Platforms & Matching with Noisy Signals

Carlos Cañón*

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Abstract

A platform offers an environment where heterogenous buyers and sellers have one opportunity to meet a partner and form a match; no match is feasible outside the environment. We evaluate and compare the profit maximizing and the surplus maximizing platforms. There are two important features: buyers and sellers only observe (inside the environment) a noise of their partner's type and do not observe their own noisy signal, and the platform can only charge a fixed (but possibly different) access fee to buyers and sellers. The platform do not run the match, but he influences the matching equilibrium through the effect on the probability of being matched, and through the expected type of buyers and sellers willing to participate. We compute and interpret the platform's optimal pricing rule under the light of the two-sided market literature, and show which matching equilibrium emerges under several model's primitives. Finally, we show the profit maximizing platform overprovides information compared to the benchmark case.

Keywords: Matching; Search; Noisy Signals; Two-Sided Markets; Optimal Pricing Rule; Discrimination

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*Toulouse School of Economics Ph.D. student. Contact information: 21 Allée de Brienne, Batiment F, Bureau 003, 31000 Toulouse - France. Webpage: canoncic.freeshell.org. E-mail: cicenator [at] gmail.com (work), canoncic [at] sdf.lonestar.org (personal)

1 Introduction

Platforms are often regarded as "central planners" because they own an environment in which, for example, buyers and sellers enter to meet each other and engage in transactions that otherwise could not be carried out. The literature of two-sided markets so far had payed great attention in understanding how the network effects shape the platform's pricing scheme, see for example [1], [6], [19], [21]. An interesting task we face is to understand how the behavior of the platform changes (e.g., its pricing scheme or any other tool), under different environments that are rather relevant for many industries.

The specific situation this paper is interested in is the one of a platform that owns an environment were buyers and sellers have imperfect information about their partner. We will consider both the profit maximizing and the surplus maximizing platforms. To simplify even more, we assume the motivation to participate into the environment is because this is the only way to match with a trading partner, we also assume the platform has a technology that only allows him to charge an access fee. This environment has three important features: that buyers and sellers have imperfect information about their trading partner's type, also that they cannot control the information their partner receives about them, and that the platform cannot directly control de match but he controls the information buyers and sellers have from each other.

We have two questions in mind. The main question we ask is how the platform's behavior (i.e. pricing scheme and optimal information provision to buyers and sellers) changes in this particular environment vis-a-vis the standard two-sided market literature. The second question we want to answer is how the matching equilibrium is affected by the platform's behavior. To our knowledge this is the first paper that studies the impact of a profit maximizing and also a surplus maximizing central planner on the matching equilibrium with noisy signals.

Why is this environment interesting in the first place? The main application we have in mind are platforms that cannot directly control which buyers and sellers will engage into trading, but that can indirectly shape the equilibrium matching patterns observed inside the environment through its pricing scheme and its optimal provision of information. For example, imagine a platform that cannot force buyers and sellers to search for a matching partner in a specific way, but that can determine the expected "quality" of buyers (seller) that will participate and/or the equilibrium probability of forming a match. To wrap-up, the platforms we are interested in are not capable to directly run the match between buyers and sellers, but can *induce* it in ways that latter will be explained in greater detail.

Our results hinges on an assumption that guarantees the better is the type, e.g. of buyers and sellers, the more likely is that he receives a high signal. We show there exists a matching equilibrium, among those buyers and sellers that participate, where the participants use strategies that are increasing in types. Then we proceed to characterize the matching equilibria, we show that they depend on the mass of buyers and sellers willing to participate. In particular, we determine several thresholds on the mass of participants that determine which equilibrium will emerge. In the next step we compute the optimal pricing rules for the profit maximizing platform, and for the surplus maximizing platform; nice intuitions will emerge out from their comparison. We show through numerical simulation which matching equilibrium will be picked by the profit maximizing platform. Finally, we compute the optimal noise for the profit maximizing platform, and for the surplus maximizing platform. We show that, compared to the benchmark case, the profit maximizing platform overprovide information to both buyers and sellers.

This paper is feeding from two strands of literature. On one hand, we are clearly related to the two-sided market literature cited at the beginning. The novelty we propose to this literature is to understand the platform's behavior under the particular environment we just described. The closest paper to ours we can identify is [8], the main difference is that network participants have imperfect information and that the platform can affect the equilibrium probability of finding a match. On the other hand, our paper is related to the matching literature such as [4], [5], [7]. The novelty we propose here is to understand how matching equilibrium is affected by the platforms' activities given the particularities of the environment we propose. The closest papers to ours are [12] and [18], but neither of them study the effect of a profit maximizing platform.

The paper proceeds as follows. Section 2 presents the model, there we will show there exist a matching equilibrium, and then we will characterize it. Section 3 discuss the optimal pricing rule. Section 4 discuss the optimal noise. Section 5 concludes.

2 Static Model

The model has four features. First, buyers and sellers only perceive utility from forming a match. Second, there are some buyers and sellers whose type is negative. Third, its impossible to form a match outside the searching environment hosted by the platform. Finally, inside the searching environment buyers and sellers only receive one matching partner to choose from.

2.1 Environment

Time. One period.

Players. One profit maximizing platform endowed with an environment. On the other hand, two disjoint groups of *ex-ante* heterogeneous network users, i.e. buyers (B) and sellers (S), that want to meet with each other to engage in trading activities. More formally, buyer's type is defined by the usage benefit $b \sim F_B(b)$ and $f_B(b) > 0$ on the support $[\underline{b}, \overline{b}]$, where $\underline{b} < 0 < \overline{b}$. Seller's type is also determined by a usage benefit $s \sim F_S(s)$ and $f_S(s) > 0$ on the support $[\underline{s}, \overline{s}]$, where $\underline{s} < 0 < \overline{s}$. Buyers and sellers' type is private information.

Information Structure. *Inside the environment.* Buyers and sellers only directly observe a signal from their partner's type. Seller s will observe a signal $\theta \in \{\underline{\theta}, \overline{\theta}\}$ st $\underline{\theta} < \overline{\theta}$, with the conditional probability mass function (pmf) $\tilde{m}(\theta \mid b) = m(b)\mathbb{1}_{\theta=\overline{\theta}} + (1 - m(b))\mathbb{1}_{\theta=\underline{\theta}}$, where $m(b) = Prob\{\overline{\theta} \mid b\}$, and the conditional discrete probability distribution (cdpd) $\tilde{M}(\theta \mid b)$. Analogously, buyer b will observe a signal $\omega \in \{\underline{\omega}, \overline{\omega}\}$ st $\underline{\omega} < \overline{\omega}$ with the conditional pmf $\tilde{n}(\omega \mid s) = n(s)\mathbb{1}_{\omega=\overline{\omega}} + (1 - n(s))\mathbb{1}_{\omega=\underline{\omega}}$, where $n(s) = Prob\{\overline{\omega} \mid s\}$, and the conditional cdpd $\tilde{N}(\omega \mid s)$. Neither sellers or buyers will observe their own signal. Finally, and to simplify, assume $n(s)$ satisfy $n(\underline{s}) \geq 0, n(\overline{s}) \geq 1, n'(s) > 0$, and $m(b)$ satisfy $m(\underline{b}) \geq 0, m(\overline{b}) \geq 1, m'(b) > 0$[1]

Outside the environment. Buyers and sellers type is private information.

Payoffs *Transaction.* If buyer b and seller s match they receive a payoff equivalent to their partner's type, e.g. b obtains s, and s obtains b. If buyer b and seller s do not match they receive zero profit. Finally, everybody has to pay a search cost proportional to $c > 0$[2]; in particular, we will have that the cost of searching and accepting any partner is higher than the cost of searching and only accepting partners with a high signal.

Participation. Platform will charge a linear price to all buyers (P^b), and another to all sellers (P^s). Also will offer pmf's $n(s)$ and $m(b)$. Buyer b will obtain a expected trade surplus $\mathbb{E}_\omega w(\omega, b)$, and will pay an entry fee P^b. Finally, seller s will obtain a expected trade surplus $\mathbb{E}_\theta v(\theta, s)$, and will pay an entry fee P^s.

Matching Game. Random meeting. Simultaneously buyer b observes ω, and seller s observes θ. If both accepts their partner they form a match, otherwise, no match is formed. To simplify I will assume that each buyer (seller) will encounter a maximum one partner.

Strategies. *Buyers.* The accept/reject decision is governed by $\Lambda_b : \{\underline{\omega}, \overline{\omega}\} \to \{\{\overline{\omega}\}, \{\underline{\omega}, \overline{\omega}\}\}$. The enter/stay out decision is governed by $\sigma_b : [\underline{b}, \overline{b}] \to \{0, 1\}$. Finally define, $\Lambda_B = (\Lambda_b)_{b\in[\hat{b},\overline{b}]}$ and $\sigma_B = (\sigma_b)_{b\in[\underline{b},\overline{b}]}$.

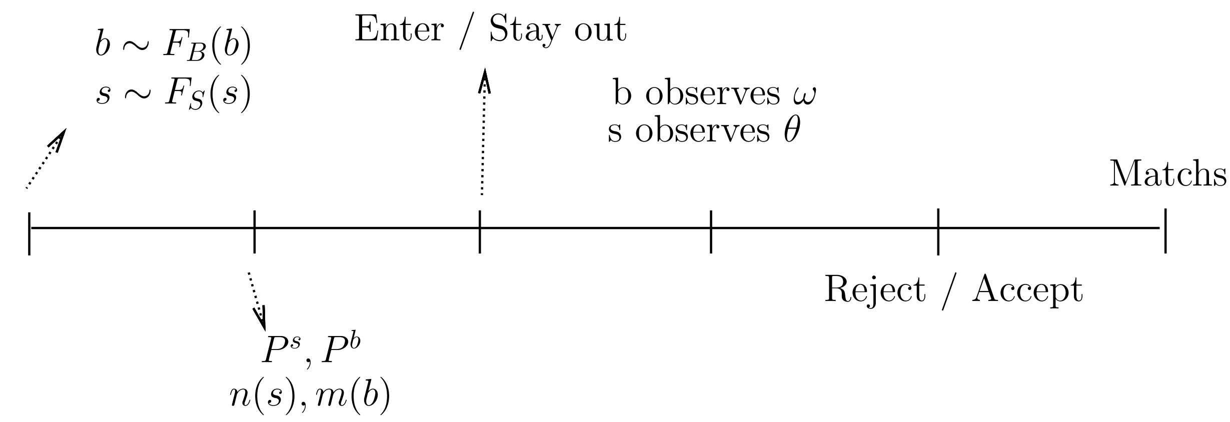
Sellers. The accept/reject decision is governed by $\Lambda_s : \{\underline{\theta}, \overline{\theta}\} \to \{\{\overline{\theta}\}, \{\underline{\theta}, \overline{\theta}\}\}$, where $\hat{b}$ is the threshold seller indifferent between participating and staying out. The enter/stay out decision is governed by $\sigma_s : [\underline{s}, \overline{s}] \to \{0, 1\}$. Finally define, $\Lambda_S = (\Lambda_s)_{s\in[\hat{s},\overline{s}]}$, and $\sigma_S = (\sigma_s)_{s\in[\underline{s},\overline{s}]}$.

Platform. His strategy will specify access fees, e.g. P^s, P^b, and signal's conditional pmf's, e.g. $n(s)$, $m(b)$.

Timing. At the beginning, (i) Buyer b privately learn $b \sim F_B(b)\ \forall b$, seller s privately learn $s \sim F_S\ \forall s$. Then, (ii) the platform determines P^s, P^b, $n(s)$, $m(b)$. At the next step, (iii) all buyers and sellers decide to enter or stay out. Among those that participate, (iv) buyer b and seller s randomly meet in pairs. Any buyer b privately learn ω, and any seller s privately learn θ. Now within each pair, (v) buyers and sellers accept or reject their partner. Finally, (vi) if buyer b and seller s accept they form a match and receive the corresponding payoff, otherwise they stay single and receive zero payoff.

[1] In case θ and ω where a continuous random variables this assumption is equivalent to assume the families of functions $\{\tilde{m}(\theta \mid b) : b \in [\underline{b}, \overline{b}]\}$ and $\{\tilde{n}(\omega \mid s) : s \in [\underline{s}, \overline{s}]\}$ are strict log-supermodular (satisfy MLRP). See [7] for additional details.

[2] This cost in the two-sided market literature is the membership cost (see [21])



2.2 Matching Equilibrium

Lets define what a matching equilibrium is. Denote $\Upsilon(b, \Lambda_b, \Lambda_S; \sigma_B, \sigma_S)$ as the expected utility of buyer b using Λ_b, given all sellers using Λ_S and given the entry decisions σ_B and σ_S. Similarly denote $\Upsilon(s, \Lambda_B, \Lambda_s; \sigma_B, \sigma_S)$ for seller s. A matching equilibrium is a strategy profile $(\Lambda_B^*, \Lambda_S^*)$ such that $\forall b \in [\hat{b}, \bar{b}]$

$$\Upsilon(b, \Lambda_b^*, \Lambda_S^*; \sigma_B, \sigma_S) \quad \geq \quad \Upsilon(b, \Lambda_b', \Lambda_S^*; \sigma_B, \sigma_S)$$

where Λ_b' is al alternative strategy, and $\forall s \in [\hat{s}, \bar{s}]$

$$\Upsilon(s, \Lambda_B^*, \Lambda_s^*; \sigma_B, \sigma_S) \geq \Upsilon(s, \Lambda_B^*, \Lambda_s'; \sigma_B, \sigma_S)$$

and where Λ_s' is al alternative strategy.

This model can replicate qualitatively the results of a dynamic matching model that assume the distribution of types (of buyers and sellers) is stationary, the essential assumption is that types could be negative. In particular, this model can be viewed as a static reformulation of [7].

2.2.1 Optimal Accept/Reject Strategy

Consider the case of seller s that receives a partner with probability $\mathcal{P}^s$[3]. After observing signal θ the seller updates his beliefs about his partner's type (b) using bayes rule, e.g. $k(b \mid \theta) = \frac{\tilde{m}(\theta|b)(1-F_B(\hat{b}))f_B(b)}{\int_{\hat{b}}^{b} \tilde{m}(\theta|b)f_B(b)db}$ where $\tilde{m}(\theta \mid b) = m(b)\mathbb{1}_{\theta=\bar{\theta}} + (1-m(b))\mathbb{1}_{\theta=\underline{\theta}}$. He must decide whether to accept him or not, and if he does not he will obtain zero payoff. On the other hand, if he accepts his partner two things might happen: he gets accepted or rejected. His partner (e.g. buyer b) will accept those sellers belonging the set Λ_b, then the probability his partner accepts him is $\Delta_s(\Lambda_b \mid s) = Prob\{\omega \in \Lambda_b \mid s\}$. Then, the expected payoff of announcing accept, conditional on having a partner type b, is $b\Delta_s(\Lambda_b \mid s)$. Finally, to obtain the unconditional expected payoff from accepting his partner one must integrate out the type b using $k(b \mid \theta)$.

The objective function for seller s, conditional on θ, will be,

$$\begin{aligned} v(\theta, s) &= \max\left\{\mathcal{P}^s \int_{\hat{b}}^{\bar{b}} b\Delta_s(\Lambda_b \mid s) \frac{k(b \mid \theta)}{1-F_B(\hat{b})} db, 0\right\} - c \\ &= [\mathcal{P}^s a(\theta, s)\gamma(\theta, s)]^+ - c \qquad (1) \end{aligned}$$

[3] Latter we will assume that $\mathcal{P}^s = \{\frac{N_B}{N_S}, 1\}$ and $\mathcal{P}^b = \mathcal{P}^s \frac{N_S}{N_B}$, where N_B and N_S are respectively the mass of buyers and sellers that participate. Potential participants are small enough to regard it as a parameter.

where $a(\theta,s) = \int_{\hat{b}}^{\overline{b}} \Delta_s(\Lambda_b \mid s)\frac{k(b|\theta)}{1-F_B(\hat{b})}db$ is the probability seller s is accepted, and $\gamma(\theta,s) = \int_{\hat{b}}^{\overline{b}} b\frac{\Delta_s(\Lambda_b|s)}{a(\theta,s)}\frac{k(b|\theta)}{1-F_B(\hat{b})}db$ is the expected buyer's type conditional on being accepted by him, and conditional on him deciding to participated. Finally, the unconditional expected surplus from entering will be $\Psi(s) = \mathbb{E}_\theta v(s,\theta)$.

A similar expression can be constructed for buyer b. He will encounter a seller s with probability $\mathcal{P}^b$, and he must decide whether to accept him or not. If he doesn't payoff is zero. If he does, the unconditional expected payoff will be $\mathcal{P}^b d(\omega \mid b)\alpha(\omega,b)$, where $k(s \mid \omega) = \frac{\tilde{n}(\omega|s)(1-F_S(\hat{s}))f_S(s)}{\int_{\hat{s}}^{s}\tilde{n}(\omega|s)f_S(s)ds}$ and $\tilde{n}(\omega \mid s) = n(s)\mathbb{1}_{\omega=\overline{\omega}} + (1-n(s))\mathbb{1}_{\omega=\underline{\omega}}$. Analogously define $\Delta_b(\Lambda_s \mid b) = Prob\{\theta \in \Lambda_s \mid b\}$, $d(\omega,b) = \int_{\hat{s}}^{\overline{s}} \Delta_b(\Lambda_s \mid b)\frac{k(s|\omega)}{1-F_S(\hat{s})}ds$, and $\alpha(\omega,b) = \int_{\hat{s}}^{\overline{s}} s\frac{\Delta_b(\Lambda_s|b)}{d(\omega,b)}\frac{k(s|\omega)}{1-F_S(\hat{s})}ds$.

The objective function for buyer b, conditional on ω, will be,

$$w(\omega,b) \quad = \quad \left[\mathcal{P}^b d(\omega,b)\alpha(\omega,b)\right]^+ - c \tag{2}$$

and the unconditional expected surplus from entering will be $\Phi(b) = \mathbb{E}_\omega w(b,\omega)$.

The optimal accept/reject strategy can be easily characterized by a threshold condition and is very simple in this setup. In the general case with a continuum of possible signals [7] showed that if the family of functions $\{m(\theta \mid b) : b \in [\underline{b},\overline{b}]\}$ and $\{n(\omega \mid s) : s \in [\underline{s},\overline{s}]\}$ satisfy a strict log-supermodularity assumption, then the optimal strategy is completely characterized by a threshold signal such that only those partners with a signal higher or equal will be accepted. In this setup the strategy greatly simplifies because we only consider two possible signals; e.g. for seller s, if the threshold $\hat{\theta}(s)$ is equal to the low signal then any buyer will be accepted, and if the threshold is greater to the low signal then only buyers with $\overline{\theta}$ will be accepted. The following lemma formalizes this paragraph.

Lemma 2.1. *Let the set $\Lambda_b = \emptyset$ and the set $\Lambda_s = \emptyset$ has no positive measure. (I) The optimal accept/reject strategy for a seller $s \in [\hat{s},\overline{s}]$, is $\Lambda_s = \{\underline{\theta},\overline{\theta}\}$ if $\hat{\theta}(s) = \underline{\theta}$, and $\Lambda_s = \{\overline{\theta}\}$ otherwise, for $\underline{\theta} \leq \hat{\theta}(s) \leq \overline{\theta}$. (II) The optimal accept/reject strategy for a buyer $b \in [\hat{b},\overline{b}]$, is $\Lambda_b = \{\underline{\omega},\overline{\omega}\}$ if $\hat{\omega}(b) = \underline{\omega}$, and $\Lambda_b = \{\overline{\omega}\}$ otherwise, for $\underline{\omega} < \hat{\omega}(b) \leq \overline{\omega}$.*

Finally, using the previous lemma, the expected surplus seller s and buyer b can be rewritten as

$$\Psi(s) \quad = \quad \frac{1}{1-F_B(\hat{b})}\left\{\sum_{\theta\in\Lambda_s}\mathcal{P}^s a(\theta,s)\gamma(\theta,s)\tilde{m}(\theta) - c\sum_{\theta\in\Lambda_s}\tilde{m}(\theta)\right\} \tag{3}$$

$$\Phi(b) \quad = \quad \frac{1}{1-F_S(\hat{s})}\left\{\sum_{\omega\in\Lambda_b}\mathcal{P}^b d(\omega,b)\alpha(\omega,b)\tilde{n}(\omega) - c\sum_{\omega\in\Lambda_b}\tilde{n}(\omega)\right\} \tag{4}$$

where $\tilde{m}(\theta) = \int_{\hat{b}}^{b}\tilde{m}(\theta \mid b)f_B(b)db$ and $\tilde{n}(\omega) = \int_{\hat{s}}^{s}\tilde{n}(\omega \mid s)f_S(s)ds$.

2.2.2 Existence of (Monotone Pure Strategy) Equilibrium

As types are not perfectly observed we will follow [7] strategy to prove existence. The author shows that we can reinterpret this matching model with two heterogeneous populations as a two-player simultaneaous game with incomplete information and a continuum of types, and where each player chooses a type-contingent strategy. In this case, as the type-contingent strategy is fully characterized by a threshold signal, e.g. $\hat{\theta}(s), \hat{\omega}(b)$, then each player will need to choose a scalar.

Formally, we will have two players, i.e. Player 1 and Player 2. The first player's type $s \in [\underline{s}, \overline{s}]$ is distributed according to the cdf $F_S(s)$ and pdf $f_S(s) > 0$ over all the support, and his action space $\{\underline{\theta}, \overline{\theta}\}$. Similarly, Player 2's type $b \in [\underline{b}, \overline{b}]$ is distributed according to the cdf $F_B(b)$ and pdf $f_B(b) > 0$ over all the support, and his action space $\{\underline{\omega}, \overline{\omega}\}$. Players' utility functions will be $U_1(\hat{\theta} \mid s, \hat{\omega}(\cdot)) \equiv \Psi(s)$ and $U_2(\hat{\omega} \mid b, \hat{\theta}(\cdot)) \equiv \Phi(b)$.

A pure strategy equilibrium of the bayesian game is defined by a pair of strategies $(\hat{\theta}^*(s), \hat{\omega}^*(b))$ such that, (i) $\hat{\theta}^*(s) = argmax_{\hat{\theta} \in \{\underline{\theta}, \overline{\theta}\}} U_1(\hat{\theta} \mid s, \hat{\omega}(\cdot))$ given $\hat{\omega}^*(b)$, and (ii) $\hat{\omega}^*(b) = argmax_{\hat{\omega} \in \{\underline{\omega}, \overline{\omega}\}} U_2(\hat{\omega} \mid b, \hat{\theta}(\cdot))$ given $\hat{\theta}^*(s)$. This definition its important as long as there is a relationship between a matching equilibrium and the existence of a pure strategy equilibrium of the two-players bayesian game. We can directly use Proposition 2 from [7] to argue that $(\hat{\theta}^*(s), \hat{\omega}^*(b))$ is a matching equilibrium iff there exist a pure strategy equilibrium of the two-players bayesian game.

To show existence of a monotone pure strategy equilibrium for this two-players bayesian game, following [2], we need to show that player's objective functions satisfy the single crossing of incremental returns (SCP-IR) condition in own actions and type. If this is the case, we know that buyers' and sellers' best response strategy is increasing whenever his partner use increasing strategies. The following proposition confirms that we have such situation.

Proposition 2.1. *There exists a matching equilibrium in strategies that are increasing in types.*

2.2.3 Characterization of the Equilibria

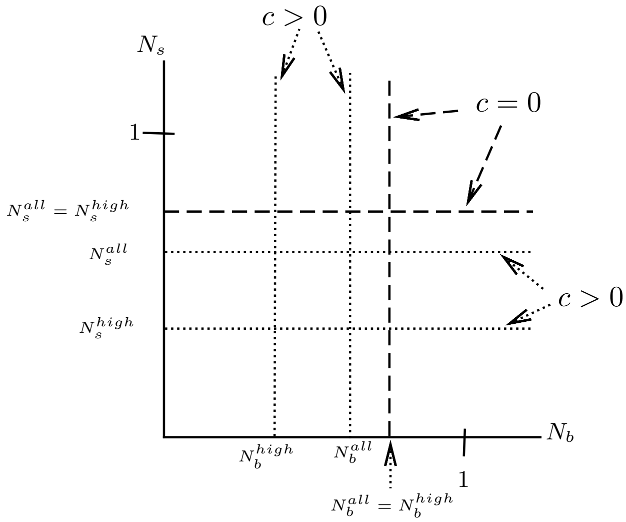
The main difference with the matching literature is that search frictions (i.e. search cost, discount rate, informational asymmetries), that determine which equilibrium will emerge (e.g. conditions to obtain positive or negative assortative matching), are no longer assumed exogenous and can be explained by the actions of a central planner that in this case is the profit maximizing platform. For this simple environment the mass of buyers and sellers that decide to participate, which in turn depends on the access fee established by the platform, will determine the equilibrium observed ex-post.

For this environment there are three possible equilibria: either all buyers and sellers accept any partner disregarding their signal, or everybody only accept partners with the high signal, or some buyers (sellers) accept any partners and the rest of them only accept partners with a high signal[4]. The following lemma shows that the mass of buyers and sellers that participate completely determine which equilibrium will emerge.

Lemma 2.2. *For a fixed mass of buyers (N_B) and sellers (N_S) that decide to participate, the symmetric matching equilibria will be: (i) All buyers and sellers accept every partner they receive if $N_i < (>) N_i^{all}$ for $i \in \{b, s\}$, (ii) All buyers and sellers accept only partners with the high signal if $N_i > (<) N_i^{high}$ for $i \in \{b, s\}$, (iii) Some buyers and sellers will accept only partners with the high signal and the rest will accept any partner they receive if $N_i^{both} \in [N_i^{all}, N_i^{high}]$ for $i \in \{b, s\}$.*

Before continuing we want to build more on the interpretation of "c". The literature of matching that analyzed the impact of search frictions in the matching assortativeness had considered discount factors and search cost. In our model, parameter "c" is not a search cost like in [3], [4] or [7], it is must be understood as the cost of searching conditional on being selective or not. To wit, the seach cost if a seller (or a buyer) accepts any partner will be equal to c, but will equal to $cm(b)$ if he decides only to accept partners with the high signal.

[4]The possibility everybody rejects their partner is not an option because at the stage when they decide to participate or not they should have concluded that the expected surplus once inside was non-negative.



Example 1. *Consider the case where $s, b \sim U_{[-1,1]}$, $m(b) = F_B(b)$ and $n(s) = F_S(s)$; see figure bellow. We obtained that if $c = 0$ the characterization of the matching equilibria is very simple, in particular the two thresholds N_i^{all}, N_i^{high} for $i \in \{b, s\}$ are equal. Indeed, if the search cost is equal to zero to all buyers and sellers, there must be only one threshold level of participation (i.e. $N_i^{all} = N_i^{high}$ for $i \in \{b, s\}$) such that any participation bellow it will induce buyers and sellers to accept any partner, and any participation above it will induce buyers and sellers to only accept partners with the high signal.*

In case $c > 0$ we obtain a situation where the thresholds are different (i.e. $N_i^{all} \neq N_i^{high}$ for $i \in \{b, s\}$). The reason is that the search cost affects more those buyers and sellers with a low type compared to those with a high type. What's interesting here is that the platform can pick and equilibrium where rather than observing "all" buyers and sellers accept every partner, or only accept partners with the high signal, we will observe that "some" buyers and sellers will only accept partners with the high signal, and the rest will accept any partner they encounter.

For this example, and after some algebra, we obtain that (i) all buyers and sellers accept their partner irrespective of their signal if $N_i < N_i^{all}$ for $i \in \{b, s\}$, (ii) all buyers and sellers accept only partners with a high signal if $N_i > N_i^{high}$ for $i \in \{b, s\}$, and (iii) some buyers and sellers will only accept high signal partners and the rest will accept any partner if $N_i^{both} \in [N_i^{all}, N_i^{high}]$ for $i \in \{b, s\}$.

Most of these results are robust to other unimodal distributions of types. For example, assuming that $b, s \sim Logistic(\alpha, \beta)$ all results hold if the mean of the distribution is not close to the right of the support, and if the the distribution has enough dispersion around the mean. Mathematical files are available upon request.

3 Pricing

We expect that the platform is big enough to modify the probability that any seller (buyer) meets one buyer, e.g. $\mathcal{P}^s$, (seller, e.g. $\mathcal{P}^b$) once he decides to participate. This assumption is interesting because the platform knows that buyers and sellers care about the mass of network users that decide to participate, e.g., a seller will be tempted to participate the higher the mass of potential partners and the lower the mass of potential sellers willing to participate.

Until this point of the research we deliberately prevent this from happening for two reasons. First, in the only attempt we know to deal with similar issues, [8] allows that the pricing scheme affects the average type of partner he can encounter inside the environment, but assumes that the probability of forming a match is unaffected. In our model we are considering both effects, then we prefer to keep it simple and do not include a third effect. And second, by including this additional effect the objective function is now not differentiable. This could be easily solved by replacing this it with the probability of meeting *at least* one partner, but again we consider this is not the main effect and certainly will complicate the analysis.

The probability a seller (buyer) meets one buyer (seller) will be,

$$\begin{aligned}\mathcal{P}^s &= \min\{1/\zeta, 1\}\\ \mathcal{P}^b &= \mathcal{P}^s\zeta\end{aligned}$$

where N_S is the mass of sellers that participate, and N_B is the mass of buyers that participate, and $\zeta = \frac{N_S}{N_B}$. Naturally, for the symmetric pure strategy equilibrium we will have that $\mathcal{P}^s = \mathcal{P}^b = 1$.

3.1 Profit Maximizing Pricing Rule

Platform demands are determined by the mass of buyers and sellers which type exceeds a threshold value, e.g. $\hat{b}$ and $\hat{s}$ respectively, these thresholds are coming from the buyer and the seller with net expected payoff equal to zero. Formally, the mass of buyers and sellers that will participate is $N_S = Prob\{s \geq \hat{s}\}$ and $N_B = Prob\{b \geq \hat{b}\}$, where $\hat{s}$ satisfy $P^s = \mathbb{E}_\theta v(\theta, \hat{s})$, and $\hat{b}$ satisfy $P^b = \mathbb{E}_\omega w(\omega, \hat{b})$. Using that $\hat{s} = F_S^{-1}(1 - N_S)$ and $\hat{b} = F_B^{-1}(1 - N_B)$, the following lemma shows both access fee depend on the number of buyers and sellers willing to participate.

Lemma 3.1. *Both access fees are uniquely determined and are a function on the mass of buyers and sellers willing to participate. In particular,*

$$\begin{aligned}P^s(N_S, N_B) &\equiv \frac{1}{N_B}\int_{F_B^{-1}(1-N_B)}^{\bar{b}} \left[\mathcal{P}^s bn(F_S^{-1}(1-N_S)) - c\right] m(b)f_B(b)db\\ P^b(N_S, N_B) &\equiv \frac{1}{N_S}\int_{F_S^{-1}(1-N_S)}^{\bar{s}} \left[\mathcal{P}^b sm(F_B^{-1}(1-N_B)) - c\right] n(s)f_S(s)ds\end{aligned}$$

For the platform is equivalent to choose the entry fees (P^s, P^b) or the mass of buyer and sellers that enter the environment (N_S, N_B) to search a matching partner. The optimization program for the platform will be

$$\begin{aligned}\max_{N_S, N_B} \quad & N_B P^b(N_B, N_S) + N_S P^s(N_B, N_S) - \phi N_B N_S\\ s.t. \quad & N_S, N_B \geq 0\end{aligned}$$

It is easy to show this program has a solution. Indeed, the support of (N_S, N_B) is bounded from below by $(0, 0)$, and from above by the total number of buyers and sellers that can potentially participate; also its clear the profit function is continuous in both arguments. Then, by Weierstrass theorem there is a solution for the program.

The contribution to the two-sided market literature is to understand the platform's optimal pricing rule in a searching environment like this. Most of the literature so far emphasizes on the differences between the Lerner conditions coming from a two-sided market vis-a-vis the one from a one-sided market, and do not study how the optimal pricing rule changes with different assumptions on the way trade surplus is generated[5]; see [21] for a comprehensive analysis. But indeed this is a serious drawback because we believe that agents (e.g. buyers and sellers) that participate in a two-sided market industry are just searching for a trading partner. The following proposition represents a first step in filling this gap.

Proposition 3.1. *(i) The optimal pricing rule, associated to an interior solution*[6]*, is described by*

$$\frac{P^b - \left[\frac{N_S}{N_B}(P_s + \Xi_B) + \phi N_S\right]}{P_b} = \frac{1}{\xi_{B,P^b}}$$

$$\frac{P^s - \left[\frac{N_B}{N_S}(P_b + \Xi_S) + \phi N_B\right]}{P_s} = \frac{1}{\xi_{S,P^s}}$$

where ξ_{i,P^i} is the own price elasticity of demand of side i for $i \in \{B, S\}$, and Ξ_B (Ξ_S) is the change in the trade surplus of the worst seller (buyer) that participates $\hat{s}$ ($\hat{b}$)) due to an increase of the participation of buyers (sellers). (ii) The own price elasticity of demand is inversely related to the change in the equilibrium probability that the worst type in one side (e.g. $\hat{s}$ or $\hat{b}$) forms a match, due to an increase of participants in the same side of the market, e.g. $\xi_{B,P^b} \propto \left(\frac{\partial m(F_B^{-1}(1-N_B))}{\partial N_B}\right)^{-1}$ and $\xi_{S,P^s} \propto \left(\frac{\partial n(F_S^{-1}(1-N_S))}{\partial N_S}\right)^{-1}$.

This last proposition contributes in two ways to the literature of two-sided markets. On one hand, shows that the two-sided marginal revenue *on the other side of the market* can be decomposed in two terms; take for example the FOC with respect to N_B, there such revenue is $\frac{\partial P^s(N_S,N_B)}{\partial N_B} \equiv \frac{N_S}{N_B}[P^s + \Xi_B]$. We obtain that an increase in N_B will marginally increase the expected trade surplus for the worst seller $\hat{s}$ in Ξ_B, but will marginally decrease his expected trade surplus in P^s because now more sellers are willing to participate due to the network effects. This implies that the two-sided marginal revenue on the other side of the market in a search environment can be positive of negative.

On the other hand, in a search environment the own price elasticity of demand will be inversely related to the sensitivity of the equilibrium probability the worst type forms a match. This implies that the mark-up the platform has on one side is proportional to the sensitivity of the equilibrium probability the worst type, on that side of the market, forms a match. In other words, the mark-up on one side will be higher the easiest it is to the worst type to form a match after more people, on that side, decides to participate.

3.2 Surplus Maximizing Pricing Rule

Platforms we observe in reality not always maximize profits, indeed there are many that care about the trading surplus. For example, governments usually "facilitate" simultaneously the seeking activities of unemployed workers and of employers that post vacancies by creating job agencies that reduce search costs, in addition, these agencies have incentives to synthesize the information they recollect in order to reduce evenmore the search costs. These kind of platforms are interesting at least for two reasons. First, because they serve as a benchmark for welfare comparisons. And second, because they are not just theoretically interesting, many non-profit agencies or government agencies can be understood as surplus maximizing platforms.

[5] In particular, many authors in this literature assume that trade surplus of agents in one side of the market depends linearly on the mass of agents that participate on the other side of the market; see [19].

[6] We will assume the second order conditions are satisfied, see [21].

Two construct the objective function we must determine the mass of buyers and sellers willing to participate, and the total social value of the platform. The mass of buyers that participate is equal to the mass of them whose type is above a threshold value $\hat{b}$, i.e. $N_B = Prob\{b \geq \hat{b}\}$; and the mass of sellers is determined analogously, i.e. $N_S = Prob\{s \geq \hat{s}\}$. On the other hand, the total social value of the platform is equal to the expected trading surplus from those buyers and sellers willing to participate minus the cost of providing the service, i.e. $\int_{\hat{s}}^{\overline{s}} \frac{\Psi(s)}{1-F_S(\hat{s})} f_S(s)ds + \int_{\hat{b}}^{\overline{b}} \frac{\Phi(b)}{1-F_B(\hat{b})} f_B(b)db - \phi N_B N_S$.

As with the profit maximizing platform, we can either find the optimal mass of buyers and sellers willing to participate, e.g. (N_B, N_S), or find the optimal entry fees, e.g. (P^s, P^b), that maximize the social value of the platform. In this case, the entry fee for the buyers is equal to the marginal change of the expected trading surplus of buyers, this guarantee buyer $\hat{b}$ will obtain zero surplus from participating; the entry fee for sellers is determined analogously.

The optimization program for the profit maximizing platform, which is fully derived in next proposition's proof, will be

$$\begin{array}{ll} \max\limits_{N_S, N_B} & \dfrac{1}{N_S N_B} \displaystyle\int_{F_S^{-1}(1-N_S)}^{\overline{s}} \int_{F_B^{-1}(1-N_B)}^{\overline{b}} [(\mathcal{P}^s b + \mathcal{P}^b s) n(s) m(b) - c(m(b) + n(s))] f_B(b) f_S(s) db ds - \phi N_S N_B \\ st & N_S, N_B \geq 0 \end{array}$$

and its interpretation is straightforward. The first element is the net value of all matches between buyers b and sellers s, conditional on them willing to participate, and second is the cost of running the searching environment. The optimal pricing rule we will find echoes [21] because the optimal social price will include the marginal private cost and the marginal external benefit of including and additional participant. But our contribution in this particular subject relies on disentangling the marginal external benefit.

Proposition 3.2. *(i) The optimal pricing rule, associated to an interior solution, is described by the following equations*

$$P^b = \phi N_S + \frac{\partial}{\partial N_B} \left(\int_{\hat{s}}^{\overline{s}} \frac{\Psi(s)}{1 - F_S(\hat{s})} f_S(s) ds \right) \tag{5}$$

$$P^s = \phi N_B + \frac{\partial}{\partial N_S} \left(\int_{\hat{b}}^{\overline{b}} \frac{\Phi(b)}{1 - F_B(\hat{b})} f_B(b) db \right) \tag{6}$$

(ii) Both $\frac{\partial}{\partial N_B}\left(\int_{\hat{s}}^{\overline{s}} \frac{\Psi(s)}{1-F_S(\hat{s})} f_S(s)ds\right)$ *and* $\frac{\partial}{\partial N_S}\left(\int_{\hat{b}}^{\overline{b}} \frac{\Phi(b)}{1-F_B(\hat{b})} f_B(b)db\right)$ *can be split in two, a positive network effect due to an increase in the number of potential partners with whom to match, and a marginal cost associated to a reduction in the probability of being accepted.*

The optimal social price, as [21] remarked, follows the Pigouvian Standard, to wit, the entry fee must account for the private marginal private cost and the external (e.g. other's side of the platform) marginal benefit[7]. Vis-a-vis to what the literature on two-sided markets had already found, we now have a precise characterization of the marginal external benefit under the undeniable fact that who ever participates in the searching environment will receive a noisy signal from his matching partner. In particular, we are able to distinguish two different mechanisms through which the platform affects the environment, e.g. through the expected partner's type, and through the equilibrium probability of forming a match.

[7] See for example [19], [1], [6].

To understand better the external marginal benefit take equation (6) in the previous Proposition. In the appendix we show that,

$$\begin{aligned}\frac{\partial}{\partial N_B}\left(\int_{\hat{b}}^{\bar{b}}\frac{\Phi(b)}{1-F_B(\hat{b})}f_B(b)db\right) &= \frac{1}{N_S N_B^2}\int_{F_S^{-1}(1-N_S)}^{\bar{s}}\int_{F_B^{-1}(1-N_B)}^{\bar{b}}[\mathcal{P}^s b n(s)-c]m(b)f_B(b)dbf_S(s)ds \\ &+ \frac{1}{N_S N_B}\int_{F_S^{-1}(1-N_S)}^{\bar{s}}[\mathcal{P}^s F_B^{-1}(1-N_B)n(s)-c]m(F_B^{-1}(1-N_B))\times \\ &\quad f_B(F_B^{-1}(1-N_B))\frac{\partial F_B^{-1}(1-N_B)}{\partial N_B}f_S(s)ds\end{aligned}$$

Each of the elements at the right hand side represents a different channel through which the platform affects the environment. The first one holds fix the social value coming from the sellers and quantifies the impact on the equilibrium probability of forming a match. Intuitively, if the platform allows more buyers to participate, the probability a sellers ends-up with a "worst-type" buyer increases; recall we assume all sellers have a positive probability of forming a match. The second element at the right hand side holds fix the impact on this equilibrium probability and quantifies the impact on the social value coming from sellers. Intuitively, this elements shows the change in seller's social value if the number of buyers participating increases.

Finally, we are in shape to compare the optimal pricing rule from the profit maximizing and surplus maximizing platform. To avoid confusions, denote $P^{sm,J}$ and $P^{pm,J}$ respectively as the entry fee of the surplus maximizing platform and profit maximizing platform at side J, where J could denote buyers or sellers.

Lemma 3.2. *Denote μ^J, for $J \in \{B, S\}$, as the market power distortion at the side of the market J. We can decompose the difference between $P^{sm,J}$ and $P^{pm,J}$ as,*

$$P^{pm,B} - P^{sm,B} = \mu^B + \frac{\partial}{\partial N_B}\left(N_S\Psi(\hat{s}) - \int_{\hat{s}}^{\bar{s}}\frac{\Psi(s)}{1-F_S(\hat{s})}f_S(s)ds\right) \tag{7}$$

$$P^{pm,S} - P^{sm,S} = \mu^S + \frac{\partial}{\partial N_S}\left(N_B\Phi(\hat{b}) - \int_{\hat{b}}^{\bar{b}}\frac{\Phi(b)}{1-F_B(\hat{b})}f_B(b)db\right) \tag{8}$$

Equations (7) and (8) have a nice interpretations. Without loss of generality just consider the first equation. The first element that explains the gap between is the market power the platform has en the buyers side. As we showed in Proposition (3.1) this market power is determined by the change in the equilibrium probability the worst type buyer, e.g. $\hat{b}$, is accepted when the number of buyers that participate increases. Though the market power is a well known element in industrial organization, when we focus on platforms that offer search & match environments it is interesting to find that this object is in turn determined by the equilibrium probability (its change to be precise) the worst type forms a match. On the other hand, is difficult to find a parallel in the search & match literature because there authors usually assume participants are always willing to participate.

The second element in equation (7) is what [21] call *spence distortion*. Notice that $N_S\Phi(\hat{s})$ is the social value from the worst type seller times the mass of sellers participating, equivalently it is the revenue the profit maximizing platform obtains from those sellers willing to participate. In addition, $\int_{\hat{s}}^{\bar{s}}\frac{\Psi(s)}{1-F_S(\hat{s})}f_S(s)ds$ is the social value from the sellers willing to participate. Then, the *spence distortion* effect under this environment can be understood as the difference between the platform's marginal revenue on the sellers and the marginal social value also on the sellers' side.

To wrap-up equation (7) interpretation. While the market power on side B is explained from changes within the side of the buyers, i.e. captures the effect of more buyers on the equilibrium probability the worst type buyer forms a match, the second element is explained from changes on the other side of the market, i.e. captures the effect of more buyers on the difference between the "private and social" value of those sellers that choose to participate.

3.3 Matching Equilibrium with Unimodal Distribution of Types: A Numerical Approach

Until now we had studied the case where, for some given pdf's $f_B(b)$, $f_S(s)$ and some give conditional pmf's $\tilde{(m)}(\theta \mid b)$ and $\tilde{n}(\omega \mid s)$, the platform chooses the entry fees that maximize his profit function. Even with this simplified environment[8] the solution depends both on the distribution of types and on the conditional distribution of the signals. Thus, we require to make additional assumptions to get a flavor of the solution of the program.

We will assume for simplicity that the conditional pmf's of the signals are $m(b) = F_B(b)$ and $n(s) = F_(s)$. Then, we only need to pick the distribution of types. Additionally, we will assume that $\underline{b} = \underline{s} = -1$, $\overline{b} = \overline{s} = 1$, and that $F_B(b) = F_S(s)$. Then, we just need to focus on a symmetric matching equilibrium. To begin, we will assume that the distribution of types is a Uniform(-1,1), latter we will assume what happens with a Logistic(α, β), where $\alpha \in [-1, 1]$ and $\beta \geq 0$.

The distribution of types is a Uniform with support $[-1, 1]$, e.g. $s, b \sim U_{[-1,1]}$. Figures (1) and (2) show in the horizontal axis the mass of buyers and sellers willing to participate (i.e. $N_S = N_B = N \in [0, 1]$), the blue line (on top) is the equation that describes the restriction such that all participants choose to accept any partner they receive, in particular when the function crosses zero we will find N^{all}. We showed before that for this equilibrium to exist $N^{platform} \leq N^{all}$, where $N^{platform}$ is the mass of buyers and sellers that solve the platform's optimization program. The red line (the one below), is the equation that describes the restriction such that all participants choose to only accept partners with the high signal, and in particular when it crosses the horizontal axis we will find N^{high}. We already showed that for this equilibrium to exist $N^{platform} \geq N^{all}$. Finally, in Figure (3) we include an additional equation that represents the FCO coming from the platform'optimization program, the point where is crosses the horizontal axis determines $N^{platform}$.

These first set of figures show that the platform always picks the equilibrium where every buyer and seller decides to accept any partner they get. The first two figures show that if $c > 0$ is no longer the case that $N^{all} = N^{high}$, and moreover, they show that each of these threshold point moves to the left. The third figure shows that even if $c = 0$, the platform will pick a participation level bellow N^{high} implying that everyone will accept their current partner. Then, we conclude here that if the unimodal distribution of types has a constant pdf (like with the uniform distributions), the platform will decide let in "enough" high type agents such that buyers and sellers realize that the probability of meeting someone with a negative type is very small.

The distribution of types is logistic, e.g. $s, b \sim (\alpha, \beta)$. Figures (4) - (7) show several combinations of (α, β), and in each we plot one small figure similar to Figure (3) and another small figure with the pdf of the logistic with the particular parameter values.

We find two regularites worth to be mentioned. The first is that the results obtained with the uniform distribution holds except when at the same time the mean of the distribution is "positive enough" and the dispersion of the distribution is "small enough" (e.g. *beta* close to zero). Indeed, in Figures (4) - (6) we observe that the profit maximizing platform will choose a participation level (e.g. $N^{platform}$) such that every buyer and seller will accept any partner they receive.

[8] In a general environment the platform can also choose the joint distribution of θ and ω such that its marginal distributions are $\tilde{M}\theta$ and $\tilde{N}(\omega)$. This task is the natural next step of the research.

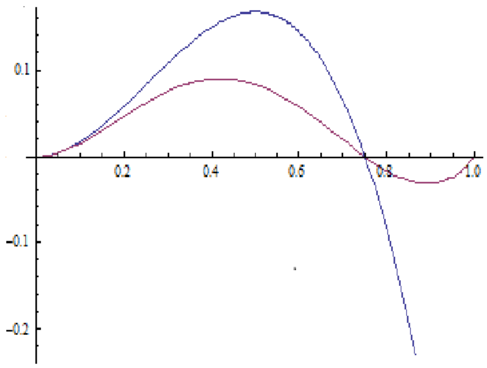


Figure 1: Restriction $U_{[-1,1]}$ and $c = 0$

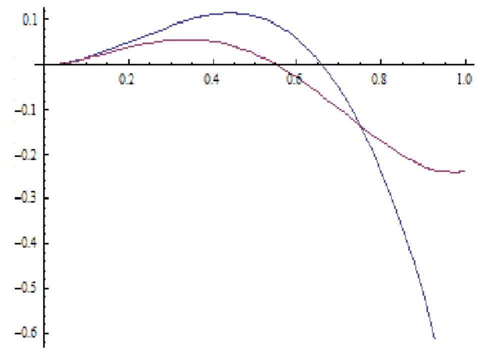


Figure 2: Restrictions $U_{[-1,1]}$ and $c > 0$

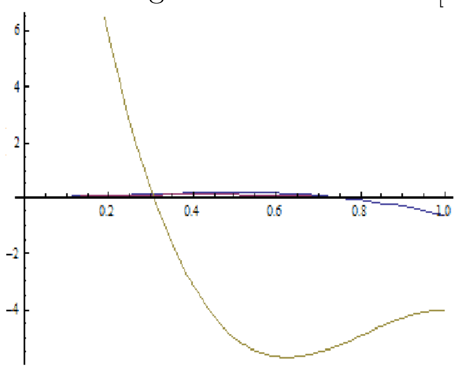


Figure 3: Equilibrium $U_{[-1,1]}$ and $c = 0$

The second case is associated to distributions where the mean is positive enough and where the dispersion around the mean is small. This situation represent those cases where there are few bad buyers and sellers in each population. Figure (7) shows that the profit maximizing platform will choose a participation level (e.g. $N^{platform}$) such that every buyer and seller will only accept partners with a high signal. [7] identified this issues and called it the "acceptance curse", and refers to the fact that being accepted reduces the potential partner's type. Then, if most of the population is of good type, then buyers and sellers will become selective to reduce the impact of the acceptance curse.

4 Optimal Noise

The initial formulation of the model allowed the platform both to pick the entry fee and the noise, for buyers and sellers, that maximize its objective function. Previously we found the optimal pricing rules for the profit maximizing and surplus maximizing platforms taken as given the noise, e.g. $m(b) = Prob\{\theta = \overline{\theta} \mid b\}$ and $n(s) = Prob\{\omega = \overline{\omega} \mid s\}$, and only imposing the assumption that buyers (sellers) with a higher type will be more likely to receive a high signal (e.g. $\overline{\theta}$ for buyers and $\overline{\omega}$ for sellers) than lower type buyers (seller), to wit, $m(b)' > 0$ and $n(s)' > 0$. In this part we will tackle this issue under the previous assumption.

The monotonicity assumption is a strong one but greatly simplifies the analysis. The strict monotonicity of functions $m(b)$ and $n(s)$ on one hand guarantees that the buyers and sellers willing to participate will use strategies (inside the environment) increasing in types, and provides a simple characterization of the matching equilibria. On the other hand, as [7] already showed, this assump-

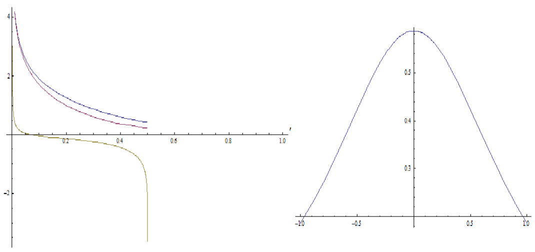


Figure 4: Logistic[0, 0.42] and $c = 0$

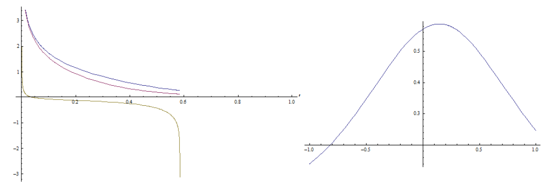


Figure 5: Logistic[0.14, 0.42] and $c = 0$

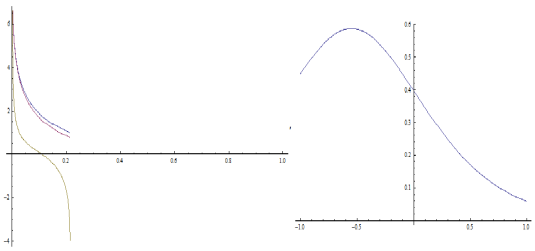


Figure 6: Logistic[−0.55, 0.42] and $c = 0$

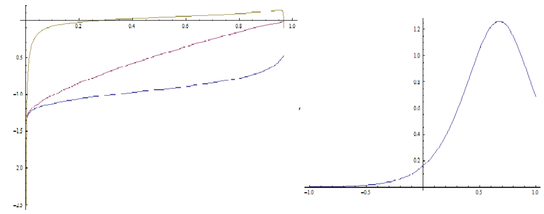


Figure 7: Logistic[0.67, 0.19] and $c = 0$

tion guarantees the matching equilibrium show a stochastic positive assortativeness. Then we are ruling out the possibility the platform decides to implement stochastic negative assortativeness, or even a combination of both. To wrap-up, the strict monotonicity assumption while greatly simplify the analysis, also restrict us to consider a stochastic positive assortativeness.

To begin the analysis fix the mass of buyers and sellers willing to participate. Informally speaking, these platforms do not have the technology to determine the mass of people willing to participate through the access fee.

4.1 Profit Maximizing Noise

Lemma 4.1. *Assume that the functions $m(b)$ and $n(s)$ are strictly increasing, and the mass of buyers and sellers willing to participate (e.g. N_S and N_B) is given. Then, the optimal conditional probability mass functions that maximize profits will be,*

$$Prob\{\theta = \overline{\theta} \mid b\} = \frac{1}{f_B(b)} \quad if \quad b > \tilde{b}, \dot{f}_B(b) < 0$$

$$Prob\{\omega = \overline{\omega} \mid s\} = \frac{1}{f_S(s)} \quad if \quad s > \tilde{(}s), \dot{f}_S(s) < 0$$

where $\tilde{s} = \frac{c}{\mathcal{P}^b m(F_B^{-1}(1-N_B))}$ *and* $\tilde{b} = \frac{c}{\mathcal{P}^s n(F_S^{-1}(1-N_S))}$.

The previous lemma shows several features. The optimal noise in one side of the market is just the inverse of the unconditional density function of types, at that side of the market, for those types above a certain threshold. In addition, the threshold depends on the mass of participants on the other side of the market. The caveat is that this optimal path is only optimal for those segments of the support such that the density function has a strictly decreasing slope. Moreover, we cannot say something about the other segments of the support of types because the monotonicity assumption is strict.

Finally, given the relationship between $Prob\{\Theta = \overline{\theta} \mid B = b\}$ and the joint density function of (Θ, B), e.g. $g_{\Theta,B}(\theta, b)$, if we can determine the optimal path for the former, we also can determine an optimal path for the latter. Lemma (4.1) implies that the optimal path for the joint density of (Θ, B), at the point $\Theta = \overline{\theta}$, is one for those buyers above the threshold $\tilde{b}$, and zero for the rest. Similarly, the optimal path for the joint density of (Ω, S), at the point $\Omega = \overline{\omega}$, is one for those sellers above the threshold $\tilde{s}$, and zero for the rest.

4.2 Surplus Maximizing Noise

Lemma 4.2. *Assume that the functions $m(b)$ and $n(s)$ are strictly increasing, and the mass of buyers and sellers willing to participate (e.g. N_S and N_B) is given. Then, (i) the optimal conditional probability mass functions that maximize surplus will be,*

$$Prob\{\theta = \overline{\theta} \mid b\} = \frac{\mathcal{P}^s}{f_B(b)(\mathcal{P}^s b + \mathcal{P}^b s) Ln\left(\frac{\mathcal{P}^s \bar{b} + \mathcal{P}^b s}{\mathcal{P}^s \hat{b} + \mathcal{P}^b s}\right)}\left[1 - c\int_{\hat{b}}^{\bar{b}} \frac{f_B(z)}{\mathcal{P}^s z + \mathcal{P}^b s} dz\right] + \frac{c}{\mathcal{P}^s b + \mathcal{P}^b s}$$

$$Prob\{\omega = \overline{\omega} \mid s\} = \frac{\mathcal{P}^b}{f_S(s)(\mathcal{P}^s b + \mathcal{P}^b s) Ln\left(\frac{\mathcal{P}^s b + \mathcal{P}^b \bar{s}}{\mathcal{P}^s b + \mathcal{P}^b \hat{s}}\right)}\left[1 - c\int_{\hat{s}}^{\bar{s}} \frac{f_S(z)}{\mathcal{P}^s b + \mathcal{P}^b z} dz\right] + \frac{c}{\mathcal{P}^s b + \mathcal{P}^b s}$$

and (ii) the strict monotonicity assumption holds if $\dot{f}_B(b), \dot{f}_S(s) < 0$, or if $c > 0$ is "sufficiently high".

Two features should be remarked. The optimal path for the surplus maximizing platform is a smooth function on the buyer's and seller's type. This imply that the conditional probability a buyer (seller), willing to participate, of receiving a high signal is different for all buyers, and evenmore, for each buyer this probability depends on the seller (buyer) he encounters. In other words, the optimal conditional probability a participant receives a high signal is a function on the types of both participants, and not only on one of them.

The second feature is that the strict monotonicity assumption is not trivially satisfied. Lemma (4.2) shows that the unitary transaction cost (e.g. $c > 0$) and the slope of the pdf of types (e.g. $\dot{f}_B(b)$ and $\dot{f}_S(s)$) determine whether the optimal path satisfies the assumption or not. This issue should be analyzed in greater detail. Finally, it is trivial to show that without a unitary transaction cost (e.g. $c = 0$), the optimal path is only determined for those regions in the support of types where the slope of the pdf is negative.

Comparing lemmas (4.1) and (4.2) we observe that while the profit maximizing platform treats all buyers (sellers) in the same fashion, the surplus maximizing platform does not. Focusing on the case of sellers, the profit maximizing platform will assign $n(s) = \frac{1}{f_S(s)}$ if $s > \tilde{s}$ and $\dot{f}_S(s) < 0$. The role buyers play here is through the threshold $\tilde{s}$ because it depends on the mass of buyers willing to participate. Then, all sellers above the threshold are equally treated. On the other hand, this is not true for the surplus maximizing platform because the optimal conditional probability seller with type s receives a high signal (e.g. $\overline{\omega}$) is also a function of the buyer's type he encounters. Then, the surplus maximizing platform will treat each seller differently depending on the buyer he encounters inside the searching environment.

5 Conclusion

This papers analyse a profit maximizing platform that offers an environment to two populations of vertically differentiated buyers and sellers that are willing to engage in trading activities that can only occur within the environment. We assume that those buyers and sellers that participate, although they privately know their type, only observe a noisy signal about their partner's type; additionally we assume no one observe's the noisy signal their partner is receiving from them. Finally we make two assumption about the platform, on one hand we assume they can only charge a (possibly different) access fee to buyers and sellers, and on the other hand, that the platform cannot directly run the match.

The platform, by setting the access fees, can indirectly determine the matching equilibrium through two channels: through the equilibrium probability of forming a match, and through the expected partner's type as in [8]. To simplify the exposition we propose an static model, and to avoid any trivial solution we assume the support of types can take negative values.

Our findings are relevant to two strands of literature. In the two-sided market literature we show that with a search environment as ours the optimal pricing rule of the profit maximizing platform changes in two aspects. First, the own price elesticity of demand is determined by the sensitivity of the equilibrium probability of forming a match. In particular, we show that the mark-up in any side of the market is proportional to the change in the equilibrium probability of forming a match (for the worst type on the same side) due to an increase of the participants on the same side of the market. Second, the two-sided marginal revenue on the other side of the market (see [21]) is not a strictly positive value. In particular, if network effects are strong enough its likely its value becomes negative.

In the literature on matching, to our knowledge, we are the first one to analyze the role of a profit maximizing central planner in the determination of the matching equilibrium. Borrowing the insights from [7] we show that in our environment there are three possible matching equilibria. Using a numerical approach we study the symmetric matching equilibrium case with unimodal distribution of types. We find that in general the profit maximizing platform chooses a participation level such that every buyer and seller that participates will accept any partner they encounter irrespective of their signal. Finally, we obtain that if the populations of buyers and sellers is mainly composed by "good" types, then the profit maximizing platform will choose a level of participation such that every buyer and sellers that participates will only accept high signal partners.

Appendix

Proof of Lemma 2.1

Proof. For this we must show that $\mathcal{P}^s a(\theta,s)\gamma(\theta,s) = \mathcal{P}^s \int_{\hat{b}}^{\overline{b}} b\Delta_s(\Lambda_b \mid s)\frac{k(\theta|b)}{1-F_B(\hat{b})}db$ is continuous and strictly increasing in θ. As in [7], the continuity is obtained by applying the Lebesgue Dominated Convergence Theorem (LDCT) as all functions where assumed to be measurable to the appropriate Borel σ-algebra.

On the other hand, the strictly increasing condition is a consequence of assuming $n(s)$ and $m(b)$ are strictly increasing functions. More formally, we must show that the conditional density of b on θ and on being accepted by his current partner, e.g. $h(b \mid \theta, s) = \Delta_s(\Lambda_b \mid s)\frac{k(b|\theta)}{1-F_B(\hat{b})}$, satisfy the MLRP. Indeed this is guaranteed from $m'(b) > 0$, i.e. for $\overline{\theta} > \underline{\theta}$ and $b' > b$

$$\begin{aligned} \frac{h(b' \mid \overline{\theta}, s)}{h(b \mid \overline{\theta}, s)} > \frac{h(b' \mid \underline{\theta}, s)}{h(b \mid \underline{\theta}, s)} \quad &\Leftrightarrow \quad \frac{k(b' \mid \overline{\theta})}{k(b \mid \overline{\theta})} > \frac{k(b' \mid \underline{\theta})}{k(b \mid \underline{\theta})} \\ &\Leftrightarrow \quad \frac{m(b')}{m(b)} > \frac{1-m(b')}{1-m(b)} \\ &\Leftrightarrow \quad m(b') > m(b) \end{aligned}$$

Then, according to Milgrom (1981) proposition 4, we conclude $\mathcal{P}^s a(\theta,s)\gamma(\theta,s)$ is strictly increasing in θ.

Finally, focusing in the interesting case where $\mathcal{P}^s a(\overline{\theta},s)\gamma(\overline{\theta},s) > 0$ and $\mathcal{P}^s a(\underline{\theta},s)\gamma(\underline{\theta},s) < 0$ there must be a $\underline{\theta} \leq \hat{\theta}(s) < \overline{\theta}$ such that $\mathcal{P}^s a(\hat{\theta}(s),s)\gamma(\hat{\theta}(s),s) = 0$, such that if $\hat{\theta}(s) = \underline{\theta}$ then the seller s will accept any buyer, and will only accept partners with the high signal otherwise.

A similar argument can be elaborated for each buyer. That is, there must be a $\underline{\omega}(b) \leq \hat{\omega}(b) < \overline{\omega}(b)$ such that $\mathcal{P}^b d(\hat{\omega}(b),b)\alpha(\hat{\omega}(b),b) = c$, such that if $\hat{\omega}(b) = \underline{\omega}$ then the buyer b will accept any seller, and will only accept partners with the high signal otherwise. □

Proof of Proposition 2.1

Proof. To show there exists a pure strategy equilibrium in increasing strategies we must show some regularity conditions hold (see [2]), and the utility function of all player should satisfy the SCP-IR condition in own type and actions. The regularity conditions are satisfied because all types' densities are atomless, bounded, and the actions sets are closed and bounded. Moreover, applying the Lebesgue Dominated Convergence Theorem its true that if $\hat{\theta}_n(s) \to \hat{\theta}(s)$ and $\hat{\omega}_n(b) \to \hat{\omega}(b)$, then $U_1(\hat{\theta}_n(s) \mid s, \hat{\omega}_n(\cdot)) \to U_1(\hat{\theta}(s) \mid s, \hat{\omega}(\cdot))$ and $U_2(\hat{\omega}_n(b) \mid b, \hat{\theta}_n(\cdot)) \to U_2(\hat{\omega}(b) \mid b, \hat{\theta}(\cdot))$. These conditions guarantee expected payoffs are continuous and well defined objects when their partners use increasing strategies.

To show SCP-IR condition holds on Player 1's objective function we must show that $\forall \overline{\theta} > \underline{\theta}$ and $\forall s_H > s_L$, if $U_1(\overline{\theta}(s_L) \mid s_L, \hat{\omega}(\cdot)) \geq U_1(\underline{\theta}(s_L) \mid s_L, \hat{\omega}(\cdot))$ holds, then $U_1(\overline{\theta}(s_H) \mid s_H, \hat{\omega}(\cdot)) \geq U_1(\underline{\theta}(s_H) \mid s_H, \hat{\omega}(\cdot))$ also holds.

Then,

$$\begin{aligned} U_1(\overline{\theta}(s_L) \mid s_L, \hat{\omega}(\cdot)) &\geq U_1(\underline{\theta}(s_L) \mid s_L, \hat{\omega}(\cdot)) \\ -c\tilde{m}(\overline{\theta}) + \mathcal{P}^s a(\overline{\theta}, s_L)\gamma(\overline{\theta}, s_L)\tilde{m}(\overline{\theta}) &\geq -c(\tilde{m}(\overline{\theta}) + (1-\tilde{m}(\overline{\theta}))) + \mathcal{P}^s a(\overline{\theta}, s_L)\gamma(\overline{\theta}, s_L)\tilde{m}(\overline{\theta}) \\ &+ \mathcal{P}^s a(\underline{\theta}, s_L)\gamma(\underline{\theta}, s_L)(1-\tilde{m}(\overline{\theta})) \\ c &\geq \mathcal{P}^s a(\underline{\theta}, s_L)\gamma(\underline{\theta}, s_L) \end{aligned}$$

and,

$$\begin{aligned} U_1(\overline{\theta}(s_H) \mid s_H, \hat{\omega}(\cdot)) &\geq U_1(\underline{\theta}_L(s_H) \mid s_H, \hat{\omega}(\cdot)) \\ -c\tilde{m}(\overline{\theta}) + \mathcal{P}^s a(\overline{\theta}, s_H)\gamma(\overline{\theta}, s_H)\tilde{m}(\overline{\theta}) &\geq -c(\tilde{m}(\overline{\theta}) + (1-\tilde{m}(\overline{\theta}))) + \mathcal{P}^s a(\overline{\theta}, s_H)\gamma(\overline{\theta}, s_H)\tilde{m}(\overline{\theta}) \\ &+ \mathcal{P}^s a(\underline{\theta}, s_H)\gamma(\underline{\theta}, s_H)(1-\tilde{m}(\overline{\theta})) \\ c &\geq \mathcal{P}^s a(\underline{\theta}, s_H)\gamma(\underline{\theta}, s_H) \end{aligned}$$

All boils down to show that $\mathcal{P}^s a(\underline{\theta}, s)\gamma(\underline{\theta}, s)$ is decreasing in s. After simple manipulations the last expression can be written as $\int_{\hat{b}}^{\bar{b}} b\Delta_s(\Lambda_b \mid s)\mathcal{P}^s \frac{k(b|\underline{\theta})}{1-F_B(\hat{b})}db$. Finally, taking the derivative with respect to s we obtain

$$\frac{\mathcal{P}^s}{1-F_B(\hat{b})} \int_{\hat{b}}^{\bar{b}} b \frac{\partial \Delta_s(\Lambda_b \mid s)}{\partial s} k(b \mid \underline{\theta}) db$$

and thus we need that $\frac{\partial \Delta_s(\Lambda_b|s)}{\partial s} < 0$. Notice that $\omega \in \Lambda_b$ iff $\omega \geq \hat{\omega}(b)$, thus $\Delta_s(\Lambda_b \mid s)$ is equivalent to $Prob\{\omega \geq \hat{\omega}(b) \mid s\} = 1 - \tilde{N}(\hat{\omega}(b) \mid s)$, then what we need is that $\frac{\partial \tilde{N}(\hat{\omega}(b)|s)}{\partial s} > 0$. This condition is satisfied because we assumed that $n(s)$ is strictly increasing in s. To wrap-up, we showed that Player 1's utility function satisfy the SCP-IR in own type and action if $n(s)' > 0$. Using similar arguments, if $m(b)' > 0$ we can guarantee Player 2's utility function satisfy the SCP-IR in own type and action.

We can conclude that there exists a monotone pure strategy equilibrium of the two-player bayesian game, then this implies there exists a matching equilibrium $(\hat{\theta}^*(s), \hat{\omega}^*(b))$ with strategies increasing in types. □

Proof of Lemma 2.2

Proof. We just need to compare the seller's (buyer's) expected utility from participating in the platform if they decide to only accept partners with high signal, with the expected utility if they decide to accept any partner they might encounter. We will concentrate on the case of seller s, the argument for any buyer holds analogously.

The expected payoff of a seller from accepting only the high signal is obtained by replacing $\hat{\theta} = \overline{\theta}$ into $U_1(\overline{\theta} \mid s, \hat{\omega}(\cdot))$, and the expected payoff from accepting any buyer is obtained by computing $\mathbb{E}_\theta[U_1(\theta \mid s, \hat{\omega}(\cdot))] = U_1(\overline{\theta} \mid s, \hat{\omega}(\cdot))\tilde{m}(\overline{\theta}) + U_1(\underline{\theta} \mid s, \hat{\omega}(\cdot))(1-\tilde{m}(\overline{\theta}))$.

Now we can establish the condition that determines seller s optimal decision,

$$\begin{aligned} U_1(\overline{\theta} \mid s, \hat{\omega}(\cdot)) &\gtreqless \mathbb{E}_\theta[U_1(\theta \mid s, \hat{\omega}(\cdot))] \\ \mathcal{P}^s a(\overline{\theta}, s)\gamma(\overline{\theta}, s)\tilde{m}(\overline{\theta}) - c\tilde{m}(\overline{\theta}) &\gtreqless \mathcal{P}^s a(\overline{\theta}, s)\gamma(\overline{\theta}, s)\tilde{m}(\overline{\theta}) + \mathcal{P}^s a(\underline{\theta}, s)\gamma(\underline{\theta}, s)(1-\tilde{m}(\overline{\theta}))^2 - c(1-\tilde{m}(\overline{\theta})\tilde{m}(\underline{\theta})) \\ c &\gtreqless \mathcal{P}^s a(\underline{\theta}, s)\gamma(\underline{\theta}, s) \end{aligned} \tag{9}$$

The symmetric matching equilibria in increasing strategies is characterized by a threshold (η_s, η_b), where $\eta_i \in [\hat{i}, \bar{i}]$ for $i \in \{b, s\}$, such that any buyer with a higher type $(b > \eta_b)$ will only accept the high signal $\overline{\omega}$, and any buyer with a lower type $(b < \eta_b)$ will accept both signals $\{\underline{\omega}, \overline{\omega}\}$. Similarly for the sellers, any seller whose type is high enough $(s > \eta_s)$ will only accept high signals $\overline{\theta}$, and any seller with a lower type $(s < \eta_s)$ will accept both signals $\{\underline{\theta}, \overline{\theta}\}$. Following [7],

$$\begin{aligned}
\gamma(\overline{\theta}, s) &= \frac{\int_{\hat{b}}^{\eta_b} bm(b)f_B(b)db + n(s)\int_{\eta_b}^{\overline{b}} bm(b)f_B(b)db}{\int_{\hat{b}}^{\eta_b} m(b)f_B(b)db + n(s)\int_{\eta_b}^{\overline{b}} m(b)f_B(b)db} \\
\gamma(\underline{\theta}, s) &= \frac{\int_{\hat{b}}^{\eta_b} b(1-m(b))f_B(b)db + n(s)\int_{\eta_b}^{\overline{b}} b(1-m(b))f_B(b)db}{\int_{\hat{b}}^{\eta_b} (1-m(b))f_B(b)db + n(s)\int_{\eta_b}^{\overline{b}} (1-m(b))f_B(b)db} \\
a(\overline{\theta}, s) &= \frac{\int_{\hat{b}}^{\eta_b} m(b)f_B(b)db + n(s)\int_{\eta_b}^{\overline{b}} m(b)f_B(b)db}{\int_{\hat{b}}^{\overline{b}} m(b)f_B(b)db} \\
a(\underline{\theta}, s) &= \frac{\int_{\hat{b}}^{\eta_b} (1-m(b))f_B(b)db + n(s)\int_{\eta_b}^{\overline{b}} (1-m(b))f_B(b)db}{\int_{\hat{b}}^{\overline{b}} (1-m(b))f_B(b)db}
\end{aligned}$$

analogous expressions can be obtained for $d(\omega, b)$ and $\alpha(\omega, b)$ for $\omega \in \{\underline{\omega}, \overline{\omega}\}$, and all depend on η_s.

Equilibrium (i). If every buyer and seller that participates accepts his partner, then $\eta_b = \overline{b}$ and $\eta_s = \overline{s}$. Using equation (9) and the fact that $\hat{b} = F_B^{-1}(1-N_B)$ and $\hat{s} = F_S^{-1}(1-N_S)$, we obtain for sellers,

$$\begin{aligned}
c &< \mathcal{P}^s a(\underline{\theta}, s)\gamma(\underline{\theta}, s) \\
c &< \mathcal{P}^s \frac{\int_{\hat{b}}^{\overline{b}} b(1-m(b))f_B(b)db}{\int_{\hat{b}}^{\overline{b}} (1-m(b))f_B(b)db} \equiv A_S(N_B)
\end{aligned}$$

and for buyers,

$$c < \mathcal{P}^b \frac{\int_{\hat{s}}^{\overline{s}} s(1-n(s))f_S(s)ds}{\int_{\hat{s}}^{\overline{s}} (1-n(s))f_S(s)ds} \equiv A_B(N_S)$$

Finally, define (N_S^{all}, N_B^{all}) as the mass of buyers and sellers that satisfy the above inequalitites. This equilibrium will exist if $N_i < N_i^{all}$ for $i \in \{b, s\}$.

Equilibrium (ii). If every buyer and seller that participates only accepts partners with the high type, then $\eta_b = \hat{b}$ and $\eta_s = \hat{s}$. Using equation (9) we obtain for sellers,

$$\begin{aligned}
c &> \mathcal{P}^s a(\underline{\theta}, s)\gamma(\underline{\theta}, s) \\
c &> \mathcal{P}^s n(\hat{s}) \frac{\int_{\hat{b}\overline{b}} b(1-m(b))f_B(b)db}{\int_{\hat{b}\overline{b}} (1-m(b))f_B(b)db} \\
c &\equiv B_B(N_B, N_S)
\end{aligned}$$

and for buyers,

$$c > \mathcal{P}^b m(\hat{b}) \frac{\int_{\hat{s}\overline{s}} s(1-n(s))f_S(s)ds}{\int_{\hat{s}\overline{s}} (1-n(s))f_S(s)ds} \equiv B_S(N_B, N_S)$$

Define (N_S^{high}, N_B^{high}) as the mass of buyers and sellers that satisfy the above inequalities. This equilibrium will exist if $N_i > N_i^{high}$ for $i \in \{b, s\}$.

Equilibrium (iii). This is the situation where some buyers (sellers) only accepts partners with the high signal, and the rest accepts any partner; then, $\eta_b \in (\hat{b}, \overline{b})$ and $\eta_s \in (\hat{s}, \overline{s})$. Using equation

(9) we obtain for sellers,

$$\begin{aligned} c &= \mathcal{P}^s a(\underline{\theta},\eta_s)\gamma(\underline{\theta},\eta_s) \\ c &= \mathcal{P}^s \frac{\int_{\hat{b}}^{\eta_b} b(1-m(b))f_B(b)db + n(\eta_s)\int_{\eta_b}^{\overline{b}} b(1-m(b))f_B(b)db}{\int_{\hat{b}}^{\overline{b}}(1-m(b))f_B(b)db} \end{aligned}$$

and for buyers,

$$c = \mathcal{P}^b \frac{\int_{\hat{s}}^{\eta_s} s(1-n(s))f_S(s)ds + m(\eta_b)\int_{\eta_s}^{\overline{s}} s(1-n(s))f_S(s)ds}{\int_{\hat{s}}^{\overline{s}}(1-n(s))f_S(s)ds}$$

Define (η_s,η_b) as the threshold values that solve the desired equalities. Moreover, this equilibrium (N_S^{both}, N_B^{both}) will exist if $N_i^{both} \in [N_i^{all}, N_i^{high}]$ for $i \in \{b, s\}$.

□

Proof of Lemma 3.1

Proof. We will focus on the case of a seller s. Using the fact that $\mathcal{P}^s = \mathbb{E}_\theta v(\theta,\hat{s})$ and $\hat{s} = F_S^{-1}(1 - N_S)$, we obtain that

$$\begin{aligned} P^s &= \mathbb{E}_\theta v(\theta, F_S^{-1}(1-N_S)) \\ &= \sum_{\theta\in\Lambda_s} \mathcal{P}^s \int_{F_B^{-1}(1-N_B)}^{\overline{b}} b\Delta_s(\Lambda_b \mid F_S^{-1}(1-N_S)) \frac{k(b\mid\theta)}{1-F_B(\hat{b})}\frac{\tilde{m}(\theta)}{1-F_B(\hat{b})}db - \frac{c}{1-F_B(\hat{b})}\sum_{\theta\in\Lambda_s}\tilde{m}(\theta) \\ &= \frac{1}{1-F_B(\hat{b})}\left\{\int_{F_B^{-1}(1-N_B)}^{\overline{b}} \mathcal{P}^s b\Delta_s(\Lambda_b \mid F_S^{-1}(1-N_S))\Delta_b(\Lambda_s\mid b)f_B(b)db - c\int_{\hat{b}}^{\overline{b}}\sum_{\theta\in\Lambda_s}\tilde{m}(\theta\mid b)f_B(b)db\right\} \\ &= \frac{1}{1-F_B(\hat{b})}\int_{F_B^{-1}(1-N_B)}^{\overline{b}} \left[\mathcal{P}^s b\Delta_s(\Lambda_b\mid F_S^{-1}(1-N_S)) - c\right]\Delta_b(\Lambda_s\mid b)f_B(b)db \\ &= \frac{1}{1-F_B(\hat{b})}\int_{F_B^{-1}(1-N_B)}^{\eta_b(N_S,N_B)} \left[\mathcal{P}^s b\Delta_s(\Lambda_b\mid F_S^{-1}(1-N_S)) - c\right]\Delta_b(\Lambda_s\mid b)f_B(b)db \\ &+ \frac{1}{1-F_B(\hat{b})}\int_{\eta_b(N_S,N_B)}^{\overline{b}} \left[\mathcal{P}^s b\Delta_s(\Lambda_b\mid F_S^{-1}(1-N_S)) - c\right]\Delta_b(\Lambda_s\mid b)f_B(b)db \end{aligned}$$

now you can use the fact that $\Lambda_b = \{\underline{\omega},\overline{\omega}\}$ iff $\hat{\omega}(b) = \underline{\omega}$, and $\Lambda_b = \{\overline{\omega}\}$ iff $\hat{\omega}(b) > \underline{\omega}$. Thus, you can replace $\Delta_s(\Lambda_b \mid s)$ with $(1-\tilde{N}(\hat{\omega}(b)\mid s))$. Using similar arguments, you can replace $\Delta_b(\Lambda_s\mid b)$ with $(1-\tilde{M}(\hat{\theta}(s)\mid b))$.

$$\begin{aligned} P^s &= \frac{1}{1-F_B(\hat{b})}\int_{F_B^{-1}(1-N_B)}^{\eta_b(N_S,N_B)} \left[\mathcal{P}^s b(1-\tilde{N}(\hat{\omega}(b)\mid F_S^{-1}(1-N_S))) - c\right](1-\tilde{M}(\hat{\theta}(\hat{s})\mid b))f_B(b)db \\ &+ \frac{1}{1-F_B(\hat{b})}\int_{\eta_b(N_S,N_B)}^{\overline{b}} \left[\mathcal{P}^s b(1-\tilde{N}(\hat{\omega}(b)\mid F_S^{-1}(1-N_S))) - c\right](1-\tilde{M}(\hat{\theta}(\hat{s})\mid b))f_B(b)db \end{aligned}$$

finally we can use the fact that $\hat{\theta}(\hat{s}) = \underline{\theta}$, and that $\hat{\omega}(b) = \underline{\omega}$ when $b \in [F_B^{-1}(1-N_B), \eta_b(N_S,N_B)]$ and that $\overline{\omega} > \hat{\omega}(b) > \underline{\omega}$ when $b \in [\eta_b(N_S,N_B), \overline{b}]$.

Then,

$$
\begin{aligned}
P^s &= \frac{1}{1-F_B(\hat{b})}\int_{F_B^{-1}(1-N_B)}^{\eta_b(N_S,N_B)} \left[\mathcal{P}^s b(1-\tilde{N}(\underline{\omega} \mid F_S^{-1}(1-N_S))) - c\right](1-\tilde{M}(\underline{\theta} \mid b))f_B(b)db \\
&+ \frac{1}{1-F_B(\hat{b})}\int_{\eta_b(N_S,N_B)}^{\bar{b}} \left[\mathcal{P}^s b(1-\tilde{N}(\underline{\omega} \mid F_S^{-1}(1-N_S))) - c\right](1-\tilde{M}(\underline{\theta} \mid b))f_B(b)db \\
&= \frac{1}{1-F_B(F_B^{-1}(1-N_B))}\int_{F_B^{-1}(1-N_B)}^{\bar{b}} \left[\mathcal{P}^s b(1-\tilde{N}(\underline{\omega} \mid F_S^{-1}(1-N_S))) - c\right](1-\tilde{M}(\underline{\theta} \mid b))f_B(b)db \\
&= \frac{1}{1-(1-N_B)}\int_{F_B^{-1}(1-N_B)}^{\bar{b}} \left[\mathcal{P}^s b(1-\tilde{n}(\underline{\omega} \mid F_S^{-1}(1-N_S))) - c\right](1-\tilde{m}(\underline{\theta} \mid b))f_B(b)db \\
&= \frac{1}{N_B}\int_{F_B^{-1}(1-N_B)}^{\bar{b}} \left[\mathcal{P}^s b\tilde{n}(\overline{\omega} \mid F_S^{-1}(1-N_S)) - c\right]\tilde{m}(\overline{\theta} \mid b)f_B(b)db \\
&= \frac{1}{N_B}\int_{F_B^{-1}(1-N_B)}^{\bar{b}} \left[\mathcal{P}^s bn(F_S^{-1}(1-N_S)) - c\right]m(b)f_B(b)db \\
&\equiv P^s(N_S,N_B)
\end{aligned}
$$

and analogously we can construct the fees for the buyers as,

$$
\begin{aligned}
P^b &= \frac{1}{N_S}\int_{F_S^{-1}(1-N_S)}^{\bar{s}} \left[\mathcal{P}^b sm(F_B^{-1}(1-N_B)) - c\right]n(s)f_S(s)ds \\
&\equiv P^b(N_S,N_B)
\end{aligned}
$$

□

Proof of Proposition 3.1

Proof. The FOC with respect to N_B will be,

$$
\begin{aligned}
0 &= P^b(N_S,N_B) + \frac{N_B\mathcal{P}^b}{N_S}\int_{F_S^{-1}(1-N_S)}^{\bar{s}} sn(s)f_S(s)ds\frac{\partial m(F_B^{-1}(1-N_B))}{\partial N_B} \\
&- \frac{N_S}{N_B^2}\int_{F_B^{-1}(1-N_B)}^{\bar{b}} [\mathcal{P}^s bn(F_S^{-1}(1-N_S)) - c]m(b)f_B(b)db \\
&- \frac{N_S}{N_B}m(F_B^{-1}(1-N_B))f_B(F_B^{-1}(1-N_B))\frac{\partial F_B^{-1}(1-N_B)}{\partial N_B}\left(\mathcal{P}^s n(F_S^{-1}(1-N_S))F_B^{-1}(1-N_B) - c\right) \\
&- \phi N_S
\end{aligned}
$$

noticing that

$$
\begin{aligned}
\frac{\partial P^b(N_S,N_B)}{\partial N_B} &= \frac{\mathcal{P}^b}{N_S}\int_{F_S^{-1}(1-N_S)}^{\bar{s}} sn(s)f_S(s)ds\frac{\partial m(F_B^{-1}(1-N_B))}{\partial N_B} \\
\Xi_B &= m(F_B^{-1}(1-N_B))f_B(F_B^{-1}(1-N_B))\frac{\partial F_B^{-1}(1-N_B)}{\partial N_B}(\mathcal{P}^s n(F_S^{-1}(1-N_S))F_B^{-1}(1-N_B) - c)
\end{aligned}
$$

the first order condition is reduced to,

$$\begin{aligned} P^b(N_S,N_B) - \frac{N_S}{N_B}\left(P^s(N_S,N_B)+\Xi_B\right) - \phi N_S &= -N_B\frac{\partial P^b(N_S,N_B)}{\partial N_B} \\ \frac{P^b(N_S,N_B) - \left[\frac{N_S}{N_B}(P^s(N_S,N_B)+\Xi_B)+\phi N_S\right]}{P^b(N_S,Nb)} &= -\frac{N_B}{P^b(N_S,N_B)}\frac{\partial P^b(N_S,N_B)}{\partial N_B} \end{aligned}$$

Then,

$$\frac{P^b - \left[\frac{N_S}{N_B}(P^s+\Xi_B)+\phi N_S\right]}{P^b} = \frac{1}{\xi_{B,P^b}}$$

and analogously the seller's optimal pricing rule is,

$$\frac{P^s - \left[\frac{N_B}{N_S}(P^b+\Xi_S)+\phi N_B\right]}{P^s} = \frac{1}{\xi_{S,P^s}}$$

where $\Xi_S = n(F_S^{-1}(1-N_S))f_S(F_S^{-1}(1-N_S))\frac{\partial F_S^{-1}(1-N_S)}{\partial N_S}(\mathcal{P}^b m(F_B^{-1}(1-N_B))F_S^{-1}(1-N_S)-c)$. □

Proof of Proposition 3.2

Proof. Lets start with the optimization program. The platform's social value is,

$$\begin{aligned} SV(N_S,N_B) &= \int_{\hat{s}}^{\bar{s}} \frac{1}{1-F_B(\hat{b})}\sum_{\theta\in\Lambda_s}[\mathcal{P}^s a(\theta,s)\gamma(\theta,s)-c]\tilde{m}(\theta)\frac{f_S(s)}{1-F_S(\hat{s})}ds \\ &+ \int_{\hat{b}}^{\bar{b}} \frac{1}{1-F_S(\hat{s})}\sum_{\omega\in\Lambda_b}[\mathcal{P}^b d(\omega,b)\alpha(\omega,b)-c]\tilde{n}(\omega)\frac{f_B(b)}{1-F_B(\hat{b})}db \\ &- \phi N_S N_B \\ SV(N_S,N_B) &= \int_{\hat{s}}^{\bar{s}}\int_{\hat{b}}^{\bar{b}} \frac{\mathcal{P}^s b}{(1-F_B(\hat{b}))(1-F_S(\hat{s}))}\Delta_s(\Lambda_b \mid s)\sum_{\theta\in\Lambda_s}\tilde{m}(\theta\mid b)f_B(b)f_S(s)dbds \\ &+ \int_{\hat{b}}^{\bar{b}}\int_{\hat{s}}^{\bar{s}} \frac{\mathcal{P}^b s}{(1-F_B(\hat{b}))(1-F_S(\hat{s}))}\Delta_b(\Lambda_s \mid b)\sum_{\omega\in\Lambda_b}\tilde{n}(\omega\mid s)f_S(s)f_B(b)dsdb \\ &- \frac{c}{(1-F_B(\hat{b}))(1-F_S(\hat{s}))}\left[\int_{\hat{s}}^{\bar{s}}\int_{\hat{b}}^{\bar{b}}\tilde{m}(\theta\mid b)f_B(b)dbf_S(s)ds + \int_{\hat{b}}^{\bar{b}}\int_{\hat{s}}^{\bar{s}}\tilde{n}(\omega\mid s)f_S(s)dsf_B(b)db\right] \\ &- \phi N_S N_B \end{aligned}$$

now we can replace $\hat{s} = F_S^{-1}(1-N_S)$ and $\hat{b} = F_B^{-1}(1-N_B)$, also can replace $\Delta_s(\Lambda_b \mid s)$ with $1-\tilde{N}(\hat{\omega}(b)\mid s)$ and $\Delta_b(\Lambda_s \mid b)$ with $1-\tilde{M}(\hat{\theta}(s)\mid b)$, and using Fubini's Theorem the new expression for the platform's social value is,

$$\begin{aligned} SV(N_S,N_B) &= \frac{1}{N_S N_B}\left[\int_{F_S^{-1}(1-N_S)}^{\bar{s}}\int_{F_B^{-1}(1-N_B)}^{\bar{b}}(\mathcal{P}^s b+\mathcal{P}^b s)(1-\tilde{N}(\hat{\omega}(b)\mid s))(1-\tilde{M}(\hat{\theta}(s)\mid b))\right] \\ &- \frac{c}{N_S N_B}\left[(1-\tilde{N}(\hat{\omega}(b)\mid s))+(1-\tilde{M}(\hat{\theta}(s)\mid b))\right] \\ &- \phi N_S N_B \end{aligned}$$

finally, using the fact $\hat{\theta}(s) = \underline{\theta}$ if $s \in [\hat{s}, \eta_s]$ and $\underline{\theta} < \hat{\theta}(s) < \overline{\theta}$ if $s \in [\eta_s, \overline{s}]$, and $\hat{\omega}(b) = \underline{\omega}$ if $b \in [\hat{b}, \eta_b]$ and $\underline{\omega} < \hat{\omega}(b) < \overline{\omega}$ if $b \in [\eta_b, \overline{b}]$, then the optimization program of the surplus maximizing platform will be,

$$\begin{array}{ll} \max\limits_{N_S, N_B} & \frac{1}{N_S N_B}\left[\int_{F_S^{-1}(1-N_S)}^{\overline{s}} \int_{F_B^{-1}(1-N_B)}^{\overline{b}} (\mathcal{P}^s b + \mathcal{P}^b s) n(s) m(b) - c(m(b) + n(s))\right] - \phi N_S N_B \\ st & N_S, N_B \geq 0 \end{array}$$

Now continue with the FOC, without loss of generality just consider the one wrt N_B.

$$\begin{aligned} \phi N_S &= -\frac{1}{N_S N_B^2} \int_{F_S^{-1}(1-N_S)}^{\overline{s}} \int_{F_B^{-1}(1-N_B)}^{\overline{b}} [\mathcal{P}^s b n(s) - c] m(b) f_B(b) db f_S(s) ds \\ &- \frac{1}{N_S N_B} \int_{F_S^{-1}(1-N_S)}^{\overline{s}} [\mathcal{P}^s F_B^{-1}(1-N_B) n(s) - c] m(F_B^{-1}(1-N_B)) f_B(F_B^{-1}(1-N_B)) \frac{\partial F_B^{-1}(1-N_B)}{\partial N_B} f_S(s) ds \\ &- \frac{1}{N_S N_B^2} \int_{F_B^{-1}(1-N_B)}^{\overline{b}} \int_{F_S^{-1}(1-N_S)}^{\overline{s}} [\mathcal{P}^b s m(b) - c] n(s) f_S(s) ds f_B(b) db \\ &- \frac{1}{N_S N_B} \int_{F_S^{-1}(1-N_S)}^{\overline{s}} [\mathcal{P}^b s m(F_B^{-1}(1-N_B)) - c] n(s) f_B(F_B^{-1}(1-N_B)) \frac{\partial F_B^{-1}(1-N_B)}{\partial N_B} f_S(s) ds \end{aligned}$$

notice that the last two elements at the right hand side are nothing that the derivate of $\int_{\hat{b}}^{\overline{b}} \frac{\Phi(b)}{1-F_B(\hat{b})} f_B(b) db$ wrt to the mass of buyers willing to participate, e.g. N_B. Thus, they are equal to the buyers' entry fee, e.g. P^b. Similarly, the first two elements of the right hand side are the derivative of $\int_{\hat{s}}^{\overline{s}} \frac{\Psi(s)}{1-F_S(\hat{s})} f_S(s) ds$ wrt to N_B. Thus,

$$\begin{aligned} P^b &= \phi N_S - \frac{1}{N_S N_B^2} \int_{F_S^{-1}(1-N_S)}^{\overline{s}} \int_{F_B^{-1}(1-N_B)}^{\overline{b}} [\mathcal{P}^s b n(s) - c] m(b) f_B(b) db f_S(s) ds \\ &- \frac{1}{N_S N_B} \int_{F_S^{-1}(1-N_S)}^{\overline{s}} [\mathcal{P}^s F_B^{-1}(1-N_B) n(s) - c] m(F_B^{-1}(1-N_B)) f_B(F_B^{-1}(1-N_B)) \frac{\partial F_B^{-1}(1-N_B)}{\partial N_B} f_S(s) ds \\ &= \phi N_S + \frac{\partial}{\partial N_B}\left(\int_{\hat{s}}^{\overline{s}} \frac{\Psi(s)}{1-F_S(\hat{s})} f_S(s) ds\right) \end{aligned}$$

and analogously,

$$\begin{aligned} P^s &= \phi N_B + \frac{1}{N_S^2 N_B} \int_{F_B^{-1}(1-N_B)}^{\overline{b}} \int_{F_S^{-1}(1-N_S)}^{\overline{s}} [\mathcal{P}^b s m(b) - c] n(s) f_S(s) ds f_B(b) db \\ &+ \frac{1}{N_S N_B} \int_{F_B^{-1}(1-N_B)}^{\overline{b}} [\mathcal{P}^b F_S^{-1}(1-N_S) m(b) - c] n(F_S^{-1}(1-N_S)) f_S(F_S^{-1}(1-N_S)) ds f_B(b) db \\ &= \phi N_B + \frac{\partial}{\partial N_S}\left(\int_{\hat{b}}^{\overline{b}} \frac{\Phi(b)}{1-F_B(\hat{b})} f_B(b) db\right) \end{aligned}$$

□

Proof of Lemma 3.2

Proof. Starting with the profit maximizing pricing rule add and substract the surplus maximizing price,

$$\begin{aligned}
P^{pm,b} - P^{sm,b} &= -P^b \frac{N_B}{P^b}\frac{\partial P^b}{\partial N_B} + \frac{N_S}{N_B^2}\int_{F_B^{-1}(1-N_B)}^{\bar{b}} [\mathcal{P}^s bn(F_S^{-1}(1-N_S)) - c]m(b)f_B(b)db \\
&+ \frac{N_S}{N_B} m(F_B^{-1}(1-N_B))f_B(F_B^{-1}(1-N_B))\frac{\partial F_B^{-1}(1-N_B)}{\partial N_B}[\mathcal{P}^s n(F_S^{-1}(1-N_S))F_B^{-1}(1-N_B) - c] \\
&- \frac{1}{N_S N_B^2}\int_{F_S^{-1}(1-N_S)}^{\bar{s}}\int_{F_B^{-1}(1-N_B)}^{\bar{b}} [\mathcal{P}^s bn(s) - c]m(b)f_B(b)db f_S(s)ds \\
&- \int_{N_S N_B}\int_{F_S^{-1}(1-N_S)}^{\bar{s}} [\mathcal{P}^s F_B^{-1}(1-N_B)n(S) - c]m(F_B^{-1}(1-N_B))f_B(F_B^{-1}(1-N_B)) \times \\
&\quad \frac{\partial F_B^{-1}(1-N_B)}{\partial N_B} f_S(s)ds
\end{aligned}$$

denoting $\mu^B = -P^b \frac{N_B}{P^b}\frac{\partial P^b}{\partial N_B}$ as the platform's market power,

$$\begin{aligned}
P^{pm,b} - P^{sm,b} &= \mu^B + \frac{1}{N_B}\{\frac{N_S}{N_B}\int_{F_B^{-1}(1-N_B)}^{\bar{b}} [\mathcal{P}^s bn(F_S^{-1}(1-N_S)) - c]m(b)f_B(b)db \\
&- \frac{1}{N_S N_B}\int_{F_S^{-1}(1-N_S)}^{\bar{s}}\int_{F_B^{-1}(1-N_B)}^{\bar{b}} [\mathcal{P}^s bn(s) - c]m(b)f_B(b)db f_S(s)ds\} \\
&+ \frac{1}{N_B}\{N_S[\mathcal{P}^s n(F_S^{-1}(1-N_S))F_B^{-1}(1-N_B) - c]m(F_B^{-1}(1-N_B))f_B(F_B^{-1}(1-N_B))\frac{\partial F_B^{-1}(1-N_B)}{\partial N_B} \\
&- \frac{1}{N_S}\int_{F_S^{-1}(1-N_S)}^{\bar{s}} [\mathcal{P}^s F_B^{-1}(1-N_B)n(s) - c]m(F_B^{-1}(1-N_B))f_B(F_B^{-1}(1-N_B)) \times \\
&\quad \frac{\partial F_B^{-1}(1-N_B)}{\partial N_B} f_S(s)ds\} \\
&= \mu^B + \frac{1}{N_B}\left(N_S P^{pm,s} - \int_{F_S^{-1}(1-N_S)}^{\bar{s}} \frac{\Psi(s)}{1-F_S(\hat{s})} f_S(s)ds\right) \\
&+ \{\frac{N_S}{N_B}[\mathcal{P}_s n(F_S^{-1}(1-N_S))F_B^{-1}(1-N_B) - c] \\
&- \frac{1}{N_S N_B}\int_{F_S^{-1}(1-N_S)}^{\bar{s}} [\mathcal{P}^s n(s)F_B^{-1}(1-N_B) - c]f_S(s)ds\} \times \\
&\quad m(F_B^{-1}(1-N_B))f_B(F_B^{-1}(1-N_B))\frac{\partial F_B^{-1}(1-N_B)}{\partial N_B}
\end{aligned}$$

thus finally,

$$P^{pm,b} - P^{sm,b} = \mu^B + \frac{\partial}{\partial N_B}\left(N_S P^{pm,s} - \int_{F_S^{-1}(1-N_S)}^{\bar{s}} \frac{\Psi(s)}{1-F_S(\hat{s})} f_S(s)ds\right)$$

and analogously,

$$P^{pm,s} - P^{sm,s} = \mu^S + \frac{\partial}{\partial N_S}\left(N_B P^{pm,b} - \int_{F_B^{-1}(1-N_B)}^{\bar{b}} \frac{\Phi(b)}{1-F_B(\hat{b})} f_B(b)db\right)$$

□

Proof of Lemma 4.1

Proof. There is relationship between the conditional pmf of $\overline{\theta}$, conditional on b (e.g. $Prob\{\theta = \overline{\theta} \mid b\} = m(b)$), and the joint density of (Θ, B) (e.g. $g_{\Theta,B}(\theta, b)$),

$$\begin{aligned} g_{\Theta,B}(\theta, b) &= Prob\{\Theta = \theta \mid B = b\} f_B(b) \\ &= \tilde{m}(\theta \mid b) f_B(b) \end{aligned}$$

similarly we can define a relationship between the conditional pmf of $\overline{\omega}$, conditional on s (e.g. $Prob\{\omega = \overline{\omega} \mid s\} = n(s)$), and the joint density of (Ω, S) (e.g. $g_{\Omega,S}(\omega, s)$),

$$\begin{aligned} g_{\Omega,S}(\omega, s) &= Prob\{\Omega = \omega \mid S = s\} f_S(s) \\ &= \tilde{n}(\omega \mid s) f_S(s) \end{aligned}$$

Then we can either pose the optimization program in terms of $\{\tilde{m}(\overline{\theta} \mid b), \tilde{n}(\overline{\omega}, s)\} = \{m(b), n(s)\}$, or in terms of $\{g_{\Theta,B}(\overline{\theta}, b), g_{\Omega,S}(\overline{\omega}, s)\}$. In this version I will pick the latter because we think its more natural that the platform chooses the joint density functions between the noise and the type for both buyers and sellers. To simplify notation define $g_{\Theta,B}(\overline{\theta}, b) = g_B(b)$ and $g_{\Omega,S}(\overline{\omega}, s) = g_S(s)$.

The optimization program is,

$$\begin{aligned} \max_{g_B(b), g_S(s)} \quad & \frac{N_B}{N_S} \int_{F_S^{-1}(1-N_S)}^{\overline{s}} \left[\mathcal{P}^b s \frac{g_B(F_B^{-1}(1-N_B))}{f_B(F_B^{-1}(1-N_B))} - c \right] g_S(s) ds + \\ & \frac{N_S}{N_B} \int_{F_B^{-1}(1-N_B)}^{\overline{b}} \left[\mathcal{P}^s b \frac{g_S(F_S^{-1}(1-N_S))}{f_S(F_S^{-1}(1-N_S))} - c \right] g_B(b) db \\ st \quad & g_S(s), g_B(b) \in [0, 1] \end{aligned}$$

the solution of this problem is quite simple because it must be at each pair (b, s), and because its a bang-bang solution. Thus,

$$g_S(s) = \begin{cases} 1 & \text{if } s > \tilde{s} \\ \text{anything} & \text{if } s = \tilde{s} \\ 0 & \text{if } s < \tilde{s} \end{cases}$$

$$g_B(b) = \begin{cases} 1 & \text{if } b > \tilde{b} \\ \text{anything} & \text{if } b = \tilde{b} \\ 0 & \text{if } b < \tilde{b} \end{cases}$$

where $\tilde{s} = \frac{c f_B(F_B^{-1}(1-N_B))}{\mathcal{P}^b g_B(F_B^{-1}(1-N_B))}$ and $\tilde{b} = \frac{c f_S(F_S^{-1}(1-N_S))}{\mathcal{P}^s g_S(F_S^{-1}(1-N_S))}$. This imply that

$$n(s) = \begin{cases} \frac{1}{f_S(s)} & \text{if } s > \tilde{s} \ \& \ \dot{f}_S(s) < 0 \\ \text{anything} & \text{if } s = \tilde{s} \\ 0 & \text{if } s < \tilde{s} \end{cases}$$

$$n(b) = \begin{cases} \frac{1}{f_B(b)} & \text{if } b > \tilde{b} \ \& \ \dot{f}_B(b) < 0 \\ \text{anything} & \text{if } b = \tilde{b} \\ 0 & \text{if } b < \tilde{b} \end{cases}$$

notice that the additional condition $\dot{f}_S(s), \dot{f}_B(b) < 0$ is coming from the strict monotonicity assumption of functions $\dot{m}(b), \dot{n}(s) > 0$. □

Proof of Lemma 4.2

Proof. As we mentioned in the previous proof, we can either write the optimization program in terms of $\{\tilde{m}(\overline{\theta} \mid b), \tilde{n}(\overline{\omega}, s)\} = \{m(b), n(s)\}$, or in terms of $\{g_{\Theta,B}(\overline{\theta}, b), g_{\Omega,S}(\overline{\omega}, s)\}$. In this version I will pick the latter because we think its more natural that the platform chooses the joint density functions between the noise and the type for both buyers and sellers. To simplify notation define $g_{\Theta,B}(\overline{\theta}, b) = g_B(b)$ and $g_{\Omega,S}(\overline{\omega}, s) = g_S(s)$.

The optimization program is,

$$\begin{array}{cl} \max\limits_{G_B(b),G_S(s)} & \frac{1}{N_S N_B} \int_{F_S^{-1}(1-N_S)}^{\overline{s}} \int_{F_B^{-1}(1-N_B)}^{\overline{b}} [(\mathcal{P}^s b + \mathcal{P}^b s) g_S(s) g_B(b) - c(g_B(b) f_S(s) + g_S(s) f_B(b))] db ds \\ st & g_B(b), g_S(s) \in [0,1] \end{array}$$

where $\frac{dG_B(b)}{db} = g_B(b)$ and $\frac{dG_S(s)}{ds} = g_S(s)$. The couple of euler equations yield,

$$\begin{aligned} \frac{1}{N_S N_B} \left[\left(\mathcal{P}^s b + \mathcal{P}^b s \right) g_B(b) - c f_B(b) \right] &= K_1 \\ \frac{1}{N_S N_B} \left[\left(\mathcal{P}^s b + \mathcal{P}^b s \right) g_S(s) - c f_S(s) \right] &= K_2 \end{aligned}$$

where K_1, K_2 are two constants of integration. Then,

$$\begin{aligned} g_B(b) &= \frac{K_1 N_S N_B + c f_B(b)}{\mathcal{P}^s b + \mathcal{P}^b s} \\ g_S(s) &= \frac{K_2 N_S N_B + c f_S(s)}{\mathcal{P}^s b + \mathcal{P}^b s} \end{aligned}$$

Taking the first euler equation one obtains,

$$\begin{aligned} \int dG_B(b) &= \int \frac{K_1 N_S N_B + c f_B(b)}{\mathcal{P}^s b + \mathcal{P}^b s} db \\ G_B(b) &= \frac{K_1 N_S N_B}{\mathcal{P}^s} Ln(\mathcal{P}^s b + \mathcal{P}^b s) + c \int \frac{f_B(b)}{\mathcal{P}^s b + \mathcal{P}^b s} db + K_3 \end{aligned}$$

where K_3 is another constant of integration. Using the boundary conditions, i.e. $G_B(\overline{b}) = \overline{G}_B$ and $G_B(\hat{b}) = \hat{G}_B$, we obtain that $K_1 = \frac{\mathcal{P}^s}{N_S N_B Ln\left(\frac{\mathcal{P}^s \overline{b} + \mathcal{P}^b s}{\mathcal{P}^s \hat{b} + \mathcal{P}^b s}\right)} \left[\overline{G}_B - \hat{G}_B - \int_{\hat{b}}^{\overline{b}} \frac{f_B(b)}{\mathcal{P}^s b + \mathcal{P}^b s} db \right]$. Finally, the optimal $g_B(b)$ will be

$$g_B(b) \equiv g_{\Theta,B}(\overline{\theta}, b) = \frac{\mathcal{P}^s}{(\mathcal{P}^s b + \mathcal{P}^b s) Ln\left(\frac{\mathcal{P}^s \overline{b} + \mathcal{P}^b s}{\mathcal{P}^s \hat{b} + \mathcal{P}^b s}\right)} \left[1 - c \int_{\hat{b}}^{\overline{b}} \frac{f_B(b)}{\mathcal{P}^s b + \mathcal{P}^b s} db \right] + c \frac{f_B(b)}{\mathcal{P}^s b + \mathcal{P}^b s}$$

and finally,

$$m(b) \equiv Prob\{\theta = \overline{\theta} \mid b\} = \frac{\mathcal{P}^s}{f_B(b)(\mathcal{P}^s b + \mathcal{P}^b s) Ln\left(\frac{\mathcal{P}^s \overline{b} + \mathcal{P}^b s}{\mathcal{P}^s \hat{b} + \mathcal{P}^b s}\right)} \left[1 - c \int_{\hat{b}}^{\overline{b}} \frac{f_B(z)}{\mathcal{P}^s z + \mathcal{P}^b s} dz \right] + \frac{c}{\mathcal{P}^s b + \mathcal{P}^b s}$$

and analogously,

$$n(s) \equiv Prob\{\omega = \overline{\omega} \mid s\} \quad = \quad \frac{\mathcal{P}^b}{f_S(s)(\mathcal{P}^s b + \mathcal{P}^b s) Ln\left(\frac{\mathcal{P}^s b + \mathcal{P}^b \overline{s}}{\mathcal{P}^s b + \mathcal{P}^b \hat{s}}\right)} \left[1 - c \int_{\hat{s}}^{\overline{s}} \frac{f_S(z)}{\mathcal{P}^s b + \mathcal{P}^b z} dz\right] + \frac{c}{\mathcal{P}^s b + \mathcal{P}^b s}$$

Finally we need to check the conditions such that the strict monotonicity assumption holds. We have that,

$$\dot{m}(b) > 0 \quad iff \quad 0 > \mathcal{P}^s + \frac{\dot{f}_B(b)}{f_B(b)}(\mathcal{P}^s b + \mathcal{P}^b s) + \frac{c f_B(b) Ln\left(\frac{\mathcal{P}^s \overline{b} + \mathcal{P}^b s}{\mathcal{P}^s \hat{b} + \mathcal{P}^b s}\right)}{\left[1 - c \int_{\hat{b}}^{\overline{b}} \frac{f_B(z)}{\mathcal{P}^s z + \mathcal{P}^b s} dz\right]}$$

$$\dot{n}(s) > 0 \quad iff \quad 0 > \mathcal{P}^b + \frac{\dot{f}_S(s)}{f_S(s)}(\mathcal{P}^s b + \mathcal{P}^b s) + \frac{c f_S(s) Ln\left(\frac{\mathcal{P}^s b + \mathcal{P}^b \overline{s}}{\mathcal{P}^s b + \mathcal{P}^b \hat{s}}\right)}{\left[1 - c \int_{\hat{b}}^{\overline{b}} \frac{f_S(z)}{\mathcal{P}^s b + \mathcal{P}^b z} dz\right]}$$

notice the strict monotonicity assumption holds if $\dot{f}_B(b), \dot{f}_S(s) < 0$, or if $c > 0$ is "sufficiently high". □

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