



## **Optimum and Adequate Level of International Reserves**

**By:**  
**Gerencia Técnica**  
**Banco de la República**

### **Summary**

When managing international reserves, central banks generally face the problem of determining what their optimum or adequate level is. A critical review of some methodologies for calculating the optimum amount of reserves is presented in this document. Also, a combination of international liquidity indicators is shown to shed light on the proper level of international reserves, based on a method recently proposed by the International Monetary Fund (IMF). Different exercises are used to illustrate the high sensitivity of the optimum level or reserves when feasible variations in the models' parameters are considered. In addition, these models rely on the questionable assumption that the country has a level of short term external liabilities that is independent of the level of reserves. These factors significantly limit the practical usefulness of these models in assessing the adequate level of international reserves.

**Key Words:** international reserves, optimum level

**JEL classification:** E58, F32

## I. Introduction

When managing international reserves, central banks generally face the problem of determining their optimum or adequate level. Whether there is too much or not enough, an inadequate level of international reserves can generate substantial costs for the economy as a whole. Currently there are multiple indicators and methodologies that generate different criteria which could be used to determine an adequate level. Nevertheless, it is important to bear in mind the limitations and potential problems that using a specific methodology or indicator implies. High sensitivity to key parameters and certain assumptions used in the different methodologies causes them to arrive at very different “optimum” levels of international reserves. A critical review of some methodologies for calculating the optimum amount of reserves is presented in this document along with a combination of indicators of international liquidity which shed light on a suitable level of international reserves based on a method that was recently proposed by the International Monetary Fund (IMF).

## II. Optimum International Reserve Models

The models used to determine the optimum level of international reserves are based on a cost-benefit analysis. The benefit of maintaining international reserves comes from a lower probability that an external crisis will occur or that its cost will be lower in terms of output or consumption. A sufficient level of reserves makes the economy more stable and less vulnerable. The (opportunity) cost of keeping international reserves rises due to the fact that these are invested in low risk liquid assets that have a lower return than other alternative uses (e.g., physical investment, payment of foreign debt).

The pioneering work in this area is that of Heller (1966). Later Ben Bassat and Gottlieb (1992) postulated a model in which the international reserves reduce the probability of an external or exchange rate crisis. In this context, the optimum level of international reserves,  $R^*$ , minimizes the following function of expected costs,  $C$ :

$$C = p(R)C_o + (1 - p(R))Rr \quad (1)$$

$$s.t \quad D = D_n + R$$

Where  $p = p(R)$  is the probability of an external crisis that depends inversely on the level of the international reserves,  $C_o$ , the cost of the external crisis measured as a proportion of the GDP and  $r$  is the opportunity cost of maintaining reserves. In the wealth restriction,  $D$  is the gross foreign debt and  $D_n$  is a constant equivalent to foreign debt net of reserves.

More recently Jeanne (2007) proposed a model for a small, open economy in which a sudden stop in the capital flows into the country hinders access to international financing to make payments on the foreign debt. In this context, the international reserves can be used to mitigate the drop in output and stabilize household consumption. Assuming a policymaker, who is risk averse, Jeanne (2007) found that the optimum reserve level ( $R^*$ ) is the following:

$$R^* = L + C_o - \left[ 1 - \left( 1 + \frac{r}{p} \right)^{-1/\sigma} \right]$$

As can be seen, this level covers the shortfall in foreign financing in the case of a sudden stop ( $L$ )<sup>1</sup> and the associated loss of consumption or output ( $C_o$ ), with an upward adjustment that depends on the probability of a external crisis ( $p$ ) and the degree of risk aversion ( $\sigma$ ), and a downward adjustment that depends on the opportunity cost of keeping reserves ( $r$ ).<sup>2</sup>

Lastly, in a recent study by Calvo et al. (2012), the optimum level of the international reserves is that which minimizes an equation of expected total cost which is similar to (1), but recognizes that reserves can reduce both the probability of an external crisis and its cost

### III. Comments on the Models of Optimum International Reserves and Applications to the Colombian Case

The practical application of the models described in the section above for calculating the optimum level of reserves for a country faces two types of difficulties: 1) the models are very sensitive to the value of the parameters and 2) they assume the country has a given value of short term foreign liabilities. To illustrate the first point, the optimum level of reserves for different parameters using Jeanne's methodology (2007) is shown in Table 1. One of the first results (first line of Table 1) corresponds to the parameters used by Mejia (2012). These are based on estimates using data from the crises in various emerging and developing countries during the last century. As can be seen, the optimum level of reserves for Colombia would have been more than US\$53 billion in 2012.

A second result (second line in Table 1) considers the Colombian economy of the 90s to be different from today's economy. There are huge differences with respect to the size of the currency mismatches, the credibility of the inflation target, the pass through of exchange rate movements to domestic prices, exchange rate flexibility, and the possibility of carrying out counter-cyclical monetary and fiscal policies. An alternative for incorporating the effects of the new policy framework is to normalize the observed responses of the Colombian economy to the shock of the 2008 so that they correspond to the size of the exogenous shock in the 90s.

A simple exercise using this method is obtained when the output losses resulting from the sudden capital stops in 1998-1999 and 2008-2009 in Colombia are compared. Between 1998 and 1999 net capital outflows were 5.69% of 1997 GDP. The accumulated difference between the observed GDP and a calculation of the potential GDP for 1999 and 2000 was -8.8%. On the other hand, between 2008 and 2009, there were net capital outflows of 2% of 2007 GDP while the accumulated difference between the GDP and its potential level for 2009 and 2010 was -1.74%. Based on these data, a sudden stop of a percentage point of the GDP for 1997 produced an output loss of 1.54%

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<sup>1</sup> Foreign debt that cannot be rolled over.

<sup>2</sup> The notation was modified to be compatible with the rest of the document.

of the potential GDP in the 1998-1999 episode, in contrast to an output loss of 0.88% due to a sudden stop of one percentage point of 2007 GDP in the 2008-2009 episode.

In other words, the recent sudden stop was much less costly per unit of shock than the one in the 90s. Even if the nature of the shocks in the two periods was different, the internal conditions of the economy were also different and this is what should be captured in an alternative calculation of the optimum reserves. For example, if a sudden stop size of 10% of GDP is assumed, then the output loss under the new conditions in the economy would be 9%. With this value, the optimum reserves for 2012 would fall from US\$53.6 billion to US\$42.5 billion assuming that the rest of the parameters are the same as those used in the exercise presented in the first line of Table 1. A third result (third line of Table 1) is based on the fact that in 1998-1999 capital outflows were 6% of GDP and thus, the resulting output loss under current conditions would be 5% of potential output. With these parameters, the optimum reserves would be US\$13 billion.

A fourth scenario (fourth line in Table 1) considers the possibility that the probability of a sudden stop could depend on the conditions in the economy or in a group of economies that are similar to Colombia's. In a world where these emerging economies show greater robustness, the probability of a sudden stop originating from this group could be lower, as would be the probability of contagion effects due to crises in other places. For example, with the rest of the parameters being equal to those in the first line of Table 1, a 5% probability of a sudden stop takes the optimum reserves to US\$31.4 billion.

A fifth result (fifth line of Table 1) interprets Jeanne's (2007) theoretical model literally and defines the size of the sudden stop as the value of the payments on Colombia's short term debt in 2012 which in a crisis scenario would not be rolled over. With this size of a shock and a 10% probability of a sudden stop, the optimum amount of reserves would be zero both when the 1998-1999 impact is employed and when the 2008-2009 one is used. Finally, the sixth line of Table 1 presents a configuration (out of many possible ones) that generates a level of optimum reserves that is similar to what Colombia had in June 2012.<sup>3</sup> In this configuration, the assumptions from the exercise in the first line of Table 1 are maintained except for the size of the sudden stop which is fixed at 8% of the GDP and the output loss which is assumed to be equal to 9% of the potential GDP. To summarize, reasonable variations in the parameters produce large changes in the optimum level of reserves.

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<sup>3</sup> US\$ 33.876 billion.

**Table 1**  
**Level of optimum reserves for Colombia in 2012 based on Jeanne's methodology (2007)**

Risk aversion coefficient <sup>1</sup>	Size of the external crisis*	Output loss*	External crisis probability	Opportunity cost of reserves <sup>2</sup>	Optimum level of reserves (US billion)
2	10	12	0,1	0,0168	53,6
	10	9	0,1		42,5
	6	5	0,1		13,0
	10	12	0,05		31,4
	2,5	3.85 o 2.2	0,1		0,0
	8	9	0,1		33,8

1 Corresponds to the economic literature tradition of real cycles and models of small, open economies

2 Colombia EMBi. A return of the reserves equal to zero es assumed.

\*As percentage of GDP

The Ben Bassat and Gottlieb model (1992) is also sensitive to the assumed parameters. For example, in its application to the current Colombian case, the probability of a crisis was defined as a logistic function<sup>4</sup> with parameter  $f$  defined by:<sup>5</sup>

$$f = \alpha_0 + \alpha_1 \ln(R/A) + \alpha_2 \ln(VIX) + \alpha_3(m) + \alpha_4 \ln(Spread) + \alpha_5 \exp(D/X) + \varepsilon$$

This equation implies that the probability of an external crisis depends on the indicator of foreign currency liquidity ( $R/A$ = reserves/next year's payments of foreign debt), global and regional risk indicators ( $VIX$  and  $Spread$ ), capital adequacy indicator ( $(D/X$ = ratio of foreign debt to exports) and the degree to which the economy is open ( $m$ = ratio of imports to GDP). A drop in the level of reserves raises the probability of a crisis when the indicators of external liquidity deteriorate.

The results of the model based on an estimate of the logistic function presented in the Appendix are given in Table 2. Each line in the table assumes different external crisis costs. As can be seen, the results are sensitive to the assumption made about this parameter. The current level of reserves turns out to be optimum with a crisis cost of 10% of the GDP.

<sup>4</sup>  $p=e^f/1+e^f$ .

<sup>5</sup> Ben Bassat and Gottlieb's methodology (1992) has been used on different occasions by the Banco de la República. In 1994 Olivieros and Varela (1994) estimated the optimum level of reserves for 1993 using the following equation:

$f = \alpha_0 + \alpha_1 \ln(R/M) + \alpha_2 \exp(D/X) + \alpha_3 m + \varepsilon$ . Where  $M$  are imports.

In 2003, the Gerencia Técnica updated the calculations and estimated  $f$  to be:  $f = \alpha_0 + \alpha_1 \ln(R/A) + \alpha_2 \exp(D/X) + \alpha_3 m + \alpha_4 \ln(spread) + \varepsilon$ .

**Table 2**  
**Level of optimum reserves for Colombia in 2012 based on the Ben Bassat and Gottlieb's methodology (1992)**

<b>Cost of the crisis (% del PIB)</b>	<b>Optimum reserves First quarter of 2012</b>
5%	US\$ 23.77 b
10%	US\$ 34.09 b
12%	US\$ 37.47 b

As was mentioned at the beginning of this section, the second weakness the models of the optimum reserve level have is that they assume a given value for the short-term foreign debt. That is, if the purchases of international reserves are accompanied by greater short term indebtedness for the country, the net international liquidity position for the economy will not rise as much as the international reserves and the protection against a sudden stop ends up being less than planned. This point becomes particularly relevant when the purchase of reserves is sterilized. In this case, the monetary expansion that results from the purchase of foreign currency is offset by the issuance of government or central bank bonds. This issuance may attract foreign capital which would reduce the country's net short-term foreign assets and the level of protection against a sudden stop.<sup>6</sup>

The size of this effect depends on the degree of substitution between internal and external assets. At one end of the scale, if that substitution is perfect, the sterilized purchase of reserves will prompt capital inflows (possibly short-term ones) for an equivalent amount and completely offset the protection sought through the purchase of reserves. At the other end, if the substitution is null, then each dollar purchased will constitute net protection. Between these two extremes, the net protection achieved will be less than the purchase of reserves. In this case, if the authority decides to reach the level of initial optimum protection, they will have to purchase more international reserves. These additional purchases may be very high if the internal and external assets are close substitutes. At this point, it is possible that the level of initial objective protection may not be optimal given the higher opportunity cost that the additional purchases of reserves imply.

As an illustration, a simple modification of Jeanne's model (2007) in which the level of short-term foreign financing,  $L$ , is no longer fixed but depends on the amount of international reserves,  $L(R)$ , is useful to get an idea of the sensitivity of the model. In this case, the authority recognizes that their optimum choice for international reserves could affect the foreign financing decisions of the

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<sup>6</sup> It is also possible that a higher accumulation of international reserves would reduce the perception of the country's risk and the corresponding risk premiums and, as a consequence, it attracts more capital.

rest of the agents in the economy and incorporates this factor into their optimization problem. With this modification, the optimum level of international reserves is:

$$R^* = L(R^*) + C_0 - \left[ 1 - \left( 1 + \frac{L'(R^*) + r}{p(1 - L'(R^*))} \right)^{-1/\sigma} \right]$$

A simple specification of  $L(R^*)$  could be  $L(R^*) = L_0 + \alpha R^*$  in which the  $\alpha$  parameter measures the response of short term foreign financing in the event of a rise in international reserves. If  $\alpha = 1$ , then each additional dollar in international reserves (purchased with sterilized intervention) will attract an additional dollar of short-term foreign financing. If  $\alpha = 0$ , then the sterilized purchases of reserves will not attract additional foreign financing. Therefore,  $\alpha$  is a measurement of the degree of substitution between internal and external assets. When this specification of  $L(R^*)$  is replaced in the formula of optimum reserves, the result is:

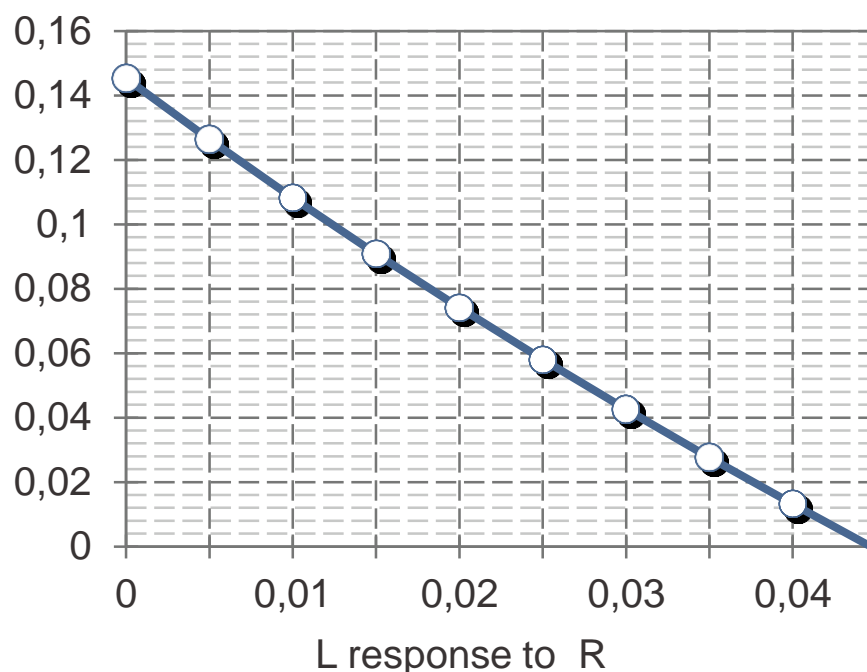
$$R^* = \frac{L_0 + C_0 - \left[ 1 - \left( 1 + \frac{\alpha + r}{p(1 - \alpha)} \right)^{-1/\sigma} \right]}{1 - \alpha}$$

Assuming the parameters in line 1 of Table 1, Graph 1 shows the optimum level of international reserves for different values of the  $\alpha$  parameter. When  $\alpha=0$ , the optimum reserve level is equal to line 1 in Table 1. This result is to be expected since, in Jeanne's model (2007), the optimum amount depends on a level of indebtedness that does not change when reserves are purchased. However, to the extent that the fraction of additional foreign financing induced by sterilized purchases of reserves ( $\alpha$ ) rises, the optimum amount of the latter declines. In fact, for  $\alpha \geq 0.045$  the optimum balance of reserves is lower than or equal to zero. This means that if US\$100 million in sterilized purchases attracts US\$4.5 million or more in capital, the amount of optimum reserves derived from this methodology is zero.

Again, this occurs because sterilized purchases of reserves attract additional capital and the central bank would have to purchase even more reserves to reach a given level of protection against a sudden stop. However, in the process, the central bank will incur in a higher opportunity cost that will dissuade them.



**Graph 1**  
**Optimum Reserves (% GDP)**



In general, to maintain the value of the insurance, it would be necessary for additional capital inflows to be null (or very low). This would be equivalent to a rise in macroeconomic savings or, at least to a situation in which these inflows are long term. The latter is difficult to ensure given that debts can generally be prepaid and, as a result of that, a liability that was originally long term can become a short term one and generate unexpected pressure on the FX market.

In summary, the application of models for calculating optimum international reserves has significant weaknesses. Their substantial sensitivity to changes in the parameters and the assumption of constant foreign liabilities subtract from their usefulness as tools for guiding a reserve policy.

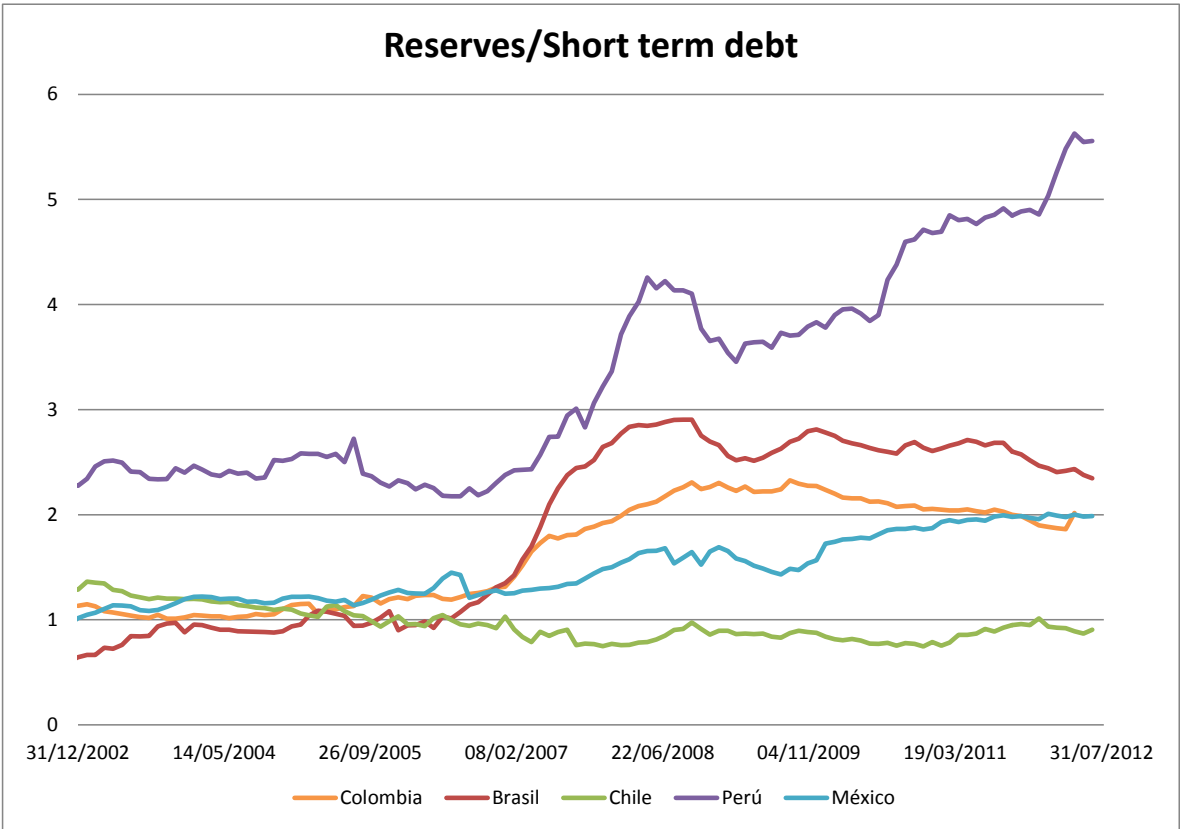
#### **IV. International Reserves Indicators**

Since international reserves are used to shield the country from external shocks that may stem from the current account or the capital account of the balance of payments, the reserves indicators should be related to variables that measure those kinds of shocks. Current account shocks may be the result of a drastic reduction in exports that makes it more difficult to pay for imports. Capital account shocks are caused by difficulties in getting external financing such as restricted access to international loans, reduced foreign investment or higher capital outflows by both foreign citizens and residents.

The reserves to imports indicator is frequently used to measure the ability of a country to meet its current external payments, while the indicators of reserves to monetary aggregates or to short-term debt seek to measure the capital account's vulnerability to shocks. Although the reserves to GDP indicator is used as a benchmark in some cases, it has less relevance because the vulnerability to external shocks does not depend as much on the size of the economy as it does on a country's trade and financial integration with the rest of the world.

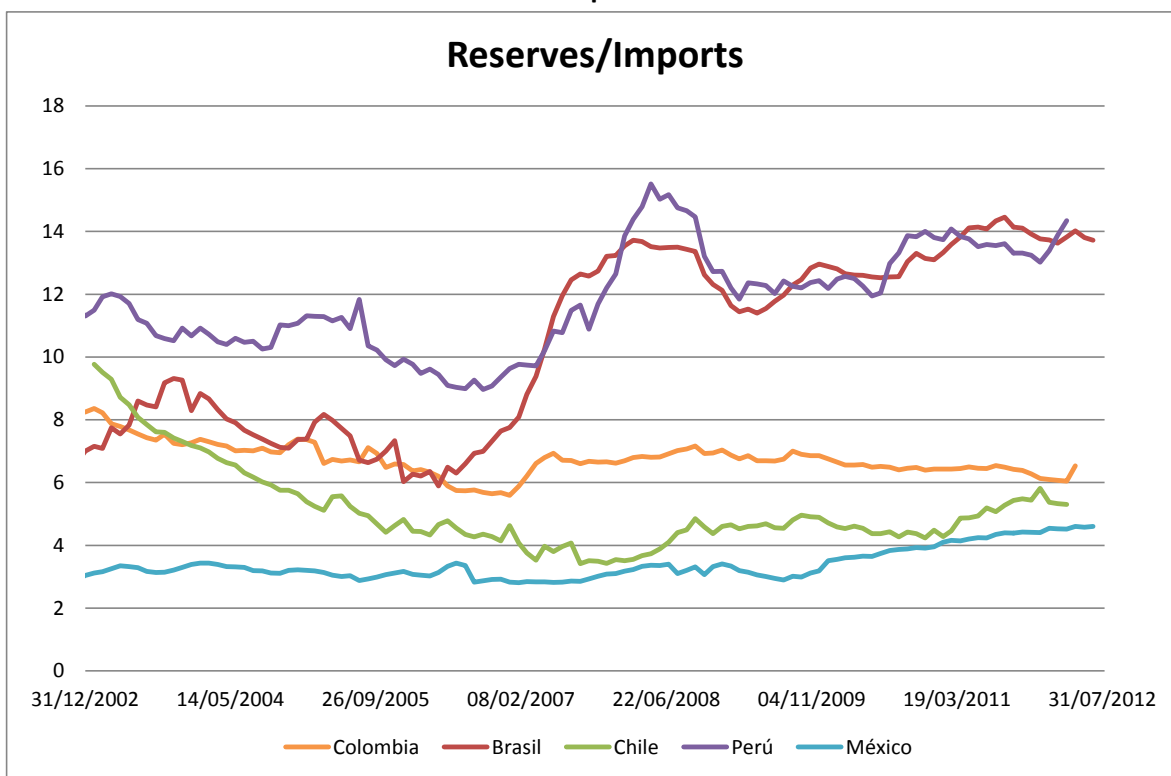
Graphs 2 to 5 show the reserves indicators for Colombia and for some countries in the region<sup>7</sup> calculated by using the long-term component of the denominators. This method makes it possible to filter out fluctuations in the indicators caused by temporary shocks and leads to a better understanding of their trends. As can be seen, the indicators of Colombia's reserves have intermediate values for the region and are not far from their values in recent years.

**Graph 2**

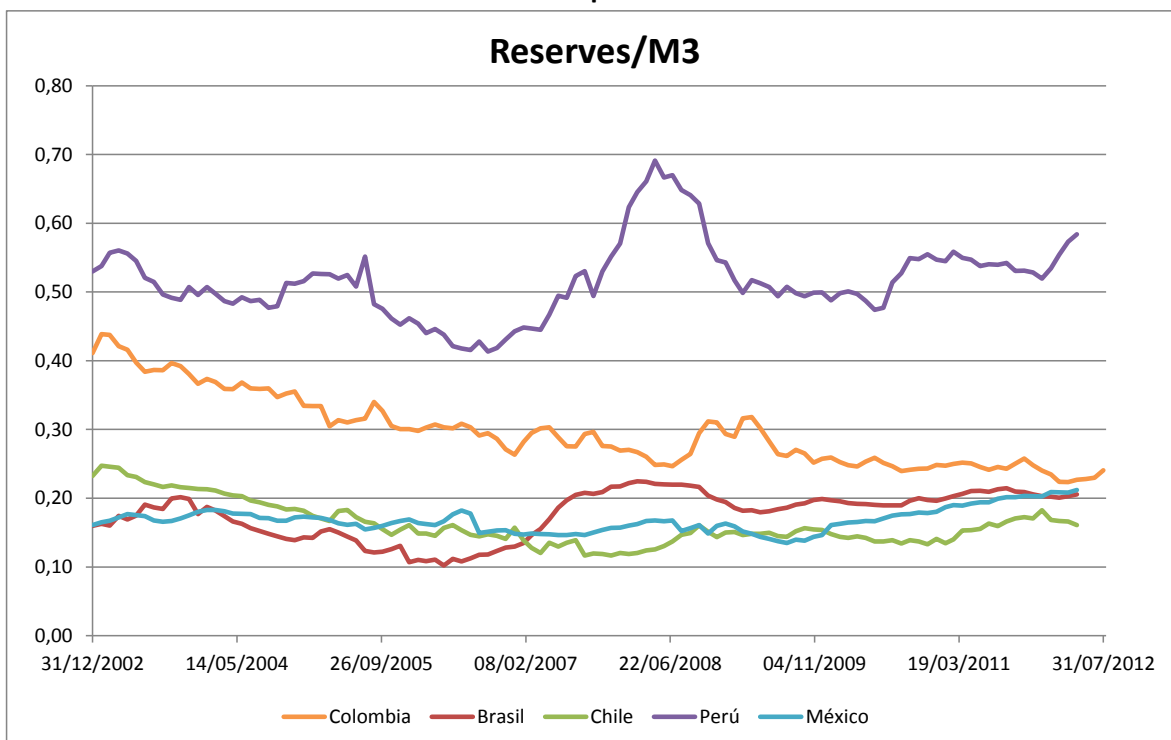


<sup>7</sup> The reserve indicators for Peru are not fully comparable with those of other countries because Peru has a partially dollarized economy and includes a legal reserve in dollars as part of the reserves.

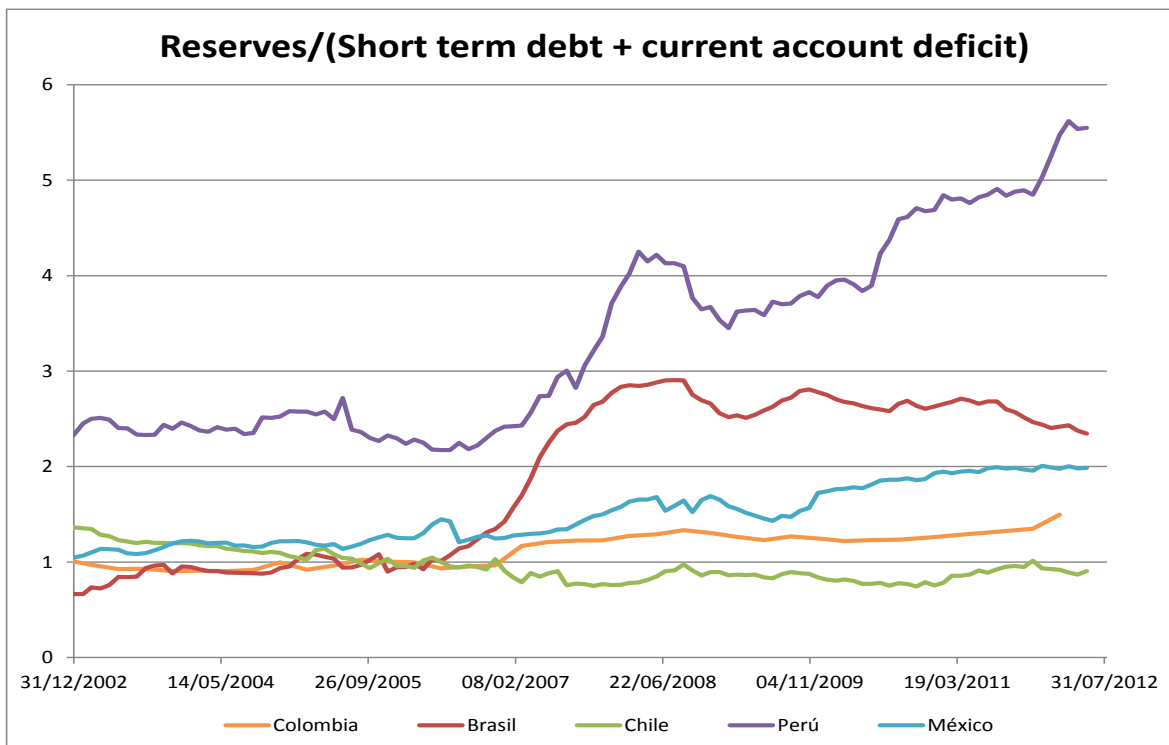
Graph 3



Graph 4



Graph 5



## V. Adequate Level of Reserves

In 2011, the IMF published the paper "Assessing Reserve Adequacy" in which it offers an alternative methodology to the models of optimal reserves. These models are limited by their strong sensitivity to changes in their parameters and the use of isolated indicators that show the coverage of the international reserves in response to individual shocks to the balance of payments (such as those mentioned in the section above).

The methodology proposed by the IMF seeks to determine the level of international reserves that offers protection from all potential sources of risk. Four main sources of risk are identified as follows:

1. Export revenues may be reduced in the event of an unexpected drop in the foreign demand or a negative shock to the terms of trade.
2. A cut off and/or reduction in foreign financing may hinder a rollover of the short-term debt.
3. Outflows may occur due to portfolio flows.
4. There might be unexpected outflows of domestic capital from the country which could be absorbed by the monetary aggregates.

Once the sources of risk and the variables used to measure them are determined, a weighting of the relative risk of each one of the sources is estimated. To this end, distributions of annual losses

associated with periods of exchange market pressure are calculated for each one of the variables by taking a sample of several emerging and low-income countries for the period between 1990 and 2009. In order to obtain the weighting of each variable, the tenth percentile of the distribution is used as this reflects a critical scenario.

Thus, the IMF found that the weightings for different exchange rate regimes are as follows:

$$\textit{Fixed exchange rate:} \quad 30\% \textit{ STD} + 15\% \textit{ PP} + 10\% \textit{ M2} + 10\% \textit{ X}$$

$$\textit{Flexible exchange rate:} \quad 30\% \textit{ STD} + 10\% \textit{ PP} + 5\% \textit{ M2} + 5\% \textit{ X}$$

Where STD corresponds to the short-term debt, PP is foreign portfolio investment, M2 is the monetary aggregate chosen, and X represents exports. For example, the IMF study found that during periods of stress in the FX market, countries with floating exchange rates recorded, on average, 30% reductions in short-term debt, drops of 10% in the balance of foreign portfolio investment, M2 decreases of 5%, and declines of 5% in the level of annual exports. Note that the IMF found that, everything else being constant, a country with a fixed exchange rate system should have a higher level of reserves than a country with a floating exchange regime.

Calculations based on this methodology for several economies<sup>8</sup> (assuming a floating exchange rate system) are presented in Table 3.<sup>9</sup> The results show that no country holds lower than adequate reserve levels.

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<sup>8</sup> The data on portfolio liabilities and exports correspond to the International Financial Statistics published by the International Monetary Fund. The M2 data were obtained from the World Bank while the short-term debt data correspond to the estimates calculated by the Economist Intelligence Unit (EIU) for Brazil, Chile, Mexico, and Peru. Data from Banco de la República were used in the case of Colombia and figures from the World Bank were used for Argentina, India, and Thailand.

<sup>9</sup> The data on reserves for each country correspond to 2011. Regarding the calculation of adequate reserves, data from 2011 are used whenever possible. However, certain data for some countries were only available for 2010.

**Table 3**

Country	Reserves (USD Million)	Adequate Reserves (USD Million)
ARGENTINA	43,373	33447
BRAZIL	350,738	202401
CHILE	41,971	32187
COLOMBIA	31,909	20854
MEXICO	144,307	91418
PERU	47,310	13932
KOREA	306,935	167258
PHILIPPINES	75,123	18612
INDIA	272501	133431
THAILAND	167808	55032
TURKEY	87937	79121

This methodology makes it possible to consider several indicators simultaneously and its results are intuitive since the adequate level of reserves becomes higher as a country's exposure to external variables increases.

## **VI. Conclusion**

The models for the optimum level of international reserves make it possible to identify some relevant general elements for a discussion of a reserve policy. Nevertheless, their practical usefulness is limited due to the sharp sensitivity of their results to feasible variations in the parameters and the fact that they assume a given level of short-term foreign liabilities for the country. Therefore, central banks evaluate different indicators of reserves (or combination of such as suggested by the IMF) to determine their adequate level.

## Appendix

### Estimate of the logistic function for the Ben-Bassat and Gottlieb's model (1992)

#### Econometric Results

Dependent Variable: LRATIO  
 Method: Two-Stage Least Squares  
 Date: 07/24/12 Time: 09:31  
 Sample (adjusted): 1995Q2 2012Q1  
 Included observations: 68 after adjustments  
 White heteroskedasticity-consistent standard errors & covariance  
 Instrument specification: C LSLATIN M\_Y LVIX LGASTOY LRAM(-1)  
 EDTX

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-7.928866	0.827189	-9.585308	0.0000
LRAM	-0.956067	0.092006	-10.39138	0.0000
LVIX	0.667546	0.124125	5.378020	0.0000
M_Y	6.904479	3.438920	2.007746	0.0490
LSLATIN	0.354734	0.105379	3.366275	0.0013
EDTX	4.28E-08	3.81E-08	1.124995	0.2649
R-squared	0.769945	Mean dependent var	-3.595949	
Adjusted R-squared	0.751392	S.D. dependent var	0.560377	
S.E. of regression	0.279407	Sum squared resid	4.840246	
F-statistic	40.92478	Durbin-Watson stat	0.510022	
Prob(F-statistic)	0.000000	Second-Stage SSR	5.064809	
J-statistic	20.10454	Instrument rank	7	
Prob(J-statistic)	0.000007			

Estimate done for TSLS with robust errors for the 1995 Q2–2012 Q1 sample. Instrumental variables: EMBI, imports/GDP, VIX, international reserves/short term debt, lagged one period, government expenditures/GDP, EXP(external debt/exports).

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