The Optimal Monetary Policy Instrument, Inflation vs. Asset Price Targeting, and Financial Stability *

C.A.E. Goodhart[†], C. Osorio[‡] and D.P. Tsomocos [§]

August, 2009

Abstract

This paper assesses the choice of policy instruments for crisis management and prevention and whether Central Banks should target consumer and asset prices to maintain financial stability. Our results suggest that the interest rate is preferable to the money supply instrument because in times of financial distress the Central Bank automatically satisfies the increased demand for money, and that monetary policy aimed at stabilizing consumer inflation, but not asset price inflation, can promote financial instability. However, we also show that Central Banks' financial stability objective should be primarily achieved by regulatory measures.

Keywords: general equilibrium, securitization, financial fragility, monetary policy, regulation

JEL Classification: D5, E43, E44, E5, G01, G2

* We acknowledge helpful comments from Udara Peiris and Alexandros Vardoulakis. However, all remaining errors are ours.

† Financial Markets Group, London School of Economics.
E-mail: caegoodhart@aol.com
Phone: +44 (0) 20 7955 7555
Address: Room R414, London School of Economics and Political Science, Houghton Street, London WC2A 2AE, UK

‡ Department of Economics, University of Oxford.
E-mail: carolina.osorio@economics.ox.ac.uk
Phone: +44 (0) 78 9695 836
Address: St. Hilda's College, Cowley Place, Oxford OX4 1DY, UK

§ Said Business School and St. Edmund Hall, University of Oxofrd.
E-mail: dimitrios.tsomocos@sbs.ox.ac.uk
Phone: + 44 (0) 1865 288 932
Address: Sad Business School, Park End Street, Oxford OX1 1HP, UK

1 Introduction

Over the last couple of years the global financial system has undergone a period of unprecedented turmoil initiated by problems in the U.S. mortgage market, which then spread to securitized products and a wide range of credit markets. Interbank markets have struggled to provide liquidity across the banking sector, thereby failing to act as a conduit for monetary policy, and systemically important financial institutions have collapsed calling for public intervention on a scale not seen for decades. We argue that the current crisis is a bona fide general equilibrium example whereby various interacting channels in the financial markets affect and are affected by the real economy. The purpose of this paper is twofold; first, to explain the current U.S. financial crisis by modeling a contagion phenomenon that commences with increased default in the mortgage sector and then hinges upon the rest of the nominal sector of the economy; and second, to assess the choice of policy instruments and whether Central Banks should target consumer and asset prices to maintain financial stability.

To this end we construct a two-period, rational expectations, monetary general equilibrium model with commercial banks, default, and collateral along the lines of Goodhart, Tsomocos and Vardoulakis (2008). However, we extend this framework by introducing an investment bank and a hedge fund, allowing mortgage debt to be securitized, and separating the interbank from the repo market. This way, we succeed in focusing more closely on the transmission mechanism of monetary policy and its impact on financial stability. Moreover, we model two types of default; in the mortgage market default is highly discontinuous as in Geanakoplos (2003) and Geanakoplos and Zame (1995), whereas is in credit markets where financial institutions interact with each other, default is modeled as a continuous phenomenon as in Shubik and Wilson (1977) and Dubey et al. (2005). Unlike Goodhart et. al. (2006), we abstract from modeling capital adequacy requirements explicitly since we are not considering a wide range of asset markets; and for simplicity, we assume commercial banks not issuing shares of stock.

The economy experiences an initial adverse productivity shock, and the Central Bank reacts by changing its monetary policy stance. Both shocks lead to increased default in the mortgage market, thus affecting the financial system as a whole through the derivatives markets. In this set-up the U.S.'s 2004-2008 experience is well illustrated since western economies were subject to an adverse supply shock in the guise of rising energy and commodity prices. This induced Central Banks worldwide to increase interest rates to contain inflationary expectations, and hence, the well-known second-round effects of rising oil prices. However, these shocks caused house prices to fall in the U.S., thereby triggering the mortgage crisis in that country and the subsequent contagious global financial turmoil.

Owing to the non-linearity of the system and its large size, which amounts to 68 equations, we solved the model numerically. Similarly, the comparative statics exercises are numerical simulations. One of the main strengths of this approach is that a vast number of simulations can be performed to examine a huge variety of potential shocks and policies, as well as the multiple interacting channels. All the shocks we investigate, we reckon, are akin to the current financial crisis and we attempt to draw relevant policy implications. We explore the effects of expansionary policy, government subsidies, increased default penalties, direct liquidity assistance to poor households, and a shock that makes some banks more risk-averse.

Although Dynamic Stochastic General Equilibrium (DSGE) models have gained popularity as tools for policy discussion and analysis among academics and central banks in recent years, they are inappropriate, at best, for financial stability analysis. The benchmark DSGE model is a fully micro-founded representative agent model with real and nominal rigidities that incorporates elements of the Real Business Cycle approach and the New Keynesian paradigm. The latter has its cornerstone in the work of Woodford (2003), which explains why his neo-Wicksellian theory of monetary policy, whereby an interest rate (rule-based) approach is the optimal policy to stabilize the rate of inflation, is used by most central banks around the world rather than the quantity theory of money approach favored by Irving Fisher.

However, Woodford's model may not provide a sound basis for the foundations of monetary theory and policy. He builds a model based on a cashless economy by describing money as a friction and then providing a theoretical foundation for monetary policy in a setting where the distortion generated by money disappears. Therefore, money and credit arise as inessential additions to the non-monetary version of the model, which implies that a trivial quantity theory of money holds. Moreover there is no role for Central Banks because the means for credit settlement are unspecified and the nominal interest rate is undefined. Furthermore, Woodford's model can only produce an inter-temporal relative commodity price vector that cannot be affected by the Central Bank because it results from the representative agent's optimization problem given his preferences and the economy's endowments and technology. This implies that the price of money is indeterminate and conceptually undefined (Buiter, 1999). Put differently, money is treated as a unit of account rather than as an asset and/or as a means of payment that can be used to transfer wealth inter-temporally.

DSGE models are useful for identifying sources of economic fluctuations, forecasting, and predicting the effects of policy interventions; however, since these are representative agent models, they rule out trade between agents, and hence, the possibility of default. Liquidity has real effects only when it affects and is affected by the potentiality of default. Moreover, in these models money doesn't have an essential role and financial frictions such as default risk, banks, contagion and incomplete financial markets, which are essential for financial stability analysis, are not included (Barsden et.al., 2008).

The financial accelerator has been the most common approach to incorporate financial frictions into a DSGE framework (Bernanke, Gertler and Gilchrist, 1999). This is a representative agent model with asymmetric information where a partial equilibrium model for the credit market is embedded into the standard new Keynesian framework. The model captures the effects of firms' balance sheet on investment by relying on a one-period stochastic optimal debt contract with costly-state verification. However, banks do not play an active role in this model and equilibrium outcomes are constrained efficient, which implies that regulatory policies are not relevant in this framework (Geanakoplos and Polemarchakis, 1986).

To explore contagious financial crises, a model of heterogeneous banks with different portfolios is needed to allow for the existence of an interbank market and contagion. A set-up that allows for default is also required; otherwise there would be no crises. Furthermore, money, banks and interest rates must play an essential role, since we are concerned with financial crises. And finally, financial markets cannot be complete, otherwise all eventualities could be hedged and equilibrium outcomes would be constrained efficient, thus limiting the scope for welfare improving economic policy.

The framework presented here incorporates all these elements. Moreover, in this model monetary policy is non-neutral, a non-trivial quantity theory of money holds, the term structure of interest rates depends on aggregate liquidity and default risk in the economy, and the Fisher effect, whereby nominal rates are a function of real rates and inflation expectations, holds. Additionally, financial fragility arises as an equilibrium outcome which is characterized by reduced aggregate bank profitability and increased aggregate default (Tsomocos, 2003).

The paper proceeds as follows. Section 2 presents the model. In section 3 the equilibrium of the model is defined, its properties are derived, and the benchmark equilibrium outcome is discussed. The comparative statics results are reported in Section 4, and section 5 presents the implications for inflation targeting based on the models' results. Finally, section 6 concludes.

2 The Baseline Model

2.1 The Economy

Consider a canonical General Equilibrium with Incomplete Markets (GEI) model in which time extends over two periods ($t \in T = \{0, 1\}$). The first period consists of a single initial state and the second period consists of S possible states. At the initial period, households, commercial banks, financial institutions and the authorities make their decisions expecting (rationally) the realization of any of the possible states in the next period. In the second period, one of the S states occurs and agents make their choices accordingly. Suppose there are 2 possible states of the world in the second period ($s \in \{1, 2\}$),and let the set of all states be denoted by $s^* \in S^* = \{0\} \cup S = \{0, 1, 2\}$.

The (endowment) economy has two goods, a basket of consumption goods and housing, which are denoted by subscripts 1 and 2 respectively. Housing is a durable good, which provides utility in every period after its purchase, and for tractability purposes, it is assumed to be infinitely divisible. There are two households $h \in H = \{\alpha, \theta\}$,two commercial banks $j \in J = \{\gamma, \delta\}$, an investment bank ($\psi$), and a hedge fund ($\phi$). The economy has other three players, a Central Bank which can inject (withdraw) money into (from) the system, the Government which can increase or decrease the level of private monetary or commodity endowments, and a Financial Supervisory Agency (FSA) which imposes penalties on defaults. We do not seek to model the actions of these official players, which is why they operate as strategic dummies. There are 10 active markets in this economy: the goods, housing, mortgage, short term loans, consumer deposit, repo, interbank, Mortgage Backed Securities (MBS's), Collateralized Debt Obligation (CDO's), and wholesale money markets.

Households are risk-averse agents who maximize their expected utility over their consumption stream of housing and goods. We use a CRRA utility function for $h \in H$ to capture the wealth effects of price and interest rates movements. Households are heterogeneous in their endowments of goods and money; α is endowed with goods at all states and with a small amount of money at the initial period, whereas θ is endowed with housing only at $t = 0$ and with a large amount of cash in the first period.

Commercial banks are also risk-averse agents who maximize their expected second period profits. We suppose commercial banks have quadratic preferences over their profits[1],which implies they face a portfolio allocation problem whereby they try to diversify idiosyncratic risks. Commercial banks are heterogeneous in their endowments of capital; while bank γ has a large endowment of capital at the initial period, bank δ is poorly capitalized at all states.

Following Goodhart et al. (2004, 2006) an important friction is introduced in the short term consumer credit markets. Individual borrowers are assigned, by history or by informational constraints, to a single bank over the two periods of the model.[2] By assumption, households cannot default on short term loans; hence, without loss of generality, let α borrow from bank γ and θ borrow from bank δ, in the short term credit market. In the case of inter-period loans and deposits, we assume households make transactions with the bank offering the best rate, which is the highly capitalized bank (γ). Since α is poor and θ is rich, in monetary endowments, the former takes out a mortgage with bank γ, while the latter makes a long term deposit in that bank.

In contrast to commercial banks, the investment bank and the hedge fund are assumed to have linear preferences over their expected second-period profits, which implies that these agents are risk neutral and do not seek to accumulate profits.

[1] A CARA utility function.

[2] Restricted participation can also arise as an outcome of banks aiming to outperform each other by introducing a relative performance criterion into their objective functions (see Bhattacharya et al., 2007).

2.2 Money

Money is introduced by a cash-in-advance constraint, whereby all commodities and assets can be traded only for money, and all asset deliveries are paid in money. Cash-in-advance models aim to illustrate the importance of liquidity for transactions. There are many versions of cash-in-advance models in the monetary theory literature (e.g. Lucas and Stokey, 1983 and 1987; Svensson, 1985; Bloise, Dreze and Polemarchakis, 2005); we follow the model of Dubey and Geanakoplos (1992), where multiple facets of money are captured. Money is fiat and is the stipulated medium of exchange; it doesn't give utility to agents, it cannot be privately produced, and is perfectly durable. Moreover, money enters the system as outside or inside money.

Outside money enters the system free and clear of any offsetting obligations, i.e. private sectors' aggregate monetary endowments, which can be interpreted as a government transfer or as an inheritance from the (unmodeled) past. Inside money enters the system accompanied by an offsetting obligation; it is the stock of money supplied by the Central Bank which is matched by individual borrowers' debt obligation to commercial banks. Since money is fiat, it must exit the system at the final period. Hence, inside and outside money exit the economy via loan repayments by households/investors to commercial banks, loan repayments by commercial banks to the Central Bank, or by the Central Bank's liquidation of commercial banks.

2.3 Default and Collateral

The model incorporates two types of (endogenous) default. In the mortgage market default is highly discontinuous since agents default on their mortgage when the endogenous value of collateral is lower than the mortgage's amount due. In this case, the bank seizes the amount of housing pledged as collateral and offers it for sale in the next period; the proceeds from this sale determine the effective mortgage rate (or equivalently the mortgage's repayment rate) (Geanakoplos, 2003, Geanakoplos and Zame, 1995).

By assumption, in addition to mortgages, only interbank and wholesale money market loans are defaultable. However, these loans are unsecured which is why we model default in these markets as a continuous phenomenon following Shubik and Wilson (1977) and Dubey et al. (2000). In this case, the fraction of (defaultable) loans that agents repay is a choice variable. By defaulting, agents face a penalty which reduces their utility by a scalar τ_s^k, for $s = \{1, 2\}$ and $k = \{\delta, \psi, \phi\}$, per monetary unit of account not repaid. In equilibrium, agents will equalize the marginal utility of defaulting with the marginal disutility of the bankruptcy penalty.[3] The vector $\left\{\tau_s^k\right\}$ represents the default penalties set by the FSA.

Since rational expectations are assumed throughout, in equilibrium, expected rates of delivery for mortgage, interbank and wholesale money market loans are equal to actual rates of delivery. For this reason, default can be established as an equilibrium outcome without destroying the orderly functioning of the financial system. This result contrasts the multitude of papers following the work of Bryant (1982) and Diamond and Dybvig (1983), in which financial instability is rationalized by modeling bank runs and panics based on some type of co-ordination failure. Tsomocos (2003) shows that bank runs are a particular case of the monetary general equilibrium model with commercial banks and default, which arises when commercial banks are homogeneous.

2.4 Securitization

Geanakoplos and Zame (2002) argue that the reliance on collateral to secure loans can distort households' consumption plans because collateral is scarce. Furthermore, the scarcity of collateral incentivizes agents to create innovations to economize it. Akin to this model, one way of stretching collateral is by allowing the same physical collateral to be used many times, which motivates the existence and growth of securitization and derivatives markets. In this framework, the mortgage's collateral is securitized twice; first, in the MBS's

[3]In the literature this requirement is known as the "on-the-verge" condition; see Dubey and Geanakoplos (2005).

market, where the investment bank purchases the mortgage asset from the commercial bank that extended the loan; and second, in the CDO's market.

The investment bank (ψ) buys the mortgage asset from bank γ at a price p^{α}.Then, it structures a CDO by attaching a Credit Default Swap (CDS) to the MBS. The CDS protects the CDO buyer (ϕ) against default in the mortgage market, in which case ϕ delivers the mortgage's collateral to ψ, and ψ reimburses ϕ with the amount of cash it invested. $\tilde{q}^{\alpha}$ is the price of the CDO, which is higher than the MBS's price because it includes the CDS's cost of insurance.

Assume α honors his mortgage if $s = 1$ and defaults if $s = 2$. Thus, the mortgage can be regarded as an asset with the following vector of payoffs across states:

$$R^{\alpha} = \begin{bmatrix} 1 + \bar{r}^{\gamma\alpha} \\ 1 + \bar{r}_s^{\gamma\alpha} \end{bmatrix}$$

where $(1 + \bar{r}^{\gamma\alpha})$ is the interest rate offered by bank γ at the initiation of the mortgage contract, and $(1 + \bar{r}_s^{\gamma\alpha})$ is the effective mortgage rate in case of default. This implies that the CDO has the following payoffs:

$$R^{CDO} = \begin{bmatrix} (1 + \bar{r}^{\gamma\alpha}) / \tilde{q}^{\alpha} \\ 1 \end{bmatrix}$$

In the bad states of the world ψ pays ϕ the total amount of cash it invested in the CDO, whereas in the good states ϕ earns the monetary payoff of the mortgage asset net of the premium paid to ψ.

2.5 The Time Structure of Markets

Initially, commercial banks ($j \in J$) organize a short term credit market with the Central Bank, which operates as a strategic dummy in the repo market at $t \in T$ by providing liquidity through open market operations $\left(M_{s^*}^{CB}\right)$ or by entering into (reversal) repurchase agreements with commercial banks.[4] Since bank γ is assumed to be highly capitalized, it enters into a reverse repurchase agreement with the Central Bank (makes a deposit), while δ, the poor bank enters into a repurchase agreement (borrows).

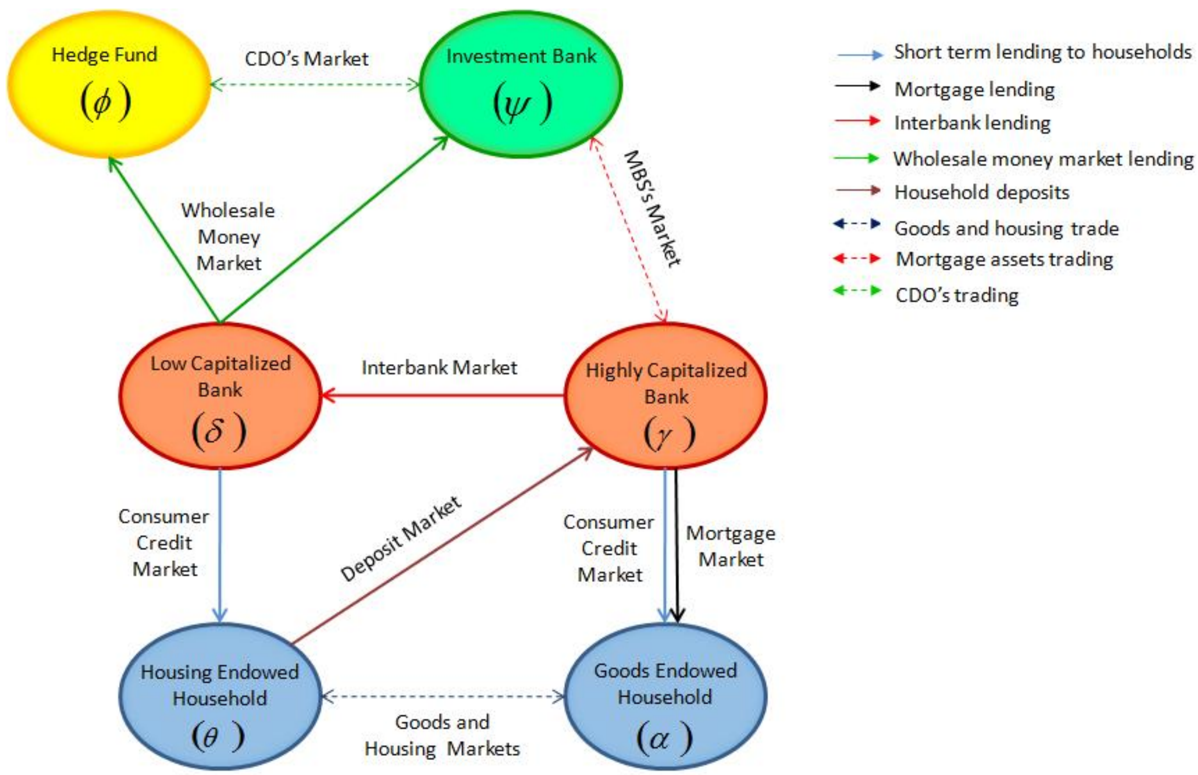
Long term credit markets meet at the initial period after short term consumer credit and repo markets close. α and θ take out short term loans at $s^* \in S^*$ because cash-in-advance is needed for all market transactions. Then, α takes out a mortgage with bank, while θ makes a long term deposit at that bank. Also, given that bank δ is poorly capitalized, it must borrow from bank γ in the interbank market before extending credit to investors.

The investment bank (ψ) buys the mortgage asset from γ in the MBS's market, and securitizes it into a CDO containing the mortgage backed security and CDS. Since ϕ has no capital and ψ has a small endowment of capital, both borrow from bank δ in the wholesale money market before making their respective investments in the derivatives markets. At the end of the first period, consumption and settlement of one-period loans take place.

In the second period, the repo and consumer short term credit markets meet before settlement and defaults take place in the mortgage, MBS's, CDO's, interbank and wholesale money markets. At the end of this period, consumption and settlement of one-period loans take place, and the Central Bank liquidates commercial banks by taking over their profits. A diagram of the economy's nominal flows and its time structure are presented below.

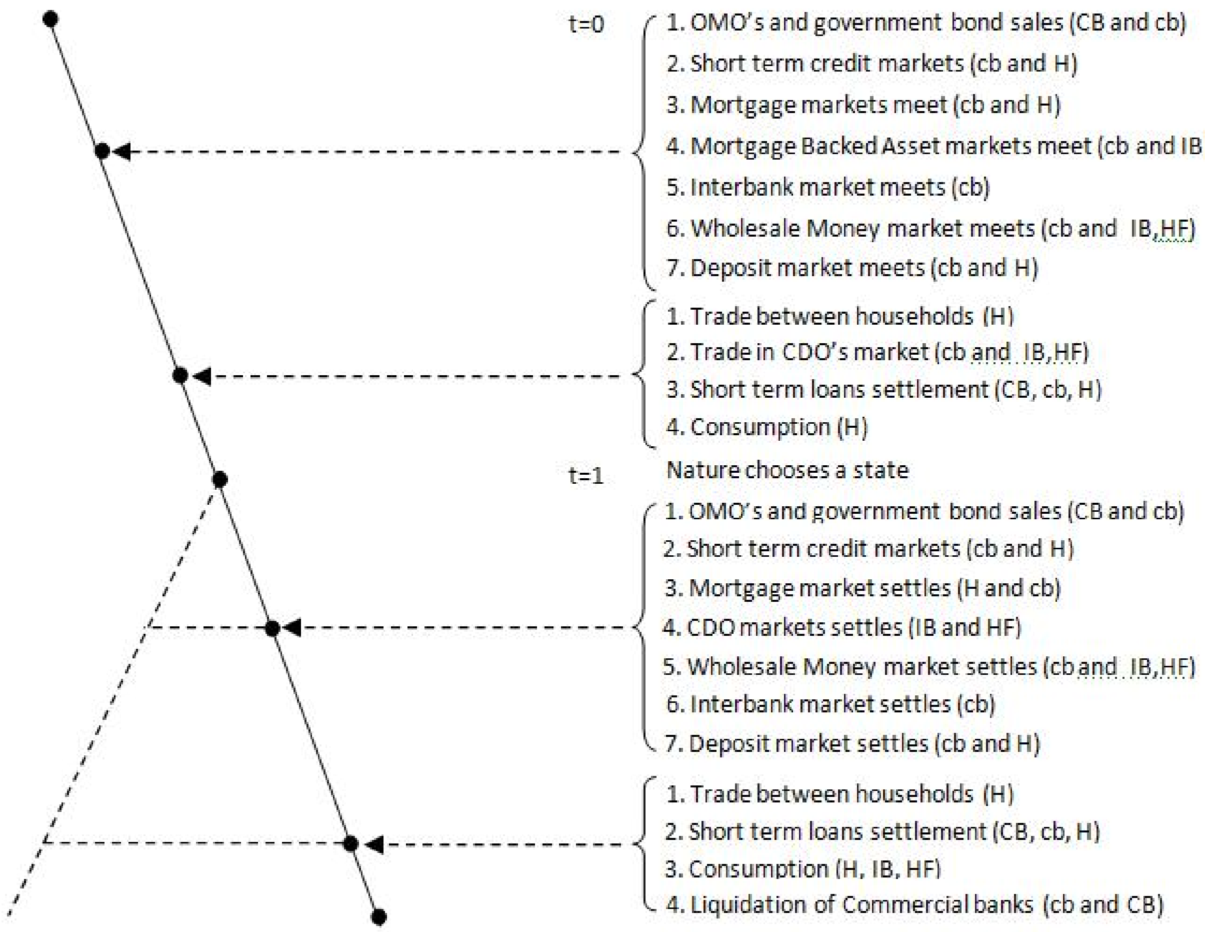
[4]In practice, these repurchase agreements are very short term collateralized loans, where the collateral is a very liquid and safe asset that is exchanged for cash when the loan is acquired and when it is repaid. we will abstract from this collateralization feature for simplicity.

Figure 1: Nominal Flows of the Economy



The straight lines and their direction represent lending flows. The dashed lines indicate trade.

Figure 2: Time Structure



2.6 Agents' Behavior

2.6.1 Household α's Optimization Problem

Consumer α maximizes his utility which depends on his consumption of goods and housing. He is endowed with goods and money in every state, and takes out a short term loan and a mortgage with bank γ to purchase housing in the first period. At $t = 0$. α uses the cash obtained from the mortgage loan, the short term loan, and his monetary endowment to buy housing units, and he pledges these as collateral to the mortgage.

If a good state is realized in the second period, he takes out a short term loan to buy more housing and pay back his mortgage. In the bad states of the world, defaults on his mortgage and his house is seized by bank γ. Nevertheless,α still needs housing services; therefore he takes out a short term loan to purchase housing in the second period. At the end of each period, household α repays his short term obligations with the proceeds of goods sales.

Denote by $S_1^\alpha \subset S$ the set of states in which α honors his mortgage.

$$S_1^\alpha = \left\{ s \in S : \frac{b_{02}}{p_{02}} \geq \bar{\mu}^\alpha \right\}$$

where $(b_{02}/p_{02})$is the amount of housing purchased at $t = 0$, p_{02} is the price of housing at $s \in S_1^\alpha$ in the second period, and $\bar{\mu}^\alpha$ is the value of outstanding mortgage debt. The maximization problem is as follows.

$$\begin{aligned} \max_{q^\alpha_{s*1}, b^\alpha_{s*2}, \mu^\alpha_{s*}, \bar{\mu}^\alpha} U^\alpha \quad &= \quad u\left(e^\alpha_{01} - q^\alpha_{01}\right) + u\left(\frac{b^\alpha_{02}}{p_{02}}\right) + \sum_{s \in S} \omega_s u\left(e^\alpha_{s1} - q^\alpha_{s1}\right) \\ &\quad + \sum_{s \in S_1^\alpha} \omega_s u \left(\frac{b^\alpha_{02}}{p_{02}} + \frac{b^\alpha_{s2}}{p_{s2}}\right) + \sum_{s \notin S_1^\alpha} \omega_s u\left(\frac{b^\alpha_{s2}}{p_{s2}}\right) \end{aligned} \tag{1}$$

$s.t.$

$$b^\alpha_{02} \leq \frac{\bar{\mu}^\alpha}{(1 + \bar{r}^{\gamma\alpha})} + \frac{\mu^\alpha_0}{(1 + r^\gamma_0)} + e^\alpha_{m,0} \tag{2}$$

(i.e. housing expenditure at t=0 $\leq$ mortgage loan + short-term borrowing + private monetary endowments at t=0)

$$\mu^\alpha_0 \leq p_{01} q^\alpha_{01} \tag{3}$$

(i.e. short term loan repayment at t=0 $\leq$ goods sales revenues at t=0)

$$b_{s2}^{\alpha}+\bar{\mu}^{\alpha}\leq\frac{\mu_{s}^{\alpha}}{(1+r_{s}^{\gamma})}+e_{m,s}^{\alpha}\qquad\text{for}\quad s\in S_{1}^{\alpha}\tag{4}$$

(i.e. housing expenditure at $s\in S_1^\alpha$+ mortgage repayment $\leq$ short-term borrowing+private monetary endowments at $s\in S_1^\alpha$)

$$b_{s2}^{\alpha}\;\leq\frac{\mu_{s}^{\alpha}}{(1+r_{s}^{\gamma})}+e_{m,s}^{\alpha}\qquad\text{for}\quad s\notin S_{1}^{\alpha}\tag{5}$$

(i.e. housing expenditure at $s\notin S_1^\alpha$ $\leq$ short-term borrowing+private monetary endowments at $s\notin S_1^\alpha$)

$$\mu_{s}^{\alpha}\leq p_{s1}q_{s1}^{\alpha}\tag{6}$$

(i.e. short term loan repayment$\leq$ goods sales revenues at t=0)

$$q_{s^*1}^{\alpha}\leq e_{s^*1}^{\alpha}\tag{7}$$

(i.e. quantity of goods sold at $s\in S^*$ $\leq$ goods endowments at$s\in S^*$)

where

$b_{s^*1}^{\alpha}\equiv$ amount of fiat money spent by α to trade in the housing market in s^*
$q_{s^*1}^{\alpha}\equiv$ amount of goods offered for sale by α in s^*
$\bar{\mu}^{\alpha}\equiv$ repayment value of the mortgage credit that γ extends to α
$\bar{r}^{\gamma\alpha}\equiv$ mortgage rate offered to α by bank γ
$r_{s^*}^{\gamma}\equiv$ short term rate offered to α by bank γ in s^*
$\mu_{s^*}^{\alpha}\equiv$ short term loan that γ extends to α in s^*
$p_{s^*2}\equiv$ price of housing in state s^*
$p_{s^*1}\equiv$ price of goods in state s^*
$e_{s^*1}^{\alpha}\equiv$ α's endowment of goods at state s^*
$e_{m,s^*}^{\alpha}\equiv$ α's monetary endowments in state s^*
$\omega_s\equiv$ probability of state s

$u(x)=\frac{x^{1-c^h}}{1-c^h}\equiv$ households have a CRRA utility function, where c^h is the risk aversion coefficient of $h\in H=\{\alpha,\theta\}$

2.6.2 Household θ's Optimization Problem

θ is endowed with money in every state and with a large amount of housing at $t=0$. He sells houses and buys goods in both periods. At the beginning of the first period, uses his cash inflows (monetary endowment and a short term loan), to buy goods and to make a long term deposit in bank γ. In the second period, θ uses the gross return of his deposit, his monetary endowment and a short term loan to purchase consumption goods.

Finally, θ repays his short term obligations with the proceeds from housing sales at the end of each period. The maximization problem is as follows.

$$\max_{q^\theta_{s^*2}, b^\theta_{s^*1}, \mu^\theta_{s^*}, \bar{d}^\theta} U^\theta = u\left(\frac{b^\theta_{01}}{p_{01}}\right) + u\left(e^\theta_{02} - q^\theta_{02}\right) + \sum_{s \in S} \omega_s u\left(\frac{b^\theta_{02}}{p_{02}}\right) + \sum_{s \in S} \omega_s u\left(e^\theta_{s2} - q^\theta_{s0} - q^\theta_{s2}\right) \tag{8}$$

s.t.

$$b^\theta_{01} + \bar{d}^\theta \leq \frac{\mu^\theta_0}{1 + r^\delta_0} + e^\theta_{m,0} \tag{9}$$

(i.e. goods expenditure at t=0 + inter-period deposits $\leq$ short-term borrowing + private monetary endowments at t=0)

$$\mu^\theta_0 \leq p_{02} q^\theta_{02} \tag{10}$$

(i.e. short term loan repayment at t=0 $\leq$ housing sales revenues at t=0)

$$b^\theta_{s1} \leq \frac{\mu^\theta_s}{1 + r^\delta_s} + \bar{d}^\theta\left(1 + \bar{r}^\gamma_d\right) + e^\theta_{m,s} \qquad \text{for} \quad s \in S \tag{11}$$

(i.e. goods expenditure at $s \in S \leq$ short-term borrowing + deposits and interest payment+private monetary endowments at $s \in S$)

$$\mu^\theta_s \leq p_{s2} q^\theta_{s2} \tag{12}$$

(i.e. short term loan repayment at $s \in S \leq$ housing sales revenues at $s \in S$)

$$q^\theta_{s^*2} \leq e^\theta_{s2} - q^\theta_{02} \tag{13}$$

(i.e. number of housing units sold at $s \in S \leq$endowment of housing at t=0 - units of housing sold at $s \in S$)

where
$b^\theta_{s^*1}$ $\equiv$amount of fiat money spent by θ to trade in the goods market in s^*
$q^\theta_{s^*2}$ $\equiv$amount of housing offered for sale by α in s^*
$\bar{d}^\theta$ $\equiv$amount deposited by θ in bank γ
$\mu^\theta_{s^*}$ $\equiv$short term loan that δ extends to θ in s^*
$r^\delta_{s^*}$ $\equiv$short term interest rate offered by δ to θ in s^*
$\bar{r}^\gamma_d$ $\equiv$deposit rate on $\bar{d}^\theta$, offered by bank γ
$e^\theta_{s^*2}$ $\equiv$ θ's endowment of housing

2.6.3 Bank γ's Optimization Problem

Bank γ is a risk averse agent that maximizes the utility provided by its second period expected profits, after which it is liquidated by the Central Bank. γ has quadratic preferences over its expected profits, and a high level of capital endowments in the first period. Initially, γ interacts with the Central Bank in the repo market by entering into a reverse repurchase agreement; it also makes a short term loan and a mortgage extension to α. Then it sells its mortgage asset to ψ, receives a deposit from θ, and makes a deposit in the long term interbank market.

In the second period, γ uses the profits accumulated from the first period and the repayment of the interbank deposit, to extend a short term loan to α, make a deposit in the repo market, and pay back its depositor (θ). γ's second period profits are the sum of the reverse repurchase agreement gross returns and α's repayment on the short term loan. The maximization problem is as follows.

$$\max_{m^{\gamma}_{s*2}, \bar{m}^{\alpha}, d^{G\gamma}_{s^*}, \bar{d}^{\gamma}, \pi^{\gamma}_{s}} \Pi^{\gamma} = \sum_{s \in S} \omega_s \left(\pi^{\gamma}_{s} - c^{\gamma} \left(\pi^{\gamma}_{s} \right)^2 \right) \tag{14}$$

s.t.

$$d^{G\gamma}_{0} + m^{\gamma}_{0} + \bar{m}^{\alpha} + \bar{d}^{\gamma} \leq e^{\gamma}_{0} + \frac{\bar{\mu}^{\gamma}_{d}}{1 + \bar{r}^{\gamma}_{d}} \tag{15}$$

(i.e. deposits in the repo market + short-term lending +mortgage extension + interbank lending $\leq$ capital endowment at t=0 + consumer deposits)

$$d^{G\gamma}_{s} + m^{\gamma}_{s} + \bar{\mu}^{\gamma}_{d} + \leq e^{\gamma}_{s} + \pi^{\gamma}_{0} + \bar{R}^{\delta}_{s} \bar{d}^{\gamma} \left(1 + \bar{\rho}\right) \tag{16}$$

(i.e. short-term lending + deposits in the repo market at $s \in S$ + consumer deposits repayment $\leq$ capital endowment at $s \in S$ + accumulated profits from previous period + interbank loan repayments at $s \in S$)

$$\pi^{\gamma}_{0} = m^{\gamma}_{0} \left(1 + r^{\gamma}_{0}\right) + d^{G\gamma}_{0} \left(1 + \rho^{CB}_{0}\right) + p^{\alpha} \bar{m}^{\alpha} \tag{17}$$

(i.e. profits at t=0 = short term loan repayment + repo deposits and interest payment at t=0 + MBS's sales revenues)

$$\pi^{\gamma}_{s} = m^{\gamma}_{s} \left(1 + r^{\gamma}_{s}\right) + d^{G\gamma}_{s} \left(1 + \rho^{CB}_{s}\right) \tag{18}$$

(i.e. profits at $s \in S$ = short term loan repayment + repo deposits and interest payment at $s \in S$)

where
$\pi^{\gamma}_{s^*}$ $\equiv$bank γ's profits at s^*
$\bar{m}^{\alpha}$ $\equiv$mortgage extension α
m^{α}_{s} $\equiv$short term credit extension to α in state s^*
$\bar{d}^{\gamma}$ $\equiv$long term deposits in the interbank market by bank γ in s^*
$d^{GJ}_{s^*}$ $\equiv$cash sent by bank $j \in J$ to enter a reverse repurchase agreement in s^*
$\bar{\mu}^{\gamma}_{d}$ $\equiv$long term borrowing by bank γ from household θ
$\bar{\rho}$ $\equiv$long term interbank market rate
$\rho^{CB}_{s^*}$ $\equiv$short term interest rate on government bonds in state s^*
$e^{J}_{s^*}$ $\equiv$capital endowment of bank $j \in J$ in state s^*
p^{α} $\equiv$price of MBS's sold to psi
$\bar{R}^{\delta}_{s}$ $\equiv$delivery rate on the inter-period interbank deposit that γ made in bank δ

2.6.4 Bank δ's Optimization Problem

δ is also a risk averse bank with quadratic preferences that maximizes the utility provided by its second period expected profits. At the end of the second period the bank is liquidated by the Central Bank.

At every $s \in S^*$, δ enters into a repurchase agreement with the Central Bank and uses θ's short term credit repayment to meet its repo market obligation. In the first period, δ borrows money in the long term interbank market, and extends long term credit to ψ and ϕ in the wholesale money market. In the second period, it uses wholesale money market repayments to meet its obligation in the interbank market. His second period profits are given by θ's short term loan repayment less the amount due in the repo market. The maximization problem is as follows.

$$\max_{m^{\delta}_{s*2}, \bar{m}, \mu^{G\delta}_{s^*}, \bar{\mu}^{\delta}, \mu^{\delta}_{s^*}, \bar{v}^{\delta}_{s}, \pi^{\gamma}_{s}} \Pi^{\delta} = \sum_{s \in S} \omega_s \left(\pi^{\delta}_s - c^{\delta} \left(\pi^{\delta}_s \right)^2 \right) - \sum_{s \in S} \omega_s \bar{\tau}^{\delta}_s \left[\bar{D}^{\delta}_s \right]^+ \tag{19}$$

s.t.

$$m^{\delta}_0 + \bar{m} \leq e^{\delta}_0 + \frac{\mu^{G\delta}_0}{1 + \rho^{CB}_0} + \frac{\bar{\mu}^{\delta}}{1 + \bar{\rho}} \tag{20}$$

(i.e. short-term lending at t=0 + wholesale money market credit extension $\leq$ capital endowment + short-term borrowing in the repo market at t=0 + interbank borrowing)

$$\mu^{G\delta}_0 \leq m^{\delta}_0 \left(1 + r^{\delta}_0 \right) \tag{21}$$

(i.e. repo loan repayment at t=0 $\leq$ short-term loan repayment at t=0)

$$m^{\delta}_s + \bar{v}^{\delta}_s \bar{\mu}^{\delta} \leq e^{\delta}_s + \frac{\mu^{G\delta}_s}{1 + \rho^{CB}_s} + \bar{R}_s \bar{m} \left(1 + \bar{r} \right) \tag{22}$$

(i.e. short-term lending + interbank loan repayment at $s \in S \leq$ capital endowment + wholesale money market loan repayment short-term loan repayment at $s \in S$)

$$\pi^{\delta}_s = m^{\delta}_s \left(1 + r^{\delta}_s \right) - \mu^{G\delta}_s \tag{23}$$

(i.e. profits at $s \in S$ = short term loan repayment - repo loan repayment at $s \in S$)

where
$\pi^{\delta}_s \equiv$bank δ's profits in state s
$m^{\delta}_{s^*} \equiv$short term credit extended by δ to θ in state s^*
$\bar{m} \equiv$overall long term credit extension to financial institutions $\{\psi, \phi\}$
$\mu^{Gj}_{s^*} \equiv$amount due by bank $j \in J$ in the repo-market in state s^*
$\bar{\mu}^{\delta} \equiv$long term borrowing by δ in the interbank market
$\bar{v}^{\delta}_s \equiv$re-payment rate to γ on long term interbank loans
$\bar{R}_s \equiv$delivery rate on the wholesale money market credit extension
$\bar{\tau}^{\delta}_s \equiv$marginal disutility to δ for defaulting on the interbank loan in state s
$\bar{D}^{\delta}_s = \left(1 - \bar{v}^{\delta}_s \right) \bar{\mu}^{\delta} \equiv$ δ's nominal value of long term interbank debt due to default in state s. Also, define $[x]^+ = \max\{0, x\}$

2.6.5 Investment Bank's Optimization Problem

ψ has risk neutral preferences over its expected second period profits. It buys mortgage assets from γ and securitizes them as explained in section 2.4. Then it sells the structured asset (CDO) to ϕ. ψ finances the purchase of mortgage assets with an inter-period loan from δ. In the second period, ψ repays ϕ after the CDO's market has settled. In the bad states of the world, ψ ends up owning the mortgage's collateral and selling it in the housing market [5]. Furthermore, the CDS leg of the CDO contract forces ψ to return to ϕ its initial investment in the CDO security [6]. The maximization problem is as follows.

$$\max_{\tilde{m}^{\alpha},\bar{\mu}^{\psi},\bar{v}_s^{\psi},} \Pi^{\psi} = \sum_{s\in S} \omega_s \pi_s^{\psi} - \sum_{s\in S} \omega_s \bar{\tau}_s^{\psi} \left[\bar{D}_s^{\psi}\right]^{+} \tag{24}$$

s.t.

$$\tilde{m}^{\alpha} \leq e_0^{\psi} + \frac{\bar{\mu}^{\psi}}{1+\bar{r}} \tag{25}$$

(i.e. expenditure in the MBS's market $\leq$ capital endowments at t=0 + wholesale money market borrowing)

$$\bar{v}_s^{\psi}\bar{\mu}^{\psi} \leq \frac{\tilde{m}^{\alpha}}{p^{\alpha}}\tilde{q}^{\alpha} \quad \text{for} \quad s \in S_1^{\alpha} \tag{26}$$

(i.e. whole sale money market loan repayment at $s \in S_1^{\alpha} \leq$ CDO's sales revenues + capital endowments at $s \in S_1^{\alpha}$)

$$\tilde{m}^{\alpha}\tilde{q}^{\alpha} + \bar{v}_s^{\psi}\bar{\mu}^{\psi} \leq e_s^{\psi} + \left(\tilde{q}^{\alpha} + \frac{b_{02}^{\alpha}p_{22}}{\bar{m}^{\alpha}p_{02}}\right)\frac{\tilde{m}^{\alpha}}{p^{\alpha}} \quad \text{for} \quad s \notin S_1^{\alpha} \tag{27}$$

(i.e. CDS settlement payment + wholesale money market loan repayment at $s \notin S_1^{\alpha} \leq$ capital endowment at $s \notin S_1^{\alpha}$ + CDO's sales revenues + collateral sales revenues)

$$\pi_s^{\psi} = \frac{\tilde{m}^{\alpha}}{p^{\alpha}}\tilde{q}^{\alpha} - \bar{v}_s^{\psi}\bar{\mu}^{\psi} \quad \text{for} \quad s \in S_1^{\alpha} \tag{28}$$

$$\pi_s^{\psi} = e_s^{\psi} + \left(\tilde{q}^{\alpha} + \frac{b_{02}^{\alpha}p_{22}}{\bar{m}^{\alpha}p_{02}}\right)\frac{\tilde{m}^{\alpha}}{p^{\alpha}} - \tilde{m}^{\alpha}\tilde{q}^{\alpha} - \bar{v}_s^{\psi}\bar{\mu}^{\psi} \quad \text{for} \quad s \notin S_1^{\alpha} \tag{29}$$

where
π_s^{ψ} $\equiv$bank ψ's profits at state s
$\tilde{m}^{\alpha}$ $\equiv$amount of money sent by ψ to purchase mortgage assets from γ
$\bar{\mu}^{\psi}$ $\equiv$inter-period borrowing from δ
$\bar{r}$ $\equiv$interest rate offered by δ on the inter-period loan
$\bar{v}_s^{\psi}$ $\equiv$ ψ's repayment rate on the loan extended by δ in state s
$\tilde{q}^{\alpha}$ $\equiv$price at which the CDO is sold to ϕ
$\bar{\tau}_s^{\psi}$ $\equiv$marginal disutility to ψ for defaulting on the wholesale market loan in s
$\bar{D}_s^{\psi} = \left(1-\bar{v}_s^{\psi}\right)\bar{\mu}^{\psi}$ $\equiv$ ψ's nominal value of wholesale debt due to default in s
$(1+\bar{r}_s^{\gamma\alpha}) = (p_{22}b_{02}^{\alpha}/p_{02}\bar{m}^{\alpha})$ $\equiv$effective mortgage rate in state $s \notin S_1^{\alpha}$

[5] This will help us capture the underwriting effects that have affected financial institutions in the current crisis.
[6] For simplicity we have abstracted from allowing the investment bank to default on its CDS obligation, which would capture counterparty risk in the derivatives markets.

2.6.6 Hedge Fund's Optimization Problem

ϕ has risk neutral preferences over its expected second period profits. It buys CDO's from ψ and finances this purchase with an inter-period loan from δ in the wholesale money market. After the state of the world is realized in the second period, ϕ repays δ with the gross returns of its CDO investment. The maximization problem is as follows.

$$\max_{\bar{\mu}^{\phi},\hat{m}^{\alpha},\bar{v}^{\phi}_{s^*}} \Pi^{\phi} = \sum_{s\in S} \omega_s \pi^{\phi}_s - \sum_{s\in S} \omega_s \bar{\tau}^{\phi}_s \left[\bar{D}^{\phi}_s\right]^+ \tag{30}$$

s.t.

$$\hat{m}^{\alpha} \leq \frac{\bar{\mu}^{\phi}}{1+\bar{r}} \tag{31}$$

(i.e. expenditure in the CDO's market $\leq$ wholesale money market borrowing)

$$\bar{v}^{\phi}_s \bar{\mu}^{\psi} \leq \frac{\hat{m}^{\alpha}}{\tilde{q}^{\alpha}}\left(1+\bar{r}^{\gamma\alpha}\right) \quad \text{for} \quad s \in S^{\alpha}_1 \tag{32}$$

(i.e. wholesale money market loan repayment $\leq$ CDO's payoffs at $s \in S^{\alpha}_1$)

$$\bar{v}^{\phi}_s \bar{\mu}^{\psi} \leq \hat{m}^{\alpha} \quad \text{for} \quad s \notin S^{\alpha}_1 \tag{33}$$

(i.e. wholesale money market loan repayment $\leq$ CDO's payoffs at $s \notin S^{\alpha}_1$)

$$\pi^{\phi}_s = \frac{\hat{m}^{\alpha}}{\tilde{q}^{\alpha}}\left(1+\bar{r}^{\gamma\alpha}\right) - \bar{v}^{\phi}_s \bar{\mu}^{\psi} \quad \text{for} \quad s \in S^{\alpha}_1 \tag{34}$$

$$\pi^{\phi}_s = \hat{m}^{\alpha} - \bar{v}^{\phi}_s \bar{\mu}^{\psi} \quad \text{for} \quad s \notin S^{\alpha}_1 \tag{35}$$

where
$\pi^{\phi}_s \equiv$bank ϕ's profits at state s
$\hat{m}^{\alpha} \equiv$amount of money sent by ϕ to purchase CDO's from ψ
$\bar{\mu}^{\phi} \equiv$inter-period borrowing from δ
$\bar{v}^{\phi}_s \equiv \phi$'s repayment rate on the loan extended by δ
$\bar{\tau}^{\phi}_s \equiv \phi$'s marginal disutility of default on wholesale money market loans at s
$\bar{D}^{\phi}_s = \left(1-\bar{v}^{\phi}_s\right)\bar{\mu}^{\phi} \equiv \phi$'s nominal value of long term interbank debt in state s

2.7 Market Clearing Conditions

There are 10 markets in the economy: the goods, housing, mortgage, short term loans, consumer deposit, repo, interbank, MBS's, CDO's and wholesale money markets. In each of these markets the price equating demand and supply is determined.

2.7.1 Goods Market

In every state-period, the goods market clears when the amount of money offered for goods is exchanged for the quantity of goods offered for sale.

$$p_{01} = \frac{b^{\theta}_{01}}{q^{\alpha}_{01}} \tag{36}$$

$$p_{s1} = \frac{b^{\theta}_{s1}}{q^{\alpha}_{s1}} \quad \text{for} \quad s \in S \tag{37}$$

2.7.2 Housing Market

In every state-period, the housing market clears when the amount of money offered for housing is exchanged for the quantity of housing offered for sale. In every $s \notin S^{\alpha}_{1}$, since agent α defaults on his mortgage, the amount of housing he pledged as collateral in the previous period is also offered for sale by the investment bank.

$$p_{02} = \frac{b^{\alpha}_{02}}{q^{\theta}_{02}} \tag{38}$$

$$p_{s2} = \frac{b^{\alpha}_{s2}}{q^{\theta}_{s2}} \quad \text{for} \quad s \in S^{\alpha}_{1} \tag{39}$$

$$p_{s2} = \frac{b^{\alpha}_{s2}}{q^{\theta}_{s2} + b^{\alpha}_{02}/p_{0}2} \quad \text{for} \quad s \notin S^{\alpha}_{1} \tag{40}$$

2.7.3 Mortgage Market

The mortgage market clears when the amount offered to be repaid in the second period is exchanged for the mortgage extension offered in the first period.

$$(1 + \bar{r}^{\gamma\alpha}) = \frac{\bar{\mu}^{\alpha}}{\bar{m}^{\alpha}} \tag{41}$$

The effective return on the mortgage at any state in the second period is definde as

$$(1 + \bar{r}^{\gamma\alpha}_{s}) = \frac{min \left\{ \text{collateral's value, mortgage amount due} \right\}}{\text{initial mortgage extension}}$$

$$= \frac{min \left\{ p_{22} \left(b^{\alpha}_{02}/p_{02} \right), \bar{\mu}^{\alpha} \right\}}{\bar{m}^{\alpha}}$$

therefore, the clearing conditions for effective returns on mortgages is given by

$$(1 + \bar{r}^{\gamma\alpha}_{s}) = \begin{cases} (1 + \bar{r}^{\gamma\alpha}) & \text{for} \quad s \in S^{\alpha}_{1} \\ \\ \left(\dfrac{p_{22} b^{\alpha}_{02}}{p_{02}} \right) \left(\dfrac{\bar{\mu}^{\alpha}}{1 + \bar{r}^{\gamma\alpha}} \right)^{-1} & \text{for} \quad s \notin S^{\alpha}_{1} \end{cases} \tag{42}$$

2.7.4 Short-term Consumer Credit Markets

For any state-period, short-term consumer credit markets clear when the amount offered to be repaid at the end of the period is exchanged for the short term credit extension offered at the beginning of that period.

$$(1 + r^{\gamma}_{s^*}) = \frac{\mu^{\alpha}_{s^*}}{m^{\gamma}_{s^*}} \tag{43}$$

$$\left(1 + r^{\delta}_{s^*}\right) = \frac{\mu^{\theta}_{s^*}}{m^{\delta}_{s^*}} \tag{44}$$

2.7.5 Consumer Deposit Market

The consumer deposit market clears when the amount commercial banks offer to repay to households in the second period is exchanged for the amount of savings offered to deposit in the first period.

$$(1 + \bar{r}^{\gamma}_{d}) = \frac{\bar{\mu}^{\gamma}_{d}}{\bar{d}^{\theta}} \tag{45}$$

2.7.6 Wholesale Money Market

The wholesale money market clears when the aggregate amount offered to be repaid in the second period is exchanged for the long term credit extension offered in the first period.

$$(1 + \bar{r}) = \frac{\bar{\mu}^{\psi} + \bar{\mu}^{\phi}}{\bar{m}} \tag{46}$$

2.7.7 Repo Market

In every state-period, the repo market clears when the amount offered to be repaid at the end of the period is exchanged for the short term credit extension and the liquidity provided by the Central Bank (through OMO's) at the beginning of the period.

$$\left(1 + \rho^{CB}_{s^*}\right) = \frac{\mu^{G\delta}_{s^*}}{M^{CB}_{s^*} + d^{G\gamma}_{s^*}} \tag{47}$$

2.7.8 Interbank Market

The interbank market clears when the amount offered to be repaid in the second period is exchanged for the long term credit extension in the first period.

$$(1 + \bar{\rho}) = \frac{\bar{\mu}^{\delta}}{\bar{d}^{\gamma}} \tag{48}$$

2.7.9 MBS's Market

The MBS's market clears when the amount of money offered for these securities is exchanged for the quantity of MBS's offered for sale.

$$p^{\alpha} = \frac{\tilde{m}^{\alpha}}{\bar{m}^{\alpha}} \tag{49}$$

2.7.10 CDO's Market

The CDO's market clears when the amount of money offered for these securities is exchanged for the quantity of CDO's offered for sale.

$$\tilde{q}^{\alpha} = \frac{\hat{m}^{\alpha}}{\tilde{m}^{\alpha}} \tag{50}$$

2.8 Conditions on Expected Delivery Rates (Rational Expectations)

Rational expectations conditions imply that commercial banks are correct in their expectations about the fraction of loans that will be repaid to them.

2.8.1 Wholesale Money Market

δ's expected rate of wholesale money market loan delivery is given by

$$\bar{R}_s = \begin{cases} \dfrac{\bar{v}_s^{\psi}\bar{\mu}^{\psi} + \bar{v}_s^{\phi}\bar{\mu}^{\phi}}{\bar{\mu}^{\psi} + \bar{\mu}^{\phi}} & \text{if} \quad \bar{\mu}^{\psi} + \bar{\mu}^{\phi} > 0 \\ & \\ \text{arbitrary} & \text{if} \quad \bar{\mu}^{\psi} + \bar{\mu}^{\phi} = 0 \end{cases} \qquad \forall s \in S \tag{51}$$

2.8.2 Interbank Market

γ's expected rate of interbank loan delivery is given by

$$\bar{R}_s^{\delta} = \begin{cases} \dfrac{\bar{v}_s^{\delta}\bar{\mu}^{\delta}}{\bar{\mu}^{\delta}} = \bar{v}_s^{\delta} & \text{if} \quad \bar{\mu}^{\delta} > 0 \\ & \\ \text{arbitrary} & \text{if} \quad \bar{\mu}^{\delta} = 0 \end{cases} \qquad \forall s \in S \tag{52}$$

3 Equilibrium

3.1 Definition

Let

$$\begin{aligned}
\sigma^{\alpha} &= (q_{s1}^{\alpha}, b_{s2}^{\alpha}, \mu_s^{\alpha}, \bar{\mu}^{\alpha}) \in \Re^{s+1} \times \Re^{s+1} \times \Re^{s+1} \times \Re \\
\sigma^{\theta} &= \left(q_{s2}^{\alpha}, b_{s1}^{\alpha}, \mu_s^{\theta}, \bar{d}^{\theta}\right) \in \Re^{s+1} \times \Re^{s+1} \times \Re^{s+1} \times \Re \\
\sigma^{\gamma} &= \left(\phi_s^{\gamma}, m_s^{\gamma}, d_s^{G\gamma}, \bar{m}^{\alpha}, \bar{\mu}_d^{\gamma}, \bar{d}^{\gamma}\right) \in \Re^{s+1} \times \Re^{s+1} \times \Re^{s+1} \times \Re \times \Re \times \Re \\
\sigma^{\delta} &= \left(\phi_s^{\delta}, m_s^{\delta}, \mu_s^{G\gamma}, \bar{v}_s^{\delta}, \bar{m}, \bar{\mu}^{\delta}\right) \in \Re^{s+1} \times \Re^{s+1} \times \Re^{s+1} \times \Re^{s} \times \Re \times \Re \\
\sigma^{\psi} &= \left(\bar{v}_s^{\psi}, \bar{\mu}^{\psi}, \tilde{m}^{\alpha}\right) \in \Re^{s} \times \Re \times \Re \\
\sigma^{\phi} &= \left(\bar{v}_s^{\phi}, \bar{\mu}^{\phi}, \hat{m}^{\alpha}\right) \in \Re^{s} \times \Re \times \Re
\end{aligned}$$

Also, let the vector of macroeconomic variables be represented by

$$\eta = \left(p_{s1}, p_{s2}, \rho_s^{CB}, r_s^{\gamma}, r_s^{\delta}, \bar{r}^{\gamma\alpha}, \bar{r}_d^{\gamma}, \bar{r}, \bar{\rho}, p^{\alpha}, \tilde{q}^{\alpha}\right)$$
$$\in \Re^{s+1} \times \Re^{s+1} \times \Re^{s+1} \times \Re^{s+1} \times \Re^{s+1} \times \Re \times \Re \times \Re \times \Re \times \Re \times \Re$$

and the budget set of all agents denoted by

$$B^{\alpha}(\eta) = \{\sigma^{\alpha} : (2) - (7)\,\text{hold}\}$$
$$B^{\theta}(\eta) = \left\{\sigma^{\theta} : (9) - (13)\,\text{hold}\right\}$$
$$B^{\gamma}(\eta) = \{\sigma^{\gamma} : (15) - (16)\,\text{hold}\}$$
$$B^{\delta}(\eta) = \left\{\sigma^{\delta} : (20) - (22)\,\text{hold}\right\}$$
$$B^{\psi}(\eta) = \left\{\sigma^{\psi} : (25) - (27)\,\text{hold}\right\}$$
$$B^{\phi}(\eta) = \left\{\sigma^{\phi} : (31) - (33)\,\text{hold}\right\}$$

Then $\left(\sigma^{\alpha}, \sigma^{\theta}, \sigma^{\gamma}, \sigma^{\delta}, \sigma^{\psi}, \sigma^{\phi}, \eta\right)$ is a monetary equilibrium with commercial banks, collateral, securitization, and default (MEBCSD) iff:

1. All agents maximize given their budget sets:
   (a) $\sigma^h \in \arg\max_{\sigma^h \in B^h(\eta)} U^h\left(\chi_{s^*}^h\right)$, for $h \in H = \{\alpha, \theta\}$, $s^* \in S^*$
   (b) $\sigma^j \in \arg\max_{\sigma^j \in B^j(\eta)} \Pi^j\left(\chi_{s^*}^j\right)$, for $j \in J = \{\gamma, \delta\}$, $s^* \in S^*$
   (c) $\sigma^k \in \arg\max_{\sigma^k \in B^k(\eta)} \Pi^k\left(\chi_{s^*}^k\right)$, for $k = \{\psi, \phi\}$, $s^* \in S^*$

   Where $\chi_{s^*}^h$ is the vector of quantities of housing and goods consumed by agent h at state $s^* \in S^*$, $U^h(.)$ is households' utility function over consumption streams of goods and houses, and $\Pi(.)$ is the commercial banks and investors' utility function over their second period profits.

2. All markets clear. Hence, equations (36) – (50) hold.

3. Expectations are rational. Thus, conditions (51) – (52) are satisfied.

3.2 Properties of the MEBCSD

At each market meeting, money is exchanged for another commodity or security. Hence, the traditional transaction motive for holding money and the standard Hicksian IS/LM determinants of money demand, namely interest rates and income, are at work in this model.

3.2.1 The Transmission Mechanism of Monetary Policy, Credit Spreads, and the Term Structure of Interest Rates Proposition

At all states monetary policy is transmitted to the economy through the repo market via credit extension by commercial banks in the short term credit markets; however, in the first period long term credit markets create an additional channel for the transmission of monetary policy. After making a deposit in the repo market and extending short term credit to households, the rich bank takes consumer deposits, extends mortgage loans, sells its mortgage assets in the MBS's market, and makes a credit extension in the inter-period interbank market. On the other hand, after borrowing at the repo market and extending short term credit to households, the poor bank raises funds in the interbank market and makes long term credit extensions to investors in the wholesale money market.

Thus, banks portfolio decisions and default in the mortgage, interbank, short term credit, repo and wholesale money markets determine the money multiplier in the economy, as well as credit spreads between lending and borrowing interest rates. The model can encompass monetary propositions about credit spreads that hold because ex-ante interest rates are considered, and these do not incorporate a default premia. Therefore, in the presence of default, lending rates have to be at least as high as borrowing rates to preclude arbitrage opportunities.

Proposition 3.1 *At any MEBCSD, $r^{\delta}_{s^*}, \rho^{CB}_{s^*} \geq 0$ and, since household θ cannot default on short term credit loans, $r^{\delta}_{s^*} = \rho^{CB}_{s^*} \quad \forall s^* \in S^*$.*[7]

Proposition 3.2 *At any MEBCSD, $r^{\gamma}_{s^*}, \rho^{CB}_{s^*} \geq 0$ and, since household α cannot default on short term credit loans, $r^{\gamma}_{s^*} = \rho^{CB}_{s^*} \quad \forall s^* \in S^*$.*

Proposition 3.3 *At any MEBCSD, $\bar{r}^{\gamma}_{d}, \rho^{CB}_{0} \geq 0$ and, since bank γ cannot default on short term credit loans, $\bar{r}^{\gamma}_{d} = \rho^{CB}_{0}$.*

Proposition 3.4 *At any MEBCSD, $p^{\alpha}, \rho^{CB}_{0} \geq 0$ and $p^{\alpha} = 1 + \rho^{CB}_{0}$.*

Proposition 3.5 *At any MEBCSD, $\bar{r}$,$\bar{\rho}$,$\bar{r}^{\gamma}_{d} \geq 0$ and $\bar{r} \geq \bar{\rho} \geq \bar{r}^{\gamma}_{d}$.*

The term structure of interest rates is affected by both, liquidity provision by banks and default by households, investors and banks. The economy's overall liquidity affects the determination of interest rates because in this finite horizon model, money is fiat and must exit the system at the final period. This implies that both inside and outside money exit the economy via loan repayments by households/investors to commercial banks, loan repayments by commercial banks to the Central Bank, or by the Central Bank's liquidation of commercial banks. Moreover, default emerges as an equilibrium phenomenon that affects interest rates because these price-in anticipated default rates (default premium).

Put formally, $\forall s \in S$ aggregate ex-post interest rate payments to commercial banks adjusted by default equal the economy's total amount of outside money plus interest payments of commercial banks' accumulated profits. This is not the case in the first period, where uncertainty induces commercial banks to accumulate profits and/or make indirect investments in the derivatives markets; thus aggregate interest payments will be less than or equal to aggregate initial monetary endowments.

Proposition 3.6 *At any MEBCSD for $s \in S^{\alpha}_{1}$,*

$$\sum_{j \in J} \left(m^{j}_{0} r^{j}_{0} \right) + \rho^{CB}_{0} \bar{m}^{\alpha} + \sum_{j \in J} \left(\pi^{j}_{s} \right) + \rho^{CB}_{s} M^{CB}_{s} + \rho^{CB}_{0} \bar{r}^{\gamma\alpha} \bar{m}^{\alpha} =$$

$$\sum_{h \in H} \left(e^{h}_{m,0} + e^{h}_{m,s} \right) + \sum_{\tilde{k} = \{\gamma, \delta, \psi\}} \left(e^{k}_{0} + e^{k}_{s} \right) + \frac{r^{\gamma}_{0}}{1 + r^{\gamma}_{0}} \pi^{\gamma}_{0}$$

[7]See the Appendix 1, for the proofs of the propositions.

For $s \notin S_1^{\alpha}$,

$$\sum_{j \in J}\left(m_0^j r_0^j\right) + \rho_0^{CB}\bar{m}^{\alpha} + \sum_{j \in J}\left(\pi_s^j\right) + \rho_s^{CB} M_s^{CB} + \rho_0^{CB}\bar{m}^{\alpha}\left(\tilde{q}^{\alpha} - (1+\bar{r}_s^{\gamma\alpha})\right) =$$

$$\sum_{h \in H}\left(e_{m,0}^h + e_{m,s}^h\right) + \sum_{\tilde{k}=\{\gamma,\delta,\psi\}}\left(e_0^k + e_s^k\right) + \frac{r_0^{\gamma}}{1+r_0^{\gamma}}\pi_0^{\gamma}$$

For $t = 0$[8]

$$\sum_{j \in J}\left(m_o^j r_o^j\right) < \sum_{h \in H}\left(e_{m,0}^h\right) + \sum_{\tilde{k}=\{\gamma,\delta,\psi\}}\left(e_0^k\right)$$

3.2.2 Monetary Policy Non-Neutrality Proposition

I have introduced two nominal frictions to the model, private monetary endowments and default on credit markets, which ensure positive nominal interest rates by pining down the price of money (Dubey and Geanakoplos, 1992; Shubik and Wilson, 1977; Shubik and Tsomocos, 1992; Espinoza et al., 2008). Therefore, the essential role for money is at work in this model and is represented by a monetary transaction technology (cash-in-advance constraints) that captures the importance of liquidity for transactions.

Lemma 3.7 *Assume agent h borrows from bank j in the short term credit market. Furthermore, let $\left\{\chi_{s^*,l}^h, \chi_{s^*,m}^h\right\}$ denote traded quantities of two distinct goods $\{l, m\}$, and suppose that h purchases good l, and sells and has an endowment $\left(e_{s^*,m}^h\right)$ of good m at $s^* \in S^*$. If $r_{s^*}^j > 0$, then*

$$\frac{p_{s^*l}\left(1+r_{s^*}^j\right)}{p_{s^*m}} = \frac{u'\left(\chi_{s^*l}^h\right)}{u'\left(e_{s^*m}^h - \chi_{s^*m}^h\right)}$$

i.e. there is a wedge between selling and purchasing prices.

Proposition 3.8 *If nominal interest rates are positive, then monetary policy is non-neutral.*

3.2.3 The Quantity Theory of Money Proposition

The model has a non-trivial quantity theory of money. An agent will not hold idle cash he does not want to spend; instead, he will lend it out to someone who is willing to use it. It follows that if all the interest rates are positive, then in equilibrium the quantity theory of money holds with money velocity equal to one. Moreover, since quantities supplied in the markets are chosen by agents (unlike the representative agent model's sell-all assumption), the real velocity of money is endogenous. Consequently, nominal changes (i.e. monetary policy shocks) affect both prices and quantities.

At each state in the second period, nominal income equals the stock of money because all the liquidity available in the economy is channeled to commodity markets. However, at uncertainty and the inability of agents to complete the asset span will induce commercial banks to accumulate profits and/or make indirect investments in the derivatives markets (through credit extensions in the interbank and wholesale money markets).

Proposition 3.9 *In a MEBCSD, if $\rho_{s^*}^{CB} > 0$ for some $s^* \in S^*$, then aggregate income at $s \in S_1^{\alpha}$ is equal to the stock of money at that period, namely the total amount of outside and inside money, plus commercial banks' accumulated profits from the previous period, plus the banking financial sector's net payoffs from its indirect investments in the derivatives markets. When there is no default in the mortgage market, the*

[8] This condition holds with strict inequality when the system has an interior solution and with weak inequality otherwise.

mortgage's repayment is forgone income to commercial banks and is used by the hedge fund to repay its wholesale money market obligation.

$$\sum_{h\in H,l=\{1,2\}}\left(p_{sl}q_{sl}^{h}\right)=\sum_{h\in H}e_{m,s}^{h}+\sum_{j\in J}e_{s}^{j}+M_{s}^{CB}+\pi_{0}^{\gamma}+\bar{R}_{s}\bar{m}\left(1+\bar{r}\right)-\bar{m}^{\alpha}\left(1+\bar{r}^{\gamma\alpha}\right)$$

For $s\notin S_{1}^{\alpha}$, the banking financial sector's loss due to default on the mortgage and derivatives markets is embedded in the expected repayment rates of wholesale money market loans.

$$\sum_{h\in H,l=\{1,2\}}\left(p_{sl}q_{sl}^{h}\right)=\sum_{h\in H}e_{m,s}^{h}+\sum_{j\in J}e_{s}^{j}+M_{s}^{CB}+\pi_{0}^{\gamma}+\bar{R}_{s}\bar{m}\left(1+\bar{r}\right)$$

For $s=0$, national income is equal to the stock of money in the economy less indirect expenditures by commercial banks in the derivatives markets.

$$\sum_{h\in H,l=\{1,2\}}\left(p_{0l}q_{0l}^{h}\right)=\sum_{h\in H}e_{m,0}^{h}+\sum_{j\in J}e_{0}^{j}+M_{0}^{CB}-\bar{m}$$

3.2.4 The Fisher Effect Proposition

The model has an integral monetary sector where equilibrium interest rates are determined in nominal terms. Therefore, long term nominal interest rates equal their corresponding real interest rate plus the expected rate of inflation and a risk premium.

Proposition 3.10 *Suppose agent α chooses $b_{02}^{\alpha},b_{12}^{\alpha}>0$ and has money left over when the mortgage loan comes due, then at a MEBCSD the following equation must hold*

$$\left(1+\bar{r}^{\gamma\alpha}\right)=\left(1+\frac{u'\left(\chi_{02}^{\alpha}\right)}{u'\left(\chi_{02}^{\alpha}+\chi_{12}^{\alpha}\right)}\right)\left(\frac{p_{12}}{p_{02}}\right)$$

Similarly, assume agent θ chooses $b_{s^{*}2}^{\theta}>0\ \ \forall s^{*}\in S^{*}$, and has money left over when the consumer deposit market meets, then at a MEBCSD

$$\left(1+\bar{r}_{d}^{\gamma}\right)=\frac{u'\left(\chi_{01}^{\theta}\right)/p_{01}}{E_{s}\left\{u'\left(\chi_{s1}^{\theta}\right)/p_{s1}\right\}}=\frac{u'\left(\chi_{01}^{\theta}\right)/p_{01}}{\sum_{s\in S}\omega_{s}u'\left(\chi_{s1}^{\theta}\right)/p_{s1}}$$

Hence, nominal long term interest rates are approximately equal to real interest rates (which are linear in the inter-temporal marginal rates of substitution) plus expected inflation and a risk premium.

3.3 Discussion of Equilibrium

Hereafter, we analyze a parameterized version of the model (a numerical solution), where the chosen vector of parameter values allows an illustration of how default in the mortgage market hinges upon the nominal sector of the economy (see Table 1 of Appendix 2). We assumed two possible states of nature in the second period, and that a state 1 realization is more likely than a state 2 realization. In the first period, agent θ is relatively richer than α in monetary endowments; at all states bank γ is more capitalized than bank δ, the investment bank (ψ) has a very small amount of capital and the hedge fund (ϕ) has no capital.

The economy is supposed to experience an adverse productivity shock in the goods sector that is moderate in the first state and severe in the second state. Moreover, the Central Bank reacts by loosening monetary policy in state 1 and by tightening it in state 2; hence, relative to the first period the repo rate is lower at $s=1$ and higher at $s=2$.

Note that repo rates equal short term interest rates at all states, and that in the first period the deposit rate equals the repo rate (as the no arbitrage conditions for default-free loans hold). Furthermore, the wholesale money market rate is higher than the interbank rate, which in turn is higher than the deposit rate, thus confirming that the no arbitrage conditions for long term defaultable loans maintain.

In the benchmark equilibrium house and goods prices move in opposite directions; the relative price of houses drops from the first to the second period, and it's lower in state 2 than in state 1. This is a consequence of the negative supply shock in the goods market. Intuitively, agent α defaults on his mortgage when the value of his house is low, and house prices fall when goods endowments are scarce because α is forced to demand less housing due to lower goods sales revenues. Hence, lower demand in the housing market reduces house prices, while lower supply in the goods market raises the price of goods.

Moreover, as the relative price of houses drops across time, household α's marginal rate of substitution of housing over goods consumption decreases (lemma 7); hence, the volume of trade in the housing market is lower in the second period. Similarly, as the Fisher effect proposition holds, positive inflation rates of goods in both states of the second period imply that household θ's marginal utility of goods consumption is higher at $t = 1$, or equivalently, that the quantity of traded goods is higher at $t = 0$ than at either state of the second period.

Household α is rich in the endowment of goods at the first period; thus he can finance a large percentage of his desired housing expenditure with sales revenues, and he is not required to have a large loan-to-value mortgage. However, due to falling house prices, α defaults on his mortgage at state 2; consequently, the mortgage's effective return decreases and default rates in the wholesale and interbank markets increase at that state. Since rational expectations are assumed throughout, this induces bank γ to offer a very high mortgage rate in the first period.

In state 1, there is no default in the wholesale money market because α honors his mortgage obligation, which is the underlying asset to MBS's and CDO's securities; ψ and ϕ repay their wholesale money market obligations fully with the proceeds from the securitization premium and the mortgage's payoff respectively. However, bank δ defaults on a small percentage of its interbank liability, because that's the repayment rate that equates its marginal utility of default to the default penalty (δ's marginal disutility of default).

In contrast, at state 2 default in the mortgage market creates significant losses in the non-banking financial sector. The CDS contract forces the hedge fund to deliver the mortgage's collateral to ψ, and in return ϕ receives the total amount of its investment. ψ assumes a write-down loss because it has to go to the housing market and sell the collateral, which pushes house prices further down. Although, ψ and ϕ had undertaken hedging strategies and have no incentives to accumulate profits, their overall revenues are not enough to cover their obligations with bank δ. Thus, default increases in the wholesale money market, which reduces bank δ's revenues and forces it to default significantly on its interbank loan.

Finally, notice that since monetary policy is non-neutral and a non-trivial quantity theory of money maintains, expansionary monetary policy in state 1 offsets partially the adverse effects of the productivity shock on trade, while tighter monetary policy at state 2 exacerbates them.

The values of endogenous variables at the benchmark equilibrium are presented in Table 2 of Appendix 2.

3.4 Remarks on Welfare

There are two states and two assets (the MBS or mortgage and the CDO). However markets are not complete because there is default in the mortgage, interbank and wholesale money markets as well as limited participation of agents in the derivatives, repo, interbank, short term credit, consumer deposit, and wholesale money markets. For instance, only γ and ψ can trade in the MBS's market, and only ϕ and ψ are allowed to trade in the CDO's market. These restrictions prevent each agent from completing the asset span, which implies that financial markets are incomplete and that any MEBCSD is constrained inefficient. Therefore, there is scope for welfare improving economic policy, both regulatory and monetary (Geanakoplos and Polemarchakis, 1986).

4 Comparative Statics

In this section we describe how endogenous variables react to shocks by analyzing their directional response to changes in the vector of exogenous variables. The Newton's method is used to calculate numerically how the initial equilibrium changes as we vary each parameter at a time. We conducted several experiments, but we only report those we reckon more interesting: expansionary policy, government subsidies, tighter default penalties for commercial banks, investment banks and hedge funds in the bad states of the world, capital injections to commercial banks and investment banks in the bad states of nature, direct liquidity assistance to poor households in the bad states of the world, and an increase of the rich bank's risk aversion.

The purpose of these exercises is twofold; first, to show how certain measures contribute to financial fragility; and second, to assess the efficiency of different policies at improving welfare and promoting financial stability and market discipline. Hereafter, we will use the Goodhart-Tsomocos (2006) measure[9] to determine whether a policy promotes financial stability or not, and we will refer to policies that contribute to market discipline if they are successful at reducing the investment bank and hedge fund's borrowing (leverage). This analysis is based on the propositions derived in the previous sections.

4.1 Policies that Contribute to Financial Fragility

4.1.1 Expansionary Monetary Policy in the First Period

Let the Central Bank engage in expansionary monetary policy by increasing the money supply in the initial period (see column 1 of Table 3 in Appendix 2). This reduces the repo rate at that period, which induces bank δ to borrow more from the repo market, and bank γ to deposit less in it. To preclude arbitrage opportunities, both commercial banks increase their supply of short term loans, which reduces r_0^{γ} and r_0^{δ} until they equal the repo rate. Lower short term interest rates provide incentives to households to borrow and spend more in the goods and housing markets. This increases prices and the aggregate quantity of trade in the first period as predicted by the quantity theory of money proposition.

Household θ uses some of the additional liquidity available in the first period to increase his deposits with bank γ, thereby reducing the deposit rate. Consequently, bank γ takes additional deposits until the deposit rate is as low as the repo rate in order to exclude arbitrage.

Increased access to households' first-period savings, allows bank γ to extend more credit in the interbank and mortgage markets reducing their corresponding interest rates and the price of MBS's. A lower interbank rate induces bank δ to borrow more from γ. Consequently, δ extends more credit in the wholesale money market, thus reducing its interest rate. This provides incentives to ψ and ϕ to increase their leverage and spend more in the MBS's and CDO's markets. However, since demand in the latter rises more than in the former, the price of CDO's falls.

[9]According to these authors, an economy is financially unstable whenever substantial default of a "number" of households and banks occurs, and the aggregate profitability of the banking sector decreases significantly.

In the first period the purchasing price of houses increases relative to the selling price of goods; this leaves α with a sub-optimally low marginal rate of substitution (MRS) of housing over goods consumption, which induces him to reduce the supply of goods. The opposite is true for θ, who increases the supply of houses at $t = 0$ to bring down his MRS of goods over housing consumption from a sub-optimally high level.

By the term structure of interest rates proposition, short term rates fall at state 1 and increase at state 2. Intuitively, in the good state θ's deposit interest repayment increases relative to his desired level of spending, which induces him to borrow less in the short term credit market; this reduces r_1^δ. Hence, bank δ borrows less in the repo market putting downward pressure on its interest rate until $\rho_1^{CB} = r_1^\delta$. Similarly, bank γ reallocates its portfolio by shifting his short term deposits away from the repo market and into the short term credit market, which reduces r_1^γ until it reaches the repo rate level, thus precluding arbitrage opportunities. Mortgage and deposit rates fall, but lower goods and housing inflation at $s = 1$ have different offsetting effects on their corresponding real rates; housing inflation decreases less than the nominal mortgage rate, whereas goods inflation falls more than the deposit rate. Hence, by the Fisher effect proposition α anticipates consumption, while θ postpones it.

In the bad state, booming house prices at $t = 0$ decrease the collateral's worth, thereby increasing default by the investment bank. The opposite is true for the hedge fund because its CDS insurance repayment is worth more than its loan obligation. Thus, overall default in wholesale money market remains unchanged. This implies that bank γ's revenues from its long term lending and borrowing activities decrease at $s = 2$ since the spread between the interbank and deposit rate narrows. Moreover, to meet its increasing obligation in the deposit market, γ is forced to reduce lending in the repo and short term credit markets at $s = 2$ so as to make ρ_2^{CB} and r_2^δ rise equally and exclude arbitrage opportunities. A higher repo rate induces bank δ to borrow less from that market, which limits its ability to extend short term credit in the bad states of nature. Therefore, δ decreases its short term lending until $r_2^\delta = \rho_2^{CB}$.

In the second state, higher short term interest rates increase the purchasing price of houses and goods, leaving both α and θ, with sub-optimally low marginal rates of substitution. Thus, households reduce their spending in the goods and housing markets (by proposition 7), which decreases their respective prices,(p_{21}andp_{22}). Lower house prices at this state intensify the housing crisis.

Expansionary monetary policy at the initial period induces households to substitute consumption efficiently across time since their overall consumption stream rises. However, according to the Goodhart-Tsomocos (2006) financial stability measure, this policy contributes to financial fragility, because in the bad states of nature the poor household and the investment bank default more, and the rich bank's profits decrease. Bank γ is worse off because although default in the interbank market remains unchanged, the spread between the interbank and deposit rates narrows. Moreover, this policy exacerbates the mortgage crisis by inflating house prices in the initial period and lowering them in the bad states of nature of the subsequent period. This implies that market discipline also deteriorates as the investment bank and the hedge fund increase their leverage while MBS's and CDO's become riskier.

4.1.2 Government Subsidies (the Transfer Paradox)

Let the Government engage in an initiative that seeks to promote home ownership by exogenously increasing the endowment of houses in the economy (see column 2 of Table 5 in Appendix 2). Such a policy leaves θ with a sub-optimally high MRS of goods over housing consumption, which induces him to increase the supply of houses at all states, thereby reducing their price. This allows household α to spend less while still purchasing a larger amount of housing. Since the relative price of houses declines and consumption of housing rises at all states, α reduces his supply of goods to increase his MRS of housing over goods consumption from a sub-optimally low level (proposition 7); thus, the price of goods increases at all states. Consequently,

θ makes fewer deposits, borrows less and reduces his consumption of goods at all states but $s = 1$, where interest rates are low and allow him to finance his goods purchases with short term loans.

As household θ deposits less, the deposit rate rises. Hence, to preclude arbitrage opportunities, γ reallocates its first-period portfolio by reducing its deposits in the repo and short term credit markets until their corresponding interest rates rise equally and reach the deposit rate level. By the same non-arbitrage argument, the price of MBS's rises, thus incentivizing γ to extend more mortgage credit.

Similarly, bank δ borrows less in the repo market and reduces its short term credit assets until $r_0^\delta = \rho_0^{CB}$. The bank also substitutes short for long term lending by extending more credit in the wholesale money market; this requires δ to take out a larger loan in the interbank market, thereby increasing its interest rate and motivating γ to deposit more in that market. However, since the interbank rate increases slightly, the interbank-deposit credit spread narrows.

Higher short term interest rates in the initial period induce α to substitute short term for mortgage borrowing, which increases the mortgage rate. As mortgage lending and the price of MBS's rise, derivatives become expensive and induce ψ and ϕ to increase their leverage. Since mortgage rates are higher and the prices of houses falls more in the bad state than at the initial period, the effective mortgage repayment and the collateral's worth in the bad states of nature decrease. Hence, earnings in the shadow banking system decrease dramatically, forcing the investment bank and the hedge fund to default more on their wholesale money market obligations. Since rational expectations are assumed throughout, lower expected rates of repayment increase the wholesale money market interest rate (by proposition 5), but the wholesale money market-interbank credit spread remains unchanged.

By the term structure of interest rates proposition, short term interest rates fall at $s = 1$ and increas at $s = 2$. In the good states of nature investors do not default on their wholesale money market obligations, so δ's revenues increase and allow it to default less in the interbank market. This induces γ to extend more credit in the short term and repo markets, thereby reducing their corresponding interest rates equally so as to preclude arbitrage opportunities. A lower repo rate incentivizes bank δ to borrow more at that market and extend more credit to household θ at $s = 1$; this reduces r_1^δ until it reaches its non-arbitrage level. In the bad states of the world the opposite happens. Default increases in the wholesale money market while the wholesale money market-interbank credit spread remains unchanged; this erodes bank δ's revenues at $s = 2$ and force it to reduce its consumer short term credit extensions.

This policy improves α's welfare at the expense of household θ because the relative price of houses falls significantly at all states; therefore, household α's capacity to finance house purchases with goods sales improves, while θ's capacity to finance goods purchases with housing sales deteriorates. This result provides an example of the so-called Transfer Paradox, whereby an agent can be hurt by accepting a gift, the donor of which is made better off (see Mas-Collel and Leontieff)[10]

This measure promotes financial instability because in the bad states of nature default increases in the mortgage, interbank and wholesale money markets and the rich bank's profits decreases. Bank γ is worse-off as it assumes large losses due to increased default in the interbank market, a narrower interbank-deposit spread, and lower short term credit demand in the bad states of nature. On the other hand, bank δ's profits increase marginally because under the prevailing bankruptcy code, the bank is better off by accumulating profits than by defaulting less on its interbank obligation since the wholesale money market-interbank spread does not change.

[10]Initially, it was demonstrated that in a two agent, two good economy this transfer paradox could only occur at Walrasian unstable equilibria (see Samuelson (1947, 1952) and Balasko, (1978)) . However, subsequently Chichilnisky's (1980, 1983) observed that in a three agent two good economy, the transfer paradox could, indeed, occur at a globally Walrasian stable equilibrium. This point was further examined by Geanakoplos and Heal (1982).

Moreover, the policy fails to promote market discipline because it encourages the investment bank and the hedge fund to increase their leverage in an environment where MBS's and CDO's become riskier due to increased default in the mortgage market in the bad states of nature.

4.2 Policies for Crises Management and Prevention

4.2.1 Expansionary Monetary Policy in the Bad States of Nature

In principle, central banks can use the monetary base or the nominal (repo) interest rate as monetary policy instruments. The choice between adopting either of these instruments may have implications for the Central Bank's ability to maintain financial stability. To explore this issue, we describe and compare the effects of an expansionary monetary policy in the bad states of nature under both regimes.

Monetary Base Instrument (the Localized Liquidity Trap). Let the Central Bank engage in expansionary monetary policy at state 2 in the second period by increasing the monetary base and letting the repo rate clear the market (see column 2 of Table 3 in Appendix 2). This policy reduces the repo rate at $s = 2$, thus inducing bank δ to borrow more and bank γ to deposit less in that market. Consequently, both commercial banks increase their supply of short term loans in order to preclude arbitrage opportunities; this reduces r_2^{γ} and r_2^{δ} until they equal the repo rate. Lower short term interest rates provide incentives to households to borrow and spend more in the goods and housing markets. Hence, prices and the aggregate quantity of trade increase at state 2, as predicted by the quantity theory of money proposition.

Higher spending and inflation in the goods market at $s = 2$ leave θ with a sub-optimally high MRS of present over future consumption of goods. Hence, θ increases his expenditure in the goods market at the initial period (Fisher Effect proposition), for which he requires to make fewer deposits in bank γ and to borrow more in the short term credit market. This increases the price of goods and puts upward pressure on the deposit rate and the short term interest rate offered by bank δ.

Furthermore, higher spending and prices in the goods market in the first period, leave θ with a sub-optimally low MRS of goods over housing consumption, which induces him to reduce the supply of houses, thus increasing p_{02}. By the same arguments, α responds to higher goods prices by increasing his supply of goods market at $t = 0$, and then as house prices rise, he reduces his housing expenditures.

The relative price of goods at the initial period falls, thereby incentivizing θ to increase further his expenditure in the goods market. This improves α's sales revenues at that period and induce him to substitute mortgage for short term borrowing, which reduces the mortgage rate. Since the mortgage rate decreases and house prices increase more in the bad state than in the initial period, the collateral's worth and effective mortgage rate rise at $s = 2$. Therefore, ψ and ϕ default less on their respective wholesale money market obligations, thereby improving bank δ's profits and allowing it to increase its repayment rate in the interbank market. Lower levels of expected default reduce the interbank and wholesale money market interest rates (by proposition 5).

Nonetheless, these interest rates rise back to their original levels. As θ makes fewer deposits, the deposit rate increases significantly. This represents a shortage of long term funds for bank γ, and hence for bank , which limits commercial banks' capacity to lend in the second period. Therefore, default in the interbank market increases (although in the bad state the net effect is still negative), inducing γ to take fewer deposits from households and re-allocate its portfolio away from risky-assets (mortgage and interbank lending) and into (safe) short-term credit assets. This brings the deposit and interbank rates back to their original levels and puts downward pressure on first-period short term interest rates.

However, since the credit spread between the interbank and repo interest rates fails to narrow, δ demands even more credit in the repo market, which pushes the repo rate back up to its initial value. Therefore, δ provides less short term credit to household θ, thereby raisin r_0^{δ} up to its original level. Finally, to preclude arbitrage opportunities bank γ moderates its extension of consumer credit to households and reduces its deposits in the repo market until r_0^{γ} and ρ_0^{CB} reach their initial values.

Anticipated expansionary policy in the bad states of nature fails to improve households' welfare. Household α is just as well-off because he is able to smooth consumption efficiently across time due to a lower mortgage rate and higher relative goods prices at $s = 2$; whereas, household θ is worst-off because he is credit constrained, and hence, unable to substitute future for present consumption efficiently. Furthermore, this policy is unsuccessful in promoting financial stability because, although default in the mortgage, interbank, and wholesale money markets decreases, commercial banks' profitability falls in the bad states of nature as credit conditions in the interbank and wholesale money markets fail to ease.

Thus, once a crisis unravels the economy falls into a localized liquidity trap [11] as monetary policy becomes ineffective at easing credit conditions between financial institutions. This result provides evidence that the transmission mechanism of monetary policy is distorted as commercial banks reallocate their portfolios away from risky assets and into default-free assets, which by assumption include consumer loans in this model. Moreover, although liquidity is channeled to consumer credit markets, interest rates remain unchanged and households are subject to credit rationing.

Repo Rate Instrument. Let the Central Bank engage in expansionary monetary policy at state 2 in the second period by reducing the repo rate and letting the money supply clear the repo market (see column 3 of Table 3 in Appendix 2). The effects of this measure lead broadly to the same results as a money supply expansion, but the crucial differences rely on the better functioning of the transmission mechanism of monetary policy, which has *important* implications for financial stability.

Bank γ is subject to a shortage of long term funds after the deposit rate increases significantly; however, when the Central Bank fixes the repo rate at lower level in $s = 2$ these additional funds are quasi-automatically [12] supplied to commercial banks, because their lending capacity at the second period is less affected. Consequently, and in contrast to the money supply setting, bank γ re-allocates its portfolio away from the repo market deposits and into short-term consumer, mortgage and interbank lending. This reduces the interbank rate, and pushes the mortgage rate further down, thereby improving the wholesale and interbank market repayment rates for the bad states of nature.

A lower interbank rate allows bank δ to borrow more long term funds and extend more credit in the second period consumer credit and wholesale money markets. Note that in contrast to the monetary supply setting, the interbank-repo and wholesale-repo credit spreads narrow.

A reduction of the repo rate in the bad states of nature doesn't improve financial stability fully either; default rates fall in the mortgage, interbank and wholesale money markets, but the banking sector's profits drop as credit conditions between financial institutions do not ease enough. However, households' welfare

[11] The well-known liquidity trap is an extreme case of financial instability, where the latter is coupled with monetary policy ineffectiveness. Various authors provide formalizations of the liquidity trap based on non-rational expectations (Tobin ,1982, Grandmont and Laroque ,1973, and Hool ,1976). However, in this model this phenomenon is related to the explanation proposed by Dubey Geanakoplos, 2003 and Tsomocos 2003, whereby a liquidity trap occurs because of the incompleteness of asset markets, and it manifests itself when the government employs an expansionary monetary policy and commercial banks do not channel the increased liquidity to the consumer credit markets but the asset market. The proof of a liquidity trap proposition for this model is quite technical and out of the scope of this paper. Nonetheless, the comparative statics exercise shows that a localized version of this phenomenon is at work in this model because expansionary monetary policy fails to ease credit transactions between financial institutions.

[12] See Goodhart et.al. (2008) and Steiger (2006).

improves because they are able to substitute consumption across time and goods efficiently as short term consumer and mortgage lending increase.

Comparison of Expansionary Monetary Policies in the Bad States. The simulations results suggest that the repo interest rate is preferable to the monetary base as policy instrument. In times of financial distress agents lose confidence in the banking system, which increases significantly the demand for safe and liquid assets. If the Central Bank uses the interest rate as policy instrument, it will quasi-automatically satisfy the additional demand for money, whereas if the money supply is used, that extra demand for money will drive key interest rates up, thereby leading the economy to a (localized) liquidity trap.

4.2.2 Tighter Default Penalties in the Bad States of Nature

Tighter Default Penalties for the Low Capitalized Commercial Bank. Let the FSA set a stricter bankruptcy code that affects the poor commercial bank by increasing its default penalty in the bad states of the world (i.e. $\bar{\tau}_2^{\delta}$ rises) (see see column 1 of Table 4 in Appendix 2) This policy induces bank δ to accumulate fewer profits and default less on its interbank obligation at that state. Higher expected repayment rates reduce the interbank rate (by proposition 5) and they encourage bank γ to extend more credit in that market. Hence, γ takes more consumer deposits, thereby raising the deposit rate. To preclude arbitrage opportunities in the first period, the bank also reduces short term consumer credit and makes fewer deposits in the repo market until ρ_0^{CB} and r_0^{γ} reach the deposit rate level.

Higher short term interest rates at the initial period provide incentives to household α to substitute short term for mortgage borrowing, which puts upward pressure on the mortgage rate and induces bank γ to extend more mortgage loans. Since the additional extension of mortgage credit is significant, the mortgage rate drops back to its original level.

On the other hand, a lower interbank rate allows bank δ to extend more credit in the wholesale money market while still reducing its liabilities with bank γ. Therefore, the wholesale money market rate drops inducing ψ and ϕ to increase their leverage and spending in the derivatives markets. Higher leverage moderates the fall of the wholesale money market rate and causes the wholesale money market-interbank credit spread to widen, whereas higher demand for structured products increases the price of MBS's and CDO's.

By the term structure of interest rates proposition, short term interest rates drop at $s = 1$ and rise at $s = 2$. In the good state, household α cannot increase its housing expenditure despite lower short term interest rates, because his mortgage debt burden has increased. On the contrary, household θ is able to increase his goods spending because of lower short term interest rates and higher deposit interest payments. At the initial period and the bad states of nature, higher short term interest rates induce households to spend and trade less in the goods and housing markets, thereby reducing their corresponding prices.

Furthermore, since the price of houses drops more at $s = 2$ than in the first period, the collateral's worth and effective mortgage repayment in the bad states of the world fall, which forces the investment bank to default more on its wholesale money market obligation. The hedge fund's level of default remains unchanged because the balance sheet effects of lower funding costs in the wholesale money market and higher leverage offset each other.

Tighter Default Penalties for the Investment Bank. Let the FSA set a stricter bankruptcy code that affects the investment bank by increasing its default penalty in the bad states of the world (i.e. $\bar{\tau}_2^{\psi}$) (see column 2 of Table 4 in Appendix 2). This policy induces ψ default less at that state by reducing its leverage and spending in the MBS's market. Consequently, the price of MBS's drops while the price of CDO's increases. By proposition 5, as default by ψ decreases, the wholesale money market rate drops and the wholesale money market-interbank credit spread narrows. This allows the hedge fund to make a larger CDO's investment while still reducing its leverage, which induces bank δ to extend less credit in that market and borrow less in the interbank market.

Although, lower demand for interbank funding puts downward pressure on the interbank rate, the latter remains unchanged. The intuition is as follows: as the price of MBS's and interbank rate fall, bank γ re-allocates its portfolio at $t = 0$ by substituting long for short term assets; to exclude arbitrage, the bank extends more credit in the repo and short term credit markets, and reduces mortgage and interbank credit extensions until ρ_0^{CB} and r_0^{γ} decrease equally. Consequently, the mortgage rate increases and the interbank rate rises up to its original level. Since γ reduces its long term lending activities, it requires fewer deposits from household θ, which reduces the deposit rate until it reaches a level consistent with the no arbitrage condition; thus the interbank-deposit credit spread widens. Similarly, bank δ ncreases its borrowing in the repo market and extends more short term credit to households at the initial period until r_0^{δ} falls to the prevailing repo rate level.

Even though short term interest rates drop at the initial period, their corresponding credit flows rise, thereby leaving aggregate interest payments unchanged. Thus, by the term structure of interest rates proposition, short term interest rates in both states of the second period, as well as the mortgage rate, remain unchanged. The latter doesn't change because lower short term interest rates at the initial period and a higher mortgage rate induce household α to substitute mortgage for short term borrowing, which brings the mortgage rate back down to its original level.

Lower levels of default in the wholesale money market at $s = 2$ allow commercial banks to extend more credit to households, who increase their goods and housing expenditure, thus inflating their corresponding prices. Since house prices increase at $s = 2$, the collateral's worth and effective mortgage rate in the bad states of nature rise, allowing ψ and ϕ to increase further their wholesale money market repayment rates.

Tighter Default Penalties for the Hedge Fund. Let the FSA set a stricter bankruptcy code that affects the hedge fund by increasing its default penalty in the bad states of the world (i.e. $\bar{\tau}_2^{\phi}$) (see column 3 of Table 4 in Appendix 2). This policy induces ϕ to default less at that state by decreasing its leverage and spending in the CDO's market, which reduces the price of this security. As ϕ defaults less, the difference between the wholesale money market and interbank rates narrows because the former drops. Lower funding costs at the wholesale money market allow the investment bank to spend more on the MBS's market while still reducing its leverage.

Higher demand increases the price of MBS's, thereby providing incentives to bank γ to extend more mortgage credit, which reduces the mortgage rate. To preclude arbitrage opportunities, γ re-allocates its first period portfolio by making fewer deposits in the repo market and lending less in the short term credit market such that ρ_0^{CB} and r_0^{γ} rise equally to their non-arbitrage level. Consequently, bank δ substitutes short for long term assets in the initial period; a higher repo rate incentivizes δ to borrow less in that market, extend fewer short term loans to households, and increase its wholesale money market lending. The latter requires bank δ to borrow more at the interbank market, thus raising its interest rate. This incentivizes γ to extend more interbank loans for which it requires to takes more deposits from households. Hence, the deposit rate increases until it reaches its non-arbitrage level, and the interbank-deposit spread narrows.

Higher short term interest rates at the initial period reduce households' spending in the goods and housing markets. However, the relative price of houses increases because a higher deposit rate induces household θ to reduce the supply of houses, while a lower mortgage rate induces household α to increase the supply of goods (Fisher Effect proposition). Moreover, since short term credit extension falls, aggregate interest payments decrease. Thus, by the term structure of interest rates proposition, short term interest rates decrease at $s = 1$ (as mortgage interest payments rise) and increase at $s = 2$.

In the bad states of nature, the collateral's worth falls because of higher house prices in the first period. Therefore, the investment bank defaults more on its wholesale money market loan, thereby reducing bank δ's revenues and forcing it to default more on its interbank obligation. Moreover, since δ runs short on liquidity, it extends fewer short loans to households, which increases r_2^δ. To preclude arbitrage, bank γ reduces its deposits in the repo market and extends fewer short term loans to households. Because of higher short term interest rates in the bad states, households decrease their spending. This reduces the prices of goods and houses at that state, thereby exacerbating the mortgage crisis.

Comparison of Tighter Bankruptcy Code Policies. Households' welfare improves (weakly) only in the case where the regulator increases the default penalty to the investment bank because agents are able to substitute consumption efficiently across time; while α postpones consumption, θ anticipates it. With the other two policies the rich household is better off and the poor household is worse off; in the case of tighter default penalties for the commercial bank, this happens because deposit and mortgage interest payments rise, whereas in the case of a stricter bankruptcy code for the hedge fund, a severe fall in the relative price of goods in the first period reduces the poor household's consumption stream by limiting his capacity to finance house purchases with goods sales.

Similarly, commercial banks' profits in the bad states of nature only improve with the policy affecting investment banks. This measure reduces default in the interbank and wholesale money markets as ψ and ϕ lower their leverage levels, which enables γ and δ to extend more credit to households. Hence, commercial bank's profits rise because short term interest rates in the bad state remain unchanged and consumers cannot default on their short term loans.

With the other two policies, either bank is worse off. Imposing a higher default penalty to the low capitalized bank reduces default and stimulates lending in the interbank market. However, this policy encourages ψ and ϕ to increase their leverage and investment positions in the derivatives markets, which in case of default in the mortgage market, erodes the rich bank's profits. The policy that imposes higher default penalties on the hedge fund fails to reduce default by the investment bank and the low capitalized bank; moreover, it doesn't discourage the investment bank from increasing its leverage, which in an environment of narrower wholesale money market-interbank credit spreads lowers the poor bank's profits. In sum, tighter bankruptcy code policies affecting the investment bank, the agent engaged in securitization activities, are the most effective at improving the banking sector and household's welfare as well as promoting financial stability.

4.2.3 Remarks on Direct Liquidity Assistance to Poor Households

We also conducted an experiment whereby the Government provides direct liquidity assistance to the poor household in the bad states of nature by raising his private monetary endowments $\left(e_{m,2}^{\alpha}\right)$ (see column 1 of Table 5 in Appendix 2).

This policy resembles a fiscal expansion because it stimulates spending, but by the term structure of interest rates proposition, it also requires households to increase their interest payments at the final period. Providing direct liquidity assistance to α in the bad states of nature improves this household's welfare at the expense of the rich one, who is forced to anticipate consumption inefficiently because his deposit interest payments fall and relative house prices drop, thus reducing his income stream.

The policy's effect on the financial sector is very positive because it promotes financial stability and market discipline by improving commercial banks' profitability in the bad states of nature, reducing default in the interbank and wholesale money markets, and encouraging investment banks and hedge funds to moderate their leverage ratios.

4.2.4 Increased Risk-Aversion of the Rich Commercial Bank

Let bank γ's risk aversion coefficient increase; with this shock we proxy the effects of a regulatory change, whereby assuming commercial banks are subject to a Basel I type regulation, the FSA imposes higher risk weights for interbank loans and mortgages (see column 3 of Table 5 in Appendix 2). This policy widens the interbank-deposit credit spread because γ demands a higher compensation for bearing the risk of lending to bank δ. This implies that γ re-allocates its portfolio away from interbank loans and into repo and short term credit assets, which induces it to take fewer consumer deposits. Consequently, $\bar{r}_s^{\gamma}$, r_0^{γ} and ρ_0^{CB} fall equally so as to preclude arbitrage opportunities.

Lower short term interest rates at the initial period have two effects; on the one hand, they induce household α to substitute mortgage for short term borrowing, thereby reducing the mortgage rate and discouraging bank γ from extending mortgage loans; on the other hand, they increase households spending in the goods and housing markets, which raises the price of these goods.

Since the mortgage rate falls and housing inflation decreases at the second period, α increases his housing expenditure at $s = 1$ to raise his MRS of present over future housing consumption from a sub-optimally low level. This increases the relative price of houses at that state, which induces α to lower the supply of goods. Thus, goods prices increase and discourage θ from spending in the goods market at that state.

On the other hand, as the deposit rate falls by less than goods inflation in the second period, θ spends more in the goods market at $s = 2$ to raise his inter-temporal MRS of present over future goods consumption from a sub-optimally low level. This increases the relative price of goods at that state and α's revenues at that state, which induces him to spend more in the housing market. Consequently, house prices rise at $s = 2$.

Higher spending in the goods and housing markets in the bad states of nature induce households to borrow more in the short term credit markets, thereby increasing short term interest rates and providing incentives to commercial banks to increase their lending activities in those markets. Moreover, since house prices rise more in the bad state than at the initial period, the effective mortgage rate at $s = 2$ increases, thus inducing the investment bank to default less on its long term loan.

Since overall default in the wholesale money market decreases, bank δ's revenues in the second period improve, which allows it to default less on its interbank obligation in the bad states of the world. Nonetheless, the interbank and wholesale money market rates increase because γ reduces its supply of interbank loans; this induces bank δ to borrow less at that market and to extend fewer wholesale money market loans. Moreover, the wholesale-interbank credit spread narrows. Higher funding costs in the wholesale money market induce ψ and ϕ to reduce their leverage and spending in the MBS's and CDO's market, thereby reducing their respective prices.

This policy improves households' welfare; the household affiliated with the rich bank is able to increase his levels of consumption at all states, while the other one anticipates consumption efficiently. The intuition is that as the rich bank's risk aversion increases, its preference for consumer loans over interbank loans rises, which makes credit conditions more favorable to households.

This measure improves the rich bank's profits in the bad states of nature because wider credit spreads between the deposit and the interbank rate increase its second period revenues and its ability to extend short term credit. The opposite is true for the poor bank; the policy narrows the credit spread between the interbank rate and the wholesale money market rate, thereby hurting δ's revenues and lending capacity in the final period despite lower default in the wholesale money market.

Overall this policy is successful at promoting financial stability and market discipline because it reduces default in the mortgage, interbank and wholesale money markets, improves rich commercial banks' profitability, and induces investment banks and hedge funds to reduce the leverage ratio on their derivatives investments.

5 Implications for Inflation Targeting

Central banks around the world operate under an inflation targeting regime, whereby the short term interest rate is set to stabilize the rate of inflation of goods and services. However, the current financial crisis has reminded us all that, in addition to achieving price stability, central banks are responsible for maintaining financial stability. One of the problems of central banking is that these two objectives may often be conflicting. Our model portrays this disjuncture.

We chose a parameterization for the benchmark equilibrium of the model that allowed the Central Bank to follow actions consistent with an inflation targeting strategy. Note that goods have a positive inflation rate while house prices fall, and that the Central Bank tightens monetary policy when goods inflation is very high. Nevertheless, in those states of nature default rates in the mortgage, interbank and wholesale money markets also reach their highest levels.

Although the initial equilibrium is driven by demand (monetary policy) shocks and supply shocks, it is possible to assess the contribution of monetary policy to financial instability through the comparative statics exercises. Our simulations show that expansionary monetary policy in the first period promotes financial instability by increasing aggregate default and reducing commercial banks' profits; but in the bad states of nature of the second period, expansionary monetary policy (if effective) reduces default in the mortgage, interbank and wholesale money markets despite failing to improve the banking sector's profits.

Hence, we can argue that by taking into account house prices to conduct monetary policy (e.g. by widening the targeted Consumer Price Index to include an appropriate measure of housing prices), the Central Bank can contribute to financial stability.

However, one of the reasons why central banks are subject to trade-off price and financial stability is the fact that they have only one instrument, the short term interest rate. Thus, the objective of financial stability could be better achieved by the development and application of separate instruments designed for that purpose. This proposal is fully developed by Brunnermeier et.al (2009), who stress the importance of designing a *counter-cyclical* regulatory mechanism aimed at reducing the systemic risks that are threatening to financial stability.

Akin to this idea, our results show that regulation of systemic financial institutions is more effective than monetary policy at promoting financial stability. Although we do not model explicitly a regulatory institution, our simulations show that by implementing measures that induce bank γ (the interbank lender) and the investment bank (the CDS seller), to behave more prudently ex-ante, reduces default and improves the banking sector's profitability without deteriorating households' welfare.

6 Concluding Remarks

This model overcomes some of the limitations that DSGE models have for undertaking financial stability analyses, which have become extremely relevant under the current juncture. We present a framework that incorporates heterogeneous agents, endogenous default, an essential role for money, and incomplete financial markets; these elements ensure that financial fragility arises as an equilibrium outcome, thereby justifying the role of economic policy. Moreover, to understand and explore policy issues related to the current financial crisis, we introduced collateralization and securitization to the model, because these elements capture financial markets' innovations over the recent past.

The results of the simulations provide evidence that the propositions describing the properties of the equilibrium hold as well as common-sense insights about the efficiency of different policies for crisis prevention and management. Changes to money supply feed into prices and quantities as predicted by the quantity theory of money proposition. Although we have assumed an endowment economy whereby the volume of goods is fixed by definition, the comparative statics exercises show that the remaining variables of the real sector (traded quantities of goods and assets and real interest rates) are non-neutral to monetary (or regulatory) policy changes. It is also observed that the Fisher effect is incorporated in the model because nominal long-term interest rates change in response to changes in real rates and expected inflation. Finally, the simulations show that interest rate differentials move in response to changes in aggregate liquidity and default risk as predicted by the term structure of interest rates and credit spreads propositions.

Regarding policy analysis, our results suggest that government subsidies and expansionary monetary policy in the first period are crisis catalysts. This result is consistent with the United States recent experience; on the one hand, extremely accommodative monetary policy in a world where regulation could not keep up with emerging financial innovations, was a main contributor to the emergence of the current financial crisis as argued by (Calomirirs, 2008); on the other hand, the U.S. government subsidized home ownership in ways that rewarded mortgage leverage, thus promoting financial fragility. These measures include deductibility of owned home mortgage interest, government funding subsidies that lower mortgage rates [13], and government initiatives that induce banks to increase credit access for low income individuals (Calomiris, 1989, 1990, 1992, Caprio and Klingebiel 1996a, 1996b, and Dermirguc-Kunt, et.al. 2008).

The comparative statics exercises also show that in times of financial distress expansionary monetary policy implemented by means of the money supply is ineffective. This is due to the fact that once a crisis unravels, the transmission mechanism of monetary policy is distorted and the economy falls into a localized liquidity trap: commercial banks reallocate their portfolios away from risky assets and into default-free assets, credit flows between financial institutions fail to recover, and households are subject to credit rationing.

However, when the interest rate is used as the monetary policy instrument, the Central Bank automatically satisfies the additional demand for money that arises in times of crisis, thus succeeding to ease credit conditions between financial institutions and allowing households to have more access to credit. As advocated by some economists (e.g. Calomiris, 2008), these results suggest that in times of financial distress monetary authorities should deliver targeted assistance through the discount window.

Due to agent heterogeneity, the effects of some policies depend on the particular agent, or part of the economy, on which they fall because the distribution of income and welfare between agents is affected differently (Goodhart 2004). This is evidenced by the comparative statics exercises. Firstly, tighter default penalties worked better when imposed to the investment bank because this agent bears the risk of mortgages and CDS's, which are the riskiest assets in the economy; secondly, if the FSA could implement a policy whereby large commercial banks were incentivized to behave more prudently, then financial stability and market discipline would improve.

Finally, we suggest that monetary policy conduct should take into account the behavior of housing prices in order to promote financial stability. However, our results show that regulatory measures, which are implemented by means of policy instruments different from those used to achieve price stability, are more effective. This is consistent with economists' common view that a key factor contributing to the current

[13]Via Federal Home Loan Bank lending and liability protection for Fannie Mae and Freddie Mac.

financial turmoil were the regulatory flaws that weakened market discipline; these include the procyclicality of capital requirements (Catarineu et.al., 2004), and the lack of liquidity requirements and leverage limits.

References

[1] Balasko, Y. (1978), 'The Transfer Problem and the Theory of Regular Economies', *International Economic Review*, 19, pp. 687-694.

[2] Barsden, G., K. Linduist, and D. Tsomocos (2008), 'Evaluation of Macroeconomic Models for Financial Stability Analysis,' *Journal of World Economic Review*, Volume 3:1.

[3] Bhattacharya, S., C.A.E. Goodhart, P. Sunirand, and D.P. Tsomocos, (2007), 'Relative Performance, Banks and Sequential Contagion', *Economic Theory*, 32, 2, pp. 381-398.

[4] Bernanke, B., M. Gertler, and S. Gilchrist (1999), 'The Financial Accelerator in a Quantitative Business Cycle Framework' in J. Taylor and M. Woodford (eds) *Handbook of Macroeconomics*, Volume 1C, North Holland.

[5] Bloize G., J.H. Dreze and H. Polemarchakis (2005), 'Monetary Economy over Infinite Horizon', *Journal of Economic Theory*, 25(1), pp. 51-74.

[6] Brunnermeier, M. et.al.,(2009), 'The Fundamental Principles of Financial Regulation', *Geneva Reports on the World Economy – Preliminary Conference Draft*, 11.

[7] Bryant, J., (1982), 'A Model of Reserves, Bank Runs, and Deposit Insurance', *Journal of Banking and Finance*, 4, pp. 335-344.

[8] Buiter,W. H. (1999), 'The Fallacy of the Fiscal Theory of the Price Level', NBER Working Paper No. 7302.

[9] Calomiris, C. (2008), 'The Subprime Turmoil: What's Old, What's New, and What's Next', Federal Reserve Bank of Kansas City Symposium, Federal Reserve Bank of Kansas City.

[10] Caprio, G. and D. Klingebiel (1996a), 'Bank Insolvencies: Cross-Country Experience', Working Paper No. 1620, World Bank.

[11] Caprio, G. and D. Klingebiel (1996b), 'Bank Luck, Bad Policy, or Bad Banking?', Annual Bank Conference on Development Economics, World Bank.

[12] Catarineu-Rabell, E., Jackson, P. and Tsomocos, D.P. (2004), 'Procyclicality and the new Basel Accord-banks' choice of loan rating system'. *Economic Theory*, vol. 26, pp. 537-557.

[13] Dem Demirguc-Kunt, et. al (2008), 'Deposit Insurance Around the World', Cambridge, MA: MIT Press.

[14] Diamond, D. and P. Dybvig, (1983), 'Bank Runs', *Journal of Political Economy*, 91, pp. 401-419.

[15] Dubey, P. and J. Geanakoplos, (1992), 'The Value of Money in a Finite-Horizon Economy: a Role for Banks', in P. Dasgupta, D. Gale et al. (eds), *Economic Analysis of Market and Games*, Cambridge MA: MIT Press.

[16] Dubey, P., J. Geanakoplos, and, M. Shubik, (2005), 'Default and Punishment in General Equilibrium', *Econometrica*, 73 (1) , pp. 1-37.

[17] Dubey, P., Shubik, M., (1978), 'The Non-cooperative Equilibria of a Closed Trading Economy with Market Supply and Bidding strategies', *Journal of Economic Theory*, 17, pp. 1-20.

[18] Eagle, D. (2005). 'Multiple Critiques of Woodford's Model of a Cashless Economy,' Macroeconomics 0504028, EconWPA database , accessed on November 16, 2006. http://ideas.repec.org/p/wpa/wuwpma/0504028.html

[19] Espinoza R., C. Goodhart and D. Tsomocos (2008), 'State Prices, Liquidity and Default', *Economic Theory*, 39 (2), pp. 174-199.

[20] Geanakoplos, J. (2003), 'Liquidity, Default, and Crashes: Endogenous Contracts in General Equilibrium, Advances in Economics and Econometrics: Theory and Applications',8th World Conference, Volume II, *Econometric Society Monographs.*

[21] Geanakoplos, J. and G.M. Heal, (1983), 'A Geometric Explanation of the Transfer Paradox in a Stable Economy', *Journal of Development Economics.* 13, pp. 223-236.

[22] Geanakoplos, J.D., Polemarchakis, H.M., (1986), 'Existence, Regularity and Constrained Suboptimality of Competitive Allocations when the Asset Market is Incomplete'. In: Heller,W., Starret, D. (Eds.), *Essays in Honor of K. Arrow*, vol. III. Cambridge University Press.

[23] Geanakoplos J., and W. R. Zame, (2002), 'Collateral and the Enforcement of Contracts', Cowles Foundation Discussion Paper.

[24] Goodhart CAE, P. Sunirand P. and D. Tsomocos, (2004), 'A Model to Analyse Financial Fragility: Applications', *Journal of Financial Stability*, 1, pp. 1-30.

[25] Goodhart CAE, P. Sunirand P. and D. Tsomocos, (2006), 'A Model to Analyse Financial Fragility', *Economic Theory* (27), pp. 107-142.

[26] Goodhart CAE, P. Sunirand P. and D. Tsomocos, (2008), 'The Optimal Monetary Instrument for Prudential Purposes' (forthcoming). Working Paper *ORFC fe-26*, University of Oxford, Said Business School.

[27] Goodhart CAE, D. Tsomocos and A. Vardoulakis, (2009), 'Modelling a Housing and Mortgage Crisis', in *Financial Stability, Monetary Policy and Central Banking.* Edited by C. Echavarria, R. Fuentes and D. Gray, Series on Central Banking, Analysis and Economics Policies, Central Bank of Chile, forthcoming, 2009.

[28] Grandmont, J.M., and G. Laroque, (1973), 'On Money and Banking'. *Review of Economic Studies*, pp. 207-236.

[29] Hool, R.B., (1976), 'Money, Expectations and the Existence of a Temporary Equilibrium', *Review of Economic Studies* (40), pp. 439-445.

[30] Leontief, W., (1936), 'Note on the Pure Theory of Capital Transfers', in *Explorations in economics: notes and essays contributed in honor of F.W. Taussig.* McGraw-Hill, New York, pp. 84-91.

[31] Lucas R. and N. Stokey, (1983), 'Optimal Fiscal and Monetary Policy in an Economy Without Capital', *Journal of Monetary Economics*, 12, pp. 55-93.

[32] Lucas R. and N. Stokey, (1987), 'Money and Interest in a Cash-in-Advance Economy', *Econometrica*, 55(3), pp. 491-513.

[33] Mas-Colell ,A., M. Whinston, and J.R. Green, *Microeconomic Theory*, Oxford University Press, Oxford, p. 542.

[34] Samuelson, P.A., (1947), *Foundations of Economic Analysis.* Harvard University Press, Cambridge, MA.

[35] Samuelson, P.A., (1952), 'The Transfer Problem and Transport Costs: the Terms of Trade when Impediments are Absent'. *The Economics Journal*, 62, pp. 278-304.

[36] Shubik M. and D.P. Tsomocos, (1992), 'A Strategic Market Game with a Mutual Bank with Fractional Reserves and Redemption in Gold (A Continuum of Traders)', *Journal of Economics*, 55(2), pp. 123-150.

[37] Shubik M., Wilson C., (1977), 'The Optimal Bankruptcy Rule in a Trading Economy Using Fiat Money', *Journal of Economics*, 37, pp. 337-354.

[38] Svensson L. (1985), 'Money and Asset Prices in a Cash-in-Advance Economy', *Journal of Political Economy*, 93 (5), pp. 919-944.

[39] Steiger, O. (2006), 'The Endogeneity of Money and the Eurosystem: A Contribution to the Theory of Central Banking', *Complexity, Endogenous Money and Macroeconomic Theory: Essays in Honour of Basil J. Moore*, pp. 150-169.

[40] Tobin, J., (1982), 'The Commercial Banking Firm: a Simple Model', *Scandinavian Journal of Economics*, 84 (4), pp. 495-530.

[41] Tsomocos, D.P., (2003), 'Equilibrium Analysis, Banking and Financial Instability', *Journal of Mathematical Economics*, 39 (5), pp. 619-655.

[42] Tsomocos, D.P., (2004), 'Equilibrium Analysis, Banking, Contagion and Financial Fragility', Bank of England Working Paper Series No. 175.

[43] Woodford, M. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton, NJ, Princeton University Press.

Appendix 1

Proof of Proposition 3.1 If $r^{\delta}_{s^*} < \rho^{CB}_{s^*}$ for some $s^* \in S^*$, then let δ borrow less in the repo market by an amount Δ and extend an amount Δ less of credit in the short term credit market. Consequently δ would realize additional profits of $\Delta\left(\rho^{CB}_{s^*} - r^{\delta}_{s^*}\right)$ and improve its utility, which is a contradiction that the agent has optimized. Similarly, if $r^{\delta}_{s^*} > \rho^{CB}_{s^*}$ for some $s^* \in S^*$, then let δ borrow more in the repo market by an amount Δ and extend an amount Δ of credit in the short term credit market. Thus, δ would realize additional profits of $\Delta\left(r^{\delta}_{s^*} - \rho^{CB}_{s^*}\right)$ and improve its utility, which is a contradiction that the agent has optimized.

Proof of Proposition 3.2 If $r^{\gamma}_{s^*} < \rho^{CB}_{s^*}$ for some $s^* \in S^*$, then let γ lend an amount Δ more in the repo market and lend an amount Δ less in the short term credit market. Therefore, γ would realize additional profits of $\Delta\left(\rho^{CB}_{s^*} - r^{\gamma}_{s^*}\right)$ and improve its utility, a contradiction. Similarly, if $r^{\gamma}_{s^*} > \rho^{CB}_{s^*}$ for some $s^* \in S^*$, then let γ deposit an amount Δ less in the repo market and lend more by an amount of Δ in the short term credit market. Then γ would realize additional profits of $\Delta\left(r^{\gamma}_{s^*} - \rho^{CB}_{s^*}\right)$ and improve its utility, which is a contradiction that the agent has optimized.

Proof of Proposition 3.3 If $\bar{r}^{\gamma}_{d} < \rho^{CB}_{0}$ then let γ take more consumer deposits by an amount Δ and extend an amount Δ more of credit in the repo market. Consequently, γ would realize additional first period profits of $\Delta\left(\rho^{CB}_{0} - \bar{r}^{\gamma}_{d}\right)$. If $\bar{r}^{\gamma}_{d} > \rho^{CB}_{0}$ then let γ take less consumer deposits by an amount Δ and extend an amount Δ less of credit in the repo market. Therefore, γ would realize additional first period profits of $\Delta\left(\bar{r}^{\gamma}_{d} - \rho^{CB}_{0}\right)$, which would improve its utility by allowing it to extend more short-term credit in the second period, a contradiction.

Proof of Proposition 3.4 If $p^{\alpha} < 1 + \rho^{CB}_{0}$, then let γ reduce its mortgage extension by Δ, sell the remaining mortgage loans in the MBS's market and increase its deposits in the repo market by an amount of Δ. Consequently, γ would realize additional first period profits of $\Delta\left(1 + \rho^{CB}_{0} - p^{\alpha}\right)$, which would improve its utility by allowing it to extend more short-term credit in the second period, a contradiction. If $p^{\alpha} > 1 + \rho^{CB}_{0}$, then let γ increase its mortgage extension by Δ , sell the additional mortgage loan in the MBS market, and decrease its deposits in the repo market by an amount of Δ. Thus, γ would realize additional first period profits of $\Delta\left(p^{\alpha} - 1 - \rho^{CB}_{0}\right)$, which would improve its utility by allowing it to extend more short-term credit in the second period, a contradiction.

Proof of Proposition 3.5 From bank γ's first order conditions it follows that:

$$\left(\sum_{s\in S} \omega_s \left(1+\rho^{CB}_{s}\right)\left(1-2c^{\gamma}\pi^{\gamma}_{s}\right)\right)\left(1+\bar{r}^{\gamma}_{d}\right) =$$
$$\left(\sum_{s\in S} \omega_s \bar{R}^{\delta}_{s}\left(1+\rho^{CB}_{s}\right)\left(1-2c^{\gamma}\pi^{\gamma}_{s}\right)\right)\left(1+\bar{\rho}\right)$$

Since rational expectations are assumed throughout and there is default in the interbank market, i.e. $\bar{R}^{\delta}_{s} \in [0,1] \quad \forall s \in S$, then $\bar{\rho} \geq \bar{r}^{\gamma}_{d}$. Intuitively, if $\bar{\rho} < \bar{r}^{\gamma}_{d}$, then let γ take an amount Δ less of consumer deposits and extend an amount Δ less of credit in the interbank market. Then γ's second period revenues would increase by at least $\Delta\left(\bar{r}^{\gamma}_{d} - \bar{\rho}\right)$, allowing it to extend more short-term credit in the second period and improve its utility, which is a contradiction. This proves the first part of the proposition.

From bank δ's first order conditions it follows that:

$$\sum_{s\in S} \omega_s \bar{\tau}_s \left(1+\bar{\rho}\right) = \sum_{s\in S} \omega_s \bar{\tau}_s \bar{R}_s \left(1+\bar{r}\right)$$

Since rational expectations are assumed throughout and there is default in the wholesale money market, i.e. $\bar{R}_s \in [0,1] \quad \forall s \in S$, then $\bar{r} \geq \bar{\rho}$. Intuitively, if $\bar{r} < \bar{\rho}$, then let δ borrow less in the interbank market by an amount Δ and extend an amount Δ less of credit in the wholesale money market. Then δ's second period revenues would increase by at least $\Delta\left(\bar{\rho}-\bar{r}\right)$, allowing it to extend more short-term credit in the second period and improve its utility, a contradiction. This proves the second part of the proposition.

Proof of Proposition 3.6 Since $\rho_{s^*}^{CB}$,$r_{s^*}\gamma$,$r_{s^*}\delta$, $\bar{r}^{\gamma\alpha}$, $\bar{r}_s^{\gamma}$, $\bar{r}$, $r\bar{h}o>0$, and all elements in η stay bounded, then agents don't hold idle cash in the second period, and all unused cash at $t=0$ is preserved and spent in the next period. If bank γ, had an amount Δ of unused money at $t=1$, it would deposit it in the repo market or extend more short term credit to households, and improve its profits by at least $\Delta\rho_s^{CB}$. Alternatively, it could take more consumer deposits at $t=0$ by an amount of, say Δ, and extend more credit in the mortgage, interbank or short term credit markets. Hence γ's second period profits would increase by at least $\Delta\rho_s^{CB}$, since either first period profits are larger, or interbank repayments rise. Then, γ would repay consumer deposits with the cash left-over at the second period. By the same argument, bank γ's first period profits, are preserved and spent in the next period.

If bank δ had unused cash at $t=1$, it wouldn't have borrowed, say Δ, in the repo market, in which case it would have saved $\Delta\rho_s^{CB}$ of interest payments. Alternatively, δ could extend more short term credit to households by an amount Δ, and improve its profits by at least $\Delta\rho_s^{CB}$. δ doesn't hold idle cash at $t=0$ either. Otherwise, it could borrow an amount of, say Δ, in the interbank or repo markets and save $\Delta\rho_0^{CB}\left(\Delta\bar{\rho}\right)$ of interest payments. Similarly, if δ held idle cash in the first period, it would have used it to extend more credit in the wholesale money market, say by an amount of Δ, and improve its second period by at least $\Delta\rho_s^{CB}$. If default occurs in the interbank or derivatives markets, then adjust the previous arguments by $\Delta\bar{\tau}_s^{\delta}\left(1-\bar{v}_s^{\delta}\right)$ and $\Delta\bar{\tau}_s^{k}\left(1-\bar{v}_s^{k}\right)$, $k=\{\psi,\phi\}$, respectively to induce the profits improvement.

If households had unused cash at some $s^*\in S^*$ they wouldn't have borrowed in the short term credit markets. Moreover, if α had an amount Δ of unused cash at $t=1$, he could have borrowed $\left(\Delta/1+\bar{r}^{\gamma\alpha}\right)$ more in the mortgage market, thereby increasing his expenditure in the housing market by $\left(\Delta/1+\bar{r}^{\gamma\alpha}\right)$ and improving his utility at $t=0$ by $\Delta\left(u'\left(\chi_{02}^{\alpha}\right)+\omega_1 u'\left(\chi_{02}^{\alpha}+\chi_{12}^{\alpha}\right)\right)/\left(1+\bar{r}^{\gamma\alpha}\right)$, and at $s=1$ by $\Delta\left(\omega_1 u'\left(\chi_{02}^{\alpha}+\chi_{12}^{\alpha}\right)\right)/\left(1+\bar{r}^{\gamma\alpha}\right)$. Then at $s=1$, α could have used his left over cash to defray his loans.

If θ had unused cash at $t=0$, he could have deposited an amount of, say Δ, with bank γ, receive additional interest payments for $\Delta\bar{r}_d^{\gamma}$ at $t=1$ and increase his spending in the goods market by $\Delta\bar{r}_d^{\gamma}$; this would improve his utility by $\Delta\bar{r}_d^{\gamma}u'\left(\chi_{s1}^{\theta}\right)$ at $s\in S$.

Investor $k\in\{\psi,\phi\}$ does not hold idle cash at $t=0$, otherwise it wouldn't have borrowed in the wholesale money market. Furthermore, if $k\in\{\psi,\phi\}$ had unused cash at $t=1$, it would have borrowed $\Delta/\left(1+\bar{r}\right)$ in the wholesale money market and increase its spending in the derivatives markets at $t=0$; then in the second period, it could use its left over cash to defray, or default less, on its wholesale money market loan

Finally, note that households, commercial banks, and investors never repay more than what they owe. We have shown that in $t=1$ agents don't hoard money and that all unused cash from $t=0$ is spent in the second period; this implies that all agents' budget constraints are binding. Hence, the stock of money in the economy is returned to commercial banks and equality follows for $s\in S$ after adjusting for default, and the weak inequality follows for $s=0$.

Proof of Lemma 3.7 This result follows immediately from the model's first order conditions.

Proof of Proposition 3.8 For some $s^*\in S^*$ let agent i purchase good l, sell good m and borrow from bank j, and let agent h purchase good m, sell good and borrow from bank k. Then by lemma 3.7

$$\frac{p_{s^*l}\left(1+r_{s^*}^{j}\right)}{p_{s^*m}}=\frac{u'\left(\chi_{s^*l}^{i}\right)}{u'\left(e_{s^*m}^{i}-\chi_{s^*m}^{i}\right)}$$

$$\frac{p_{s^*m}\left(1+r_{s^*}^k\right)}{p_{s^*l}}=\frac{u'\left(\chi_{s^*m}^h\right)}{u'\left(e_{s^*l}^h-\chi_{s^*l}^h\right)}$$

Consider an expansionary monetary policy shock at some $s^* \in S^*$ $\left(M_{s^*}^{CB}\text{ increases}\right)$. Then by propositions 1 and 2, $r_{s^*}^j$ and $r_{s^*}^k$ decrease. Assume monetary policy is neutral; then i and h's consumption of goods l and m remain unchanged. From the first equality this implies that p_{s^*l}/p_{s^*m} increases, but from the second equality it implies that p_{s^*l}/p_{s^*m} decreases, which is a contradiction.

Proof of Proposition 3.9 If $\rho_{s^*}^{CB} > 0$ for some $s^* \in S^*$ then, by the same arguments provided in the proof of proposition 6, agents don't hold idle cash in the second period and all unused cash from $t = 0$ is preserved and spent at $t = 1$. This implies the budget set of all agents is binding and equalities of the proposition follow $\forall\ \ s^* \in S^*$.

Proof of Proposition 3.10 The equations of the propositon follow immediately from the model's first order conditions. Taking logarithms of the first equality the desired result obtains.

$$\bar{r}^{\gamma\alpha} \approx \frac{u'\left(\chi_{02}^{\alpha}\right)}{u'\left(\chi_{02}^{\alpha}+\chi_{12}^{\alpha}\right)}+\Pi_{12}$$

where $\Pi_{12} = (p_{12} - p_{02})/p_{02}$ represents housing inflation at state 1. This equation does not hold at $s = 2$ since no mortgage interest payments take place because of default. Note that the real interest rate is adjusted by the probability of a state 1 realization, thereby embedding a risk premium term. This proves the proposition for the mortgage rate.

Now, define $\lambda_1 = \frac{\omega_1 u'\left(\chi_{11}^{\theta}\right)/p_{11}}{\sum_{s\in S}\omega_s u'\left(\chi_{01}^{\theta}\right)/p_{s1}}$ and $\lambda_2 = \frac{\omega_2 u'\left(\chi_{21}^{\theta}\right)/p_{21}}{\sum_{s\in S}\omega_s u'\left(\chi_{01}^{\theta}\right)/p_{s1}}$. These parameters can be interpreted as risk-neutral probabilities since $\lambda_1 + \lambda_2 = 1$.

Dividing the second equality of the proposition by λ_s gives

$$(1+\bar{r}_d^{\gamma})=\frac{u'\left(\chi_{01}^{\theta}\right)/p_{01}}{u'\left(\chi_{01}^{\theta}\right)/p_{s1}}\left(\frac{\lambda_s}{\omega_s}\right)$$

Taking logarithms to the equation above, the desired result obtains:

$$\bar{r}_d^{\gamma} \approx \frac{u'\left(\chi_{01}^{\theta}\right)}{u'\left(\chi_{01}^{\theta}\right)}+\Pi_{s1}+\log\left(\frac{\lambda_s}{\omega_s}\right)$$

where $\Pi_{s1} = (p_{s1} - p_{01})/p_{01}$ represents goods inflation at state $s \in S$, and (λ_s/ω_s) denotes a macroeconomic risk premium term. This proves the proposition for the consumer deposit interest rate.

Appendix 2

Table 1: Exogenous Variables

Risk Aversion Coefficients		Goods Endowments		Housing Endowments		Monetary Endowments		Default Penalties		Others	
c^{α}	1.30	e_{01}^{α}	30	e_{02}^{θ}	20	$e_{m,0}^{\alpha}$	10	$\bar{\tau}_1^{\delta}$	1.00	M_0^{CB}	25
c^{θ}	1.30	e_{11}^{α}	20			$e_{m,1}^{\alpha}$	1	$\bar{\tau}_2^{\delta}$	0.05	M_1^{CB}	28
c^{γ}	0.03	e_{21}^{α}	4			$e_{m,2}^{\alpha}$	1	$\bar{\tau}_1^{\psi}$	2.00	M_2^{CB}	0.1
c^{δ}	0.03					$e_{m,0}^{\theta}$	60	$\bar{\tau}_2^{\psi}$	0.00001	ω_1	0.85
						$e_{m,1}^{\theta}$	1	$\bar{\tau}_1^{\phi}$	0.1	ω_2	0.15
						$e_{m,2}^{\theta}$	1	$\bar{\tau}_2^{\phi}$	0.00005		
						e_0^{γ}	60				
						e_1^{γ}	1.0				
						e_2^{γ}	1.0				
						e_0^{δ}	1.0				
						e_1^{δ}	0.1				
						e_2^{δ}	0.1				
						e_0^{ψ}	0.00001				
						e_1^{ψ}	0.00001				
						e_2^{ψ}	0.00001				

Table 2: Initial Equilibrium

		Housholds		*Financial Sector*						*Trade and Spending*			
Prices		Lending Borrowing		Lending Borrowing		Repayment Rates		Goods		Housing		Derivatives	
p_{01}	3.23	μ_0^α	53.43	$d_0^{G\gamma}$	14.62	$\bar{v}_1^\alpha$	100%	q_{01}^α	16.53	q_{02}^θ	4.47	$\tilde{m}^\alpha$	14.10
p_{11}	11.46	μ_1^α	106.46	$d_1^{G\gamma}$	8.18	$\bar{v}_2^\alpha$	85.3%	q_{11}^α	9.29	q_{12}^θ	4.34	$\hat{m}^\alpha$	31.51
p_{21}	53.59	μ_2^α	85.70	$d_2^{G\gamma}$	7.64	$\bar{v}_1^\delta$	98.5%	q_{21}^α	1.60	q_{22}^θ	4.30		
p_{02}	12.75	$\bar{\mu}^\alpha$	34.07	m_0^γ	37.16	$\bar{v}_2^\delta$	58.6%	b_{01}^θ	53.43	b_{02}^α	56.96		
p_{12}	11.53	μ_0^θ	56.96	m_1^γ	83.09	$\bar{v}_1^\psi$	100%	b_{11}^θ	106.46	b_{12}^α	50.02		
p_{22}	6.50	μ_1^θ	50.02	m_2^γ	56.02	$\bar{v}_2^\psi$	88.8%	b_{21}^θ	85.70	b_{22}^α	57.02		
r_0^γ	0.44	μ_2^θ	27.96	$\bar{m}^\alpha$	9.81	$\bar{v}_1^\phi$	100%						
r_1^γ	0.28	$\bar{d}^\theta$	46.19	$\bar{\mu}_d^\gamma$	66.42	$\bar{v}_2^\phi$	64.3%						
r_2^γ	0.53			$\bar{d}^\gamma$	44.61								
r_0^δ	0.44			$\mu_0^{G\delta}$	56.96								
r_1^δ	0.28			$\mu_1^{G\delta}$	46.36								
r_2^δ	0.53			$\mu_2^{G\delta}$	11.84								
$\bar{r}_d^\gamma$	0.44			m_0^δ	39.62								
$\bar{r}^{\gamma\alpha}$	2.47			m_1^δ	39.04								
ρ_0^{CB}	0.44			m_2^δ	18.28								
ρ_1^{CB}	0.28			$\bar{m}$	45.61								
ρ_2^{CB}	0.53			$\bar{\mu}^\delta$	69.17								
$\bar{\rho}$	0.55			$\bar{\mu}^\psi$	21.92								
$\bar{r}$	0.56			$\bar{\mu}^\phi$	48.98								
p^α	1.44												
$\tilde{q}^\alpha$	2.23												

Table 3: Expansionary Monetary Policy

	Increase Money Supply $t=0$	Increase Money Supply $s=2$	Decrease Repo Rate $s=2$		Increase Money Supply $t=0$	Increase Money Supply $s=2$	Decrease Repo Rate $s=2$
p_{01}	+	+	+	$\bar{d}^{\theta}$	+	-	-
p_{11}	-	-	-	$\bar{m}^{\alpha}$	+	-	+
p_{21}	-	+	+	$d_0^{G\gamma}$	-	+	-
p_{02}	+	+	+	$d_1^{G\gamma}$	-	+	-
p_{12}	-	-	-	$d_2^{G\gamma}$	-	-	-
p_{22}	-	+	+	$\bar{d}^{\gamma}$	+	-	+
$\bar{r}^{\gamma\alpha}$	-	-	-	m_0^{γ}	+	+	+
r_0^{γ}	-	$\approx$	$\approx$	m_1^{γ}	+	-	-
r_1^{γ}	-	$\approx$	$\approx$	m_2^{γ}	-	+	+
r_2^{γ}	+	-	-	$\bar{\mu}_d^{\gamma}$	+	-	-
r_0^{δ}	-	$\approx$	$\approx$	$\bar{m}$	+	-	+
r_1^{δ}	-	$\approx$	$\approx$	$\mu_0^{G\delta}$	+	+	+
r_2^{δ}	-	-	-	$\mu_1^{G\delta}$	-	+	-
ρ_0^{CB},M_0^{CB}	-	$\approx$	+	$\mu_2^{G\delta}$	-	+	-
ρ_1^{CB},M_1^{CB}	-	$\approx$	+	m_0^{δ}	+	+	+
ρ_2^{CB},M_2^{CB}	+	-	+	m_1^{δ}	-	+	-
$\bar{r}_d^{\gamma}$	-	$\approx$	$\approx$	m_2^{δ}	-	+	+
$\bar{r}$	-	$\approx$	-	$\bar{\mu}^{\delta}$	+	-	-
$\bar{\rho}$	-	$\approx$	-	$\bar{v}_1^{\delta}$	+	-	-
p^{α}	-	$\approx$	$\approx$	$\bar{v}_2^{\delta}$	$\approx$	-	-
$\tilde{q}^{\alpha}$	-	-	-	$\tilde{m}^{\alpha}$	+	-	-
q_{01}^{α}	-	+	+	$\bar{\mu}^{\psi}$	+	-	-
q_{11}^{α}	-	$\approx$	+	$\bar{v}_1^{\psi}$	$\approx$	$\approx$	$\approx$
q_{21}^{α}	+	-	-	$\bar{v}_2^{\psi}$	-	+	+
b_{02}^{α}	+	-	-	$\hat{m}^{\alpha}$	+	-	-
b_{12}^{α}	-	+	+	$\bar{\mu}^{\phi}$	+	-	-
b_{22}^{α}	-	+	+	$\bar{v}_1^{\phi}$	$\approx$	$\approx$	$\approx$
$\bar{\mu}^{\alpha}$	+	-	-	$\bar{v}_2^{\phi}$	+	+	+
μ_0^{α}	+	+	+	U^{α}	+	$\approx$	$\approx$
μ_1^{α}	-	-	-	U^{θ}	+	-	+
μ_2^{α}	-	+	+	U_0^{α}	+	-	-
b_{01}^{θ}	+	+	+	U_1^{α}	-	-	-
b_{11}^{θ}	-	-	-	U_2^{α}	-	+	+
b_{21}^{θ}	-	+	+	U_0^{θ}	-	+	+
q_{02}^{θ}	+	-	-	U_1^{θ}	+	+	+
q_{12}^{θ}	-	+	+	U_2^{θ}	+	-	-
q_{22}^{θ}	-	+	+	π_1^{γ}	-	-	-
μ_0^{θ}	+	+	+	π_2^{γ}	-	-	-
μ_1^{θ}	-	+	-	π_1^{δ}	-	+	$\approx$
μ_2^{θ}	-	+	+	π_2^{δ}	$\approx$	-	-

Table 4: Stricter Default Penalties

	Increase δ's Default Penalty $s=2$	Increase ψ's Default Penalty $s=2$	Increase ϕ's Default Penalty $s=2$		Increase δ's Default Penalty $s=2$	Increase ψ's Default Penalty $s=2$	Increase ϕ's Default Penalty $s=2$
p_{01}	-	$\approx$	-	$\bar{d}^{\theta}$	-	+	-
p_{11}	-	+	-	$\bar{m}^{\alpha}$	+	-	+
p_{21}	-	+	-	$d_0^{G\gamma}$	-	+	-
p_{02}	-	+	+	$d_1^{G\gamma}$	-	-	+
p_{12}	-	+	-	$d_2^{G\gamma}$	+	+	-
p_{22}	-	+	-	$\bar{d}^{\gamma}$	+	-	+
$\bar{r}^{\gamma\alpha}$	$\approx$	$\approx$	-	m_0^{γ}	-	+	-
r_0^{γ}	+	-	+	m_1^{γ}	+	-	+
r_1^{γ}	$\approx$	$\approx$	-	m_2^{γ}	-	+	-
r_2^{γ}	+	$\approx$	+	$\bar{\mu}_d^{\gamma}$	+	-	+
r_0^{δ}	+	-	+	$\bar{m}$	+	-	+
r_1^{δ}	$\approx$	$\approx$	-	$\mu_0^{G\delta}$	-	+	-
r_2^{δ}	+	$\approx$	+	$\mu_1^{G\delta}$	-	-	+
ρ_0^{CB}	+	-	+	$\mu_2^{G\delta}$	+	+	-
ρ_1^{CB}	$\approx$	$\approx$	-	m_0^{δ}	-	+	-
ρ_2^{CB}	+	$\approx$	+	m_1^{δ}	-	-	-
$\bar{r}_d^{\gamma}$	+	-	+	m_2^{δ}	-	+	-
$\bar{r}$	$\approx$	-	-	$\bar{\mu}^{\delta}$	-	-	+
$\bar{\rho}$	-	$\approx$	+	$\bar{v}_1^{\delta}$	+	$\approx$	+
p^{α}	+	-	+	$\bar{v}_2^{\delta}$	+	$\approx$	-
$\tilde{q}^{\alpha}$	+	+	-	$\tilde{m}^{\alpha}$	+	-	+
q_{01}^{α}	-	+	-	$\bar{\mu}^{\psi}$	+	-	+
q_{11}^{α}	+	-	+	$\bar{v}_1^{\psi}$	$\approx$	$\approx$	$\approx$
q_{21}^{α}	$\approx$	$\approx$	+	$\bar{v}_2^{\psi}$	-	+	-
b_{02}^{α}	-	+	-	$\hat{m}^{\alpha}$	+	+	+
b_{12}^{α}	-	+	-	$\bar{\mu}^{\phi}$	+	-	+
b_{22}^{α}	-	+	-	$\bar{v}_1^{\phi}$	$\approx$	$\approx$	$\approx$
$\bar{\mu}^{\alpha}$	+	-	+	$\bar{v}_2^{\phi}$	$\approx$	+	+
μ_0^{α}	-	+	-	U^{α}	-	$\approx$	-
μ_1^{α}	+	-	+	U^{θ}	+	$\approx$	+
μ_2^{α}	-	+	-	U_0^{α}	-	-	-
b_{01}^{θ}	-	+	-	U_1^{α}	-	+	-
b_{11}^{θ}	+	-	+	U_2^{α}	-	+	-
b_{21}^{θ}	-	+	-	U_0^{θ}	$\approx$	+	-
q_{02}^{θ}	-	$\approx$	-	U_1^{θ}	+	-	+
q_{12}^{θ}	$\approx$	$\approx$	+	U_2^{θ}	+	-	+
q_{22}^{θ}	-	+	-	π_1^{γ}	-	-	+
μ_0^{θ}	-	+	-	π_2^{γ}	+	+	-
μ_1^{θ}	-	+	-	π_1^{δ}	-	+	-
μ_2^{θ}	-	+	-	π_2^{δ}	-	$\approx$	+

Table 5: Public Spending, Government Subsidies, and Increased Risk Aversion

	Direct Liquidity Assistance to α, $s=2$	Increase θ's Housing Endowment $t=0$	Increase γ's Risk Aversion Coefficient		Direct Liquidity Assistance to α, $s=2$	Increase θ's Housing Endowment $t=0$	Increase γ's Risk Aversion Coefficient
p_{01}	+	+	+	$\bar{d}^{\theta}$	-	-	+
p_{11}	-	+	+	$\bar{m}^{\alpha}$	-	+	-
p_{21}	+	+	+	$d_0^{G\gamma}$	+	-	+
p_{02}	+	-	+	$d_1^{G\gamma}$	+	+	+
p_{12}	-	-	+	$d_2^{G\gamma}$	+	-	+
p_{22}	+	-	+	$\bar{d}^{\gamma}$	-	+	-
$\bar{r}^{\gamma\alpha}$	-	+	-	m_0^{γ}	+	-	+
r_0^{γ}	-	+	-	m_1^{γ}	-	+	-
r_1^{γ}	+	-	$\approx$	m_2^{γ}	-	-	+
r_2^{γ}	+	+	-	$\bar{\mu}_d^{\gamma}$	-	+	-
r_0^{δ}	-	+	-	$\bar{m}$	-	+	-
r_1^{δ}	+	-	$\approx$	$\mu_0^{G\delta}$	+	-	+
r_2^{δ}	+	+	-	$\mu_1^{G\delta}$	+	-	+
ρ_0^{CB}	-	+	-	$\mu_2^{G\delta}$	+	-	+
ρ_1^{CB}	+	-	$\approx$	m_0^{δ}	+	-	+
ρ_2^{CB}	+	+	-	m_1^{δ}	+	-	+
$\bar{r}_d^{\gamma}$	-	+	-	m_2^{δ}	+	-	+
$\bar{r}$	-	+	+	$\bar{\mu}^{\delta}$	-	+	-
$\bar{\rho}$	-	+	+	$\bar{v}_1^{\delta}$	-	+	$\approx$
p^{α}	-	+	-	$\bar{v}_2^{\delta}$	+	-	+
$\tilde{q}^{\alpha}$	-	+	-	$\tilde{m}^{\alpha}$	-	+	-
q_{01}^{α}	+	-	+	$\bar{\mu}^{\psi}$	-	+	-
q_{11}^{α}	-	-	-	$\bar{v}_1^{\psi}$	$\approx$	$\approx$	$\approx$
q_{21}^{α}	-	-	$\approx$	$\bar{v}_2^{\psi}$	+	-	+
b_{02}^{α}	+	-	+	$\hat{m}^{\alpha}$	-	+	-
b_{12}^{α}	+	-	+	$\bar{\mu}^{\phi}$	-	+	-
b_{22}^{α}	+	-	+	$\bar{v}_1^{\phi}$	$\approx$	$\approx$	$\approx$
$\bar{\mu}^{\alpha}$	-	+	-	$\bar{v}_2^{\phi}$	+	-	$\approx$
μ_0^{α}	+	-	+	U^{α}	+	+	+
μ_1^{α}	-	+	-	U^{θ}	+	-	$\approx$
μ_2^{α}	+	-	+	U_0^{α}	-	+	+
b_{01}^{θ}	+	-	+	U_1^{α}	-	+	+
b_{11}^{θ}	-	+	-	U_2^{α}	+	+	+
b_{21}^{θ}	+	-	+	U_0^{θ}	+	-	+
q_{02}^{θ}	-	+	+	U_1^{θ}	+	-	-
q_{12}^{θ}	+	+	$\approx$	U_2^{θ}	-	-	-
q_{22}^{θ}	+	+	+	π_1^{γ}	-	+	+
μ_0^{θ}	+	-	+	π_2^{γ}	+	-	+
μ_1^{θ}	+	-	+	π_1^{δ}	+	-	+
μ_2^{θ}	+	-	+	π_2^{δ}	+	+	-