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Output gap and Neutral interest measures for Colombia*

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Abstract

In this paper two new measures of the Colombian output gap and the real neutral interest rate are proposed. Instead of relying only on statistical filters, the proposed measures use semi-structural New-Keynesian models, adapted for a small open economy. The output gap measures presented are in line with previous works for Colombia and capture all the turning points of the Colombian business cycle, as measured by Alfonso et al. (2011). They are also strongly correlated with inflation and precede its movements along the sample. The neutral interest rate computed indicates that the monetary policy stance has been overall countercyclical, but has failed to anticipate the output gap's movements, or at least react strongly enough to them.

Keywords: Output Gap, New-keynesian model, Neutral interest rate.
JEL Classification: E23, E32, E43.

The conduct of monetary policy requires information on the current state of the economy and a measure of the monetary stance. This information is crucial for policy makers but is by nature unobservable, and thus subject to great uncertainty, implying the need for methodologies capable to account for both things (Taylor (1999) and Woodford (2003)). This document uses semi-structural New Keynesian models to obtain such information for the Colombian economy in the 1994-2011 period.

The state of the economy is summarized in the output gap, defined as the difference between observed and potential output, the latter understood as the level of economic activity in absence of inflationary pressures. The output gap is therefore an indicator of inflation pressures and the dynamics of the aggregate demand.

The monetary policy stance is measured by the difference between the real interest rate and the neutral interest rate (Blinder, 1999), defined as an interest rate level at which the monetary authority exerts no influence over the behavior of the aggregate demand, in other words: “Any higher real interest rate constitutes “tight money” and will eventually imply falling inflation; and any lower real rate is “easy money” and signals eventually rising inflation” (Blinder, 1999, pp 33). Note that the neutral rate

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is not equal to the natural rate of interest, for the latter is, "the real rate of interest required to keep aggregate demand equal at all times to the natural rate of output" (Woodford, 2003, pp 248). The natural rate is interpreted as a desirable level for the real interest rate, whereas the neutral rate only indicates the effect of the real interest rate over the output gap.

The output gap and the neutral interest rate must be inferred from the macroeconomic information available. For the output gap, the techniques to do so rely on the use of statistical tools such as filters, VARs, factor models, among others, that allow the decomposition of output in its trend component (associated with the potential output) and its cyclical component (associated with the output gap).[1] The neutral interest rate is more difficult to extract because its value is not necessarily related to a trend or smooth component of the real interest rate, moreover, this last variable is also unobservable, for it depends on the agents' inflation expectations.

In order to jointly estimate the desired variables it is necessary to account for the structural relationships between them and other variables as the inflation rate, as well as variables that affect a small open economy, as the real exchange rate, the foreign interest rate, etc. Because of this, we expand a statistical model, the local linear trend model, with a New-Keynesian model adapted for a small open economy. Two alternative specifications of the model, which differ in the way expectations are formed, are considered. This is done in order to present different measures of the output gap and the monetary stance, recognizing that its measurement is subject to model uncertainty (Orphanides and Williams, 2002).

In the first specification of the model, agents are assumed to follow pre-determined rules when forming expectations. These rules are a function of current and lagged values of the variable over which the expectation is formed. In this way the model has a direct state space representation and the output gap can be extracted by means of the Kalman filter. In the second specification, agents are assumed to have rational expectations about the future, taking into account all information available. In order to extract the output gap, the solution to the rational expectations equilibrium of the model has to be computed, and then the state space representation can be formulated.

The approach taken here is similar to the previous work of Echavarría et al. (2007) and Berg et al. (2006), and seeks to complement a literature already existing for Colombia, noting the works of González et al. (2010), Torres (2007), Rodríguez et al. (2006), Gómez and Julio (1998) and Cobo (2004) among many others. It is also closely related to various articles that seek to jointly estimate the dynamics of the output gap and the natural interest rate. This is the case of Laubach and Williams (2003), Garnier and Wilhelmsen (2009), Mesonnier and Renne (2007) and Castillo et al. (2006).

The description of the models is covered in Section 1. Both models are estimated with Colombian data, this is described in Sections 2 and 3. Afterward both models are used to extract the output gap measures for Colombia, this is discussed in Section 4. Finally, results for neutral interest rate estimates are presented in Section 5.

1 Models

Two models are used to extract information about the output gap and the natural interest rate for Colombia in the 1994-2011 period. Both models are built on top of a local linear trend model, introducing the neutral interest rate, and a more elaborate definition of the output gap, using a semi-structural New-Keynesian model for a small open economy. One of the models has adaptative expectations and the second one rational expectations. The gap is extracted using the Kalman filter. The idea is to give economic structure to the output gap, and introduce the notion of a neutral interest rate, as opposed to the use of a purely statistical model. This allows to extract information from series other than the GDP when computing the output gap, and infer the dynamics of the neutral rate. This same strategy was used by González et al. (2010) for computing a measure of the Colombian natural interest rate, showing the differences between purely statistical and macroeconomic models. Appendix A presents

[1] Most of this techniques imply unwanted results over the relations of output's permanent and transitory component, making them completely correlated or orthogonal, depending on the method (Canova, 2007, Ch 3).

the equations for each model.

1.1 Local Linear Trend Model

The local linear model will be used as a base for building the more elaborate macroeconomic models that are shown below. It is a purely statistical model that decomposes output (y) into a trend component with an stochastic drift ($\overline{y}_t$) and the output gap ($\tilde{y}_t$).

The output gap is given by:

$$\tilde{y}_t = y_t - \overline{y}_t \tag{1.1}$$

The output trend component is assumed to follow a random walk with a stochastic drift:

$$\bar{y}_t = \bar{y}_{t-1} + g_t + \epsilon_t^{\overline{y}} \tag{1.2}$$

The drift is the growth rate of the trend component of output and is given by:

$$g_t \quad = \quad (1-\tau)\,\bar{g}_{ss} + \tau g_{t-1} + \epsilon_t^g \tag{1.3}$$

both $\epsilon_t^{\overline{y}}$ and ϵ_t^g are *iid* Gaussian disturbances. The shocks' variances $\left(\sigma_{\overline{y}}^2, \sigma_g^2\right)$ and τ are parameters to be estimated.

Note that $\epsilon_t^{\overline{y}}$ and ϵ_t^g account for permanent shocks to the level of potential output, providing an explanation for movements in that variable. This feature allows to use data on the GDP level when estimating the output gap. However, the local linear model does not give any economic structure to the output gap, and does not include other variables, also relevant for monetary policy. Because of that, this model is complemented with economic structural relationships as described in the adaptative and rational expectations models.

1.2 Adaptative Expectations Semi-Structural Model

The model consists in equations (1.1), (1.2), (1.3), an IS curve, a Phillips curve, a UIP condition, and equations for the dynamics of the real interest rate and the real exchange rate.

The IS curve is given by:

$$\tilde{y}_t = \beta_1 \tilde{y}_{t-1} - \beta_2\left(r_{t-1} - \bar{r}_{t-1}\right) + \beta_3 \tilde{q}_{t-1} + z_t^y \tag{1.4}$$

According to this representation, the output gap depends on its past value, the real interest rate gap (being $\overline{r}_t$ the neutral rate of interest), the real exchange rate gap ($\tilde{q}_t$) and an exogenous variable z_t^y that stands for the effects of demand shocks in the IS curve. z_t^y is assumed to follow an AR(1) process:

$$z_t^y = \rho_y z_{t-1}^y + \epsilon_t^y \tag{1.5}$$

Note that when the real interest rate r_t is equal to $\overline{r}_t$ the term of the IS curve involving the real interest rate is canceled, thus eliminating the effect of the real interest rate over the output gap. This is why the variable $\overline{r}_t$ is taken as the neutral interest rate.

The Phillips curve for the annualized quarterly inflation rate, is given by:

$$\pi_t = \pi_{t+1|t}^e + \lambda_2 \tilde{y}_t + \lambda_3\left(q_t - q_{t-1}\right) + z_t^\pi \tag{1.6}$$

where $\pi_{t+1|t}^e$ denotes the period t expectations over period $t+1$ inflation, q_t is the real exchange rate level, and z_t^π is an exogenous variable that stands for the effects of supply shocks over the Phillips curve. As before, z_t^π is assumed to follow an AR(1) process:

$$z_t^\pi = \rho_\pi z_{t-1}^\pi + \epsilon_t^\pi \tag{1.7}$$

Inflation expectations are defined as an average between the inflation target ($\bar{\pi}$) and lagged annual inflation ($\pi_{4,t-1}$), this is:

$$\pi^e_{t+1|t} = \lambda_1 \bar{\pi} + (1-\lambda_1)\,\pi_{4,t-1} \tag{1.8}$$

as for the annual inflation ($\pi_{4,t}$) it follows from the definition of π_t that:

$$\pi_{4,t} = \frac{1}{4}\left(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}\right) \tag{1.9}$$

The model is complemented by three sets of equations characterizing the dynamics of the real interest rate, the foreign real interest rate and the real exchange rate.

The real interest rate must satisfy two equations. The Fisher equation (1.10), and an uncovered interest parity condition (1.11):

$$r_t = i_t - \pi^e_{t+1|t} \tag{1.10}$$

$$r_t - r^\star_t = (\bar{r}_t - \bar{r}^\star_t) + 4\left(q^e_{t+1|t} - q_t\right) + \epsilon^r_t \tag{1.11}$$

where $r^\star_t$ is the foreign real interest rate, $\bar{r}^\star_t$ its neutral value at period t, and $q^e_{t+1|t}$ is the one period ahead expected value of the real exchange rate.

The neutral interest rate is assumed to follow an AR(1) process, this means that it is an exogenous factor for the model, nevertheless its value can be extracted from the model, since the relation between the neutral rate and other variables is well defined by IS curve (1.4), and the UIP condition (1.11). Since all equations operate simultaneously in the equilibrium, the value of the neutral rate depends implicitly on the foreign interest rate, the real exchange rate and the overall state of the economy.[2]

$$\bar{r}_t = \rho_r \bar{r}_{t-1} + (1-\rho_r)\,\bar{r}_{ss} + \epsilon^{\bar{r}}_t \tag{1.12}$$

The real exchange rate gap is defined as the difference between its realized and its trend value:

$$\tilde{q}_t = q_t - \bar{q}_t \tag{1.13}$$

its trend is assumed to follow a random walk:

$$\bar{q}_t = \bar{q}_{t-1} + \epsilon^q_t \tag{1.14}$$

and the expected real exchange rate is assumed to be an average between the trend, and the lagged value of the exchange rate:

$$q^e_{t+1|t} = \varphi \bar{q}_t + (1-\varphi)\,q_{t-1} \tag{1.15}$$

Finally, the nominal interest rate responds to a contemporaneous Taylor rule,[3] the rule's intercept is given by the neutral interest rate plus the inflation target, following Taylor (1993) and Woodford (2003), and it is assumed that the foreign neutral interest rate and the foreign interest rate gap evolve exogenously following AR(1) processes:

$$i_t = \gamma_1 i_{t-1} + (1-\gamma_1)\left((\bar{r}_t + \bar{\pi}) + \gamma_2(\pi_{4,t} - \bar{\pi}) + \gamma_3 \tilde{y}_t\right) + \epsilon^i_t \tag{1.16}$$

$$\bar{r}^\star_t = \rho_{r^\star} \bar{r}^\star_{t-1} + (1-\rho_{r^\star})\,\bar{r}^\star_{ss} + \epsilon^{\bar{r}^\star}_t \tag{1.17}$$

$$r^\star_t - \bar{r}^\star_t = \kappa\left(r^\star_{t-1} - \bar{r}^\star_{t-1}\right) + \epsilon^{r^\star}_t \tag{1.18}$$

[2] The relation between the neutral interest rate and the potential output's growth rate (g_t) is not included explicitly, as is done by Laubach and Williams (2003), Mesonnier and Renne (2007) and Echavarría et al. (2007). Nevertheless, an extra exercise was carried out modifying the definition of the neutral interest rate. The potential output's growth rate recovered was very stable and implied little changes over the neutral rate, with respect to the results presented in Section 5.

[3] As in Laubach and Williams (2003) and Mesonnier and Renne (2007) the equilibrium is well defined in the absence of a Taylor rule, and the nominal interest rate can be taken as a exogenous variable. The Taylor rule is included for comparison with the rational expectations model, where it plays a crucial role for equilibrium determinacy (see Taylor (1999) and Woodford (2003)).

All ϵ^j variables, with $j \in \{y, \pi, r, q, \overline{r}, \overline{r}^\star, r^\star\}$, are assumed to be *iid* Gaussian disturbances with mean zero and constant variance.

1.3 Rational Expectations Semi-Structural Model

The second model is built on top of the adaptative expectations model and differs from it in the way inflation and real exchange rate expectations are formed, and in the dynamics of the nominal interest rate, for which it is now possible to assume a forward looking Taylor rule. Additionally a forward looking component is introduced into the IS curve.

The IS curve (1.4) is modified and is given by:

$$\tilde{y}_t = \beta_1 \tilde{y}_{t-1} - \beta_2 \left(r_{t-1} - \overline{r}_{t-1}\right) + \beta_3 \tilde{q}_{t-1} + \beta_4 E_t \left\{\tilde{y}_{t+1}\right\} + z_t^y \tag{1.19}$$

Inflation expectations (1.8) are also modified and are now given by the average between expected and lagged annual inflation:

$$\pi^e_{t+1|t} = \lambda_1 E_t \left\{\pi_{4,t+4}\right\} + \left(1 - \lambda_1\right) \pi_{4,t-1} \tag{1.20}$$

Exchange rate expectations formulation is also modified, and is the average between expected and lagged exchange rate. The relative importance of each component is given by the parameter φ. The equation that characterizes the expectations is:

$$q^e_{t|t+1} = \varphi E_t \left\{q_{t+1}\right\} + \left(1 - \varphi\right) q_{t-1} \tag{1.21}$$

The Fisher equation (1.10) is defined in terms of the expected inflation corresponding to rational expectations:

$$r_t = i_t - E_t \left\{\pi_{t+1}\right\} \tag{1.22}$$

Finally, the Taylor rule is modified to include the 4-periods expected value of inflation, taking into account the lagged effect of monetary policy:

$$i_t = \gamma_1 i_{t-1} + \left(1 - \gamma_1\right) \left(\left(\overline{r}_t + \overline{\pi}\right) + \gamma_2 E_t \left(\pi_{4,t+4} - \overline{\pi}\right) + \gamma_3 \tilde{y}_t\right) + \epsilon_t^i \tag{1.23}$$

2 Data

A set of 5 macroeconomic variables is used for the estimation and filtering process. All variables are used in quarterly frequency with a sample that ranges from the first quarter of 1994 to the last quarter of 2011, thus the sample has 72 observations.

The series used are the natural logarithm of the seasonally adjusted GDP, total CPI inflation (seasonally adjusted), and the nominal interest rate, taken as the average rate of the 90 days certificate of deposit (CDT). As for foreign variables, the real interest rate is taken as the 90 days certificate of deposit rate for the US,[4] and the real exchange rate corresponds to the bilateral exchange rate between Colombia and the US, computed with the average bilateral nominal exchange rate and the CPI indexes for both countries (all items included).

Two things are worthwhile mentioning. The first is that, as in Mesonnier and Renne (2007), the real interest rate is computed in-model, in a way consistent with the models' inflation expectations. The second is that the Colombian economy experienced a disinflation period in the 2000's, with a decreasing inflation target. Since the models take the nominal series as stationary, I shall work with the domestic inflation and nominal interest rate series relative to the inflation target, this eliminates the trend from the series and makes them compatible with the models definitions. Two parallel exercises were conducted incorporating a time varying inflation target, assuming AR(1) and random walk dynamics, the results are robust to this changes.

[4] The real rate is computed ex-post with the US CPI inflation, the CPI is seasonally adjusted and all items are included.

3 Parametrization

The parameters are divided in two sets. One is fixed and is composed mainly by those of the steady state, and the other one is to be estimated. The estimation is done by means of bayesian techniques.

3.1 Fixed Parameters

The parameters that determine the long run values of the variables in the models are fixed according to the characteristics of Colombian data. The long run rate of output growth is fixed at 4% in annual terms ($\bar{g}_{ss} = 0.04$). The inflation target is set at 3% ($\bar{\pi} = 0.03$) accordingly to the mid point of the long run target band for inflation of the Banco de la República. Since Colombia is a small open economy, its real interest rate is given in the long run by the foreign interest rate, hence the domestic and foreign real interest rates are set to 2.5% in the steady state ($\bar{r}_{ss} = \bar{r}^{\star}_{ss} = 0.025$). This fact along with the absence of drift in the equilibrium exchange rate process imply that there is no depreciation in steady state.

3.2 Estimation

Parameters that are not fixed are estimated by means of bayesian techniques, combining prior information with the model's likelihood function (computed with the Kalman filter). Two chains of 100.000 draws are used when computing the parameters' posterior distributions. There are three types of prior distributions used. For bounded parameters (between 0 and 1) a Beta distribution is used, the mean is set to the mid point of the interval. For unbounded parameters a Gamma distribution is used, the mean is set to 0.3 in accordance to previous estimations of semi-structural models. Finally the shocks' variances are all associated with an Inverse-Gamma prior distribution. Appendix B summarizes the prior distributions used for the estimation of the models. The results of the estimation procedure for each model are presented in Appendices C and D respectively. The estimation was made using the Dynare software (Adjemian et al., 2011).

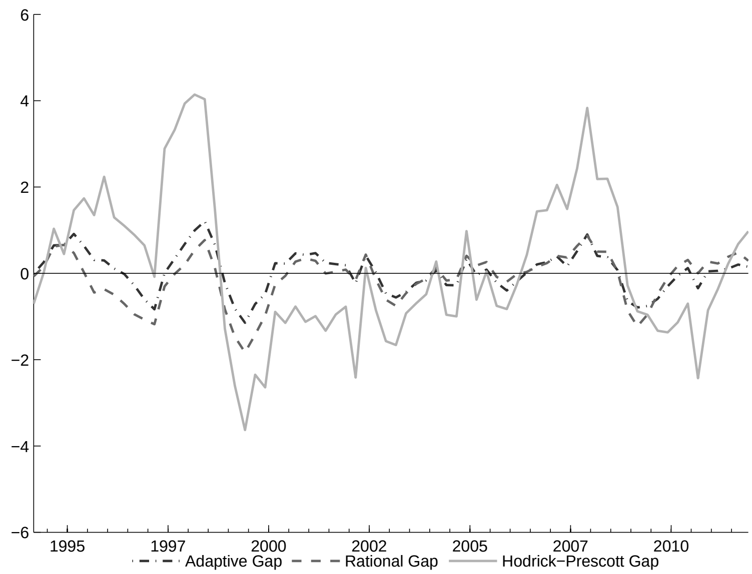
4 The Output Gap

After the estimation the parameters are set to their posterior mode values. Then each model is used to extract the output gap from the data. The output gap measure that is proposed is obtained with the Kalman smoother for variable $\tilde{y}$ in each model. Since the Hodrick-Prescott filter (henceforth HP filter) can be represented as a special case of the local linear trend model it is used as a benchmark for the results (see Harvey and Jaeger (1993) and Canova (2007)). Figure 1 presents the results for both models. Note that, although all three measures co-move they are not equal, showing that the economic models have additional information when compared to the statistical filter. The most notorious differences are in the 2000-2004 and 2006-2009 periods. In the first period the models identify a closed output gap whereas the HP filter still has a negative cyclical component. In the second period the models, specially the adaptative expectations model, fail to recognize a great increase in the output gap, as opposed to the HP filter, which identifies a strong positive cycle.

Besides the differences between the proposed measures for the output gap and the one given by the HP filter, there are also differences between those measures and the consensus among the experts. According to them, the gap should have been positive at the beginning of the sample and more negative at the 1998-1999 recession. The models fail to reproduce these facts because of two reasons. First, the Kalman filter is initialized at an arbitrary point, that does not necessarily reflect the true value of the states. In the previous exercise the filter was initialized as if the gap was equal to zero -its steady state value- in 1994Q1.[5] Second, the local linear trend model, on top of which the proposed models are built, understands the data in the 1998-1999 period as a change in output's trend, this means that

[5] In Figure 1 the output gap is not equal to zero at the first period because the gap measure is given by the Kalman filter smoother, which takes into account the whole sample for determining the gap value at each period.

Fig. 1: Output Gap Measures



Output gap measures given by the adaptative and rational expectations models, and the cyclical component of output obtained from the Hodrick-Prescott filter with $\lambda = 1600$. The series are all of quarterly frequency for the period 1994-2011. Calculations were made using Dynare.

Tab. 1: Output Gap Prior Information

Period	Value	Period	Value	Period	Value	Period	Value	Period	Value
1994Q1	1.35%	1994Q2	1.62%	1994Q3	2.03%	1994Q4	2.38%	1999Q4	−5.72%

the model is attributing part of the recession to a decrease in potential output, thus generating a "less negative" output gap. It is important to note that most of the models designed to extract output's cyclical component fail to recognize a strong negative output gap in the 1998-1999 period,[6] but, unlike most of them, the use of the Kalman filter allows us to incorporate additional information about the output gap, for the estimation and filtering process.[7]

Because of the above discussion a second exercise is carried out. The models are now estimated using the same database and prior distributions for the parameters, while allowed to observe the output gap level given by the experts for the first four observations of the sample and the fourth quarter of 1999 (Table 1). This information is subject to measurement error, whose variance is estimated along with all the other parameters. The results of the estimation are summarized in Appendices C.2 and D.2. Both models turn out to assign little variance to the measurement error of the output gap additional information, as reflected in the recovered output gap measure, Figure 2.

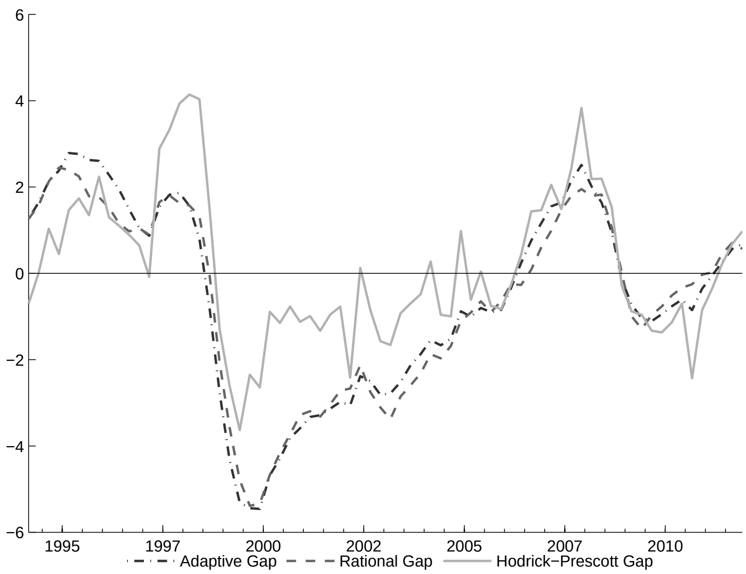
The output gap measures recovered from this exercise are able to recognize both a positive gap between 1994 and 1998, as well as a more negative and persistent gap following the 1998-1999 recession, up until the mid 2000's. They also present a somewhat higher gap at the end of the sample and in the 2007-2008 period. As before there is a co-movement between the two measures, with differences in the timing and magnitude of the cycles.

The gaps presented also match previous findings on the Colombian business cycle. As shown in Figure 3, both measures identify all the peaks and troughs presented by Alfonso et al. (2011), who use

[6] An exception to this is the measure proposed in Cobo (2004), based on the production function approach.

[7] The methodology presented in Julio (2011) represents an exception to this, allowing the introduction of 'Priors' as linear restrictions on the Hodrick-Prescott filter.

Fig. 2: Output Gap Measures



Output gap measures given by the adaptative and rational expectations models with prior information about the output gap level in 1994 and 1999Q4, and the cyclical component of output obtained from the Hodrick-Prescott filter with $\lambda = 1600$. The series are all of quarterly frequency for the period 1994-2011. Calculations were made using Dynare.

an accumulated diffusion index, computed with 24 Colombian series,[8] in order to obtain a chronology of the business cycle. Figure 3 also makes clear that there is a strong positive correlation between the output gap measures and the core inflation, defined as the CPI less food items inflation. Moreover, the output gap precedes the movements in the core inflation to some extent. Note for example the inflation's peaks after the 1995, 1997 and 2007 peaks in the output gap, as well as the falls in inflation after the 1998 and 2009 falls in the output gap.

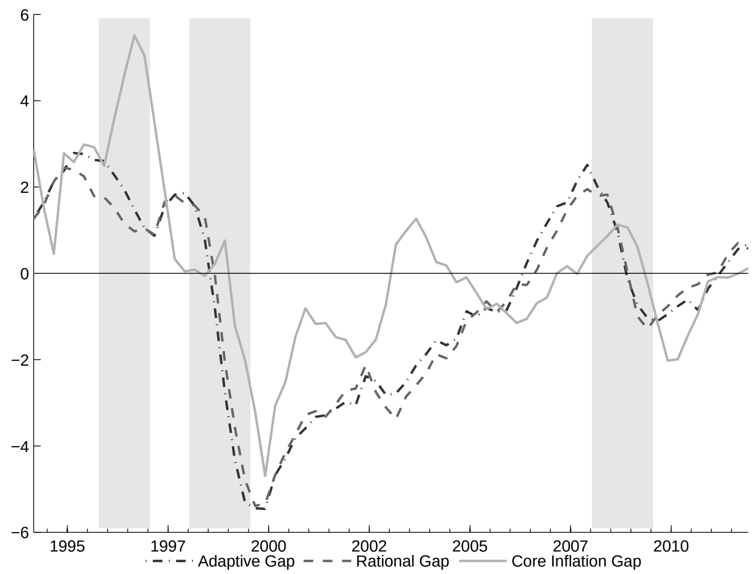
Finally, the output gap can be decomposed into the effects of the shocks by using the state space representation of the model (Canova, 2007). The historical decomposition for the output gap measures is computed and reported in Figure 4. The exercise consists in identifying which shocks affected the economy in the sample period using the observed macroeconomic series, along with the economic structure of the model. After the shocks have been identified it is possible to compute their individual impact over the output gap.

Since the identification and impact depend on the model's structure, the decomposition is different for the adaptative expectations and the rational expectations model. Nevertheless there are common features between the two models. The most important one is that the output gap is explained mostly by the effect of shocks to the IS curve (demand shocks). This is very useful if one wishes to interpret the output gap as a measure of demand pressures in the economy. Another common characteristic is the low and short-lived effect of the filter's initial values over the output gap, it can be seen that this effect is only determinant in the first period and that only lasts for approximately 12 periods. Another common feature is the effect of the Phillips curve shock (supply shocks) after the 1999 recession. Because of the large drop in inflation that followed the first quarters of 1999, the models identify a Phillips curve shock that helps to explain such drop, as a consequence positive pressures over the output gap were created.

There are three other shocks that appear significant in the historical decomposition. The first is to the foreign interest rate, this shock is more relevant in Panel 4b and has a negative effect over the output gap for the 2000's period. During this period the foreign interest rate was low and the models

[8] The only variable in common between this exercise and the one of Alfonso et al. (2011) is the nominal interest rate.

Fig. 3: Output Gap Measures



Output gap measures given by the adaptative and rational expectations models with prior information about the output gap level in 1994 and 1999Q4, and the CPI less food items inflation relative to the inflation target. Grey areas correspond to peak-to-trough periods of the Colombian business cycle according to Alfonso et al. (2011). The series are all of quarterly frequency for the period 1994-2011. Calculations were made using Dynare.

identify this as a negative shock, associated to a real appreciation of the exchange rate. Nevertheless there must be caution over this result, for the models are biased towards the negative effects of the shock, since they do not take into account the positive effects of appreciation and a cheaper debt over the aggregate demand.

The second shock is to the real exchange rate trend, it is expansive in 2004 and 2010, both periods of real exchange rate appreciation. The reason for this is that the model interprets these appreciations as changes in the real exchange rate trend. When the trend is lowered the exchange rate gap becomes positive, hence increasing the output gap. However this effect is not of great magnitude, relative to the effect of other shocks.

The third shock is to the neutral interest rate. Note that for the model this variable is completely exogenous and only influenced by this shock. Because the model is able to extract both the level of the real interest rate and of the interest rate gap, the neutral rate of interest can be computed. The negative effect over the output gap of the neutral interest rate shock in the early 2000's is explained by a decrease in the neutral rate from the high levels of the late 1990's, which lowered the interest rate gap. More about the neutral rate is discussed in the following Section.

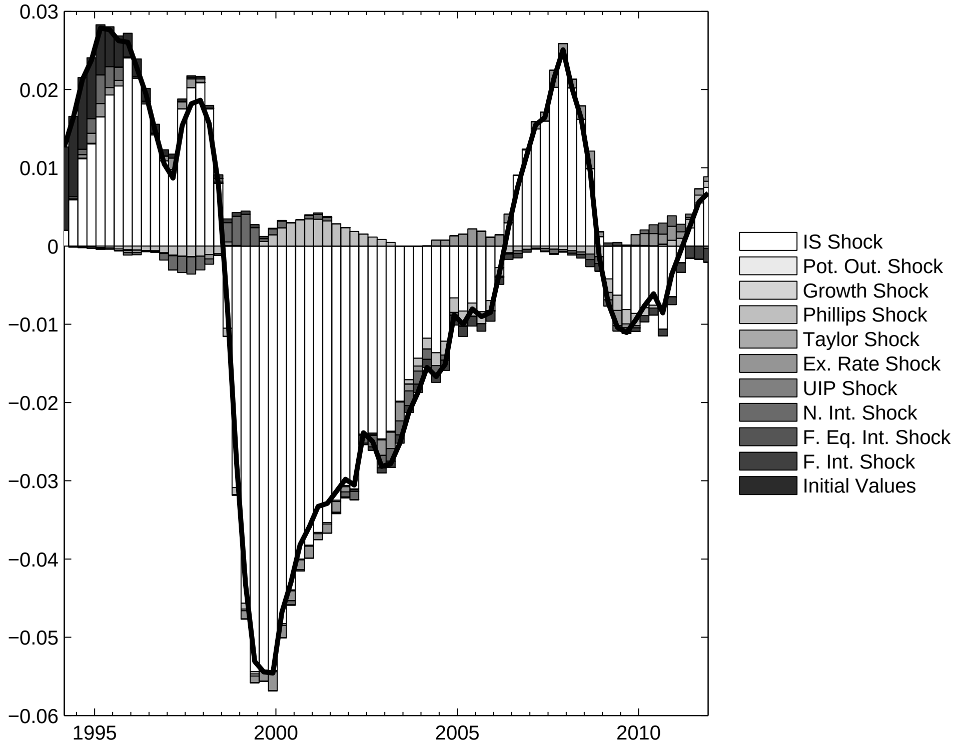
5 The Neutral Interest Rate

Before discussing the models' implications over the neutral interest rate, it is important to examine the results on the real interest rate measures implied by the models, recalling that this variable is computed in-model, given the nominal interest rate and the inflation, and conditioned to the models' structure. Figure 5 reports the models' real interest rates with the ex-post interest rate, computed as the difference between the current nominal interest rate and the inflation rate of total CPI. It can be seen that all three variables are very similar, and that the real interest rate measures given by the models are smoother than the ex-post interest rate.

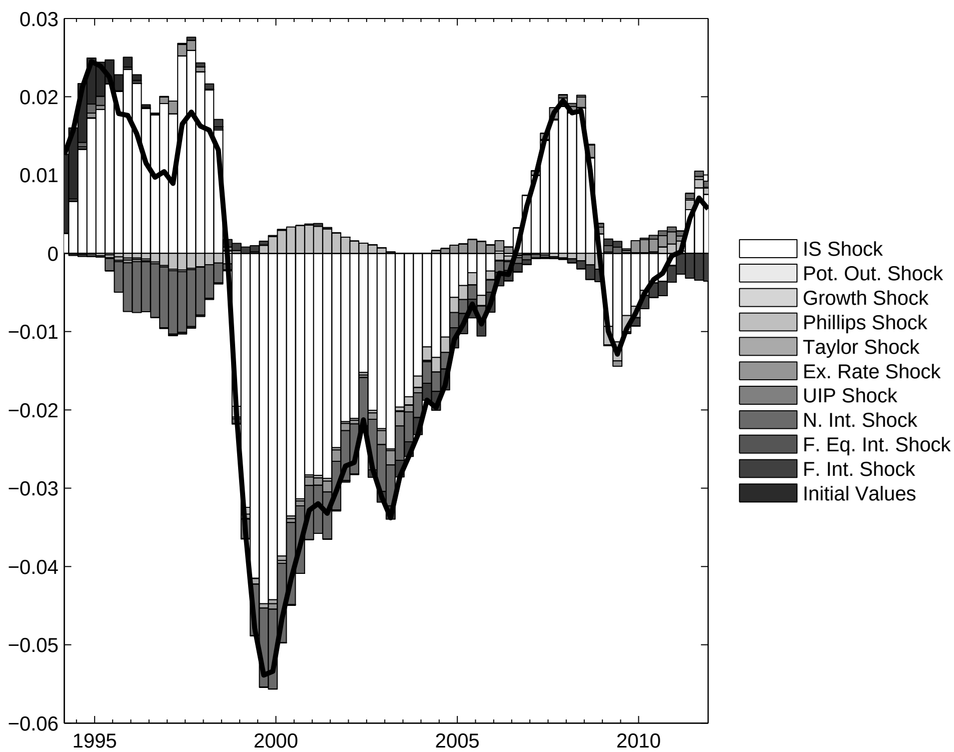
The period under consideration is characterized by high and volatile levels of the real interest rate

Fig. 4: Output Gap Historical Decomposition

(a) Adaptative Model

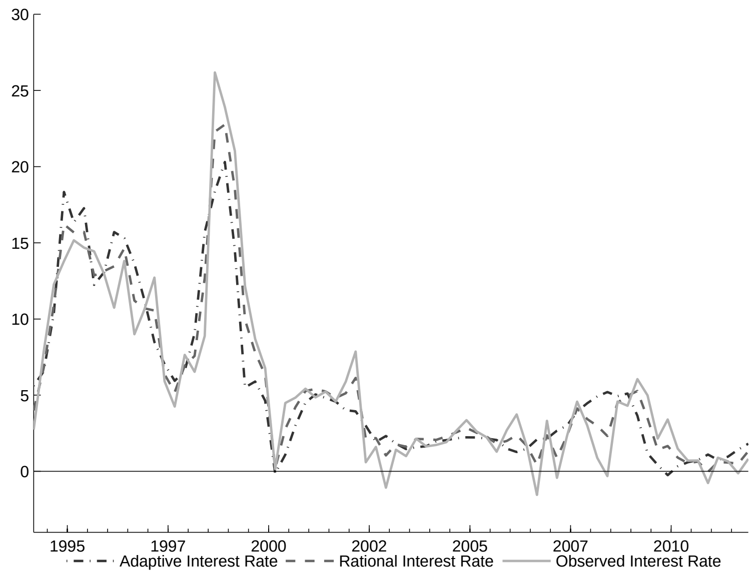


(b) Rational Model



Output gap historical decomposition in shocks given by the adaptative and rational expectations models with prior information about the output gap level in 1994 and 1999Q4. The series are all of quarterly frequency for the period 1994-2011. Calculations were made using Dynare.

Fig. 5: Real Interest Rate



Real interest rate measures given by the adaptative and rational expectations models with prior information about the output gap level in 1994 and 1999Q4, and the ex-post real interest rate for 90 days certificate of deposit. The series are all of quarterly frequency for the period 1994-2011. Calculations were made using Dynare.

before the 2000, followed by a more stable period with a lower interest rate level. The volatility, and subsequent stabilization, of the real interest rate is probably explained by changes in the Colombian monetary policy, I refer to Giraldo et al. (2011) and the references therein for a review of Colombia's recent monetary history. Overall there are no drastic changes in the real interest rate, save from the great increase that coincides with the 1998-1999 recession, which is explained by the large drop in inflation that followed the crisis (see Figure 4).

Turning to the neutral interest rate, Figure 6 presents the neutral and real interest rate that the adaptative model (Panel 6a) and the rational expectations model (Panel 6b) recovered from the data. Note that both measures of the neutral rate are even more volatile than the real interest rate measures. The volatility of the real interest rate, although present only before the 2000, influences the neutral interest rate in the whole sample, generating a changing measure of neutrality for the last decade.

Both models imply that there was a positive interest rate gap before the 1999 recession and a negative gap afterward, with a slow convergence of the neutral rate to the levels that the real interest rate has presented after the crisis. Then, the interest rate gap turns positive in the 2007-2008 period, although considerably more in the adaptative expectations model. At the end of the sample the interest rate gap becomes negative, again in a larger amount in the adaptative expectations model.

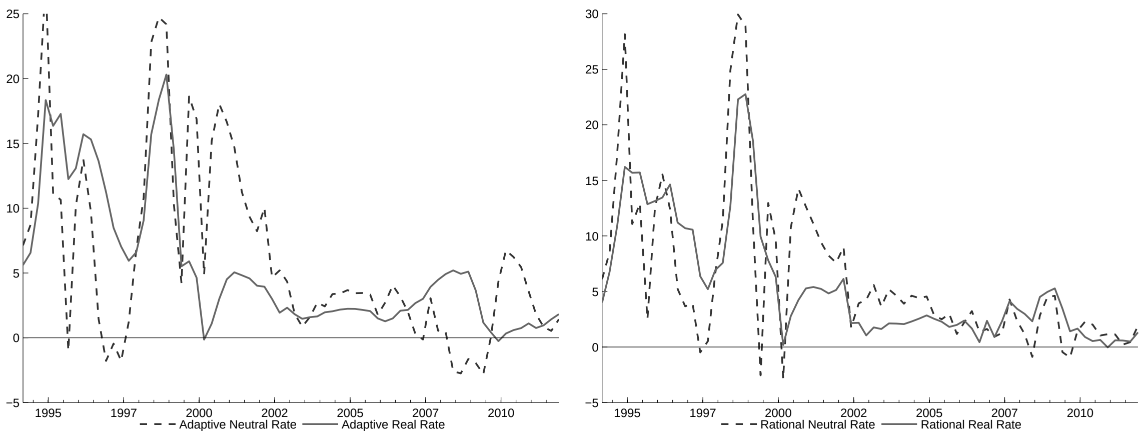
Note, from Figure 7, that the behavior of the interest rate gap is countercyclical almost everywhere. It exerts a negative pressure over the output gap while it was positive in the pre-1999 period, and has expansive effects afterward, up until the 2007-2008 period, in which the output gap is again positive. Finally, the interest rate gap has positive effects after 2008, when there is a drop in the output gap, associated to the international turbulence that followed the recent US financial crisis.[9] Yet, it must be mentioned that the interest gap reaction is lagged with respect to the output gap movements,[10] this

[9] The countercyclality of the interest rate gap is clearly interrupted in the Rational expectations model between 1999 and 2001 (Panel 7b). In this period the interest rate gap turns positive while the output gap remains negative. This is attributed to a drop in the neutral interest rate (see Panel 6b), since this variable is exogenous for the model, this means that the model identifies the need of a positive interest gap in order to explain the drop in the output gap in those periods.

[10] Recall that the interest rate gap presented in Figure 7 is smoothed with a fourth order moving average. This is done

Fig. 6: Neutral Interest Rate

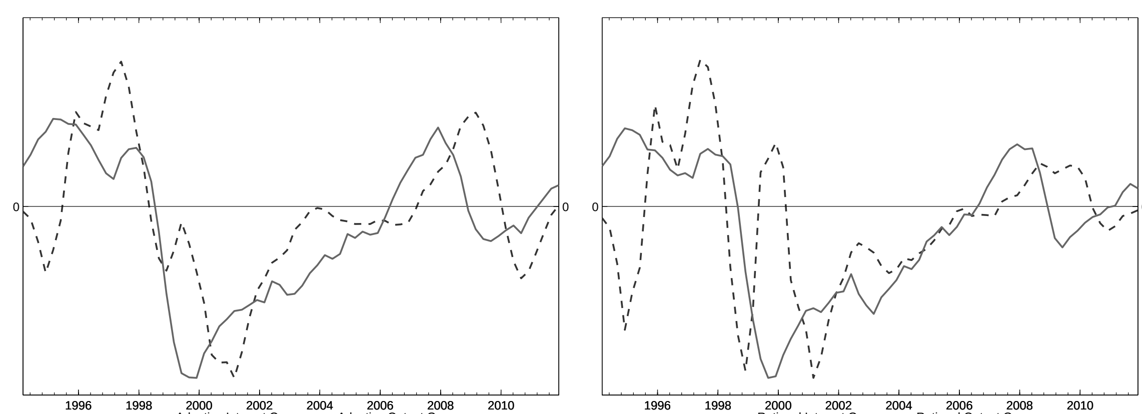
(a) Adaptative Model (b) Rational Model



Neutral interest rate and real interest rate given by the adaptative and rational expectations models with prior information about the output gap level in 1994 and 1999Q4. The series are all of quarterly frequency for the period 1994-2011. Calculations were made using Dynare.

Fig. 7: Neutral Interest Rate

(a) Adaptative Model (b) Rational Model



Smoothed real interest rate gap and output gap given by the adaptative and rational expectations models with prior information about the output gap level in 1994 and 1999Q4. The real interest rate gap is smoothed with a fourth order moving average.The series are all of quarterly frequency for the period 1994-2011. Calculations were made using Dynare.

is clearer in Panel 7a for the adaptive expectations model, and post-2008 period in Panel 7b. In other words, given that the monetary authority can influence the real interest rate, the monetary policy, although countercyclical, has failed to anticipate the changes of the output gap, or at least to react strongly enough to them.

6 Final Remarks

This document presents two new measures of the Colombian output gap and neutral interest rate. Both variables are crucial for the conduct of monetary policy and their measurement is subject to a great deal of uncertainty. Because of this the results presented here are not to be taken as final, but as an extra input, useful for policy evaluation and academic research.

The models deliver an output gap coherent with previous works for Colombia, as Echavarría et al. (2007)), and is capable to identify all the turning points of the Colombian business cycle, as measured by Alfonso et al. (2011). The Colombian output gap begins with a positive, although variable, level from 1994 to 1997, when there is a large drop that starts with the 1998-1999 crisis, after this drop the output gap remains negative until 2006. The gap turns positive in the 2006-2008 period and then drops in 2009, after the international turmoil that followed the US financial crisis of 2008. Both models imply that the gap has recovered from its last drop and is positive since 2011, although still close to zero.

As for the neutral interest rate, the models are more heterogenous in the results, but both imply a somewhat countercyclical behavior of the monetary policy during most of the sample period. They also imply a delay between the movements of the output gap and those of the interest rate gap. This may correspond to a lack of anticipation of the monetary authority, or the need of stronger reaction to the economy's condition.

Finally, it is noted that the methodology presented relies in semi-structural models to take into account the relations between several macroeconomic aggregates, and there are still efforts to be done in order to compute a micro-founded measure of the output gap, and the natural interest rate, in the spirit of Woodford (2003) and Christiano et al. (2010a,b). These new measures can potentially improve our understanding of the shocks that affect the economy, and the design of monetary policy.

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for clarity since the neutral rate measures are too volatile, and does not affect the findings.

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A Equations

A.1 Adaptative Expectations Model

$$y_t = \tilde{y}_t + \bar{y}_t \tag{A.1}$$
$$\bar{y}_t = \bar{y}_{t-1} + g_t + \epsilon_t^{\overline{y}} \tag{A.2}$$
$$g_t = (1-\tau)\,\bar{g}_{ss} + \tau g_{t-1} + \epsilon_t^g \tag{A.3}$$

$$\tilde{y}_t = \beta_1 \tilde{y}_{t-1} - \beta_2 \left(r_{t-1} - \bar{r}_{t-1}\right) + \beta_3 \tilde{q}_{t-1} + z_t^y \tag{A.4}$$

$$\pi_t = \pi_{t+1|t}^e + \lambda_2 \tilde{y}_t + \lambda_3 \left(q_t - q_{t-1}\right) + z_t^\pi \tag{A.5}$$
$$\pi_{t+1|t}^e = \lambda_1 \bar{\pi}_t + (1-\lambda_1)\,\pi_{4,t-1} \tag{A.6}$$
$$\pi_{4,t} = \frac{1}{4}\left(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}\right) \tag{A.7}$$

$$i_t = \gamma_1 i_{t-1} + (1-\gamma_1)\left(\left(\bar{r}_t + \overline{\pi}\right) + \gamma_2\left(\pi_{4,t} - \overline{\pi}\right) + \gamma_3 \tilde{y}_t\right) + \epsilon_t^i \tag{A.8}$$
$$r_t = i_t - \pi_{t+1|t}^e \tag{A.9}$$
$$r_t - r_t^\star = 4\left(q_{t+1|t}^e - q_t\right) + \left(\bar{r}_t - \bar{r}_t^\star\right) + \epsilon_t^r \tag{A.10}$$
$$\bar{r}_t = \rho_r \bar{r}_{t-1} + (1-\rho_r)\,\bar{r}_{ss} + \epsilon_t^{\overline{r}} \tag{A.11}$$

$$r_t^\star = \bar{r}_t^\star + \kappa\left(r_{t-1}^\star - \bar{r}_{t-1}^\star\right) + \epsilon_t^{r^\star} \tag{A.12}$$
$$\bar{r}_t^\star = \rho_{r^\star} \bar{r}_{t-1}^\star + (1-\rho_{r^\star})\,\bar{r}_{ss}^\star + \epsilon_t^{\overline{r}^\star} \tag{A.13}$$

$$q_t = \tilde{q}_t + \bar{q}_t \tag{A.14}$$
$$\bar{q}_t = \bar{q}_{t-1} + \epsilon_t^q \tag{A.15}$$
$$q_{t+1|t}^e = \varphi \bar{q}_t + (1-\varphi)\,q_{t-1} \tag{A.16}$$

$$z_t^y = \rho_y z_{t-1}^y + \epsilon_t^y \tag{A.17}$$
$$z_t^\pi = \rho_\pi z_{t-1}^\pi + \epsilon_t^\pi \tag{A.18}$$

A.2 Rational Expectations Semi-Structural Model

$$y_t = \tilde{y}_t + \bar{y}_t \tag{A.19}$$
$$\bar{y}_t = \bar{y}_{t-1} + g_t + \epsilon_t^{\overline{y}} \tag{A.20}$$
$$g_t = (1-\tau)\,\bar{g}_{ss} + \tau g_{t-1} + \epsilon_t^{g} \tag{A.21}$$

$$\tilde{y}_t = \beta_1 \tilde{y}_{t-1} - \beta_2\left(r_{t-1} - \bar{r}_{t-1}\right) + \beta_3 \tilde{q}_{t-1} + \beta_4 E_t\left\{\tilde{y}_{t+1}\right\} + z_t^{y} \tag{A.22}$$

$$\pi_t = \pi_{t+1|t}^{e} + \lambda_2 \tilde{y}_t + \lambda_3\left(q_t - q_{t-1}\right) + z_t^{\pi} \tag{A.23}$$
$$\pi_{t+1|t}^{e} = \lambda_1 E_t\left\{\pi_{4,t+4}\right\} + (1-\lambda_1)\,\pi_{4,t-1} \tag{A.24}$$
$$\pi_{4,t} = \frac{1}{4}\left(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}\right) \tag{A.25}$$

$$i_t = \gamma_1 i_{t-1} + (1-\gamma_1)\left((\bar{r}_t + \overline{\pi}) + \gamma_2 E_t\left(\pi_{4,t+4} - \overline{\pi}\right) + \gamma_3 \tilde{y}_t\right) + \epsilon_t^{i} \tag{A.26}$$
$$r_t = i_t - E_t\left\{\pi_{t+1}\right\} \tag{A.27}$$
$$r_t - r_t^{\star} = 4\left(q_{t+1|t}^{e} - q_t\right) + \left(\bar{r}_t - \bar{r}_t^{\star}\right) + \epsilon_t^{r} \tag{A.28}$$
$$\bar{r}_t = \rho_r \bar{r}_{t-1} + (1-\rho_r)\,\bar{r}_{ss} + \epsilon_t^{\overline{r}} \tag{A.29}$$

$$r_t^{\star} = \bar{r}_t^{\star} + \kappa\left(r_{t-1}^{\star} - \bar{r}_{t-1}^{\star}\right) + \epsilon_t^{r^{\star}} \tag{A.30}$$
$$\bar{r}_t^{\star} = \rho_{r^{\star}} \bar{r}_{t-1}^{\star} + (1-\rho_{r^{\star}})\,\bar{r}_{ss}^{\star} + \epsilon_t^{\overline{r}^{\star}} \tag{A.31}$$
$$\tag{A.32}$$
$$q_t = \tilde{q}_t + \bar{q}_t \tag{A.33}$$
$$\bar{q}_t = \bar{q}_{t-1} + \epsilon_t^{q} \tag{A.34}$$
$$q_t^{e} = \varphi E_t\left\{q_{t+1}\right\} + (1-\varphi)\,q_{t-1} \tag{A.35}$$

$$z_t^{y} = \rho_y z_{t-1}^{y} + \epsilon_t^{y} \tag{A.36}$$
$$z_t^{\pi} = \rho_\pi z_{t-1}^{\pi} + \epsilon_t^{\pi} \tag{A.37}$$

B Prior Distributions

Tab. 2: Prior Distributions

Parameter	Description	Distribution	Mean	Std. Dev.
σ_i	Shock "i" standard deviation	Inv. Gamma	0.0125	∞
τ	Persistence of the growth process	Beta$(0, 1)$	0.5	0.15
λ_1	Inflation Expectations	Beta$(0, 1)$	0.5	0.15
λ_2	Elasticity of inflation to output gap	Gama	0.30	0.25
λ_3	Elasticity of inflation to depreciation	Gama	0.30	0.25
β_1	Elasticity of output gap to its lag	Beta$(0, 1)$	0.5	0.15
β_2	Elasticity of output gap to real interest gap	Gama	0.30	0.25
β_3	Elasticity of output gap to exchange rate gap	Gama	0.30	0.25
β_4	Elasticity of output gap to expectations	Gama	0.30	0.25
φ	Exchange rate expectations	Beta$(0, 1)$	0.5	0.15
κ	Persistence of foreign interest rate gap	Beta$(0, 1)$	0.5	0.15
ρ_r	Persistence of natural interest rate	Beta$(0, 1)$	0.5	0.15
$\rho_{r^\star}$	Persistence of foreign natural interest rate	Beta$(0, 1)$	0.5	0.15
ρ_y	Persistence of IS shock	Beta$(0, 1)$	0.5	0.15
ρ_π	Persistence of Phillips curve shock	Beta$(0, 1)$	0.5	0.15

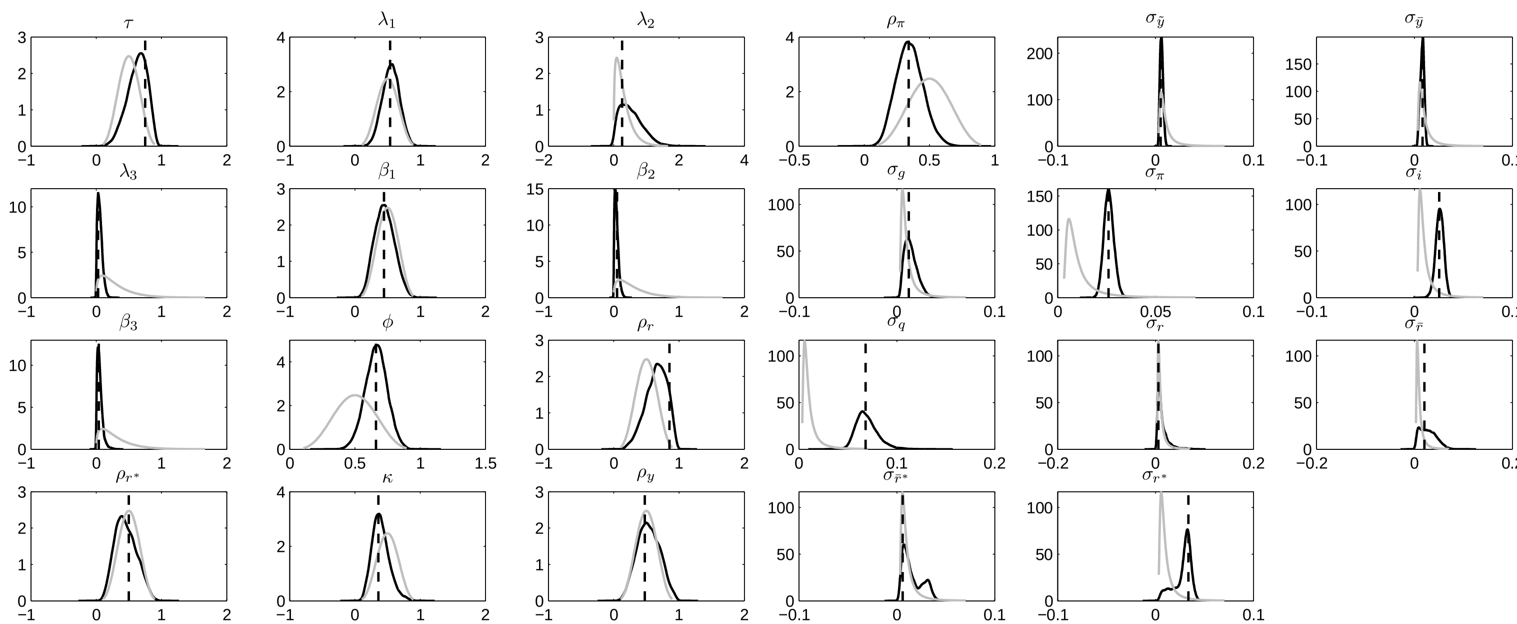
C Estimation Results - Adaptative Expectations Model

C.1 Unconditioned Estimation

Tab. 3: Estimation Results - Adaptive Expectations Model

Parameter		Prior		Posterior			HPD 90%	
		Mean	SD	Mean	Mode	SD	Lower	Upper
Persistence of the growth process	τ	0.50	0.15	0.62	0.75	0.13	0.39	0.87
Inflation expectations	λ_1	0.30	0.25	0.55	0.54	0.14	0.34	0.76
Elasticity of inflation to output gap	λ_2	0.30	0.25	0.54	0.26	0.42	0.01	1.03
Elasticity of inflation to depreciation	λ_3	0.30	0.25	0.06	0.03	0.04	0.00	0.11
Elasticity of output gap to its lag	β_1	0.50	0.15	0.45	0.44	0.17	0.20	0.69
Elasticity of output gap to real interest gap	β_2	0.30	0.25	0.04	0.05	0.04	0.00	0.08
Elasticity of output gap to exchange rate gap	β_3	0.30	0.25	0.05	0.04	0.04	0.00	0.11
Exchange rate expectations	φ	0.50	0.15	0.66	0.66	0.09	0.52	0.80
Persistence of natural interest rate	ρ_r	0.50	0.15	0.65	0.85	0.13	0.41	0.92
Persistence of foreign natural interest rate	$\rho_{r^\star}$	0.50	0.15	0.45	0.50	0.18	0.19	0.72
Persistence of foreign interest rate gap	κ	0.50	0.15	0.39	0.36	0.09	0.18	0.61
Persistence of IS shock	ρ_y	0.50	0.15	0.53	0.48	0.20	0.25	0.81
Persistence of Phillips curve shock	ρ_π	0.50	0.15	0.35	0.34	0.10	0.18	0.51
Std. Dev. IS Curve Shock	σ_y	0.0125	∞	0.01	0.01	0.00	0.00	0.01
Std. Dev. Potential Output Shock	$\sigma_{\bar{y}}$	0.0125	∞	0.01	0.01	0.00	0.00	0.01
Std. Dev. Growth Shock	σ_g	0.0125	∞	0.01	0.01	0.01	0.00	0.02
Std. Dev. Phillips Curve Shock	σ_π	0.0125	∞	0.03	0.03	0.00	0.02	0.03
Std. Dev. Nominal Interest Rate Shock	σ_i	0.0125	∞	0.03	0.03	0.00	0.02	0.03
Std. Dev. Potential Exchange Rate Shock	σ_q	0.0125	∞	0.07	0.07	0.01	0.05	0.09
Std. Dev. UIP Shock	σ_r	0.0125	∞	0.01	0.01	0.00	0.00	0.03
Std. Dev. Natural Real Interest Rate Shock	$\sigma_{\bar{r}}$	0.0125	∞	0.03	0.02	0.01	0.00	0.05
Std. Dev. Foreign Natural Interest Rate Shock	$\sigma_{\bar{r}^\star}$	0.0125	∞	0.02	0.01	0.00	0.00	0.03
Std. Dev. Foreign Interest Rate Shock	$\sigma_{r^\star}$	0.0125	∞	0.03	0.03	0.00	0.01	0.04

Fig. 8: Prior & Posterior - Adaptive Expectations Model



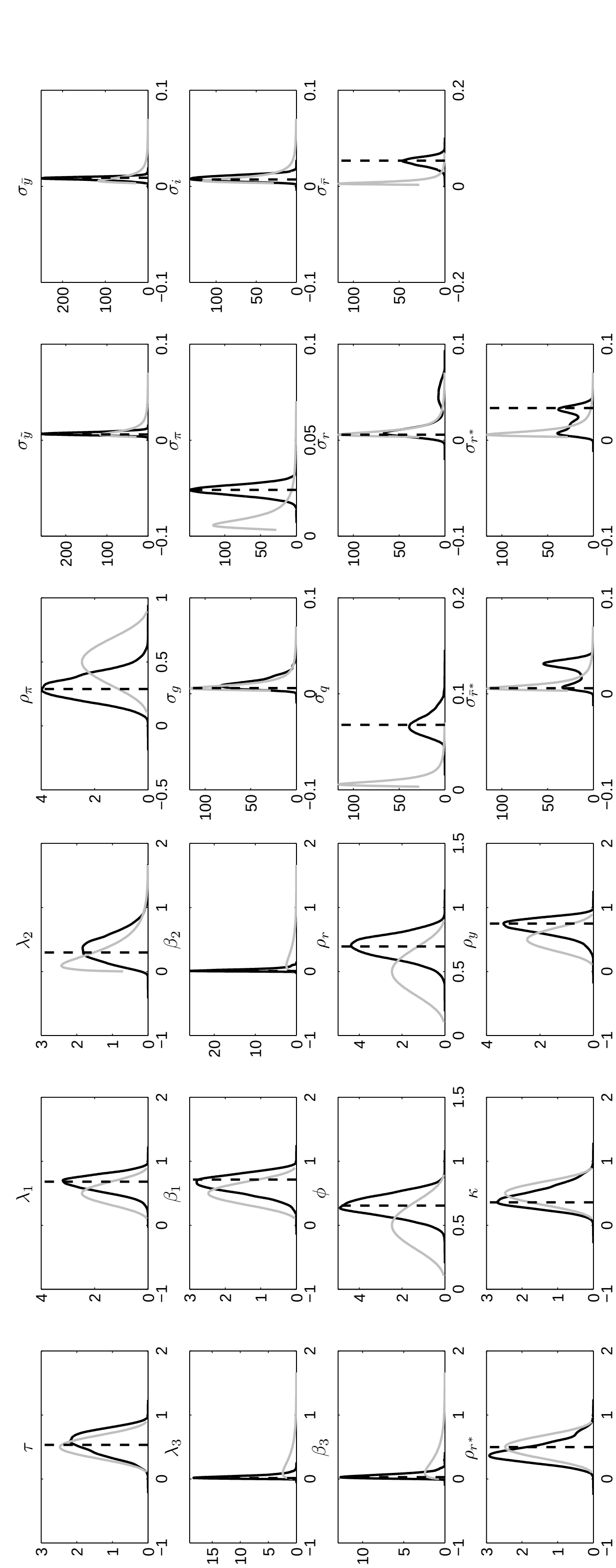
Prior and Posterior density functions for the estimated parameters, Prior functions are in gray and Posteriors functions in black, the dashed vertical line indicates the parameter posterior mean. Calculations were made with Dynare.

C.2 Conditioned Estimation

Tab. 4: Estimation Results - Adaptive Expectations Model

Parameter		Prior		Posterior			HPD 90%	
		Mean	SD	Mean	Mode	SD	Lower	Upper
Persistence of the growth process	τ	0.50	0.15	0.56	0.53	0.20	0.30	0.82
Inflation expectations	λ_1	0.30	0.25	0.65	0.68	0.13	0.46	0.85
Elasticity of inflation to output gap	λ_2	0.30	0.25	0.39	0.30	0.21	0.06	0.70
Elasticity of inflation to depreciation	λ_3	0.30	0.25	0.04	0.01	0.02	0.00	0.07
Elasticity of output gap to its lag	β_1	0.50	0.15	0.64	0.71	0.14	0.43	0.86
Elasticity of output gap to real interest gap	β_2	0.30	0.25	0.03	0.01	0.01	0.00	0.06
Elasticity of output gap to exchange rate gap	β_3	0.30	0.25	0.05	0.03	0.03	0.00	0.10
Exchange rate expectations	φ	0.50	0.15	0.65	0.65	0.08	0.52	0.78
Persistence of natural interest rate	ρ_r	0.50	0.15	0.69	0.70	0.08	0.56	0.84
Persistence of foreign natural interest rate	$\rho_{r^\star}$	0.50	0.15	0.42	0.50	0.18	0.17	0.65
Persistence of foreign interest rate gap	κ	0.50	0.15	0.43	0.36	0.09	0.20	0.69
Persistence of IS shock	ρ_y	0.50	0.15	0.69	0.75	0.12	0.52	0.89
Persistence of Phillips curve shock	ρ_π	0.50	0.15	0.30	0.29	0.09	0.14	0.45
Std. Dev. IS Curve Shock	σ_y	0.0125	∞	0.01	0.01	0.00	0.00	0.01
Std. Dev. Potential Output Shock	$\sigma_{\bar{y}}$	0.0125	∞	0.01	0.01	0.00	0.01	0.01
Std. Dev. Growth Shock	σ_g	0.0125	∞	0.01	0.01	0.00	0.00	0.02
Std. Dev. Phillips Curve Shock	σ_π	0.0125	∞	0.02	0.02	0.00	0.02	0.03
Std. Dev. Nominal Interest Rate Shock	σ_i	0.0125	∞	0.01	0.01	0.00	0.00	0.01
Std. Dev. Potential Exchange Rate Shock	σ_q	0.0125	∞	0.07	0.07	0.01	0.05	0.08
Std. Dev. UIP Shock	σ_r	0.0125	∞	0.02	0.01	0.00	0.00	0.05
Std. Dev. Natural Real Interest Rate Shock	$\sigma_{\bar{r}}$	0.0125	∞	0.05	0.05	0.01	0.04	0.07
Std. Dev. Foreign Natural Interest Rate Shock	$\sigma_{\bar{r}^\star}$	0.0125	∞	0.02	0.01	0.00	0.00	0.04
Std. Dev. Foreign Interest Rate Shock	$\sigma_{r^\star}$	0.0125	∞	0.02	0.03	0.00	0.00	0.04
Std. Dev. Measurement Error	σ	0.0125	∞	0.008	0.005	0.002	0.003	0.013

Fig. 9: Prior & Posterior - Adaptive Expectations Model



Prior and Posterior density functions for the estimated parameters, Prior functions are in gray and Posteriors functions in black, the dashed vertical line indicates the parameter posterior mean. Calculations were made with Dynare.

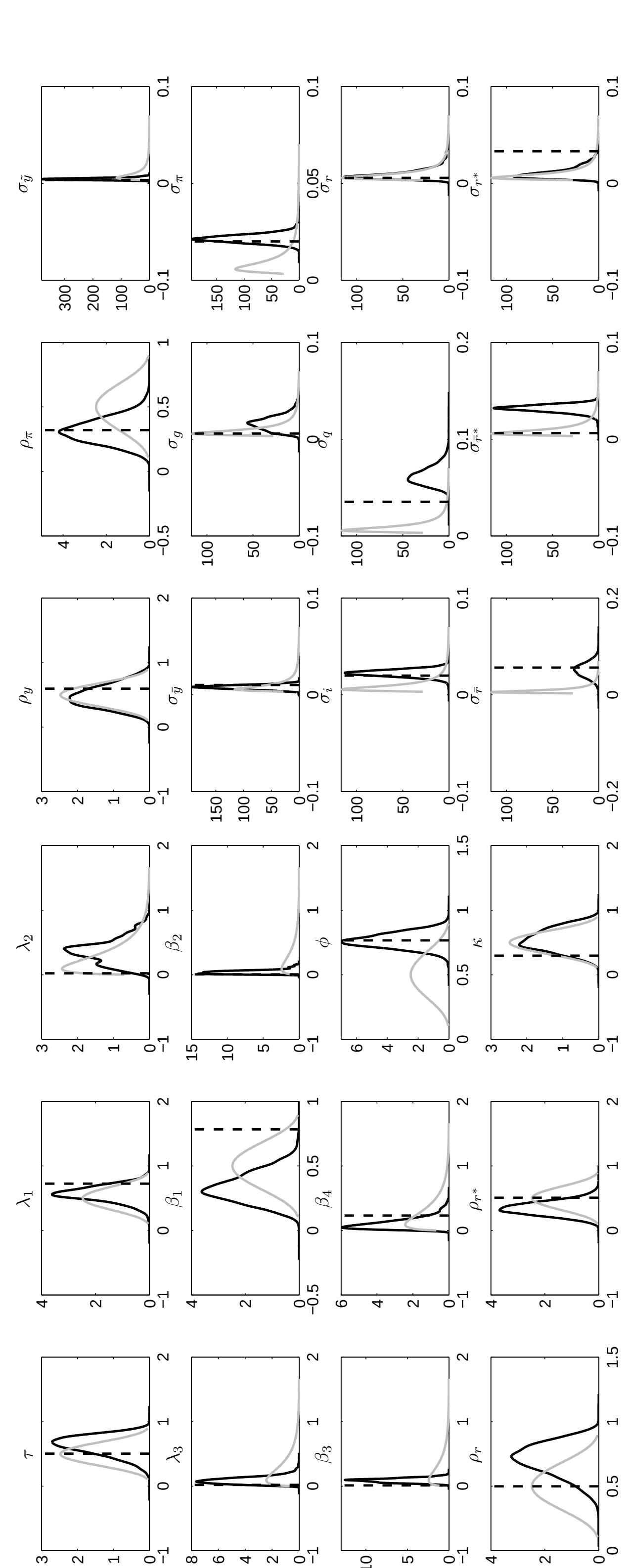
D Estimation Results - Rational Expectations Model

D.1 Unconditioned Estimation

Tab. 5: Estimation Results - Rational Expectations Semi-Structural Model

Parameter		Prior		Posterior			HPD 90%	
		Mean	SD	Mean	Mode	SD	Lower	Upper
Persistence of the growth process	τ	0.50	0.15	0.63	0.51	0.18	0.39	0.85
Inflation expectations	λ_1	0.30	0.25	0.57	0.73	0.08	0.39	0.77
Elasticity of inflation to output gap	λ_2	0.30	0.25	0.36	0.02	0.03	0.06	0.66
Elasticity of inflation to depreciation	λ_3	0.30	0.25	0.09	0.02	0.03	0.00	0.16
Elasticity of output gap to its lag	β_1	0.50	0.15	0.33	0.78	0.08	0.14	0.51
Elasticity of output gap to real interest gap	β_2	0.30	0.25	0.04	0.01	0.01	0.00	0.08
Elasticity of output gap to exchange rate gap	β_3	0.30	0.25	0.09	0.01	0.01	0.04	0.14
Elasticity of output gap to expectations	β_4	0.30	0.25	0.11	0.23	0.10	0.00	0.23
Exchange rate expectations	φ	0.50	0.15	0.76	0.77	0.06	0.67	0.86
Persistence of natural interest rate	ρ_r	0.50	0.15	0.70	0.50	0.15	0.52	0.91
Persistence of foreign natural interest rate	$\rho_{r^\star}$	0.50	0.15	0.34	0.51	0.19	0.16	0.51
Persistence of foreign interest rate gap	κ	0.50	0.15	0.54	0.30	0.08	0.28	0.82
Persistence of IS shock	ρ_y	0.50	0.15	0.47	0.60	0.16	0.20	0.74
Persistence of Phillips curve shock	ρ_π	0.50	0.15	0.32	0.32	0.10	0.16	0.47
Std. Dev. IS Curve Shock	σ_y	0.0125	∞	0.00	0.00	0.00	0.00	0.01
Std. Dev. Potential Output Shock	$\sigma_{\bar{y}}$	0.0125	∞	0.01	0.01	0.00	0.00	0.01
Std. Dev. Growth Shock	σ_g	0.0125	∞	0.02	0.01	0.00	0.00	0.03
Std. Dev. Phillips Curve Shock	σ_π	0.0125	∞	0.02	0.02	0.00	0.02	0.03
Std. Dev. Nominal Interest Rate Shock	σ_i	0.0125	∞	0.02	0.02	0.01	0.02	0.03
Std. Dev. Potential Exchange Rate Shock	σ_q	0.0125	∞	0.06	0.04	0.02	0.05	0.08
Std. Dev. UIP Shock	σ_r	0.0125	∞	0.01	0.01	0.00	0.00	0.02
Std. Dev. Natural Real Interest Rate Shock	$\sigma_{\bar{r}}$	0.0125	∞	0.05	0.06	0.02	0.03	0.08
Std. Dev. Foreign Natural Interest Rate Shock	$\sigma_{\bar{r}^\star}$	0.0125	∞	0.03	0.01	0.00	0.03	0.04
Std. Dev. Foreign Interest Rate Shock	$\sigma_{r^\star}$	0.0125	∞	0.01	0.03	0.00	0.00	0.02

Fig. 10: Prior & Posterior - Rational Expectations Semi-Structural Model



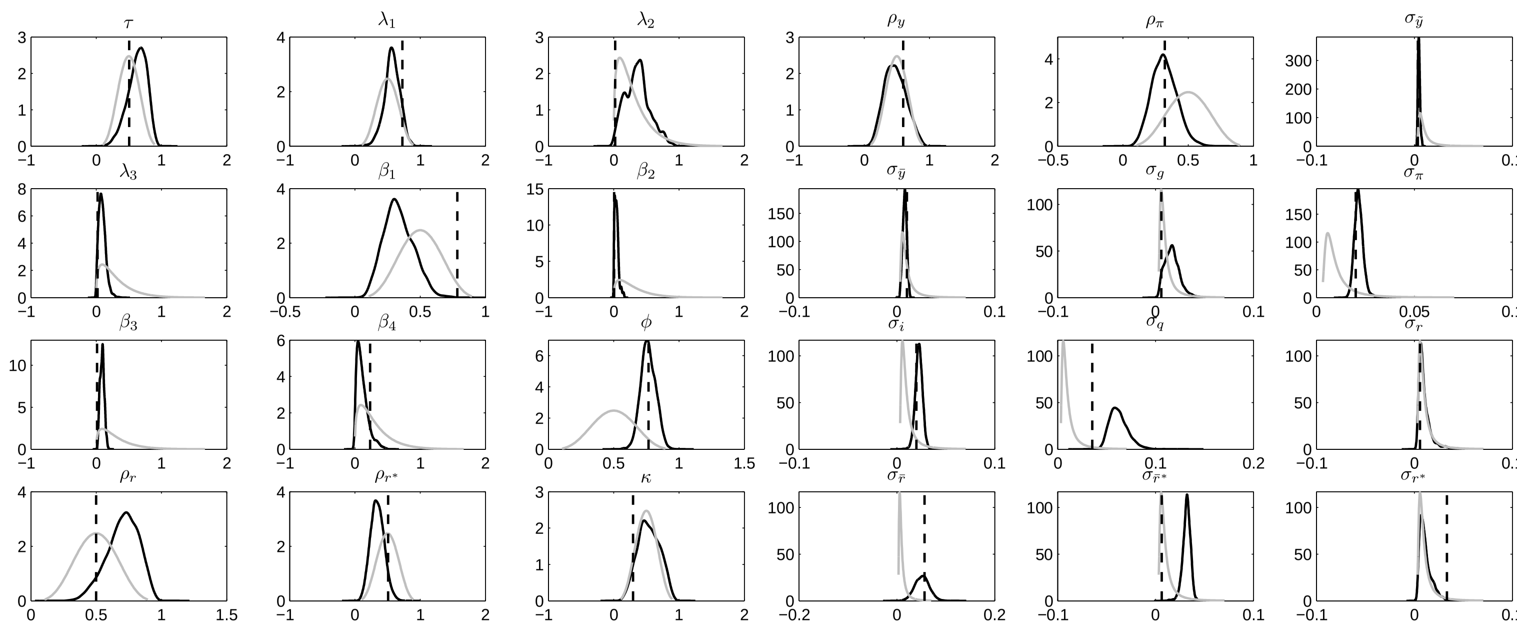
Prior and Posterior density functions for the estimated parameters, Prior functions are in gray and Posteriors functions in black, the dashed vertical line indicates the parameter posterior mean. Calculations were made with Dynare.

D.2 Conditioned Estimation

Tab. 6: Estimation Results - Rational Expectations Semi-Structural Model

Parameter		Prior		Posterior			HPD 90%	
		Mean	SD	Mean	Mode	SD	Lower	Upper
Persistence of the growth process	τ	0.50	0.15	0.56	0.52	0.19	0.29	0.81
Inflation expectations	λ_1	0.30	0.25	0.74	0.75	0.07	0.63	0.88
Elasticity of inflation to output gap	λ_2	0.30	0.25	0.18	0.07	0.07	0.01	0.35
Elasticity of inflation to depreciation	λ_3	0.30	0.25	0.04	0.01	0.02	0.00	0.08
Elasticity of output gap to its lag	β_1	0.50	0.15	0.62	0.72	0.10	0.41	0.83
Elasticity of output gap to real interest gap	β_2	0.30	0.25	0.02	0.01	0.01	0.00	0.05
Elasticity of output gap to exchange rate gap	β_3	0.30	0.25	0.05	0.03	0.02	0.01	0.09
Elasticity of output gap to expectations	β_4	0.30	0.25	0.13	0.14	0.14	0.00	0.26
Exchange rate expectations	φ	0.50	0.15	0.75	0.77	0.05	0.66	0.84
Persistence of natural interest rate	ρ_r	0.50	0.15	0.64	0.65	0.08	0.50	0.78
Persistence of foreign natural interest rate	$\rho_{r^\star}$	0.50	0.15	0.34	0.51	0.20	0.16	0.51
Persistence of foreign interest rate gap	κ	0.50	0.15	0.52	0.28	0.08	0.26	0.78
Persistence of IS shock	ρ_y	0.50	0.15	0.62	0.70	0.14	0.41	0.84
Persistence of Phillips curve shock	ρ_π	0.50	0.15	0.34	0.32	0.10	0.17	0.50
Std. Dev. IS Curve Shock	σ_y	0.0125	∞	0.01	0.00	0.00	0.00	0.01
Std. Dev. Potential Output Shock	$\sigma_{\bar{y}}$	0.0125	∞	0.01	0.01	0.00	0.01	0.01
Std. Dev. Growth Shock	σ_g	0.0125	∞	0.01	0.01	0.00	0.00	0.02
Std. Dev. Phillips Curve Shock	σ_π	0.0125	∞	0.02	0.02	0.00	0.01	0.02
Std. Dev. Nominal Interest Rate Shock	σ_i	0.0125	∞	0.01	0.01	0.00	0.00	0.01
Std. Dev. Potential Exchange Rate Shock	σ_q	0.0125	∞	0.06	0.06	0.01	0.04	0.08
Std. Dev. UIP Shock	σ_r	0.0125	∞	0.01	0.01	0.00	0.00	0.02
Std. Dev. Natural Real Interest Rate Shock	$\sigma_{\bar{r}}$	0.0125	∞	0.06	0.06	0.01	0.04	0.07
Std. Dev. Foreign Natural Interest Rate Shock	$\sigma_{\bar{r}^\star}$	0.0125	∞	0.03	0.01	0.00	0.02	0.04
Std. Dev. Foreign Interest Rate Shock	$\sigma_{r^\star}$	0.0125	∞	0.01	0.03	0.00	0.00	0.02
Std. Dev. Measurement Error	σ	0.0125	∞	0.009	0.005	0.002	0.004	0.015

Fig. 11: Prior & Posterior - Rational Expectations Semi-Structural Model



Prior and Posterior density functions for the estimated parameters, Prior functions are in gray and Posteriors functions in black, the dashed vertical line indicates the parameter posterior mean. Calculations were made with Dynare.