Constant-Interest-Rate Projections and Its Indicator Properties

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Abstract

This paper propose indicator variables for the implementation of monetary policy in an inflation targeting regime. Using constant interest rate projections, the notion of a target-compatible interest rate is presented. This variable allows to extract some characteristics that the expected future path of the interest rate have to fulfill in order to be compatible with the target. The specific formulation of the target-compatible interest rate is presented under alternative assumptions over the forecasting horizon (unconditional or conditional forecasts) and the objective of the monetary authority (inflation target or a loss function). The empirical counterpart of the various formulations is shown using a DSGE model for Colombia; a small open economy with an inflation targeting regime.

Keywords: monetary policy, constant-interest-rate projections, modest policy interventions, inflation targeting.
JEL Codes: E37, E47, E52.

1 Introduction

In an inflation targeting regime it's always important for the central bank to have an indicator about its monetary policy stance when adjusting the nominal interest rate. One of the indicators usually used for this purpose is the natural interest rate (NIR). The gap between the nominal interest rate and the natural rate measures the policy stance. When this gap is zero it is said that there is a *neutral* monetary policy. When the gap is positive, the monetary policy is said to be contractive and the opposite occurs for a negative gap.

The standard definition of the natural interest rate states that it is a rate which makes output to converge to its potential keeping inflation stable (Bomfim, 1997). It is possible to think about the natural interest rate as the intercept of a standard interest rate rule like the Taylor rule. However, as Woodford (2003) pointed, the natural rate of interest embedded in a Taylor rule is subject to different disturbances, generating a time-varing natural interest rate.

Most of the literature about the natural interest rate estimation uses one of two approaches: statistical filters[1] (Basdevant et al., 2004; Cuaresma et al., 2004) and semi-structural models (Laubach and Williams, 2003; España, 2008; Castillo et al., 2006; Echavarría et al., 2007). A survey of both approaches to estimate the NIR is presented by González

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[1] These estimations do not have a particular model, for example, linear detrending, moving averages, unobserved components models, multivariate Hodrick-Prescott filters, etc.

et al. (2010). With Colombian data they show how differences between the different estimations are explained not only by the methodology, but by the natural rate definition. The authors point out that in the statistical filters estimation, the NIR is defined as the trend component of the observed interest rate, while the semi-structural model considers the natural rate as a medium term anchor for monetary policy.

Nevertheless, Svensson (2002) argues that this type of models do not have the required structure to estimate the natural rate of interest and lack of a solid economic theory to support it.[2] Similarly, Larsen and McKeown (2003) emphasize the importance of a fully structural model, because the statistical and semi-structural models are less useful in a policy context and lack of structural interpretation of the interest rate gap and its variations. Neiss and Nelson (2001) for the UK and Giammarioli and Valla (2003) for the euro area, estimate the interest rate gap using dynamic stochastic general equilibrium models (DSGE). The definition of the NIR in this context states that is the real short term rate of interest that equates aggregate demand with flexible prices output at all times. But the natural rate of interest generated by Neiss and Nelson (2001) and Giammarioli and Valla (2003) appears to be more volatile than the actual real rate, which makes its use quite difficult.

In this paper we suggest the constant-interest-rate (CIR) projections as an alternative indicator of the monetary policy stance.[3] The concept of CIR forecasts was introduced by Leitemo (2003) and has been used by the central banks of England and Sweden (see Goodhart, 2009). According to Leitemo (2003), the CIR is defined by a level of the interest rate consistent with having the inflation forecast k periods ahead on target. This policy can be denoted by:

$$\pi_{t+k}\left(i_t\right) = \overline{\pi} \tag{1}$$

with

$$\left\{i_{t+j}\right\}_{j=0}^{k} = i \tag{2}$$

where π_t is the inflation rate prevailing at t, i_t is the interest rate at t, $\overline{\pi}$ is the inflation target and i the CIR level. The parameter k is the chosen forecast targeting horizon.

The CIR projections have been subject to different critiques. One of them points to the inconsistency of the constant-interest-rate underlying assumptions with the existence of a unique equilibrium in forward-looking models.[4] On this respect, Galí (2010) shows, based on the concept of "modest policy interventions" of Leeper and Zha (2003), that adding a sequence of unexpected (contemporaneous) monetary policy shocks to the central bank's rule allows to generate the desired interest rate path, conditional on a switch to the endogenous policy rule after the forecast horizon. Under this framework, each period, the central bank surprises the private sector with deviations from a known instrument rule, guaranteeing the existence of a unique equilibrium.

Another critique states that the CIR is time-inconsistent , this means that the constant-interest-rate level determined on t to $t+k$ cannot be the same level determined on the next period, i.e. on $t+1$ to $t+k+1$, even if there is no new information. In that case, agents in the model observe how the monetary authority is postponing the return of inflation on target, which diminishes the credibility of the monetary policy. It is clear that with CIR projections a central bank will face this problem, but CIR forecasts can still be useful

[2] According to Svensson (2002) a minimum set-up is conformed by a dynamic general equilibrium model with time preferences shocks, consumption shocks, population growth, technology shocks, capital stock, etc.

[3] As in Galí (2010), we use the term projection to refer to forecasts conditional on a given interest rate path.

[4] If a dynamic forward-looking model is fed with an exogenous interest rate path (constant or not) will have an equilibrium indeterminacy. See Galí (2010).

instruments when deriving monetary policy. Svensson (1999) argues that it would bring important information when taking monetary policy decisions:

> An indicator of "risks to price stability" [...], should be useful when discussing monetary policy that aims to maintain price stability. [...] The obvious candidate is a conditional inflation forecast, conditional upon unchanged monetary policy in the form of an unchanged interest rate. [...] This indicator signals whether and in which direction the inflation target is likely to be missed, if policy is not adjusted, and thereby it also signals in which direction the instrument needs to be adjusted. (Svensson, 1999, pp. 26)

However, this approach has two major disadvantages in addition to the mentioned critics. First it does not offer a direct measure of the magnitude in which the instrument should be adjusted. Second, precisely when there are the higher risks on price stability a policy consisting in a fixed interest rate at the current level imply a deeper problem of time inconsistency and is not under the Leeper and Zha (2003) framework, since the interventions are far from modest. Our proposed approach focus on the instrument given the target (or targets; we allow for a central bank that have a loss function between inflation and output growth). We answer the question of which should be the level of the interest rate to accomplish a certain objective, in particular the inflation target, for the real time expected evolution of the economy and also for possible contingencies in the future.

Leitemo (2003) suggest a method which involves to derive the inflation forecast function k periods ahead and minimize the difference of the inflation forecast function and the inflation target given a constant interest rate path. The evolution of the model is determined substituting the CIR path in the model state space form. However, the previous procedure have the property that the implied movements required of the interest rate have to be accompanied by a credible announcement of a change in the policy regime where the agents are fully informed about the constant interest rate policy that is going to be implemented. This is a shortcoming for the application in policy of this measure. Furthermore such a policy every quarter cannot be think to be credible.

Time-inconsistency is not a problem for the method proposed here, because the model structure does not consider an announced CIR policy by the central bank, but we keep constant the interest rate by unanticipated (and modest) monetary policy shocks which deviates temporary the central bank of its endogenous policy rule as in Galí (2010). This approach, were the fixed interest rate is consistent with the monetary target imply that the implied policy shocks belong to the Leeper and Zha (2003) framework.

The article is organized as follows: Section 2 discusses and defines different measures of monetary policy stance using CIR projections. To illustrate the approach, in Section 4, we study the case of Colombia, a small open economy with an inflation targeting regime, and compute for the period 2001Q2 - 2011Q1 a real time measure of the monetary policy stance and an ex-post measure of the stance using the contingent CIR path for the actual contingency realized along the sample. We use a New-Keynesian DSGE model for a small open economy.

The procedure used for the exercise is described in Section 3, the "modesty" degree of the policy interventions needed to compute the CIR projections is tested too. Section 5 considers some extensions to the standard CIR exercise. One of them tries to incorporate the inflation-output trade-off faced by a central bank on a flexible CIR projections which weights the two objectives. In addition, we propose and evaluate a commitment scenario under a constant-interest rate policy which requires a modified policy rule with anticipated policy shocks. Section 6 concludes.

2 The target-compatible interest rate

Let $\bar{\pi}_t$ the inflation target for period t and $\mathbf{x}_t$ a vector that contains a set of sufficient variables of the economy that characterize the state of the economy at time t. Then the expected target-compatible interest rate is defined as

$$i: \quad \mathrm{E}\left(\pi_{t+k}|\mathbf{x}_t, i_{t+1}=i,\ldots,i_{t+k}=i\right)=\bar{\pi}_{t+k} \tag{3}$$

This level of the interest rate assures that a policy consisting of a constant-interest-rate will attain that the expected value of inflation for the central bank is equal to the target. The final realization of inflation at time $t+k$ will depend also on the different contingencies that may arrive between period t and $t+k$. The contingencies are set to their unconditional expected value in the policy path because by definition they are orthogonal to $\mathbf{x}_t$. A conditional approach is to consider the contingency target-compatible interest rate defined as

$$i_z: \quad \Pr\left(\pi_{t+k}=\bar{\pi}_{t+k}|\mathbf{x}_t, z_t,\ldots,z_{t+k}, i_{t+1}=i_z,\ldots,i_{t+k}=i_z\right)=1 \tag{4}$$

where z_t is a realization of the sources uncertainty in the economy, i.e the shocks that arrive every period. With such a policy of fixed interest rate the monetary authority is certain that if the future outcomes of the economy are driven by $z_t,\ldots,z_{t+k}$ then the inflation will meet the target. Then, the monetary policy deviation $\tilde{i}_t$ is defined as

$$\tilde{i}_t = i_t - i$$

or given a possible realization of the shocks, the contingency deviation of monetary policy is

$$\tilde{i}_{z,t} = i_t - i_z$$

This deviations of monetary policy do not correspond to policy errors because the benchmark is a constant-interest-path and the monetary authority might use a different path to achieve the same objective. However, the information contained in $\tilde{i}_t$ is very useful: if it is different from zero, the interest rate have to be changed and the sign describes if the subsequent path should have higher or lower interest rates.

For the magnitude of the change, the CIR i sets a critical level that any path consistent a priori with the target have to cross, it is also a lower bound for the maximum level of the interest rate in the corresponding path. Clearly the path with the lowest maximum is the constant interest rate. Then the monetary authority using the CIR can identify some properties that must suffice the path of the interest rate if the objective is to set the conditional or unconditional expected value of inflation in the target in k periods.

The question the CIR leaves open is which path of the interest rate is optimal given that in general the constant interest rate is not the unique path to achieve the inflation target. We cannot answer that question because it will depend on particular characteristics of each economy. Nevertheless, a deeper question emerge: should the central bank necessarily implement one of those paths of interest rate? Even with inflation targeting regime the central banks are committed with other objectives, such as financial stability or to reduce the business cycle volatility.

Therefore, depending on the objectives of the central bank the CIR might be instead of a policy prescription only an indicator of the efforts required to obtain price stability. Nevertheless we can generalize the CIR to be applied to any objective function (or loss function L in this case) in the following form

$$i^L = \arg\min_i L\left(\mathbf{y}_{t+k}|\mathbf{x}_t, i_t=i,\ldots,i_{t+k}=i\right)$$
$$i^L_z = \arg\min_i L\left(\mathbf{y}_{t+k}|\mathbf{x}_t, z_t,\ldots,z_{t+k}, i_t=i,\ldots,i_{t+k}=i\right)$$

where $\mathbf{y}_{t+k}$ is a vector of variables relevant for the central bank loss function. We call this measure the flexible CIR, since in principle is compatible with deviations from the inflation target. For the Colombian case we show how this measure behaves relative to the standard case if the loss function L includes the deviations of the GDP growth rate relative to its long run trend in addition to inflation target deviations.

Up to this point we have relied on the fact that the expectations that define the CIR are those of the central bank conditional on the state of the economy $\mathbf{x}_t$ and the constant-interest-rate path. We are explicit that the expectations are those of the central bank because the agents in the economy do not incorporate such path in forming expectations about the evolution of the economy and, in particular, with the policy behavior of the central bank. These deviations of the central bank from the expected path of interest rate are taken by the agents as the modest interventions described by Leeper and Zha (2003). As long as these interventions are small enough such that do not affect the way the agents form expectations, the policy is potentially implementable and is a valid benchmark. In an inflation targeting regime, the CIR will imply a deviation from the expected path of the interest rate but since it is a path consistent with the objective of the central bank, is expected that such path does not incorporate extremely high policy shocks as might be the case with the approach discussed by (Svensson, 1999, pp. 26). For the Colombian case we evaluate statistically if effectively the implied interventions of our approach are modest. They turn out to be in fact modest.

Then, the difference in the expectation of the central bank and the agents is the private information the central bank has about the policy shocks implemented in the CIR path. An alternative is then to move to the case where the central bank announces to the public the future monetary policy shocks consistent with a constant-interest-rate path that ensures the inflation target. In this case, the monetary policy shocks are decided in a way that takes into account the effect on the agents' decision of such information. However, the central bank can adjust the interest rate when new information arrives. Let $\varepsilon_t^{t,k}$ be the vector of policy interventions between t and $t+k$ announced at time t. Then the CIR policy with anticipated shocks equalizes the expectations of the central bank and the agents in the economy and it is defined as

$$\left\{i, \varepsilon_t^{t,k}\right\}: \quad \tilde{\mathrm{E}}\left(\pi_{t+k} | \mathbf{x}_t, \varepsilon_t^{t,k}, i_t = i\right) = \bar{\pi}_{t+k}$$

Here, $\tilde{\mathrm{E}}$ is used to make explicit that the expectations are not only those of the central bank. For the Colombian case we show how this alternative policy imply a less volatile CIR and less effort for the monetary authority to achieve the inflation target at time t. Nevertheless, if such a policy were actually implemented the variance of the distribution of inflation increases and the commitment can be very costly for the monetary authority. We compute the cost of such commitment under this set-up and for the case where the central bank maintains their privately decided CIR for the whole period for every contingency.

The next section presents a detailed description of the method to calculate the empirical version of the CIR and include the relevant algorithms for implementation.

3 Computing constant-interest-rate projections

So far the alternative specifications of the CIR rely on the assumption that the expectations are given by $\mathbf{x}_t$, the state of the economy. However, there have not being addressed directly how from $\mathbf{x}_t$ the monetary authority can forecast π_{t+k} or in general $\mathbf{y}_{t+k}$. First, the elements of $\mathbf{x}_t$ might not be observable, then the expectations cannot be computed. Second, we need to establish the relationship between $\mathbf{x}_t$ (and i_t) with the probability distribution of π_{t+k}. To solve this two issues we introduce the concept of the information available for the central bank and a economic model for the evolution of the economy.

There is an economic model with rational expectations whose equilibrium can be represented by a covariance stationary stochastic process of the form

$$\mathbf{x}_{t+1} = \mathbf{T}\mathbf{x}_t + \mathbf{R}\eta_{t+1} \tag{5}$$

where the vector $\mathbf{x}_t = (c_t, z_t)'$ is a $n \times 1$ vector of variables of the structural model with c_t being a vector of endogenous variables, and z_t a vector of exogenous variables that follow univariate first-order processes. Also, we have that $\eta_t \sim \mathrm{N}(\mathbf{0}, \mathbf{Q})$ is a vector of r independent-over-time structural shocks. The $\mathbf{T}$ and $\mathbf{R}$ matrices are function of the parameters of the model and summarizes all the relations in the structural model and the evolution of the economy. We assume that such relations are stable through time and that the exogenous processes are the only ones subject to innovations.

The relationship between the model and the set of observable variables is represented by:

$$\mathbf{y}_t = \mathbf{Z}_t\mathbf{x}_t + \epsilon_t \tag{6}$$

where $\mathbf{y}_t$ is a vector of $m \times 1$ observable variables, $\epsilon_t \sim \mathrm{N}(\mathbf{0}, \mathbf{H}_t)$ is a vector of measurement errors, and $\mathbf{Z}_t$ is a selection matrix. Letting $\mathbf{Z}_t$ and $\mathbf{H}_t$ be a time-varing matrices allows to have different information sets at different periods. It also allows to have missing observations. Equations (5) and (6) define the state space representation of the model.

Let I_t be the information available for the central bank at time t, where I_t includes the structure of the economy described by (5) and the data available. The expected state of the economy at time k with information up to time t is

$$\hat{\mathbf{x}}_{s|t} = \mathrm{E}(\mathbf{x}_s|I_t)$$

and, in particular, for $s = t + 1$ we have

$$\begin{aligned}\hat{\mathbf{x}}_{t+1|t} &= \mathrm{E}(\mathbf{x}_{t+1}|I_t)\\ &= \mathbf{T}\hat{\mathbf{x}}_t\end{aligned}$$

Under this framework, the empirical counterparts of the definitions in (3) and (4) are

$$\hat{i}_t : \mathrm{E}\left(\pi_{t+k}|\hat{\mathbf{x}}_{t|t}, i_{t+1} = i, \ldots, i_{t+k} = i\right) = \bar{\pi}_{t+k} \tag{7}$$

$$\hat{i}_{z,T} : \Pr\left(\pi_{t+k} = \bar{\pi}_{t+k}|\hat{\mathbf{x}}_{t|T}, \hat{z}_{t|T}, \ldots, \hat{z}_{t+s|T}, i_{t+1} = i, \ldots, i_{t+k} = i\right) = 1 \tag{8}$$

This two measures are relevant benchmarks for policy evaluation: equation (7) for the real time evaluation of the policy deviation, and equation (8) as an ex-post evaluation of the contingency deviation given that $T \geq t + k$.

The intuition behind the procedure to compute these measures is very simple: starting with the rational expectations solution of a dynamic general equilibrium model with forward looking variables and its state space representation, one can implement the recursions of the Kalman filter to compute a conditional forecast. The forecast is conditioned: (i) to a constant-interest-rate path without measurement error and (ii) to the presence of only one new shock, namely the monetary policy shock.

Associated with the information set I_s for any period s there is a set of observable variables $\{\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_t, \ldots, \mathbf{y}_s\}$. Usually, the latter set has data for variables like GDP, inflation, interest rates, among others. We define $\mathbf{x}_0$ as the Kalman filter and smoother initial values. Let $\hat{\mathbf{x}}_{t|s} = \mathrm{E}(\mathbf{x}_t|I_s, \mathbf{x}_0)$ denote the sequence of the filtered values of $\mathbf{x}_t$ for all $s \leq T$ and $\tilde{\mathbf{x}}_{t|T} = \mathrm{E}(\mathbf{x}_t|I_T, \mathbf{x}_0)$ its smoothed values sequence.

The CIR measure in (7) is computed using the Kalman filter, this means that to determine the level of the interest rate at t that should be fixed by k periods onwards, the information set only contains data until t. We call it the up-to-date CIR or UD-CIR. The

CIR measure in (8) uses the Kalman smoother for the same purpose, but, because the use of the smoother, the information set is extended to include the future path of shocks, i.e. the shocks identified by the smoother from period s onwards. Henceforth we will call this the full information CIR or FI-CIR.

The following algorithm depicts the necessary steps to compute the up-to-date CIR projection for any period s.

Algorithm 1 Computing the up-to-date CIR

1. Compute the Kalman smother $\hat{\mathbf{x}}_{t|s}$ for $t = 1, \ldots, s+k$. Note that this implies to compute the smoother for $t = 1, 2, \ldots, s-1$, and the filter for $t = s, \ldots, s+k$.

2. From $\hat{\mathbf{x}}_{t|s}$ extract the vector $\hat{\mathbf{z}}_t^-$ with the smoothed values of the exogenous states excluding the monetary policy shock $\hat{\varepsilon}_t^0$. Without loss of generality, we consider ε_t^0 as a state of the model.

3. Define a new observable variable set $\left\{\mathbf{y}_1^z, \mathbf{y}_2^z, \ldots, \mathbf{y}_t^z, \ldots, \mathbf{y}_{s+k}^z\right\}$ where $\mathbf{y}_t^z = \left(\hat{\mathbf{z}}_t^-, i_t\right)$ for $t = 1, 2, \ldots, s-1$ and $\mathbf{y}_t^z = \left(\hat{\mathbf{z}}_t^-, i\right)$ for $t = s, \ldots, s+k$. The value for i will be found on step 4.

4. Adjust i to solve the following problem

$$\min_{i} \quad \left(\pi_{s+k} - \overline{\pi}\right)^2 \tag{9}$$

   subject to the model structure and the CIR policy in (1) and (2). In the previous problem π_{s+k} is taken from $\hat{\mathbf{x}}_{t|s}^z$ for $t = 1, \ldots, s+k$, the vector with the new smoothed states, which is found using the Kalman smoother over the new observable variable set and fixing $\mathbf{H}_t = \mathbf{0}$ for all t.

As we are fixing both the same initial values to compute the Kalman smoother and the same sequence over the exogenous variables (which jointly accounts for all the fluctuations) with zero measurement error, the step 4 will identify exactly the same sequence of exogenous shocks (even the policy shock path), and thus the same fluctuations on all the other variables, until period $s-1$. Since for periods $t = s, \ldots, s+k$ we don't have any idea about the future realizations of structural shocks, we set all of them equal to their unconditional mean letting free the monetary policy shock ε_t^0. However, at the same time we are conditioning the filter to retrieve $\{i_t\}_{t=s}^{s+k} = i$, which implies that the Kalman smoother will find the sequence for $\left\{\varepsilon_t^0\right\}_{t=s}^{s+k}$ that satisfies the constant-interest-rate path. The latter sequence is the same that Galí (2010) propose as a solution to compute CIR projections.

The computations needed to find the full information CIR path are very similar to the UD-CIR but with a slightly modification to account for the previously identified future path of shocks.

Algorithm 2 Computing the full information CIR

1. Compute the Kalman smoother for $\tilde{\mathbf{x}}_{t|T}$ for $t = 1, \ldots, s+k$ where $T = s+k$.

2. From $\tilde{\mathbf{x}}_{t|T}$ extract the vector $\tilde{\mathbf{z}}_t^-$ with the smoothed values of the exogenous states excluding $\tilde{\varepsilon}_t^0$.

3. Define a new observable variable set $\left\{\mathbf{y}_1^z, \mathbf{y}_2^z, \ldots, \mathbf{y}_t^z, \ldots, \mathbf{y}_{s+k}^z\right\}$ where $\mathbf{y}_t^z = \left(\tilde{\mathbf{z}}_t^-, i_t\right)$ for $t = 1, 2, \ldots, s-1$ and $\mathbf{y}_t^z = \left(\tilde{\mathbf{z}}_t^-, i\right)$ for $t = s, \ldots, s+k$. The value for i will be found on the step 4.

4. Adjust i to solve the problem in (9) subject to the model structure and the CIR policy in (1) and (2). The value for π_{s+k} is taken from $\tilde{\mathbf{x}}_{t|T}^z$ for $t = 1, \ldots, s+k$, the vector with the new smoothed states, which is found using the Kalman smoother over the new observable variable set and fixing $\mathbf{H}_t = \mathbf{0}$ for all t.

Note that we are fixing the paths for all the exogenous variables minus ε_t^0 with zero measurement error to the paths found by the Kalman smoother, so we are supposing that we know "perfectly" the future.

4 A standard constant-interest-rate projection for Colombia

Here we apply the methodology presented above on a CIR estimation for the Colombian economy using a standard small open economy model with information between 2001Q2 and 2011Q1. The model economy is described by a standard New-Keynesian model based on a neoclassical growth model in which agents decisions are made in an optimizing framework. The model summarizes a small open economy that is affected by changes in the international interest rate and foreign demand. It has one exogenous source of growth: technological progress. Also, it considers real rigidities such as consumption habit and investment adjustment cost, as well as nominal rigidities in prices and wages following the works of Erceg et al. (2000), Christiano et al. (2005) and Smets and Wouters (2007). A more detailed description of the model is found on Appendix A. The parameters' values are presented on Appendix B.

As is shown in the model description, we consider the following monetary policy rule:

$$i_t = \rho_s i_{t-1} + (1-\rho_s)\left(\bar{i} + \varphi_\pi\left(\pi_t - \overline{\pi}\right) + \varphi_y\left(\Delta y_t/\overline{\Delta y} - 1\right)\right) + \varepsilon_t^0 \tag{10}$$

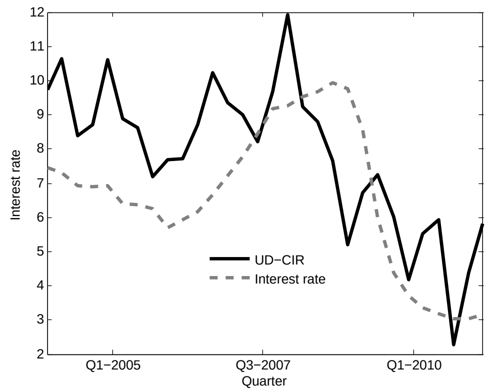
where i_t is the nominal gross interest rate, ρ_s is the smoothing coefficient, $\bar{i}$ is the steady state value of the nominal interest rate, φ_π determines the response to deviations of the inflation from its target, φ_y is the response to deviations of the yearly GDP growth Δy_t from its long run value $\overline{\Delta y}$, and ε_t^0 denotes an iid contemporaneous monetary policy shock with $\varepsilon_t^0 \sim \mathrm{N}\left(0, \sigma_0^2\right)$. The monetary policy shock is unanticipated, it means that each period the central bank surprises the private sector with deviations from a known instrument rule.

Using this model, we compute at each period the CIR measures in (7) and (8). For all exercises we choose $k = 6$ following evidence about the transmission of monetary policy in Colombia (see Huertas et al., 2005).

4.1 The CIR exercise

We compute the two CIR measures proposed on the previous section. Figure 1 shows the constant-interest-rate path consistent with a sequence of contemporaneous policy shocks and an information set which goes until period t for each period t. Every point of the solid

Fig. 1: Up-to-date CIR with contemporaneous shocks



line represents the interest rate that the central bank would have kept constant by $k = 6$ periods without any information about the future. The dashed line is the observed interest rate path.

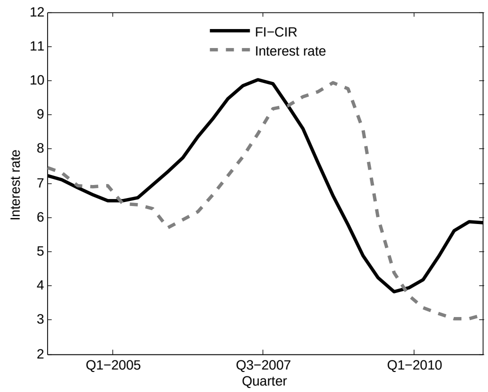
The estimated UD-CIR oscillates between 2.3 percent and 12 percent. As is stated by the monetary policy deviation definition, one can interpret the differences between the CIR and the observed interest rate path as a measure of the expansionary or contractionary position of the central bank. If the nominal interest rate is above the CIR, the interest rate has a negative effect on the economic activity considering a neutral scenario, because a lower policy rate would have guaranteed an inflation rate on its target and a higher GDP growth. In the other case, one says that the interest rate has a positive effect on the GDP and is an expansionary interest rate.

In general, the movements in the observed interest rate are similar to the UD-CIR dynamics, evidencing how the monetary policy seeks to approximate a neutral interest rate. We can see how in the first part of the sample, the nominal interest rate was around 1 percent point above the UD-CIR, with a positive spread between the two rates, and the implied expansionary stance, expanding gradually until 2006. Between 2006Q1 and 2008Q2, a period when Colombia was moving to an inflationary episode with an important acceleration of the GDP growth rate, the central bank takes a less expansive (but not yet contractive) policy, setting an interest rate close to the UD-CIR. On the last two quarters of 2008, with an inflation rate reaching 7.6 percent, the central bank strength its monetary policy position, setting a contractive monetary policy. After that, since 2009, when the inflationary pressures disappeared, the yearly GDP growth was close to zero, the world economy experienced a downturn and the world interest rates were at its historical minimum value , the monetary authorities decided to take an expansive position, as shown by the positive spread between the observed interest rate and the UD-CIR. This expansionary stance was wider at the end of the sample.

Nevertheless, the up-to-date CIR scenario differs from the real context faced by the central bank. To contrast these results in terms of the availability of the information, below we compute the FI-CIR, which allows to make an ex-post evaluation of the stance of monetary policy. This is reported on Figure 2. The path depicted by the solid line is the full information CIR at every t implied by an information set which contains data until $t + k$. It means that to find the FI-CIR the central bank knows perfectly all shocks that actually will hit the economy in the near future. Again, the dashed line is the observed interest rate path.

For the horizon considered, the FI-CIR fluctuates between 3.8 percent and 10 percent and it's smoother than the UD-CIR. The results show some differences with the up-to-

Fig. 2: CIR with full information and contemporaneous shocks



date CIR in determining the stance of monetary policy. For instance, in the first part of the sample, and until 2005Q2, the FI-CIR indicates that the central bank had a slightly contractionary monetary policy. However, according to the FI-CIR, between 2005Q3 and 2008Q1, the economy experienced a highly expansive interest rate policy with a gap widening continuously and reaching levels of -3 percentage points. The FI-CIR estimation also indicates that the maximum *neutral* policy rate was 10 percent at 2007Q2, the same value that the rate at 2008Q3, its maximum observed value. It could mean two things: (i) with perfect information about the future, the FI-CIR would have indicated a faster reaction of the monetary policy to what happened in the second semester of 2008, and (ii) the FI-CIR would have suggested an anticipated reaction.

Moving forward, the FI-CIR estimation shows that with a lower interest rate than the observed one between 2008Q1 and 2009Q4 the inflation would have been on target six periods ahead, evidencing a contractionary interest rate policy. At the end of the sample, after two quarters of neutral monetary policy (2010Q1:2010Q2), the FI-CIR would have pointed towards an increase in the policy rate, unlike the observed interest rate, indicating a expansionary monetary policy.

At this point it is important to make a remark: usually a central bank is between the two scenarios presented before. The central bank does not know perfectly the future and the sequence of shocks that will hit the economy, however it does have partial (and diffuse) information about the future behavior of some key variables (for example, through leading indicators). Therefore, despite not being in any of the two scenarios, a central bank is more close to the up-to-date information scenario, which implies that an estimated UD-CIR path embody a lot of uncertainty about upcoming economic events.[5]

4.2 Are CIR projections modest policy interventions?

The previous exercise needs to conditioning the path of one structural shock of the model. However, Leeper and Zha (2003) (henceforth LZ) and Adolfson et al. (2005) (henceforth ALLV) warn that such conditional forecast may be subject to the Lucas (1976) critique since the shocks that need to be adjusted may behave differently from what is assumed in

[5] When we say that the UD-CIR embodies a lot of uncertainty we are not strict enough. Actually, in the computation of the UD-CIR path the model “knows perfectly” the sequence of future shocks: we are supposing that the future values of shocks are zero, because we do not know anything about the future. Note that setting the shocks values at zero is the same as setting them in their unconditional mean, thus implying that there is no relevant information that affects the prediction of its future value. However, the above sentence refers to the uncertainty faced by the policy-maker when deriving monetary policy.

the model. When the above happens, the agents' beliefs about the model's structure can change.

For this reason, we compute the modesty statistic proposed by LZ for structural VARs, adapted by ALLV for a DSGE model, to verify the plausibility of CIR projections. The modesty statistic measures how unusual a conditional forecast of a variable is relative to an unconditional forecast. The modest statistic proposed by LZ states that when the absolute value of the univariate statistic is larger than 2 (because its distribution is standard normal), it is possible to say that the intervention is immodest. The LZ modesty statistic for a DSGE model at time t for variable x_i at horizon k is defined as

$$M_{i,t}^{k} = \frac{x_{i,t+k}\left(\overline{\eta}\right) - \hat{x}_{i,t+k|t}}{\mathrm{Std}\left(x_{i,t+k}\left(\eta\right)\right)} \tag{11}$$

where $\hat{x}_{i,t+k|t} = \mathrm{E}_t\left(x_{i,t+k}\right)$ is the unconditional forecast of x_i at t, $x_{i,t+k}\left(\overline{\eta}\right)$ is the CIR projection and $\mathrm{Std}\left(x_{i,t+k}\left(\eta\right)\right)$ accounts for the standard deviation of the x_i forecast on $t+k$ given the distribution of the model structural shocks in η. The statistic in (11) allows us to determine if the conditional and the unconditional forecasts differ too much at the end of CIR target horizon.

Figure 3 presents the LZ statistics for up-to-date and full information CIR projections for a selected set of model variables. The solid and dashed lines represents the UD-CIR and FI-CIR modesty statistics. The shaded area represents the modesty area, i.e. the values of $M_{i,t}^{k}$ for which an intervention is considered modest. Given that under up-to-date projections we are setting the future shocks values to its unconditional mean, the standard deviation of $x_{i,t+k}$ is computed taking into account the probability distribution functions of all model's structural shocks. The results suggest that the UD-CIR projections were read by the model as modest interventions, implying that the sequences of $\left\{\varepsilon_{t+i}^{0}\right\}_{i=0}^{k}$ needed to compute the CIR projections at all t did not change the agents' beliefs about the model's structure. Therefore, the UD-CIR projections are model-consistent.

For the full information projections, unlike the LZ statistics for the UD-CIR projections, the standard deviation of $x_{i,t+k}$ is computed taking into account only the probability distribution function of the monetary policy shock, because we are using with zero measurement error the shock identification done by Kalman smoother for the future disturbances affecting the economy until period $t+k$.

The results indicate that for all considered variables the FI-CIR projections are modest policy interventions for all periods. This result is conditional on the model specification and it's parameter values.

If we had some variables showing immodest policy interventions for some periods, it would be useful a statistic to get an overall result, so we also compute the ALLV generalization of the univariate statistic. It is a multivariate modesty statistic which allows to measure the intervention's effects on all variables' forecasts taken together. The statistic is given by:

$$M_{t}^{k}\left(\overline{\eta}\right) = \left[\mathbf{x}_{t+k}\left(\overline{\eta}\right) - \hat{\mathbf{x}}_{t+k|t}\right]' \mathbf{S}'\mathbf{P}_{t+k|t}^{-1}\mathbf{S}\left[\mathbf{x}_{t+k}\left(\overline{\eta}\right) - \hat{\mathbf{x}}_{t+k|t}\right] \tag{12}$$

where $\mathbf{P}_{t+k|t} = \mathrm{Cov}\left[\mathbf{x}_{t+k}\left(\eta\right)\right]$ and $\mathbf{S}$ is a $p \times n$ selection matrix. According to ALLV, the multivariate modest statistic is distributed χ_p^2 , where p is the number of variables considered in the computation of $M_t^k\left(\overline{\eta}\right)$. The total number of variables considered was 9, so $p = 9$. Define $\overline{M}_{\chi}$ as the values at which the χ_p^2 cdf are equal to $1-\alpha$, with α being some confidence level, say 0.05. To determine if an intervention is modest with the ALLV statistic, we consider as the null hypothesis that $M_t^k\left(\overline{\eta}\right) = 0$, so if $M_t^k\left(\overline{\eta}\right) \geq \overline{M}_{\chi}$ we say that the intervention is immodest. The same restrictions about the shock's distribution functions considered for the UD-CIR and FI-CIR in the univariate statistic apply.

Figure 4 shows the multivariate ALLV modesty statistic for both the UD-CIR and the FI-CIR. The solid line is the modesty statistic for the UD-CIR and the dashed line is

Fig. 3: Univariate modest policy statistics

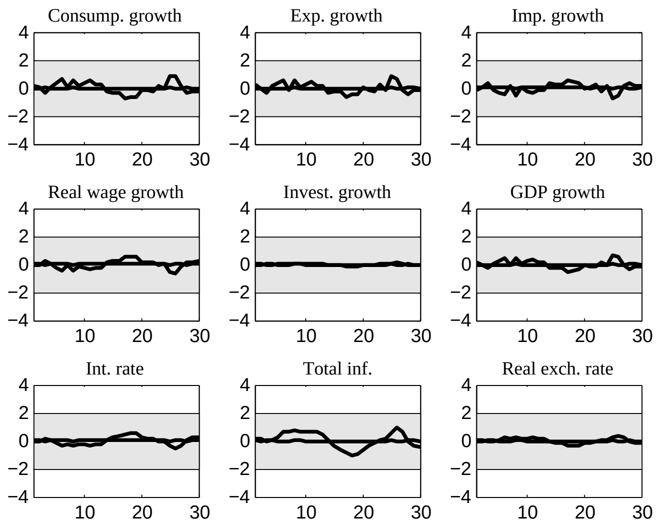
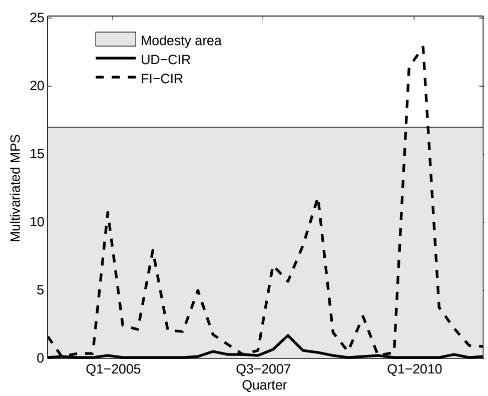


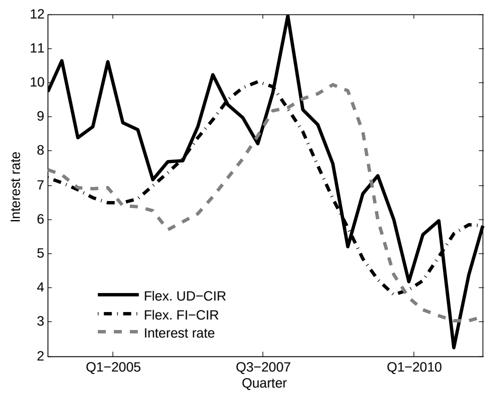
Fig. 4: Multivariate modest policy statistics



the statistic for the FI-CIR projections. The shaded area represents the modesty area at $\alpha = 0.05$. As the univariate statistic suggest for the UD-CIR projections, the ALLV statistic indicates that there were not immodest policy interventions. The multivariate statistic for the FI-CIR projections indicates that there were two periods with immodest interventions, so these two FI-CIR projections could have some problems. In the other cases, it does not happen and the FI-CIR results are reliable according to the ALLV statistic.[6]

[6] There are two additional modesty statistics that could be computed, and which pretend to measure the differences between the two forecast from t to $t+k$, i.e. during all CIR target horizon. The first one is a multi-period modification of the univariate LZ statistic given by $\tilde{M}_{i,t} = (1+k)^{-1} \sum_{h=t}^{t+k} \left[x_{i,h}(\overline{\eta}) - \hat{x}_{i,h|t} \right] \mathrm{Std}\left(x_{i,h}(\eta) \right)^{-1}$. If $(1+k)^{1/2} |\tilde{M}_{i,t}| > 2$ we say that the policy intervention is immodest at a 0.05 confidence level. The other one is a multi-period adaptation of the ALLV multivariate statistic which could be computed as $\tilde{M}_t(\overline{\eta}) = (1+k)^{-1} \sum_{h=t}^{t+k} \left[\mathbf{x}_h(\overline{\eta}) - \hat{\mathbf{x}}_{h|t} \right]' \mathbf{S}' \mathbf{P}_{h|t}^{-1} \mathbf{S} \left[\mathbf{x}_h(\overline{\eta}) - \hat{\mathbf{x}}_{h|t} \right]$ where $\tilde{M}_t(\overline{\eta}) \sim \Gamma(a,b)$ with $a = (1+k)p/2$ and $b = 2/(1+k)$.

Fig. 5: Flexible CIR



5 Some extensions

After the results of the standard CIR with full and partial information some questions about its operability and its flexibility show up. Some of them are related with the fact that usually policy-makers are monitoring not only the inflation rate, but also several economic variables. Another issue is concerned with the consequences of a commitment of the central bank to an announced interest rate path, more precisely to a constant-interest-rate path. The following exercises pretend to give some insights in this respect.

5.1 Flexible constant-interest-rate projection

The standard CIR projections only cares about minimizing the difference between the inflation k periods ahead and the inflation target, but it does not say anything about GDP growth rate. The strict standard CIR makes the GDP growth take any value necessary to reach the objective. However, the monetary authorities are usually monitoring both inflation and GDP growth. For this reason we calculate a flexible CIR using the same methodology presented in Section 3 but with a different objective function. It implies to minimize a loss function which includes the deviations of the GDP growth rate relative to its long run trend in addition to inflation target deviations. Therefore, the algorithm seeks to minimize the weighted quadratic differences of a certain set of individual objectives of its targets:

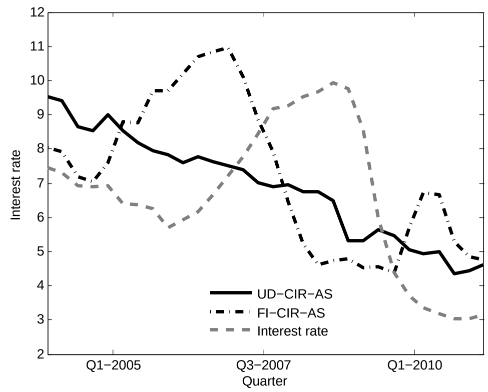
$$\min_{i} \quad \left(\mathbf{q}-\mathbf{q}^{obj}\right)' \mathbf{w} \left(\mathbf{q}-\mathbf{q}^{obj}\right)$$

subject to the model structure and the CIR policy in (1) and (2). Here, $\mathbf{q} = (\pi_{t+k}, \Delta y_{t+k})$, $\mathbf{q}^{obj} = (\overline{\pi}, \Delta\overline{y})$, and $\mathbf{w}$ is a conformable square matrix with subjective weights for the different objectives. As a measure for Δy_{t+k} we consider the yearly accumulated output growth rate, which ensures a smooth path in output dynamics.

To illustrate the use of flexible CIR projections, we consider that the objectives are that inflation reaches its target after k periods, and that yearly accumulated output will be growing at its long run rate. We also suppose that an hypothetically policy-maker assigns an importance of $3/4$ to the inflation objective and $1/4$ to its output objective. This means that $\mathbf{w} = \mathrm{diag}\,(0.75, 0.25)$. Figure 5 shows the results of this exercise.

Comparing the flexible UD-CIR with the standard UD-CIR we observe that the former has a lower variability, i.e. when the UD-CIR suggests a more expansive (contractive) policy, the flexible UD-CIR shows a higher (lower) rate. There is a simple reason for this: on the flexible CIR estimation the central bank is taking in account the GDP growth and,

Fig. 6: CIR with anticipated shocks



because of the inflation-output trade-off, the monetary policy has to be less stringent than the standard CIR in expansionary and contractionary times.

Likewise, the flexible FI-CIR indicates smaller interest rate changes, although it reacts similarly to the standard FI-CIR showing both stronger and anticipated reactions when comparing with flexible UD-CIR.

5.2 Constant-interest-rate with anticipated shocks

As Laséen and Svensson (2009) argue, interest rate projections with anticipated policy-rate paths would in many cases seem more relevant for the transparent flexible inflation targeting. To approach to that question we explore the implications of constant-interest-rate projections with a commitment policy. The scenario is simple: the central bank commits to keep a constant-interest-rate from period t by k periods. This implies that the rule followed by the monetary authority needs the inclusion of k anticipated shocks. Thus, we extend the standard Taylor rule on (10) to include a set of k anticipated shocks, resulting in the following interest rate rule:

$$i_t = \rho_s i_{t-1} + (1-\rho_s)\left(\bar{i} + \varphi_\pi\left(\pi_t - \overline{\pi}\right) + \varphi_y\left(y_t/\overline{y} - 1\right)\right) + \varepsilon_t^0 + \varepsilon_{t-1}^1 + \cdots + \varepsilon_{t-k}^k \tag{13}$$

where ε_{t-j}^j for $j = 0, 1, 2, \ldots, k$, denotes j-period anticipated shocks to i_t. To implement the procedure explained on Section 3 the modified model with (13) must be solved.

As in the baseline exercise, we present the computation of an up-to-date CIR and a full information CIR, both of them with anticipated policy shocks. Figure 6 shows the UD-CIR with anticipated shocks. Our results show a negative trend since the beginning of the sample, a period where the Colombian economy experienced an almost continuous decrease on its inflation rate. At the same time, the central bank diminished year by year its inflation target.

On Figure 6 we present the full information CIR with anticipated shocks. The FI-CIR with anticipated shocks fluctuates around the up-to-date CIR with anticipated shocks, but it shows more variability than the companion with up-to-date information, because when the monetary authority has full information about the future it can react in a better way than in the UD-CIR scenario. This means that when the central bank has a wider information set the FI-CIR reacts continuously to the future shocks.

The previous results indicate that, for both the up-to-date and the full information CIR the commitment scenario (i.e. lower variability with anticipated shocks) demands less effort on the monetary policy that the standard CIR because the agents of the model are incorporating the credible commitment acquired by the central bank when taking its decisions. Below, we try to figure out what would be the consequences of implementing such policy.

5.2.1 The cost of commitment

Now we know that a constant-interest-rate policy with anticipated shocks demand less effort than a standard CIR policy with a sequence of contemporaneous policy shocks but, which would be the cost of implementing it? What is the cost for the central bank to commit to a fixed interest rate path by k periods? To calculate it we consider a coherent measure of cost for a central bank under an inflation targeting regime: what would be the inflation after k periods if the central bank commits to a CIR policy having no information about the future? The procedure to compute it requires to use the CIR path calculated previously and "implement" it on an extended information set using the Kalman smoother.

Figure 7a shows the total inflation at period $t + k$ if the central bank had announced its CIR policy on each quarter t. The dotted line is the trend of the inflation target and the shaded band represents an interval of inflation target ± 1 pp. The results suggest that if the central bank would have kept its interest rate fixed in a myopic way, the inflation would have had greater variations, which is equivalent to say that the differences between the inflation and the inflation target target would have been larger than the observed ones. To illustrate it, consider 2008Q4. The UD-CIR with anticipated shocks suggests an interest rate of 7 percent (approximately). However, the cost of commit to this rate would have been a deflation slightly higher than 2 percent in 2010Q2. The reason to this is that such an interest rate would have been a more contractive monetary policy than the observed one, which achieved to bring inflation to 2.3 percent in 2010Q2.

A second, and less stringent, evaluation considers how a central bank is committing to keep a CIR policy, more specifically, we consider a adaptive commitment policy. Under this assumption, the announcement would be that the interest rate will be fixed for the next k periods if no unexpected shocks arrive to the economy. However, a lot of unexpected shocks arrives continuously and the central bank has to react to them changing its policy rate. On Figure 7b we present where the inflation rate would arrive if the central bank commits to a determinate policy rate on t by k periods (implying k unanticipated shocks different from zero on t), but later it realizes about new information and reacts endogenously through its policy rule (13). The inflation at $t + k$ path is close to the inflation target range and is not too deviated as in the extreme exercise because the monetary policy between $t+1$ and $t+k$ is reacting to the same shocks, but the observed one is slightly closer to the inflation target on average.

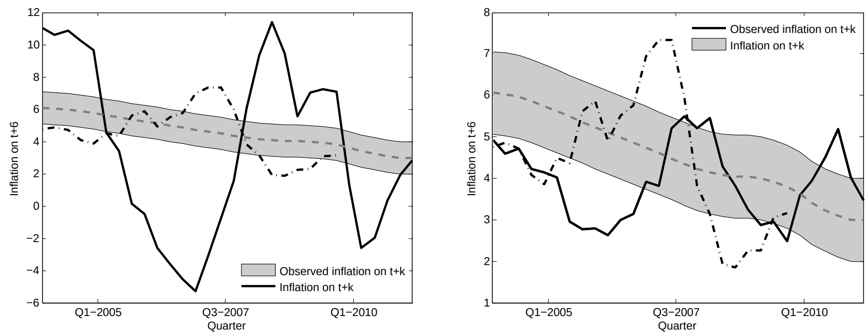
The previous two measures of cost of a commitment policy allows to say that a myopic policy, which does not react to the new shocks hitting the economy, has a huge cost in terms of deviations of the inflation from its target. The results show that an "adaptive" commitment policy also would has larger deviations than a standard policy.

6 Conclusions

We propose indicator variables for the implementation of monetary policy in an inflation targeting regime. These measures are used as indicators of the monetary policy stance and tries to overcome the difficulties of the natural interest rate estimations. We use constant interest rate projections to present the notion of a target-compatible interest rate. This rate is the level of the interest rate that assures that a policy consisting of a constant-interest-rate will attain that the expected value of inflation for the central bank is equal to the target. This variable allows to extract some characteristics that the expected future path of the interest rate have to fulfill in order to be compatible with the target.

Fig. 7: Cost of commitment

(a) Extreme case **(b)** Adaptive case



When the the monetary policy deviation, i.e. the spread between the CIR and the observed interest rate, is equal to zero one can say that the monetary authorities are implementing a *neutral* interest rate policy. A positive monetary policy deviation is an indicator of an expansionary monetary policy and the opposite occurs for a negative deviation.

The specific formulation of the target-compatible interest rate is presented theoretically under alternative assumptions over the forecasting horizon (unconditional or conditional forecasts) and the objective of the monetary authority (inflation target or a loss function).

The CIR projections are used as the empirical counterpart of the target-compatible interest rate. We have shown an alternative way to compute constant-interest-rate projections using the Kalman filter and the Kalman smoother. The basic idea consists in computing a conditional forecast where we only consider a sequence of unanticipated monetary policy shocks in order to find the constant interest rate level which guarantees that the inflation forecast k periods ahead will be on target.

Using the proposed methodology to compute the CIR and a standard small open economy DSGE model calibrated and estimated for the Colombian economy, we estimate the constant-interest-rate path with up-to-date and full information for the period 2003Q2 to 2010Q1. The results showed that the movements in the observed interest rate was similar to the UD-CIR dynamics, evidencing how the monetary policy seeks to drive inflation to its target with a stance close to neutral. Although this similarity, the UD-CIR shows a gap with the observed interest rate, allowing to determine when the monetary policy had a contractive or expansive stance. On the other hand, with perfect information about the future shocks that actually hit the economy, the FI-CIR would have indicated a stronger and anticipated reaction of the monetary policy to different shocks hitting the economy.

We also worked on some extensions of the CIR definition with the purpose of making it more flexible and evaluate a different monetary policy with it, i.e. a commitment policy. In the first case, we showed how the proposed algorithm allows to reconcile the inflation-output trade-off with the CIR projections assigning subjective weights for different objectives. The estimation of flexible constant-interest-rate projections indicates a less stringent monetary policy in expansionary and contractionary times compared to the standard CIR.

In the second case, we consider the effect of a monetary policy where the central bank commits to keep the interest rate fixed by k periods. To evaluate this policy we use a coherent measure of cost for a central bank under an inflation targeting regime: the inflation rate after k periods if the central bank commits to a CIR policy having no information about the future. Our results point out that a “myopic” monetary policy has a huge cost in terms of deviations of the inflation from its target, and that an “adaptive” commitment policy has slightly worst results than a standard policy.

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A The model

Here we depict the model used in our computations. It is a standard New-Keynesian model for an open economy in which agents decisions are made in an optimizing framework. It is assumed that the economy is affected by changes in the international interest rate and foreign demand. It also considers real rigidities such as consumption habit and investment adjustment costs, as well as nominal rigidities in prices and wages following the works of Erceg et al. (2000), Christiano et al. (2005) and Smets and Wouters (2007). The model economy includes six different agents: households, intermediate good firms, final good firms, investment firms, central bank, and external sector. Below we describe in detail the problem of each agent.

Growth in the model is determined by the trend productivity growth per worker, per hour worked. The evolution of the technological progress A_t is given by $A_t = (1 + g_t) A_{t-1}$ where

$$g_t = \rho_g g_{t-1} + (1 - \rho_g)\,\overline{g} + \varepsilon_t^g, \qquad \varepsilon_t^g \sim \mathrm{N}\left(0, \sigma_g^2\right),\ \rho_g \in (0, 1)$$

A.1 Households

The economy is populated by a continuum of households indexed by j. The household problem is to maximize the discounted sum of its future utility subject to a budget constraint. The intertemporal preferences of the representative household are given by

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{z_t^u}{1-\sigma} \left(c_{j,t} - bc_t\right)^{1-\sigma} - \frac{\chi}{1+\eta} h_{j,t}^{1+\eta} \right\}$$

where $c_{j,t}$, and $h_{j,t}$ denote the j household's levels of aggregate consumption and labor supply. Consumption is subject to external habit formation measured by b. The variable z_t^u is an exogenous process which represent a preference shock to the marginal utility of consumption.

The household buys the consumption good at a price p_t^c, and invest in capital stock by buying investment goods x_t at a price p_t^x. We assume that there are investment adjustment costs, so the capital accumulation equations is given by

$$k_{j,t} = (1 - \delta)\, k_{j,t-1} + \left(1 - \frac{\kappa_x}{2}\left(\frac{x_{j,t}}{x_{j,t-1}} - 1\right)^2\right) x_{j,t} \tag{14}$$

where k_t is the stock of capital and δ is the capital depreciation rate. The representative household also has to pay the external debt service at the nominal interest rate $i_t^\star$, which is in foreign currency. In addition, the household buys Arrow-Debreu securities $a_{j,t+1}$ at a real price $p_{j,t+1,t}^a$ in order to insure himself against idiosyncratic shocks. The household derives its income from the rent of capital r_t^k, the supply of its differentiated labor services, new external debt $b_t^\star$ and the profits of the local firms Π_t, which it owns. Finally, it also buys internal bonds which earn a nominal interest rate i_t. The budget constraint on equation (15) summarizes the above.

$$\begin{aligned} c_{j,t} + \frac{p_t^x}{p_t^c} x_{j,t} + b_{j,t} + q_t \left(\frac{i_{t-1}^\star}{\pi_t^\star}\right) \frac{b_{t-1}^\star}{g_t} & \\ + \int_0^1 p_{j,t+1,t}^a a_{j,t+1} d\omega_{j,t+1,t} &= r_t^k k_{t-1} + w_{j,t} h_{j,t} + \left(\frac{i_{t-1}}{\pi_t^c}\right) b_{t-1} \\ &\quad + q_t b_{j,t}^\star + \Pi_t + a_{j,t} \end{aligned} \tag{15}$$

with $q_t = s_t p_t^\star / p_t^c$ the real exchange rate, s_t the nominal exchange rate, $p_t^\star$ the foreign price index, and $\pi_t^\star = p_t^\star / p_{t-1}^\star$ the gross foreign inflation rate.

A.1.1 Domestic and imported consumption

Households consume a composite consumption good. This composite good is produced using domestically produced and imported goods. We assume the following consumption bundle

$$c_t = \left[(\gamma_c)^{\frac{1}{\omega_c}} \left(c_t^d\right)^{\frac{\omega_c - 1}{\omega_c}} + (1-\gamma_c)^{\frac{1}{\omega_c}} \left(c_t^m\right)^{\frac{\omega_c - 1}{\omega_c}} \right]^{\frac{\omega_c}{\omega_c - 1}} \tag{16}$$

where c_t^d and c_t^m are consumption of the domestic and imported goods, $1-\gamma_c$ is the share of imports in total consumption and ω_c the elasticity of substitution across consumption goods. The household seeks to minimize the cost of its consumption expenditure by choosing an optimal combination of c_t^d and c_t^m. Deriving the optimally conditions for this problem, we obtain an expression for the inflation of total consumption price:

$$\pi_t^c = \left[\gamma_c \left(\pi_t^d p_{t-1}^{cd}\right)^{1-\omega_c} + (1-\gamma_c) \left(\pi_t^m p_{t-1}^{cm}\right)^{1-\omega_c} \right]^{\frac{1}{1-\omega_c}} \tag{17}$$

where π_t^d and π_t^m stand for the price inflation of domestic and imported consumption goods. Also, p_t^{cd} and p_t^{cm} stand for the relative prices of domestic and imported consumption goods.

A.1.2 Wage setting

The households offer differentiated labor services in a monopolistically competitive labor market. We assume that wages are rigid in nominal terms à la Calvo (1983). Each period a fraction $1-\xi_w$ of the households receive a stochastic signal that allows them to optimally adjust their nominal wage. The households that don't receive the signal adjust their real wage according to the following rule

$$w_{j,t}^{rule} = w_{j,t-1} \left(\frac{A_t}{A_{t-1}} \right) \left(\frac{\pi_{t-1}^c}{\pi_t^c} \right) \tag{18}$$

The problem of the households that are allowed to adjust their wages is to choose the nominal wage that maximizes its expected stream of utilities, given that it will only be allowed to change its price optimally on receipt of a random signal and the demand function for its labor service variety.

As in Smets and Wouters (2007), we suppose the existence of a labor aggregator firm whose problem is to minimize its costs subject to the Kimball (1995) aggregator:

$$\min_{h_{j,t}} \int_0^1 w_{j,t} h_{j,t} dj$$
$$\text{s.t.} \int_0^1 \Upsilon_w \left(h_{j,t}/h_t\right) dj = 1$$

with $\Upsilon_w(1) = 1$, $\Upsilon'_w(\cdot) > 0$ and $\Upsilon''_w(\cdot) < 0$. The real wage w_t is a function of expected and past real wages, expected, current, and past inflation, the wage mark-up, and an additive wage mark-up disturbance denoted by z_t^w.

A.2 Investment goods producers

The homogenous investment good x_t is produced using a domestic and imported inversion goods x_t^d and x_t^m, respectively. The production technology is given by

$$x_t = z_t^x \left[(\gamma_x)^{\frac{1}{\omega_x}} \left(x_t^d\right)^{\frac{\omega_x - 1}{\omega_x}} + (1-\gamma_x)^{\frac{1}{\omega_x}} \left(x_t^m\right)^{\frac{\omega_x - 1}{\omega_x}} \right]^{\frac{\omega_x}{\omega_x - 1}} \tag{19}$$

where z_t^x is a shock to the investment efficiency, $1-\gamma_x$ is the share of imports in investment and ω_x the elasticity of substitution across investment goods.

A.3 Firms

The domestic production of intermediate goods combines labor and capital and is exposed to unit-root technology process A_t and to an economy-wide productivity shock z_t. These firms rent capital and labor in perfectly competitive factor markets. The production function of the intermediate firms is given by

$$y_{i,t} = z_t A_t^{1-\alpha} k_{i,t-1}^{\alpha} h_{i,t}^{1-\alpha} \tag{20}$$

ans z_t follows

$$z_t = \rho_z z_{t-1} + (1-\rho_z)\,\overline{z} + \varepsilon_t^z, \qquad \varepsilon_t^z \sim \mathrm{N}\left(0, \sigma_z^2\right),\ \rho_z \in (0,1)$$

Each of the domestic goods firms is subject to price stickiness as in Calvo (1983). Each intermediate firm faces, in any period, a probability $1-\xi_d$ that it can reoptimize its price. The firms that can't adjust optimally their prices update them according to the following rule

$$p_{i,t}^{rule} = p_{i,t-1}\pi_{t-1}^{d} \tag{21}$$

where π_t^d represents the inflation of the domestic good. The problem of the firms that are allowed to adjust their price is to choose the price $p_{i,t}^{\star}$ that maximizes their expected stream of profits, given that it will only be allowed to change its price optimally on receipt of a random signal and the demand function for its intermediate good variety.

The final domestic good is a composite of a continuum of i differentiated intermediate goods, each supplied by a different firm. The problem of the producer of the final good is to minimize its costs subject to the Kimball (1995) aggregator

$$\begin{aligned} &\min_{y_{i,t}} \int_0^1 p_{i,t} y_{i,t} di \\ &\text{s.t.} \int_0^1 \Upsilon_p\left(y_{i,t}/y_t\right) di = 1 \end{aligned}$$

with $\Upsilon_p(1) = 1$, $\Upsilon_p'(\cdot) > 0$ and $\Upsilon_p''(\cdot) < 0$. As Smets and Wouters (2007) point out, the Kimball aggregator is a more general form of the Dixit-Stiglitz aggregator because it allows for a time-varying demand elasticity. For this reason, the inflation in the Phillips curve for the domestically produced good depends on past and future inflation (by the price stickiness), the current price mark-up and an additive price mark-up disturbance denoted by z_t^{π}.

A.4 External sector

We assume that the demand for exports of domestically produced goods is given by:

$$e_t = \left(\frac{p_t^d}{s_t p_t^{c\star}}\right)^{-\mu} y_t^{\star} \tag{22}$$

where p_t^d is the aggregated price of domestically produced final goods, $p_t^{c\star}$ is the external consumption bundle price, and $y_t^{\star}$ is a measure of the external demand. The variables $p_t^{c\star}$ and $y_t^{\star}$ are exogenous variables following an autoregressive process.

As in Schmitt-Grohe and Uribe (2003) we close the model imposing a condition that ensures that the external debt to GDP ratio converge to a predetermined value. Therefore,

Tab. 1: Calibration results

Ratios	**Data**	**Model**
Total investment / GDP	0.22	0.23
Imported investment / Total investment	0.36	0.37
Total consumption / GDP	0.80	0.75
Imported consumption / Total consumption	0.12	0.15
Capital / GDP	6.82	6.68
Exports / GDP	0.20	0.19
Total imports / GDP	0.23	0.20

the effective external nominal interest rate has a premium which depends on the deviation of debt from this target ratio. This is

$$i_t^\star = \overline{i}^\star z_t^{i^\star} \exp\left\{\phi_b \left(\frac{q_t b_t^\star}{y_t} - \overline{b}^\star\right)\right\} \tag{23}$$

where $\overline{i}^\star$ is the nominal risk free international interest rate, $z_t^{i^\star}$ is a shock to the interest rate, $\overline{b}^\star$ is the target value of the debt-output ratio, and ϕ_b is the elasticity of the external interest rate to deviations of debt ratio from this target. The shock to the foreign interest rate follows an autoregressive process

$$z_t^{i^\star} = \rho_{i^\star} z_{t-1}^{i^\star} + (1 - \rho_{i^\star})\,\overline{z}^{i^\star} + \varepsilon_t^{i^\star}, \qquad \varepsilon_t^{i^\star} \sim \mathrm{N}\left(0, \sigma_{i^\star}^2\right),\ \rho_{i^\star} \in (0,1)$$

A.5 Central bank

The central bank follows a Taylor rule which reacts to deviations of the total inflation from its target, and to the growth rate of output:

$$i_t = \rho_s i_{t-1} + (1-\rho_s)\left(\overline{i} + \varphi_\pi \left(\pi_t^c - \overline{\pi}\right) + \varphi_y \left(\Delta y_t / \overline{\Delta y} - 1\right)\right) + \varepsilon_t^0 \tag{24}$$

where i_t is the nominal gross interest rate, ρ_s is a smoothing coefficient, $\overline{i}$ is the steady state value of the nominal interest rate, φ_π determines the response to deviations of the inflation from its target, φ_y is the response to deviations of the yearly GDP growth Δy_t from its long run value $\overline{\Delta y}$, and ε_t^0 denotes an iid contemporaneous monetary policy shock with $\varepsilon_t^0 \sim \mathrm{N}\left(0, \sigma_0^2\right)$.

A.6 Market clearing

To clear the final goods market, and the foreign debt market, the following equation must hold in equilibrium:

$$y_t + q_t b_t^\star = c_t + \frac{p_t^x}{p_t^c} x_t + q_t \left(\frac{i_{t-1}^\star}{\pi_t^\star}\right) \frac{b_{t-1}^\star}{g_t} \tag{25}$$

B Parameters

We calibrate some parameters of the model by minimizing the difference of some observed ratios for Colombia and the same ratios obtained from the model. The objective ratios and the model ratios are presented in Table 1. The shock variances were estimated through bayesian techniques. The remaining parameters were borrowed from Bonaldi et al. (2010). Table 2 presents the parameter values.

Tab. 2: Model parameters

Parameter	Value	Parameter	Value	Parameter	Value
Bonaldi et al. (2010)					
β	0.9939	ϕ_b	0.0051	$\overline{z}^x$	0.9790
η	3.4322	b	0.2572	$\overline{z}^u$	0.1331
$\overline{i}^\star$	1.0123	ξ_d	0.3615	$\overline{b}^\star$	1.2355
μ	1.5131	ξ_w	0.4547	$\rho_{c^\star}$	0.8000
$\overline{\pi}$	1.0074	ρ_s	0.7000	ρ_g	0.4000
$\overline{\pi}^\star$	1.0074	φ_π	2.5000	$\rho_{i^\star}$	0.5000
σ	3.9971	φ_y	0.8000	ρ_{z^u}	0.5000
δ	0.0250	$\overline{z}^{i^\star}$	1.0000	ρ_{z^x}	0.2500
χ	104.34				
Calibrated					
α	0.5143	ω_c	6.2000	$\overline{p}^{c\star}$	0.9800
γ_x	0.8180	θ_w	2.8000	$\overline{y}^\star$	0.8000
γ_c	0.9900	θ_d	1.7000	$\overline{g}$	0.0100
ω_x	2.8000				
Estimated					
$\sigma_{c^\star}$	0.1325	σ_{z^u}	0.9048	σ_{z^w}	0.2171
σ_g	0.1325	σ_{z^x}	0.2183	$\sigma_{p^{c\star}}$	0.0589
$\sigma_{i^\star}$	0.0033	σ_{z^π}	0.0193	σ_0	0.0181