Non-Parametric and Semi-Parametric Asset Pricing: An Application to the Colombian Stock Exchange

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Non-parametric and Semi-parametric Asset Pricing: An Application to the Colombian Stock Exchange

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Abstract

We estimate a non-parametrical Capital Asset Pricing Model (CAPM) and find strong evidence rejecting the classical linear CAPM. Furthermore, we find inconsistent linear betas for a series of stocks in the Colombian stock exchange (BVC), supporting the hypothesis of a better and consistent fitting of non-parametrical versions of the CAPM.

Keywords: CAPM, Non-parametrics, Kernel estimation, bootstrapping, SML
JEL classification: G12, C14, C15.

1. Introduction

The linear one-factor Capital Asset Pricing Model (CAPM) proposed by Sharpe (1964) is the most widely used equilibrium model by financial practitioners. However, there are many studies questioning its validity (see, for instance, Banz (1981), Basu (1983), and Fama and French (1995)). Different alternatives have been proposed in the literature to overcome the shortages of the linear one-factor CAPM[2]. These alternatives include multifactor extensions such as Merton's ICAPM (1973), Ross' APT (1976) or Fama and

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[2]For more on the shortages of the CAPM see Erdős et al. (2011)

French's three-factor-model (1996), models that allow for time-variation in the market risk or in the betas (see, for instance, Chen (1991), Ferson and Harvey (1991), and Jagannathan and Wang (1996)), and nonlinear asset pricing models such as the threshold CAPM proposed by Akdeniz et al. (2002).

Following a recent study by Erdős et al. (2011), we propose and estimate a non-parametric one-factor asset pricing model using information on all the stocks listed in the Colombian stock exchange (BVC) between 2002 and 2011. Using our model, we estimate parametric and non-parametric characteristic lines (CL) for every stock listed at the BVC and test whether or not the assumptions of linearity and stability of the betas are adequate for the data. We find evidence favorable to rejecting these assumptions in almost all of the cases at the 10% significance level. We show that when linearity of the CL does not hold the estimated linear betas are inconsistent. We estimate the betas semi-parametrically providing mean average square error optimal estimates. We show that these estimates, calculated as the average slope coefficient of the non-parametric CLs, are not constant when extreme market movements occur. Our non-parametric asset pricing model outperforms the linear one-factor CAPM as the estimated R^2 is significantly higher for all the stocks in our database.

Finally, we estimate the security market line (SML) for the returns of the firms listed in the Colombian BVC returns by regressing the expected asset return on its market risk approximated by the estimated semi-parametric betas. While the linearity of the stocks SMLs cannot be rejected at standard significance levels, the slope of the SMLs is negative under both linear and non-linear estimations[3]. This suggests there might be a small company effect, as proposed by Banz (1981) and Basu (1983), among others. Unfortunately, given that the BVC is a relatively small stock market in which only large Colombian firms participate, in this study we are not able to identify significant size-effects in order to estimate a non-parametric extension of Fama and French's three-factor-model (1996).

2. Methodology

For the estimation of the non-parametric CAPM we use local polynomial kernel regression. Given the differentiability ,the estimation of the derivative

[3]For the linear estimation the slope coeficiente is statistically equal to zero.

is straightforward. Minimizing:

$$\min_{\beta_0,\ldots,\beta_p} \sum_{i=1}^{n} \{Y_i - \beta_0 - \beta_1(X_i - \chi) - \ldots - \beta_p(X_i - \chi)^p\} K_H(\chi - X_i) \quad (1)$$

Yields:

$$\hat{\beta}^*(\chi) = \left(\hat{\beta}_0(\chi), \hat{\beta}_1(\chi), \ldots, \hat{\beta}_p(\chi)\right)^T \quad (2)$$

Where Y_i represents the excess return of $stock_i$[4], X_i stands for the excess return of the stock market[5], and $\beta(\chi)$ is the unknown vector of parameters we wish to estimate. Härdle et al. (2004) this can be solved by weighted least squares regression:

$$\hat{\beta}^*(\chi) = \left(X^T W X\right) X^T W Y \quad (3)$$

For this type of regression the using a polynomial of odd-order is more convinient than using one of even-order(Fan and Gijbels, 1996). For local linear regressions we use p=1.

The weighting matrix is defined by the Nadaraya-Watson weighting matrix $W_{Hi}(\chi)$:

$$W_{Hi}(\chi) = \frac{K_H(\chi - X_i)}{\frac{1}{n}\sum_{j=1}^{n} K_H(\chi - X_j)} \quad (4)$$

The appropiate bandwidth is estimated by a rule of thumb, which minimizes the average squared error (ASE) by cross-validation, where $\hat{m}(X_i)$ is the estimation of the local polynomial kernel regression at point X_i, and $m(X_i)$ is the true value of the function:

$$ASE(h) = ASE(\hat{m}_h) = \frac{1}{n}\sum_{i=1}^{n} \{\hat{m}_h(X_i) - m(X_i)\}^2 \quad (5)$$

As shown by Härdle et al. (2004) the selection of the kernel is secondary. For the estimation of the derivatives it is convenient to have differentiable a differentiable kernel. Therefore we use a Gaussian kernel.

[4]The excess return of $stock_i$ is calculated as the difference between the return of $stock_i$ and the risk-free interest rate

[5]The excess return of the stock market is calculated as the difference between the average stock market return and the risk-free interest rate.

For a measure of goodness of fit we use the classical definition of R-squared:

$$R^2 \equiv 1 - \frac{SSE}{SST} \tag{6}$$

Where $SST = \sum_{i=1}^{n}\{Y_i - \bar{Y}\}$ and $SSE = \sum_{i=1}^{n}\{Y_i - \hat{m}_h(X_i)\}$

In order to test the hypothesis of validity of the linear CAPM we use the following statistic:

$$\tau = \sum_{i=1}^{n}\{\hat{m}_h(X_i) - \hat{m}_{\hat{\theta}}(X_i)\}^2 \tag{7}$$

Where:

$$\hat{m}_{\hat{\theta}}(X_i) = \frac{\sum_{j=1}^{n} K_H(X_i - X_j) m_{\hat{\theta}}(X_j)}{\sum_{j=1}^{n} K_H(X_i - X_j)} \tag{8}$$

Which is simply the sum of the distance between the non-parametric estimator and the linear one . Because the rate of convergence of the linear and non-parametric estimator is different, we use the correction proposed by Härdle and Mammen (1993) which consist in the addition of an artificial bias to the linear estimation. For the estimation of τ's confidence intervals we use the wild bootstrapping approach proposed by Wu and Jeff (1986).

Finally, for the estimation of the betas we use the following formula, which is simply the average of the derivative estimation, as shown by Blundell and Duncan (1991):

$$\hat{\beta}^* = E(\hat{m}_h(\chi)) \approx \frac{1}{n}\sum_{i=1}^{n}\hat{\beta}_1(X_i) \tag{9}$$

The derivative estimation is simply the local polynomial regression of degree p=1 solved by weighted least squares.

3. Data

We collect daily data on the return of all the stock in the BVC for the 10-year period between 2002 and 2011. As a proxy of the market return we use the COLCAP ndex which measures capitalization weighted and adjusted using the stocks dividend. For the estimation of the risk-free rate we compute the sum of the return of the US one-month Treasury bill and the Colombian five-year credit default swap. All data was collected from Bloomberg.

4. Results and Discussion

In this study we estimate parametric and non-parametric CLs for every stock of the BVC (Figure 1 illustrates an example for one stock listed in the BVC), and test whether or not the assumption of linearity holds, using the test statistic proposed in Equation (7). We find evidence for the rejection of linearity in almost every stock. The null hipothesys of equality of the linear and non-parametric estimators can be rejected at the 10% significance level for all but seven stocks (see the fourth column labeled "NL(p-value)" in Table1). This finding provides evidence favoring the hypothesis that the linear especification of the CAPM is not adequate in this context, as linear betas are biased and inconsisted estimators of the true betas. The sign of the bibasof the linear estimators is not clear however, being positive for some stocks and for negative for others (compare $\hat{\beta}_{Kernel}$ with $\hat{\beta}_{Linear}$ in Table1).

There are significant gains in explanatory power and fit of using a non-linear CAPM. Notice in Table1 that the gain les between one and four percentage points. Expected returns presented in the third columna of Table1. were estimated using an E-GARCH, which allows for uneven tails in the distribution of the error terms.

Finally, we estimate the security market line (SML) for the returns of the firms listed in the Colombian BVC returns by regressing the expected asset return on its market risk approximated by the estimated semi-parametric betas (see Figure 2). While the linearity of the stocks SMLs cannot be rejected at standard significance levels, the slope of the SMLs is negative under both linear and non-linear estimations. This suggests there might be a small company effect. However, given that the BVC is a relatively small stock market in which only large Colombian firms participate, in this study we are not able to identify significant size-effects

Figures and tables

Figure1

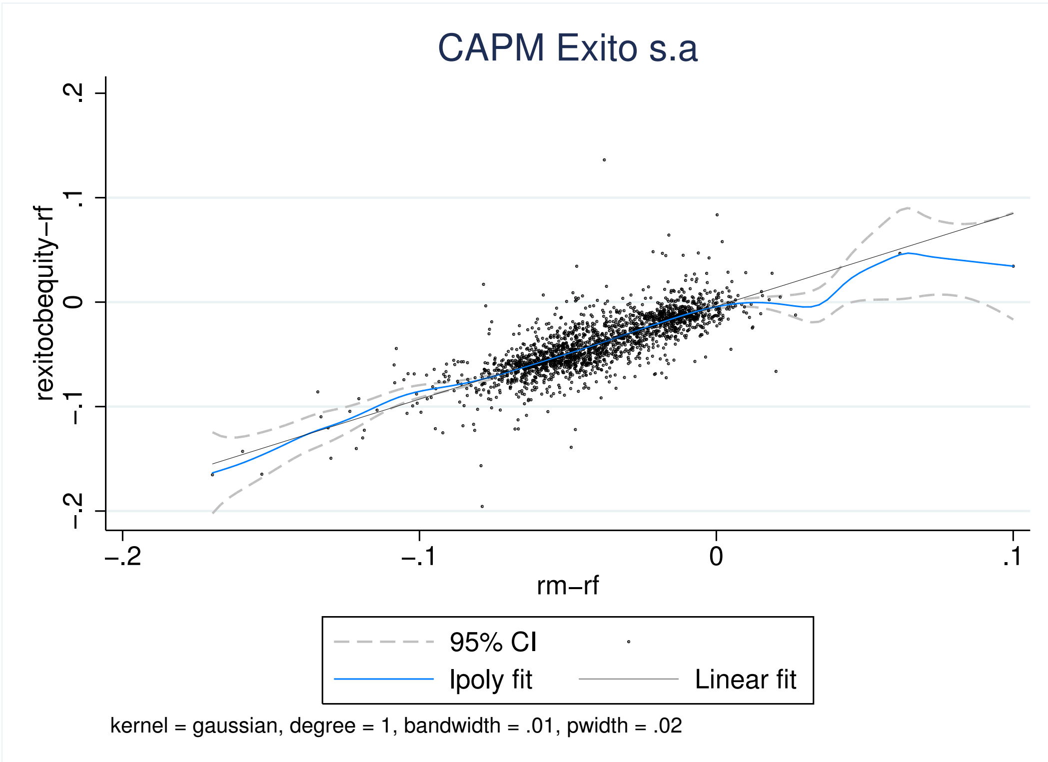
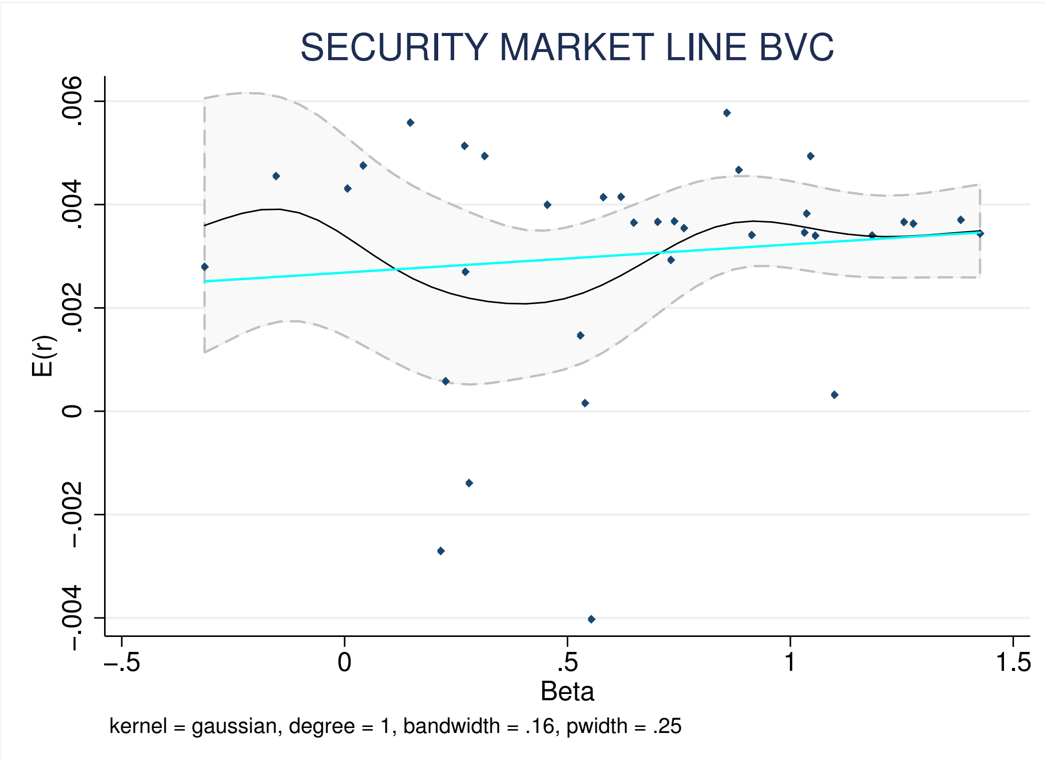


Figure2



SECURITY MARKET LINE BVC
E(r)
.006
.004
.002
0
−.002
−.004
−.5
0
.5
1
1.5
Beta
kernel = gaussian, degree = 1, bandwidth = .16, pwidth = .25

Table 1

Variable	Obs	E(r)	NL(p-value)	h^*	R^2_{Kernel}	$\hat{\alpha}_{Kernel}$	$\hat{\beta}_{Kernel}$	R^2_{Linear}	$\hat{\alpha}_{Linear}$	$\hat{\beta}_{Linear}$
AVAL	2015	0.13%	***0.000	0.014054	0.660208	-0.00978	0.740574	0.653351	-0.00319***	0.911***
BCOLO	2015	0.07%	0.117	0.012775	0.84236	0.018549	1.47298	0.836413	0.00178***	1.040***
BIOMAX	533	0.15%	***0.000	0.011611	0.047516	-0.00474	0.558874	0.042843	-0.00732***	0.368***
BMC	1045	0.12%	***0.007	0.011156	0.240271	-0.01905	0.125098	0.199937	-0.0112***	0.471***
BOGOTA	2015	0.10%	***0.000	0.015952	0.739587	-0.00848	0.778319	0.7343	-0.00428***	0.887***
BVC	1134	-0.05%	***0.000	0.029731	0.53123	-0.00276	0.885262	0.530873	-0.00249***	0.896***
CEMARGOS	2015	-0.07%	***0.000	0.017481	0.809897	0.00356	1.092037	0.806835	0.00115**	1.030***
CNEC	344	0.15%	**0.0213	0.016478	0.295827	0.00605	1.508118	0.289816	0.00487**	1.417***
COLINV	2015	0.16%	**0.0117	0.012031	0.76511	0.017461	1.443446	0.753897	0.000960	1.017***
COLTEJ	2015	0.29%	0.122	0.011138	0.276204	-0.05263	-0.34341	0.247028	-0.00810***	0.807***
CONCONC	234	-0.10%	**0.03	0.012756	0.158775	-0.00747	0.451124	0.151431	-0.00593***	0.562***
CORFICOL	1978	0.22%	***0.000	0.011103	0.598479	0.004355	1.072386	0.587838	-0.000471	0.947***
ECOPETL	1027	0.14%	***0.000	0.017169	0.496517	0.001937	1.035812	0.493842	0.000265	0.961***
EEB	599	0.02%	***0.006	0.010972	0.034204	-0.00783	0.346834	0.030402	-0.00429*	0.604***
ENKA	1072	-0.14%	0.111	0.009218	0.271741	-0.02899	-0.22955	0.227986	-0.00594***	0.756***
ETB	2015	0.00%	***0.000	0.018711	0.515311	-0.00861	0.791106	0.510727	-0.00422***	0.904***
EXITO	2015	0.04%	***0.007	0.012793	0.643206	-0.01023	0.728847	0.635815	-0.00404***	0.889***
FABRI	2015	-0.18%	***0.000	0.017198	0.504558	0.011585	1.301197	0.501076	0.00238**	1.063***
GRUPOSUR	2015	0.24%	***0.000	0.009476	0.864126	0.005686	1.141041	0.851754	0.00331***	1.080***
INTBOL	2015	0.00%	***0.002	0.013726	0.533959	-0.00629	0.837783	0.514742	-0.00579***	0.851***
INVARGOS	2015	0.07%	***0.007	0.010151	0.811235	0.020042	1.518022	0.800829	0.00110**	1.029***
ISA	2015	0.10%	***0.006	0.011347	0.794482	0.003547	1.081264	0.791056	-0.000849*	0.968***
ISAGEN	1055	0.05%	0.206	0.012813	0.656164	-0.00181	0.923965	0.650842	-0.00470***	0.798***
MINEROS	2015	0.13%	*0.053	0.013337	0.487106	-0.01195	0.673539	0.477529	-0.00697***	0.802***
NUTRESA	2015	0.05%	**0.015	0.014459	0.770181	0.004969	1.129314	0.763398	-0.00288***	0.926***
ODINSA	1719	0.09%	***0.000	0.010706	0.358903	-0.03542	0.001498	0.306862	-0.0125***	0.633***
PFAVAL	135	-0.03%	***0.000	0.008753	0.189937	-0.01172	0.175987	0.115012	-0.00818***	0.416***
PFAVTA	137	-0.51%	***0.000	0.008761	0.316611	-0.00559	0.699603	0.27372	-0.00571***	0.692***
PFBCOLO	2015	0.07%	*0.065	0.013021	0.772331	-0.01325	0.650658	0.766818	-0.000409	0.982***
PFBHELMB	1012	-0.01%	*0.094	0.015926	0.491529	-0.00733	0.689567	0.480097	-0.00518***	0.788***
PFCORCOL	2015	0.25%	***0.000	0.011311	0.326999	-0.03603	0.034011	0.29287	-0.0110***	0.682***
PFDAVVND	292	-0.10%	***0.001	0.008768	0.14155	-0.00466	0.531355	0.120791	-0.00244	0.700***
PREC	495	0.15%	***0.000	0.008811	0.399775	0.002929	1.258215	0.391999	0.00747***	1.593***
SIE	547	-0.10%	***0.000	0.009555	0.052207	-0.01212	0.191814	0.046021	-0.00834***	0.468***
TABLEMA	2015	0.22%	*0.068	0.015028	0.454358	-0.02446	0.353636	0.446832	-0.00171	0.941***

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