Estimating financial institutions' intraday liquidity risk: a Monte Carlo simulation approach

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Estimating financial institutions' intraday liquidity risk: a Monte Carlo simulation approach[†]

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Abstract

The most recent financial crisis unveiled that liquidity risk is far more important and intricate than regulation have conceived. The shift from bank-based to market-based financial systems and from Deferred Net Systems to liquidity-demanding Real-Time Gross Settlement of payments explains some of the shortcomings of traditional liquidity risk management.

Although liquidity regulations do exist, they still are in an early stage of development and discussion. Moreover, no all connotations of liquidity are equally addressed. Unlike market and funding liquidity, intraday liquidity has been absent from financial regulation, and has appeared only recently, after the crisis.

This paper addresses the measurement of Large-Value Payment System's intraday liquidity risk. Based on the generation of bivariate *Poisson* random numbers for simulating the minute-by-minute arrival of *received* and *executed* payments, each financial institution's intraday payments time-varying volume and degree of synchrony (i.e. timing) is modeled.

To model intraday payments' uncertainty allows for (i) overseeing participants' intraday behavior; (ii) assessing their ability to fulfill intraday payments at a certain confidence level; (iii) identifying participants non-resilient to changes in payments' timing mismatches; (iv) estimating intraday liquidity buffers. Vis-à-vis the increasing importance of liquidity risk as a source of systemic risk, and the recent regulatory amendments, results are useful for financial authorities and institutions.

Keywords: Payments Systems, Intraday, Liquidity Risk, Bivariate *Poisson* process, Monte Carlo Simulation, Liquidity Buffer, Oversight.

JEL Classification: C15, C63, E47, G17, D81.

[†] The opinions and statements are the sole responsibility of the author and do not necessarily represent neither those of Banco de la República nor of its Board of Directors. Results are illustrative; they may not be used to infer credit quality or to make any type of assessment for any financial institution. The author is indebted to Clara Machado for the numerous and vital discussions that supported the model's design and the document's final version. Valuable comments and suggestions were provided by Fernando Tenjo, Joaquín Bernal, Freddy Cepeda and Fabio Ortega. LVPS data was processed with assistance from Freddy Cepeda and Fabio Ortega. As usual, any remaining errors are the author's own.

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1. Introduction

It is widely accepted that liquidity risk mismanagement played a key role in the most recent episode of global financial crisis. Literature has recommended improving liquidity risk management by imposing and monitoring liquidity requirements on systemically important banks and broker dealers (French et al., 2010), or designing liquidity buffers in order to mitigate systemic risk (Capel, 2011; IMF, 2010b; Borio, 2009; Tirole, 2009).

Although liquidity regulations and tools do exist, they are still in an early stage of development and discussion (IMF, 2010a; Tucker, 2009). Moreover, no all connotations of liquidity are equally addressed by risk literature or regulation. Liquidity has focused on market and funding liquidity, where both correspond to the ability to generate cash from balance sheets' liabilities and assets positions, respectively, whereas liquidity risk has traditionally focused on measuring mismatches between them (e.g. short-term liabilities and liquid assets).[2]

As acknowledged by Ball et al. (2011), prior to the recent financial crisis regulators did not focus on intraday liquidity risk and there were no standard monitoring or liquidity management measures in place; only after the crisis authorities have begun to focus on intraday liquidity. Two main structural shifts may explain the new emphasis on intraday liquidity: (i) the shift from bank-based to market-based financial systems, and (ii) the evolution of payment systems from Deferred Net Systems to liquidity-demanding Real-Time Gross Settlement (RTGS) systems.

It is important to acknowledge that these structural shifts have not resulted from shocks; they are the result of a continuous and protracted evolution of financial markets. However, because prudential regulation has ignored these structural shifts for decades, the regulatory challenge is substantial: designing an intraday liquidity risk management framework.

Among the four typical stages of risk management (i.e. identifying, assessing, monitoring and mitigating risk) this paper focuses on the second one. The approach herein presented for assessing Large-Value Payment System's intraday liquidity risk is based on the generation of bivariate *Poisson* random numbers for simulating the minute-by-minute arrival of received (incoming) and executed (outgoing) payments. In this sense, following Ball et al. (2011), the identified source of intraday liquidity risk herein modeled is the timing mismatch between incoming and outgoing payments, which may lead to significant intraday liquidity needs.

[2] For instance, Tirole (2008) refers to funding liquidity as how much can be raised on the liability side of the balance sheet, whilst market liquidity refers to the asset side, with prudential measurements of liquidity usually consisting of measuring some mismatch between short-term liabilities and liquid assets.

This Monte Carlo simulation procedure is capable of modeling the intraday-dependency governing the joint arrival of both types of payments, along with their value; this is, the simulation procedure is able to capture the degree of synchrony (i.e. the timing) attained by each financial institution when receiving and executing payments (i.e. the virtuous circle of coordinated actions by settlement agents), where such synchrony and the volume of payments varies throughout the day.

The main outcome of the proposed procedure is estimating a risk measure or metric such as an intraday liquidity Value at Risk (VaR) that is able to answer a rather specific question: **what is an institution's maximum intraday liquidity needs at a defined confidence level?** An answer to this question may be suitable for (i) overseeing participants' intraday behavior; (ii) assessing their ability to fulfill intraday payments at a certain confidence level; (iii) identifying participants non-resilient to changes in payments' timing mismatches; (iv) estimating intraday liquidity buffers. Vis-à-vis the most recent amendments to financial regulation and the increasing importance of liquidity risk as a source of systemic risk, results are useful for financial authorities and institutions tackling the challenge of managing intraday liquidity risk.

This document is structured as follows. The second section briefly addresses the increasing relevance of intraday liquidity risk management. The third section describes the intuition and procedure behind the proposed approach to simulating the minute-by-minute liquidity; Exhibit A contains a comprehensive technical explanation on the Monte Carlo simulation algorithm. The fourth section presents preliminary results and analysis for a set of institutions participating in Colombia's large-value payment system (LVPS). The fifth section presents some final comments on the approach, its usefulness and the challenges ahead.

2. The increasing relevance of liquidity and intraday liquidity risk management

The recent financial crisis highlighted the need to improve liquidity risk management, including the management of intraday liquidity risk (Ball et al., 2011). Liquidity is by no means a new concept; however, it is still an elusive one. It comprises several dissimilar connotations, such as market, funding or intraday liquidity. Although these connotations of liquidity allow for a fairly clear theoretical distinction, in practice they are entangled, particularly under stress scenarios. In this sense, all connotations of liquidity should be equally addressed by prudential regulation.

Notwithstanding the importance of properly addressing liquidity risk and its connotations, related regulation is scarce when compared to solvency's. The contemporary momentum of liquidity and intraday liquidity regulation emerges from the recent global financial crisis, which has unveiled financial markets' structural shifts that have increased the relevance of designing a proper prudential regulatory framework.

Two such structural shifts are commonly acknowledged by literature: (i) the shift from bank-based to market-based financial systems, and (ii) the shift from Deferred Net Systems to liquidity-demanding Real-Time Gross Settlement (RTGS) of payments. As explained below, the former has increased the importance of all connotations of liquidity risk, whereas the latter has emphasized the relevance of intraday liquidity risk.

2.1. The relevance of liquidity risk in market-based financial systems

The underdevelopment of liquidity regulation results from traditional focus on solvency, which is the legacy of banking runs dominating systemic risk origins since the outbreak of the Great Depression. Consequently, liquidity risk has evaded prudential regulation.

The best example of the absence of liquidity risk management is the regulatory approach by the Basel Committee on Banking Supervision (BCBS-BIS). As documented by Goodhart (2008), the BCBS-BIS's original goal was to reverse the downward trend in capital and liquidity for the main international commercial banks; however, for reasons yet-to-discover, this downward trend was reversed for capital, but not for liquidity. Thus, according to Eichengreen (2008), some Basel Accord's critics argue that its focus on capital adequacy (i.e. lack of liquidity requirements) encouraged regulators to neglect the importance of liquidity in their supervisory activities.

Additionally, not only BCBS's regulation disregards liquidity risk management, but it is dedicated to banks only, where its scope is measuring mismatches between short-term liabilities (e.g. deposits) and liquid assets (e.g. loans and investment portfolios). In today's financial markets, with many functions that defined banks' traditional domain increasingly performed by securities markets and non-bank market participants (Kambhu et al., 2007)[3], focusing on solvency and ignoring liquidity is highly unsafe from a prudential point of view. Therefore, the structural shift from bank-based to market-based systems and the evolution of financial infrastructures, where markets' and assets' liquidity has become as important as banks' solvency, explains to some extent the increasing relevance of liquidity risk management.

Hence, as a consequence of the nature of the global financial crisis, Ackermann (2008) concludes that (i) in our capital-based financial system, which has developed as a result of disintermediation and credit risk transfer, liquidity is far more important than in a purely bank-based financial system, and (ii) better liquidity management –rather than higher capital buffers- is likely to provide the right answer.[4]

[3] This scheme, in which markets and non-bank participants involve in credit intermediation activities traditionally performed (and regulated to be performed) by banks, is commonly referred as the "parallel banking system" or "shadow banking system" (Krugman, 2009; Acharya et al. 2009).

[4] Ackerman (2008) goes further to state that higher capital requirements may have an adverse effect: if they are too onerous, more activities will be pushed to unregulated parts of the financial system.

2.2. The relevance of intraday liquidity risk in RTGS payment systems

A second structural change behind the increasing relevance of liquidity risk is the evolution of financial markets from Deferred Net Systems to liquidity-demanding Real-Time Gross Settlement (RTGS) of payments.[5] As recognized by the Committee on Payment and Settlement Systems (CPSS-BIS, 1997), this structural shift was encouraged by banking authorities in an attempt to limit settlement and systemic risk in the interbank settlement process, and to contribute to the reduction of the settlement risk in securities and foreign exchange transactions. However, as pointed out by Bernal (2009) and Bech and Soramäki (2002), in RTGS systems the reduction in settlement risk is traded off against increased liquidity requirements so that the payment system becomes more reliant on the virtuous circle of coordinated actions by participating settlement agents and, therefore, increased liquidity risk.[6] Following Ball et al. (2011), this means that whereas RTGS systems financial institutions can re-use liquidity from incoming payments to fund outgoing payments, timing mismatches between incoming and outgoing payments can lead to significant –intraday- liquidity needs.

Consequently, as documented by Leinonen and Soramäki (2004), interest in intraday liquidity results from payment systems' development and shrinking delivery times: before the 1990s operations were strictly on the daily level and intraday liquidity had no significance; as the processing of payments has been speeded up and central banks have converted to RTGS, intraday liquidity has received increasing emphasis.

For instance, the emergence of intraday liquidity risk is rather clear in the evolution of BCBS-BIS's approach to liquidity risk. What the BCBS-BIS (2000) regarded as the "Principles for the assessment of liquidity management in banking organizations" referred to short-term liquidity management in a day-to-day basis for banks reliant on short-term funding, and in a one-to-three-months-ahead basis for other banks non-reliant on short-term funding; intraday liquidity risk was mentioned but wasn't regarded as being decisive.

Recently, the "Principles for the management and supervision of liquidity risk" (BCBS-BIS, 2008) explicitly included the management of intraday liquidity risk as a principle on its own (Principle 8), where its purpose is meeting payment and settlement obligations on a timely basis under both normal and stressed conditions in order to contribute to the smooth

[5] A DNS system effects the settlement of obligations or transfers between or among counterparties on a net basis at some later time. A RTGS system consists of the continuous (real-time) settlement of funds or securities transfers individually on an order-by-order basis (without netting); the processing of instructions is carried out on an individual basis at the time they are received rather than at some later time (CPSS-BIS, 2003). Bech (2008) documents that the number of central banks that implemented RGTS systems increased from three in 1985 to 93 at the end of 2006. According to World Bank (2011), from a total of 142 countries surveyed as of December 2010, 116 (83%) have an RTGS system for their LVPS.

[6] Such increasing demand for intraday liquidity is also documented by Bech (2008), Rochet (2008) and CPSS-BIS (1997).

functioning of payment and settlement systems.[7] Furthermore, even more recently, the document entitled “Basel III: International framework for liquidity risk, measurement, standards and monitoring” (BCBS-BIS, 2010) acknowledges that intraday liquidity needs may not be covered by Basel III’s new Liquidity Coverage Ratio (LCR)[8], and states that the BCBS-BIS is reviewing if (and how) intraday liquidity should be addressed.

Another example of the nowadays increasing focus of regulation on intraday liquidity risk is the UK’s Financial Services Authority (FSA) novel liquidity adequacy regulation (FSA PS 09/16), which considers intraday liquidity as a key risk driver in its new liquidity regime. As in BCBS-BIS’s (2008) principles, FSA’s aim is to ensure that firms are able to meet their payment and settlement obligations on a timely basis in both normal and stressed conditions, emphasizing that this is important for the firm, the firm’s counterparties and the smooth functioning of payment and settlement systems as a whole. It is worth noting that unlike BCBS-BIS’s, FSA’s liquidity regulation is not intended for banks only, and refers to “firms”, with this term comprising banks, building societies, and some types of investment firms; according to J.P. Morgan (2010), this new regulatory regime includes for the first time all FSA-regulated broker-dealer firms under formal liquidity resource supervision.

In the author’s view, concurring with Hervo (2008), structural developments in the financial industry have led to a clear trend in shortening time horizon of liquidity risk and liquidity management. As Hervo quotes regarding nowadays financial markets, “my short-term is intraday, my medium-term is overnight and my long-term is one week”.

Even though payment and settlement systems have received relatively little attention from financial market researchers (Leionen and Soramäki, 2004), intraday liquidity literature has gained momentum, especially with works by Bech (2008), Leionen (2007) and Koponen and Soramäki (1998). In the Colombian case LVPS’ intraday liquidity has been addressed by Bernal et al. (2011), Bernal (2009) and Bernal and Merlano (2005), whereas some models based on LVPS’ payments data for payments simulation, connectedness assessment and the identification of systemic importance, have been recently developed (Cepeda, 2008; Machado et al., 2009; León et al., 2011; León and Machado, 2011).

However, literature acquired by the author does not address intraday liquidity risk explicitly, and lacks of risk measures or metrics (such as an intraday liquidity Value at Risk or Expected Shortfall) that are able to answer a rather specific question: what is an institution’s maximum intraday liquidity needs at a defined confidence level? Next two sections deal with how to address this type of question.

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[7] The BCBS-BIS (2000) document only referred to intraday liquidity four times, with only one related to liquidity management (Principle 5). The BCBS-BIS (2008) document makes about sixty referrals to intraday liquidity (Principles 3,5,9,10), and devotes Principle 8 to recognizing its importance within a bank’s broader liquidity management strategy and its contribution to systemic risk via the smooth functioning of the payment system.

[8] The LCR is a standard measure that aims to ensure that a bank maintains an adequate level of unencumbered, high-quality liquid assets that can be converted into cash to meet its liquidity needs for a 30 calendar day time horizon under a significantly severe liquidity stress scenario specified by supervisors. (BCBS-BIS, 2010)

3. Monte Carlo intraday liquidity simulation

Monte Carlo simulation methods are suitable for addressing problems of almost any degree of complexity, and can easily address factors that most other approaches have difficulty with, and become more attractive as the complexity and/or dimensionality of the problem increases (Dowd, 2005). Therefore, as the case in hand comprises several factors to be simultaneously modeled in order to deal with a financial institution's intraday liquidity uncertainty, Monte Carlo provides a compelling approach. Next two subsections contain an explanation on the intuition behind using the Monte Carlo simulation approach to deal with intraday liquidity uncertainty and on the designed procedure, respectively.

3.1. Dealing with intraday liquidity uncertainty

According to Principle 8 of BCBS-BIS's (2008) "Principles for Sound Liquidity Risk Management and Supervision", a strategy to achieve intraday liquidity management objectives comprises six elements. Elements one, two and six are:[9]

- Financial institutions should have the capacity to measure expected daily gross liquidity inflows and outflows, anticipate the intraday timing of these flows where possible, and forecast the range of potential net funding shortfalls that might arise at different points during the day.
- Financial institutions should have the capacity to monitor intraday liquidity positions against expected activities and available resources.
- Financial institutions should be prepared to deal with unexpected disruptions to its intraday liquidity flows.

Additionally, according to BCBS-BIS's (2008) Principles 10 and 11, liquidity stress tests and contingency plans should observe the following elements:

- Stress tests should consider the implication of the scenarios across different time horizons, including on an intraday basis.
- Financial institutions' stress tests should consider how the behavior of counterparties would affect the timing of cash flows, including on an intraday basis.
- The size of financial institutions' liquidity cushion also should reflect the potential for intraday liquidity risks.

A common approach suitable for tackling the quantitative demands imposed by these elements is Monte Carlo simulation. All elements –specially the underlined- converge to

[9] The reader should be aware that these principles by BCBS-BIS (2008) are limited to banks. Taking into account the importance of the non-banking institutions in financial markets, the author avoids limiting the application of these principles to banks; authors refer to "financial institutions" instead of banks. Please note that all underlined emphasis is the author's.

modeling financial institutions' intraday payments uncertainty (i.e. expected liquidity, potential shortfalls, liquidity scenarios, etc.), whereas dealing with uncertainty is the whole point of building a Monte Carlo model (Hubbard, 2009).[10]

Traditional Monte Carlo methods in Finance are aimed at repeatedly simulating the random process governing the returns of an asset or instrument, where the governing process is the Geometric Brownian Motion, along with some other variations to this customary process.[11] Such typical application of Monte Carlo is rather straightforward and flexible since the random process is easily simulated (i.e. there is only one stochastic process for each asset), even when considering the dependence across different assets.

However, simulating intraday liquidity is more intricate. In order to measure expected intraday liquidity inflows and outflows (BCBS-BIS's Principle 8) two different stochastic processes are to be simulated: one governing the inflows (arrival of received payments), other governing the outflows (arrival of executed payments). Because of the type of behavior to be modeled (arrival of payments), Geometric Brownian Motion is unsuitable, and an arrival-type process has to be used: each process has to be generated with a *Poisson* process.

Furthermore, since the degree of synchrony between the arrival of received and executed payments is critical for modeling the virtuous circle of coordinated actions by agents typical of RTGS systems (Bernal, 2009), the simulation of the random processes has to capture executed and received payments' dependence: each process has to be generated from a bivariate *Poisson* process. Paraphrasing Ball et al. (2011), this would allow for modeling the timing mismatches between incoming and outgoing payments that lead to an increase in the amount of intraday liquidity required to continue making payments in a timely fashion.

Additionally, the size (i.e. monetary value) of the payments has to be modeled along with the frequency of arrival, where the size of payments may not be distributed as a Normal variable and where samples may not be large enough to make –parametric- distributional assumptions. Finally, since the behavior of the arrivals, their dependence and their monetary value are not constant throughout the intraday, the simulation's factors or parameters have to be time-dependent.

These considerations, altogether, demand a Monte Carlo simulation model significantly different from its standard implementation in Finance. Next subsection addresses the procedure behind the implementation of the Monte Carlo model suitable for the paper's purposes; Exhibit A contains a comprehensive technical explanation on the Monte Carlo

[10] Please note that the term "uncertainty" is not used in the Knightian sense (Knight, 1921), where uncertainty is related to cases in which quantifying probabilities is not possible. In this paper the term "uncertainty" is as in Hubbard (2009), where it corresponds to the lack of complete certainty about the true outcome, with uncertainty being measurable (contrary to Knight's use of the term) by the assignment of probabilities to various outcomes.

[11] Some of the most popular variations are jump-diffusion models (Merton, 1976; León, 2009) and the Fractional Brownian Motion (Mandelbrot and van Ness, 1968; León and Reveiz, 2011).

simulation algorithm, with emphasis on the simulation of bivariate *Poisson* random variables for the intraday arrival of executed and received payments.

3.2. Implementation

The model could be concisely described as the joint and time-dependent simulation of a bivariate *Poisson* processes for intraday executed and received payments, and their monetary value. The core of the model is the Monte Carlo simulation of bivariate *Poisson* random variables for the intraday arrival of executed and received payments, whereas simulating their monetary value by means of bootstrap historical simulation is subordinated to their arrival. The implementation of the herein proposed model is done in Matlab®. It consists of an algorithm executing the procedure depicted in Figure 1.

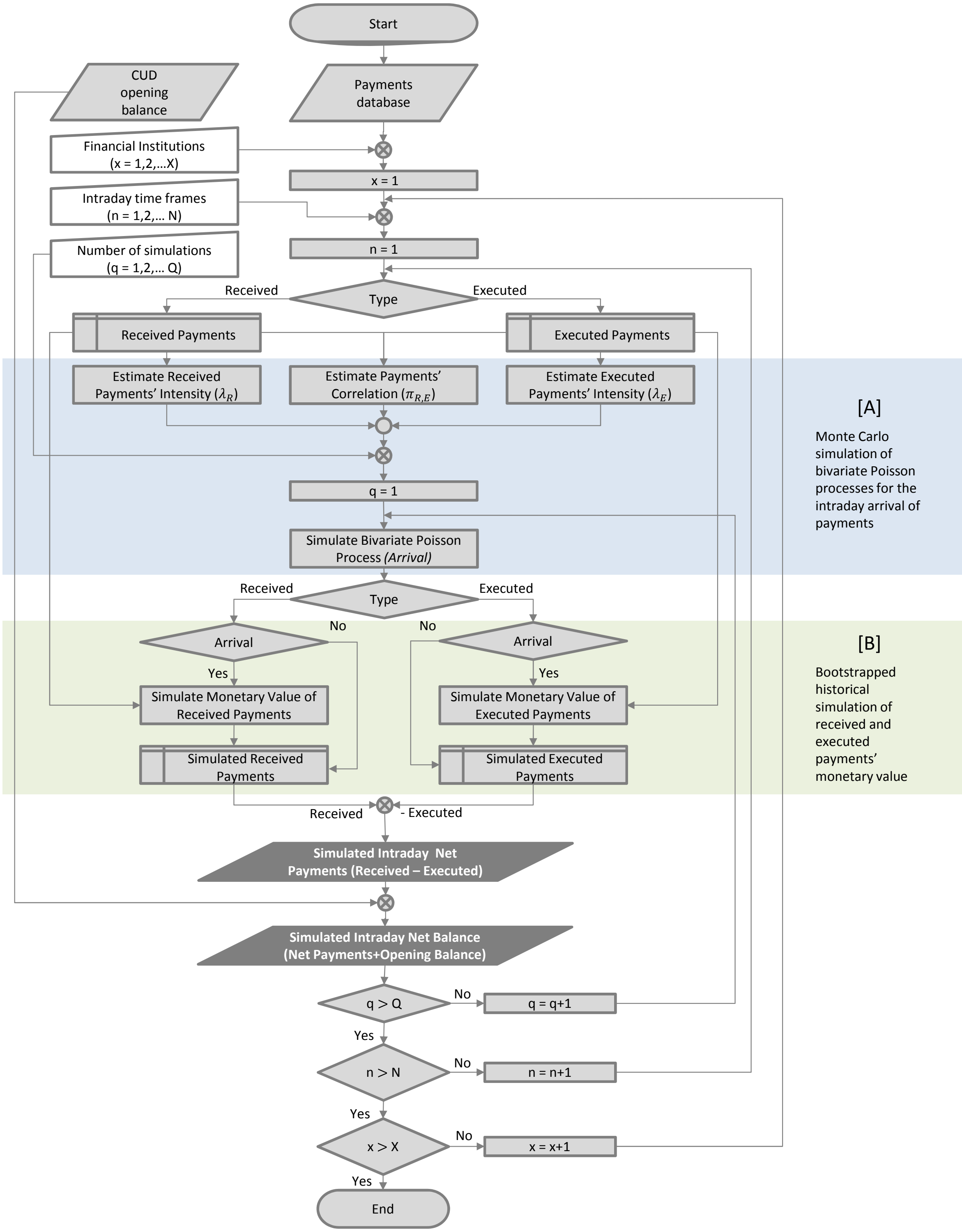
The algorithm consists of five main inputs: two databases, for LVPS' payments and opening balances, which contain information for all participating financial institutions during one day; three manual inputs, which select the financial institutions to analyze, define the intraday time frames to use, and the number of simulations to generate.

After reading the inputs, the algorithm selects the first financial institution (e.g. Bank A) and the first time frame to use (e.g. from 7:00am to 8:00am). According to this selection the LVPS' orders and opening balances databases are filtered. Afterwards the algorithm classifies both types of payments (i.e. executed and received) for the selected institution and time frame.

Afterwards (Section [A] of Figure 1), the Monte Carlo simulation of the payments' arrival starts by estimating the intensity of the executed and received processes (λ_E and λ_R), along with their correlation ($\pi_{(E,R)}$). After estimating the parameters required for the simulation of the bivariate *Poisson* process for the intraday arrival of payments, the algorithm generates the first of the simulations to make for this financial institution, for the selected time frame. Based on the algorithm designed by Yahav and Shmueli (2011) for simulating bivariate *Poisson* processes, the algorithm yields a minute-by-minute two-dimensional vector where the simulated joint-occurrence of executed and received payments is registered.[12]

[12] Exhibit A contains a comprehensive technical explanation on the Monte Carlo simulation algorithm.

Figure 1
Algorithm's Procedure (Flow Chart)



Source: author's design

Subsequently, after simulating the first path of arrivals for the selected financial institution and time frame, the algorithm employs the bootstrapped historical simulation method for generating the monetary value of each of the arrivals previously simulated (Section [B] of Figure 1). This is, each time the algorithm generated the arrival of an executed (received) order the algorithm employs a Uniform random number generator to resample –with replacement- from the historical record of monetary values of executed (received) payments that the selected financial institution made during the selected time frame. This process yields a minute-by-minute two-dimensional vector where the simulated value of executed and received orders is registered.

Next, the monetary value of received and executed payment orders is subtracted. The result is the simulated intraday net payments. If the opening balance is added the result is the simulated intraday net balance for the selected financial institution and time frame. Both results are the main building blocks of the model: a single simulation of the minute-by-minute liquidity balance (with and without opening balance) for a selected financial institution and time frame. In order to make all the simulations defined by the user, and to cover all financial institutions and time frames, the algorithm performs three loops.[13]

4. Preliminary results[14]

Based on a day of transactions from the February 2012 LVPS's database, this section applies the herein proposed model to simulate the intraday liquidity of two selected financial institutions. The financial institutions selected belong to the top-ten systemically important institutions according to León and Machado (2011), and they correspond to a commercial banking firm (henceforth referred as CBF1) and a broker-dealer firm (BDF1). The selected time frame corresponds to an hour-by-hour breakdown of the Colombian LVPS working hours (i.e. 7:00 – 20:00). The selected number of simulations for all calculations is 1000, but figures display 100 simulations for comprehensibility purposes.

Figure 2 displays the observed minute-by-minute intraday payments for selected institutions CBF1 and BDF1; executed (received) payments appear with negative (positive) sign. It is clear that modeling the intraday liquidity as a non-time-varying process would be inconvenient. The intraday liquidity of financial institutions is heavily dependent on (i) the schedule or timeline designed by the administrator of the LVPS and by all other infrastructures that use the LVPS as their settlement system, and (ii) the design of the LVPS, which may be DNS or RTGS, and may also include liquidity saving mechanisms and other types of rules that affect decision-making by the system's participants.

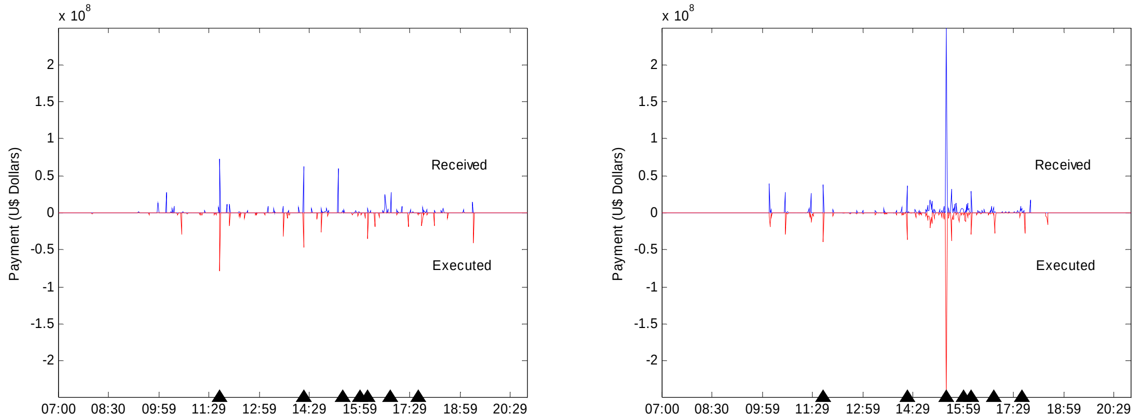
[13] This is a technical drawback of the proposed model. In the case of Colombia's LVPS, where more than a hundred of financial institutions participate directly in the LVPS, with 13 time frames (i.e. hourly, from 7:00 to 20:00), with 1000 simulations, the whole procedure for a single day consists of about 2 million registers.

[14] *Due to disclosure issues the numerical data of this section corresponds to an escalated version of actual data. The escalating procedure consisted of using a multiplying factor in the* 1 ± 0.1 *range to multiply all data in order to assure financial institutions' anonymity without sacrificing congruence and comparability.*

Figure 2

CBF1
Opening balance: US $ 750.0 million
Total executed/received: US $ 498.7 / 538.1 million

BDF1
Opening balance: US $ 0.3 million
Total executed/received: US $ 872.8 / 871.3 million



Received (executed) payments have a positive (negative) sign. Triangles along the x-axis identify the presence of Central Bank's Central Securities Depository (CSD) and LVPS liquidity saving mechanisms.

Source: author's calculations, data from the LVPS

Figure 2 confirms that the pattern of intraday payments is determined in most part by the liquidity saving mechanisms of the Central Bank's Central Securities Depository (CSD) and the LVPS (triangles along the x-axis); the former consists of optimization algorithms running at 11:50, 14:20, 15:30, 16:15, 16:55, 17:45, while the latter running at 16:00.[15] The CSD's optimization algorithms are key to the intraday liquidity of financial institutions since the central government local bond market (i.e. the TES market) is the most active and liquid in the Colombian financial system, where CSD TES-related payments account for about 50% of LVPS' total payments (BR, 2011).

It is also clear that for the selected date both institutions display a distinctive pattern of intraday liquidity. Beyond any particularity arising from the choice of the date, these results arise from their characteristic business and regulatory framework. For instance, commercial banks have to comply with reserve requirements, whereas broker-dealer firms don't have to; commercial banks trade bonds and foreign exchange on their own account only, whereas broker-dealer firms trade on their own account and on behalf of clients, profiting from brokerage via commissions; broker-dealer firms are allowed to trade stocks, whereas commercial banks are not.

It is important to stress that such characteristics may explain two distinctive features of the selected financial institutions. First, the intraday liquidity pattern is more concentrated at the end of the day for BDF1. This pattern results from (i) the lack of reserve requirement and the corresponding low level of opening balance; (ii) its reliance on the liquidity arriving from the virtuous circle of coordinated actions by other settlement agents; and (iii) on the

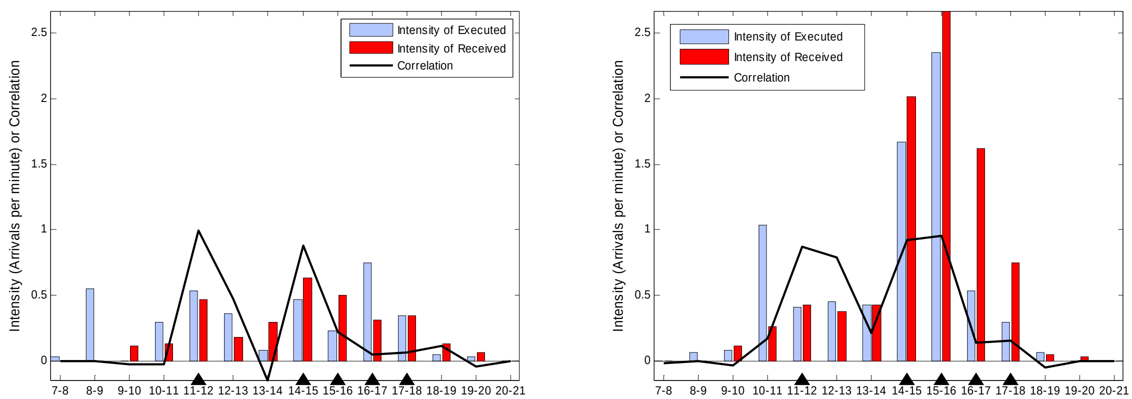
[15] For an introduction to the design and functioning of the Colombian RTGS payment system and its timeline please refer to Bernal et al. (2011), BR (2011) and Bernal and Merlano (2005).

prominence of liquidity saving mechanisms provided by the CDS for TES-related transactions. In the other hand, CBF1, which holds high levels of opening balance due to reserve requirements (i.e. around 2,700 times the BDF1's), may execute payments earlier[16]. Second, the size of payments is also distinctive, with payments executed and received by the BDF representing about 1.8 times the CBF's.

The estimated parameters for the Monte Carlo simulation of bivariate *Poisson* process for the intraday arrival of payments for both selected financial institutions are displayed in Figure 3.

Figure 3
Estimated intensities (λ_E and λ_R) and correlation ($\pi_{(E,R)}$) parameters

CBF1 BDF1



Triangles along the x-axis identify the presence of Central Bank's Central Securities Depository (CSD) and LVPS liquidity saving mechanisms.

Source: author's calculations, data from the LVPS

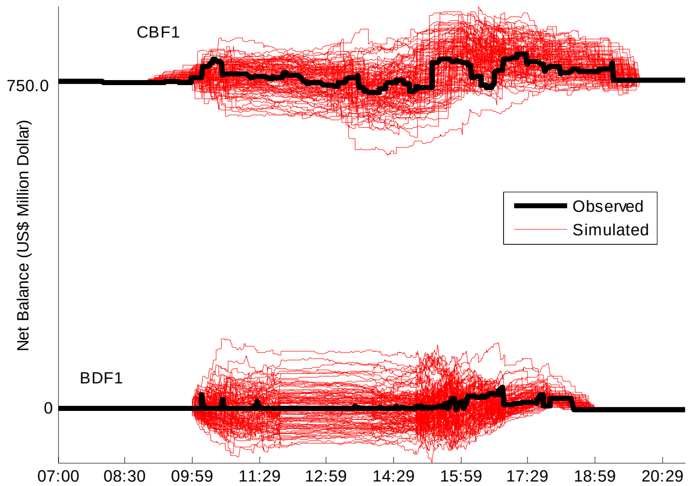
Based on the estimated parameters, the result of simulating 100 times the minute-by-minute intraday liquidity of the two selected financial institutions with hourly time frames is exhibited in Figure 4[17]. This figure corresponds to the simulated intraday net balance; this is, it considers the opening balance of each institution.[18]

[16] As in 52% of the countries using an RTGS system (World Bank, 2011), local participants may use all their reserve requirements balance for intraday settlement purposes. Reserve requirements in Colombia are based on the daily averaging of reserve requirements within a two-week reserve maintenance period; according to Gray (2011), averaging of reserve requirements is an effective way of enhancing liquidity management, but the reserve maintenance period need to be at least two weeks long.

[17] To achieve a fair approximation of the correlation of the simulated bivariate Poisson series to the target correlation is the mainstay of the bivariate Poisson process and the model. As presented in Exhibit B, the mean of the correlation of the simulated bivariate Poisson series replicates the target correlation, whereas the simulated correlation of each series disperses around the target correlation as expected.

[18] Under certain circumstances it would be useful not to consider the opening balance; for example, to analyze the ability of an institution to face executed payments with received payments (i.e. the virtuous circle of liquidity). Other analysis may be available by excluding some intraday funding sources; for instance, this would allow for analyzing the reliance of an institution on Central Bank's or on monetary market's intraday liquidity. In forthcoming papers the authors expect to implement such variations to the model.

Figure 4
Observed and simulated intraday net balance paths for CBF1 & BDF1



CBF1 opening balance: US $ 750.0 million. BDF1 opening balance: US $ 0.3 million.

Source: author's calculations

Taking into account that the Colombian LVPS is a RTGS system where no intraday overdraft is allowed, it is meaningful to find that intraday liquidity paths simulated for the CBF1 remain positive for any scenario; this is, under the herein proposed model and assumptions, the CBF would not exhaust its liquidity, and will be able to fulfill its intraday payments without delays or queuing. The rationale behind this finding is the existence of the reserve requirement for commercial banks, which serves as an important source of liquidity for this type of financial institutions, as in Bernal et al. (2011).

Meanwhile, because the opening balance of BDF1 is significantly lower than CBF1's (about 0.04% of CBF1's), BDF1's simulated paths where its liquidity is exhausted is representative. This also concurs with findings by Bernal et al. (2011) regarding the reliance of non-banking institutions (i.e. with no reserve requirements) on the virtuous circle of intraday liquidity, along with the presence of significantly higher turnover ratios for BDFs when compared to CBFs; for the two selected financial institutions, the turnover ratio (i.e. the ratio of payments and opening balance) reached 0.7 and 3,181.2 for CBF1 and BDF1, correspondingly.

Since the simulated minute-by-minute balance of received and executed orders is available, it is possible to estimate a Value at Risk (VaR) type-of-measure of intraday liquidity risk[19]. This measure would answer the following question: what is an institution's maximum intraday liquidity needs at a defined confidence level? Figure 5 displays the answer to that question for three different intraday scenarios[20]: the maximum intraday

[19] Estimating other risk measures (e.g. Expected Shortfall) is straightforward under the herein proposed model.
[20] Please note that these scenarios correspond to the *time horizon* in a typical VaR model.

liquidity needs (i) within the day; (ii) by the end of the day (i.e. 17:00); and at a significant moment within the day (e.g. 15:30). A 99% confidence level and 1000 simulations are used for all calculations.

Figure 5
CBF1 & BDF1 - Intraday liquidity Value at Risk
(US $ Million)

Institution	Opening Balance	Executed Payments	Turnover Ratio	Scenarios (99% VaR)					
				Intraday (Upper bound)		15:30		17:00 (Lower bound)	
				Net Balance	Opening Balance Utilization	Net Balance	Opening Balance Utilization	Net Balance	Opening Balance Utilization
CBF1	750.0	498.7	0.7	570.6	23.9%	606.3	19.2%	678.5	9.5%
BDF1	0.3	872.8	3181	-126.7	46294%	-94.1	34389%	-34.8	12782%

Source: author's calculations.

The first scenario (i.e. within the day) corresponds to the "upper bound". Bech and Soramäki (2002) define the upper bound as the amount of liquidity required to settle all payments immediately. Under this bound the liquidity need is maximized, as in a RTGS payment system; this is the most conservative (i.e. liquidity demanding) intraday scenario.

The second scenario (i.e. by the end of the day) corresponds to the "lower bound". Bech and Soramäki (2002) define the lower bound as the liquidity required by the system if all payments are to be settled collectively at the close of the day, as in a DNS system. The author's choice of the "by the end of the day" minute for the Colombian case corresponds to the time where the settlement of securities and cash has reached about 95%-98% and 85%-90%, respectively.

Finally, the choice of 15:30 as a significant moment within the day for the Colombian LVPS corresponds to the middle of both institutions' executed payments most active time frame (i.e. 15:00-14:00). During this hour the payments executed by both institutions correspond to 37.5% of their executed payments –the highest of the intraday-, where the accumulated executed payments reach 67.8% and 79.8% of each institutions' total, for CBF1 and BDF1, respectively. Furthermore, the 15:30 minute corresponds to half an hour before the closing of the access to the monetary (Lombard) liquidity window of the Central Bank.

However, as previously mentioned, because the Colombian LVPS relies on a RTGS system where overdrafts are not allowed, all paths below the zero net balance level (i.e. negative net balances) are simply unfeasible. Yet, simulating those paths allows for estimating the resilience of a financial institution when facing an unexpected and extreme change in its intraday liquidity patterns. In this sense, because if an institution is heavily reliant on incoming payments it may be vulnerable to a liquidity stress should its counterparties decide to delay or stop making payments to it (Ball et al., 2011), the results from the simulation would help to identify non-resilient institutions.

A financial institution displaying net balance paths significantly below zero could be considered as non-resilient and a potential source of delays and interruptions for the functioning of the LVPS under extreme but plausible circumstances. The overall resilience of such institution would depend –for instance- on its stock of eligible and unencumbered collateral for accessing central bank's liquidity facilities, or for accessing the monetary market.

The previously presented Value at Risk (VaR) type-of-measure of intraday liquidity risk is replicated for a wider set of CBFs and BDFs[21]. Based on the same database and assumptions (e.g. 99% confidence interval, 1000 simulations, three different intraday scenarios) Figure 6 exhibits the 99% net balance VaR and the percentage of opening balance that would have been used to surmount such scenario for an average CBF and BDF. [22]

Figure 6
Average CBF & BDF* - Intraday liquidity Value at Risk
(US $ Million)

Type of Institution	Average Opening Balance	Avg. Executed Payments	Average Turnover Ratio	Scenarios (Average 99% VaR)					
				Intraday (Upper bound)		15:30		17:00 (Lower bound)	
				Net Balance	Opening Balance Utilization	Net Balance	Opening Balance Utilization	Net Balance	Opening Balance Utilization
CBFs	305.8	618.3	2.0	18.1	94.1%	59.2	80.6%	92.4	69.8%
BDFs	1.3	257.2	199.6	-58.9	4671.3%	-43.3	3462.3%	-23.8	1945.6%

(*) Non-weighted averages for 11 CBFs and 8 BDFs.
Source: author's calculations.

The mean VaR by type of institution confirms the findings for the two selected CBF1 and BDF1. The average CBF holds a significant amount of funds at the beginning of the day (about 49.5% the average executed payments), which allows withstanding a 99% confidence level adverse setting at any moment within the day; even at the most severe scenario (i.e. the upper bound) the average CBF holds a positive net balance, and requires a fraction of its opening balance (94.1%). As before, such significant amount of funding at the beginning of the day results from reserve requirements for CBFs.

[21] The LVPS database for the selected date comprised 19 CBFs and 26 BDFs, among other types of financial institutions. The institutions included in the set used in this exercise (Figure 6) were 11 CBFs and 8 BDFs; the criterion for inclusion was a threshold of at least ten payments per hour on average within the day.
[22] Figure 6's results correspond to non-weighted averages. When using weighted averages the intraday liquidity requirements increase for both types of financial institutions, but the analysis remains.

In the other hand, the average BDF, which is not covered by reserve requirements, holds a significantly lower opening balance, about 0.04% the opening balance of the average CBF, and equivalent to 0.5% the average executed payments. Because of this low level of funds at the beginning of the day the average BDF would be unable to fulfill its intraday payments at a 99% confidence level adverse setting, not even at the less adverse scenario (i.e. the lower bound).[23] The resilience of the average BDF would depend mainly on its stock of eligible and unencumbered collateral for accessing central bank's liquidity facilities or for accessing the monetary market.

Results for the selected individual institutions (i.e. CBF1 and BDF1) and for an average CBF and BDF concur with results obtained by Bernal and Merlano (2005), Machado et al. (2009) and León et al. (2011). For instance, based on the comparison of the upper bound and the available balances of financial institutions, Bernal and Merlano (2005) found that liquidity shortages existed for BDFs, even at the aggregated level, whereas CBFs' required reserve balance held at the central bank was enough to settle the totality of obligations in the payments system. Likewise, based on a mix of network theory and simulation of payments, Machado et al. (2009) and León et al. (2011) found that BDFs are prone to exhausting their liquidity and queuing payments when the LVPS' network faces an attack (i.e. failure-to-pay by a too-connected selected institution). However, as previously mentioned, the herein presented approach improves the measurement of intraday liquidity risk by allowing for estimating standard metrics such as VaR or Expected Shortfall.

The results of the model are important for financial authorities. Despite any comparison between both types of institutions should consider that their business and regulation differ significantly, financial authorities in charge of prudential regulation, supervision and oversight may use this type of intraday liquidity VaR in order to assess the resilience of financial institutions when confronting intraday liquidity shocks. This information is key for authorities since, as emphasized by Kodres (2009), failure or insolvency are not the only sources of systemic shocks, but mere failure-to-pay or non-payment of transactions can gridlock the entire financial system.

Furthermore, as acknowledged by CPSS-BIS (2005), a higher than optimal degree of systemic risk in key payment and settlement systems may result from negative externalities, with RTGS-related negative externalities coming in the form of insufficient incentives to consider the full impact on others of delaying outgoing payments. In this sense, the model's results are an approximation to (i) the impact of changing timing mismatches on an institution's intraday liquidity, (ii) its ability to avoid delaying outgoing payments and, ultimately, (iii) its share of systemic risk in the LVPS.

Additionally, taking into account recent amendments to financial regulation (e.g. from BCBS-BIS and FSA), this model may be a starting point for assessing financial institutions' liquidity and intraday liquidity adequacy. Accordingly, being able to contrast financial

[23] A similar conclusion is obtained by Bernal and Merlano (2005) regarding delays due to insufficient intraday funds by BDFs and other non-banking firms in the Colombian market.

institutions' real-time observed intraday liquidity with the model's resulting intraday liquidity uncertainty may be a valuable input for an overseer trying to identify abnormal intraday liquidity stances.

5. Final remarks

As the most recent financial crisis revealed, nowadays non-banking institutions are as important as banking institutions, and liquidity is as important as solvency, where financial infrastructures such as the LVPS play a key role for financial stability. This evolution of financial systems, resulting from the shift to market-based financial systems and to RTGS of payments, has been protracted but ignored to some extent, especially by prudential regulation.

As aforementioned, prior to the recent financial crisis, regulators did not focus on intraday liquidity risk and there were no standard monitoring or liquidity management measures in place (Ball et al., 2011). Regulation is working hard in order to catch up with risks arising from increasingly important non-banking institutions and liquidity. Regarding the latter, it is clear that regulators (e.g. BCBS-BIS, FSA) are updating their regulatory framework in order to enhance liquidity risk management, where intraday liquidity is one of the major concerns and challenges. These efforts concur with the documented trend in shortening time horizons of liquidity risk and liquidity management, with intraday liquidity as the new standard for what should be considered as short term.

The proposed model addresses a key question for intraday liquidity risk management: what is an institution's maximum intraday liquidity needs at a defined confidence level? The chosen approach allows for modeling the minute-by-minute liquidity of any financial institution, where the main risk factors to be modeled are the arrival of executed and received payments (i.e. their intensity), their synchrony (i.e. their timing or correlation), and their size (i.e. their monetary value). As aforementioned, following Ball et al. (2011), the identified source of intraday liquidity risk herein modeled is the timing mismatch between incoming and outgoing payments, which may lead to significant intraday liquidity needs.

Besides answering that key question, the model may be suitable for quantitatively supporting analysis regarding three main issues: (i) overseeing participants' intraday behavior; (ii) assessing their ability to fulfill intraday payments at a certain confidence level; (iii) identifying participants non-resilient to changes in payments' timing mismatches; (iv) and estimating intraday liquidity buffers[24]. These four issues, as demonstrated by the most recent financial crisis, are critical for mitigating liquidity and systemic risk.

Finally, as previously stated, the model's results are an approximation to the main negative externality arising from a financial institution within a RTGS-based LVPS: an institution's

[24] As documented and discussed by Ball et al. (2011), introducing intraday liquidity buffers should make institutions more resilient to any potential liquidity stress; however, their introduction also makes intraday liquidity more costly, and may result in incentives to delay payments.

insufficient regard to the potential costs or loss that others would incur in the event of its failure to fulfill its payments in a timely manner. In this sense the model assesses (i) the impact of varying timing mismatches on an institution's intraday liquidity; (ii) its ability to avoid delaying outgoing payments, and (iii) its contribution to systemic risk.

Despite the advantages of the model are rather apparent, some drawbacks are worth mentioning. First, as with any other model, its outcomes should be analyzed and used with care; they intend to provide a fair explanation of reality based on their assumptions, and they are by no means a substitute for sound judgment, or the sole metric to use to measure intraday liquidity risk. Second, the author considers this model a novel approximation to a long-ignored problem, but recognize that its usefulness depends on (i) the ability of financial authorities to articulate the measurement of intraday liquidity risk with the other stages of risk management (i.e. monitoring and mitigating risk), and (ii) to understand the business line of each type of institution. Third, the methodology is demanding in terms of computational resources.

The author also recognizes several challenges ahead. The first is to develop an appropriate back-testing method. The second is to run the model for a long period (e.g. a month), which may discard any particularities and biases in the selection of a specific date and would allow for a comprehensive characterization of the intraday patterns of financial institutions. Despite results concur with other related models and papers that use longer periods, it is a well-known fact that the daily averaging of reserve requirements within the two-week reserve maintenance period results in opening balances' cyclic patterns.

The third refers to analyzing the effects of excluding some major liquidity sources from the estimation of the model's parameters. Author's first choice would be to exclude the systemically most important financial institution, or each institution's most liquidity-contributing counterparty. This variant would allow for estimating the change in intraday payments synchrony and uncertainty due to the absence or failure-to-pay by a relevant counterparty.

The fourth challenge consists of a wide-ranging joint simulation of all participants' received and executed payments. This entails capturing and modeling the dependence between received and executed payments across all participants, and not only the dependence between received and executed payments for a single institution.[25] The author expects to address these challenges in forthcoming papers.

[25] The fourth challenge is particularly demanding. It requires shifting from bivariate to multivariate Poisson processes, where the dimension of the problem would escalate from independently generating 2 joint series of length q for each participant (i.e. received and executed payments) to jointly generating $N \times 2$ series of length q for the whole system, where N and q stand for the number of participants and the number of simulations, respectively. The most appealing feature of such shift is to explicitly model institutions' connectedness (*via* the dependence between received and executed payments across institutions), whereas the herein proposed model does it implicitly.

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7. Exhibit A

7.1. Monte Carlo simulation of intraday payments based on bivariate *Poisson* processes and bootstrap historical simulation

The model could be described as the joint and time-dependent simulation of a bivariate *Poisson* processes for intraday executed and received payments, and their monetary value. The core of the model is the Monte Carlo simulation of bivariate *Poisson* random variables for the intraday arrival of executed and received payments, whereas simulating their monetary value by means of bootstrap historical simulation is subordinated to their arrival. Both simulation procedures are addressed next.

7.1.1. Monte Carlo simulation of bivariate *Poisson* processes for the intraday arrival of payments[26]

The bivariate *Poisson* generation is based on the algorithm proposed by Yahav and Shmueli (2011). Their algorithm is based on the NORTA (*NORmal To Anything*) procedure for generating multivariate *Poisson* (P) data with a target correlation structure (Π_P) and arrival rates ($\lambda_1, \lambda_2, \dots \lambda_x$), which is based on simulating data from a multivariate Normal (N) distribution and converting it into an arbitrary continuous distribution with specific correlation matrix. Let $\Phi(x)$ be the Normal cumulative distribution function and $\Omega(x)$ any target cumulative distribution function, the generalized NORTA procedure is as follows:

a) Generate a *q*-dimensional Normal vector $\overrightarrow{X_N}$ with mean $\vec{\mu} = 0$, variance $\vec{\sigma} = 1$, and a correlation matrix Π_N.[27]

b) For each value X_N, $i \in 1,2, \dots q$, calculate the Normal CDF ($\Phi(x)$):

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\sigma^2}} e^{\frac{-u^2}{2}} du \qquad F1$$

c) For each $\Phi(x)$, calculate the target cumulative distribution function ($\Omega(x)$):

$$X_{y_i} = \Omega^{-1}(\Phi(X_{N_i})) \qquad F2$$

[26] This section is based on Yahav and Shmueli (2011). Several references were omitted in order to preserve readability.
[27] Generating Normal multivariate random numbers (i.e. with correlation matrix Π_N) is straightforward by means of the Cholesky decomposition. The reader may refer to Cuthbertson and Nitzsche (2004) and León and Reveiz (2011).

d) The resulting vector $\overrightarrow{X_Y}$ is then a *q*-dimensional vector, distributed according to the target cumulative distribution function ($\Omega(x)$), with correlation Π_N.

Despite the simplicity of the NORTA procedure, generating bivariate or multivariate probability distributions when the target distribution is a discrete probability distribution (i.e. random variable can assume only a certain clearly separated values) is more complicated. Such is the case for the *Poisson* distribution.

The *Poisson* distribution describes the number of times some event occurs during a specified time, space, area or volume interval. It is a discrete probability distribution since it is formed by counting (Lind et al., 2006), and is based on two assumptions: (i) the probability of an event occurring is proportional to the length of the interval and (ii) the probability of an event occurring in an interval is independent of its occurrence in other intervals.

The *Poisson* distribution is defined by a single parameter (λ) that expresses the probability of a number of events occurring in a fixed interval of time (i.e. the mean number of occurrences in a particular interval), where λ is commonly referred as the intensity of the process. The *Poisson* cumulative distribution function ($\Xi(x)$) corresponds to:

$$\Xi(x) = \sum_{i=0}^{x} \frac{e^{-\lambda}\lambda^{i}}{i!} \qquad \textit{F3}$$

With λ sufficiently large, the Normal distribution is a fair approximation to the *Poisson* distribution, where λ is its mean and variance. Conveniently, as the *Poisson* distribution converges asymptotically to a Normal distribution, attaining multivariate *Poisson* distributed random variables with correlation Π_P is straightforward since the dependence between both distributions would also converge asymptotically ($\Pi_N \approx \Pi_P$).

However, as pointed out by Yahav and Shmueli (2011), when the Normal distribution is not a fair approximation to the *Poisson* distribution (i.e. when λ is small), the convergence in correlation ceases to exist ($\Pi_N \neq \Pi_P$). The main consequence of such lack of convergence in distribution is that the feasible correlation between two random *Poisson* variables is no longer in the traditional range [-1,1], but in a narrower range [$\underline{\pi}_P \geq -1, \overline{\pi}_P \leq 1$].

Furthermore, the smaller the intensity of any of the *Poisson* processes, the narrower the correlation range, and the more difficult it is to obtain a target correlation; as demonstrated by Shin and Pasupathy (2009), as any of intensity rates λ_1 and λ_2 approximate to zero ($\lambda_1, \lambda_2 \rightarrow 0$), the minimum feasible correlation approximates to zero ($\underline{\pi}_P \rightarrow 0$); as exhibited in figures 2 and 3, this is the case at the beginning and the end of the day, when payments are rather scarce.[28]

Therefore, in order to attain bivariate *Poisson* random variables with intensity rates λ_1 and λ_2, this paper follows the functional approximation developed by Yahav and Shmueli (2011) to estimate the relationship between the desired *Poisson* correlation (π_P) and the actual (Normal) correlation (π_N). The procedure is the following:

a) Let U be a vector of Uniform randomly distributed variables, compute the correlation mapping [$\underline{\pi}_P \geq -1, \overline{\pi}_P \leq 1$], where

$$\underline{\pi}_p = corr\left(\Xi_{\lambda_1}^{-1}(U), \Xi_{\lambda_2}^{-1}(1-U)\right) \qquad \overline{\pi}_p = corr\left(\Xi_{\lambda_1}^{-1}(U), \Xi_{\lambda_2}^{-1}(U)\right) \qquad \textit{F4}$$

b) Compute the coefficients of the exponential function estimated by Yahav and Shmueli (2011)[29]:

$$a = -\frac{\overline{\pi}_P \times \underline{\pi}_P}{\overline{\pi}_P - \underline{\pi}_P} \qquad b = log\left(\frac{\overline{\pi}_P + a}{a}\right) \qquad \textit{F5}$$

c) Based on the previously computed coefficients, compute the correlation required to generate bivariate Normal random variables (π_N) that approximate the target correlation (π_P):

$$\pi_N = \frac{log\left(\frac{\pi_P + a}{a}\right)}{b} \qquad \textit{F6}$$

d) Generate bivariate Normal random variables with $\mu_1 = \mu_2 = 0$, $\sigma_1 = \sigma_2 = 1$ and correlation π_N.

e) Based on the bivariate Normal random variables ($X_N \sim N(0, \pi_N)$), follow NORTA procedure with the *Poisson* distribution as the target cumulative distribution function (CDF), with intensity rates λ_1 and λ_2.

$$X_{P_i} = \Xi^{-1}(\Phi(X_{N_i})) \qquad \textit{F7}$$

[28] Since the minimum feasible correlation approximates to zero ($\underline{\pi}_P \to 0$) when intensity rates approximate to zero ($\lambda_1, \lambda_2 \to 0$), the implemented algorithm includes an instruction to round any intensity below 0.0167 (i.e. one arrival per hour) to zero, and to simulate the two Poisson processes as non-correlated ($\underline{\pi}_P = 0$).

[29] Based on simulations, Yahav and Shmueli (2011) find that the relationship between the desired correlation (π_P) and the actual correlation (π_N) is best approximated by an exponential function:

$$\pi_p = a \times e^{b \times \pi_N} - a$$

The resulting vector $\overrightarrow{X_P}$ is then a two-dimensional vector, distributed according to the *Poisson* cumulative distribution function ($\Xi(x)$), with intensity rates λ_1 and λ_2, and correlation π_P. For the problem at hand, this procedure yields two vectors: (i) a minute-by-minute vector of executed payments and (ii) a minute-by-minute vector of received payments. Both vectors contain the minute-by-minute arrival of payments (their occurrence, not their value), where both vectors approximate the target correlation corresponding to the estimated synchrony between received and executed payments.[30]

Due to the time-variant characteristics of the LVPS's intraday payments, which displays time frames with distinctive intensity rates and correlations, the aforementioned procedure is not applied to the whole day; instead, it is applied to one-hour windows, which allows for capturing the intraday seasonality of executed and received payments.

7.1.2. Bootstrapped historical simulation of received and executed payments' value

Previous subsection presented the method for simulating the occurrence or arrival of both received and executed payments, but not their monetary value. The author's choice for simulating the monetary value of payments is based on bootstrapped historical simulation.

Based on Dowd (2005), the author designs a bootstrap procedure (resampling with replacement[31]) to simulate the monetary value of received and executed payments once they occur as a result of the arrival simulation method previously described. Each time a received (executed) payment arrives the model draws a sample from the received (executed) historical records, and takes its monetary value as the received (executed) payment's value for that occurrence.

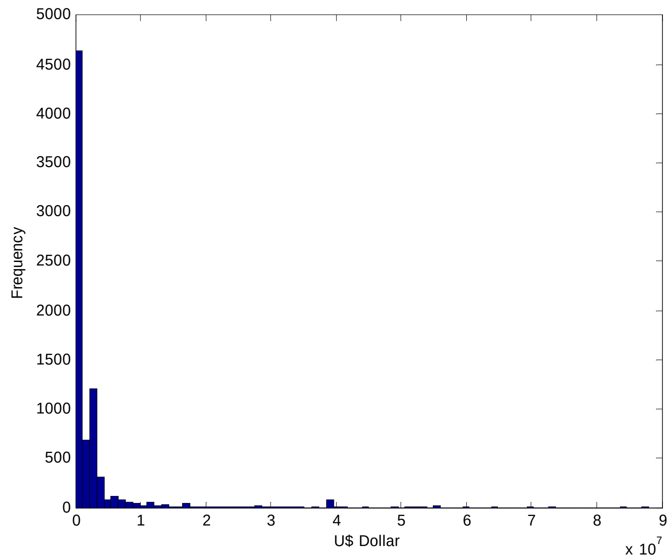
Compared to other methods available for simulating the monetary value of the payments, the bootstrap avoids unreliable assumptions such as Normality of the data set or the existence of large samples (Dowd, 2005). Regarding the Normality of payments, it is clear

[30] In order to attain comparability between simulations the algorithm is instructed to always obtain the same monetary value of received (executed) payments as in the observed data. Such restriction excludes the effect of payments size, and permits focusing on the analysis of changes in payments' synchrony.

[31] The bootstrap procedure consists of sampling from a data set of size n. Each sampling requires the generation of Uniform random numbers between 1 and n to randomly draw observations from the data set; drawn observations are returned to the data set (i.e. observations are replaced into the data set). Since Monte Carlo may be broadly defined as a method that provides approximate solutions by performing statistical sampling experiments on a computer (Fishman, 1995) or a random number generator that is useful for forecasting, estimation, and risk analysis (Mun, 2006), the bootstrap procedure may be considered as involving a Monte Carlo procedure within; therefore, the author regard the whole method herein presented as an implementation of a Monte Carlo simulation.

that they do not converge to a Gaussian distribution (Figure A1), while it is also common to find intraday periods characterized by small samples to work with (Figure 2 and 3)[32].

Figure A1
Distribution of LVPS payments
(selected day)



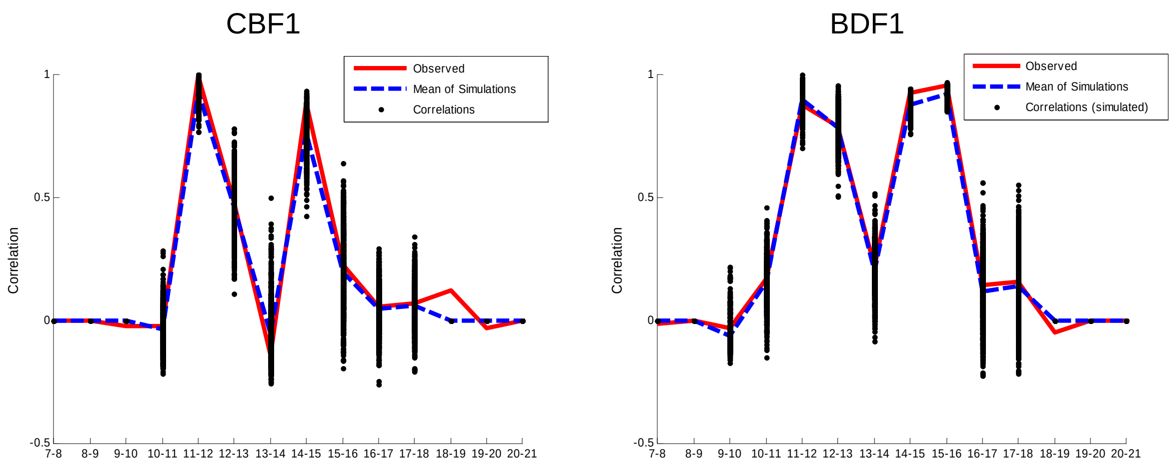
Source: author's calculations, data from the LVPS

[32] Small samples of payment orders are a significant problem for non-very-active financial institutions, which tend to make payments infrequently.

8. Exhibit B

To achieve a fair approximation of the correlation of the simulated bivariate *Poisson* series to the target correlation is the mainstay of the bivariate *Poisson* process and the model. Figure B1 displays that the mean of the correlation of the simulated bivariate *Poisson* series replicates the target correlation, whereas the simulated correlation of each series disperses around the target correlation.

Figure B1



Source: author's calculations

It is worth mentioning that since the minimum feasible correlation approximates to zero ($\underline{\pi}_p \rightarrow 0$) when intensity rates approximate to zero ($\lambda_1, \lambda_2 \rightarrow 0$), the implemented algorithm includes an instruction to round any intensity below 0.0167 (i.e. one arrival per hour) to zero, and to simulate the two *Poisson* processes as non-correlated ($\underline{\pi}_p = 0$). The author regards this as a safe practice because of the theoretical support behind such assumption (Yahav and Shmueli, 2011; Shin and Pasupathy, 2009), and because during low-intensity intervals (e.g. 7:00-9:00, 19:00-20:00) the frequency of the payments are non-significant relative to the rest of the intervals.