

From Hyperinflation to Stable Prices: Argentina's Evidence on Menu Cost Models*

Fernando Alvarez

University of Chicago
f-alvarez1 AT uchicago.edu

Martin Gonzalez-Rozada

UTDT
mrozada AT utdt.edu

Andy Neumeyer

UTDT
pan AT utdt.edu

Martin Beraja

UTDT
martin beraja AT hotmail.com

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Abstract

We review and extend several comparative statics results of fixed menu cost models of price setting firms facing real idiosyncratic shocks, such as the type studied by Golosov and Lucas (2007) as well as by many others. These results are confronted with their empirical counterparts using the micro data underlying Argentina's consumer price index for a time period where inflation rates vary from almost 5000% per year to less than zero (1988-1997). We find that the following theoretical predictions are verified in the data : (i) the steady state frequency of price changes is unresponsive to inflation for low inflation rates; (ii) the elasticity of the steady state frequency of price changes with respect to inflation converges to 2/3 as inflation becomes large; (iii) the frequency of price increases is unresponsive to inflation and equal to the frequency of price decreases for small inflation rates; (iv) the frequency of price decreases converges to zero as inflation increases; (v) the average magnitude of price changes is symmetric for price increases and decreases at low inflation rates; (vi) for high rates of inflation the magnitude of price increases and decreases should be increasing with the inflation rate (in the data this is not verified for price decreases that stay roughly constant, and decrease for inflation rates exceeding 500% per year); (vii) the steady state dispersion of relative prices is unresponsive to inflation for low rates; (viii) the steady state dispersion of relative prices is an increasing function of inflation for high rates of inflation.

We also show that the Argentine data on the relation between the frequency of price changes and the rate of inflation is consistent with the cross country evidence available in the literature.

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1 Introduction

Infrequent nominal price adjustment are at the center of a large number of models of aggregate fluctuations and monetary policy analysis. In this paper we use a unique data set to examine some of the predictions of models of monopolistic firms setting prices subject to a fixed cost of adjustment. These models have been introduced by Barro (1972) and Sheshinski and Weiss (1977), and augmented to include idiosyncratic firm level shocks by Bertola and Caballero (1990), Danziger (1999), Golosov and Lucas (2007), Gertler and Leahy (2008), among others.

We concentrate on simple comparative static results which apply as long as the fixed cost are small enough. The results compare the effect of different constant inflation rates on the average frequency of price changes and on the dispersion of relative prices, keeping everything else fixed. The first type of result is that in the neighborhood of zero inflation, the average frequency of price changes as well as the dispersion of relative prices are approximately unresponsive to inflation. Likewise, under the same conditions we found that at zero inflation the frequency of price increases should be the same as of price decreases, as well as the absolute value of price increases should be the same as the one for price decreases. The second type of result is that for inflation rates that are large relative to the idiosyncratic shocks, the elasticity of the average frequency of price changes with respect to inflation is approximately $2/3$. In the case of high inflation the elasticity of relative price dispersion with respect to inflation should be positive. Furthermore, as inflation diverge most price changes should be increases, but conditional on a decrease, the size of the decrease should be large. The result for high inflation are from Sheshinski and Weiss (1977) and especially Benabou and Konieczny (1994), properly interpreted. The results for low inflation are new, but closely related to some results in Alvarez, Lippi, and Paciello (2010) and Alvarez and Lippi (2011) for models with permanent idiosyncratic shocks.

These theoretical predictions underlie the cost of inflation in models with sticky prices. First, the frequency of adjustments is a direct source of welfare cost of inflation for the society, since these resources are wasted. Second, the “extra” price dispersion created by nominal variation in prices is the other avenue for inefficiency in models with sticky prices, as explained in chapter 6 of Woodford (2003) or Burstein and Hellwig (2008), among many others.

In this paper we illustrate some of the theoretical predictions reported above using the micro-data underlying the construction of the argentinean CPI index from 1989 to 1997. The unique feature of this data is that in the initial years inflation was extremely high, almost 5000% during 1989 and almost 1500% during 1990. After the stabilization plan of 1991 there is a quick reduction of inflation, and after 1992 there is virtually price stability. We examine

the time series of the average frequency of price changes, the dispersion of the frequency of price changes across industries, and the dispersion of relative prices. We compare how these statistics vary with the rate of inflation, in particular how they covary with inflation at low rates of inflation, and how they covary with inflation at high rates of inflation. We believe that the large variation of inflation in our data set makes it a good laboratory to test features of different models of price adjustment, and as such we view our finding interesting well beyond the effect of large inflation.

We report three type of findings. First, we find that the frequency of price changes is approximately uncorrelated with contemporaneous inflation for inflation rates below 10%, and that this frequency has an elasticity close to $2/3$ for higher inflation rates. These finding are robust to different treatment of sales, product substitutions, and missing values in the estimation of the frequency of price changes, as well as robust with respect to the level of aggregation of price changes. We also find them robust to whether we use contemporaneous inflation or an estimate of expected future inflation, for the relevant time frame. Second, we find that the cross-industry dispersion on the frequency of price changes diminishes as inflation increases. We interpret this to be consistent with our hypothesis that as inflation increases, the determinants for price changes become more similar across industries, since aggregate inflation is common to all goods. Third, we find that the dispersion of relative prices is approximately uncorrelated with inflation for low value of inflation, but it is tightly related to inflation for large values, with an elasticity below $1/3$.

The paper is organized as follows. [Section 2](#) contains the theoretical analysis of the effect of inflation on the frequency of price changes and on the dispersion of relative prices. [Section 3](#) describes our data set. [Section 4](#) has an explanation of the method used to compute the frequency of price changes (adapted mostly from [Klenow and Kryvtsov \(2008\)](#) and to less extent from [Nakamura and Steinsson \(2008\)](#)) to allow for time varying frequency. [Section 5](#) presents the estimates of the time series of the frequency of price changes and inflation, including extensive sensitivity analysis. This section also presents the analysis of the time series of the cross industry dispersion of the frequency of price changes vs inflation. [Section 6](#) presents a decomposition of inflation into its extensive and intensive margins for positive and negative price changes and analyzes how this decomposition varies with the level of inflation. [Section 7](#) presents the estimates of the dispersion of relative prices vs. inflation. [Section 8](#) discusses the relation with other studies that analyze the frequency of price changes and inflation. Several appendices discuss other methodological issues, more details of the estimates, as well as of the data-base. [Section G](#) in the appendix contains a short description of the history of economic policy and inflation for the years before and during our sample.

2 Comparative Static Properties of Menu Cost Models

In this section we review several theoretical predictions of models where price adjustment is subject to a fixed cost. We study the average frequency of price changes of a single monopolist firms facing a fixed cost of changing its nominal price in the presence of idiosyncratic shocks and constant inflation. We highlight two comparative static result of a class of models which apply as long as the fixed cost are small enough. The results compare the effect of different constant inflation rates on the average frequency of price changes and on the dispersion of relative prices, keeping everything else fixed. The first result is that in the neighborhood of zero inflation, the average frequency of price changes as well as the dispersion of relative prices are unresponsive to inflation. Additionally, the frequency of price increases and price decreases should be similar, as well as the average size of price increases and price decreases. The second result is that for inflation rates that are large relative to the idiosyncratic shocks, the elasticity of the average frequency of price changes with respect to inflation converges to $2/3$, provided the idiosyncratic shocks are persistent. Likewise in this case, the elasticity of relative price dispersion with respect to inflation is positive, converging approximately $1/3$. The inflation rate required for behavior of the average frequency of price changes, and the relative prices to converge to these values may be different, depending on the persistence of the idiosyncratic shocks. As the inflation rate becomes very large, price decreases become less frequent, or equivalently most adjustment are price increases. Also, as inflation rate increases, the absolute value of the average price increase as well as price decrease becomes larger. We will first write down a simple set up where these results are obtained, explain the nature of the assumptions needed for the results, and remark in which form these results are already present in the literature. We will discuss the pros and cons of applying these comparative statics results to the study of time series properties of inflation and frequency of price changes and also illustrated with a case that we characterized and solve numerically for a range of inflations similar to the one in our data set. We will then characterize the solution of a particular version of the model and illustrate the results of the propositions. We find these theoretical prediction interesting because they underlie the cost of inflation in model with sticky prices. First, the frequency of adjustment, when adjustment carries a cost, is an obvious source of welfare lost due to inflation for the society. Second, the “extra” price dispersion created by nominal variation in prices is the other avenues for inefficiency in models with sticky prices, since it creates distortions in relative prices. See, for example, chapter 6 of [Woodford \(2003\)](#) and referees therein for the analysis of the second effect. See ? and [Burstein and Hellwig \(2008\)](#) for earlier and recent examples of analysis that takes both effects of inflation into account, the latter using heterogeneous consumers that search for

products and homogenous firms, and the latter using differentiated products in the demand side and heterogeneity in the firms cost.

2.1 Sensitivity to inflation at low and high inflation

In this section we explore the sensitivity to inflation of the frequency of price changes and of price dispersion to inflation, at both very low and very high inflation. We write a simple model of a monopolist adjusting the nominal price of its product subject to a possible time varying fixed cost C_t , i.e. a cost that it is independent of the size of the price change. We assume that the instantaneous profit of the monopolist depend on its price relative to the economy (or industry) wide average price and on an idiosyncratic shock. We let $F(p - \bar{p}, z)$ be the real value of the profit per period as a function of the log of the nominal price charged by the firm, denoted by p , relative to the log of the nominal economy (or industry) wide price, denoted by \bar{p} , and the idiosyncratic shock z . We assume that the economy wide price grows at a constant inflation rate π so that $\bar{p}(t) = \pi t + \bar{p}(0)$. The variable $z \in Z$ is a shifter of the profit function. We also allow the fixed cost to depend on z , in which we write $C_t = \zeta(z_t)$. We assume that $\{z_t\}$ is a diffusion with coefficients $a(\cdot)$ and $b(\cdot)$:

$$dz = a(z) dt + b(z) \sigma dW$$

where $\{W(t)\}$ is a standard Brownian Motion so $W(t) - W(0) \sim N(0, t)$. We keep the parameter σ separately from $b(\cdot)$ so that when $\sigma = 0$ the problem is deterministic. We use $r \geq 0$ for the real discount rate of profits and adjustment costs. We let $\{\tau_i\}$ be the stopping times at which prices are adjusted and $\{\Delta p(\tau_i)\}$ the corresponding price changes, so that the problem of the firm can be written as

$$V(p - \bar{p}, z) = \max_{\{\tau_i, \Delta p_i\}_{i=0}^{\infty}} \mathbb{E} \left[\int_0^{\infty} e^{-rt} F(p(t) - \bar{p}(0) - \pi t, z(t)) dt + \sum_{i=0}^{\infty} e^{-r\tau_i} \zeta(z(\tau_i)) \mid z(0) = z \right] \quad (1)$$

with $p(t) = p + \sum_{i=0}^{\tau_i < t} \Delta p(\tau_i)$ for all $t \geq 0$ and $\bar{p}(0) = \bar{p}$.

We think of this problem as similar to the firm's problem in [Golosov and Lucas \(2007\)](#). To describe the solution of this problem the state space $\mathbb{R} \times Z$ can be divided into a control set $\mathcal{C}(\pi, \sigma^2)$ and an inaction set $\mathcal{I}(\pi, \sigma^2)$, so that if $(p - \bar{p}, z) \in \mathcal{C}(\pi, \sigma^2)$, then it is optimal to adjust prices, and otherwise if $(p - \bar{p}, z) \in \mathcal{I}$, inaction is optimal. Since $\{z(t)\}$ has continuous paths, with additional regularity conditions, all the adjustment will occur at the boundary of the inaction set, which we denote as $\partial\mathcal{I}$. Conditional on adjustment, the firm will change

prices so that its nominal price just after adjustment is given by $p = \bar{p} + \psi(z; \pi, \sigma^2)$. Thus the solution of the problem consists on finding the control \mathcal{C} and inaction \mathcal{I} sets, as well as the optimal adjustment function ψ . We are including π and σ^2 as explicit arguments of the decisions rules to conduct some comparative statics. Using the optimal decision rules we can compute the density of the invariant distribution of the state, $g(p - \bar{p}, z; \pi, \sigma^2)$, as well as the the expected time between adjustments $\mathcal{T}(p - \bar{p}, z; \pi, \sigma^2)$ starting from the state $(p - \bar{p}, z)$. Note that using $g(\cdot)$ we can readily find the distribution of relative prices in the economy or industry. Using these objects we can compute the expected time elapsed between consecutive adjustments under the invariant distribution, and its reciprocal, the expected number of adjustments per unit of time, which we denote by $\lambda_a(\pi, \sigma^2)$. We also denote $\lambda_a^+(\pi, \sigma^2)$ and $\lambda_a^-(\pi, \sigma^2)$ the frequencies of price increases and decreases respectively. Furthermore, we let $\Delta_p^+(\pi, \sigma^2)$ the average size of price change, conditional of having an increase, and $\Delta_p^-(\pi, \sigma^2)$ the the corresponding average size of price changes, conditional of having a decrease. interest is to study $\lambda_a, \bar{\sigma}$, etc. as a function of π .

Our first result is that if we assume that $F(\cdot)$ is symmetric in the log of the static profit maximizing relative price p^* as well as in its shifter z , and if the process for the shocks is symmetric, then inflation has only a second order effect on the frequency of price changes at zero inflation. To state our symmetry assumption we define $p^*(z) = \arg \max_x F(x, z)$. We say that $a(\cdot), b(\cdot)$ and $F(\cdot)$ are *symmetric* if $Z = [-\bar{z}, \bar{z}]$, we normalize $p^*(0) = 0$, and

$$a(z) = -a(-z) \leq 0 \text{ and } b(z) = b(-z) > 0 \text{ for all } z \in [0, \bar{z}] , \quad (2)$$

$$p^*(z) = -p^*(-z) \geq 0 \text{ for all } z \in [0, \bar{z}] \text{ and} \quad (3)$$

$$F(\hat{p} + p^*(z), z) = F(-\hat{p} + p^*(-z), -z) + f(z) \text{ for all } z \in [0, \bar{z}] \text{ and } \hat{p} \geq 0 , \quad (4)$$

for some function $f(z)$. We let $\mu(z)$ the density of the invariant distribution of z , when it exists. **Equation (2)** implies that the invariant distribution μ as well as the transition densities of the exogenous process $\{z_t\}$ are symmetric around $z = 0$. **Equations (3) -(4)** state that the profit function is symmetric around the (log) maximizing price and its cost shifter. Thus if the price is \hat{p} *higher* than the optimal for a firm with z , profits deviate from its optimal value by the same amount as with prices \hat{p} *lower* than the optimal when the shifter is $-z$. The function f allows to have an effect of the shifter z on the level of the profits that is independent of the price. An example of a symmetric case is

$$a(z) = -a_0 z , b(z) = b_0 , F(p, z) = d_0 - c_0 (p - z)^2 - f_0 z \text{ so } p^*(z) = z , \quad (5)$$

for non-negative constants a_0, b_0, c_0, d_0 and f_0 . One way to think about the symmetry as-

sumption is to consider a second order approximation of the profit function around the profit maximizing price, so that

$$F(p, z) = F(p^*(z), z) + \frac{1}{2}F_{pp}(p^*(z), z)(p - p^*(z))^2 + o((p - p^*(z))^2) . \quad (6)$$

We note that when the fixed adjustment cost C is small, then the firm will adjust the prices frequently enough so that $(p - p^*(z))^2$ will be small, and hence the quadratic approximation should be increasingly accurate as C become small.

We let $h(\hat{p}; \pi, \sigma^2) = \int_{\mathcal{Z}} g(\hat{p}, z; \pi, \sigma^2) dz$ the invariant distribution of the relative prices \hat{p} , for an economy, or industry, with (π, σ) . Using h we can compute several statistic of interest, such as $\hat{\sigma}(\pi, \sigma^2)$ the standard deviation of the relative prices $\hat{p} = p - \bar{p}$. As in the case of the frequency of price changes, we include (π, σ^2) explicitly as arguments of this statistic. With the symmetry assumption so defined we have the following result:

PROPOSITION 1. Let $F(\cdot)$, $a(\cdot)$ and $b(\cdot)$ be symmetric as in equation (2)-equation (4). If the frequency of price changes $\lambda_a(\cdot, \sigma^2)$ is differentiable at $\pi = 0$, the $\frac{\partial}{\partial \pi} \lambda_a(0, \sigma^2) = 0$. Likewise, if the density of the invariant $h(\hat{p}; \cdot, \sigma^2)$ is differentiable at $\pi = 0$, then $\frac{\partial}{\partial \pi} \hat{\sigma}(0, \sigma^2) = 0$. Furthermore frequencies and size of price adjustment are symmetric: $\lambda_a^+(0, \sigma^2) = \lambda_a^-(0, \sigma^2)$ and $\Delta_p^+(0, \sigma^2) = \Delta_p^-(0, \sigma^2)$.

The main idea is to use the symmetry of F to show that the expected number of adjustments is symmetric around zero inflation, i.e. that $\lambda_a(\pi, \sigma^2) = \lambda_a(-\pi, \sigma^2)$ for all π . Given the symmetry of the profit function we view this property as quite intuitive: a 1% inflation should give rise to as much price changes as a 1% deflation. Additionally for the distribution of relative prices, the main idea is to show that $h(\hat{p}; \pi, \sigma^2) = h(-\hat{p}, -\pi, \sigma^2)$ for all \hat{p}, π , i.e. that a high relative prices with inflation have the same chances that low relative prices with deflation. Thus, symmetric function, are locally unchanged w.r.t. π , and so the second moment has no first order effect of inflation at $\pi = 0$.

Proof. (of Proposition 1) The value function, the optimal adjustment function and the inaction sets are all symmetric in the sense that:

$$\begin{aligned} V(\hat{p} + p^*(z), z; \pi, \sigma^2) &= V(-\hat{p} + p^*(-z), -z; -\pi, \sigma^2) + v(z) , \\ \psi(z; -\pi, \sigma^2) &= -\psi(-z; \pi, \sigma^2), \text{ and} \\ (\hat{p} + p^*(z), z) \in \mathcal{I}(\pi, \sigma^2) &\implies (-\hat{p} + p^*(-z), -z) \in \mathcal{I}(-\pi, \sigma^2) \end{aligned}$$

for all $z \in [0, \bar{z}]$, $\hat{p} \geq 0$ and $\pi \in (-\bar{\pi}, \bar{\pi})$. The symmetry of these three objects can be established using a guess and verify argument in the Bellman equation. This argument has two parts, one deals with the instantaneous return and the second with the probabilities of

different paths of z 's. For the instantaneous return we note that:

$$\begin{aligned}
F(p(t) - \bar{p}(t), z(t)) &= F(p(0) - \bar{p}(0) - \pi t - p^*(z(t)) + p^*(z(t)), z(t)) \\
&= F(-p(0) + \bar{p}(0) + \pi t - p^*(-z(t)) + p^*(-z(t)), -z(t)) + f(z(t)) \\
&= F(-p(0) + \bar{p}(0) + \pi t + p^*(z(t)) + p^*(-z(t)), -z(t)) + f(z(t)),
\end{aligned}$$

where the second equality holds by symmetry of $F(\cdot)$ setting $\hat{p}(t) = p(0) - \bar{p}(0) - \pi t - p^*(z(t)) - \pi t$. Thus fixing the path of $\{z(t)\}$ for $0 \leq t \leq \tau$, starting with $p(0) - \bar{p}(0)$ and $z(0)$ and having inflation π , gives the same profits, assuming symmetry of $F(\cdot)$, than starting with $-p(0) + \bar{p}(0)$ and having inflation $-\pi$ and $-z(0)$. Finally the probability of the path $\{z(t)\}$ for $t \in [0, \tau]$ conditional on $z(0)$, given the symmetry of $a(\cdot)$ and $b(\cdot)$ is the same as the one for the path $\{-z(t)\}$ conditional on $-z(0)$. From here one obtain that the inaction set is symmetric. Likewise, from this property it is easy to see that the optimal adjustment is also symmetric. If with inflation π a firm adjust with current shock z setting $p = \bar{p} + \psi(z; \pi, \sigma^2)$, then with inflation $-\pi$ and current shock $-z$ it will adjust to $p = \bar{p} + \psi(-z, -\pi) = \bar{p} - \psi(z, -\pi)$. To se this, let $t = 0$ be a date where an adjustment take place, let $p(0)$ the price right after the adjustment, and let τ the stopping time until the next adjustment. The value of $p(0)$ maximizes

$$\begin{aligned}
p(0) &= \arg \max_{\tilde{p}} \mathbb{E} \left[\int_0^\tau e^{-rt} F(\tilde{p} - \bar{p}(0) - \pi t, z(t)) | z(0) \right] \\
&= \arg \max_{\tilde{p}} \mathbb{E} \left[\int_0^\tau e^{-rt} F(\tilde{p} - \bar{p}(0) - \pi t - p^*(z(t)) + p^*(z(t)), z(t)) | z(0) \right] \\
&= \arg \max_{\tilde{p}} \mathbb{E} \left[\int_0^{\tau'} e^{-rt} F(-\tilde{p} + \bar{p}(0) + \pi t - p^*(-z(t)) + p^*(-z(t)), -z(t)) | -z(0) \right] \\
&= \arg \max_{\tilde{p}} \mathbb{E} \left[\int_0^{\tau'} e^{-rt} F(-\tilde{p} + \bar{p}(0) + \pi t, -z(t)) | -z(0) \right].
\end{aligned}$$

where τ' is the stopping time obtained from τ but defined flipping the sign of the z 's. Given the symmetry of the inaction set and optimal adjustment it is relatively straightforward to establish the symmetry of the expected time to adjustment \mathcal{T} and the invariant density g . With the \mathcal{T} and g symmetric, it is immediate to establish that λ_a is symmetric. Finally, if λ_a is differentiable, then $\frac{\partial}{\partial \pi} \lambda_a(\pi, \sigma^2) = -\frac{\partial}{\partial \pi} \lambda_a(-\pi, \sigma^2)$, which establish the required result.

Now we show that $\hat{\sigma}$ has no first order effect of inflation. For that we first use that the symmetry of the decision rules and of the invariant of the shocks implies that $h(\hat{p}, \pi) = h(-\hat{p}, -\pi)$ where for simplicity we suppress σ^2 as an argument. Differentiating this expression w.r.t. π and evaluating at $\pi = 0$ we obtain: $h_\pi(\hat{p}, 0) = -h_\pi(-\hat{p}, 0)$. Let $f(\hat{p}, \pi)$ be any

symmetric differentiable function in the sense that $f(\hat{p}) = f(-\hat{p})$. Then writing the expected value of f as $\mathbb{E}[f|\pi] = \int_{-\infty}^0 f(\hat{p})h(\hat{p}, \pi)d\hat{p} + \int_0^{\infty} f(\hat{p})h(\hat{p}, \pi)d\hat{p}$ and differentiating both terms w.r.t. π and evaluating it at $\pi = 0$, using the implications for symmetry for the derivatives of h and the symmetry of f we have $\frac{\partial}{\partial \pi}\mathbb{E}[f|0] = 0$. Applying this to $f(\hat{p}) = \hat{p}^2$ we obtain that inflation does not have a first order effect on the second non-centered moment of the relative prices. Finally, to examine the effect of inflation on the variance of the relative prices, we need to examine the effect of inflation on the square of the average relative price, i.e.

$$\frac{\partial}{\partial \pi} \left[\int_{-\infty}^{\infty} \hat{p} h(\hat{p}, \pi) d\hat{p} \right]^2 \Big|_{\pi=0} = \left[\int_{-\infty}^{\infty} \hat{p} h(\hat{p}, 0) d\hat{p} \right] \left[\int_{-\infty}^{\infty} \hat{p} h(\hat{p}, 0) d\hat{p} \right] = 0,$$

since by symmetry of $h(\cdot, 0)$ around $\hat{p} = 0$ we have $\int_{-\infty}^{\infty} \hat{p} h(\hat{p}, 0) d\hat{p} = 0$. Then, we have shown that inflation has no first order effect on the variance of relative prices around $\pi = 0$.

Finally, the equality of the frequency and average size of price increases with the frequency and average size of price decreases follows immediately from the symmetry. Q.E.D.

We comment that the assumption of differentiability of λ_a with respect to π is not merely a technical condition. The function $\lambda_a(\cdot, \sigma^2)$ could have a local minimum at $\pi = 0$ without being smooth, as it is in the case of $\sigma^2 = 0$ to which we will turn in the next proposition. Nevertheless, we conjecture, but we have not proved yet at this level of generality, that as long as $\sigma^2 > 0$, the problem is regular enough to become smooth, i.e. the idiosyncratic shocks will smooth out and dominates the effect of inflation. Indeed, for several examples one can either compute all the required functions or show that indeed they are smooth, given the elliptical nature of the different ode's involved. Based on this logic, as well as on computations for different models, we believe that the length of the interval for inflations around zero for which $\lambda_a(\cdot, \sigma^2)$ is approximately flat is increasing in the value of σ^2 .

The alert reader will realize that the essential assumption is the symmetry of the profit function around the profit maximizing price, and hence if this is maintained, the result should hold for a larger class of models. Indeed, in [Alvarez, Lippi, and Paciello \(2010\)](#) and [Alvarez and Lippi \(2011\)](#) a version of [Proposition 1](#), i.e. that inflation has only a second order effect on the expected frequency of price changes at $\pi = 0$ for two wider classes of models: those that have both observations and menu cost models, and those that have multi-product goods. In those papers the approximation stated in [equation \(6\)](#) is used, and furthermore $p^*(z) = z$ is assumed to follow a random walk with no drift (so that $\bar{z} = \infty, a(z) = 0$ and $b(z) = 1$).

While we don't know of other theoretical results analyzing the sensitivity of $\lambda_a(\pi)$ around $\pi = 0$ in this set-up, there is a closely related model that contain a complete analytical characterization by [Danziger \(1999\)](#). In that paper a firm faces a demand with an elasticity of demand of $\eta = 2$, constant return to scale production function subject to permanent

idiosyncratic productivity shocks (with uniform innovations), and an (equilibrium) random nominal wage (also following a random walk). The model is set-up in discrete time. The firm faces a fixed cost of changing prices proportional to a fraction of the period sales. The equilibrium, not just the maximization problem of the firms, is completely characterized in the case of random money shocks (log money growth is iid with mean m) and random aggregate productivity shocks. In equilibrium firms adjust their prices as to control the mark-up over marginal cost in a two sided sS band: when the markup hits either s or S prices are changes so that the markup is set to I , with $s < I < S$. We note that this implies than the distribution of the log of relative prices has a mass point at I and otherwise is uniform between S and s . The paper analyzes the case where the range on monetary shocks is small relative to the range of the idiosyncratic shocks, see Assumption 2 in [Danziger \(1999\)](#) for a precise definition. In equilibrium the objective function of the firm is not symmetric in the sense of [equation \(4\)](#) (see the first equation for V_t in section II and in the statement of Theorem 1 in [Danziger \(1999\)](#)), so we don't expect the conclusion of [Proposition 1](#) to hold without further conditions. Indeed the author presents results characterizing the comparative static of several objects of interest, among them the expected duration of unchanged prices, which is referred to as Ω in that paper, as a function of the expected inflation rate, referred to as m . Theorem 5 of [Danziger \(1999\)](#) states that $\frac{\partial}{\partial m}\Omega < 0$, even if $m = 0$, or in our notation, $\frac{\partial}{\partial \pi}\lambda_a > 0$ even for $\pi = 0$. Nevertheless, as explained above if the fixed costs are small, we expect that the second order expansion of the profit function dominates, and hence $\frac{\partial}{\partial \pi}\lambda_a(0) = 0$. Indeed, we show in [Appendix B](#) that

$$\lim_{c \downarrow 0} \frac{\partial \lambda_a(\pi)}{\partial \pi} = \lim_{c \downarrow 0} \frac{\partial S(\pi) - s(\pi)}{\partial \pi} = 0 \quad (7)$$

as long as π is small enough so that Assumption 2 in [Danziger \(1999\)](#) holds. Note that since inflation has no first order effect on $S - s$, then the dispersion of relative prices does not respond to inflation either. To summarize, we find that the result in [Danziger \(1999\)](#) is consistent with our view that the effect of inflation on the frequency of price changes and in the dispersion of relative prides in menu cost sS models with idiosyncratic shocks are small.

Now we turn to the analysis of the elasticity of the frequency of adjustment λ_a and dispersion of relative prices $\bar{\sigma}$ with respect to inflation π for large values of inflation. Ideally we will like to characterize the elasticity of λ_a and $\bar{\sigma}$ with respect to π for large value of π keeping fixed σ^2 . Instead we will study the elasticity of λ_a with respect to π for the deterministic case, i.e. when $\sigma^2 = 0$, a version of the problem studied by [Sheshinski and Weiss \(1977\)](#). We give a brief explanation of why this calculation of the values of the elasticity for the [Sheshinski and Weiss's \(1977\)](#) model for a $\pi > 0$ should be informative for the case of

σ^2 and very large π , esp First notice that if in the problem described in [equation \(1\)](#) we multiply r, π, σ^2 and the function $a(\cdot)$ by a constant $k > 0$, we are just changing the units at which we measure time. Thus the corresponding expected number of adjustment per unit of time λ_a will be multiplied by k , and the dispersion of relative prices $\bar{\sigma}$ will stay constant. Furthermore if the value of r and the function $a(\cdot)$ are both equal to zero, this will mean that λ_a will be homogenous of degree one in (π, σ^2) . Using this homogeneity:

$$\lim_{\sigma^2 > 0, \pi \rightarrow \infty} \frac{\pi}{\lambda(\pi, \sigma^2)} \frac{\partial \lambda(\pi, \sigma^2)}{\partial \pi} = \lim_{\kappa \rightarrow \infty} \frac{1}{\lambda(\kappa, 1)} \frac{\partial \lambda(\kappa, 1)}{\partial \pi} = \lim_{\pi > 0, \sigma^2 \rightarrow 0} \frac{\pi}{\lambda(\pi, \sigma^2)} \frac{\partial \lambda(\pi, \sigma^2)}{\partial \pi}$$

The interpretation of r being zero is that instead of maximizing the expected discounted cost the firm is maximizing the expected average cost, a case frequently analyzed in stopping time problems. If the function $a(\cdot) = 0$ and $b(\cdot)$ is bounded, then it means that the shifter z has permanent shocks. To summarize, if r is small, and if $a(\cdot)$ is small -so that shocks are very persistent-, we expect that the elasticity of λ_a with respect to inflation obtained from [Sheshinski and Weiss \(1977\)](#) to be very close to the one for the case of $\sigma^2 > 0$ and π is very large. In [Section 2.2](#) we document numerically that indeed they are very close. We note that this argument does not directly apply price dispersion $\bar{\sigma}$ since setting $a(z) = 0$ implies that there is no invariant distribution of z , and hence no invariant distribution of relative prices. Thus for the case of $\bar{\sigma}$ the idiosyncratic shocks cannot be completely permanent, and we only expect that the equality holds approximately, even for very large values of π . In [Section 2.2](#) we return to the effect of changes on π on the dispersion of relative prices when the idiosyncratic shock is very persistent.

In the [Sheshinski and Weiss's \(1977\)](#) model the time elapsed between adjustments is simply a constant, which we denote by $\mathcal{T}(\pi)$. [Sheshinski and Weiss \(1977\)](#) find sufficient conditions so that the time between adjustments decreases with the inflation rate (see their Proposition 2), and several authors have further refined the characterization by concentrating on the case where the fixed cost C is small. Let $p^* = \arg \max_p F(p, 0)$ be log the static monopolist maximization profit, where $z = 0$ is a normalization of the shifter parameter which stays constant. In the deterministic set-up the optimal policy for $\pi > 0$ is to let the log of the price reach a value s at which time it adjusts to S , where $s < p^* < S$. The time between adjustments is then $\mathcal{T}(\pi) = (S - s)/\pi$. We also note another implication, or comparative static, obtained in the [Sheshinski and Weiss's \(1977\)](#) model, i.e. the set-up with $\sigma^2 = 0$. The distribution of (log) of relative price is uniform in the interval $[s, S]$. Thus the standard deviation of the log of the relative prices in this economy, denoted by $\bar{\sigma}$, is given by $\bar{\sigma} = \sqrt{1/12}(S - s)$. As established in Proposition 1 in [Sheshinski and Weiss \(1977\)](#), the range of prices $S - s$ is increasing in the inflation rate π . Obviously the elasticities of λ_a and

of $\bar{\sigma}$ are related since $S - s = \pi\mathcal{T}$ and $\lambda_a = 1/\mathcal{T}$. The calculations in equations (A8) and (A9) of the Appendix in [Benabou and Konieczny \(1994\)](#) imply the following result:

PROPOSITION 2. Assume that $\sigma^2 = 0$ and $\pi > 0$. Then it follows immediately that $\lambda_a^-(\pi, \sigma^2) = 0$ and that $\Delta_p^+(\pi, \sigma^2) = S - s$. Furthermore assume that $F(\cdot, 0)$ is three times differentiable, then

$$\lim_{C \rightarrow 0} \frac{\pi}{\lambda_a} \frac{\partial \lambda_a}{\partial \pi} = \frac{2}{3} \quad \text{and} \quad \lim_{C \rightarrow 0} \frac{\pi}{\bar{\sigma}} \frac{\partial \bar{\sigma}}{\partial \pi} = \frac{1}{3} \quad (8)$$

To obtain this result the authors compute the value of following an sS policy assuming that the period return function $F(p, 0)$ is cubic in terms of deviations from the profit maximizing price, i.e. $p - p^*$. This allows for explicit computation of the value of the policy and to obtain the first order conditions at s and S for any value of π . Taking the limit as C in the resulting expression gives [Proposition 2](#). We include next a short discussion of higher order approximations that suggest that the elasticity is close to $2/3$ but may be smaller. To simplify the expression we report the expressions obtained for the case of $r = 0$, i.e. the sS rule that maximizes the average profits net of adjustment costs. In this case the expression leading to Proposition 1 in [Benabou and Konieczny \(1994\)](#) gives

$$\mathcal{T}(\pi) = 2 \left[-\frac{3}{2} \frac{C}{F''(p^*, 0)} \right]^{1/3} \pi^{-\frac{2}{3}} - \frac{2}{3} \left[-\frac{3}{2} \frac{C}{F''(p^*, 0)} \right]^{2/3} \left[\frac{F'''(p^*, 0)}{-2F''(p^*, 0)} \right] \pi^{-\frac{1}{3}}. \quad (9)$$

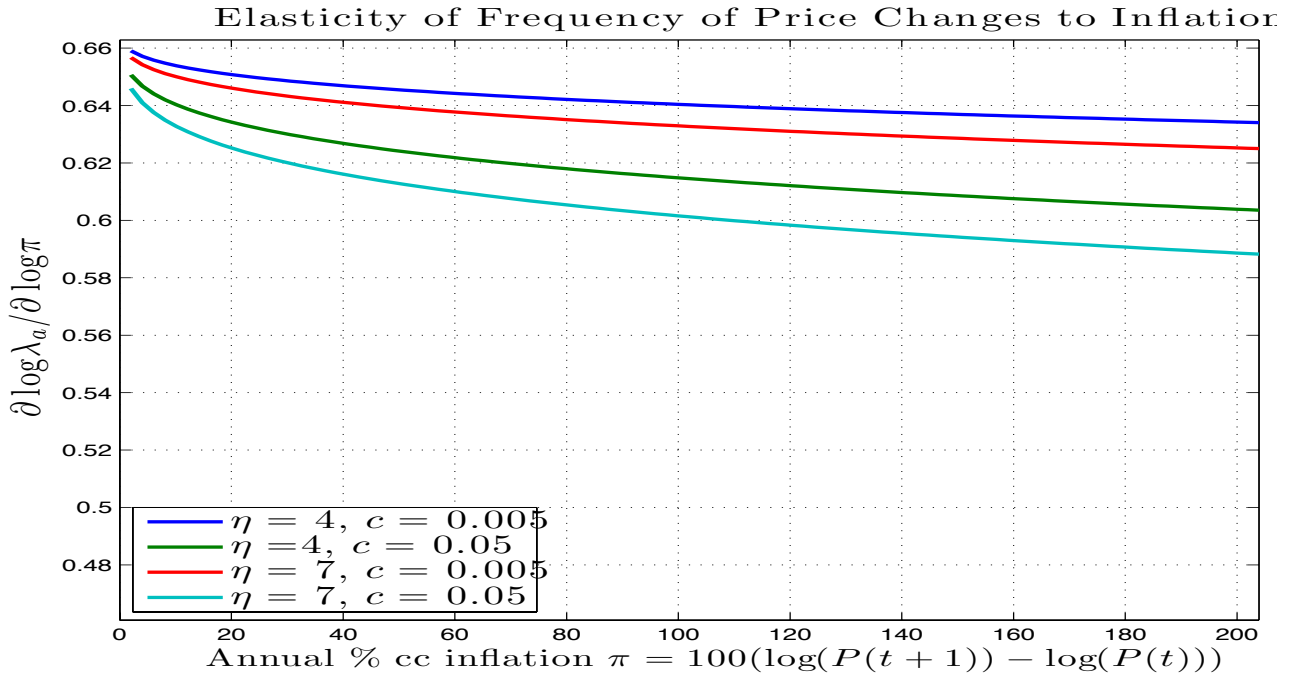
The third order approximation used to obtain this expression is accurate as long as the product of the fixed cost and inflation rate $C\pi$ is small. The point of this expression is that if $F''' > 0$, the elasticity will be smaller than $2/3$, while the opposite holds if $F''' < 0$. We note that $F''' > 0$ is indeed the sign of the third derivative in the case where $F(\cdot)$ is obtained as a third order expansion of the profit for a monopolist facing a demand with constant elasticity η and with a constant marginal cost normalized to one ($z = \log(1) = 0$). In this case, expressing the cost as fraction of the maximum profits so that $C = c F(p^*, 0)$ for some factor $c > 0$, we have

$$\mathcal{T}(\pi) = 2 \left[\frac{3}{2} \frac{c}{\eta(\eta - 1)} \right]^{1/3} \pi^{-\frac{2}{3}} + \frac{2}{3} \left[\frac{3}{2} \frac{c}{\eta(\eta - 1)} \right]^{2/3} \left[\eta - \frac{1}{2} \right] \pi^{-\frac{1}{3}}. \quad (10)$$

Thus the absolute value of the elasticity of this function may be smaller than $2/3$. Yet for the values for which the approximation makes sense, the differences are not large, as shown

in Figure 1.¹ This figure plots the elasticity of λ_a obtained from equation (10) for several combinations of the elasticity η and fixed cost c on the range of values of π for which the third order approximation to the function F is still single peaked in the range $[s, S]$. To interpret η recall that in this example the mark-up of price over marginal cost in the case of no adjustment cost is $\eta/(\eta - 1)$, so $\eta = 7$ corresponds to a mark-up of about 0.15 and $\eta = 4$ to about 1/3.

Figure 1: Theoretical Frequency of Price Changes λ_a and Inflation Rates



We finish this section with a few remarks on the applicability of these comparative static results to the time series variation in our data set. The first remark is about the interpretation of the propositions, which were obtained for extreme values of the parameters, for intermediate values of the parameters. From Proposition 2 and the analysis that it follows we conclude that in the deterministic case the elasticity of the average frequency of price changes with respect to inflation is about 2/3 if $\sigma = 0$. Combining this with the result from Proposition 1, we conclude that the elasticity is lower than 2/3 but approaches 2/3 as π becomes very large for a fixed $\sigma^2 > 0$. The second remark is that these elasticities were

¹While the absolute value of the elasticity in equation (10) goes to 1/3 as $\pi \rightarrow \infty$, the approximation is only accurate for small values of πC . Furthermore, a necessary conditions for the value of s used in the approximation to satisfies $s < p^*$ and for the approximation to F to be decreasing in the range $[p^*, S]$ is that $\pi \leq \frac{(\eta-1)\eta}{(\eta-1/2)^3} \frac{2}{3} / c$. Thus one cannot simply take the limit in equation (10) as $\pi \rightarrow \infty$ for fixed c .

obtained under the assumption that inflation is assumed to remain *constant* at the rate π , and that the frequency of price changes is computed under the invariant distribution. Thus, strictly speaking, our proposition are *not* a prediction for time series variation, but just a comparative static result. We give two comments on this respect. First, this should be less of a concern for very high inflation, since the model becomes close to static, i.e. firms plan to revise very often and the adjustment to the invariant distribution happens very fast. Second, when we analyze the argentinean data we will correlate current frequency of price changes with the current inflation rate as well as to correlate the current frequency of price changes with an average of the current and future inflation rates. We experiment with different definition of this averages, including ones where we average the inflation rates in the period between the current month and some multiple of the implied instantaneous duration given the current frequency of price changes.

2.2 Illustrating the theory with a numerical example

In this section we specify a version of the firm problem studied in [Goloso and Lucas \(2007\)](#). We characterize the value function, decisions rules, and the statics of interest analytically, up to the solution of 3 equations in 3 unknowns. We then solve for the model numerically, illustrating the theoretical results of the previous section.

The difference of the set up with the one in [Goloso and Lucas \(2007\)](#) is that we set the fixed cost proportional to current profits, and that the idiosyncratic shock is permanent, as opposed to stationary. Additionally we assume that products life is exponentially distributed that implies that there exist an ergodic distribution of relative prices (we also view this device as realistic given the rate at which product are substituted in most data sets). This version of the model is identical to the one by [Kehoe and Midrigan \(2010\)](#), if we zero out the transitory shock that gives rise to sales, and set in continuous time.

We assume that the period profits are given by a demand with constant elasticity η and with a constant return to scale technology with marginal cost given by e^z , so that

$$F(p - \bar{p}, z) = e^{-\eta p} (e^p - e^z) ,$$

so that $p - z$ is the log of the gross markup. As explained above, we assume that the shocks on the (log) of the cost are permanent in the sense that

$$dz = \mu_z dt + \sigma dW - z dN.$$

where N is the counter of a Poisson process with constant arrival rate per unit of time ρ . We

interpret this as products dying with Poisson arrival rate $\rho > 0$ per unit of time, at which time they are replaced by a new one which starts with $z = 0$ and must set its initial price. A positive value of μ_z can be interpreted as a vintage effect, i.e. the technology for new products grows at rate μ_z .

We assume that $\zeta(z) = c F(p^*(z), z)$ for some constant $c > 0$ so that $\zeta(z) = \hat{c} e^{z(1-\eta)}$. Since $p^*(z) = z + m$ where m is the log of the gross optimal static markup $m = \log(\frac{\eta}{\eta-1})$. Thus $F(p^*(z), z) = e^{(1-\eta)z} \left(\frac{\eta-1}{\eta}\right)^\eta \frac{1}{\eta-1}$. Note that $F(p^*(z), z)$ is decreasing and strictly convex on z for $\eta > 1$. We will assume that $\eta > 1$ so that the static monopolist problem has a solution and that

$$r + \rho \geq (1 - \eta) \left[\mu_z + (1 - \eta) \frac{\sigma^2}{2} \right] \quad (11)$$

This inequality is required for the profits of the problem with zero fixed cost, $\hat{c} = 0$ to be finite. In this case the expected discounted value of profits, starting with z_0 is given by

$$\begin{aligned} \mathbb{E}_0 \int_0^\infty e^{-(r+\rho)t} F(p^*(z_t), z_t) dt &= \frac{(\eta-1)^{\eta-1} e^{(1-\eta)z_0}}{\eta^\eta} \int_0^\infty e^{-(r+\rho)t} \mathbb{E}_0 [e^{(1-\eta)z_t}] dt \\ &= \frac{(\eta-1)^{\eta-1} e^{(1-\eta)z_0}}{\eta^\eta} \int_0^\infty e^{[-(r+\rho)+(1-\eta)\mu_z+(1-\eta)^2\frac{\sigma^2}{2}]t} dt. \end{aligned} \quad (12)$$

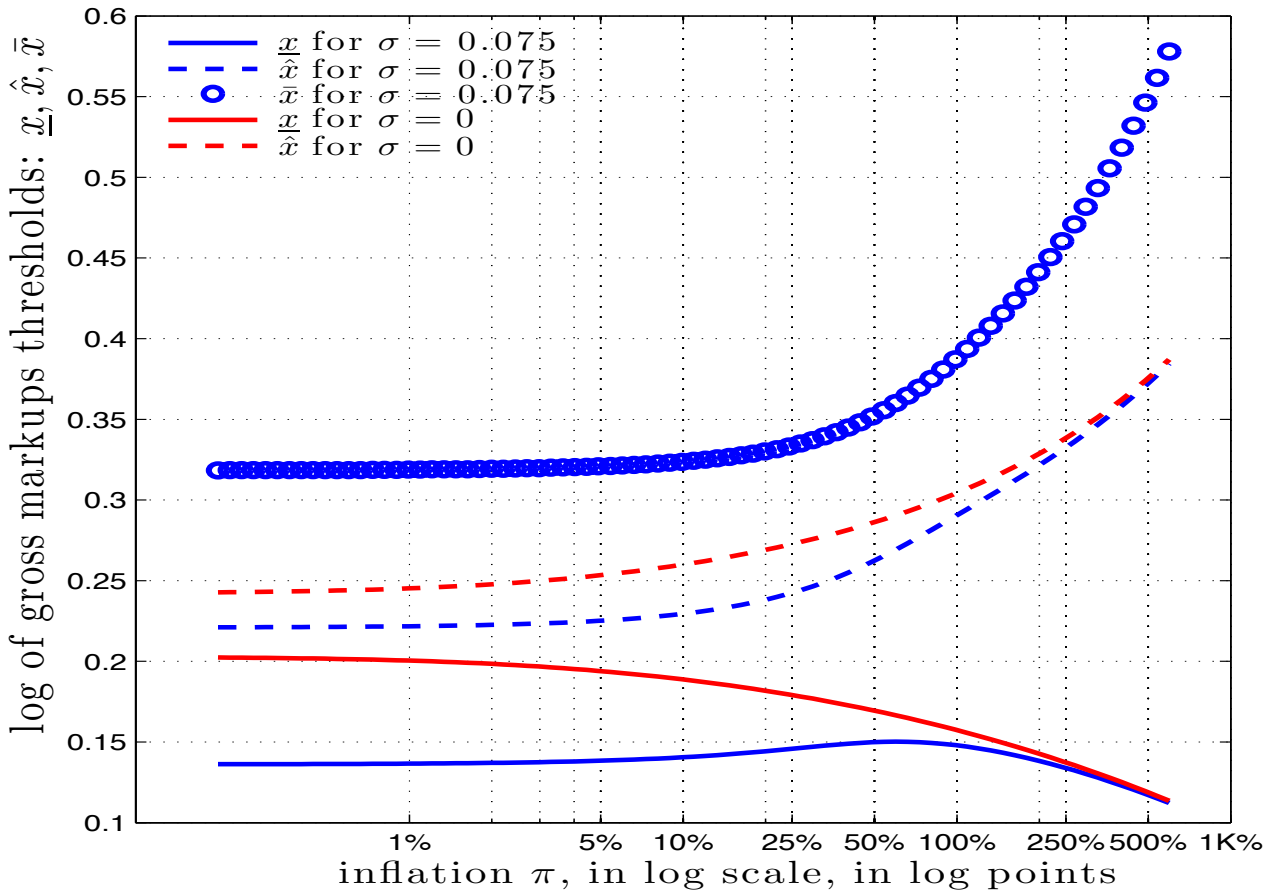
Since for $\eta > 1$ period profits are decreasing and convex on z , and hence discounted expected profits are finite if $r + \rho$ is the discount rate is high enough, or if the cost increases at a high enough rate μ_z is high (or), or if $\sigma^2/2$ is low enough.

In [Appendix C](#) we present several propositions with analytical characterization for the solution of this model. In what follows we use $x = p - z$ for the log of the real gross mark-up. In [Proposition 3](#) we show that when $\sigma > 0$, the inaction set is given by $\mathcal{I} = \{(p, z) : \underline{x} + z < p < \bar{x} + z\}$ and that the optimal return point is given by $\psi(z) = \hat{x} + z$ for three constants $X \equiv (\underline{x}, \hat{x}, \bar{x})$. This is due to the combination of assumptions of constant elasticity of demand, constant returns to scale and permanent shocks to cost while the product lasts. This means that it is optimal to keep the price unchanged when the real markup x is in the interval (\underline{x}, \bar{x}) . When prices are not changed, the real markup evolves according to $dx = -(\mu_z + \pi)dt + \sigma dW$. When the real markup hits either of the two thresholds, prices are adjusted so that the real markup is \hat{x} . [Proposition 3](#) derives a system of three equations in three unknowns for X , as well as the explicit solution to the value function, as a function of the parameters $\Theta \equiv (\pi, \mu_z, \sigma^2, \rho, r, \eta, c)$. [Proposition 4](#) derives an explicit solution for the expected number of adjustments per unit of time λ_a given a policy X and parameters $(\pi, \mu_z, \sigma^2, \rho)$. [Proposition 5](#) characterizes the density g for the invariant distribution of (p, z) implied by the policy X and the parameters $(\pi, \mu_z, \sigma^2, \rho, \eta)$.

We associate the case where $\sigma > 0$ with the models by [Golosov and Lucas \(2007\)](#) and

Kehoe and Midrigan (2010). In Proposition 6 we consider the case when $\sigma = 0, \pi + \mu_z > 0$, a version of Sheshinski and Weiss's (1977) model, in which case the optimal policy can be characterized simply by two thresholds $s \equiv \underline{x} < \hat{x} \equiv S$, that solves two equations in two unknowns. In this case $\lambda_a = \rho / \left[1 - \exp \left(-\frac{\rho}{\pi + \mu_z} (\hat{x} - \underline{x}) \right) \right] = (\pi + \mu_z) / (\hat{x} - \underline{x}) + o(\rho / (\pi + \mu_z))$, so that it coincides with the expression used for $\sigma = 0$ if ρ is small relative to π .

Figure 2: Optimal thresholds for different inflation rates



We use Proposition 3 and Proposition 6 to solve for the optimal policies characterized by the thresholds for the log of the markups \underline{x} , \hat{x} , and \bar{x} for a range of inflation rates π . We use Proposition 4 and Proposition 6 to compute the expected number of adjustment per unit of time λ_a . In both cases the inflation rates π is measure as continuously compounded and it is plotted in a log scale. For the numerical examples we follow ? and use $\eta = 3$, which implies a very large markup, but it roughly inline with marketing/IO estimates of demand elasticities. We set $\rho = 0.1$ so products have a lifetime of 10 years, and $r = 0.06$ so yearly interest rates are 6%. We let $c = 0.002$ so that adjustment cost is 20 basis point of yearly frictionless

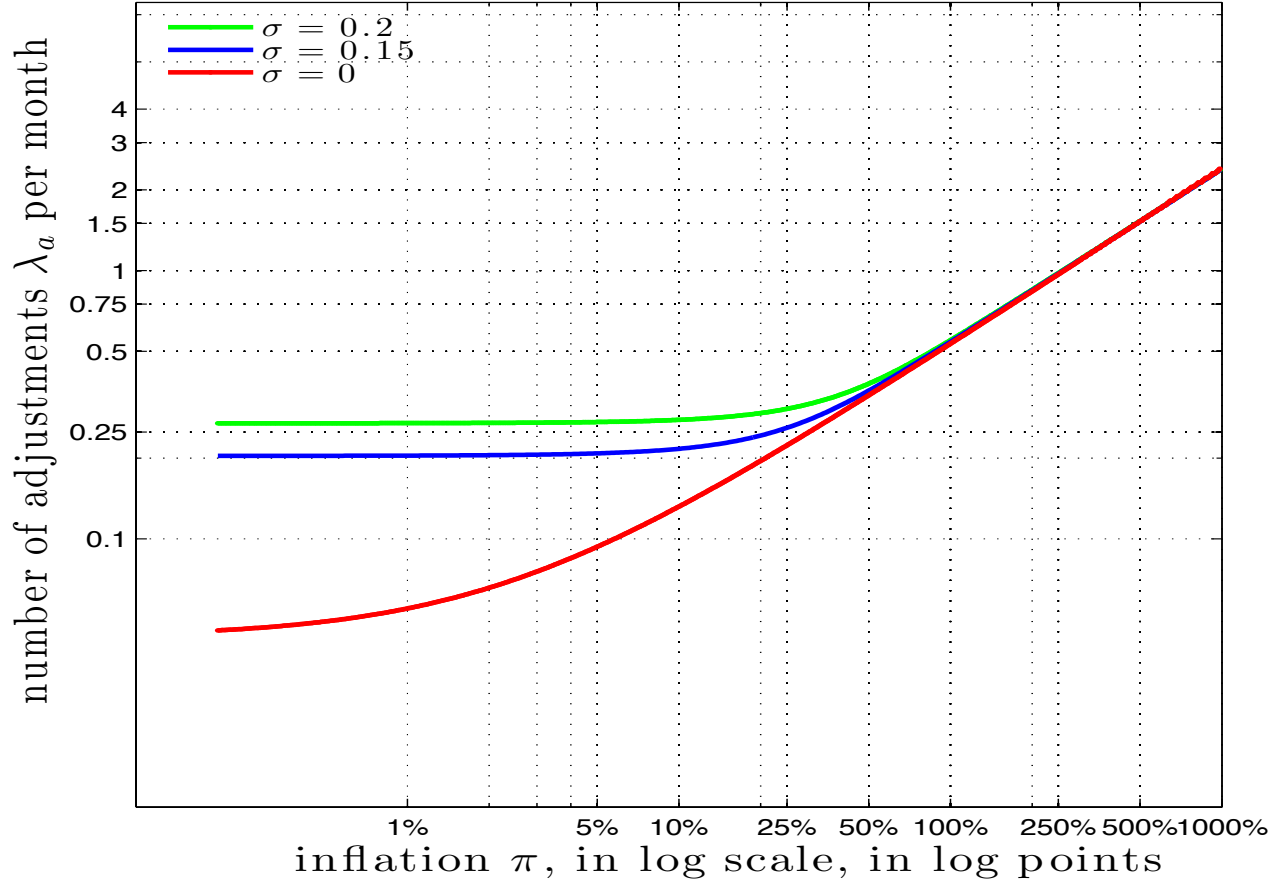
profits. We let $\mu_z = 0.02$, i.e. a 2 percent per year increase in cost (or a 2% increase in vintage productivity). We consider three values for $\sigma \in 0, 0.015, 0.20$, the first corresponds to [Sheshinski and Weiss's \(1977\)](#) model, and the others are 15% and 20% standard deviation in the change in marginal cost, at annual rates. The values of $c/(\eta(\eta - 1))$ and $\sigma = 0.15$ were jointly chosen so that at zero inflation the model matches both the average number of price changes $\lambda_a = 2.7$ and the average size of price changes of $\Delta_p^+ = \Delta_p^+ 0.10$, roughly the values corresponding to zero inflation in our data set. [Figure 2](#) illustrates the optimal threshold policies for two cases, $\sigma = 0.15$ and $\sigma = 0$. It can be seen that the threshold for the lower real markup that triggers adjustments \underline{x} corresponding to $\sigma > 0$ and the one corresponding to $\sigma = 0$ converge to each other as π increases. The same convergence is observed for the real markup at the optimal return point \hat{x} . [Figure 3](#) illustrates the frequency of price changes λ_a for three different values of σ . It can be seen that for $\sigma > 0$, the frequency λ_a is insensitive to inflation in the neighborhood of zero inflation. It can also be seen that the length interval of inflations around $\pi = 0$ for which λ_a is approximately constant is higher for higher σ . This is an illustration of the conclusion in [Proposition 1](#), even though the model does not exactly satisfies all the assumption, since the profit function F derived from a constant elasticity demand is not symmetric. Yet, as discussed above, for small cost c the terms in the quadratic expansion, which are symmetric by construction, should provide an accurate approximation. On the other hand, for very large inflation rates, the levels and slope of the three lines converge to each other. Since the graph is in log scale, it is clear that the common slope is approximately constant for large inflations, and close to $2/3$ - the actual value for this example is 0.64. Note that the conclusion of [Proposition 2](#) holds for this figure even though the model does not exactly satisfies all the assumption. Additionally the values we obtain, slightly lower than $2/3$ are completely in line with the cubic approximation developed by [Benabou and Konieczny \(1994\)](#) discussed in [equation \(10\)](#) and plotted in [Figure 1](#).

[Figure 4](#) displays the frequency of price increases λ_a^+ , together with the frequency of all adjustments λ_a , for two values of the cost volatility σ . Two features are worth mentioning: for low inflation the frequency of price increases is about half of the frequency of price changes, and as inflation become large enough all adjustment becomes price increases.

[Figure 5](#) plots the size of the "regular" price increases and "regular" price decreases, for different inflation rates. Imitating the empirical literature, we define as regular price changes those not triggered by the jump shock that reset the value of z to zero. As explained above, as inflation rises, the frequency of price decreases becomes very small, but the absolute value of the size of the price decreases becomes larger.

We now turn to the distribution of relative prices. [Figure 6](#) plots standard deviation of the the log of relative price p or different inflation rates, fixing the value of the shock

Figure 3: Frequency of Price changes for different inflation rates



z at its mode, zero. This figure has three lines, one for $\sigma = 0$ which has an elasticity of approximately $1/3$, as predicted by the quadratic approximation to Sheshinski and Weiss's (1977) in equation (10). The other two lines correspond to positive values for σ of 15% and 20% per year. It can be readily seen that, as predicted by Proposition 1 for the symmetric case, the standard deviation is insensitive to inflation around $\pi = 0$ provided that $\sigma > 0$. As inflation becomes very large, the standard deviation converged to the one for $\sigma = 0$, and hence its elasticity with respect to inflation is approximately $1/3$.

If we were to plot $g(\cdot, 0)$ for large value of π the distribution will be almost uniform between $[\hat{x}, \bar{x}]$, while if we plotted it for π close to zero it is single peaked at \hat{x} with support on $[\underline{x}, \bar{x}]$, roughly symmetric with densities decaying up to zero at the two extremes. By examining the analytical characterization of the density of the invariant distribution derived in Appendix 5-Proposition 5, one can see that shape of the distribution $g(p, z)$ is the same for other values of z . Yet, this result does not imply that the *marginal* distribution of relative

Figure 4: Frequency of Price Adjustments λ_a and of Price Increases λ_a^+

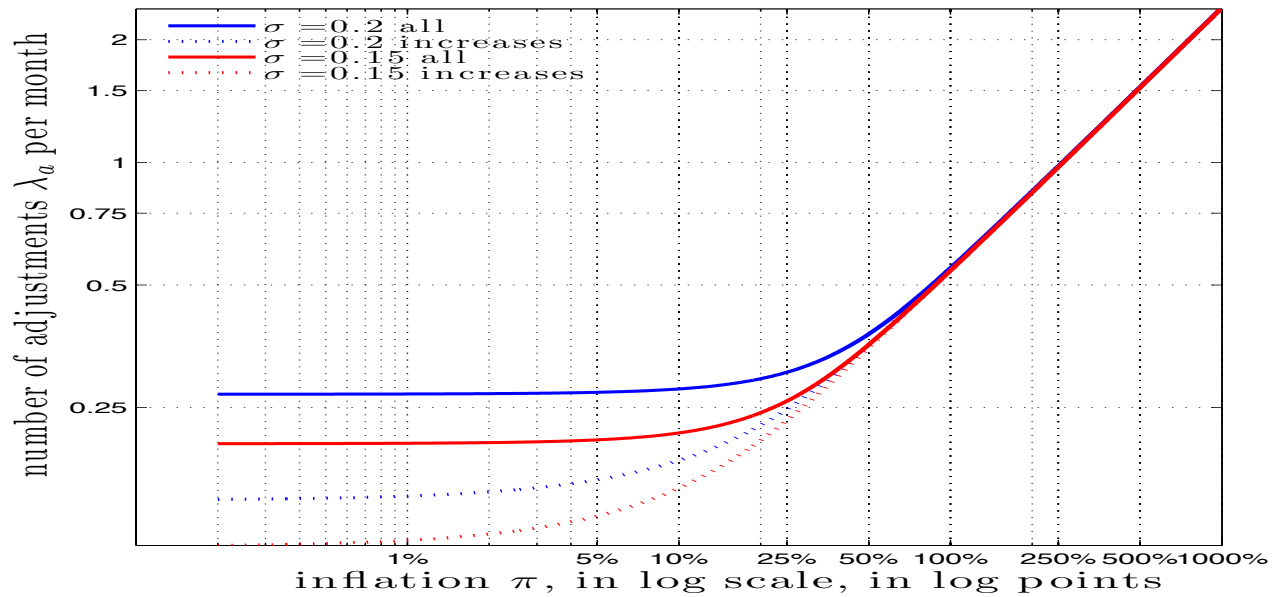
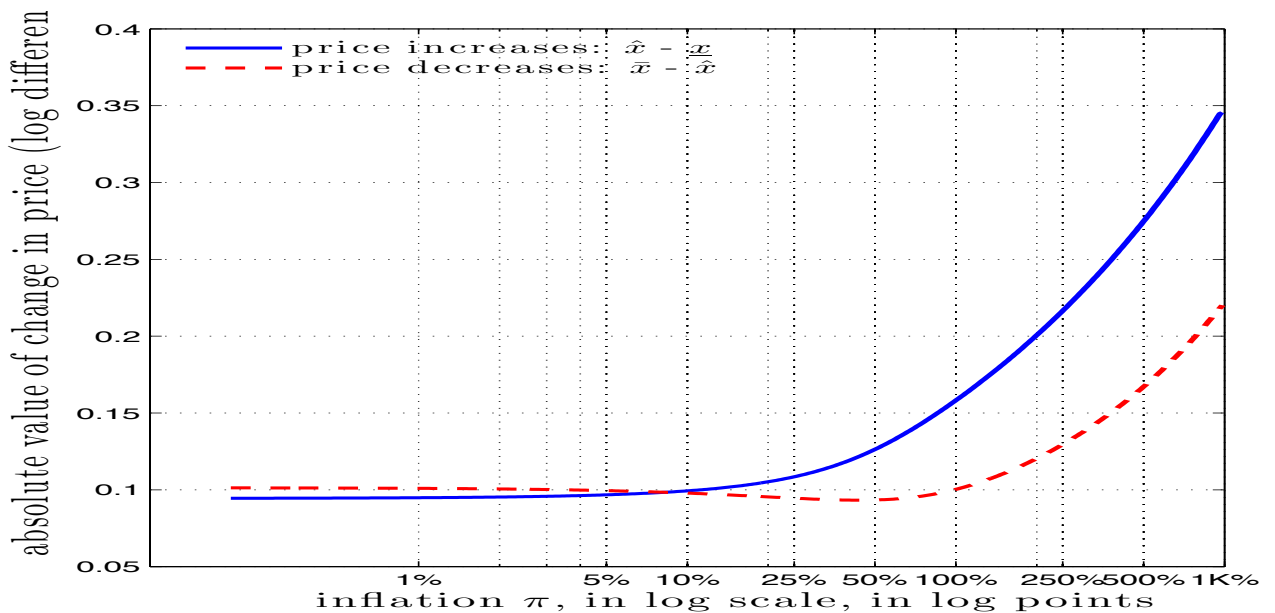
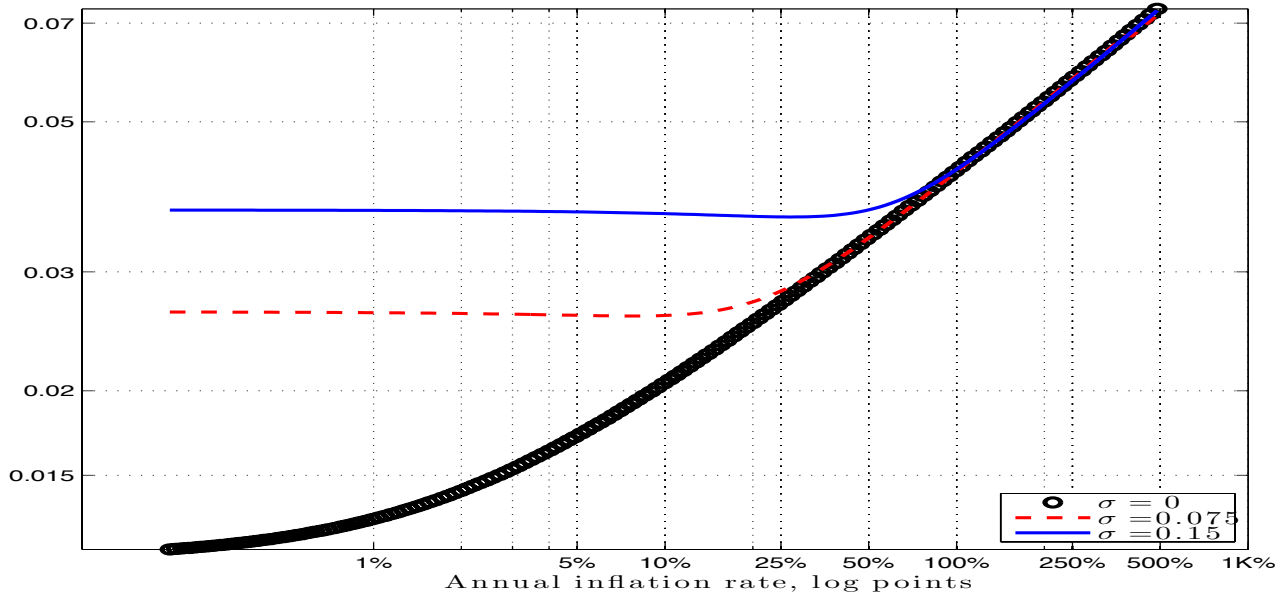


Figure 5: Average size of price increases Δ_p^+ and decreases Δ_p^-



prices $h(p)$ behaves in exactly the same way with respect to inflation. The reason for the difference is that the marginal distribution has to be integrated between values of z for

Figure 6: Standard deviation of log relative prices, conditional on $z = 0$.

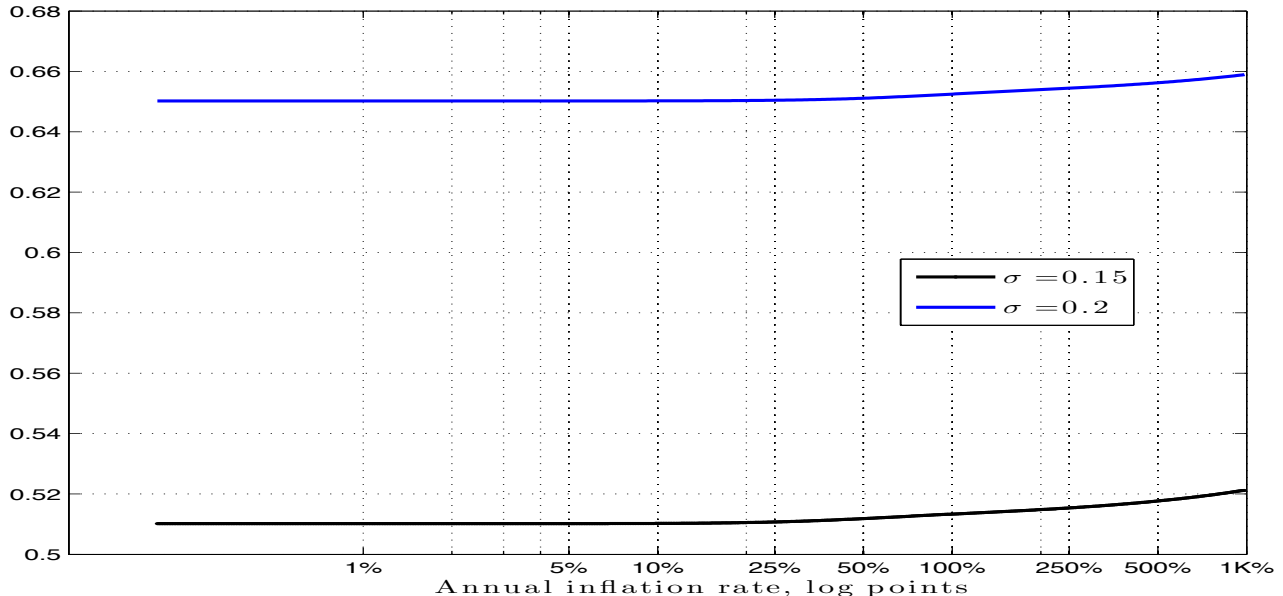


which relative prices can differ much more than then variation in relative prices within of the inaction range for a given z , which is given by $[x, \bar{x}]$. For extremely large inflation rates the width of this range can swamped the effect the variation of z but in our numerical examples it will take inflation rates even much higher than the ones observed in the peak months in argentina. Thus, the rate at which the marginal distribution of relative prices $h(\cdot)$ is slower than the rate at which each of the conditionals densities $g(\cdot, z)$ converge to a uniform as π becomes arbitrarily large. In our numerical examples the unconditional standard deviation of relative prices has an elasticity with respect to inflation much smaller than $1/3$ even for annual continuously compounded inflation rates in around $\pi = 500\%$ as Figure ?? shows, which displays the dispersion of relative prices $\bar{\sigma}$ for different inflation rates for two values of the volatility of cost σ .

We briefly comment on the difference between the behaviour of λ_a and $\bar{\sigma}$ as a function of π . In this model the value for the frequency of price changes $\lambda_a(\pi, \sigma^2)$ converges to $\lambda_a(\pi, 0)$ as inflation π increases much faster than $\bar{\sigma}(\pi, \sigma^2)$. The reason is that given the permanent nature of the shocks to a product cost, the expected time until the next adjustment \mathcal{T} is only a function of $x = p - z$. Recall that $\lambda_a = 1/\mathcal{T}$ so that the cross sectional distribution of z is essentially irrelevant for the frequency of adjustment. Instead, the standard deviation of relative prices $\bar{\sigma}$ depends on the cross sectional distribution of z on a crucial way. Indeed, if the idiosyncratic shocks were completely permanent, there will be no invariant distribution

of relative prices. In our example, the reason why there is an invariant distribution is that $\rho > 0$, so products are returned to $z = 0$ at exponentially distributed times.

Figure 7: Unconditional standard deviation of log relative prices $\bar{\sigma}$



3 Description of Data Set

Our dataset includes a total of 8,618,345 price quotes for items included in the Argentine consumer price index. An *item* is a good/service of a determined brand sold in a specific outlet in a specific period of time. The data goes from December of 1988 until September of 1997. There are a total of 545 goods/services classified according to the MERCOSUR Harmonized Index of Consumer Price (HICP) classification. The HICP uses the first four digit levels of the Classification of Individual Consumption According to Purpose (COICOP) of the United Nations plus three digit levels based on the CPI of the MERCOSUR countries. The 545 *goods/services* in the database are the seven digit level of the HICP classification; six digit level groups are called *products*; five digit level groups are called *sub-classes*; four digit level categories are called *classes*; three digit level categories are called *groups* and two digit level groups are called *divisions*.²

Goods are divided into two groups: homogenous (74.6% of price quotes) and non-homogeneous goods or differentiated goods (25.4% of price quotes). Examples of homogenous goods are:

²To simplify the exposition, when it is clear, we use goods to refer to the goods or services.

barley bread, chicken, lettuce, etc. Examples of non-homogenous goods are moccasin shoes, utilities, tourism, and professional services. Some goods, usually included in the consumer price index are excluded from our sample because their prices are gathered for any good in a basket, i.e. if one good is not available, it is substituted by any another in the basket. An example of these baskets are medicines and cigarettes. These baskets corresponds to around 9.91% of total expenditure. We also exclude fuel prices which corresponds to a 4% of total expenditure. We exclude them because they were gathered in separate data base, so they are not available for all years. In our analysis, we redefine the weights of these goods, by distributing the weights corresponding to these baskets and fuels to the other goods in the product category. As done in other studies, we exclude the cost of dueling or rents (which in our case are sampled monthly for a fixed set of representative apartment buildings and other type of properties, and include what is paid on that month, as opposed to what is paid for a new contract). Rents represent 2.33% of expenditure of households. Overall, once we exclude fuel, rents and goods in baskets we have 506 goods/services, which cover about 84% of household expenditures. Of the expenditure covered, 50.5 % corresponds to differentiated (gathered one per month) and 49.5 % corresponds to the homogenous goods (gathered twice per month).

Over the whole sample, there are 11,659 outlets. On average, there are around 3200 outlets per month where prices are recorded for homogenous goods and about the same number of outlets per month for non-homogenous goods. The distribution of goods/services between homogeneous and non-homogeneous was done to be representative of the expenditure of consumers. The data set contains 75% of observations classified as homogenous goods. Prices are collected in “traditional” outlets (i.e. small-grocery stores) and super-market chains divided according to the expenditure of consumers. The data contains an identifier of whether the good was sold in a super-market chain. Our data sets covers the geographical area of the city of Buenos Aires and the Province of Buenos Aires, which account for about 40% of the population and for about 60% of the GDP of Argentina.

Prices are gathered every two weeks for all homogenous goods and for those non-homogeneous goods gathered in super-market chains; and gathered every month for the rest of the non-homogeneous goods. The data set contains 233 of prices collected every two weeks (44%) and 302 of prices collected every month (56%). There are 29 goods gathered both monthly and biweekly.³ In average across the 9 years there are 166 outlets per good (81 outlets per product gathered monthly and 265 gathered bimonthly). Our data set has many imputed

³The outlets are divided into 20 waves, corresponding to the 20 working days of the month. Each outlet is visited, roughly in the same working day every 10 working days in the case of homogeneous goods and non-homogeneous gathered at super-markets. In any case, we have the particular day where the price was gather.

prices, as well as few price quotes that have not been recorded with no apparent reason to us. For our purposes, we will treat all these price quotes as missing observations. We give now some more detailed explanation. One circumstance where prices are imputed are stock-outs. Across the whole sample there are 10.5% of stock-out items.⁴ Across the whole sample we have 10.2% of items with prices imputed (these items are all stock-outs).⁵ Furthermore there are 2.25% of price quotes without flags for price substitution or stock-out that have no recorded prices. Summarizing, for our purposes we call missing price quotes any quote that has either a stock-out or whose price was not recorded. Using this definition, across the 9 years of our data set we have an average of 12.44% of price quotes that are missing. In addition to stock-outs, the statistical agency substitutes the price quote of an item for a similar item, typically when the good is either discontinued by the producer or not sold any longer by an outlet. Using this definition, across the 9 years of our data set we have an average of 2.39% of price quotes that have been substituted.

The data set contains an indicator of whether an item was on sale or not. The database has around 5% of items with prices associated with a sale flag. This is small compared with the 11% frequency of sales reported by [Klenow and Kryvtsov \(2008\)](#) for the US. 70% of the sales corresponds to homogeneous items (this is similar to [Klenow and Kryvtsov \(2008\)](#), who report that sales are more frequent for food items).

[Figure 8](#) displays the time series at a monthly frequency of the number of outlets per product with non missing price quotes, for all the items and for those gathered biweekly (homogenous goods) and monthly separately (differentiated goods). [Figure 8](#) displays the monthly time series of the frequency of the sale flags for all the products as well as for those gathered monthly and for those gathered biweekly and monthly separately. In [Appendix E](#) there are additional details on the time series of these frequencies.

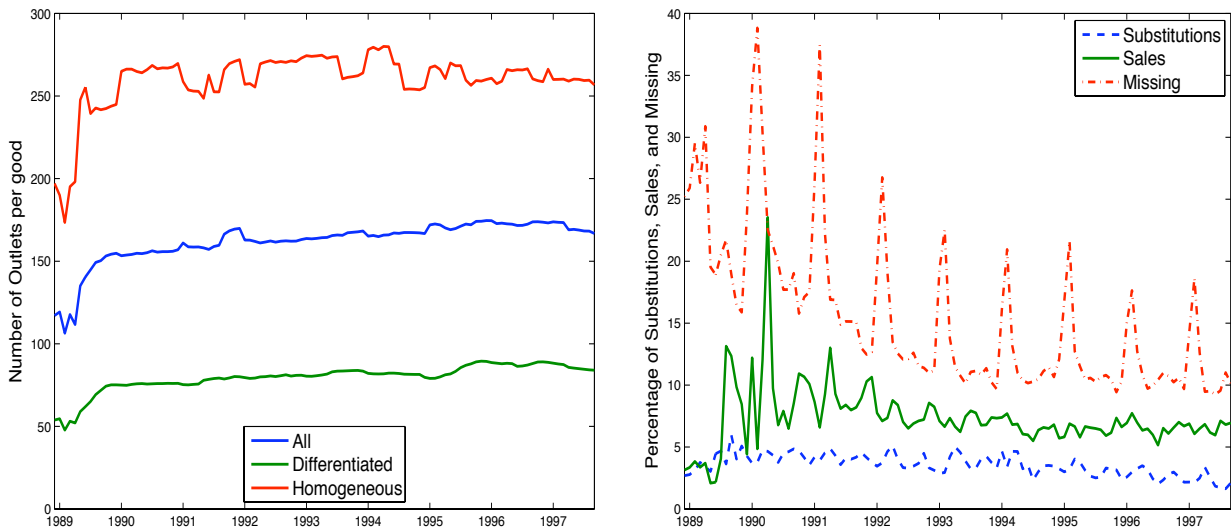
4 Estimating the Frequency of Price Changes

Here we adapt the constant hazard rate model of [Klenow and Kryvtsov \(2008\)](#) to the case where the hazard rate for price changes λ changes through time. In particular, we will assume

⁴3.2% corresponds to what we call “pure stock-out” items, that is, goods are identified in the data set as out of stock in the outlet; and the rest of the stock-outs, 7.3%, corresponds to items that were not sold by the outlet at the time the price was to be collected. Pollsters arrive to each outlet with a form that includes all items for which prices are to be collected and some outlets may not sell some of the items in the form. The price of these items appears as imputed in the database.

⁵Price imputation is very seasonal with an average of 14% during the summer months of January, February and March. Stock-outs are also very seasonal, with 35% of all stock-outs in the months of January to March. The average rate of stock-outs items between January and March is around 14% while in the rest of the months the average stock-out rate is 9.2%.

Figure 8: Number Outlets per Good, and Frequencies of Substitution and Sales



Note: For the homogenous goods during a month we count a sale or substitution if there was one such event in any of the two fifteen days subperiods. Missing includes stock-outs.

that the changes in prices during the period between $t - 1$ and t , in our case a period of two weeks, occurs with a constant probability per unit of time λ_t . We will index periods by $t = 0, 1, 2, \dots, T$, where $T = 212 \approx 9 \times 24$, since our data set covers about 9 years, and the price observations are obtained twice a month (and half of it, for those prices gathered at monthly frequency). Our goal is to estimate this probability rate for each half-month period (or for each month) in our data set. We now describe the different assumptions regarding missing price quotes, sales flags and substitution of products that we use for the estimation of the frequencies λ for each good categories separately. After establishing all the assumptions that allow us to write the likelihood function for these estimations we present four methods to estimate an aggregated frequency of price adjustment at each time period.

4.1 Main Assumptions and Definitions

We describe the assumptions used to estimate the probability of a price change. If between two observed prices there are some missing prices we use the following assumption. If the two observed prices are exactly equal we assume there has been *no* changes in prices in any times between these dates. This is the same assumption as in [Klenow and Kryvtsov \(2008\)](#). Instead, if the two observed prices are different we assume there has been *at least one change* in prices in between. The first assumption allow us to complete the missing prices in between

two observed prices that are equal. From here on, assume that the missing prices in such string of prices have been replaced.

We will refer from now on to the sequence of prices between two different observed prices as a *spell of constant prices*, or for short a *spell of prices*. Without any missing prices, a spell of constant prices is just a sequence of repeated prices ending with a different price. Notice that the last (observed) price in a spell of constant prices is the first price of the next spell.

Next we describe the possible patterns of prices, and its implications for the estimation of the probability of a price change. After following the procedure described above, all spell of prices and missing observations have only two possible patterns. The first pattern is a spell of prices ending with a price change, but with no missing observations. We consider the second pattern in the following section, where we deal with the effect of missing prices. Consider the following example for a spell for an outlet i that contains no missing prices nor substitutions:

Table 1: Example of a spell of constant prices without missing prices

p_t	10	10	10	10	10	15
	t	$t + 1$	$t + 2$	$t + 3$	$t + 4$	$t + 5$
		$\overbrace{\quad}$	$\overbrace{\quad}$	$\overbrace{\quad}$	$\overbrace{\quad}$	$\overbrace{\quad}$
		λ_{t+1}	λ_{t+2}	λ_{t+3}	λ_{t+4}	λ_{t+5}
I_t	y	0	0	0	0	1
γ_t	y	1	1	1	1	1

The braces on top of the values of λ are meant to remind the reader that λ_t refers to the constant probability of change in prices between $t - 1$ and t . The indicator I_{it} adopts the value one if, in outlet i , the price in period t is different from the price at period $t - 1$, and zero otherwise, except for the first temporal period of the first string of prices where it is missing⁶. In this example we have exactly no changes for the first four periods and at least one change in the next period. The probability of observing this completed spell of constant prices is thus:⁷

$$P = e^{-\lambda_{t+1}} \times e^{-\lambda_{t+2}} \times e^{-\lambda_{t+3}} \times e^{-\lambda_{t+4}} \times (1 - e^{-\lambda_{t+5}}) \quad (13)$$

It follows that in this simple case, assuming all the outlets selling the same good have the

⁶We also include the indicator γ_t , which we explain below, for completeness.

⁷We assume that the number of price changes between $t - 1$ and t follows a homogeneous Poisson process with arrival rate λ_t per unit of time. The probability of k occurrences is $e^{-\lambda} \lambda^k / k!$ and the waiting time between occurrences follows an exponential distribution.

same λ_j , the likelihood function of prices observed for product j is

$$L_j = \prod_{i \in O_j} [e^{-\lambda_{j,t}}]^{(1-I_{it})} \times [1 - e^{-\lambda_{j,t-\tau}}]^{I_{it}} \quad (14)$$

The maximum likelihood estimator of the arrival rate of price changes for product j between times t and $t+1$ in the simple case without missing prices and without substitutions is

$$\lambda_{j,t+1} = \log \left(\frac{\sum_{i \in O_j} 1}{\sum_{i \in O_j} (1 - I_{it})} \right) = -\log(1 - f_{jt}) \quad (15)$$

where we let O_j denote the set of the outlets i of the product j and f_{jt} is the fraction of outlets of good j that changed prices in period t . In words, $\lambda_{j,t+1}$ is the log of the ratio of the number of outlets to the number of outlets that have not changed the price between t and $t+1$. Thus $\lambda_{j,t+1}$ ranges between zero, if no outlets have change prices, and infinite if all outlets have changed prices. The probability of at least one change in prices in period t for product j is $1 - e^{-\lambda_{jt}} = f_{jt}$.

4.2 Incorporating information on Missing Prices

Now we consider the case where there are some missing prices before the price change, but we postpone the discussion of the effect of price substitutions.

In general, a spell of constant prices is a sequence of $n+1$ prices that starts with an observed price p_t , possibly followed by a series of prices all equal to p_t , then followed, possibly, by a series of missing prices, that finally ends with an observed price at p_{t+n} that differs from the value of the initial price p_t . Notice that while we also refer to this sequence of prices as a spell of constant prices, it can include more than one price change if there were missing observations, a topic to which we return in detail below.

To deal with missing prices, the interesting patterns for a spell of constant prices are those which end with a price change, but that contain some missing price(s) just before the end of the spell. For example, consider the following spell of prices for an outlet, i :

where an x means that the variable is not defined for that case, the m denotes a missing/imputed price and the y denotes that for indicator I and counter γ the first observation in the spell of prices is missing because it depends on the prices in period $t-1$ which are not in the information of the table. This example shows exactly no changes in the first four periods and at least one change sometime during the next three periods. The probability of

Table 2: Example of a spell of constant prices with observed and missing prices

p_t	10	10	10	10	10	m	m	15
	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$
		$\overbrace{\lambda_{t+1}}$	$\overbrace{\lambda_{t+2}}$	$\overbrace{\lambda_{t+3}}$	$\overbrace{\lambda_{t+4}}$	$\overbrace{\lambda_{t+5}}$	$\overbrace{\lambda_{t+6}}$	$\overbrace{\lambda_{t+7}}$
I_t	y	0	0	0	0	x	x	1
γ_t	y	1	1	1	1	x	x	3
χ_t	0	0	0	0	0	1	1	0

observing this spell is:

$$P = e^{-\lambda_{t+1}} \times e^{-\lambda_{t+2}} \times e^{-\lambda_{t+3}} \times e^{-\lambda_{t+4}} \times (1 - e^{-\lambda_{t+5}} \times e^{-\lambda_{t+6}} \times e^{-\lambda_{t+7}}) \quad (16)$$

The first four products are the probability of no change during the first four periods, and the last term is the probability of at least one change during the last three periods. The second term is the complement of the probability of no change in prices during the last three periods.

The likelihood of the sample of T periods (with $T + 1$ prices) of all the outlets for the good j —denoted by the set O_j —is the product over all outlets i of the product of all spells for outlet i of the probability [equation \(16\)](#). To write the likelihood we define an indicator, χ_{it} , and a counter γ_{it} . The indicator χ_{it} adopts the value one if a price is missing for outlet i in period t , and zero otherwise. The value of γ_{it} counts the number of periods between two non-missing prices. The counter γ_{it} is [Klenow and Kryvtsov \(2008\)](#) duration clock. Then the likelihood function of the prices observed for product j is:

$$L_j = \prod_{i \in O_j} \prod_{t=1}^T [e^{-\lambda_{j,t} \gamma_{it}}]^{(1-I_{it})(1-\chi_{it})} \times \left[1 - e^{-\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau}} \right]^{I_{it}(1-\chi_{it})} \quad (17)$$

Since the λ 's are the probability of a price change and they are indexed at the end of a period, the first temporal observation of prices at $t = 0$ does not enter the likelihood. The log likelihood is:

$$\ell_j = \sum_{i \in O_j} \left(\sum_{t=1}^T (1 - \chi_{it})(1 - I_{it}) \times (-\lambda_{j,t} \gamma_{it}) + \sum_{t=1}^T (1 - \chi_{it}) I_{it} \ln \left[1 - e^{-\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau}} \right] \right) \quad (18)$$

To compute the contribution to the likelihood of a given value of $\lambda_{j,t}$ for $t = 1, \dots, T$ we find convenient to introduce two extra counters: κ_{it} and η_{it} for any period t in which prices

are missing/imputed. The variable κ_{it} counts the number of periods since the beginning of a string of missing/imputed prices. The variable η_{it} counts the number of periods of missing/imputed prices until the next price is observed. For example, consider the string of prices in [Table 3](#).

Table 3: Example of a spell of constant prices, w/ missing prices and counters

p	10	10	m	m	m	m	m	m	15
	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	
	λ_{t+1}	λ_{t+2}	λ_{t+3}	λ_{t+4}	λ_{t+5}	λ_{t+6}	λ_{t+7}		
I	y	0	x	x	x	x	x	x	1
χ	0	0	1	1	1	1	1	1	0
γ	y	1	x	x	x	x	x	x	6
κ	x	x	1	2	3	4	5	5	x
η	x	x	5	4	3	2	1	1	x

[Table 3](#) shows an example of a spell of constant prices for a given variety and a given outlet. In [equation \(19\)](#) we highlight the contribution to the log-likelihood of the value of λ_t for a given outlet i :

$$\begin{aligned} \ell_j = & \dots + (1 - \chi_{it})(1 - I_{it}) \times (-\lambda_{j,t}\gamma_{it}) + (1 - \chi_{it})I_{it} \ln \left[1 - e^{-\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau}} \right] \\ & + \chi_{it} \ln \left[1 - e^{-\left(\sum_{\tau=0}^{\kappa_{it}-1} \lambda_{j,t-\tau} + \sum_{\tau=1}^{\eta_{it}} \lambda_{j,t+\tau}\right)} \right] + \dots \end{aligned} \quad (19)$$

The first two terms have the contribution to the likelihood of $\lambda_{j,t}$, if the price at time t is not missing. The first case corresponds to a price at the beginning of the spell of constant prices. The second to a case where the price is the last one of the spell, and uses γ_{it} to be able to write the corresponding probability. The third term, correspond to the contribution of $\lambda_{j,t}$, if the price at time t is missing, and uses κ_{it} , and η_{it} to write the corresponding probability.

Using [equation \(19\)](#) it is easy to write the FOC with respect to $\lambda_{j,t}$ of the sample as:

$$\begin{aligned} \frac{\partial \ell_j}{\partial \lambda_{j,t}} = & \sum_{i \in O_j} (1 - \chi_{it})(1 - I_{it}) \times (-\gamma_{it}) + \sum_{i \in O_j} (1 - \chi_{it}) I_{it} \frac{1}{e^{\left(\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau}\right)} - 1} \\ & + \sum_{i \in O_j} \chi_{it} \frac{1}{e^{\left(\sum_{\tau=0}^{\kappa_{it}-1} \lambda_{j,t-\tau} + \sum_{\tau=1}^{\eta_{it}} \lambda_{j,t+\tau}\right)} - 1} = 0, \end{aligned} \quad (20)$$

for $t = 1, \dots, T$.

Roughly speaking the estimator for $\lambda_{j,t}$ computes the ratio of the number of outlets i that a time t have change the prices with those that have not change prices or that have missing prices. This approximation is exact if no outlet has a missing/imputed price at period t or before. In this case $\chi_{is} = 0$ for $s = 1, 2, \dots, t$ for all $i \in O_j$, then [equation \(20\)](#) becomes

$$\sum_{i \in O_j} (1 - I_{it}) \times \gamma_{it} = \sum_{i \in O_j} \left[\frac{I_{it}}{e^{\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau}} - 1} \right],$$

which, if we make $\lambda_{j,t} = \lambda_{j,t-1} = \dots = \lambda_{j,t-\gamma_{it}+1}$ is the same expression than in [Klenow and Kryvtsov \(2008\)](#). In the case of no missing observations, and where all the λ 's are assumed to be the same, this maximum likelihood estimator coincide with the simple estimator introduced in [equation \(15\)](#).

4.3 Incorporating Missing Prices and Sales

Our data contains a flag indicating whether an item was on sale. We consider a procedure that disregards the changes in prices that occur during a sale. The idea behind this procedure is that sales are anomalies for the point of view of some models of price adjustment, and hence they are not counted as price changes. To explain this assumption we write an hypothetical example:

Table 4: Example of a spell of constant prices removing sales

p_t	10	10	10	m	8	15
	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$
	λ_{t+1}		λ_{t+2}		λ_{t+3}	
I_t	y	0	0	0	0	1
a_t	y	0	0	0	1	0

In [Table 4](#) the indicator a_t takes the value of one if the good is on sale on period t and if the price at the time t is smaller than the previous recorded price. In this case, the price in periods $t+3$ and $t+4$ is changed to 10, the value of the previous recorded price. The general principle is to consider a string of recorded prices, possible missing values, and a price that has a sale flag, and replace the price of the string of missing values and the period with a sale flag for the previous recorded price. In other words, we replace the price at the period with a sale flag for the previous recorded price and then using our first assumption on missing prices we complete the missing price in between two prices that are equal. When the sales are disregarded the number and duration of price spells can change. Once this procedure is

implemented, the likelihood is the one presented above using the modified price series -indeed in [Table 4](#) the indicator I_t is the one that corresponds to the modified price string. We refer to the corresponding estimates as those that *excludes sale quotes*. This is a procedure used by many, e.g. [Klenow and Kryvtsov \(2008\)](#). By construction with this method the estimates for λ will be smaller.

4.4 Incorporating Missing Prices and Price Substitutions

In this section we discuss different assumptions on the treatment of missing data and good substitutions that allow us to construct four estimators of the frequency of price changes that we report later on.

We use the indicators $\tilde{e}_{it}, e_{it}^*, \bar{e}_{it}$ to consider two different assumptions on how to treat a price spell that ends in some missing values or price substitutions. In particular, consider the case of a substitution of a product or a missing price. As explained above, our data set contains the information of whether the characteristic of the product sold at the outlet has changed and was subsequently substituted by a similar product. We also have information on whether the price is missing (mostly due to a stock-out). To be concrete, consider the following example of a spell of constant prices in [Table 5](#). In this table, and $s_{it} = 1$ denotes the period where a price substitution has occurred. Thus, the example has a spell of 9 prices, with two periods (3 prices) with no change in prices, then 5 periods with missing prices, and finally in the last period there is an observed price that correspond to a substitution of the good.

Table 5: Example: spell of constant prices w/ counters for substitutions

p	10	10	10	m	m	m	m	15
	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$
	λ_{t+1}	λ_{t+2}	λ_{t+3}	λ_{t+4}	λ_{t+5}	λ_{t+6}	λ_{t+7}	
I	y	0	x	x	x	x	x	1
χ	0	0	0	1	1	1	1	0
s	y	0	0	0	0	0	0	1
\tilde{e}	y	0	0	0	0	0	0	0
e^*	y	1	1	1	1	1	1	1
\bar{e}	y	0	0	1	1	1	1	1

As in the previous examples, for the first observation an y indicates that the value of the indicator cannot be decided based on the information in the table.

The issue is the interpretation of when and whether there has been a price change in the previous price spell. One interpretation is that there has been a price change somewhere between periods $t+2$ and $t+7$. A different interpretation is that, because the price spell ends with the substitution of the good, the price has not changed. The idea for this interpretation is that if the good would have not changed, the price could have been constant beyond $t+7$. The next three cases explain how to implement the first interpretation, and two ways to implement the second one. The last two cases present two simple estimators, one that treats quotes with substitutions as regular price changes, and one that exclude them.

1. We disregard the information of the substitution of a good, and proceed as we have done so far: including all price quotes as if the good have been not changed. In the previous example, it consists on assuming that the price has changed between periods $t+2$ and $t+7$. In this case we say that the probability of observing the spell in the table is given by:

$$P = e^{-\lambda_{t+1}} \times e^{-\lambda_{t+2}} \times (1 - e^{-\lambda_{t+3}-\lambda_{t+4}-\lambda_{t+5}-\lambda_{t+6}-\lambda_{t+7}}) \quad (21)$$

In this case we set $\tilde{e}_{it} = 0$ for all periods, since we don't want to exclude any part of this price spell. We refer to these assumptions as *including all price quotes*.

2. We follow [Klenow and Kryvtsov \(2008\)](#) and others and exclude completely any spell of prices that ends with a substitution of a product. We implement this by defining the indicator $e_{it}^* = 1$ for any price corresponding to this spell, i.e. we exclude all the observations –with the exception of the first price, which is relevant for the previous string. In this case we have no associated probability for this spell. The idea behind this treatment is that if there would have been no substitution of the good the price could have stayed constant even beyond $t+7$. We refer to these assumptions as *excluding substitution spells* for short.
3. We introduce a new way to handle this information based on the following underlying assumption: if a spell of constant prices ends up in a substitution we interpret that the product has changed, and hence we cannot infer from the observed price whether the price has changed or not, as in the previous case. Yet, unlike the previous case, we do not discard the information at the beginning of the string, where the product was the same and its price was not changing. In this case the associated probability for this spell is:

$$P = e^{-\lambda_{t+1}} \times e^{-\lambda_{t+2}} \quad (22)$$

In this case we use the indicator $\bar{e}_{it} = 0$ for the first two observations, among which we know that there was no price change, and $\bar{e}_{it} = 1$ for the rest of the observations where we can't conclude if there was a price change for the *same* product. We refer to these assumptions as *excluding substitution quotes* for short.

4. We present an alternative estimator to the maximum likelihood, which has the advantage of being simpler to describe and understand. This estimator imitates the one for the case where there are no missing price quotes in [equation \(15\)](#), and simply excludes the values of the missing. In this case, this estimator is:

$$\lambda_{j,t+1} = \log \left(\frac{\sum_{i \in O_j} (1 - \chi_t) (1 - \chi_{t+1})}{\sum_{i \in O_j} (1 - \chi_t) (1 - \chi_{t+1}) (1 - I_{it})} \right) \quad (23)$$

In words, $\lambda_{j,t+1}$ is a non-linear transformation of the probability of the change of prices. This probability is estimated as the ratio of the outlets that have changed prices over all the outlets, including only those price quotes that are simultaneously not missing at t and $t + 1$. While this estimator is simpler than the maximum likelihood, it does not use all the information of the missing values efficiently. We refer to this estimator as the *simple* estimator.

Finally, to completely state the notation for the likelihood function, we use the indicator ζ to deal with missing prices at the beginning of the sample. In particular, if for an outlet i the sample starts with $n + 1$ missing prices, we exclude these observations from the likelihood since we cannot determine the previous price. We do this by setting the indicator $\zeta_{it} = 1$ for $t = 0, 1, \dots, n$. Thus, depending of the assumption, the exclusion indicator e takes the values given by \tilde{e} , \bar{e} or e^* , besides the value of 1 for all the missing observations at the beginning of the sample. The log likelihood function is then:

$$\ell_j = \sum_{i \in O_j} \sum_{t=1}^T (1 - e_{it}) \left((1 - \chi_{it})(1 - I_{it}) (-\lambda_{j,t} \gamma_{it}) + (1 - \chi_{it}) I_{it} \ln \left[1 - e^{-\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau}} \right] \right) \quad (24)$$

The first order condition for $\lambda_{j,t}$, using the counters κ and η is:

$$\begin{aligned} \frac{\partial \ell_j}{\partial \lambda_{j,t}} = & \sum_{i \in O_j} (1 - \chi_{it}) (1 - e_{it}) (1 - I_{it}) \times (-\gamma_{it}) + \\ & \sum_{i \in O_j} (1 - \chi_{it}) (1 - e_{it}) I_{it} \frac{1}{e^{(\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau})} - 1} \\ & + \sum_{i \in O_j} \chi_{it} (1 - e_{it}) \frac{1}{e^{(\sum_{\tau=0}^{\kappa_{it}-1} \lambda_{j,t-\tau} + \sum_{\tau=1}^{\eta_{it}} \lambda_{j,t+\tau})} - 1} = 0, \end{aligned} \quad (25)$$

for $t = 1, \dots, T$.

4.5 Aggregation: Weighted Average, Median, Weighted Median and Pooled Maximum Likelihood

In this section we deal with the issue of aggregation. So far we have described how to estimate the frequency of price adjustment for each good category separately.

Remember that those goods that fall in the homogenous goods category are sampled bi-monthly and so will be our estimates. In this way, the first step in order to aggregate all categories is to convert them into monthly estimates, which is done simply by adding the two estimates in any given month (this results from the exponential assumption for our likelihood function).

Next, we compute three aggregated estimations. First, we calculate the weighted average of all monthly estimates (both differentiated and homogenous goods), where the weights are the corresponding expenditure shares of each good category.

$$\lambda_t = \sum_{i=1}^N \omega_i \lambda_{it}$$

For future reference, we will call this *weighted average* or simply *WA*, followed by the specific treatment of missing, sales and substitutions. For example, *Weighted average excluding sales*.

The other two estimates are the median and weighted median of all monthly estimates (both differentiated and homogenous goods). The aggregated median estimation (*median* for short) consists in taking the median λ of all products at each time period. The aggregate weighted median estimation is computed by sorting, at each time period, the expenditure weights of each product by the value of their associated λ from lowest to highest. Then we compute the accumulated sum of the weights until reaching 0.5. The aggregate weighted

median of the frequency of price adjustment is the associated λ of the product whose weight makes the accumulated sum equal to or greater than 0.5. We refer to this estimate as *weighted median*.

Finally we consider a last aggregated estimation. As mentioned, the estimates for the frequency of price changes presented allow the value of λ to depend on the time period and the good. We now consider an estimate based on the assumption that the frequency price changes is common for all goods, but that that it can change between time periods. This simply puts together the outlets for all goods in our sample. Thus, the log likelihood is:

$$\ell(\lambda_1, \dots, \lambda_T) = \sum_{j=1}^N \ell_j(\lambda_1, \dots, \lambda_T)$$

where the $\ell_j(\lambda_1, \dots, \lambda_T)$ corresponding to the log likelihood for each assumption about missing prices and or price substitutions as it has been introduced in the previous sections, and where N is the number of goods in our sample. We refer to this estimator as the *pooled maximum likelihood* or for short PML. Likewise, when we assume that all goods have the same frequency of price changes but use the simple estimator for λ , we refer to it as *simple pooled* estimator.

5 Estimated Frequency of Price Changes and Inflation

In this section we report estimates of the frequency of price changes λ_t and we describe how it co-moves with inflation π_t . We organize the presentation around the theoretical predictions of menu cost models discussed in [Section 2](#): the elasticity of the frequency of price changes with respect to inflation is low for low rates of inflation and it is close to 2/3 for high levels of inflation.

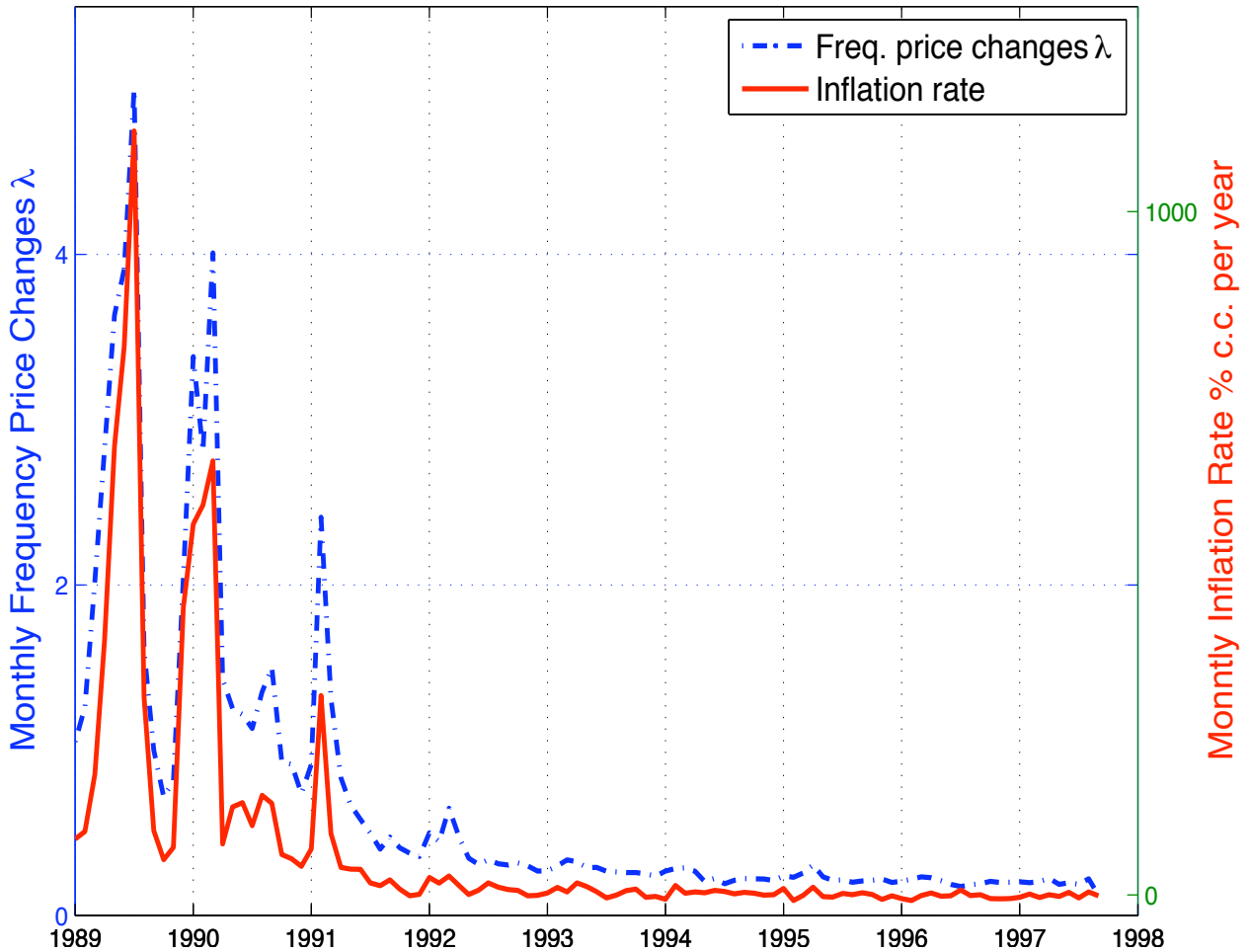
We report monthly estimates of λ_t using different sets of goods: homogenous goods, differentiated goods, all goods together, and different levels of aggregation by industry. We check the robustness of our estimates to different assumptions and exclusions regarding the presence of sales, product substitutions and missing prices (as explained in [Section 4.1](#) to [Section 4.5](#)).

We first concentrate the description of the results on the estimate that is easier to describe, which we label as *simple* estimator. In [Figure 9](#) we plot the monthly time series of the simple estimator as well as of the inflation rate. For this figure we assume that all homogeneous and all differentiated goods have the same frequency of price changes, described as pooled simple estimator in [Section 4.5](#), we aggregate the bi-weekly estimates⁸ of the homogenous goods to a

⁸The monthly frequency is the sum of the bi-weekly frequencies of each month

monthly frequency, and we plot the weighted average of these two estimators, using the share of household expenditures as weights. We remind the reader that the “simple estimator” just counts the fraction of price changes in a period of time, and transform this in a rate per unit of time λ , see [equation \(15\)](#) in [Section 4.1](#). We refer to λ as the instantaneous frequency of price changes, which has the dimension of the number of price changes per month.

Figure 9: Estimated Frequency of Price Changes λ and Inflation Rates



Note: Simple estimator of λ , $\hat{\lambda} = -\log(1 - f_t)$, where f_t is the fraction of outlets that changed price in period t . λ is estimated separately for homogeneous goods (bi-weekly sample) and for differentiated goods (monthly sample). Homogenous goods frequencies are converted to monthly by adding the bi-weekly ones for each month pair. The aggregate number is obtained by averaging with the respective expenditure shares in the Argentine CPI. Inflation is the average of the log-difference of monthly prices weighted by expenditure shares.

It is clear from [Figure 9](#) that the frequency of price changes λ and the contemporaneous inflation rate π are highly correlated. For instance, during the mid-1989 hyperinflation the

implied expected duration of a price spell is close to one week; while after 1993 the implied expected duration is close to half a year.

Next in [Figure 10](#), we produce a scatter plot using log scale for both variables.⁹ motivated by the theoretical results of [Section 2](#), which suggests no relationship between the two when the inflation rate is low, and an elasticity of λ with respect to inflation close to $2/3$ when inflation is high. On the right axis we indicate the implied instantaneous duration, i.e. $1/\lambda$. In interpreting this figure, as well as the other estimates presented below, it is worth noting that $1/\lambda_t$ is the expected duration of prices at time t if λ_t will be unchanged into the future, and provided that the probability of a price change is the same within the smaller period of observation (1 month for differentiated goods, 2 weeks for homogeneous goods).

Motivated by the theoretical considerations of [Section 2](#), as well as for the pattern we think evident in this scatter plot, we fit the following statistical model to the data:

$$\log \lambda_t = -\log(d_c) + \eta \log \max \{\pi^c, |\pi_t|\} + \varepsilon_t \quad (26)$$

where η is the elasticity of the frequency of price adjustments to inflation for high inflation, π^c is the threshold inflation rate below which the elasticity is zero, and d_c is the implied instantaneous duration at the threshold. We fit π^c , d_c , and η by Non Linear Least Squares (NLLS). We note that the comparative static of the models discussed in [Section 2](#) does not imply a kink as the one in [equation \(26\)](#), we merely use this specification because it is a low dimensional representation of interesting patterns in the data that provides a good fit and has properties at the extreme values that are consistent with our interpretation of the theory.

As indicated in [Figure 10](#) the fitted value of the elasticity is $\eta = 0.59$, which to us is surprisingly close to the theoretical value of $2/3$. The fitted value for the threshold inflation π^c is 6.1%. The line we plot in [Figure 10](#) is flat by construction for the periods with $\pi_t < \pi^c$. In [Section 5.4](#) we study more carefully the relationship between inflation $p_{i,t}$ and λ_t for low inflation periods.

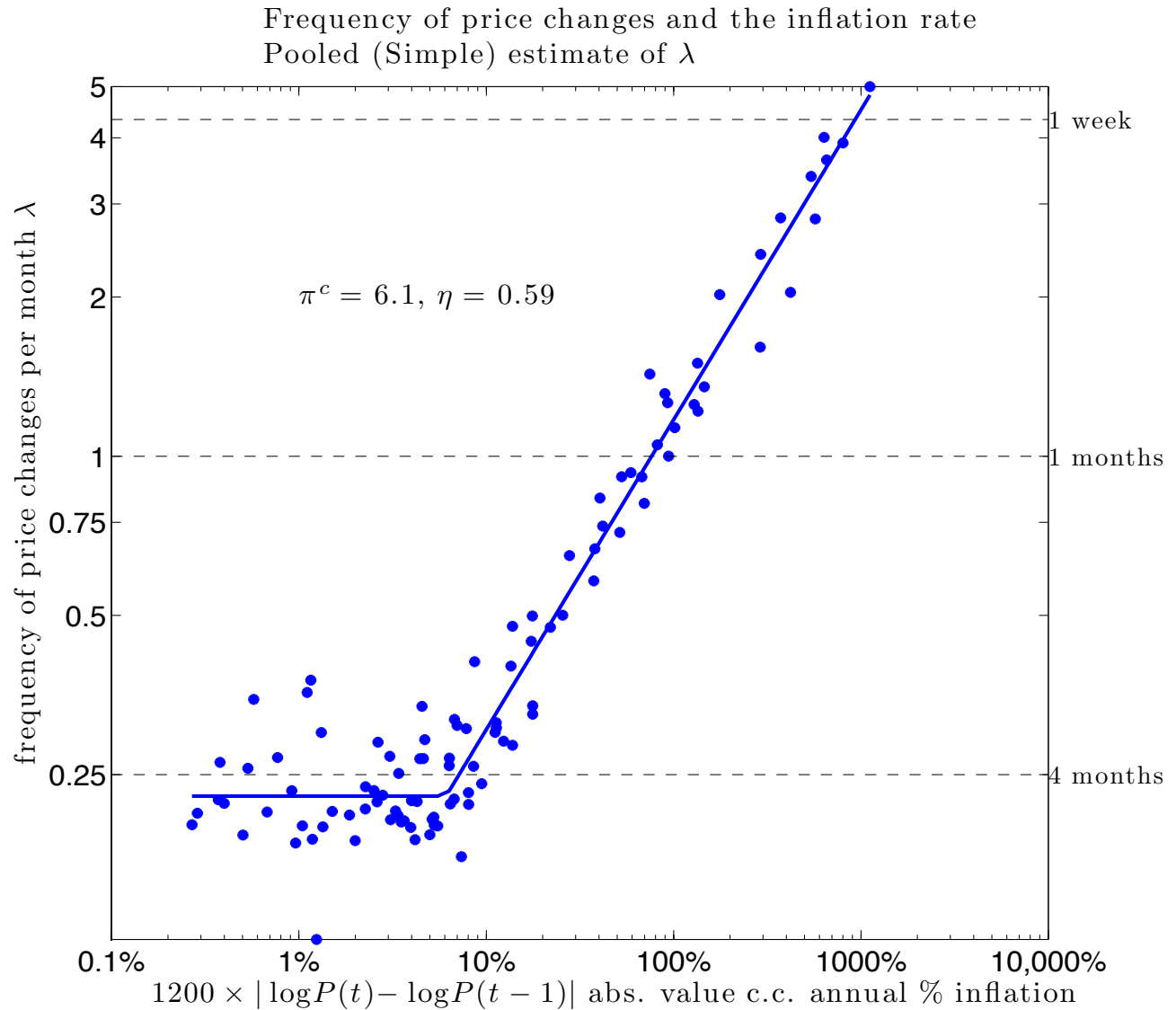
5.1 Sensitivity Analysis with Aggregate Data

This subsection reports the sensitivity of the estimates of the elasticity η , the threshold inflation π^c and the duration at low inflation d_c obtained with the simple estimator to different treatments of missing data, sales, product substitution and broad aggregation levels. The results obtained from using the different methodologies for estimating the frequencies of price adjustment described in [Section 4](#) are presented in [Table 6](#).

[Table 6](#) reports estimates of the three parameters of [equation \(26\)](#) for the sample of

⁹See [Section 2](#) for the caveats on these results and on the interpretation of contemporaneous correlations.

Figure 10: Relationship between the frequency of price changes λ and the inflation rate (pooled simple estimator)



Note: Simple estimator of λ , $\hat{\lambda} = -\log(1 - f_t)$, where f_t is the fraction of outlets that changed price in period t . λ is estimated separately for homogeneous goods (bi-weekly sample) and for differentiated goods (monthly sample). Homogenous goods frequencies are converted to monthly by adding the bi-weekly ones for each month pair. The aggregate number is obtained by averaging with the respective expenditure shares in the Argentine CPI. Inflation is the average of the log-difference of monthly prices weighted by expenditure shares. The parameters η and π^c are estimated fitting $\log \lambda_t = -\log(d_c) + \eta \log \max \{\pi^c, |\pi_t|\} + \varepsilon_t$ with non linear least squares.

differentiated goods (monthly), for the sample of homogeneous goods (bi-weekly) and for the aggregate. The latter is obtained by averaging the estimated λ s with their expenditure shares after converting the bi-weekly estimates to monthly ones.

The first and second blocks of columns show the inflation thresholds and elasticities as explained in the earlier paragraphs. The third block of columns shows the implied duration of price spells when inflation is low (below the threshold) under the assumption that the frequency of price adjustment is constant.

The first line in [Table 6](#) corresponds to the pooled simple estimator reported in [Figure 10](#). The estimates of the elasticity of the frequency of price adjustment with respect to inflation, η , are very similar for the λ s in the two samples and for the aggregate λ . The estimates for the threshold inflation and for the expected duration at the threshold differ in the two samples. Threshold inflation is around 4% per year for differentiated goods with an expected duration of eight months. For homogeneous goods the threshold inflation climbs to 12% per year and the expected duration falls to under three months. For the aggregate λ the threshold inflation is 6% per year and the expected duration is just over four months.

The other lines in the table provide estimates of the three parameters of interest for different aggregation methods and for the different treatments of missing observations, product substitutions and sales described in [Section 4](#). The values of the threshold inflation are robust to all these different estimation techniques for the λ s. The same is true for the elasticities, η , with the caveat that when we aggregate using medians the elasticity climbs to 0.8 for differentiated goods, 0.9 for homogeneous goods and 0.8 for the aggregate.

The treatment of sales and substitutions does not seem to have an effect on the estimates of π^c and η , but do affect the estimates of the expected duration of price spells when inflation is below, d_c , as in other papers in the literature (see [Klenow and Malin \(2011\)](#)). Durations increase from 4.4 months to 5.5 months when sales price quotes are replaced by the price quote of the previous regular price. In [Klenow and Kryvtsov \(2008\)](#) durations go from 2.2 months to 2.8 after the sales treatment and in [Nakamura and Steinsson \(2008\)](#) they go from 4.2 months to 3.2. Time series for frequency of substitution, sales and missing values in the sample can be seen in [Figure 8](#) and in [Figure 23](#) in [Section E](#).

What accounts for the differences in the estimates for the threshold inflation and for the implied duration at the threshold between the sample of differentiated and homogeneous goods? Expected durations are much higher for differentiated goods than for homogeneous goods. In principle, we believe that this discrepancy can be attributed to two features: an intrinsic difference between the type of goods or due to the fact that the prices of homogeneous goods are sampled bi-monthly and prices for differentiated goods once a month. For the interested reader, in [Appendix D](#) we try to elucidate this issue by conducting some further

Table 6: Estimates of Slope η , Threshold Inflation π^c , and Expected Duration d_c .

Aggregation	Annual Inflation			Elasticity			Expected Duration		
	Threshold π^c			η			at threshold d_c		
	Diff.	Hom.	Agg	Diff.	Hom.	Agg	Diff.	Hom.	Agg
A. Simple Estimator (No information from missing price quotes)									
Pooled	3.8	12.5	6.1	0.63	0.62	0.59	8.5	2.7	4.4
Weighted Average	3.8	12.4	6.1	0.64	0.64	0.60	8.0	2.7	4.3
Median	3.9	12.5	6.2	0.79	0.89	0.83	13.2	11.5	12
Weighted Median	3.9	12.6	6.1	0.79	0.87	0.82	11.3	9.5	10.6
Pooled (excludes sales)	4.0	11.8	6.0	0.63	0.65	0.61	9.3	3.6	5.5
B. All price quotes									
Pooled	3.8	12.5	6.2	0.63	0.61	0.58	8.8	2.7	4.5
Weighted Average	3.5	11.9	5.8	0.63	0.61	0.58	8.4	2.7	4.4
Median	3.9	12.0	6.1	0.78	0.88	0.80	13.3	6	8.9
Weighted Median	3.9	12.4	6.2	0.78	0.86	0.79	11.6	4.8	7.3
C. Excludes substitution quotes									
Pooled	3.8	12.6	6.4	0.67	0.63	0.61	10.6	3	5
Weighted Average	3.5	11.9	5.8	0.66	0.62	0.60	10	2.9	4.8
Median	3.9	11.9	6	0.84	0.92	0.87	16.8	14.4	15.6
Weighted Median	3.8	12.3	6	0.82	0.9	0.85	14.4	10.9	13.2
C. Excludes substitution spells									
Pooled	3.6	12.4	6.0	0.65	0.62	0.59	9.5	2.8	4.6
Weighted Average	3.5	11.7	5.6	0.65	0.61	0.58	8.7	2.8	4.5
Median	3.8	11.9	5.7	0.81	0.9	0.84	14.4	13.2	14.3
Weighted Median	3.8	12.1	5.9	0.82	0.88	0.85	13.2	10.2	11.6
D. Excludes substitution and sale quotes									
Pooled	4.3	11.7	6.0	0.64	0.64	0.60	9.5	3.6	5.6
Weighted Average									
Median									
Weighted Median									

Note: *Diff.* denotes differentiated goods, which are sampled once a month. *Hom.* denotes homogenous goods, which are sampled twice a month. *Agg* denotes the weighted average of the Differentiated and Homogenous goods, with weights given by the expenditure shares and where the Homogenous goods have been aggregated to monthly frequencies. For each case we use NLLS to fit $\log \lambda_t = -\log(d_c) + \eta \log(\max\{|\pi_t|, \pi^c\}) + \epsilon_t$. The annual expected duration at the threshold d_c is expressed in months.

exercises which point both to a violation of the assumption of the hazard rate of price changes being constant in the duration of the price spell, and thus making sampling periodicity not innocuous and homogeneous goods having higher idiosyncratic volatility as well.

5.2 Frequency of price changes and inflation: estimation at a more disaggregated level

As [Table 6](#) indicates that the frequency of price adjustment could be very different for different goods, in this section we explore the robustness of the estimates of the parameters of interest reported in the table to fitting [equation \(26\)](#) at different levels of aggregation. To do this, we estimated λ pooling the data for all the products in an industry using the *simple* estimator. We do this at five and six digits of aggregation.¹⁰ Once we estimated λ industry by industry we fit [equation \(26\)](#) pooling the estimated λ s for all industries and we also fit [equation \(26\)](#) industry by industry and analyze the distribution of the industry estimates of the triplets (η, π^c, d_c) .

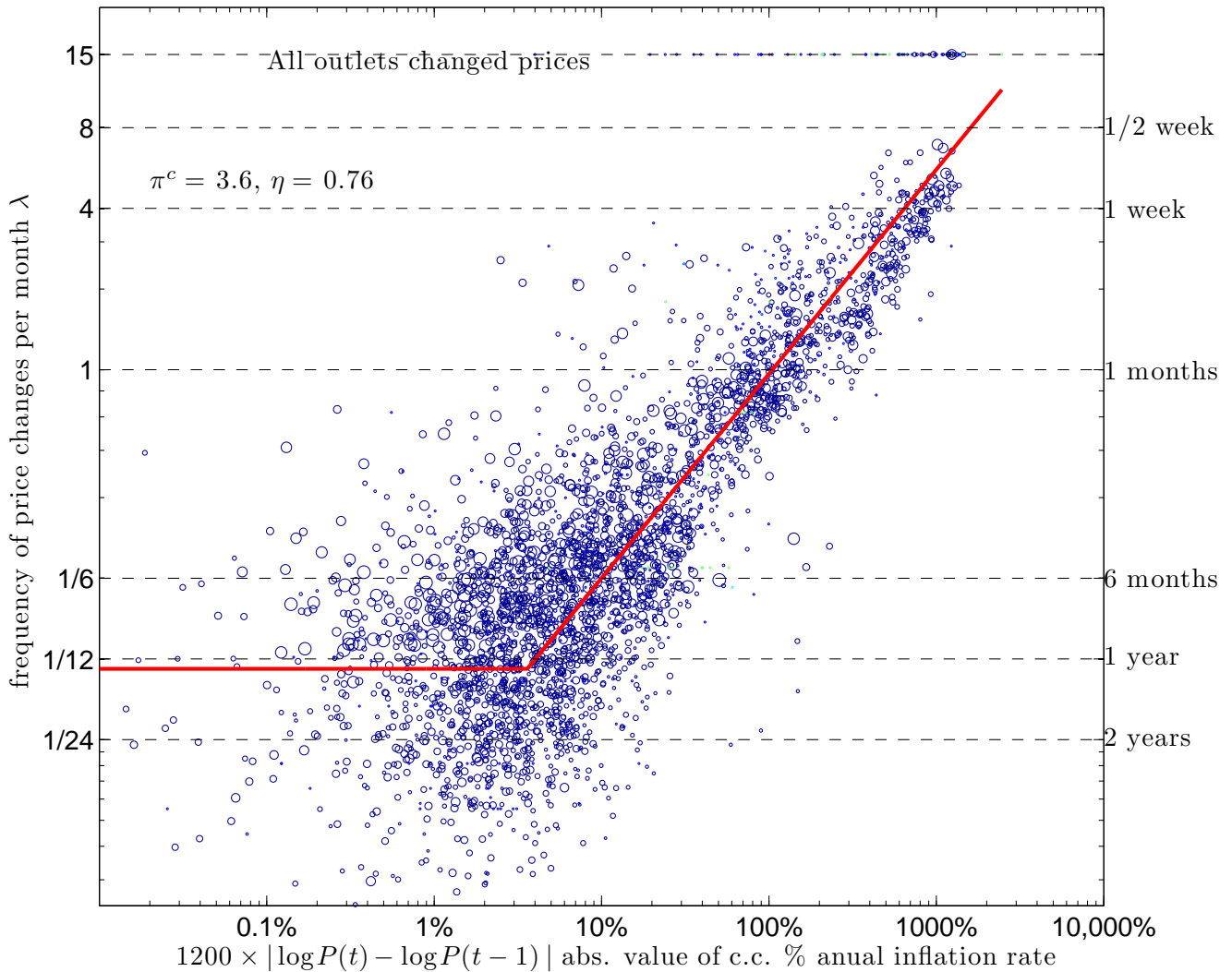
In this section we also look at the co-movements between the heterogeneity between the estimated frequencies of price adjustment and inflation. The menu cost model predicts that if the heterogeneity in the frequency of price adjustments is due to heterogeneous idiosyncratic shocks, as inflation rises and aggregate shocks become more important than idiosyncratic ones we should expect that the heterogeneity of the estimates of λ would decrease with inflation. We analyze this hypothesis at the end of the section.

[Figure 11](#) and [Figure 12](#) plot the estimated values of λ against the contemporaneous inflation of each group of goods, both in log scale. [Figure 11](#) shows a scatter plot of the estimated λ for differentiated goods and [Figure 12](#) one for the homogeneous goods, both of them at the 5 digit aggregation level. In the right axis we have also included several values of the implied instantaneous duration. The area of each circle is proportional to the square root of the number of outlets in the 5-digit group for the date of the corresponding inflation-frequency pair.

We observe that in each one, [Figure 11](#) and [Figure 12](#), there is a line of points for which all outlets changed prices. This stems from the fact that at this level of disaggregation there are several goods for which for a particular period (a month for differentiated, and two weeks for homogenous goods) all stores changed their price. The simple estimator ([equation \(23\)](#)) in this case yields an estimate of λ equal to infinity, but with an equally large standard error!. We have included these values in the scatter plots and mark them correspondingly.

¹⁰ As described in [Section 3](#) goods are classified at different level of aggregation. At five digits of aggregation the entries are categories such as dried pasta, fresh pasta, fresh pork, frozen pork. At six digits the entries are fresh gnocchi, fresh ravioli, fresh pork ribs, fresh pork legs.

Figure 11: Frequency of Price Changes and Inflation for differentiated goods sampled once a month estimated at a 5-digit disaggregation level.



Note: Each dot is a (π_t, λ_t) pair, where π_t is the inflation rate of each industry and λ is the simple estimator of the industry monthly frequency of price adjustment for each industry. When all outlets in an industry change prices in a month λ is infinity, which we set to $\lambda = 15$. We fit $\log \lambda_t = -\log(d_c) + \eta \log(\max\{\pi_t, \pi^c\}) + \epsilon_t$. The regression excludes the 1.9% of top coded observations. The size of each circle is proportional to the square root of the number of outlets used in the estimation of the corresponding λ

For differentiated goods, which are sampled monthly, we have that about 1.9% of the month-industry pairs for which all outlets have changed prices. For homogenous goods sampled every two weeks we have 1.4% of month-industry pairs for which all outlets have changed

prices. These observations are not visible in [Figure 12](#) because the circle is very small relative to the others due to the small number of outlets.

Each graph shows the fitted line of [equation \(26\)](#), for which we exclude the industry-periods for which all store change prices. The fitted values of the elasticity η and thresholds d_c, π^c are similar to the ones reported for the data aggregated at the level of all the differentiated or all the homogenous goods.

The results when we estimate the triplets (η, π^c, d_c) in [equation \(26\)](#) for each 5 or 6 digit aggregation industry separately are reported in [Table 7](#) and in [Table 8](#). We present summary statistics for the distribution of the fitted values of the elasticity η , threshold inflation π^c and duration for low inflation d_c . [Table 7](#) display the distribution of estimates using aggregate inflation as the independent variable, while [Table 8](#) reports the estimates using the inflation rate of each 5 and 6 digit aggregation industry.

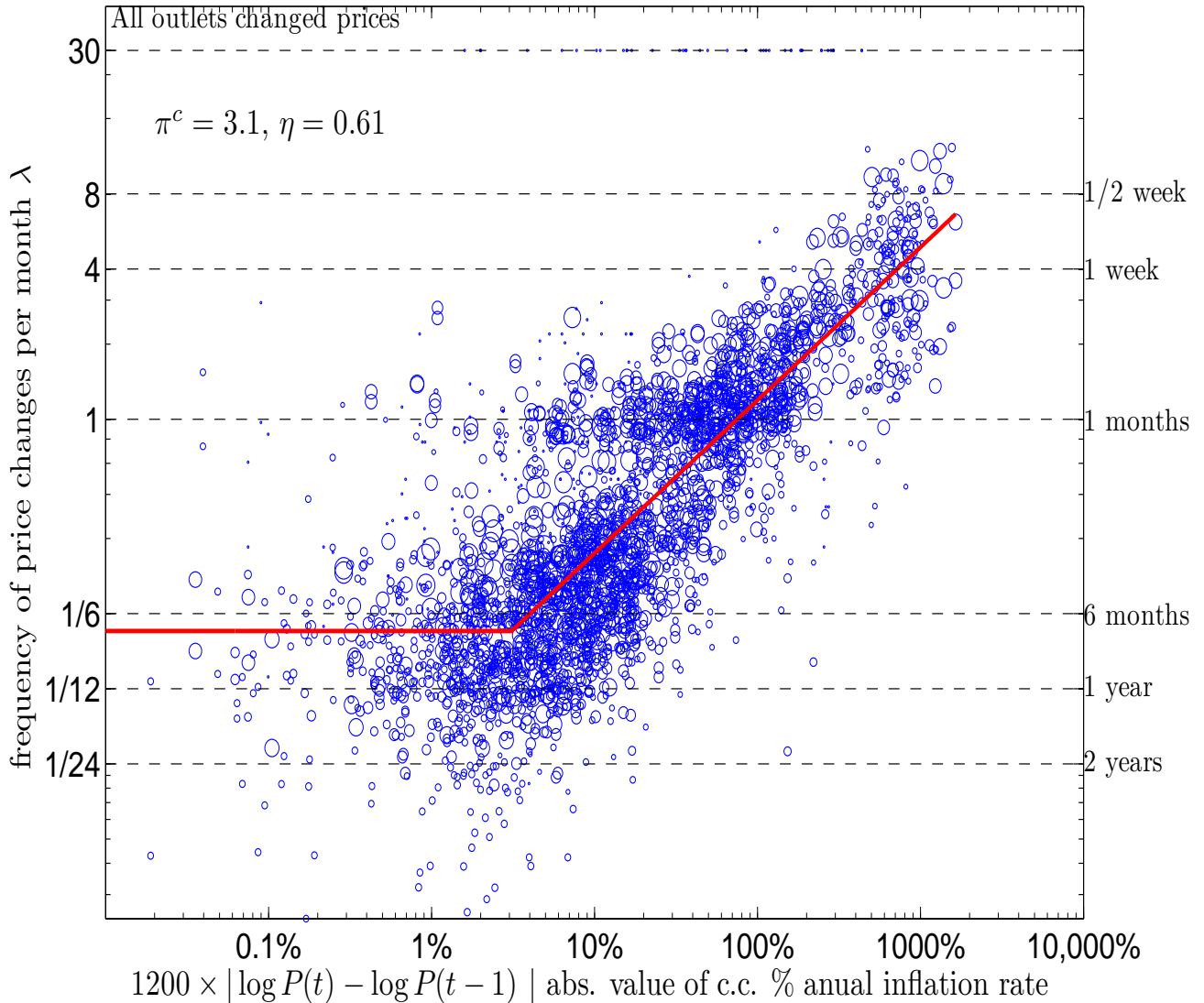
Table 7: Distribution of fitted η , π^c and d_c at 5 and 6 digit level. Using aggregate inflation

Digits	Annual Inflation Threshold π^c				Elasticity η				Duration Threshold d_c (years)			
	Diff.		Hom.		Diff.		Hom.		Diff.		Hom.	
	6	5	6	5	6	5	6	5	6	5	6	5
Mean	5.5%	5.7%	10.5%	11.6%	0.68	0.78	0.81	0.78	0.88	1.1	0.27	0.42
Median	3.8%	3.8%	11.3%	11.9%	0.68	0.78	0.86	0.82	0.79	0.99	0.21	0.42
Perc 10	2.8%	3.1%	6.7%	9.7%	0.57	0.6	0.6	0.37	0.49	0.4	0.12	0.086
Perc 25	3.3%	3.3%	9.0%	11.1%	0.66	0.67	0.71	0.77	0.71	0.6	0.18	0.22
Perc 75	4.3%	4.4%	12.1%	12.8%	0.75	0.86	0.91	0.91	0.95	1.6	0.32	0.6
Perc 90	12.2%	9.8%	12.6%	13.3%	0.77	0.91	0.91	0.92	1.5	2	0.56	0.8
Std. Dev.	6.1%	6.1%	2.4%	1.8%	0.073	0.14	0.13	0.2	0.37	0.66	0.17	0.25

Note: *Diff.* denotes differentiated goods, which are samples once a month. *Hom.* denotes homogenous goods, which are sampled twice a moth. For each case we use NLLS to fit $\log \lambda_t = -\log(d_c) + \eta \log(\max\{|\pi_t|, \pi^c\}) + \epsilon_t$. The annual expected duration at the threshold d_c is expressed in years, i.e. d_c divided by 12.

The statistics reported in [Table 7](#) and [Table 8](#) are clustered around the estimates obtained using aggregate data. For instance, the mean and median of the fitted value of the elasticity η are slightly higher than the estimates obtained pooling all the data, using the same method, as can be seen by comparing the rows of [Table 6](#) for the simple estimator aggregated using the weighted average. The procedure in [Table 6](#) averages the estimated λ across industries for each month first and then estimates the aggregate elasticity η , while the procedure in this section estimates η for each industry first and then averages the estimated elasticities. Comparing the results of [Table 7](#) with the one of [Table 8](#), the estimated elasticities η are

Figure 12: Frequency of Price Changes and Inflation for homogeneous goods sampled twice a month estimated at a 5-digit disaggregation level.



Note: Each dot is a (π_t, λ_t) pair, where π_t is the inflation rate of each industry and λ is the simple estimator of the industry monthly frequency of price adjustment for each industry. When all outlets in an industry change prices in a month λ is infinity, which we set to $\lambda = 30$. We fit $\log \lambda_t = -\log(d_c) + \eta \log(\max\{|\pi_t|, \pi^c\}) + \epsilon_t$. The regression excludes the 1.4% of top coded observations. The size of each circle is proportional to the square root of the number of outlets used in the estimation of the corresponding λ

slightly higher using the inflation of the industry, but the results are quite similar.

Turning to the relation between the dispersion of the industry λ s and inflation [Figure 11](#), [Figure 12](#) show a pattern in which as inflation increases the dispersion of the estimated in-

Table 8: Distribution of fitted η , π^c and d_c at 5 and 6 digit level. Using each good's inflation

	Annual Inflation				Elasticity				Duration Threshold			
	Threshold π^c				η				d_c (years)			
		Diff.	Hom.		Diff.	Hom.		Diff.	Hom.			
Digits	6	5	6	5	6	5	6	5	6	5	6	5
Mean	4.9%	5.7%	13.1%	10.1%	0.7	0.81	0.82	0.7	0.98	1.4	0.25	0.49
Median	3.6%	3.6%	13.1%	6.7%	0.7	0.77	0.67	0.69	0.79	1.1	0.17	0.45
Perc 10	2.7%	1.8%	5.1%	2.8%	0.59	0.6	0.61	0.43	0.51	0.5	0.12	0.094
Perc 25	3.1%	2.4%	6.8%	4.8%	0.63	0.69	0.62	0.62	0.72	0.63	0.13	0.15
Perc 75	6.5%	7.2%	17.5%	13.0%	0.8	0.91	0.89	0.73	1.1	1.9	0.32	0.76
Perc 90	9.5%	12.8%	25.0%	25.0%	0.81	1	1.5	0.78	1.9	2.4	0.63	0.91
Std. Dev.	2.7%	5.1%	7.8%	8.1%	0.087	0.17	0.39	0.25	0.56	1.1	0.22	0.33

Note: *Diff.* denotes differentiated goods, which are samples once a month. *Hom.* denotes homogenous goods, which are sampled twice a moth. For each case we use NLLS to fit $\log \lambda_t = -\log(d_c) + \eta \log(\max\{|\pi_t|, \pi^c\}) + \epsilon_t$. The annual expected duration at the threshold d_c is expressed in years, i.e. d_c divided by 12.

stantaneous frequency of price changes across industries decreases. To document this pattern we split the range of inflation about the critical value π_c fitted before into equally spaced (in logs) intervals. For each of these intervals we compute two measures of dispersion of the five-digit implied instantaneous duration: the difference between the 25th percentile and the 75th percentile (in months) and the difference between the 10th and 90th percentile. We compute these statistics, as opposed to standard deviation, because there are less sensitive to estimates of infinite duration, which happens when in a month (or a two week periods) all the outlets change prices.

Table 9 displays the estimates. Consistent with the estimates already presented, the median duration for the differentiated goods are higher than the median duration for homogeneous goods for all inflation bins. For each type of goods we can see that the dispersion falls with the level of inflation. We interpret this decrease in the dispersion of the durations across industries as reflecting that as inflation increases, the industry specific factors (such as differences in volatility, size of adjustment cost, elasticity of demand, etc) that can explain cross sectional differences in duration are swamped by the common factor given by inflation. We think that this evidence fits with the general theme of our approach.

5.3 Expected Inflation and the Frequency of Price Changes

The robustness check in this section pertains to the measure of inflation as opposed to our estimates the frequencies of price adjustment. Theoretical models of price setting behavior, such as the menu cost model, presented in Section 2, predict that firms will set prices and

Table 9: Cross Industry Dispersion of Implied Duration

Differentiated Goods, sampled monthly				
Inflation % From	% p/year To	Median Duration	15-75 pct Duration	90-10 pct Duration
0	3.6	5.8	11	24
3.6	13.3	7.9	8.7	20
13.3	49	3.9	3.5	7
49	181	1.1	0.74	1.5
181	667	0.46	0.33	0.6
667	2460	0.23	0.12	0.25

Homogeneous Goods, sampled every two weeks				
Inflation % From	% p/year To	Median Duration	15-75 pct Duration	90-10 pct Duration
0	3.5	7.7	5.9	11
3.5	12	4.8	3.7	8
12	41	2.6	2.6	4.5
41	140	0.98	0.53	1.2
140	479	0.46	0.36	0.88
479	1640	0.26	0.079	0.17

Note: Durations are calculated as $1/\lambda$, where λ is the “simple” estimator, and are expressed in months. Ranges of inflations are equally spaced in logs, above the fitted “threshold” inflation π_c . Estimated λ at the 5 digits classification.

inaction bands as a function of *expected* inflation. Our previous estimates of triplets (η, π^c, d_c) in [equation \(26\)](#), however, were estimated using *actual* inflation data. We now study the sensitivity of our results to forward looking measures of expected inflation. This might be important since our sample includes a pronounced disinflation that might have been anticipated by economic decision makers.

We will assume in this section that expected inflation is an average of the actual inflation rate of the following k_t months, where $k_t = n \frac{1}{\lambda_t}$; that is

$$\pi^e = \frac{1}{k} \sum_{s=t}^{t+k} \pi_s$$

We refer to n as the *forward looking factor*. Thus, as inflation falls (and implied durations rise) in our sample agents put an increasing weight on future inflation.

Table 10: Estimates of elasticity η , threshold Inflation π^c and implied duration d_c for different specifications of expected inflation.

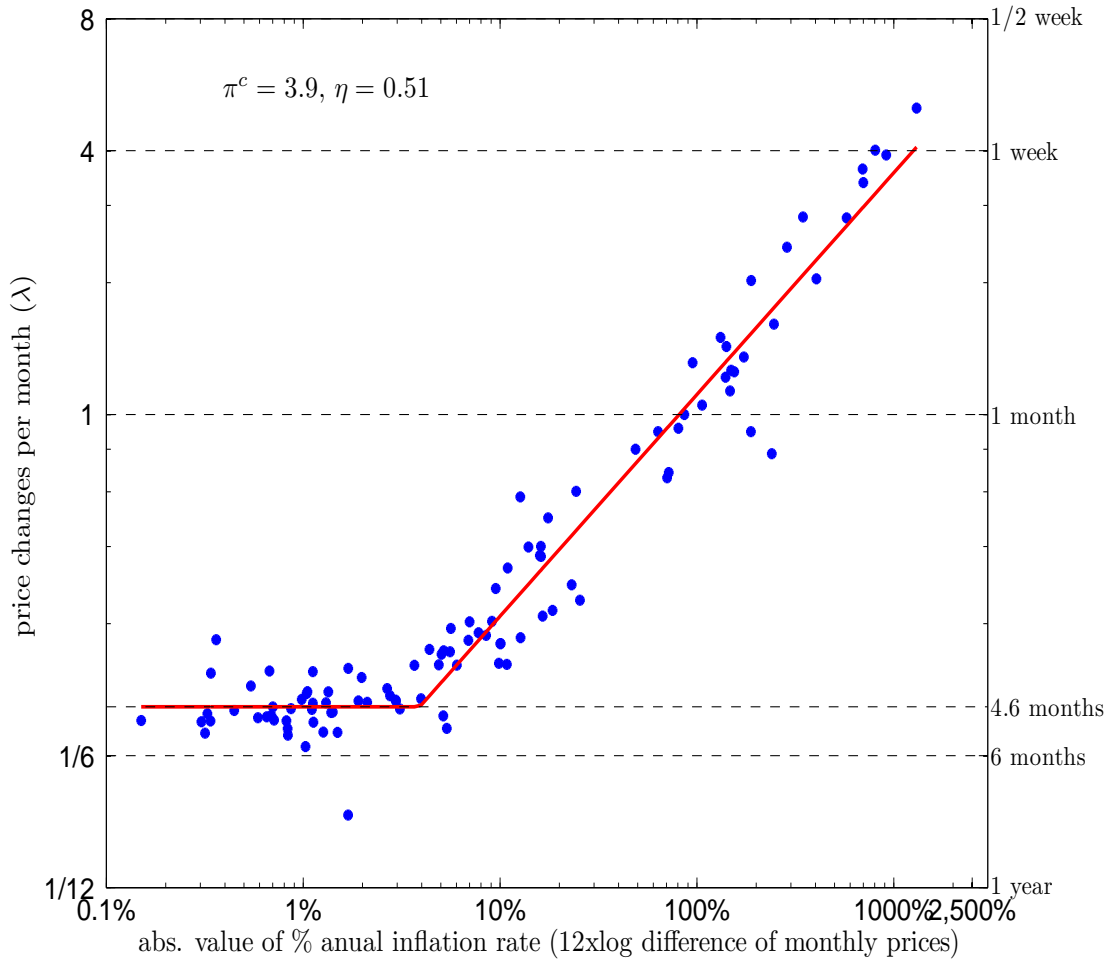
Forward looking factor	Annual Inflation Threshold π^c	Elasticity η	Implied Duration at threshold (months)	R^2
$n = 0$	6.3%	0.56	4.4	0.937
$n = 0.5$	4.4%	0.51	4.6	0.941
$n = 1$	3.9%	0.51	4.6	0.954
$n = 1.5$	3.8%	0.49	4.6	0.941
$n = 2$	3.5%	0.47	4.6	0.937

Note: Estimates of π^c , η and $\frac{1}{\lambda}$ at π^c with expected inflation $\pi^e = \frac{1}{k} \sum_{s=t}^{t+k} \pi_s$, where $k_t = n \frac{1}{\lambda_t}$ as the independent variable. The estimates of λ are the weighted (by expenditure shares) average of the simple maximum likelihood estimator for the samples of homogeneous and non-homogeneous goods. Inflation is the official inflation from INDEC. For each case we use NLLS to fit $\log \lambda_t = -\log(d_c) + \eta \log(\max\{|\pi_t|, \pi^c\}) + \epsilon_t$. The annual expected duration at the threshold d_c is expressed in months.

[Table 10](#) shows that the results presented in [Table 6](#) are not very sensitive to estimating [equation \(26\)](#) using a forward looking measure of inflation such as π^e defined above. The first row of the table shows the results when we use the actual rate of inflation. They differ slightly from those of the pooled simple estimate in [Table 6](#) because here we use the inflation rate estimated by INDEC for the whole CPI instead of the inflation rate in our sample. We use official CPI data because we need forward looking values of inflation at the end of the sample that otherwise we cannot obtain. The following four rows shows that as expectations are more forward looking the threshold inflation and the elasticity fall slightly and the implied

duration at the threshold remains constant. The R^2 of the regression is maximized for $n = 1$, so that the best fit is when agents set expected inflation equal to the average inflation over the implied duration of the fixed prices. All the estimates of the elasticity η are consistent with the theoretical prediction in Section 2. Figure 13 illustrates the results of table for the case of $n = 1$.

Figure 13: Expected Inflation and the Frequency of Price Changes.
Pooled Simple MLE of λ



Note: Expected inflation in the horizontal axis is $\pi^e = \frac{1}{k} \sum_{s=t}^{t+1/\lambda} \pi_s$. Inflation is the official inflation from INDEC. The estimates of λ are the weighted (by expenditure shares) average of the simple maximum likelihood estimator for the samples of homogeneous and non-homogeneous goods. We fit $\log \lambda_t = -\log(d_c) + \eta \log(\max\{|\pi_t|, \pi^c\}) + \epsilon_t$.

5.4 Frequency of Price changes at low inflation

In this subsection we fit alternative statistical models to the $\{\lambda_t, \pi_t\}$ data to check on the robustness of the simple representation in [equation \(26\)](#). We confirm that the elasticity of λ with respect to inflation, η , for very high inflation is about 2/3, and that at low inflation this elasticity is close to zero. We find some evidence consistent with a small *semi-elasticity* of the λ with respect to inflation at zero. The range of inflations for which the frequency of price changes are very weakly associated with inflation is imprecisely estimated, ranging from 6% to 17% per year.

Table 11: Alternative Statistical Models of $\{\lambda_t, \pi_t\}$

Using aggregate inflation of differentiated and homogeneous					
Case	ϵ^A	ν	η	R^2	$\Delta\lambda$ % when $\pi = 0$ to $\pi = 1\%^B$
$\log \lambda_t =$	$a + \epsilon \min \{\pi_t - \pi^c, 0\} + \nu(\max \{\pi_t - \pi^c, 0\})^2$ $+ \eta \max \{\log \pi_t - \log \pi^c, 0\} + \omega_t$				
$ \pi $	-5.8	-	0.59	0.96	-0.46%
includes $\pi < 0$	14.9	-	0.59	0.96	1.2%
$ \pi $	103.6	5639	0.58	0.96	8.6%
includes $\pi < 0$	71.5	1626.5	0.57	0.96	6%
Using <i>Expected</i> inflation, based on CPI ^C					
Case	ϵ^A	ν	η	R^2	$\Delta\lambda$ % when $\pi = 0$ to $\pi = 1\%^B$
$ \pi $	37	-	0.6	0.94	3.1%
includes $\pi < 0$	33	-	0.61	0.94	2.7%
$ \pi $	19	416	0.62	0.94	1.6%
includes $\pi < 0$	35.8	138	0.61	0.94	3%

A: Inflation is expressed in monthly percentage log points, i.e. $\pi(t+1) = (\log P(t+1) - \log P(t))$ where $P(t+1)$ and $P(t)$ are the price levels in consecutive months. B: Percentage change in λ when inflation changes from 0 to 1% per year. C: *Expected* inflation is an average of the CPI inflation between the current period and future periods, defined as in [Section 5.3](#). We use factor $n = 1$.

[Table 11](#) displays several of the fitted models. We use the *pooled simple* estimator for λ .

We consider two different measures of inflation. The aggregate contemporaneous inflation in the month from the goods in our sample, and the *expected* inflation, as described in [Section 5.3](#). For each case we consider statistical models that have a break at a value of inflation π^c . Below this value we fit a semi-log specification, and above this value we fit a log-log specification. Specifically we fit:

$$\log \lambda_t = a + \epsilon \min \{ \pi_t - \pi^c, 0 \} + \nu (\min \{ \pi_t - \pi^c, 0 \})^2 + \eta \max \{ \log \pi_t - \log \pi^c, 0 \} + \omega_t . \quad (27)$$

We fit this equation using NLLS. We consider a case where we replace inflation by the absolute value of inflation, as discussed above, and a version where we leave inflation with its natural sign. The motivation for using the absolute value of inflation is that in the symmetric case analyzed in [Section 2](#) the steady state effects of inflation on the frequency of price changes are symmetric around zero inflation. We consider that the absolute value of inflation will make more sense using *expected* inflation. In [Figure 14](#) we display the scatter points and the fitted line for the last case of [Table 11](#).

From [Table 11](#) we conclude the following. Allowing for a different effects of negative and positive inflation on the frequency of price changes around $\pi = 0$ has zero or a marginal effect on the fit of the points.¹¹ The fitted values make clear that the elasticity is zero at $\pi = 0$ and the estimated semi-elasticities are small. Roughly speaking, a change of annual inflation from 0 to 1% is associated with a percentage change in the frequency of price changes λ of 3% (and hence of -3% on duration).

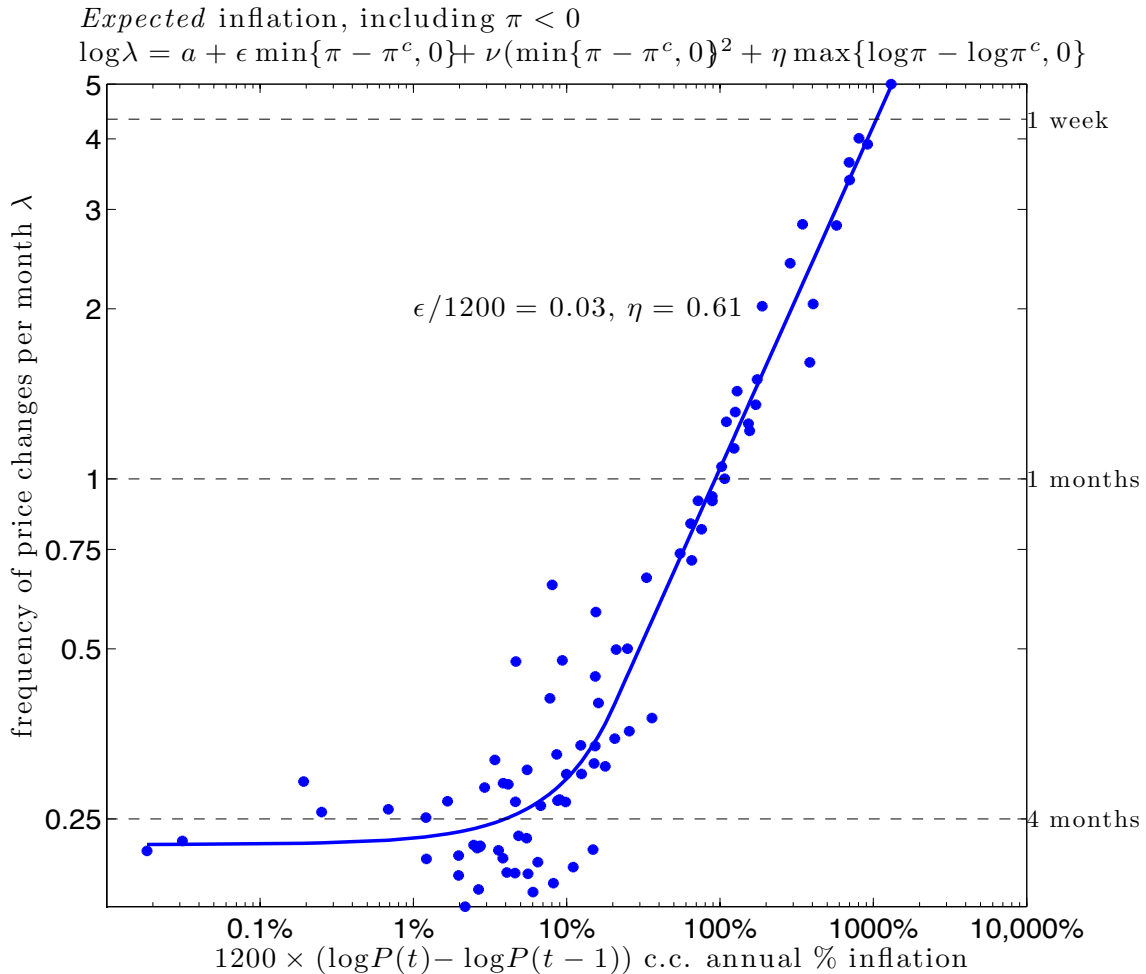
6 Inflation and the Intensive and Extensive Margins of Price Adjustments

In this section we decompose inflation into intensive and extensive margins for price increases and decreases and describe how this decomposition varies with inflation. The behavior of these variables is by and large consistent with the comparative statics of the menu cost model and empirically support to the assumption about symmetry in [Section 2](#).

The results are illustrated in [Figure 15](#) and in [Figure 16](#). [Figure 15](#) describes how the extensive margin of price adjustments, the frequencies of price increases and decreases for homogeneous and differentiated goods, varies with inflation. The theoretical counterpart of this figure from our numerical example is [Figure 4](#) where we observe that for low inflation rates

¹¹Since we don't have a proper probabilistic model for this graphs we don't report systematically standard errors on ϵ, η , etc. Nevertheless, as a way to assess the fit of the regression, we note that the t-stat of ϵ are order of magnitude smaller than those of η and frequently ϵ is not significantly different from zero.

Figure 14: Expected Inflation and the Frequency of Price Changes.
Pooled Simple MLE of λ



Fitted values including $\pi < 0$ values. Graph displays only the points for $\pi_t > 0$. Allows for non-zero quadratic term ν .

about half of the price changes are price increases and that the frequencies of price adjustment are not very sensitive to inflation. As inflation rises the frequency of price increases converges to the total frequency of price changes, implying that the frequency of price decreases must drop to zero. The data depicted in [Figure 15](#) is consistent with these theoretical findings.

The empirical behavior of the intensive margin is described in [Figure 16](#) that shows the absolute value of the magnitude of price increases and price decreases as a function of the absolute value of inflation (in semilog scale). [Figure 5](#) is the theoretical counterpart of the empirical figure in this section that we computed in our numerical example. Menu cost models imply that as the inflation rate increases, the average price changes become larger

in absolute value. If for some reason (i.e. an unusually large idiosyncratic shock) the price of a product is too high relative to the one that maximizes static profits, the firm should not incur the menu cost to reduce it since it will self correct rapidly due to inflation. As it self corrects, in expected value, it will be closer to optimal static profit maximizing price. Or more precisely, it should only incur the menu cost to decrease the price if it is extremely high. Hence, the average price decrease should be higher for higher inflation rates. [Figure 16](#) in this section and [Figure 5](#) in [Section 2](#) are qualitatively very similar, except for the behavior of price decreases at high inflation. For low inflation rates they are flat and the magnitude of price increases and decreases is the same and approximately 10%. As inflation rises, in the numerical example the magnitude of price increases and decreases rises and price increases become larger than price decreases. In the data, the magnitude of price changes stays flatter for a wider range of inflation rates than in the example and then it rises faster for the price increases. The data for price decreases departs from the theory at high inflation rates as it becomes a decreasing function of inflation above 400% per year.

7 Inflation and the Dispersion of Relative Prices

In [Section 2](#) we analyze the effect of inflation on the variance of relative prices. Summarizing our discussion there, inflation should have a very small effect on the dispersion of relative prices at low level of inflation, and for higher levels the dispersion should become an increasing function of inflation.

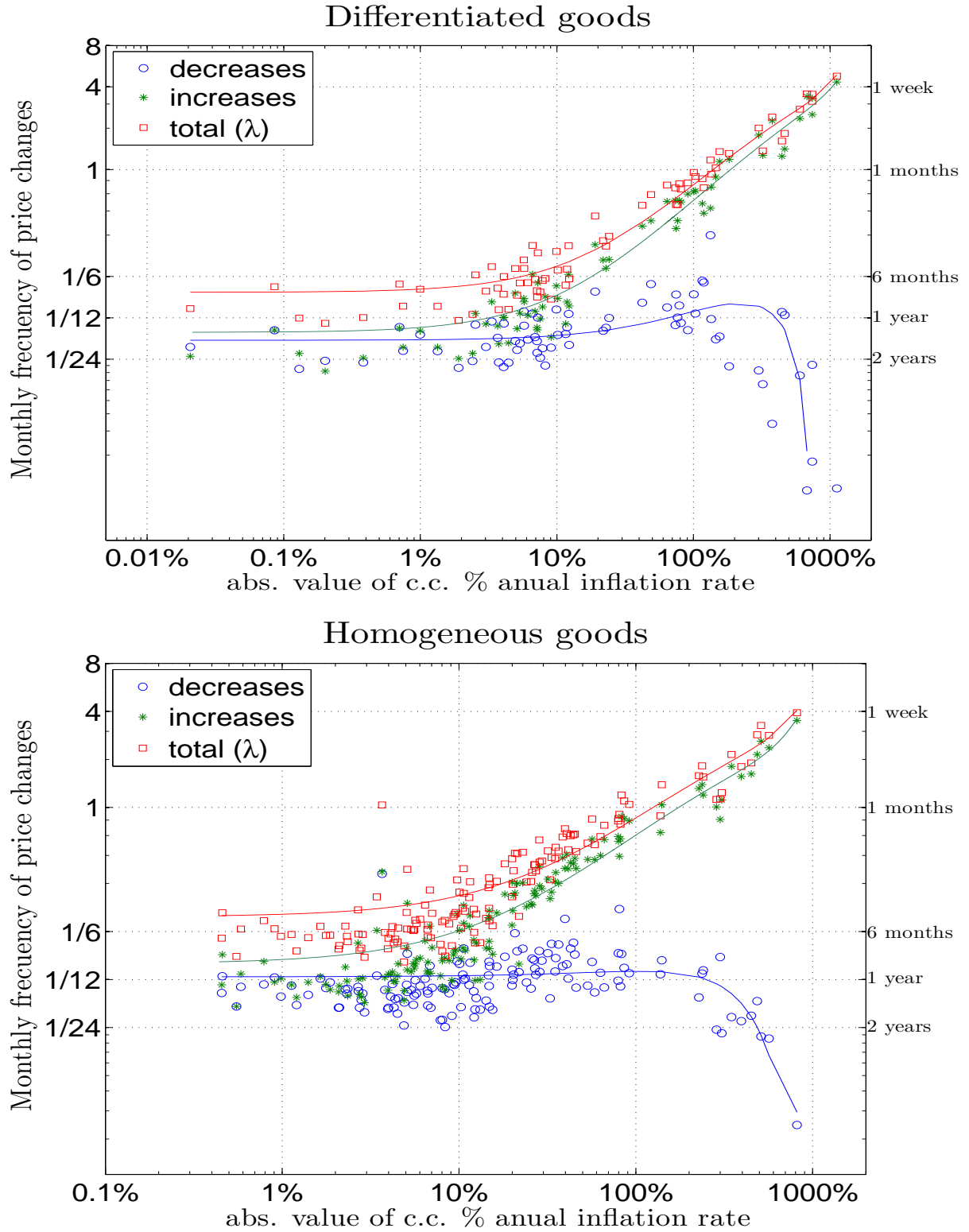
We remind the reader that the “extra” price dispersion created by nominal variation in prices is one of the main avenues for inefficiency in models with sticky prices, see for example chapter 6 of [Woodford \(2003\)](#) and [Burstein and Hellwig \(2008\)](#).

In this section we explore the association between simple measures of the average price dispersion across goods and inflation. We measure the price dispersion across outlets selling the same good or service at a given month. We then report a weighted average of this dispersion of prices, where the weights are given by each product’s expenditure share in the consumer survey. We use two measures of dispersion, the standard deviation of the log of prices, and the difference between the 90th and 10th percentile of the log of prices. To be concrete the average dispersion of relative prices at time t is given by

$$\bar{\sigma}_t = \sum_{j=1}^N \omega_j \left[\sum_{i \in O_j} \left[(\log p_{i,j,t})^2 - \left(\frac{1}{\#O_j} \sum_{i \in O_j} \log p_{i,j,t} \right)^2 \right] \right]^{\frac{1}{2}},$$

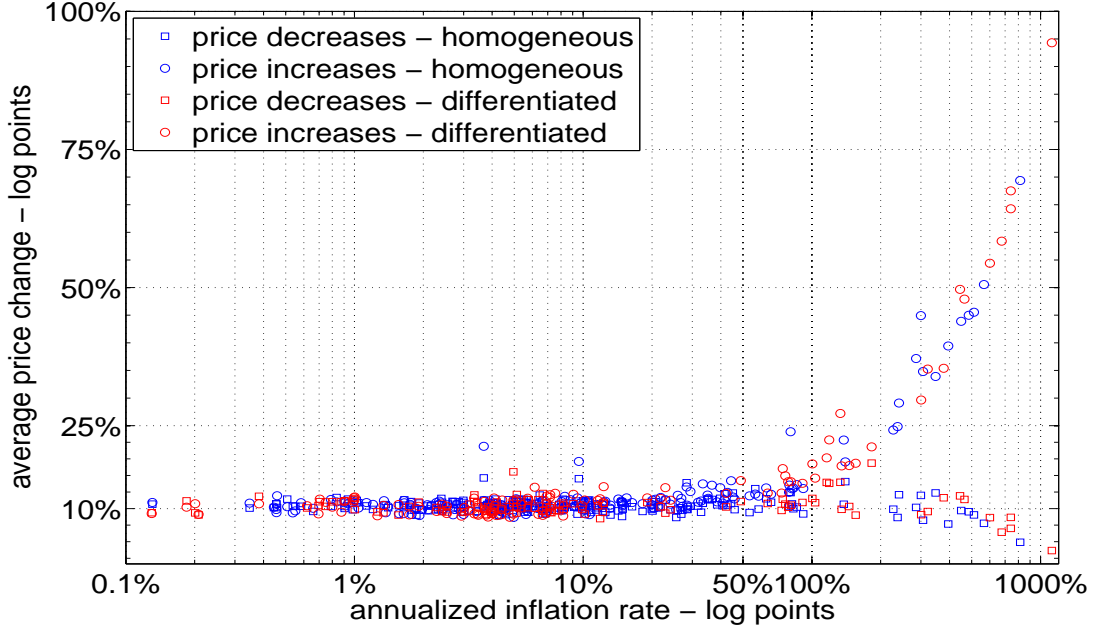
where O_j are the set of outlets that sell the good j , ω_j the expenditure share of the good

Figure 15: Inflation and the Frequency of Price Increases and Decreases



Note: The frequency of price increases and decreases is calculated as $-\log(1 - f)$, where f is the fraction of outlets increasing or decreasing price in a given date.

Figure 16: Inflation and the Frequency of Price Increases and Decreases



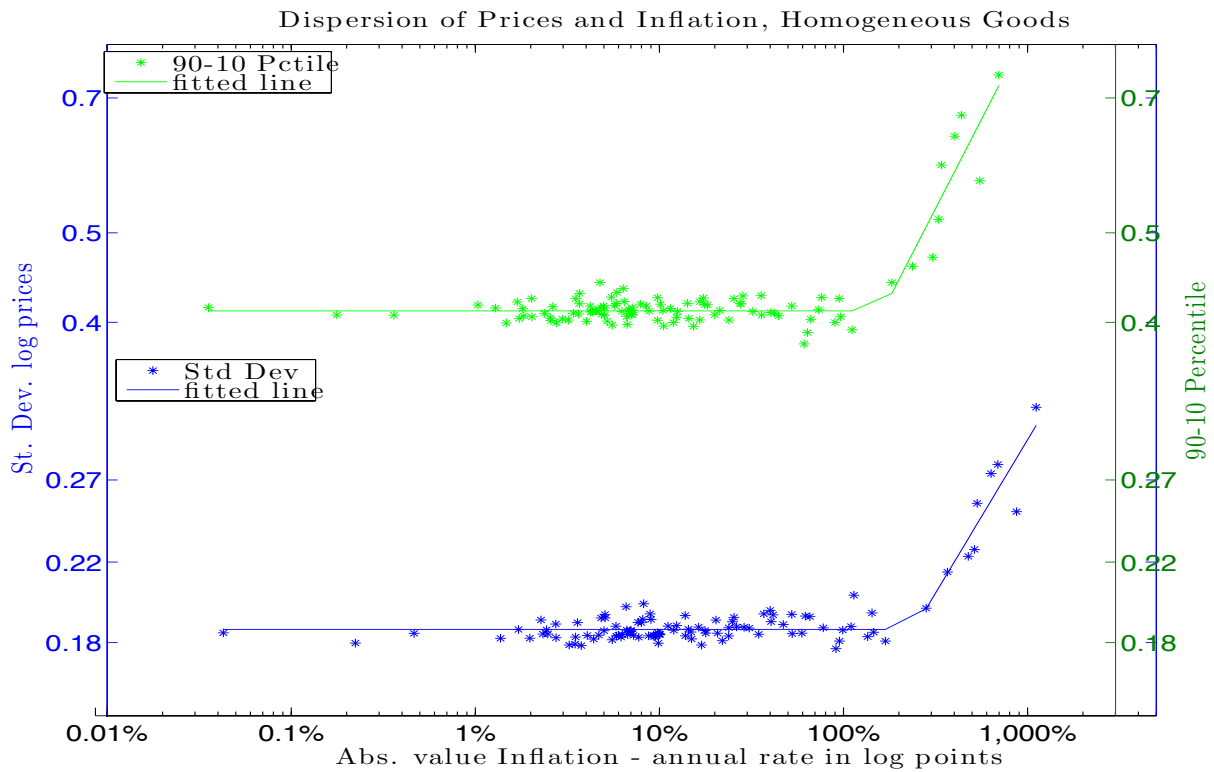
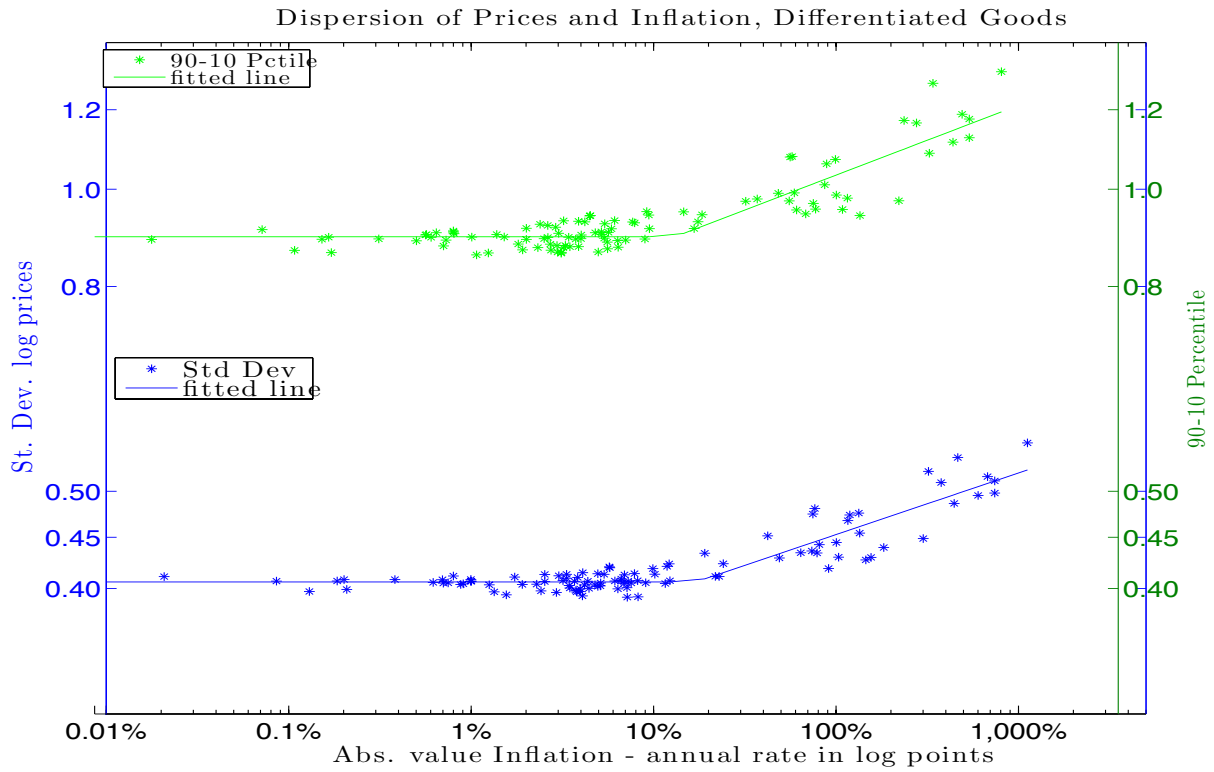
Note: The average price change is the log difference in prices, conditional on a price change taking place, averaged with expenditure weights over all homogeneous and differentiated goods in a given date.

j and $p_{i,j,t}$ is the price of the good j sold at outlet i at time t .¹² The measure of average dispersion based on the difference between the 90th and 10th percentile of log prices, denoted by \bar{q}_t , is defined in analogously. We compute the time series for $\bar{\sigma}_t, \bar{q}_t$ among differentiated goods, and among homogenous goods. In Figure 17 we plot, in a log scale, these measures of average dispersion of relative prices against the corresponding inflation for homogenous goods and differentiated goods.

From Figure 17 it is clear that, the average dispersion of relative prices for a good increases with the level of inflation, at least for high enough inflation. Notice that this pattern holds regardless of the measure of dispersion that we use. Consistent with the explanation of the difference between homogenous and differentiated goods (see Appendix F), we find that the dispersion of relative prices is about twice as large for differentiated goods than for homogeneous goods. We describe these patterns by fitting the following line using non-linear

¹²In the computation of $\bar{\sigma}$ the set of outlets that sell a given good vary across time. Also, we exclude the goods whose prices are missing

Figure 17: Average Dispersion of Relative Prices and Inflation



least squares

$$\log \bar{\sigma}_t = a + \eta \log \max \{ \pi^c, |\pi_t| \} + \varepsilon_t \quad (28)$$

and likewise for \bar{q}_t . The results are reported in the top panel of [Table 12](#). Notice that this specification imposes that for low value of inflation, the relative price dispersion and inflation are not correlated. The bottom panel of [Table 12](#) includes regressions that estimates the correlation for low inflation by fitting:

$$\log \bar{\sigma}_t = a + \epsilon \min \{ \pi_t - \pi^c, 0 \} + \eta \max \{ \log \pi_t - \log \pi^c, 0 \} + \varepsilon_t . \quad (29)$$

As can be seen from [Figure 17](#) and from the values of $\bar{\pi}$ reported in [Table 12](#) the values of inflation for which the positive correlation between $p_{i,t}$ and $\bar{\sigma}_t$ are much larger than the ones corresponding to the frequency of price changes λ_t and inflation π_t (see [Figure 10](#) and [Table 6](#)). The values of π^c are particularly large for homogeneous goods.

Table 12: Inflation and Relative Price Dispersion

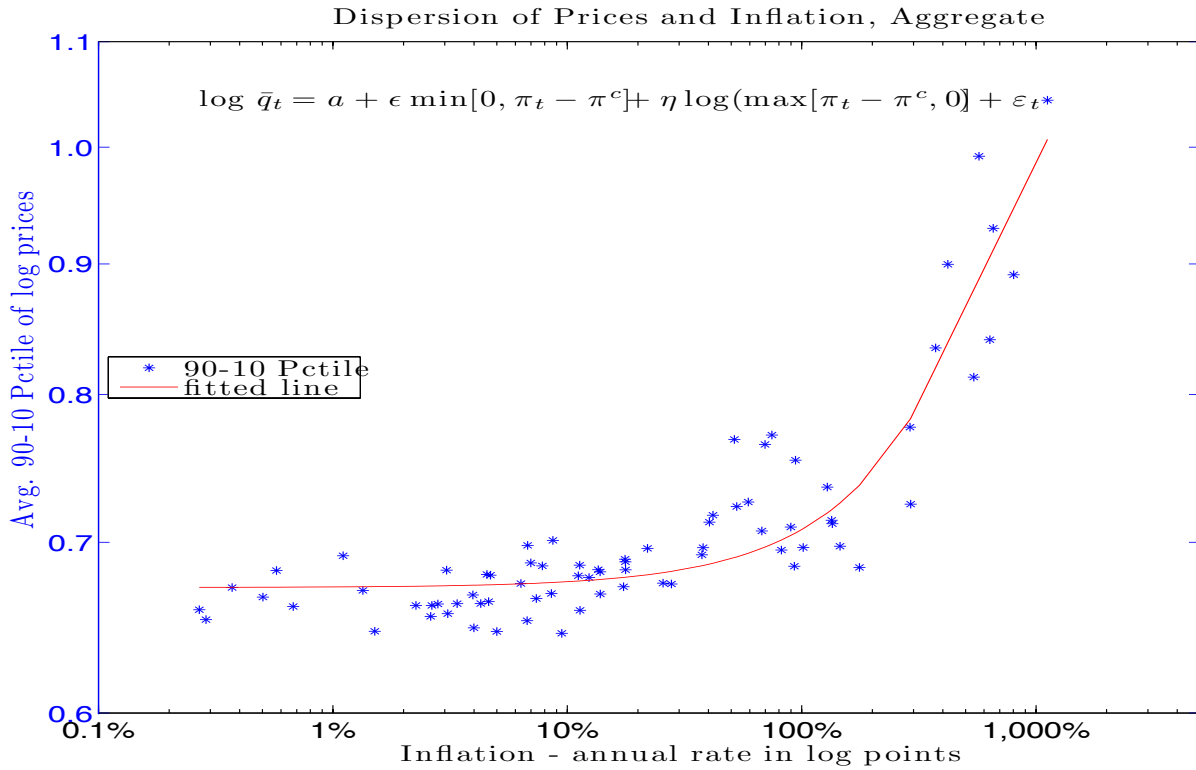
	Homogeneous		Differentiated		Aggregated	
	Std Dev	90-10 Pctile	Std Dev	90-10 Pctile	Std Dev	90-10 Pctile
$\log \bar{\sigma}_t = a + \eta \log \max \{ \pi^c, \pi_t \} + \varepsilon_t$ and likewise for \bar{q}_t .						
	$\bar{\sigma}_t$	\bar{q}_t	$\bar{\sigma}_t$	\bar{q}_t	$\bar{\sigma}_t$	\bar{q}_t
η	0.33	0.38	0.06	0.07	0.08	0.09
π^c	241%	251%	17%	17%	29%	29%
R^2	0.90	0.81	0.85	0.81	0.83	0.79
$\log \bar{\sigma}_t = a + \epsilon \min \{ \pi_t - \pi^c, 0 \} + \eta \max \{ \log \pi_t - \log \pi^c, 0 \} + \varepsilon_t$ and likewise for \bar{q}_t .						
	$\bar{\sigma}_t$	\bar{q}_t	$\bar{\sigma}_t$	\bar{q}_t	$\bar{\sigma}_t$	\bar{q}_t
ϵ	0.01	-0.01	0.11	0.13	0.05	0.05
η	0.33	0.38	0.07	0.08	0.14	0.19
π^c	256%	234%	60%	62%	250%	292%
R^2	0.90	0.89	0.85	0.83	0.86	0.79

Note: π^c is annual inflation, in log points.

The values of the elasticities η reported in [Table 12](#) are close to 1/3 for homogeneous goods, which is the theoretical limit for very large inflation, i.e. the value in [Sheshinski and](#)

Weiss's (1977) model. Instead the value of η for differentiated goods is positive but much smaller. The simple statistical model in equation (28) we fit imposes that at low inflation there is no correlation between inflation and dispersion of relative prices, and Figure 17 and Figure 18 as well as the values of ϵ in Table 12 suggest that this is the case. In Figure 18 we show a scatter plot of (\bar{q}_t, π_t) for the aggregate data where we fit equation (29).

Figure 18: Average Dispersion of Relative Prices and Inflation



We summarize the results of this section by saying that we find evidence consistent with the predictions of menu cost models emphasized in Section 2. The dispersion of prices is unresponsive to inflation for low inflation rates and it eventually increases. The range for which the dispersion of relative prices is flat with respect to inflation is flat as suggested in the numerical examples of Section 2. In the case of homogeneous goods elasticity of the standard deviation of prices with respect to inflation seems to converge to the one predicted by the Sheshinski and Weiss (1977) model, which is 1/3. We also conclude that the dispersion of relative prices caused by inflation emphasized in chapter 6 of Woodford (2003) and in Burstein and Hellwig (2008) as a welfare cost of inflation is likely to be relevant only for high rates of inflation.

8 Relation to the Literature

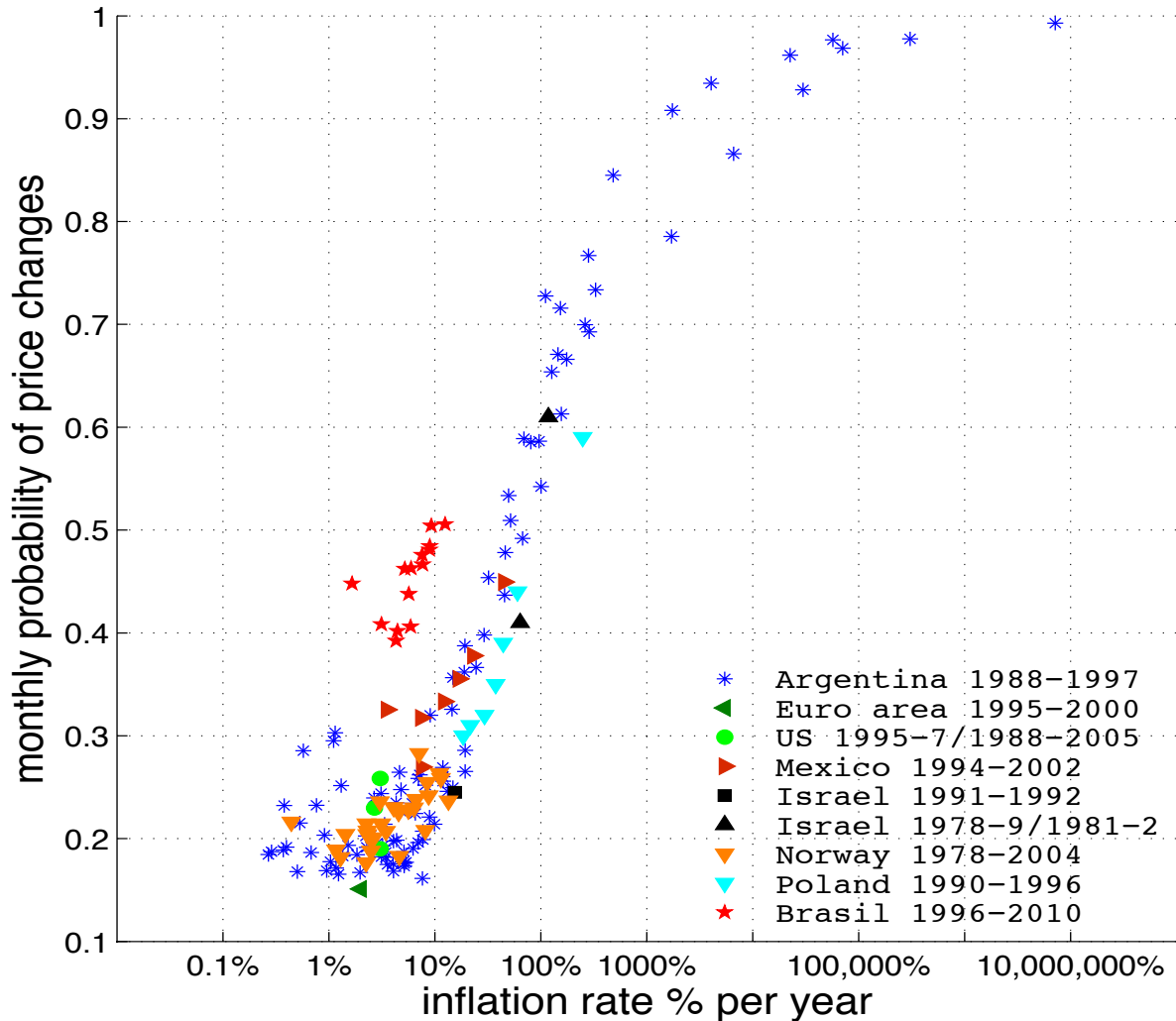
In this section we compare the results obtained in [Section 4](#) to other estimations of the frequency of price changes in the literature. [Figure 19](#) provides a visual summary of how our results compare to the existing literature and [Table 13](#) provides a succinct comparison of the data sets.

The figure illustrates the contribution of this paper to the literature and puts our results in perspective. It plots the monthly frequency of price changes against inflation for a variety of studies. First, observe how the wide range of inflation rates covered by our sample makes this paper unique: none of the other papers covers inflation rates ranging from 0 to 7.2 million per year (annualized rate of inflation in July 1989). Second, our results yield estimates of the monthly frequency of price changes that are similar to those obtained in other countries for similar inflation rates. This is remarkable since the other studies involve different economies, different goods and different time periods. It is a strong indicator that our results are of general interest, as the theory suggests, and are not a special feature of Argentina.

The studies included in the figure are all the ones we could find covering a wide inflation range. For the low inflation range we included studies for the United States by [Bils and Klenow \(2004\)](#), [Klenow and Kryvtsov \(2008\)](#) and [Nakamura and Steinsson \(2008\)](#) and for the Euro Area by [Álvarez et al. \(2006\)](#). Our estimates of the frequency of price changes are consistent with all of them. Prices in the Euro area adjust somewhat less frequently but not by much. There are other studies for low inflation countries, especially for the Euro area, but as they mostly yield estimates similar to those of [Álvarez et al. \(2006\)](#) we do not report them (see [Álvarez et al. \(2006\)](#) and [Klenow and Malin \(2011\)](#) for references to these studies). We have three data points for Israel corresponding to an inflation rate of 16% per year in 1991-2 ([Baharad and Eden \(2004\)](#)) and to inflation rates of 64% per year in 1978-1979 and 120% per year in 1981-1982 ([Lach and Tsiddon \(1992\)](#)). The frequency of price adjustment for these three points is perfectly aligned with the Argentine data. The same is true for the Polish sample that ranges from 18% to 249% per year ([Konieczny and Skrzypacz \(2005\)](#)) and for the Mexican sample that ranges from 3.5% to 45% per year ([Gagnon \(2009\)](#)). The Brazilian ([Barros et al. \(2009\)](#)) data yields higher frequencies of price changes than the other studies, but the slope of the Brazilian cloud of points is consistent with ours.

As most of these studies estimate the frequency of price changes as the fraction of outlets that change prices in any given month and then average this statistic, for comparison purposes [Figure 19](#) reports the implied probability of price adjustment $1 - e^{-\lambda}$, where λ is the simple pooled estimator. [Gagnon \(2009\)](#) and [Barros et al. \(2009\)](#) report regressions of the frequency of price changes on inflation that are misspecified as they imply that the frequency of price

Figure 19: Monthly Frequency of Price Changes and Inflation: Comparison to other Studies

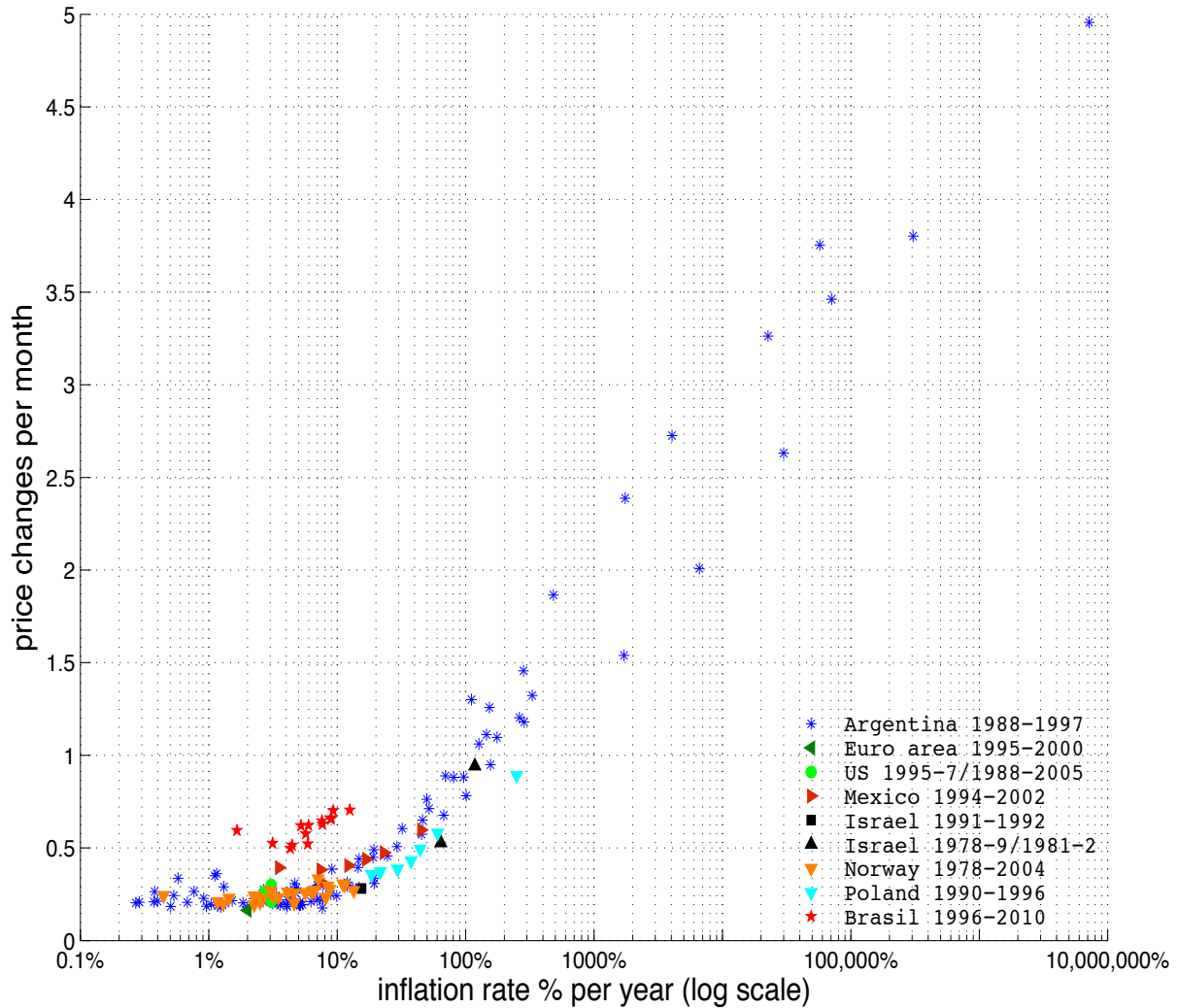


Note: Frequencies for Argentina are $1 - e^{-\lambda}$, where λ is the simple pooled estimator. Data for the Euro area is from [Álvarez et al. \(2006\)](#), for the US from [Bils and Klenow \(2004\)](#), [Klenow and Kryvtsov \(2008\)](#) and [Nakamura and Steinsson \(2008\)](#), for Mexico from [Gagnon \(2009\)](#), for Israel from [Baharad and Eden \(2004\)](#), for Norway [Wulfsberg \(2010\)](#), [Lach and Tsiddon \(1992\)](#), for Poland from [Konieczny and Skrzypacz \(2005\)](#), and for Brazil [Barros et al. \(2009\)](#). Logarithmic scale for the horizontal axis.

changes goes to infinity as inflation grows. Our specification (illustrated in [Figure 20](#)), that regresses the log of the number of price changes per month, λ , on the log of inflation is consistent with the theory presented in [Section 2](#) that predicts a constant elasticity of λ with respect to inflation.

[Table 13](#) shows that in addition to covering a wide range of inflation rates our data set

Figure 20: Price Changes per Month and Inflation: Comparison to other Studies



Note: price changes per month for Argentina are the simple pooled estimator of λ . For the other cases we plot $-\log(1 - f)$, where f is the reported frequency of price changes in each study. Data for the Euro area is from [Álvarez et al. \(2006\)](#), for the US from [Bils and Klenow \(2004\)](#), [Klenow and Kryvtsov \(2008\)](#) and [Nakamura and Steinsson \(2008\)](#), for Mexico from [Gagnon \(2009\)](#), for Israel from [Baharad and Eden \(2004\)](#), and [Lach and Tsiddon \(1992\)](#), for Poland from [Koniczny and Skrzypacz \(2005\)](#), For Brazil [Barros et al. \(2009\)](#). Logarithmic scale for the horizontal axis.

is special due to its broad coverage that includes more than 500 goods representing 85% of Argentina's consumption expenditures.

Table 13: Comparison with other Studies in Countries with High Inflation

Country	Authors	Sample product coverage	Observ. per month	Sample	Inflation (%. a.r.)	Monthly freq. (%)
Argentina	This paper	506 goods/services, representing 84% of Argentinian consumption expenditures	81,305 on average	1988-1997	$0 - 7.2 \times 10^6$	16-99
Brazil	Barros et al. (2009)	70% of Brazilian consumption expenditures	98,194 on average	1997-2010	2-13	39-50
Israel	Llach and Tsiddon (1992)	26 food products (mostly meat and alcoholic beverages)	250	1978-1979	64	41
Israel	Llach and Tsiddon (1992) Eden (2001),	26 food products (mostly meat and alcoholic beverages)	530	1981-1982	118	61
Israel	Baharada and Eden (2004)	up to 390 narrowly-defined products from the Israeli CPI	2800	1991-1992	16	24
Mexico	Gagnon (2007)	227 product categories, representing 54.1 percent of Mexican consumption expenditures	31,500	1994-2002	3.5-45	27-45
Poland	Konieczny and Skrzypacz (2005)	52 goods, including 37 grocery items, and 3 services	up to 2400	1990-1996	18-249	59-30

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A Computation of estimates and standard errors of λ

Here we describe the details on the maximum likelihood estimator and the computation of the standard errors.

We compute the maximum likelihood estimator by using an iterative procedure. In this discussion we fix a good or item. We denote the iterations by superindex j . The initial guess for λ_t^0 is the log of the ratio of the outlets that change the price between $t - 1$ and t . Then we solve for λ_t^{j+1} in [equation \(25\)](#) for the foc of λ_t taking as given the values of λ_{t-i}^j and $\lambda^{j,t+i}$ for all $i \neq 0$. Notice that this equation has a unique solution. Also notice that the solution can be ∞ , for instance if all prices change, or zero.

We have not derived expression for standard error for the different estimator of λ . Nevertheless to give an idea of the precision of our estimator, we note that for the simple pooled estimator, assuming that missing values are independent, we can use the expression for the standard error of a binomial distributed variable for the probability of a price change π , while using the estimate $\hat{\pi}_t = \text{number of outlets with a price change} / N_t$ where N_t is the number of outlet with a price quote between t and t_1 . In this case $se(\hat{\pi}) = \sqrt{\hat{\pi}(1 - \hat{\pi})/N}$. Using the delta method and $\lambda(p) = -\log(1 - p)$ then we have:

$$se(\hat{\lambda}) = \frac{\sqrt{\exp(\hat{\lambda}) - 1}}{\sqrt{N}} \quad \text{and} \quad se(\log \hat{\lambda}) = \frac{1}{\hat{\lambda}} \frac{\sqrt{\exp(\hat{\lambda}) - 1}}{\sqrt{N}}. \quad (30)$$

To give an idea of the magnitudes for our estimated parameters we use some round numbers for both homogeneous and differentiated goods. For the case where we pool *all* the differentiated goods we can take $N_d = 230 \times 80 \times 0.80 = 14720 \approx \# \text{ diff. goods} \times \text{avg. \# outlets diff. goods} \times \text{fraction of non-missing quotes}$. Pooling *all* the homogenous goods we $N_h = 60000 = 300 \times 250 \times 0.80 \approx \# \text{ homog. goods} \times \text{avg. \# outlets homog. goods} \times \text{fraction of non-missing quotes}$. So for high value of λ such as for $\log \hat{\lambda} = \log 5$, we have that the standard errors are $se(\log \hat{\lambda}_d) = 0.02$ and $se(\log \hat{\lambda}_h) = 0.010$. Instead for low values of λ such as for $\log \hat{\lambda} = \log 1/5$, we have that the standard errors are $se(\log \hat{\lambda}_d) = 0.019$ and $se(\log \hat{\lambda}_h) = 0.009$. Instead if we estimate λ at the level of each good, the standard errors should be larger by a factor $\sqrt{230} \approx 15$ and $\sqrt{330} \approx 17$ for differentiated and homogenous goods respectively.

B Frequency of Price Changes in Danziger (1999)

We want to show that [equation \(7\)](#) holds, or in the notation of [Danziger \(1999\)](#) that

$$\lim_{c \downarrow 0} \frac{\partial \Omega(m)}{\partial m} = 0 . \quad (31)$$

The rest of this section used the notation and refers to the equation and results in [Danziger \(1999\)](#). As stated in footnote 16 $\frac{\partial}{\partial \rho} \Omega = -\frac{\partial}{\partial m} \Omega$. Using the expressions in the proof of Theorem 3 we have:

$$\frac{\partial \Omega}{\partial \rho} = \frac{\Omega^2}{\phi} \frac{c}{A} \frac{\delta(BI - 4A)}{(1-c)(1-\delta B)^2} \frac{1}{1 + \delta B / (1 - c\delta B)} \quad (32)$$

Taking c to zero in the LHS of [equation \(32\)](#) and using that $\lim A = \lim B = 0$ and $\lim I = 2$ as $c \downarrow 0$ we have

$$\begin{aligned} \lim \frac{\partial \Omega}{\partial \rho} &= \frac{\Omega^2(0)}{\phi} \lim \frac{c}{A} \frac{\delta(BI - 4A)}{(1-c)(1-\delta B)^2} \lim \frac{1}{1 + \delta B / (1 - c\delta B)} \\ &= \delta \frac{\Omega^2(0)}{\phi} \left(\lim \frac{cB}{A} 2 - 4 \right) . \end{aligned}$$

Using the definition of B and and expansion of the log we have that

$$\frac{B}{A(c)} = 2 + 2c \left(\frac{I}{2} - 1 \right) + o(A)$$

Thus $\lim \frac{\partial \Omega}{\partial \rho} = 0$.

C Analytical characterization of the model in [Section 2.2](#)

PROPOSITION 3. Assume that $\sigma > 0, c > 0$ and that [equation \(11\)](#) holds. The inaction set is given by $\mathcal{I} = \{(p, z) : \underline{x} + z < p < \bar{x} + z\}$. The optimal return point is given by $\psi(z) = \hat{x} + z$. The value function in the range of inaction and the constants $X \equiv (\underline{x}, \hat{x}, \bar{x})$ with $\underline{x} < \hat{x} < \bar{x}$ solve

$$V(p, z) = e^{z(1-\eta)} V(p - z, 0) \equiv e^{z(1-\eta)} v(p - z) \quad (33)$$

$$v(x) = a_1 e^{x(1-\eta)} + a_2 e^{-x\eta} + \sum_{i=1}^2 A_i e^{\nu_i x} \quad (34)$$

where the coefficients a_i, ν_i are given by

$$\begin{aligned} 0 &= -b_0 + b_1\nu_i + b_2(\nu_i)^2 \\ a_1 &= \frac{1}{b_0 - (1-\eta)b_1 - (1-\eta)^2 b_2} \text{ and } a_2 = -\frac{1}{b_0 + \eta b_1 - (\eta)^2 b_2} \text{ where} \\ b_0 &= r + \rho - \mu_z(1-\eta) - (1-\eta)^2 \frac{\sigma^2}{2}, \quad b_1 = -\left[\mu_z + \pi + 2(1-\eta) \frac{\sigma^2}{2} \right], \quad b_2 = \frac{\sigma^2}{2}. \end{aligned}$$

and where the five values A_1, A_2, X solve the following five equations:

$$\begin{aligned} \hat{c} \quad -a_1 (e^{\hat{x}(1-\eta)} - e^{\bar{x}(1-\eta)}) - a_2 (e^{-\hat{x}\eta} - e^{-\bar{x}\eta}) &= \sum_{i=1}^2 A_i (e^{\nu_i \hat{x}} - e^{\nu_i \bar{x}}), \\ \hat{c} \quad -a_1 (e^{\hat{x}(1-\eta)} - e^{\underline{x}(1-\eta)}) - a_2 (e^{-\hat{x}\eta} - e^{-\underline{x}\eta}) &= \sum_{i=1}^2 A_i (e^{\nu_i \hat{x}} - e^{\nu_i \underline{x}}), \\ 0 &= a_1 (1-\eta) e^{\hat{x}(1-\eta)} - a_2 \eta e^{-\hat{x}\eta} + \sum_{i=1}^2 A_i \nu_i e^{\nu_i \hat{x}}, \\ 0 &= a_1 (1-\eta) e^{\bar{x}(1-\eta)} - a_2 \eta e^{-\bar{x}\eta} + \sum_{i=1}^2 A_i \nu_i e^{\nu_i \bar{x}}, \\ 0 &= a_1 (1-\eta) e^{\underline{x}(1-\eta)} - a_2 \eta e^{-\underline{x}\eta} + \sum_{i=1}^2 A_i \nu_i e^{\nu_i \underline{x}}. \end{aligned}$$

The first two equations are linear in (A_1, A_2) , given X .

The expected number of adjustments per unit of time is given in the next proposition:

PROPOSITION 4. Given a policy described by $X = (\underline{x}, \hat{x}, \bar{x})$ the expected number of adjustment per unit of time λ_a and the expected number of price increases λ_a^+ are given by

$$\begin{aligned} \lambda_a &= 1 / \left[\frac{1}{\rho} + \sum_{i=1}^2 B_i e^{q_i \hat{x}} \right] \\ \lambda_a^+ &= 1 / \left[\frac{1}{\rho} + \sum_{i=1}^2 B_{l,i} e^{q_i \hat{x}} \right] \end{aligned}$$

where q_i are the roots of $\rho = -(\pi + \mu_z)q_i + \frac{\sigma^2}{2}(q_i)^2$ and where B_i and $B_{l,i}, B_{H,i}$ solve the

following system of linear equations:

$$\begin{aligned}
0 &= \frac{1}{\rho} + \sum_{i=1}^2 B_i e^{q_i \underline{x}} = \frac{1}{\rho} + \sum_{i=1}^2 B_i e^{q_i \bar{x}}, \\
\frac{1}{\rho} &= -B_{h,1} e^{q_1 \hat{x}} - B_{h,2} e^{q_2 \hat{x}}, \quad 0 = B_{h,1} (e^{q_1 \hat{x}} - e^{q_1 \bar{x}}) + B_{h,2} (e^{q_2 \bar{x}} - e^{q_2 \hat{x}}), \\
-\frac{1}{\rho} &= B_{l,1} e^{q_1 \underline{x}} + B_{l,2} e^{q_2 \underline{x}}, \quad 0 = B_{l,1} q_1 e^{q_1 \hat{x}} + B_{l,2} q_2 e^{q_2 \hat{x}} - B_{h,1} q_1 e^{q_1 \hat{x}} - B_{h,2} q_2 e^{q_2 \hat{x}}.
\end{aligned}$$

Now we turn to the density of the invariant distribution

PROPOSITION 5. Given a policy described by $X = (\underline{x}, \hat{x}, \bar{x})$ the density of the invariant distribution $g(p, z)$ is given by

$$g(p, z) = \begin{cases} e^{\phi_1 z} [U_1^+ e^{\xi_1(p-z)} + U_2^+] & \text{if } p - z \in (\hat{x}, \bar{x}), z > 0 \\ e^{\phi_1 z} [L_1^+ e^{\xi_1(p-z)} + L_2^+] & \text{if } p - z \in [\underline{x}, \hat{x}), z > 0 \\ e^{\phi_2 z} [U_1^- e^{\xi_2(p-z)} + L_2^-] & \text{if } p - z \in (\hat{x}, \bar{x}), z < 0 \\ e^{\phi_2 z} [L_1^- e^{\xi_2(p-z)} + L_2^-] & \text{if } p - z \in [\underline{x}, \hat{x}), z < 0 \\ 0 & \text{otherwise} \end{cases} \quad (35)$$

where $\{\phi_1, \phi_2, \xi_1, \xi_2\}$ are given by

$$\begin{aligned}
\rho &= -\mu_z \phi_j + \frac{\sigma^2}{2} \phi_j^2 \quad \text{for each of the roots } j = 1, 2 \quad \text{and} \\
\xi_j &= -\frac{\pi + \mu_z - 2\phi_j \frac{\sigma^2}{2}}{\sigma^2/2}
\end{aligned}$$

and where the coefficients $\{U_i^+, L_i^+, U_i^-, L_i^-\}_{i=1,2}$ solve 8 linear equations:

$$\begin{aligned}
0 &= U_1^+ e^{\xi_1 \bar{x}} + U_2^+ = L_1^+ e^{\xi_1 \underline{x}} + L_2^+ \\
0 &= U_1^- e^{\xi_2 \bar{x}} + U_2^- = L_1^- e^{\xi_2 \underline{x}} + L_2^- \\
\frac{\phi_1 \phi_2}{\phi_1 - \phi_2} &= \frac{L_1^+}{\xi_1} [e^{\xi_1 \hat{x}} - e^{\xi_1 \underline{x}}] + L_2^+ [\hat{x} - \underline{x}] + \frac{U_1^+}{\xi_1} [e^{\xi_1 \bar{x}} - e^{\xi_1 \hat{x}}] + U_2^+ [\bar{x} - \hat{x}] \\
\frac{\phi_1 \phi_2}{\phi_1 - \phi_2} &= \frac{L_1^-}{\xi_2} [e^{\xi_2 \hat{x}} - e^{\xi_2 \underline{x}}] + L_2^- [\hat{x} - \underline{x}] + \frac{U_1^-}{\xi_2} [e^{\xi_2 \bar{x}} - e^{\xi_2 \hat{x}}] + U_2^- [\bar{x} - \hat{x}] \\
U_1^+ e^{\xi_1 \hat{x}} + U_2^+ &= L_1^+ e^{\xi_1 \hat{x}} + L_2^+ \\
U_1^- e^{\xi_2 \hat{x}} + U_2^- &= L_1^- e^{\xi_2 \hat{x}} + L_2^- .
\end{aligned}$$

Denoting the marginal density for prices h we have:

$$h(p) \equiv \int_{-\infty}^{\infty} g(p, z) dz = \int_{p-\bar{x}}^{p-\underline{x}} g(p, z) dz = \int_{p-\bar{x}}^{p-\hat{x}} g(p, z) dz + \int_{p-\hat{x}}^{p-\underline{x}} g(p, z) dz \quad (36)$$

$$h(p) = \begin{cases} \int_{p-\bar{x}}^{p-\hat{x}} e^{\phi_1 z} [U_1^+ e^{\xi_1(p-z)} + U_2^+] dz + \int_{p-\hat{x}}^{p-\underline{x}} e^{\phi_1 z} [L_1^+ e^{\xi_1(p-z)} + L_2^+] dz & \text{if } p > \bar{x} \\ \int_{p-\bar{x}}^0 e^{\phi_2 z} [U_1^- e^{\xi_2(p-z)} + U_2^-] dz + \int_0^{p-\hat{x}} e^{\phi_1 z} [U_1^+ e^{\xi_1(p-z)} + U_2^+] dz \\ + \int_{p-\hat{x}}^{p-\underline{x}} e^{\phi_1 z} [L_1^+ e^{\xi_1(p-z)} + L_2^+] dz & \text{if } p \in [\hat{x}, \bar{x}) \\ \int_{p-\bar{x}}^{p-\hat{x}} e^{\phi_2 z} [U_1^- e^{\xi_2(p-z)} + U_2^-] dz + \int_{p-\hat{x}}^0 e^{\phi_2 z} [L_1^- e^{\xi_2(p-z)} + L_2^-] dz \\ + \int_0^{p-\underline{x}} e^{\phi_1 z} [L_1^+ e^{\xi_1(p-z)} + L_2^+] dz & \text{if } p \in [\underline{x}, \hat{x}) \\ \int_{p-\bar{x}}^{p-\hat{x}} e^{\phi_2 z} [U_1^- e^{\xi_2(p-z)} + U_2^-] dz + \int_{p-\hat{x}}^{p-\underline{x}} e^{\phi_2 z} [L_1^- e^{\xi_2(p-z)} + L_2^-] dz & \text{if } p < \underline{x} \end{cases} \quad (37)$$

Using that

$$\begin{aligned} \int_a^b e^{\phi z} [D_1 e^{\xi(p-z)} + D_2] dz &= D_1 e^{\xi p} \int_a^b e^{(\phi-\xi)z} dz + D_2 \int_a^b e^{\phi z} dz \\ &= D_1 \frac{e^{\xi p + (\phi-\xi)b} - e^{\xi p + (\phi-\xi)a}}{\phi - \xi} + D_2 \frac{e^{\phi b} - e^{\phi a}}{\phi} \end{aligned}$$

we can solve the integrals for each of the branches of h where $D_i \in \{U_i^+, U_i^-, L_i^+, L_i^-\}$ for $i = 1, 2$ and $(\phi, \xi) \in \{\phi_1, \xi_1, \phi_2, \xi_2\}$ and a and b take different values accordingly.

For completeness we give the expression for the case with $\sigma = 0$, a version of [Sheshinski and Weiss \(1977\)](#) model.

PROPOSITION 6. Assume that $\sigma = 0, c > 0, \pi + \mu_z > 0$ and [equation \(11\)](#) holds. The inaction set is given by $\mathcal{I} = \{(p, z) : \underline{x} + z < p < \bar{x} + z\}$. The optimal return point is given by $\psi(z) = \hat{x} + z$. The value function in the range of inaction and the constants $X \equiv (\underline{x}, \hat{x})$ solve

$$\begin{aligned} V(p, z) &= e^{z(1-\eta)} V(p-z, 0) \equiv e^{z(1-\eta)} v(p-z) \\ v(x) &= a_1 e^{x(1-\eta)} + a_2 e^{-x\eta} + A e^{\nu x} \end{aligned}$$

where the coefficients a_i, ν are given by

$$\begin{aligned} \nu &= \frac{b_0}{b_1}, \quad a_1 = \frac{1}{b_0 - (1-\eta)b_1} \quad \text{and} \quad a_2 = -\frac{1}{b_0 + \eta b_1} \quad \text{where} \\ b_0 &= r + \rho - \mu_z(1-\eta), \quad b_1 = -[\mu_z + \pi]. \end{aligned}$$

and where the three values $A, X \equiv (\underline{x}, \hat{x})$ solve the following three equations:

$$\begin{aligned} \hat{c} &- a_1 (e^{\hat{x}(1-\eta)} - e^{x(1-\eta)}) - a_2 (e^{-\hat{x}\eta} - e^{-x\eta}) = A (e^{\nu\hat{x}} - e^{\nu x}), \\ 0 &= a_1 (1-\eta)e^{\hat{x}(1-\eta)} - a_2 \eta e^{-\hat{x}\eta} + A \nu e^{\nu\hat{x}}, \\ 0 &= a_1 (1-\eta)e^{x(1-\eta)} - a_2 \eta e^{-x\eta} + A \nu e^{\nu x}. \end{aligned}$$

Furthermore:

$$\lambda_a = \frac{\rho}{1 - \exp\left(-\frac{\rho}{\pi + \mu_z}(\hat{x} - \underline{x})\right)}$$

Proof. (of [Proposition 3](#)) To simplify the notation we evaluate all the expressions when $\bar{p} = 0$, so the relative price and the nominal price coincide.

The Bellman equation in the inaction region $(p, z) \in \mathcal{I}$ is

$$(r + \rho)V(p, z) = e^{-\eta p} (e^p - e^z) - \pi V_p(p, z) + V_z(p, z)\mu_z + V_{zz}(p, z)\frac{\sigma^2}{2}$$

for all $p \in [\underline{p}(z), \bar{p}(z)]$. The boundary conditions are given by first order conditions for the optimal return point:

$$V_p(\psi(z), z) = 0 \tag{38}$$

the value matching conditions, stating that the value at each of the two boundaries is the same as the value at the optimal price after paying the cost:

$$V(\underline{p}(z), z) = V(\psi(z), z) - \zeta(z), \quad V(\bar{p}(z), z) = V(\psi(z), z) - \zeta(z). \tag{39}$$

and the smooth pasting conditions, stating that the value function should have the same slope at the boundary than the value function in the control region (which is flat), so:

$$V_p(\underline{p}(z), z) = 0, \quad V_p(\bar{p}(z), z) = 0, \tag{40}$$

Under this conditions, the value function and optimal policies are homogeneous in the

sense that:

$$V(p, z) = e^{z(1-\eta)}V(p-z, 0) \equiv e^{z(1-\eta)}v(p-z) \quad (41)$$

$$\underline{p}(z) = \underline{z} + z, \quad \bar{p}(z) = \bar{x} + z, \quad \text{and } \psi(z) = \hat{x} + z, \quad (42)$$

where \underline{x} , \bar{x} and \hat{x} are three constant to be determined.

Using the homogeneity of the value function in [equation \(41\)](#) we can compute the derivatives

$$V_p(p, z) = e^{z(1-\eta)}v'(p-z)$$

$$V_z(p, z) = (1-\eta)e^{z(1-\eta)}v(p-z) - e^{z(1-\eta)}v'(p-z)$$

$$V_{zz}(p, z) = (1-\eta)^2e^{z(1-\eta)}v(p-z) - 2(1-\eta)e^{z(1-\eta)}v'(p-z) + e^{z(1-\eta)}v''(p-z)$$

Replacing this derivatives in the Bellman equation for the inaction region we get

$$\begin{aligned} (r + \rho)v(p-z) &= e^{-(p-z)\eta} (e^{p-z} - 1) - \pi v'(p-z) + [(1-\eta)v(p-z) - v'(p-z)]\mu_z \\ &+ [(1-\eta)^2v(p-z) - 2(1-\eta)v'(p-z) + v''(p-z)]\frac{\sigma^2}{2} \end{aligned}$$

or

$$\begin{aligned} \left[r + \rho - \mu_z(1-\eta) - (1-\eta)^2\frac{\sigma^2}{2} \right] v(p-z) &= e^{(p-z)(1-\eta)} - e^{-\eta(p-z)} \\ - v'(p-z)[\mu_z + \pi + 2(1-\eta)\frac{\sigma^2}{2}] &+ v''(p-z)\frac{\sigma^2}{2} \end{aligned}$$

We write $x = p - z$ be the log of the markup. Consider the free boundary ODE:

$$b_0 v(x) = e^{x(1-\eta)} - e^{-\eta x} + b_1 v'(x) + b_2 v''(x) \text{ for all } x \in [\underline{x}, \bar{x}]$$

$$v(\underline{x}) = v(\hat{x}) - \hat{c}, \quad v(\bar{x}) = v(\hat{x}) - \hat{c},$$

$$v'(\underline{x}) = 0, \quad v'(\bar{x}) = 0, \quad v'(\hat{x}) = 0,$$

where

$$\begin{aligned} b_0 &= \left[r + \rho - \mu_z(1 - \eta) - (1 - \eta)^2 \frac{\sigma^2}{2} \right], \\ b_1 &= - \left[\mu_z + \pi + 2(1 - \eta) \frac{\sigma^2}{2} \right], \\ b_2 &= \frac{\sigma^2}{2}. \end{aligned}$$

The value function is given by the sum of the particular solution and the solution of the homogeneous equation:

$$v(x) = a_1 e^{x(1-\eta)} + a_2 e^{-x\eta} + \sum_{i=1}^2 A_i e^{\nu_i x}$$

where ν_i are the roots of the quadratic equation

$$0 = -b_0 + b_1 \nu_i + b_2 (\nu_i)^2$$

and where the coefficients for the particular solution are

$$\begin{aligned} a_1 &= \frac{1}{b_0 - (1 - \eta) b_1 - (1 - \eta)^2 b_2} \\ a_2 &= - \frac{1}{b_0 + \eta b_1 - (\eta)^2 b_2}, \end{aligned}$$

since

$$\begin{aligned} b_0 a_1 e^{x(1-\eta)} &= e^{x(1-\eta)} + a_1 (1 - \eta) e^{x(1-\eta)} b_1 + a_1 (1 - \eta)^2 e^{x(1-\eta)} b_2 \\ b_0 a_2 e^{-x\eta} &= -e^{-x\eta} - a_2 \eta e^{-x\eta} b_1 + a_2 (\eta)^2 e^{-x\eta} b_2. \end{aligned}$$

The five constants A_1, A_2 and $X \equiv (\underline{x}, \bar{x}, \hat{x})$ are chosen to satisfies the 2 value matching conditions [equation \(39\)](#), the two smooth pasting conditions [equation \(40\)](#) and the optimal return point [equation \(38\)](#). It is actually more convenient to solve the value function in two steps. First to solve for the constants $A_i(X)$ for $i = 1, 2$ using the two value matching conditions. Mathematically, the advantage of this intermediate step is that, given X , the equations for the A_1, A_2 are linear. Conceptually, the advantage is that the solution represent the value of the policy described by the triplet $X = (\bar{x}, \underline{x}, \hat{x})$. Then we solve for $(\bar{x}, \underline{x}, \hat{x})$ using the conditions for the optimality of the thresholds, namely the two smooth pasting [equation \(40\)](#) and the f.o.c. for the return point [equation \(38\)](#).

Solving A_1, A_2 for a given policy X amount to solve the following linear system:

$$\begin{aligned}\hat{c} - a_1 (e^{\hat{x}(1-\eta)} - e^{\bar{x}(1-\eta)}) - a_2 (e^{-\hat{x}\eta} - e^{-\bar{x}\eta}) &= \sum_{i=1}^2 A_i (e^{\nu_i \hat{x}} - e^{\nu_i \bar{x}}) \\ \hat{c} - a_1 (e^{\hat{x}(1-\eta)} - e^{\underline{x}(1-\eta)}) - a_2 (e^{-\hat{x}\eta} - e^{-\underline{x}\eta}) &= \sum_{i=1}^2 A_i (e^{\nu_i \hat{x}} - e^{\nu_i \underline{x}})\end{aligned}$$

Given $A_1(X), A_2(X)$ we need to solve the following three equations:

$$\begin{aligned}0 &= a_1 (1 - \eta)e^{\hat{x}(1-\eta)} - a_2 \eta e^{-\hat{x}\eta} + \sum_{i=1}^2 A_i(X) \nu_i e^{\nu_i \hat{x}}, \\ 0 &= a_1 (1 - \eta)e^{\bar{x}(1-\eta)} - a_2 \eta e^{-\bar{x}\eta} + \sum_{i=1}^2 A_i(X) \nu_i e^{\nu_i \bar{x}}, \\ 0 &= a_1 (1 - \eta)e^{\underline{x}(1-\eta)} - a_2 \eta e^{-\underline{x}\eta} + \sum_{i=1}^2 A_i(X) \nu_i e^{\nu_i \underline{x}}.\end{aligned}$$

Proof. of [Proposition 4](#) The expected time until the next adjustment solves the following Kolmogorov equation:

$$\rho \mathcal{T}(p, z) = 1 - \pi \mathcal{T}_p(p, z) + \mathcal{T}_z(p, z) \mu_z + \mathcal{T}_{zz}(p, z) \frac{\sigma^2}{2}$$

for all p such that $\underline{p}(z) < p < \bar{p}(z)$, and all z . The boundary conditions are that time reaches zero when it hits the barriers:

$$\mathcal{T}(\bar{p}(z), z) = \mathcal{T}(\underline{p}(z), z) = 0 .$$

Given the homogeneity of the policies we look for a function satisfying

$$\mathcal{T}(p, z) = T(p - z)$$

Given the form of the expected time we have:

$$\mathcal{T}_p(p, z) = T'(p - z), \quad \mathcal{T}_z(p, z) = -T'(p - z) \text{ and } \mathcal{T}_{zz}(p, z) = T''(p - z) ,$$

so the Kolmogorov equation becomes:

$$\rho T(x) = 1 - (\pi + \mu_z) T'(x) + T''(x) \frac{\sigma^2}{2} \text{ for all } x \in (\underline{x}, \bar{x}) .$$

The solution of this equation, given \underline{x}, \bar{x} is:

$$T(x) = \frac{1}{\rho} + \sum_{i=1}^2 B_i e^{q_i x} \quad \text{for all } x \in (\underline{x}, \bar{x})$$

where q_i are roots of

$$\rho = -(\pi + \mu_z)q_i + \frac{\sigma^2}{2}(q_i)^2, \quad (43)$$

and where the B_1, B_2 are chosen so that the expected time is zero at the boundaries:

$$0 = \frac{1}{\rho} + \sum_{i=1}^2 B_i e^{q_i \underline{x}} \quad (44)$$

$$0 = \frac{1}{\rho} + \sum_{i=1}^2 B_i e^{q_i \bar{x}} \quad (45)$$

Given the solution of this two linear equations $B_1(\underline{x}, \bar{x}), B_2(\underline{x}, \bar{x})$ the expected number of adjustments per unit of time λ_a is given by

$$\lambda_a = \frac{1}{T(\hat{x})} = \frac{1}{\frac{1}{\rho} + \sum_{i=1}^2 B_i(\underline{x}, \bar{x}) e^{q_i \hat{x}}}$$

Finally, we derive the expression for the frequency of price increases. The time until the next price increase is the first time until x hits \underline{x} or the product dies while $\underline{x} < x < \hat{x}$. If x hits \bar{x} , or the product dies exogenously while $\bar{x} > x > \hat{x}$, then x then is returned to \hat{x} . Thus the expected time until the next increase in price solves the following Kolmogorov equation:

$$\rho \mathcal{T}(p, z) = \begin{cases} 1 - \pi \mathcal{T}_p(p, z) + \mathcal{T}_z(p, z) \mu_z + \mathcal{T}_{zz}(p, z) \frac{\sigma^2}{2} & \text{if } p < z + \hat{x} \\ 1 + \rho \mathcal{T}(z + \hat{x}, z) - \pi \mathcal{T}_p(p, z) + \mathcal{T}_z(p, z) \mu_z + \mathcal{T}_{zz}(p, z) \frac{\sigma^2}{2} & \text{if } p > z + \hat{x} \end{cases}$$

for all p such that $\underline{p}(z) < p < \bar{p}(z)$, and all z . The boundary conditions are that time reaches zero when it hits the barriers:

$$\mathcal{T}(\bar{x} + z, z) = \mathcal{T}(\hat{x} + z, z) \text{ and } \mathcal{T}(\underline{x} + z, z) = 0.$$

We look for a solution that is continuous and once differentiable at $(p, z) = (\hat{x} + z, z)$, and otherwise twice continuously differentiable. To do so we let $\mathcal{T}(p, z) = T_h(x)$ for $x \in [\hat{x}, \bar{x}]$

and $\mathcal{T}(p, z) = T_l(x)$ for $x \in [\underline{x}, \hat{x}]$ and

$$\begin{aligned}\rho T_l(x) &= 1 - (\pi + \mu_z)T_l'(x) + T_l''(x)\frac{\sigma^2}{2} \\ \rho T_h(x) &= 1 + \rho T_h(\hat{x}) - (\mu_z + \pi)T_h'(x) + T_h''(x)\frac{\sigma^2}{2} \\ T_l(\hat{x}) &= T_h(\hat{x}), T_l'(\hat{x}) = T_h'(\hat{x}) \\ T_h(\hat{x}) &= T_h(\bar{x}), T_l(\underline{x}) = 0.\end{aligned}$$

The solution for T_j for $j = h, l$ are:

$$T_l(x) = \frac{1}{\rho} + \sum_{i=1}^2 B_{l,i} e^{q_i x} \quad \text{and} \quad T_h(x) = \frac{1}{\rho} + \sum_{i=1}^2 B_{h,i} e^{q_i x} + \left(\frac{1}{\rho} + \sum_{i=1}^2 B_{l,i} e^{q_i \hat{x}} \right)$$

The four boundary conditions become the following four linear equations of the constants B' s:

$$\begin{aligned}\sum_{i=1}^2 B_{l,i} e^{q_i \hat{x}} &= \sum_{i=1}^2 B_{h,i} e^{q_i \hat{x}} + \sum_{i=1}^2 B_{l,i} e^{q_i \hat{x}} + \frac{1}{\rho} \\ \sum_{i=1}^2 B_{l,i} q_i e^{q_i \hat{x}} &= \sum_{i=1}^2 B_{h,i} q_i e^{q_i \hat{x}} \\ \sum_{i=1}^2 B_{l,i} e^{q_i \underline{x}} &= 0 \\ \sum_{i=1}^2 B_{h,i} e^{q_i \hat{x}} &= \sum_{i=1}^2 B_{h,i} e^{q_i \bar{x}}\end{aligned}$$

Hence, the frequency of price increases λ_a^+ is given by

$$\lambda_a^+ = 1/T_l(\hat{x}) = 1 / \left(1/\rho + \sum_{i=1}^2 B_{l,i} e^{q_i \hat{x}} \right).$$

Proof. of [Proposition 5](#) The density of the invariant distribution for (p, z) solves the forward Kolmogorov p.d.e.:

$$\rho g(p, z) = \pi g_p(p, z) - \mu_z g_z(p, z) + g_{zz}(p, z)\frac{\sigma^2}{2} \quad (46)$$

for all $(p, z) \neq (\hat{x} + z, z) = (\psi(z), z)$ and all $p : \underline{p}(z) = \underline{x} + z \leq p \leq \bar{x} + z = \bar{p}(z)$ and all z . The pde does not apply at the optimal return point, since local consideration cannot determine

g there. The other boundary conditions are zero density at the lower and upper boundaries of adjustments, and that g integrates to one:

$$\begin{aligned} g(\underline{x} + z, z) &= g(\bar{x} + z, z) = 0 \text{ for all } z \\ 1 &= \int_{-\infty}^{\infty} \int_{\underline{x}+z}^{\bar{x}+z} g(p, z) dp dz \end{aligned}$$

We will show that g can be computed dividing the state space in four regions given by whether (p, z) is such that $z > 0$ and $z < 0$ and given z whether $p \in [\underline{x} + z, \hat{x} + z]$ and $p \in [\hat{x} + z, \bar{x} + z]$.

As a preliminary step we solve for the marginal on z of the invariant distribution, which we denote by \tilde{g} . This is the invariant distribution of the process $\{z\}$ which with intensity ρ is re-started at zero and otherwise follows $dz = \mu_z dt + \sigma dW$. It can be shown that \tilde{g} is given by

$$\tilde{g}(z) \equiv \int_{\underline{x}+z}^{\bar{x}+z} g(p, z) dp = \begin{cases} \frac{\phi_1 \phi_2}{\phi_1 - \phi_2} e^{\phi_2 z} & \text{if } z < 0 \\ \frac{\phi_1 \phi_2}{\phi_1 - \phi_2} e^{\phi_1 z} & \text{if } z > 0, \end{cases} \quad (47)$$

where $\phi_1 < 0 < \phi_2$ are the two real roots of the characteristic equation

$$\rho = -\mu_z \phi + \frac{\sigma^2}{2} \phi^2. \quad (48)$$

We conjecture that g can be written as follows:

$$g(p, z) = \begin{cases} e^{\phi_1 z} k(p - z) & \text{if } z < 0 \\ e^{\phi_2 z} k(p - z) & \text{if } z > 0, \end{cases} \quad (49)$$

In this case we compute the derivatives as:

$$\begin{aligned} g_p(p, z) &= e^{\phi z} k'(p - z), \\ g_z(p, z) &= e^{\phi z} \phi k(p - z) - e^{\phi z} k'(p - z), \\ g_{zz}(p, z) &= e^{\phi z} \phi^2 k(p - z) - 2e^{\phi z} \phi k'(p - z) + e^{\phi z} k''(p - z). \end{aligned}$$

where $\phi = \phi_1$ for $z < 0$ and $\phi = \phi_2$ for $z > 0$. The p.d.e. then becomes:

$$\rho k(p - z) = \pi k'(p - z) - \mu_z [\phi k(p - z) - k'(p - z)] + [\phi^2 k(p - z) - 2\phi k'(p - z) + k''(p - z)] \frac{\sigma^2}{2}$$

for $p - z \neq \hat{x}$ or

$$\left[\rho + \phi \mu_z - \phi^2 \frac{\sigma^2}{2} \right] k(p - z) = \left(\pi + \mu_z - 2\phi \frac{\sigma^2}{2} \right) k'(p - z) + k''(p - z) \frac{\sigma^2}{2}$$

Thus

$$k(p-z) = \begin{cases} U_1 e^{\xi_{1j}(p-z)} + U_2 e^{\xi_{2j}(p-z)} & \text{if } p-z \in (\hat{x}, \bar{x}] \\ L_1 e^{\xi_{1j}(p-z)} + L_2 e^{\xi_{2j}(p-z)} & \text{if } p-z \in [\underline{x}, \hat{x}] \end{cases}$$

where ξ_{1j}, ξ_{2j} solves the quadratic equation:

$$\left[\rho + \phi_j \mu_z - \phi_j^2 \frac{\sigma^2}{2} \right] \xi + \frac{\sigma^2}{2} \xi^2 = \left(\pi + \mu_z - 2\phi_j \frac{\sigma^2}{2} \right) \xi + \frac{\sigma^2}{2} \xi^2 \quad (50)$$

for each $j = 1, 2$ corresponding to $\phi = \phi_1$ and $\phi = \phi_2$, i.e. the positive and negative values of z . We note that, by definition of ϕ in [equation \(48\)](#), the left hand side of [equation \(50\)](#) equal zero, and hence one of the two roots is always equal to zero. Thus we label $\xi_{2j} = 0$ for $j = 1, 2$. The remaining root equals:

$$\xi_{1j} = -\frac{\pi + \mu_z - 2\phi_j \frac{\sigma^2}{2}}{\sigma^2/2} \quad \text{and} \quad \xi_{2j} = 0 \quad \text{for } j = 1, 2. \quad (51)$$

We integrate $g(p, z)$ over p and equate it to $\tilde{g}(z)$ to obtain a condition for coefficients C . First we consider the case of $z > 0$:

$$\begin{aligned} \tilde{g}(z) &= \int_{\underline{x}+z}^{\hat{x}+z} e^{\phi_1 z} \sum_{i=1}^2 L_i^+ e^{\xi_{i1}(p-z)} dp + \int_{\hat{x}+z}^{\bar{x}+z} e^{\phi_1 z} \sum_{i=1}^2 U_i^+ e^{\xi_{i1}(p-z)} dp \\ &= e^{\phi_1 z} \sum_{i=1}^2 \frac{L_i^+ e^{-\xi_{i1} z}}{\xi_{i1}} [e^{\xi_{i1}(\bar{x}+z)} - e^{\xi_{i1}(\hat{x}+z)}] + e^{\phi_1 z} \sum_{i=1}^2 \frac{U_i^+ e^{-\xi_{i1} z}}{\xi_{i1}} [e^{\xi_{i1}(\hat{x}+z)} - e^{\xi_{i1}(\underline{x}+z)}] \\ &= e^{\phi_1 z} \left(\sum_{i=1}^2 \frac{L_i^+}{\xi_{i1}} [e^{\xi_{i1}\bar{x}} - e^{\xi_{i1}\hat{x}}] + \sum_{i=1}^2 \frac{U_i^+}{\xi_{i1}} [e^{\xi_{i1}\hat{x}} - e^{\xi_{i1}\underline{x}}] \right) \\ &= e^{\phi_1 z} \left(\frac{L_1^+}{\xi_{11}} [e^{\xi_{11}\hat{x}} - e^{\xi_{11}\underline{x}}] + L_2^+ [\hat{x} - \underline{x}] + \frac{U_1^+}{\xi_{11}} [e^{\xi_{11}\bar{x}} - e^{\xi_{11}\hat{x}}] + U_2^+ [\bar{x} - \hat{x}] \right) \end{aligned}$$

where the last line uses that $\xi_{2,1} = 0$. The analogous expression for $z < 0$ is

$$\tilde{g}(z) = e^{\phi_2 z} \left(\frac{L_1^-}{\xi_{12}} [e^{\xi_{12}\hat{x}} - e^{\xi_{12}\underline{x}}] + L_2^- [\hat{x} - \underline{x}] + \frac{U_1^-}{\xi_{12}} [e^{\xi_{12}\bar{x}} - e^{\xi_{12}\hat{x}}] + U_2^- [\bar{x} - \hat{x}] \right)$$

The value of the density at the boundary of the range of inaction is given by

$$g(\bar{x} + z, z) = \begin{cases} e^{\phi_1 z} \sum_{i=1}^2 U_i^+ e^{\xi_{i1} \bar{x}} & \text{for } z > 0 \\ e^{\phi_2 z} \sum_{i=1}^2 U_i^+ e^{\xi_{i2} \bar{x}} & \text{for } z < 0 \end{cases}$$

$$g(\underline{x} + z, z) = \begin{cases} e^{\phi_1 z} \sum_{i=1}^2 L_i^+ e^{\xi_{i1} \underline{x}} & \text{for } z > 0 \\ e^{\phi_2 z} \sum_{i=1}^2 L_i^+ e^{\xi_{i2} \underline{x}} & \text{for } z < 0 \end{cases}$$

If the density g at $(p, z) = (\psi(z), z) = (z + \hat{x}, z)$ is continuous on p for a given z we have:

$$g(\hat{x}, z) = \begin{cases} e^{\phi_1 z} [\sum_{i=1}^2 U_i^+ e^{\xi_{i1} \hat{x}}] = e^{\phi_1 z} [\sum_{i=1}^2 L_i^+ e^{\xi_{i1} \hat{x}}] & \text{if } z > 0, \\ e^{\phi_2 z} [\sum_{i=1}^2 U_i^- e^{\xi_{i2} \hat{x}}] = e^{\phi_2 z} [\sum_{i=1}^2 L_i^- e^{\xi_{i2} \hat{x}}] & \text{if } z < 0. \end{cases} \quad (52)$$

We summarize the results for the invariant density g here

$$g(p, z) = \begin{cases} e^{\phi_1 z} [U_1^+ e^{\xi_1(p-z)} + U_2^+] & \text{if } p - z \in (\hat{x}, \bar{x}), z > 0 \\ e^{\phi_1 z} [L_1^+ e^{\xi_1(p-z)} + L_2^+] & \text{if } p - z \in [\underline{x}, \hat{x}], z > 0 \\ e^{\phi_2 z} [U_1^- e^{\xi_2(p-z)} + L_2^-] & \text{if } p - z \in (\hat{x}, \bar{x}), z < 0 \\ e^{\phi_2 z} [L_1^- e^{\xi_2(p-z)} + L_2^-] & \text{if } p - z \in [\underline{x}, \hat{x}], z < 0 \end{cases} \quad (53)$$

where $\{\phi_1, \phi_2\}$ are the two roots of the quadratic [equation \(48\)](#) and where the use $\xi_1 \equiv \xi_{11}, \xi_2 \equiv \xi_{12}$ are given by the non-zero roots [equation \(51\)](#). The 8 values for $\{U_i^+, L_i^+, U_i^-, L_i^-\}_{i=1,2}$ solve two system of 4 linear equations, one for $\{U_i^+, L_i^+\}_{i=1,2}$ and one for $\{U_i^-, L_i^-\}_{i=1,2}$. The upper and lower boundary of the range of inaction has zero density for both positive and negative values of z :

$$0 = U_1^+ e^{\xi_1 \bar{x}} + U_2^+ = L_1^+ e^{\xi_1 \underline{x}} + L_2^+ \quad (54)$$

$$0 = U_1^- e^{\xi_2 \bar{x}} + U_2^- = L_1^- e^{\xi_2 \underline{x}} + L_2^- \quad (55)$$

The marginal distribution of the z computed using g coincides with \tilde{g} for positive and negative values of z :

$$\frac{\phi_1 \phi_2}{\phi_1 - \phi_2} = \frac{L_1^+}{\xi_1} [e^{\xi_1 \hat{x}} - e^{\xi_1 \underline{x}}] + L_2^+ [\hat{x} - \underline{x}] + \frac{U_1^+}{\xi_1} [e^{\xi_1 \bar{x}} - e^{\xi_1 \hat{x}}] + U_2^+ [\bar{x} - \hat{x}] \quad (56)$$

$$\frac{\phi_1 \phi_2}{\phi_1 - \phi_2} = \frac{L_1^-}{\xi_2} [e^{\xi_2 \hat{x}} - e^{\xi_2 \underline{x}}] + L_2^- [\hat{x} - \underline{x}] + \frac{U_1^-}{\xi_2} [e^{\xi_2 \bar{x}} - e^{\xi_2 \hat{x}}] + U_2^- [\bar{x} - \hat{x}] \quad (57)$$

The density is continuous at $(p, z) = (\psi(z), z) = (z + \hat{x}, z)$. Thus

$$U_1^+ e^{\xi_1 \hat{x}} + U_2^+ = L_1^+ e^{\xi_1 \hat{x}} + L_2^+ \quad (58)$$

$$U_1^- e^{\xi_2 \hat{x}} + U_2^- = L_1^- e^{\xi_2 \hat{x}} + L_2^- \quad (59)$$

Proof. (of [Proposition 6](#)) The expressions for the value function are obtained by setting $\sigma = 0$ and imposing the sS policy between the bands \underline{x}, \hat{x} . This problem is identical to the one in [Sheshinski and Weiss \(1977\)](#), where the discount rate is $r + \rho$. For the frequency of price adjustment we need to include the death and replacement of the products. For this we let \mathcal{T} the expected time until an adjustment:

$$\begin{aligned} \rho \mathcal{T}(p, z) &= 1 + \mathcal{T}_z(p, z) \mu_z - \mathcal{T}_p(p, z) \pi \\ \rho T(x) &= 1 - T'(x) (\mu_z + \pi) . \end{aligned}$$

with boundary conditions $T(\underline{x}) = 0$. So the solution is $T(x) = 1/\rho + B \exp\left(-\frac{\rho}{\pi + \mu_z} x\right)$ with $B = -\exp\left(\frac{\rho}{\pi + \mu_z} \underline{x}\right) / \rho$ so

$$T(x) = \frac{1}{\rho} \left[1 - \exp\left(-\frac{\rho}{\pi + \mu_z} (x - \underline{x})\right) \right]$$

and hence

$$1/\lambda_a = T(\hat{x}) = \frac{1}{\rho} \left[1 - \exp\left(-\frac{\rho}{\pi + \mu_z} (\hat{x} - \underline{x})\right) \right]$$

D Homogenous vs Differentiated goods estimation

[Figure 10](#) and [Table 6](#) show that the estimation results for the Homogenous and Differentiated goods differ. This poses the question as whether the actual goods have some idiosyncratic component or the sampling periodicity (bi-monthly vs. monthly) is affecting the estimates. To answer it we try three different exercises.

D.1 Homogenous aggregated monthly

As mentioned before, homogenous goods are sampled bi-monthly. The spirit of this section methodology is trying to mimic the results that one would have obtained if the sampling would have been monthly instead.

If this new estimation is similar to the original, then it points in the direction that sampling periodicity is not a major factor accounting for the difference in the estimates between Homogenous and Differentiated goods. I will be assumed, that for a given item no price changed occurred in a given month, if it didn't change in both the first and second fortnights. Finally, to keep things simple, we will use the Pooled simple estimator for the frequency of price adjustment.

Figure 21: Comparing two different time aggregations of homogenous goods

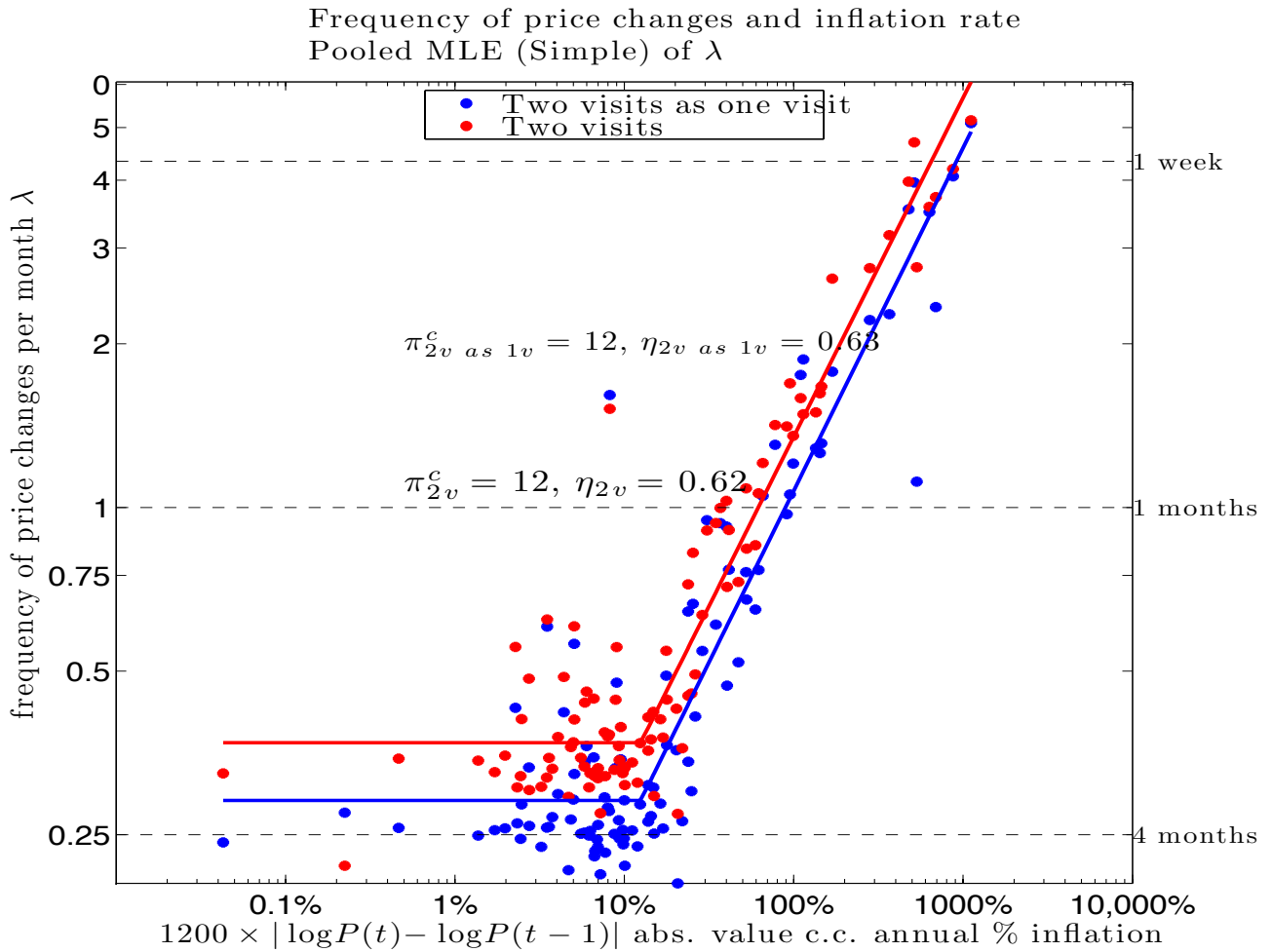
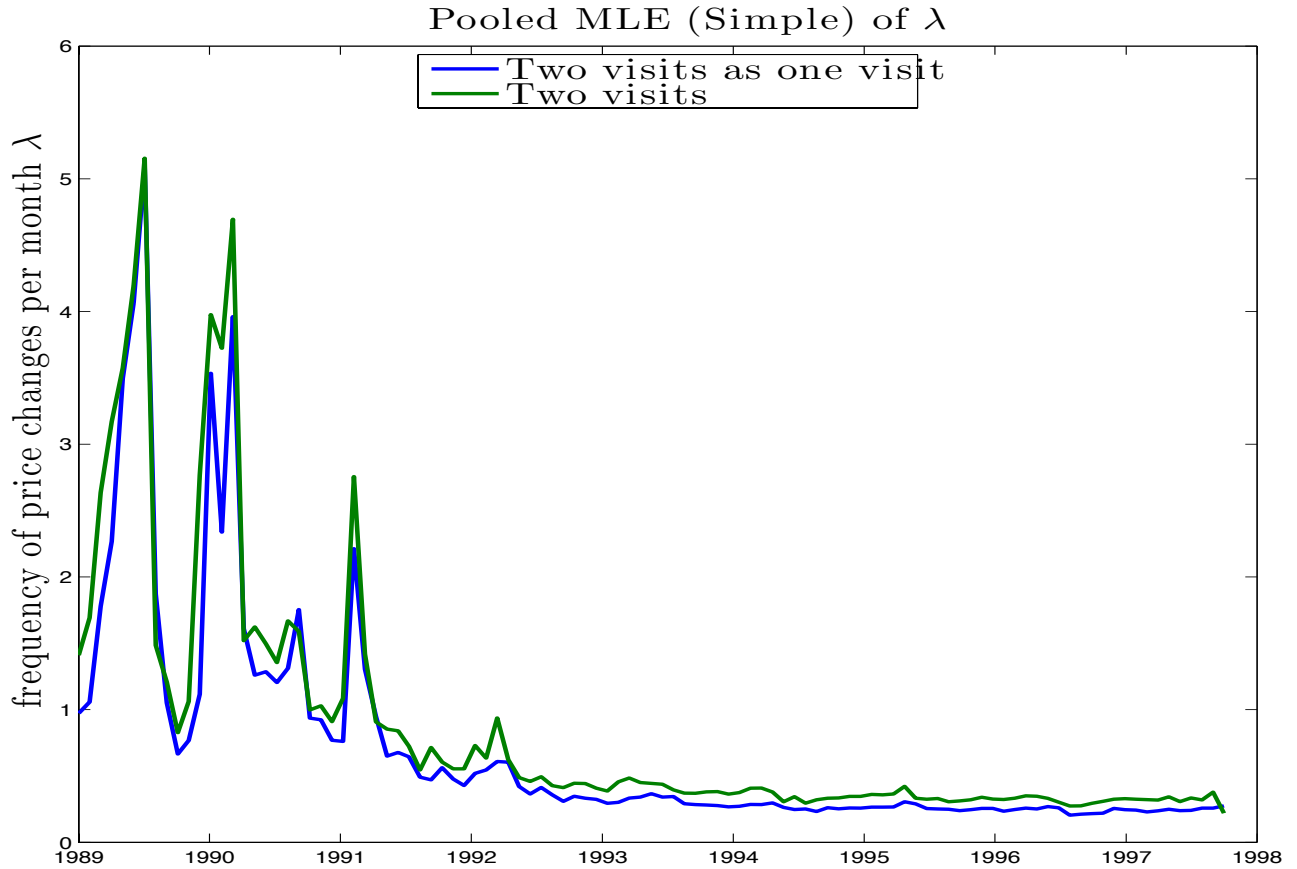


Figure D.1 above shows that the elasticity and inflation threshold are very similar in both cases. Nevertheless, this new estimate is almost always below the original as clearly seen in Figure D.1. We will argue that this could be the case by construction of both estimates, when having a decreasing hazard rate.

Figure 22: Comparing two different time aggregations of homogenous goods: time series



Define

$P_t = P(s^t)$: the probability of a price change at time t given past history s^t

$P_{t+1}^1 = P(s^{t-1}, s_t = 1, s_{t+1} = 1)$: probability of a price change at time $t + 1$ given that a price change has occurred at time t

$P_{t+1}^0 = P(s^{t-1}, s_t = 0, s_{t+1} = 1)$: probability of a price change at time $t + 1$ given that no price change has occurred at time t

Then, we have that the probability of no price adjustment in any consecutive periods can be written as

$$1 - P_t - P_t(1 - P_{t+1}^1) + (1 - P_t)(1 - P_{t+1}^0)$$

Normalizing the total number of observations in the sample to 1, the period by period

bi-monthly lambda estimation (in the simple case) would be

$$\begin{aligned}\lambda_t &= -\log(1 - P_t) \\ \lambda_{t+1} &= -\log [P_t(1 - P_{t+1}^1) + (1 - P_t)(1 - P_{t+1}^0)]\end{aligned}$$

And the monthly estimation (the one labeled Average of Homogeneous bi-weekly) would be

$$\lambda_{t,t+1}^{2v} = \lambda_t + \lambda_{t+1} = -\log [(1 - P_t) (P_t(1 - P_{t+1}^1) + (1 - P_t)(1 - P_{t+1}^0))]$$

On the other hand, if we treat the bi-monthly sample as if it were sampled monthly (Homogenous aggregated monthly) then¹ footnote (without taking into accounts the consecutive price changes that go back to the original price)

$$\lambda_{t,t+1}^{2v \text{ as } 1v} = -\log((1 - P_t)(1 - P_{t+1}^0))$$

where the expression inside the $\log(\cdot)$, is just the probability of no price change in two consecutive periods. Comparing both this estimates, it is straightforward to see that

$$\text{If } P_{t+1}^1 > P_{t+1}^0 \implies \lambda_{t,t+1}^{2v \text{ as } 1v} < \lambda_{t,t+1}^{2v}$$

In words, if we have a decreasing hazard rate then by construction one of the estimates will always be lower.

D.2 Common goods

There are 29 goods that are both in the homogenous and differentiated goods samples. These are basically sampled monthly in small shops and bi-monthly in supermarkets. Restricting attention only to these goods, we will estimate separately for each sub-sample using the Pooled simple estimator. Again, if the results are similar, it would be indicative of the relative unimportance of the sampling periodicity.

NEED TO INSERT FIGURE OF COMMON GOODS

As can be seen, the elasticities and inflation thresholds are somewhat different, but about the same order of magnitude. It is worth mentioning, that the number of observations in these subsamples is rather small, resulting in high standard error for the estimations above, and very noisy estimates.

D.3 Inflation dispersion

Calculating the Standard Deviation of the absolute value of price changes during low inflation years would give us an idea of the idiosyncratic component present in the homogenous and differentiated goods. In this way, the mean standard deviation between 1993 and 1997 is 2.44 for the Homogenous and 2.3 for the Differentiated goods, evidencing a possible greater exposure to idiosyncratic shocks. Given that homogenous goods are most of all composed by food and beverages, see description in [Appendix F](#), we find this to be in line with previous finding in the literature.

E Sales, Substitutions and Missing Values

In this section we give more information on time series for the frequency of substitution, sales and missing values, plotting separately each series for homogeneous and differentiated goods in [Figure 23](#).

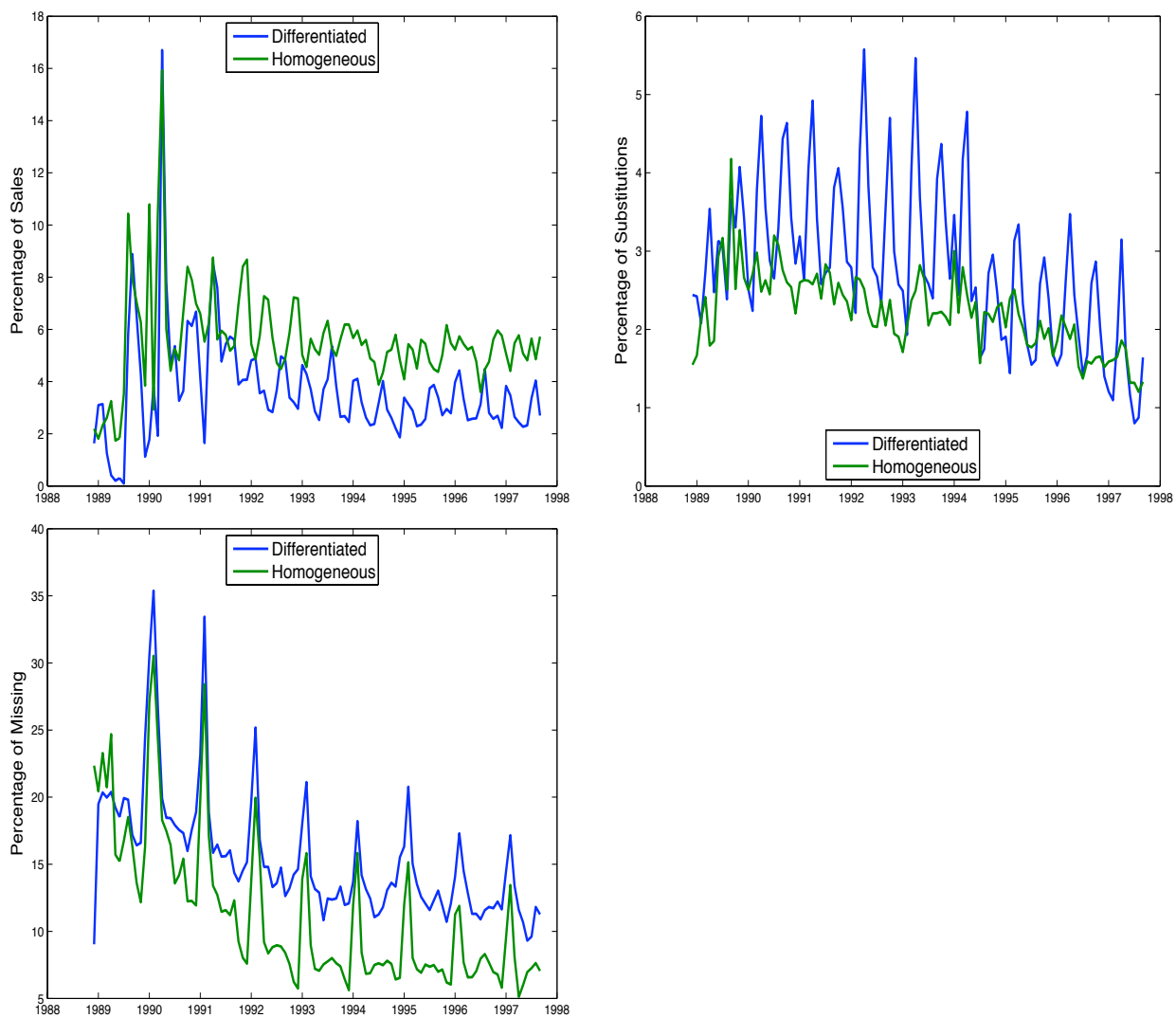
F Good/Service Classification in the CPI Database

As mentioned in the introduction, our database includes a total of 545 goods/services classified according to the MERCOSUR Harmonized Index of Consumer Price (HICP) classification. The HICP uses the first four digit levels of the Classification of Individual Consumption According to Purpose (COICOP) of the United Nations plus three digit levels based on the CPI of the MERCOSUR countries. The 545 *goods/services* in the database are the seven digit level of the HICP classification; six digit level groups are called *products*; five digit level groups are called *sub-classes*; four digit level categories are called *classes*; three digit level categories are called *groups* and two digit level groups are called *divisions*. [Table 14](#) shows two examples of this classification.

Table 14: Example of the Harmonized Index of Consumer Price Classification

Classification	Example 1	Example 2
Division	Food and Beverages	Household equipment and maintenance
Group	Food	Household maintenance
Class	Fruits	Cleaning tools and products
Sub-Class	Fresh Fruits	Cleaning products
Product	Citric Fruits	Soaps and detergents
Good	Lemons	Liquid soap

Figure 23: Frequencies of Substitution, Sales and Missing Values



Note: Missing includes stock-outs.

The precision or detail in the specification of the good depends on the degree of homogeneity of its physical characteristics. A specification is “closed” when all items belonging to that good are equivalent in its physical characteristics. A specification is “open” when the items included in that good have some different physical characteristics. The more closed the specification the more homogenous are the physical characteristics of the good. Goods with closed specifications are called homogeneous and goods with open specifications are called non-homogeneous. The main advantage of the homogenous goods is that allow to compute average prices with small standard deviation. Non homogeneous goods have more disperse prices. In the table above, lemons constitutes a homogeneous good and liquid soap is an example of a non-homogeneous good. In both cases the identification of the good includes also variable characteristics called attributes of the good. These attributes have names and values attached to them. Values can be numeric, string and/or logical expressions. For example, in the case of liquid soap, attributes names could be packaging and weight and their values pump bottle and 500ml, respectively. For most cases, the brand chosen for the product is the one most widely sold by the outlet, or the one that occupy more space in the stands, if applicable (hence brands can change from month to month or from two-weeks to two-weeks). For some cases, the brand is part of the attributes, the product is defined as one from a “top brand”.

The 545 goods are divided into two groups: homogenous (74.6% of price quotes) and non-homogeneous goods or differentiated goods (25.4% of price quotes). Examples of homogenous goods are: barley bread, chicken, lettuce, etc. Examples of non-homogenous goods are moccasin shoes, utilities, tourism, and professional services. As explained in the introduction, we excluded from our database fuel, goods in baskets and rents prices. After these exclusions the database contains 506 goods/services.

Prices are gathered every two weeks for all homogenous goods and for those non-homogeneous goods gathered in super-market chains (we group these goods in a category that we called *two visit goods*); and gathered every month for the rest of the non-homogeneous goods (we group these goods in a category that we called *one visit goods*). In the sample there are 233 goods in the two visit category, 302 in the one visit category and 29 goods belonging to both categories. As mentioned above, there are a total of 506 different goods in our sample.

Goods are weighted to construct the CPI by using the information of the National Expenditure Survey (ENGH) of 1986. Weights are computed as the proportion of the households expenditure on each good over the total expenditure of the households. In this way, the weight of a particular good is proportional to the importance of its expenditure with respect to the total expenditure without taking into account the percentage of households buying it. Our database covers about 84% of household expenditures.

Table 15 shows the top 20 goods, in terms of the importance of their weights in our sample. As it can be seen from the table, most goods whose prices are gathered twice a month are represented by food and beverages while goods whose prices are gathered monthly include services, apparel and other miscellaneous goods and services.

Table 15 about here

Table 16 shows the weight structure in our database classified by divisions. The table shows goods in terms of their weight with respect to the total weight in the sample (Total column), and with respect to the total weight of their belonging category (one or two visit goods) Food and non-alcoholic beverages represent almost 43% of the total weight in the sample and 82% of the total weight of goods whose prices are gathered twice a month. On the other hand, weights of one visit goods are less concentrated. Almost 12% of the total weight in the sample corresponds to furniture and household items and around 9% correspond to apparel. These percentages are around 24% and 19%, respectively, when computing percentages over the total weight in the one visit goods category.

Table 16 about here

F.1 Instructions to CPI's Pollsters

Pollsters record item's prices. Remember that an item is a good/service of a determined brand sold in a specific outlet in a specific period of time. Prices are transactional, meaning that the pollster should be able to buy the product in the outlet. As described above, goods are defined by its attributes. For the majority of the goods, the brand is not an attribute. The brand of a specific item is determined the first time the pollster visits an outlet. The brand is the most sold/displayed by the outlet. Once the item is completely defined, the pollster collects the price of that item next time she visits the same outlet. After the first visit, in the following visits, the pollsters arrive to each outlet with a form that includes all items for which prices are to be collected.

For example, assume the good is *soda-cola top brand* and the attributes are *package: plastic bottle and weight: 1.5 liters*. The first time the pollster goes to, say, outlet *A* she ask for the cola top brand most sold in that outlet. Assume that *Coca-Cola* is the most sold soda in outlet *A*. Then the item is completely defined: *Coca-Cola* in plastic bottle of 1.5 liters in outlet *A*. Next time the pollster goes to outlet *A* she records the price of that item.

All prices are in argentine pesos. Our dataset does not contain flags for indexed prices. In traditional outlets pollsters ask for the price of an item even when, for example it is

displayed in a blackboard, because the good has to be available in order to record its price. In supermarket chains the price recorded is collected from the shelf/counter display area.

There are a number of special situations to be taken into account:

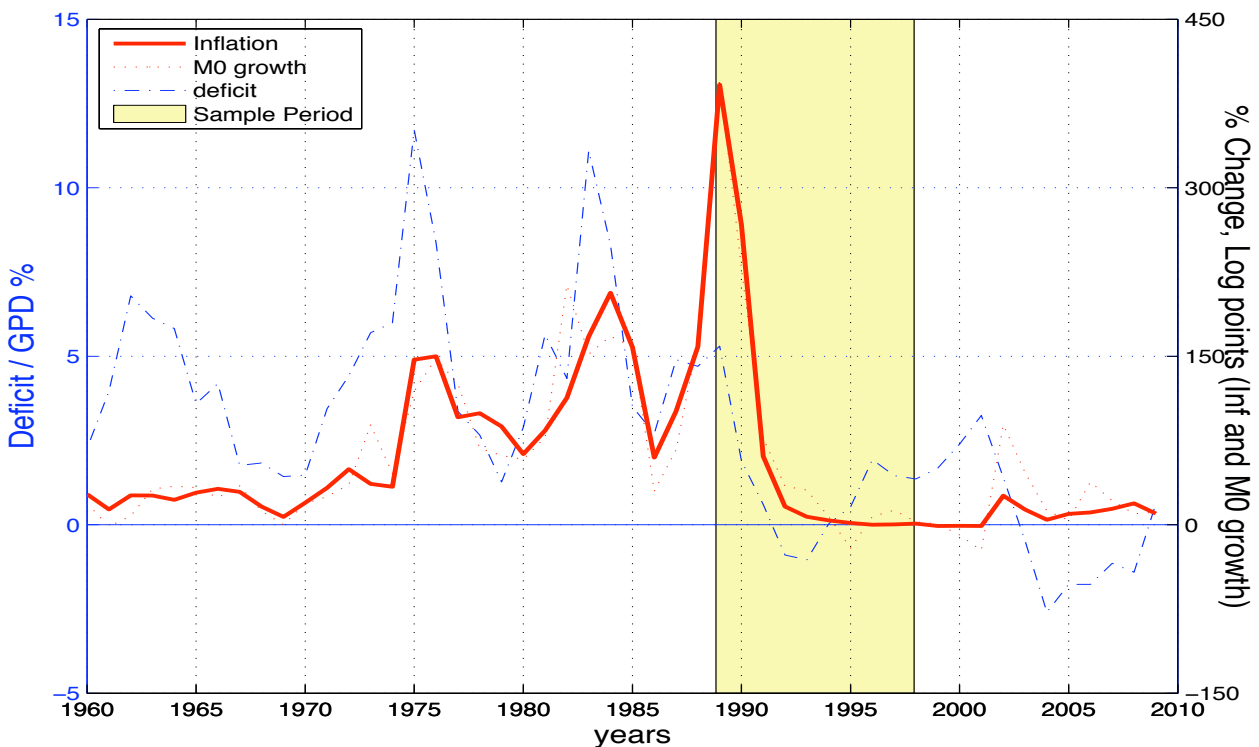
1. *Substitutions*: every time there is a change in the attributes of the good the pollster has to replace that particular good for another one. In this case, the pollster mark the price collected with a flag indicating a substitution has occurred. The goods that are substituted should be similar in terms of the type of brand and or quality.
2. *Stockouts*: every time the pollster could not buy the item, either because the good is out of stock in the outlet or because the same good or a similar one is not sold by outlet at the time the price has to be collected, she has to mark the item with a flag of stockout and she has to assign a missing price for that item. Stock-outs include what we label “pure stock-outs”, the case where the outlet has depleted the stock of the good, including end of seasonal goods/services. Stock outs also include the case where the outlet no longer carries the same good/service and it does not offers a similar good/service of comparable quality/brand. Examples of stock-outs include many fruits and vegetables not available off-season, as well as clothing such as winter coats and sweaters during summer.
3. *Sales*: every time the pollster observes a good with a sale flag in an outlet, she has to mark the price of that particular item with a sale flag.

G Background on Inflation and Economic Policy

Here we give a brief chronology of economic policy to help readers understand the economic environment in our sample period, which goes from December 1988 to September 1997. The beginning of our sample coincides with decades of high inflation culminating in two years of extremely high inflation (typically referred as two short hyperinflations) followed by a successful stabilization plan, based on a currency board, started in April of 1991 which brought price stability in about a year, and stable prices until at least three more years after the end of our sample.

The years before the introduction of the currency board witnessed several unsuccessful stabilization plans, whose duration become shorter and shorter, and that culminated in the two short hyperinflations, all of these during a period of political turmoil. Several sources describe the inflation experience of Argentina since the 1970, such as [Kiguel \(1991\)](#) and [Alvarez and Zeldes \(2005\)](#) for the period before 1991 and [Cavallo and Cottani \(1997\)](#) for descriptions right after 1991. For a more comprehensive study see [Buera and Nicolini \(2010\)](#).

Figure 24: Inflation, Money Growth and Deficits



Argentina had a very high average inflation rate since the beginning of the 1970s. In-

Table 15: Goods ordered by weight whose prices are gathered once and twice per month

One visit goods	Weight (%)	Two visits goods	Weight (%)
Lunch	2.02	Whole chicken	1.51
Lunch in the workplace	1.67	Wine	1.49
Car	1.52	French bread (less than 12 pieces)	1.38
Housemaid	1.48	Fresh whole milk	1.31
Monthly union membership	1.22	Blade steaks	0.96
Snack	0.91	Standing rump	0.93
Medical consultation	0.83	Eggs	0.87
Gas bottle	0.58	Short ribs (Roast prime ribs)	0.85
Ladies hairdresser	0.56	Striploin steaks	0.78
Labor for construction	0.54	Apple	0.73
Adult cloth slippers	0.50	Oil	0.72
Color TV	0.50	Rump steaks	0.72
Funeral expenses	0.49	Potatos	0.71
Men's dress shirt	0.49	Soda (coke)	0.70
Dry cleaning and ironing	0.48	Cheese (quartirolo type)	0.70
Sports club fee	0.47	Tomatoes	0.63
Movie ticket	0.47	Minced meat	0.60
Men's denim pants	0.45	Sugar (white)	0.59
Disposable diapers	0.43	Coffee (in package)	0.57
Shampoo	0.40	Yerba mate	0.55

Table 16: Weights for Divisions of the Harmonized Index of Consumer Price

Divisions	Weight		Weight	
	Total	One visit	Total	Two visits
Food and non-alcoholic beverages	4.94	9.91	42.67	82.39
Apparel	9.24	18.53		
Conservation and repair of housing plus gas in bottle	0.99	1.99		
Furniture and household items	11.86	23.79		
Medical products, appliances and equipment plus external medical services	3.39	6.79		
Transportation	2.60	5.22		
Recreation and culture	4.50	9.02		
Education	2.42	4.85		
Miscellaneous goods and services (Toiletries, haircut, etc.)	5.07	10.16		
Jewelry, clocks, watches plus other personal belongings	4.86	9.75		
Alcoholic beverages			3.94	7.62
Non-durable household goods			2.40	4.64
Other items and personal care products			2.77	5.36

stitutionally, the Central Bank has been part of the executive branch with no independent powers, and typically has been one of the most important sources of finance for a chronic fiscal deficit. [Figure 24](#) plots inflation, money growth and deficits between 1960 and 2010 with our sample period highlighted in yellow. The deficit as a percentage of GDP was on average well above 5% from 1975 to 1990, see [Figure 24](#). At the beginning of the 1980s fiscal deficit and its financing by the Central Bank were large even for Argentinean standards. These years coincide with a bout of high inflation that started in the second half of the last military government, 1980 to 1982, and continued during the first two years of the newly elected administration of Dr. Alfonsín, 1983 to 1984.

In June of 1985 there was a serious attempt to control inflation by a new economic team which implemented what it is referred to as the Austral stabilization plan (the name comes from the introduction of the “Austral” currency in place of the “Argentine Peso”). The core of this stabilization plan was to fix the exchange rate, to control the fiscal deficit and its financing from the Central Bank, and to introduce price and wage controls. While the Austral plan had some initial success, reducing the monthly inflation rate from 30% in June of 1985 to 3.1% in August of 1985, by mid 1986 the exchange rate was allowed to depreciate every month and inflation reached about 5% per month. By July of 1987 the monthly inflation rate was already above 10%. The same economic team started what is referred to as the “Primavera” stabilization plan in October 1988, when the inflation rate was again around 30% per month, at a time when the Alfonsín administration was becoming politically weak. The primavera plan was a new short lived exchange rate based stabilization plan that was abandoned in February of 1989¹³. Our data set starts right after the beginning of the Primavera stabilization plan, in December of 1988.

After the collapse of the “Plan Primavera” the economy lost its nominal anchor and a perverse monetary regime was in place. Legal reserve requirements for banks were practically 100% and the Central Bank paid interest on reserves (most of the monetary base) printing money. Thus a self fulfilling mechanism for inflation was in place. High inflationary expectations, led to high nominal deposit rates, which turned into high rates of money creation that validated the inflationary expectations.

[Figure 25](#) displays the yearly percent continuously compounded inflation rate and interest rate for the first years of our sample, together with references to some of the main changes in economic policy during the period. Observe how interest rates and inflation skyrocketed after the plan Primavera’s crawling peg was abandoned. In May 1989 a presidential election took place where the opposition candidate, Dr. Menem, was elected. The finance minister and the

¹³The peg started at about 12 units of Argentine currency (“the Austral”) per US dollar. To put it in perspective, at the beginning of the Austral plan, the peg was 0.8 units of Argentine currency per US dollar.

central bank president that carried the Primavera plan resigned in April 1989. Thereafter, Dr. Alfonsín's administration had two different finance ministers and two different central bank presidents, in the midst of a very weak political position and rampant uncertainty about the policies to be followed by the next administration. During the campaign for the presidential election Dr. Menem proposed economic policies that can be safely characterized as populist, with a strong backing from labor unions. Indeed, the core of his proposed economic policy was to decree a very large generalized wage increase, "el salarizado".¹⁴ In July 1989 the elected president, Dr. Menem, took office, several months before the stipulated transition date, in the midst of uncontrolled looting, riots and extreme social tension. The inflation rate at this time was the highest ever recorded in Argentina, almost 200% per month—2.3% per day.

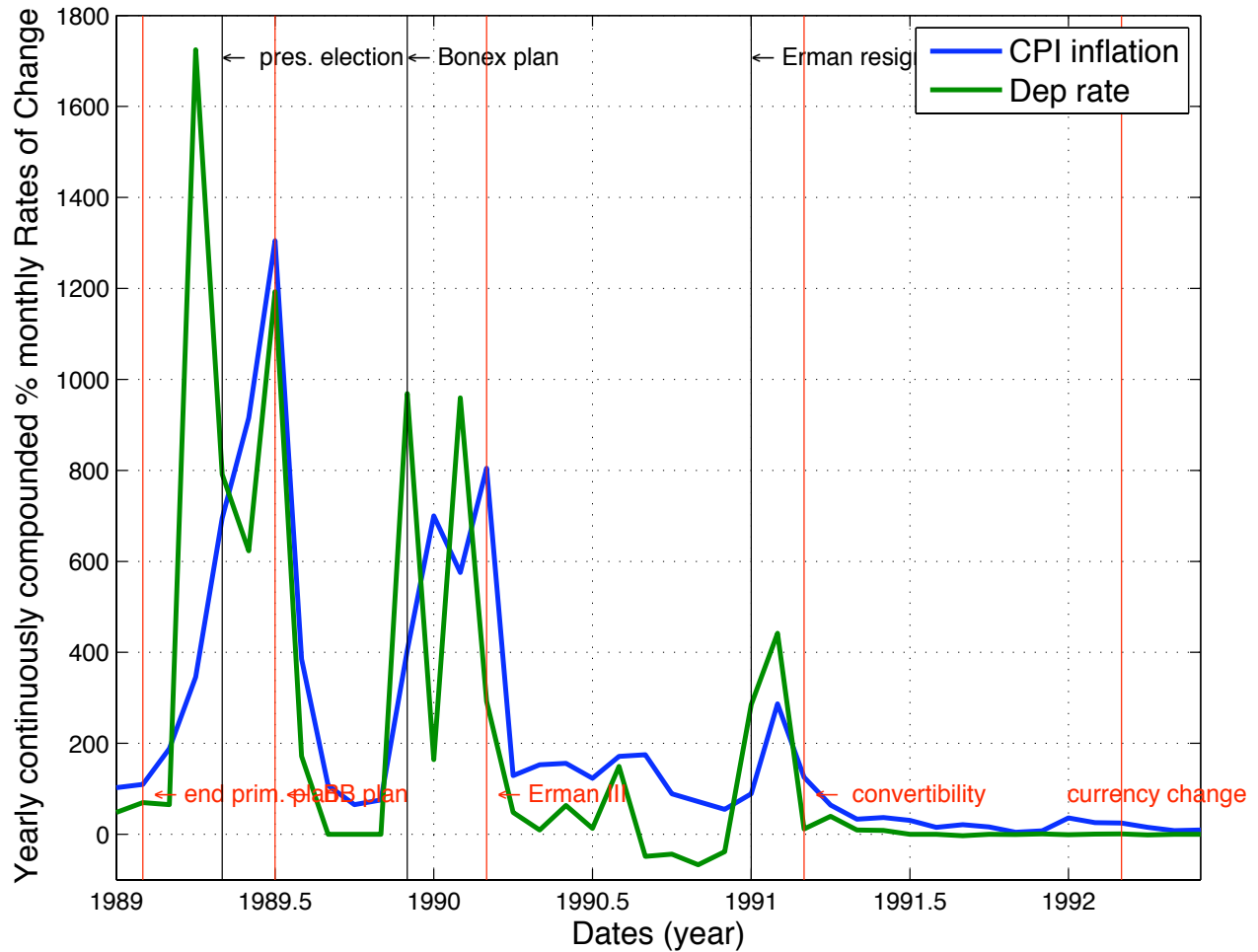
The beginning of the Menem administration started with a large devaluation of the Argentine currency, in what is known as the BB stabilization plan, for the name of the company Bunge and Born, where the two first secretaries of the treasury came from. Indeed these appointments made by the Menem administration were a surprise to most observers, given the promises made in the campaign. The announcement of tight control of the fiscal deficit, and the management of the exchange rate of this plan were also surprising for most observers. During this time inflation transitorily fell.

In December 1989, amid large losses in the value of the Argentine peso, a new finance minister was appointed, Dr. Erman González, who started yet a new "stabilization plan" (referred to as Plan Bonex). The core of this plan was a big compulsory open market operation by which the central bank exchanged all time deposits in the Banking system (mostly peso denominated time deposits with maturities of less than a month) for 10 year US Dollar denominated government bonds (Bonex 1989). This big open market operation changed the monetary regime and allowed the Central Bank to regain control of the money supply, as the government no longer had to pay interest on money (reserves on time deposits) by printing money. During Dr. González tenure there were several fiscal measures aimed at controlling the fiscal deficit. In March 1990 a renewed version of the stabilization plan was launched, with a stricter control of the money supply and of the fiscal deficit. The actual percentage inflation rates during 1989 and 1990 were 4924% and 1342% respectively!

In January of 1991, Dr. González resigned and Dr. Cavallo was appointed as finance minister. During the first two months of his tenure there was a large devaluation of the currency and a large increase in the prices of government owned public utility firms. In April 1st of 1991 there was a regime shift that lasted until 2001. The new regime was a currency board that fixed the exchange rate and enacted the independence of the Central Bank, first

¹⁴The slogan to summarize his proposed economic policy was "el salarizado", i.e. "the huge wage increase" in Spanish.

Figure 25: Inflation and Depreciation Rate during HyperInflations



by means of presidential decrees, and then by laws approved by congress. At this time the argentinean currency, the Austral, was pegged to the US dollar at 10,000 units per USD.¹⁵

On January 1st 1992 there was a currency reform that introduced a new currency (the Peso Argentino) to replace the Austral, chopping four zeros of the latter (so that one peso was pegged to one dollar, and to 10,000 australes).

There are a host of changes that were introduced at this time, both in term of deregulation and in terms of fiscal arrangements (broadening of the value added tax's base, sale of state owned firms, etc.), which in the first years reduced the size of the fiscal deficit. There was also a renewed access to the international bond markets, and a constant increase in public

¹⁵To have an idea of the average inflation rate until 1991, notice that the exchange rate when the Austral was introduced in June of 1985 was 0.8 austral per US dollars.

debt. There were also an acceleration of the trade liberalization that started in the mid 80s and a liberalization of all price and wage controls. During the years covered in our sample, GDP grew substantially, despite the short and sharp recession during 1995, typically associated with the balance of payment crisis in Mexico. The exchanged rate remained fixed until January of 2002, where the exchange rate was depreciated in the midst of a banking run that started in the last quarter of 2001, a recession that started at least a year prior, and the simultaneous default of the public debt.