para Colombia. Borradores de Economía No. 89. Banco de la República.

King, R. Plosser, C. Rebelo, S. (1988). Production, growth and business cycles. I. the Basic Neoclassical model. *Journal of Monetary Economics 21.*

Kydland, Finn E; and Prescott, Edward C. (1982). “Time to build and aggregate fluctuations”. *Econometrica*. Vol. 50, November, No. 6.

Long, John B; and Plosser, Charles I. (1983). “Real business cycles”. *Journal of Political Economy*. Vol. 91 No.1.

Mendoza, Enrique G. (1991). Real business cycles in a small open Economy. The American Economic Review, September. Vol. 81 No.4

Obstfeld; Maurice. And Rogoff, Kenneth. (1999). Foundations of internatinal macroeconomics. The MIT Press.

Posada, Carlos E. Gómez, Wilman. (2002). Crecimiento económico y gasto público: un modelo para el caso colombiano. Borradores de Economía No. 218. Banco de la República.

Rowland, Peter, and José L. Torres (2004), “Determinants of Spread and Creditworthiness for Emerging Market Sovereign Debt: A Panel Data Study”, Borradores de Economía, Banco de la República, Bogotá.

Sala-i-Martin, Xavier. (2000). Apuntes de crecimiento económico, 02 ed. Antoni Bosch Editor.

Zimmermann, Christian. (1995). International real business cycles among heterogeneous countries. Research center on employment and economic fluctuations, Université du Québec a Montréal.

Appendix 1

Deriving transition equations for foreign assets

Aggregated current account for an economy can be written as:

$$CA_{h,t} = Y_{h,t} - {}_hC_{h,t} - \tau_h C_{f,t} + \tau_f C_{h,t} - \Psi_{h,t} - I_{h,t} - C(In_{h,t}) - \delta_h K_{h,t}$$

Thus, as foreign aggregated assets evolve as:

$$B_{h,t+1} = B_{h,t}(1 + r_{h,r}) + CA_{h,t}$$

We write:

$$B_{h,t+1} = B_{h,t}(1 + r_{h,r}) + Y_{h,t} - {}_hC_{h,t} - \tau_h C_{f,t} + \tau_f C_{h,t} - \Psi_{h,t} - I_{h,t} - C(In_{h,t}) - \delta_h K_{h,t}$$

Now, it is necessary to express previous equation in term of effective labor:

$$\frac{B_{h,t+1}}{Z_{h,t}L_{h,t}} = \frac{B_{h,t}(1 + r_{h,r}) + Y_{h,t} - {}_hC_{h,t} - \tau_h C_{f,t} + \tau_f C_{h,t} - \Psi_{h,t} - I_{h,t} - C(In_{h,t}) - \delta_h K_{h,t}}{Z_{h,t}L_{h,t}}$$

Keeping in mind that $Z_{h,t+1}L_{h,t+1} = g_{hZL} Z_{h,t}L_{h,t}$ previous expression can be transformed:

$$\frac{g_{hZL}B_{h,t+1}}{Z_{h,t+1}L_{h,t+1}} = b_{h,t}(1 + r_{h,r}) + y_{h,t} - {}_hc_{h,t} - \tau_h c_{f,t} + \tau_f c_{h,t} - \psi_{h,t} - i_{h,t} - \frac{C(In_{h,t})}{Z_{h,t}L_{h,t}} - \delta_h k_{h,t}$$

$$g_{hZL}b_{h,t+1} = b_{h,t}(1 + r_{h,r}) + y_{h,t} - {}_hc_{h,t} - \tau_h c_{f,t} + \tau_f c_{h,t} - \psi_{h,t} - i_{h,t} - \frac{C(In_{h,t})}{Z_{h,t}L_{h,t}} - \delta_h k_{h,t}$$

Now, we can see that time evolution of foreign asset is:

$$g_{hZL}b_{h,t+1} = b_{h,t}(1 + r_{h,t}) + ca_{h,t} = b_{h,t}(1 + r_{h,t}) + y_{h,t} - {}_hc_{h,t} - \tau_h c_{f,t} + \tau_f c_{h,t} - \psi_{h,t} - in_{h,t} - \frac{C(In_{h,t})}{Z_{h,t}L_{h,t}} - \delta_h k_{h,t}$$

Recall that in equilibrium net investment has the form

$In_{h,t} = (q_{h,t} - 1)\frac{K_{h,t}}{\gamma_h}$, expressing this equation in terms of effective labor, is quite easy, just divide both sides of the expression by $Z_{h,t}L_{h,t}$ and we will have: $in_{h,t} = (q_{h,t} - 1)\frac{k_{h,t}}{\gamma_h}$. What can appear some little troublesome is the expression costs of adjustment. However, keeping the imposition of equilibrium, costs of investment adjustment can be written as:

$$\frac{C(In_{h,t})}{Z_{h,t}L_{h,t}} = \frac{\frac{\gamma}{2}\frac{(In_{h,t})^2}{K_{h,t}}}{Z_{h,t}L_{h,t}} = \frac{\frac{\gamma}{2}\frac{\left((q_{h,t}-1)\frac{K_{h,t}}{\gamma_h}\right)^2}{K_{h,t}}}{Z_{h,t}L_{h,t}} = \frac{\frac{\gamma}{2}\frac{(q_{h,t}-1)^2\frac{K_{h,t}^2}{\gamma_h^2}}{K_{h,t}}}{Z_{h,t}L_{h,t}} = \frac{\frac{(q_{h,t}-1)^2 K_{h,t}}{2\gamma_h}}{Z_{h,t}L_{h,t}} = \frac{(q_{h,t}-1)^2 k_{h,t}}{2\gamma_h}$$

Finally, we have the whole expression for foreign assets:

$$g_{hZL}b_{h,t+1} = b_{h,t}(1+r_{h,t}) + ca_{h,t} = b_{h,t}(1+r_{h,t}) + y_{h,t} - {}_h c_{h,t} - \tau_h c_{f,t} + \tau_f c_{h,t} - \psi_{h,t} - in_{h,t} - \frac{(q_{h,t}-1)^2 k_{h,t}}{2\gamma_h} - \delta_h k_{h,t}$$

Working in the same fashion, equation transition for foreign assets in big country is obtained.

Appendix 2

Searching for characteristic roots in the dynamic model.

The full linearized model can be written as

$$\begin{bmatrix} \tilde{c}_{h,t+1} \\ \tilde{c}_{f,t+1} \\ \tilde{q}_{h,t+1} \\ \tilde{q}_{f,t+1} \\ \tilde{k}_{h,t+1} \\ \tilde{k}_{f,t+1} \\ \tilde{b}_{h,t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & \eta_{1,1} & 0 & \eta_{1,2} \\ 0 & 0 & 0 & 0 & \eta_{1,3} & 0 & \eta_{1,4} \\ 0 & 0 & \eta_{1,8} & 0 & \eta_{1,5} & 0 & \eta_{1,6} \\ 0 & 0 & 0 & \eta_{1,9} & 0 & \eta_{1,7} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{c}_{h,t+1} \\ \tilde{c}_{f,t+1} \\ \tilde{q}_{h,t+1} \\ \tilde{q}_{f,t+1} \\ \tilde{k}_{h,t+1} \\ \tilde{k}_{f,t+1} \\ \tilde{b}_{h,t+1} \end{bmatrix} + \begin{bmatrix} \eta_{2,1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \eta_{2,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \eta_{2,3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \eta_{2,4} & 0 & 0 & 0 \\ 0 & 0 & \eta_{2,5} & 0 & \eta_{2,6} & 0 & 0 \\ 0 & 0 & 0 & \eta_{2,7} & 0 & \eta_{2,8} & 0 \\ \eta_{2,12} & 0 & \eta_{2,9} & 0 & \eta_{2,10} & 0 & \eta_{2,11} \end{bmatrix} \begin{bmatrix} \tilde{c}_{h,t} \\ \tilde{c}_{f,t} \\ \tilde{q}_{h,t} \\ \tilde{q}_{f,t} \\ \tilde{k}_{h,t} \\ \tilde{k}_{f,t} \\ \tilde{b}_{h,t} \end{bmatrix}$$

It could be noted that this model can be solved for all the gaps in $t+1$ as a function of their value in t.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & \eta_{1,1} & 0 & \eta_{1,2} \\ 0 & 0 & 0 & 0 & \eta_{1,3} & 0 & \eta_{1,4} \\ 0 & 0 & \eta_{1,8} & 0 & \eta_{1,5} & 0 & \eta_{1,6} \\ 0 & 0 & 0 & \eta_{1,9} & 0 & \eta_{1,7} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

This solution is possible because the previous matrix is diagonal which means that its determinant is different from zero. Then, matrix solution for the time path of economy's gaps is:

$$\begin{bmatrix} \vec{c}_{h,t+1} \\ \tilde{c}_{f,t+1} \\ \tilde{q}_{h,t+1} \\ \tilde{q}_{f,t+1} \\ \tilde{k}_{h,t+1} \\ \tilde{k}_{f,t+1} \\ \tilde{b}_{h,t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & -\eta_{1,1} & 0 & -\eta_{1,2} \\ 0 & 1 & 0 & 0 & -\eta_{1,3} & 0 & -\eta_{1,4} \\ 0 & 0 & 1-\eta_{1,8} & 0 & -\eta_{1,5} & 0 & -\eta_{1,6} \\ 0 & 0 & 0 & 1-\eta_{1,9} & 0 & -\eta_{1,7} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \eta_{2,1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \eta_{2,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \eta_{2,3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \eta_{2,4} & 0 & 0 & 0 \\ 0 & 0 & \eta_{2,5} & 0 & \eta_{2,6} & 0 & 0 \\ 0 & 0 & 0 & \eta_{2,7} & 0 & \eta_{2,8} & 0 \\ \eta_{2,12} & 0 & \eta_{2,9} & 0 & \eta_{2,10} & 0 & \eta_{2,11} \end{bmatrix} \begin{bmatrix} \vec{c}_{h,t} \\ \tilde{c}_{f,t} \\ \tilde{q}_{h,t} \\ \tilde{q}_{f,t} \\ \tilde{k}_{h,t} \\ \tilde{k}_{f,t} \\ \tilde{b}_{h,t} \end{bmatrix}$$

Then, to know the convergence dynamics of the time paths, we need to know what the roots values of the system are. This question can be answered by finding Eigen values of

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -\eta_{1,1} & 0 & -\eta_{1,2} \\ 0 & 1 & 0 & 0 & -\eta_{1,3} & 0 & -\eta_{1,4} \\ 0 & 0 & 1-\eta_{1,8} & 0 & -\eta_{1,5} & 0 & -\eta_{1,6} \\ 0 & 0 & 0 & 1-\eta_{1,9} & 0 & -\eta_{1,7} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \eta_{2,1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \eta_{2,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \eta_{2,3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \eta_{2,4} & 0 & 0 & 0 \\ 0 & 0 & \eta_{2,5} & 0 & \eta_{2,6} & 0 & 0 \\ 0 & 0 & 0 & \eta_{2,7} & 0 & \eta_{2,8} & 0 \\ \eta_{2,12} & 0 & \eta_{2,9} & 0 & \eta_{2,10} & 0 & \eta_{2,11} \end{bmatrix}$$

Renaming the elements of this matrix, the solution to this problem will be given by the values for λ that satisfy:

$$0=\frac{(1-\lambda)\begin{bmatrix} ag\lambda^2 oq+b\lambda q(e\lambda(l-\lambda)-f\lambda o+(j-\lambda)(l-\lambda))+ \\ +(1-\lambda)(e\lambda(l-\lambda)(\lambda-n)+f\lambda o(n-\lambda)+ \\ +g\lambda(lr-\lambda r-os)+(j-\lambda)(l-\lambda)(\lambda-n) \end{bmatrix}\left[h\lambda(\lambda-m)+ip\lambda+(k-\lambda)(\lambda-m)\right]}{(e-1)(h-1)}$$

From this expression it is possible to see that no maters how the calibration is, the model has at least one unit characteristic root, what means that as we have expressed the time evolution of the variables in terms of deviations from the steady state level, when the world economy is moved away from that state, there will be a steady state change, a structural change, and a convergence process to the new long run values of variables.

Appendix 3

The calibration process

The most important task in this thesis is not the calibration model by itself, but the construction of a model capable to capture transitional processes of small open economies which front imperfect international capital mobility in a two country model. Thus, for the Colombian case, taken as the small economy, and the US taken as the large one. Parameter values from other studies are used. Main works to cite are basically, Gómez y Posada (2004) and GRECO (2002), Hamman and Riascos (1998), Backus, Kehoe y Kydland (1992).

For both economies, the problem of the firm is a two simultaneous equation dynamic system, imposing equilibrium and steady state, the system take the form:

$$g_{ZL}k=(q-1)\frac{k}{\gamma}+k$$

$$q=q(1+r)-\left\{Pmgk+\frac{\gamma}{2}\left(\frac{(q-1)}{\gamma}\right)^2-\delta\right\}$$

And $pmgk = \alpha k^{\alpha - 1}$

Using production parameters for Colombian economy, those from Greco (2002) and Gómez and Posada (2004) and, Posada and Gómez (2002), the two equation system above was solved for k and q by using the GAUSS nonlinear equations system solver. Parameter values are shown in Table 2. Tobin's for Colombia was calculated from equation transition for capital and its resulting value was used to calculate real interest rate from equation transition for Tobin's Q, taking into account that capital-GDP ratio for Colombia is about 2.923, this give us an interest rate of 10.98%. Once these variables values are available, it is possible to calculate output, investment (net and gross), and costs of investment. Foreign debt was taken from Banco de la Rapública (Colombian central bank) in current dollars, and then it was converted into Colombian pesos and then deflated with Colombian GDP deflator to get foreign Colombian debt in 1975 Colombian pesos. This variable is used to proxy foreign assets of the small economy. Foreign assets-GDP ratio was calculated as the average value of the Colombian foreign debt-GDP ratio (in 1975 Colombian pesos), the value from 1970 to 2001 was 0.366. Another proxy for this variable is computed by using current dollars Colombian debt and GDP, which is not very different from the first one. However, the first calculation was used for the parameterization purpose. Finally consumption was calculated as a residual, from foreign assets transition equation.

For the big economy, the procedure was quite different. Because the US economy has been calibrated from a quarterly data base, it was necessary to recalculate some parameters and steady state values in this work. For real interest rate, quarterly value presented was 0.01, which means a 4% real interest rate for the USA economy. Similarly adjustment was made for capital depreciation rate; quarterly value for this parameter is 0.025, which means an annual rate depreciation of 0.1. Those quarterly values imply roughly a 10 capital-GDP ratio. After recalibrating for annual frequency and taking $\alpha_f = 0.58$, as in King, Plosser and Rebelo (1988), and Baxter (1995), capital-GDP is 4.1. This calibration not only guaranties binding restrictions about key values but also allows maintaining a very important fact: a higher capital-GDP ratio for the big economy than the one in the small economy[23].

Preference parameters from Gómez and Posada (2004), Posada and Gómez (2002) where used for the Colombian economy. And for the case of the US economy, were taken those from Backus, Kehoe y Kydland (1992), but in this last case, because USA economy is parameterized for quarterly data it was necessary recalibrate them to be consistent with annual frequency. For the case of subjective discount rate, annual value was calculated as the solution for ρ_{annual} in the equation $(1+\rho_{trim})^4 = (1+\rho_{annual})$, being $\rho_{trim} = \frac{1}{\beta_{trim}} - 1$, and $\beta_{trim} = 0.99$ is quarterly value calibrated in Backus, Kehoe y Kydland (1992), parameter θ_f was consistently calculated from Euler equation. Population growth rate was calculated from IMF financial statistics, and labor efficiency growth rate was annualized from the quarterly value in King, Plosser and Rebelo (1988), this gives an annual growth rate of 0.016 slightly greater than the one supposed for the small economy. Gamma parameter was set equal to the one for the small economy.

[23] For these calculations it was necessary to use both equations: capital and Tobin's q transition equations.