



# Systemic Importance Index for financial institutions: A Principal Component Analysis approach<sup>†</sup>

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## Abstract

As a result of the most recent global financial crisis literature has embraced *size*, *connectedness* and *substitutability* as key indicators for financial institutions' systemic importance. Despite the intuitiveness of these concepts, identifying systemic important institutions remain a non-trivial task that implies two primary challenges.

First, designing metrics for *connectedness* and *substitutability* may require, as acknowledged by literature, non-standard data sources and techniques. Second, choosing a methodology capable of aggregating the metrics designed for the three aforementioned concepts into a systemic importance index may be intricate.

The herein paper addresses the second challenge. The chosen approach is to apply Principal Components Analysis to the metrics designed by León and Machado (2011) for assessing *size*, *connectedness* and *substitutability*, where those metrics rely on a combination of balance sheet data and the application of network theory to large-value payment system's information.

Results (i) demonstrate that the three concepts and their metrics are explanatory and non-redundant for differentiating financial institutions' relative systemic importance; (ii) allow for constructing a PCA-based Systemic Importance Index, a valuable tool for financial authorities' policy and decision-making; and (iii) confirm the importance of the *too-connected-to-fail* criteria and the presence of non-banking firms among the most systemically important financial institutions in the Colombian case.

**Keywords:** Systemic Importance, Systemic Risk, Principal Components Analysis, Too-connected-to-fail, Too-big-to-fail.

**JEL Classification:** D85, C63, E58, G28.

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<sup>†</sup> The opinions and statements are the sole responsibility of the authors and do not necessarily represent neither those of Banco de la República nor of its Board of Directors. Results are illustrative; they may not be used to infer credit quality or to make any type of assessment for any financial institution. This document supplements prior work by León and Machado (2011); many of its contents and concepts are transcribed and employed, correspondingly. Valuable comments and suggestions were provided by Clara Machado, Joaquín Bernal, Pamela Cardozo and Constanza Martínez. Large-value payment system data was processed with assistance from Freddy Cepeda and Fabio Ortega. Balance sheet data was processed with assistance from Jorge Cely and Linda Mondragón.

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## Introduction

An institution may be considered as systemically important if its failure or malfunction causes widespread distress, either as a direct or indirect impact (i.e. contagion), where the main criterion for assessing systemic importance relates to their potential to have a large negative impact on the financial system and the real economy (IMF et al. ,2009).

Defining whether a financial institution<sup>3</sup> is systemically important or not may be decidedly intricate but key to the oversight, supervision and regulation of the financial system. To be able to identify systemic importance may serve the purpose of assisting financial authorities in focusing their attention and resources –the intensity of oversight, supervision and regulation- where the systemic severity resulting from a financial institution failing or near-failing is estimated to be the greatest. Identifying systemically important institutions may also serve financial authorities for enhanced policy-making (e.g. prudential regulation, oversight and supervision) and decision-making (e.g. resolving, restructuring or providing emergency liquidity).

Literature has acknowledged the existence of three key criteria for assessing and identifying the systemic importance of financial institutions: *size*, *connectedness* and *substitutability* (IMF et al., 2009; Manning et al., 2009). According to IMF et al. (2009), it is possible to relate *size* and *non-substitutability* to the direct impact of an institution failing to fulfill its role within the financial markets, whilst *connectedness* relates to the indirect impact of such event.

Despite the intuitiveness of these concepts, assessing and identifying systemic important institutions remain a non-trivial task that implies several challenges. Two challenges are particularly demanding. First, designing metrics for *connectedness* and *substitutability* may require, as acknowledged by recent literature, non-standard data sources and techniques, such as financial infrastructures' data and network theory, respectively. Second, choosing a methodology capable of robustly aggregating the metrics designed for the three aforementioned concepts into a systemic importance index may be intricate.

A consultative document by the BIS - Basel Committee on Banking Supervision introduced an early approach to both challenges (BCBS-BIS, 2011). Regarding the first challenge, the proposal relies mainly on traditional balance sheet data, against growing agreement on the convenience of using other data sources and technical approaches (Uribe, 2011a,b; León et al., 2011; ECB, 2010). About the second challenge, somewhat divergent from IMF et al. (2009) concerns and suggestions, the proposal employs an equal and fixed weighting scheme for aggregating five key indicators (i.e. each one assigned a 20% weight), where the relevance of each key indicator does not seem to follow any technique –quantitative or qualitative.<sup>4</sup>

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<sup>3</sup> For this document the authors embrace the term “financial institution” as comprising depository institutions (e.g. banks or savings associations), brokers, dealers, investment companies (e.g. mutual funds), insurance companies, investment advisers and credit unions; this is, those that may not be regarded as a “financial market utility”, where the latter is defined as in the 2010 Dodd-Frank Wall Street Reform and Consumer Protection Act: “any person that manages or operates a multilateral system for the purpose of transferring, clearing or settling payments, securities, or other financial transactions among financial institutions or between financial institutions and the person”.

<sup>4</sup> Other drawbacks of the BCBS-BIS (2011) methodological proposal are briefly discussed in León and Machado (2011), especially regarding the convenience of considering expert judgment as a key input for

Unlike BCBS-BIS (2011) proposal, León and Machado (2011) tackle the first challenge by using balance sheet data and an application of network theory to the large-value payment system's data. They design four inputs or metrics for assessing *size*, *connectedness* and *substitutability* for the Colombian financial system. *Size* is captured via the volume of deposits and money market borrowing, and the volume of financial assets under management. *Connectedness* is captured by measuring the contribution of each institution to the number and volume of the large-value payment system's transactions. *Substitutability* is captured by measuring the *betweenness centrality* (i.e. the brokerage role) of each institution within the large-value payment system's network of transactions.

Regarding the second challenge, León and Machado (2011) employ Fuzzy Logic, an Engineering-type approach based on the deconstruction of expert knowledge into a method that imitates the way experts themselves think about the decision process regarding what a systemically important financial institution is. Despite its advantages, such as capturing non-linearities in the aggregation of key indicators and incorporating IMF et al. (2009) recommendations about the importance of expert judgment and qualitative inputs, the significance of each input or metric is not tested (i.e. relies on expert knowledge), whilst the weight assigned to each input or metric is non-observable.

In order to overcome some of the drawbacks of both proposals regarding the second challenge (i.e. the aggregation of key indicators), the herein paper implements Principal Components Analysis (PCA) on León and Machado (2011) aforementioned four inputs for *size*, *connectedness* and *substitutability* for the Colombian financial market. Therefore, the purpose of this implementation is twofold: (i) identifying how significant are those inputs for differentiating the relative systemic importance for Colombian financial institutions, and (ii) obtaining an alternative Systemic Importance Index with observable but non-arbitrary weights.

Results confirm that (i) the three concepts and their metrics are explanatory and non-redundant for differentiating financial institutions' relative systemic importance; (ii) *connectedness* is the single most important input, followed by *size* and *substitutability*, in that order; (iii) Commercial Banks are the most systemically important financial institution in the Colombian case; (iv) some local non-banking institutions (i.e. Brokerage Firms), despite being insignificant by the balance sheet exposure they engender, are systemically important because of their role within the local financial system.

The document is structured as follows: based on León and Machado (2011), next section briefly introduces the systemic importance concept. Second section introduces PCA basics and documents its application to related studies. Third section presents some aggregated results for the Colombian case based on information available from financial institutions participating in the local large-value payment system (CUD)<sup>5</sup> as of May, June and July 2011. Based on the aggregated results, the fourth section exhibits the systemic importance assessment for Colombian financial institutions. The fifth section presents some final remarks. An exhibit provides further information about

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deciding the weighting scheme. Other comments to the consultative document are posted in the BIS' webpage (<http://www.bis.org/publ/bcbs201/cacomment.htm>).

<sup>5</sup> Colombia's large-value payment system (CUD) is a direct participation system where any financial institution can maintain deposits and conduct transactions with other participants without the need for an agent or intermediary. During the three months comprising the analysis (May, June, July 2011) about 147 financial institutions made transactions within the CUD.

some of the methodological approaches herein implemented for assessing *connectedness* and *substitutability*.

## 1 Systemic risk and systemic importance<sup>6</sup>

As presented by IMF et al. (2009), G20 countries embrace the following –general– definition of systemic risk: the risk of disruption of financial services that (i) is caused by an impairment of all or parts of the financial system and (ii) has the potential to have serious negative consequences for the real economy. Regarding payment systems, the Committee on Payment and Settlement Systems (CPSS-BIS, 2011) define it as the risk that the inability of a financial institution to meet its obligations could result in the inability of other system participants or of financial institutions in other parts of the financial system to meet their obligations as they become due.

Irrespective of which of these definitions is embraced, and despite there is no single definition of risk that can be completely satisfactory in every situation (Dowd, 2005)<sup>7</sup>, it is common to think of risk as a function based on two parameters: frequency and severity (Condamin et al., 2006), also referred as likelihood and impact, respectively (Gallati, 2003). Although academic effort has traditionally focused on systemic concerns based on the estimation of systemic risk (i.e. the product of frequency and impact, as in Norman et al. (2009)), there is a recent interest in focusing on systemic severity or importance.<sup>8</sup>

For example, Paul Tucker, Executive Director for Markets and member of the Monetary Policy Committee of the Bank of England, pointed out the following (Tucker, 2005):

*[T]he interesting question is not whether or not risk will crystallize, as in one form or another risks crystallize every day. Rather, the important question is whether, in the event of nasty shocks, our capital markets can absorb them or whether they have developed characteristics which may, as some suggest, leave them vulnerable.*

More recently, as a result of the most recent episode of global financial crisis, the consultative report “Principles for Financial Market Infrastructures” published by the Bank of International Settlements’ Committee on Payment and Settlement Systems (CPSS-BIS, 2011) includes several principles that aim to provide a high degree of confidence that a financial market infrastructure will continue operating and serve as a source of financial stability even in extreme market conditions. Principles 4 and 7 emphasize the importance of focusing on the severity of the systemic shocks; the latter addresses liquidity risk for financial market infrastructures:

*Principle 7 (Liquidity Risk): A financial market infrastructure should maintain sufficient liquid resources to effect same-day and, where appropriate, intraday settlement of payment obligations with a high degree of confidence under a wide range of potential stress scenarios that should include, but not be limited to, the default of [one/two]*

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<sup>6</sup> This section is transcribed from León and Machado (2011).

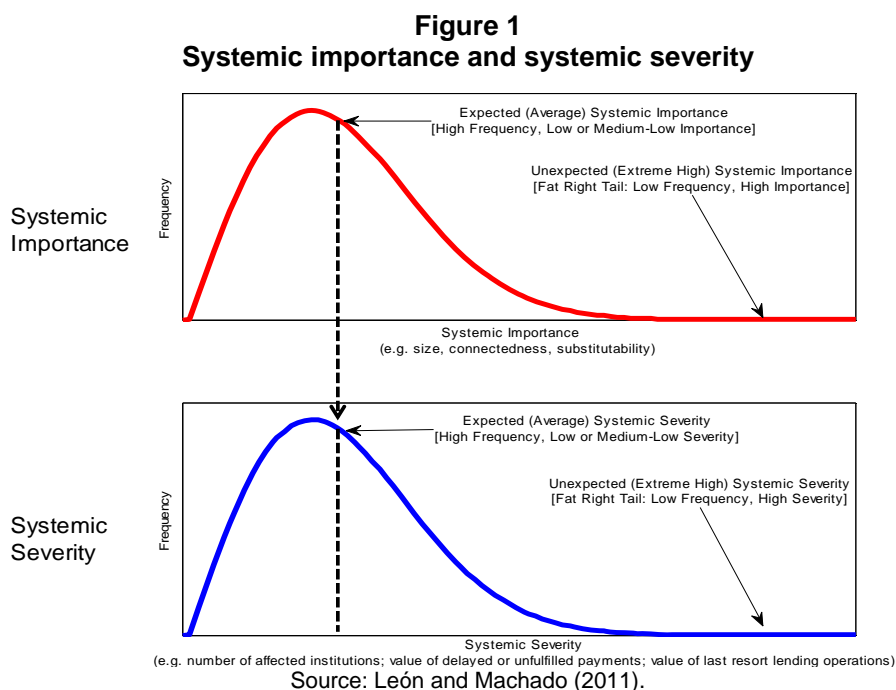
<sup>7</sup> A proper definition of risk is beyond the aim of this paper. Interesting reviews of risk definitions and their implications can be found in Hubbard (2009).

<sup>8</sup> Some authors (Rebonato, 2007; Taleb, 2007) argue that models and techniques for estimating very low probabilities of very disastrous occurrences have demonstrated to yield poor results, and even question the usefulness of those models and techniques for capturing extreme adverse events not found in historical data. Rebonato (2007) also questions the convenience of regulators using VaR-type approaches (i.e. based on estimating low probabilities) to determine prudential capital since even a high percentile (e.g. 99%) would allow a firm to incur losses equal to its regulatory capital rather often (i.e. 2-3 times a year); not to mention if extreme losses cluster, as they do.



participant[s] and [its/their] affiliates that would generate the largest aggregate liquidity need in extreme but plausible market conditions.

Such increasing interest in the impact of systemic shocks –beyond the interest in their frequency- results from the intrinsic characteristics of the financial and payments systems. As pointed out by Haldane (2009) and León et al. (2011), financial and payments networks nowadays may be described as robust to random disturbances, but highly susceptible to targeted attacks.<sup>9</sup> This results from the systemic importance of financial institutions (e.g. *size, connectedness, substitutability*) being distributed with a high degree of asymmetry (right skew) and excess kurtosis, where the average institution is of low systemic importance (Figure 1, upper panel) and the average default or failure-to-pay results in low systemic severity (Figure 1, lower panel); correspondingly, systemically important institutions and their consequent high systemic severities lurk in the extreme right tail of the distributions.



This means that traditional focus on estimating systemic risk as the sum of multiplying each participant’s estimated frequency of failure (or near failure) times its corresponding estimated impact may be dangerously diverting financial authorities from its aim of ever preserving financial stability and payment systems safety: on average the financial stability and payments system’s safety may be “guaranteed”, but not when confronted with a systemically important participant failing. This is, focusing on estimating probabilities of systemic events happening would mean preparing (i.e. overseeing, supervising, regulating) for a severe systemic shock based on the impact of a single –systemically modest- average institution.

Moreover, estimating systemic risk as the sum of multiplying each participant’s estimated frequency of failure (or near failure) times its corresponding estimated impact assumes that failures or near failures by different participants do not come together

<sup>9</sup> As mentioned by Haldane (2009), this explains why there exist long periods of apparent robustness, where peripheral –not systemically important- nodes are subject to random shocks, and short but severe episodes of systemic distress, where systemically important institutions endanger financial stability. Therefore Haldane’s characterization of the current international financial network: “robust-yet-fragile”.

(i.e. they are independent).<sup>10</sup> As recently exhibited by the concurrent episodes of distress of AIG, Lehman and Bear Sterns, such independence is by no means guaranteed.

Therefore, as financial authorities should be prepared to confront a non-average but extreme threat to financial stability or payment systems' safety, the supervision, oversight and regulation should be designed to cope with one (or even two) systemically important institution(s) failing or near failing, as suggested by CPSS-BIS (2011) when formulating Principles 4 and 7 for measuring, monitoring and managing credit and liquidity risks for financial infrastructures. In this sense financial authorities' prudential supervision, regulation and oversight (i.e. policy-making) and decision-making rely on defining what systemic importance is, and identifying institutions that comply with such definition.

According to IMF et al. (2009), G-20 members state that an institution may be considered as systemically important if its failure or malfunction causes widespread distress, either as a direct or indirect impact (i.e. contagion), where the main criterion for assessing systemic importance relates to their potential to have a large negative impact on the financial system and the real economy. This overall criterion may be conveniently explained by three more concise criteria: *size*, *connectedness* and *substitutability* (IMF et al., 2009; Manning et al., 2009).

Designing metrics which capture these criteria is complex and pertains to an ongoing debate. The authors' choice of metrics follows recent literature (Uribe, 2011a,b; León et al., 2011; ECB, 2010) about the convenience of using new data sources and techniques for measuring *size*, *connectedness* and *substitutability*.

## 1.1 Size

Some authors regard an institution as systemically important when exceeding an asset-size cutoff (Saunders et al., 2009), whilst others (IMF et al., 2009) prefer to gauge the amount of financial services it provides to the system. This is the traditional approach to systemic risk, where the systemic importance of a financial institution generally increases with its *size*, where systemically important institutions are labeled as *too-big-to-fail*.

When considering the amount of financial services provided to the system as the metric for *size* some intuitive and straightforward key indicators emerge. Because they belong to the traditional institution-centric approach to micro-prudential supervision, standard accounting data already contains relevant information, such as balance and off-balance sheet exposures (e.g. deposits, money market borrowing and lending) and volume of assets it warehouses or manages, etc.. Other relevant *size* indicators such as the volume of payments by individual institutions are not publicly disclosed, but are available for financial authorities via their involvement in large-value payment systems or via their oversight and supervision duties.

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<sup>10</sup> When estimating market risk this inconvenience is absent: it is impossible that two (or more) scenarios crystallize; there is a unique outcome (i.e. if return resulted to be 1.2% all other realizations are impossible), thus assuming independence of each realization is appropriate. For estimating systemic or credit risk, where simultaneous occurrence of outcomes is feasible (e.g. several firms may enter into default simultaneously or within a short period), this assumption may be inappropriate.



## 1.2 Connectedness

According to the European Central Bank (2010) the properties and behavior of an institution may be affected by institutions that have links to it, and also by other institutions that have no direct links, but are linked to its neighbors. Therefore, the larger the number –and volume- of the links an institution maintains with other market participants, the larger the contagion or spillovers it may generate; this is, the systemic importance of a financial institution generally increases with its degree of *connectedness*. Despite its intuitiveness this is a rather novel approach to systemic risk, where systemically important institutions are labeled as *too-connected-to-fail* (León et al., 2011; Machado et al. 2010; Chan-Lau, 2010; ECB, 2010; Clark, 2010; Zhou, 2009).

Unlike financial institutions' *size*, *connectedness* may be intricate to assess, with regulators and central banks currently lacking the resources to carry out this kind of analysis (Clark, 2010). Network theory<sup>11</sup> provides some concepts and metrics that may assist the assessment of *connectedness*. The most simple concept is the *in-degree* and *out-degree centrality*, which refer to the number of “neighbors” or “partners” an institution has within the network, where the former (latter) corresponds to incoming (outgoing) flows.

Traditional application of network theory for assessing systemic risk relies on balance sheet data such as interbank funding and lending, as in Garrat et al. (2011) or Chan-Lau (2010). Alternatively León and Machado (2011), León et al. (2011) and Machado et al. (2010) use large-value payment system's databases. The choice of *connectedness* metric and of data source (i.e. balance sheet or large-value payment system) will be addressed in the third section.

## 1.3 Substitutability

If the absence of a financial institution distorts the system because it is difficult (or impossible) to find another institution able to provide the same (or similar) type and volume of financial services (e.g. settlement, payments, interbank lending, custody, brokerage), such institution is systemically important. As pointed out by Manning et al. (2009), the severity of the impact of a payment system failure, and hence the extent of systemic risk, depends critically on whether substitutes are readily available to allow payment flows to be rerouted via another system. Consequently, the systemic importance of a financial institution generally decreases with its degree of *substitutability*, where *connectedness* and *substitutability* are both related to the *too-connected-to-fail* criteria.

Unlike financial institutions' *size*, the degree of *substitutability* may be intricate to assess. Despite there are cases in which it is easy to determine that a participant or infrastructure is *non-substitutable* (e.g. if there is a sole infrastructure in charge of all

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<sup>11</sup> Network theory (also referred as network topology or analysis) is a method used in Statistical Physics to understand and analyze the structure and functioning of complex networks. As acknowledged in authors' prior works (León and Machado, 2011; León et al., 2011; Machado et al., 2010), network theory provides appealing methods and techniques to cope with the need to change from an institution-centric to a systemic approach. The studies by Soramäki et al. (2006) and Bech and Garrat (2006) use network theory to characterize the United States (Fedwire) payment system, while Iannotta et al. (2004) apply it to the Japan case (BoJ-Net). Cepeda (2008) applies network theory to the Colombian large-value payment system (CUD) to quantify the impact of failures on its stability.

the market's clearing), it may be cumbersome to determine other participants' degree of *non-substitutability*.

For these cases network theory provides some concepts and metrics that may assist the assessment of *substitutability*. An interesting concept is *betweenness centrality* (Newman, 2010 and 2003; Buechel and Buskens, 2008; de Nooy et al., 2005), which is a measure of a network's resilience based on the assessment of the involvement of a participant in the –indirect- connection of all other participants.

As with *connectedness*, network theory for assessing *substitutability* could rely on data gathered from institutions' balance sheets (e.g. interbank funding and lending) or from large-value payment systems. The choice of *substitutability* metric and of data source (i.e. balance sheet or large-value payment system) will be addressed in the third section.

## 2 PCA basics and related applications

As previously illustrated, the characteristics that literature has identified as key determinants of systemic importance are financial institutions' *size*, *connectivity* and *substitutability*. Nevertheless, two main complications may be faced when calculating a measure associated to systemic importance of financial institutions. The first one consists of obtaining an appropriate measure of each dimension of the problem; this is how to gauge the main characteristics or properties of the system. The second one is related to how to weight these characteristics in order to construct an aggregate indicator, where this indicator may be used as a measure of systemic importance of different financial institutions. Because the first issue has been already addressed when choosing León and Machado (2011) metrics, this section addresses the second complication.

In Economics and many other fields it is quite common to deal with a set of possible correlated variables and observations. The main problem emerges when trying to summarize the common information assigning an appropriate weight for each of the variables.

The case in hand is no exception since we have four characteristics (metrics) related to systemic importance for a set of financial institutions, and the weighting scheme is unknown. Therefore, the main objective is to construct a consolidated measure of systemic importance taking into account the chosen set of characteristics. As illustrated below, this general concern has been present in many fields and sciences, and a common solution has been the use of Principal Components Analysis (PCA).

The basic intuition of this methodology consists of summarizing the information contained in a large number of correlated variables into a new set of variables, much smaller than the former one, named the "principal components". Those components represent a linear combination of an uncorrelated set of the characteristics in an ordered manner. The first principal component retains most of the common variation in all the original variables.

Campbell et al. (1997) describe PCA as a technique that permits to reduce the number of variables being analyzed without losing too much information in the covariance matrix. Our objective is to reduce the dimension form of  $N$  systemic characteristics of financial institutions into  $K$  factors. The principal components serve as these factors.

According to this, the first principal component corresponds to the linear combination of these systemic characteristics with maximum variance, and the second principal component represents the linear combination of the systemic characteristics with maximum variance of all combinations orthogonal to the first principal component.

Using the exposition of this methodology presented in Campbell et al. (1997), the first principal component corresponds to  $x_1^* R_t$  where the vector  $x_1^*$  of size  $(N \times 1)$  is the solution to the following problem:

$$\begin{aligned} & \text{Max}_{x_1} x_1' \Omega x_1 \\ \text{subject to} & \\ & x_1' x_1 = 1 \end{aligned}$$

Where  $\Omega$  represents the sample covariance matrix of a set of variables which are denoted by  $R_t$ . The solution to this problem ( $x_1^*$ ) corresponds to the eigenvector associated with the largest eigenvalue of  $\Omega$ . We denote the scoring factor of each characteristic as its respective position in this eigenvector.

If the linear combination expressed in this eigenvector can explain a representative fraction of the information of the covariance matrix, then it is possible to just use the first principal component in order to assign an appropriate weight to the different variables. As a result we can summarize in an effective way the implicit information of different characteristics and individuals in a linear form.

PCA methodology has been widely applied in many fields. Commonly, the main objective is to construct an aggregate measure combining different characteristics which can be correlated among them. Some PCA-based related applications are listed for illustrative purposes: the construction of an index for the quality of international universities (Steiner, 2006); households' wealth indexes for India (Filmer and Pritchett, 1998); stock market indexes (Feeney and Hester, 1964); an index of credit rating history of loans granted by financial institutions to particulars in Colombia (Murcia, 2007); a composite index to measure economic activity (FRB-Dallas, 2003); a Real Sector Business Confidence Index for Turkey (Ece and Hamsici, 2005); an index to measure financial markets' stress (Amol, 2010); a financial stability index for Colombia (Morales and Estrada, 2010) and financial conditions indexes for different countries (Hatzius et al., 2010; Gómez et al., 2011), among many others.

PCA approach has already been used in the systemic risk literature also. For instance, in order to capture the systemic importance of financial institutions in the United States, Billio et al. (2010) used this approach to capture the interconnectedness among the monthly returns of hedge funds, banks, brokers, and insurance.

Rodriguez-Moreno and Peña (2011) study and compare different systemic risk measures for the biggest banks in Europe and the United States. They find that simple measures based on PCA of banks' credit default swaps (CDS) and interbank rates performed better than more complicated measures based on structural credit risk models (à la Merton, 1974), collateralized debt obligations (CDO) indices and their tranches, multivariate densities and co-risk measures. Additionally, De Cadenas et al. (2010) used PCA in order to identify and evaluate different sources of risk when identifying the systemic nature of an entity. Their analysis shows that all considered institutions contribute to systemic risk, albeit to a different degree, depending on various risk factors such as size, inter-connection, unsubstitutability, balance sheet and risk quality.

Maybe the most important work on systemic risk under PCA approach is Kritzman et al. (2010). Their work studied the statistical properties and association of a big set of financial assets. In this paper they introduced a very useful concept called “absorption ratio” as a measure of financial fragility. This ratio is defined as the proportion of variance which is explained by a finite number of eigenvectors. In the words of the authors: “A high value for the absorption ratio corresponds to a high level of systemic risk, because it implies the sources of risk are more unified. A low absorption ratio indicates less systemic risk, because it implies the sources of risk are more disparate”. Kritzman et al. stress the fact that scenarios with high systemic risk do not necessarily lead to asset depreciation or financial turbulence; it could be simply an indication of market fragility since a shock is more likely to propagate quickly and broadly when sources of risk are tightly coupled.

However, up to the knowledge of the authors this methodology has not been used in order to evaluate the systematic importance for financial institutions in Colombia, nor to a combination of metrics for *size*, *connectedness* and *substitutability*. The basic idea is to combine appropriately the characteristics that the literature has identified as the determinants of systemic importance, and then to construct a PCA-based index using the scoring factors of the first principal component. The value of the index would allow for ranking different financial institutions according to their systemic importance.

### **3 PCA model for identifying systemic importance**

Based on the basic concepts introduced in the previous section, and based on the metrics designed by León and Machado (2011) following criteria defined by IMF et al. (2009) and Manning et al. (2009), this section introduces authors’ proposal for designing a PCA-based Systemic Importance Index.

#### **3.1 The inputs**

According to recent literature on systemic importance for financial institutions and payment systems (IMF et al., 2009; Manning et al., 2009), the most relevant criteria are *size*, *connectedness* and *substitutability*, where the first two relate to their potential to have a large negative impact on the financial system and the real economy, whereas the latter relates to the magnitude of the indirect impact.

Consequently, based on León and Machado (2011), the authors define four key indicators that aim to capture *size*, *connectedness* and *substitutability*, as exhibited in Table 1. Concurrent with IMF et al. (2009) the authors embrace the amount of financial services each institution provides to the system as a metric for *size*, with standard balance sheet data such as (i) volume of deposits and money market borrowing, and (ii) financial assets under management. The first one is intended to capture market exposure to credit institutions while the second captures market exposure to asset management institutions.

Concerning *connectedness* and *substitutability* the authors agree with recent literature that calls for network theory (ECB, 2010) as a mean to gain a better understanding of the financial system. However, unlike standard application of network theory, authors avoid using balance sheet data as the input for such proposal, and decide to use data from the large-value payment system as the primary source of information for assessing both criteria; this is also the choice of various authors for analyzing how financial institutions interact with each other (León and Machado, 2011; Leon et al.,

2011; Machado et al., 2010; Cepeda, 2008; Soramäki et al., 2006; Bech and Garrat, 2006; Iannotta et al., 2004).

As suggested by León and Machado (2011) and León et al. (2011), using large-value payment system data has several advantages for assessing *connectedness* and *substitutability*: (i) it is not clear whether off-balance positions are being captured or not when using claims, whilst payments comprise all transactions between payments system's participants; (ii) unlike claims, relying on payments allow for considering liquidity as a key factor in systemic risk; (iii) as emphasized by Kodres (2009), failure or insolvency are not the only sources of systemic shocks, but mere failure-to-pay or non-payment of transactions can gridlock the entire financial system; and (iv) as acknowledged by Tumpel-Gugerell (2009), a particular institution might not only be systemically relevant because other institutions are financially exposed to it via balance sheet positions, but also because other market participants rely on the continued provision of its services.

It is important to emphasize that the choice of broad key indicators follows several considerations. First and most important, broad key indicators allow for assessing systemic importance of banking and non-banking financial institutions. Unlike most models on assessing systemic importance, which are focused on banking institutions (as in BCBS-BIS (2011)), the authors consider imperative to be able to consider non-banking institutions as relevant as banking institutions; as non-banking-related systemic events have demonstrated (e.g. LTCM, Lehman, AIG, Bear Stearns, Freddie Mac and Fannie Mae), and as put forward by Ötcher-Robe et al. (2011), it is essential to improve the understanding of the shadow banking system to prevent non-banking institutions from gaining systemic importance in an unnoticed manner.<sup>12</sup>

Second, broad key indicators allow for a parsimonious model, which would allow for continuous (e.g. monthly) monitoring of systemic importance. Third, broad key indicators are convenient for comparing results across different financial systems.

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<sup>12</sup> However each key indicator may be broken down into other –more specific– key indicators; as the consultative document by the BCBS-BIS (2011) suggests. Nevertheless, such decomposition may result in an implicit preference for assessing systemic importance of some types of financial institutions (e.g. commercial banks), whilst overlooking others (e.g. brokerage firms, hedge funds). Hence, authors suggest to use wide-ranging key indicators when initially implementing the proposed model, and subsequently increasing their specificity if necessary.

**Table 1**  
**Systemic importance key indicators**

| <b>Key indicators</b>                                    | <b>Description</b>   | <b>Source / Estimation</b>   | <b>Rationale<br/>(When facing a failing or near failing institution...)</b>   |
|--|--|--|---|
| [A]<br><br>Volume of deposits and money market borrowing | Face value of liabilities a financial institution would fail to pay to the public and to other participants of the financial system in the short run.  | Balance sheet data provided by the Financial Superintendence of Colombia.  | ... the larger the deposits and money market borrowing...<br><ul style="list-style-type: none"> <li>• the larger the potential loss in confidence of the public.</li> <li>• the larger the potential impact on other institutions liquidity and solvency.</li> <li>• the larger the potential monetary impact of central bank's liquidity supply to affected financial institutions.</li> </ul>   |
| [B]<br><br>Volume of financial assets under management   | Market value of proprietary assets that may be sold in order to obtain liquidity in the short run, and the volume of assets from third parties which could be compromised or mismanaged in the short run in case of a failure or near failure. | Balance sheet data provided by the Financial Superintendence of Colombia.  | ... the larger the volume of financial assets under management...<br><ul style="list-style-type: none"> <li>• the larger the potential impact on liquidity and solvency of other financial institutions via "liquidity spirals".<sup>13</sup></li> <li>• the larger the potential impact on the real economy via market prices and portfolios' mismanagement.</li> <li>• the larger the potential monetary impact of central bank's liquidity supply to affected financial institutions.</li> </ul> |
| [C]<br><br>Contribution to the payment system            | Contribution to the total payments of the large-value payment system, weighted by the contribution to the total connections of the large-value payment system (CUD).   | Large-value payments system statistics provided by Banco de la República (CUD).  | ... the larger the volume of payments and the number of connections...<br><ul style="list-style-type: none"> <li>• the larger the number of potential institutions affected and the severity of the affectedness.</li> <li>• the larger the potential disruption in the money, capital and exchange markets.</li> <li>• the larger the potential monetary impact of central bank's liquidity supply to affected financial institutions.</li> </ul>  |
| [D]<br><br><i>Betweenness centrality</i>                 | Degree of involvement of a participant in the –indirect-connection of all other participants within the large-value payment system (CUD).  | Estimated as the change in the average number of links necessary for each participant to be connected to all other participants; if removing an institution results in a major (minor or nil) increase in the average number of links all institutions require to remain connected as before, the removed institution is to be considered as of low (high) <i>substitutability</i> . <sup>14</sup> Data provided by CUD. | ... the larger the betweenness centrality...<br><ul style="list-style-type: none"> <li>• the higher the potential efficiency and safety losses for the system.</li> <li>• the larger the potential disruption in the money, capital and exchange markets.</li> </ul>  |

Source: León and Machado (2011)

<sup>13</sup> "Liquidity spirals" refers to the internal amplifying process whereby a falling asset leads to more sales (deleveraging), which further drives down asset prices, financial intermediaries' profit and loss statements, and balance sheets' net worth (Brunnermeier et al. 2009).

<sup>14</sup> Refer to Exhibit A for a brief introduction to the measurement of *connectedness* and *betweenness centrality*.



Under the *size*, *substitutability* and *connectedness* criteria proposed by IMF et al. (2009) and Manning et al. (2009), authors consider *size* is to be captured directly by key indicators [A] and [B], and indirectly by [C]; *connectedness* is to be captured directly by [C], and indirectly by [D]; and *substitutability* is to be captured directly by key indicator [D], and indirectly by [C] (Table 2).

**Table 2**  
**How the selected key indicators of systemic importance relate to criteria from IMF et al. (2009) and Manning et al. (2009)**

| Key indicators                                    | Criteria to be captured |               |                  |
|---|-------------------------|---------------|------------------|
|   | Size                    | Connectedness | Substitutability |
| [A] Volume of deposits and money market borrowing |                         |               |                  |
| [B] Volume of financial assets under management   |                         |               |                  |
| [C] Contribution to the payment system            |                         |               |                  |
| [D] <i>Betweenness centrality</i>                 |                         |               |                  |

|                     |  |
|---------------------|--|
| Directly captured   |  |
| Indirectly captured |  |
| Non-captured        |  |

Source: León and Machado (2011).

Moreover, as presented in Table 3, the four systemic importance key indicators concur with the 2010 Dodd-Frank Wall Street Reform and Consumer Protection Act.<sup>15</sup>

**Table 3**  
**How the selected key indicators of systemic importance relate to criteria from the 2010 Dodd-Frank Wall Street Reform and Consumer Protection Act.**

| Key indicators                                    | Criteria to be captured                  |                                      |  |   |
|---|--|--------------------------------------|--|---|
|   | Aggregate monetary value of transactions | Aggregate exposure to counterparties | Interdependencies and interactions with other participants | Effect on critical markets, institutions and the system |
| [A] Volume of deposits and money market borrowing |  |                                      |  |   |
| [B] Volume of financial assets under management   |  |                                      |  |   |
| [C] Contribution to the payment system            |  |                                      |  |   |
| [D] <i>Betweenness centrality</i>                 |  |                                      |  |   |

|                     |  |
|---------------------|--|
| Directly captured   |  |
| Indirectly captured |  |
| Non-captured        |  |

Source: León and Machado (2011).

<sup>15</sup> The 2010 Dodd-Frank Wall Street Reform and Consumer Protection Act (hereafter Dodd-Frank Act) is United States of America's legislative response to the most recent episode of international financial crisis. Its main objective is to promote financial stability of the United States, whereas Section 804 of the Act addresses the main considerations to designate what systemic importance is.

The Dodd-Frank Act considers that a financial market utility or payment, clearing or settlement activity may be labeled as systemically important under the following four considerations<sup>16</sup>: (i) the aggregate monetary value of its transactions, which is to be directly captured by key indicator [C], and indirectly by [A] and [B]; (ii) its aggregate exposure of their counterparties, which is to be directly captured by key indicator [A], and indirectly by [C]; (iii) its relationship, interdependences, or other interactions with other participants, which is to be directly captured by key indicators [D] and [C], and indirectly by [A]; (iv) the effect that its failure or its disruption would have on critical markets, financial institutions, or the broader financial system, which is to be directly captured mainly by [A], [B] and [C], and indirectly by [D].

### 3.2 Testing the inputs' relevance for identifying systemic importance

The proposed model consists of evaluating the relative characteristics of each financial institution within the Colombian financial system, where the key inputs consist of the four key indicators previously presented (A, B, C, D).

Let A, B, C, D be the systemic importance key indicators as defined and described in Table 1;  $f_x^y$  the PCA's scoring factor or coefficients of the first principal component for key indicator  $x$ , for month  $y$ ; and  $\lambda^y$  the fraction of the total variance of the  $y$ -month panel that is explained by using the first component of the chosen key indicators. The main results are the following:<sup>17</sup>

| $f_x^y$     | $y = \text{May}$ | $y = \text{June}$ | $y = \text{July}$ | <b>Mean</b> |
|-------------|------------------|-------------------|-------------------|-------------|
| $x = A$     | 0.561            | 0.521             | 0.647             | 0.576       |
| $x = B$     | 0.173            | 0.150             | 0.233             | 0.185       |
| $x = C$     | 0.767            | 0.625             | 0.717             | 0.703       |
| $x = D$     | 0.259            | 0.561             | 0.114             | 0.311       |
| $\lambda^y$ | 59.2%            | 56.5%             | 50.2%             | 55.3%       |

Source: authors' calculations.

According to the results, (i) all the scoring factors pertaining to the first component of the model are different from zero; (ii) the scoring factors pertaining to the first component are able to explain more than 50% of the variance of the key indicators; (iii) on average the contribution to the payment system [C] (i.e. connectedness) is the most important key indicator, followed by the volume of deposits and money market borrowing [A], the betweenness centrality [D] and the volume of financial assets under management [B].

In addition, one important fact is that the fraction of the total variance of the  $y$ -month panel that is explained by using the chosen key indicators is decreasing in these months. Using the notion of "absorption ratio" proposed by Kritzman et al. (2010), when this proportion is decreasing, it could be an indication of decreasing market fragility since the sources of systemic risk considered were less coupled. It is important to remember that a financial or macro shock is more likely to propagate quickly and broadly when the sources of systemic risk are highly correlated.

<sup>16</sup> The Dodd-Frank Act does not limit the considerations to these four; the Act includes a fifth, which states that any other factors may be included because of their relevance.

<sup>17</sup> The four attainable components are presented and briefly discussed in Exhibit B.

If the first component's scoring factors are squared  $(f_x^y)^2$  and since by definition the sum of squared score factors must be equal to one, the results may be interpreted as each key indicator's explanatory share or weight. Table 5 exhibits  $w_x^y$ , which corresponds to the weight of key indicator  $x$ , for month  $y$ .

| $w_x^y = (f_x^y)^2$ | $y = \text{May}$ | $y = \text{June}$ | $y = \text{July}$ | <i>Mean</i> |
|---------------------|------------------|-------------------|-------------------|-------------|
| $x = A$             | 0.315            | 0.280             | 0.378             | 0.332       |
| $x = B$             | 0.029            | 0.081             | 0.136             | 0.034       |
| $x = C$             | 0.588            | 0.337             | 0.419             | 0.494       |
| $x = D$             | 0.068            | 0.302             | 0.067             | 0.097       |

Source: authors' calculations.

As before, for the three months analyzed, the most important key indicator is the contribution to the payment system [C] (i.e. *connectedness*), which on average explains by itself around 25% of the variance of the key indicators, followed by the volume of deposits and money market borrowing [A]. In this sense, the results confirm the importance of the *too-connected-to-fail* criteria as the main determinant of relative systemic importance within the Colombian financial system (as in León and Machado, 2011), where institution's *size* is also an important determinant, but secondary to *connectedness*.

#### **4 Systemic importance assessment<sup>18</sup>**

Based on the PCA approach to obtaining an index, this section uses May, June and July 2011 data to calculate the individual systemic importance of the financial institutions participating in Colombia's large-value payment system (CUD). During these three months the average of financial institutions directly participating in the CUD was 147, classified as in Table 6.<sup>19</sup>

Each financial institution's systemic importance key indicator was estimated according to Table 1. Indicators [A], [B] were obtained from statistical data from the Banking Superintendence of Colombia. [C] and [D] were obtained based on large-value payment system's databases (Banco de la República-CUD), and calculated as described in Exhibit A.

<sup>18</sup> Results are illustrative. They may not be used to infer credit quality or to make any type of assessment for any financial institution. Results do not represent an opinion or statement of Banco de la República nor of its Board of Directors. The name of each institution is not revealed due to disclosure restrictions.

<sup>19</sup> For a brief introduction to the functioning and characteristics of the Colombian large-value payments system (CUD), please refer to Banco de la República (2011), León et al. (2011), Machado et al. (2010) or Cepeda (2008).

**Table 6**  
**Main Colombian market's financial institutions directly participating in CUD**  
**(as of May 2011)<sup>d</sup>**

| Class                                 | Institution type                       | Main purpose <sup>c</sup>  |
|---------------------------------------|--|--|
| Credit Institutions (CI) <sup>a</sup> | Commercial Bank (CB)                   | Provision of deposit and loans, including mortgages. [21]  |
|                                       | Commercial Financial Corporation (CFC) | Provision of deposit and loans focused on goods and services commercialization (e.g. leasing). [20]  |
|                                       | Financial Corporation (CF)             | Provision of deposit and loans focused on medium term industrial financing; akin to an investment bank. [3]  |
| Non-Credit Institutions (NCI)         | Mutual Fund (MF)                       | Provision of investment vehicles with the purpose of investing in securities and other assets according to the risk profile of the investor. [27]              |
|                                       | Brokerage Firm (BF)                    | Provision of brokerage services with the purpose of buying and selling securities (e.g. stocks, bonds, currencies); allowed to trade for its own account. [26] |
|                                       | Pension Fund Manager (PFM)             | Provision of investment vehicles with the purpose of investing for retirement. [6]   |
|                                       | Other <sup>b</sup>                     | Insurance companies, financial cooperatives and other. [18]  |

<sup>a</sup> Financial cooperatives pertain to Credit Institutions; due to their low contribution to the CUD and their *size* they were included in the "Other" class; CIs are the only institutions able to receive last-resort lending liquidity.

<sup>b</sup> The "Other" class gathers financial institutions characterized by their particularly low (or nil) relevance for the key systemic importance indicators.

<sup>c</sup> Only the main differencing feature appears; the number of institutions as of May 2011 appears in brackets.

<sup>d</sup> Financial infrastructures and official financial institutions are excluded from this Table and were not analyzed.

Source: León et al. (2011).

Following the methodology for constructing indexes in the literature under PCA approach, let  $f_x^y$  be the coefficient from the first principal component of variance covariance matrix of systemic characteristics for key indicator  $x$ , for month  $y$  (as in Table 4), the Systemic Importance Indicator for a  $\alpha$ -financial institution ( $SII_\alpha^y$ ) will be calculated as the standardized value (i.e. in a 0 to 10 scale) of the following expression:

$$SII_\alpha^y = f_A^y \times A_\alpha + f_B^y \times B_\alpha + f_C^y \times C_\alpha + f_D^y \times D_\alpha$$

It is worth emphasizing that the Index provides a relative assessment of each institution's systemic importance. Therefore, an Index equal to zero does not correspond to the –absolute- absence of systemic importance for that institution, but a negligible importance with respect to the most important institution.

Table 7 exhibits the top-ten Systemic Importance Indicator for the three chosen periods; due to disclosure restrictions the name of the institution is replaced by its type according to Table 6, and a distinctive number.

Regarding these results, three comments are worth making. First, the types which concentrate most systemic importance in the Colombian financial market are commercial banks (CBs) and brokerage firms (BFs), as in León and Machado (2011), León et al. (2011) and Machado et al. (2010). Second, the indexes comprise twelve financial institutions, where eight appear every month, two appear in two months, and two appear in just one of them. Third, a single participant (CB1) resulted as the most systemically important financial institution every month, with CB1, CB2, CB3, CB5 ranking in the top-five every month.

**Table 7**  
**Systemic Importance Index**  
**(Top-ten financial institutions)**

| Rank | May         |       | June        |       | July        |       | Mean        |       |
|------|-------------|-------|-------------|-------|-------------|-------|-------------|-------|
|      | Institution | Index | Institution | Index | Institution | Index | Institution | Index |
| 1    | CB1*        | 10,0  | CB1*        | 10,0  | CB1*        | 10,0  | CB1*        | 10,0  |
| 2    | CB2*        | 8,1   | CB4*        | 9,2   | CB4*        | 8,5   | CB4*        | 8,5   |
| 3    | CB3*        | 8,0   | CB9         | 7,8   | CB2*        | 8,5   | CB2*        | 8,1   |
| 4    | CB4*        | 7,9   | CB2*        | 7,7   | CB3*        | 8,0   | CB3*        | 7,8   |
| 5    | BF1*        | 7,7   | CB3*        | 7,5   | CB5*        | 6,9   | BF1*        | 6,9   |
| 6    | BF2*        | 7,5   | CB10        | 6,8   | BF1*        | 6,8   | BF2*        | 6,9   |
| 7    | CB5*        | 6,6   | BF2*        | 6,6   | BF2*        | 6,6   | CB5*        | 6,6   |
| 8    | CB6*        | 6,1   | BF1*        | 6,3   | CB8         | 6,5   | CB9         | 6,3   |
| 9    | CB7         | 6,0   | CB5*        | 6,3   | CB6*        | 6,5   | CB6*        | 6,1   |
| 10   | CB8         | 5,8   | CB6*        | 5,6   | CB9         | 6,2   | CB8         | 5,8   |

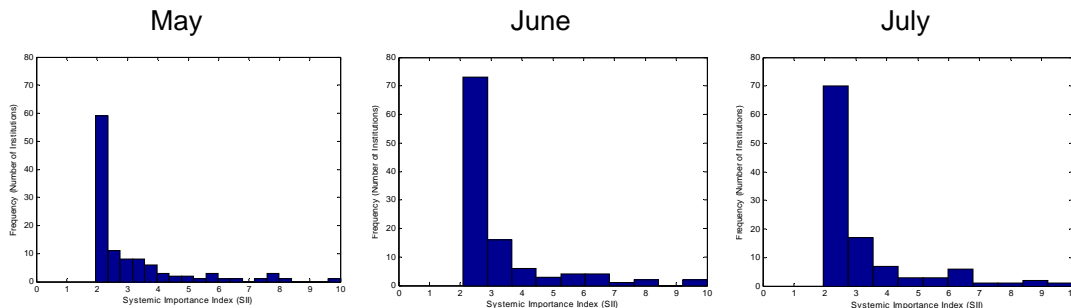
(\*) Institutions appearing in the three months analyzed.  
Source: authors' calculations.

Because the key indicators' data are the same as in León and Machado (2011), CBs emerge as particularly important due to their volume of deposits and money market borrowing, along with their contribution to the large-value payment system (CUD). The two brokerage firms (BFs) appearing in the top-ten rank convey systemic importance because of their particularly high contribution to the large-value payment system.

It also important to highlight that network theory based on payment system data allowed identifying those two BFs as systemically important due to their connectedness. This is not the result of financial system's balance sheet being exposed to BFs, but the result of the financial system relying on these BFs' continuous provision of their services; those two BFs' systemic importance would have been missed using network theory based on traditional balance sheet data.

Concurrent with León and Machado (2011), an important attribute of the Systemic Importance Index is its high level of skewness (Figure 2). Such skewness confirms the intuition regarding the high degree of asymmetry (right skew) of systemic importance, where the average institution is of low systemic importance and the average default or failure-to-pay results in low systemic severity; thus, relying on the systemic importance of the average financial institution would divert financial authorities from its aim of ever preserving financial stability and payment systems safety.

**Figure 2**  
**Distribution of the Systemic Importance Index**



Source: authors' calculations.

## 5 Final remarks

As pointed out by León et al. (2011), the most recent episode of global market turmoil exposed the limitations resulting from institution-centric metrics and the –resulting-traditional focus on *too-big-to-fail* institutions within an increasingly systemic-crisis-prone financial system. This has encouraged the appearance of the *too-connected-to-fail* concept, and has resulted in the emergence of several challenges regarding the estimation of financial institutions' systemic importance by financial authorities.

As previously documented, the main challenges relate to (i) designing measures corresponding to *size*, *substitutability* and *connectedness*, and (ii) choosing a methodology capable of robustly aggregating the metrics designed for the three aforementioned concepts.

Despite the intricacy of the first challenge, there are data sources and methodologies that may overcome some of the difficulties. Vis-à-vis data sources, unlike traditional balance sheet data, financial infrastructure data are particularly dynamic and granular and may help to identify the type, volume and risk profile of the activities and services provided by each type of institution, even at the firm level (Uribe, 2011b). Concerning methodologies, network theory has emerged as an interesting and comprehensive approach to effectively assess financial institutions' *connectedness* and *substitutability*, as suggested by ECB (2010) and Uribe (2011a,b). Together, financial infrastructures' data (i.e. large-value payment systems) and network theory, are supported by literature (Leon et al., 2011; Machado et al., 2010; Cepeda, 2008; Soramäki et al., 2006; Bech and Garrat, 2006; Iannotta et al., 2004) as a convenient approach to better approximate the way financial institutions interact with each other.

Regarding the second challenge, the choice of an aggregating method for the metrics capturing *size*, *connectedness* and *substitutability* is an ongoing topic. Each proposal has its own advantages and disadvantages, and each provides a valuable input for financial authorities.

In the case in hand, because of consisting of a different –purely quantitative- approach, the results from this paper serve the purpose of accompanying the –quantitative and qualitative- outcome of León and Machado (2011). Both methodologies should be regarded as complementary, and their outcomes should not be regarded as substitutes for sound judgment by financial authorities, or the only metrics to use when deciding the systemic importance of a financial institution.

Akin to León and Machado (2011) results based on expert knowledge, results obtained by the PCA-based proposed methodology are straightforward and grant financial authorities with the ability to acquire a comprehensive relative assessment of each financial institution's systemic importance. Both approaches share the advantages of parsimony and ease of calculation, which would allow for convenient continuous (e.g. monthly) monitoring of systemic importance, and for the ability to capture the dynamics of the financial market.

Unlike León and Machado, the PCA-based approach is able to (i) quantitatively test the significance of each metric for differentiating systemic importance within financial institutions, (ii) provide observable weights for the index, and (iii) estimate useful statistics such as the absorption ratio proposed by Kritzman et al. (2010). Unfortunately, this advantage comes at non-trivial costs: (i) the impossibility of



capturing non-linearities in the aggregation of key indicators<sup>20</sup> and (ii) ignoring IMF et al. (2009) recommendations about the importance of expert judgment and qualitative inputs. This reinforces the authors' belief regarding the convenience of developing different approaches with a complementary view.

About the results, despite the difference between both aggregation approaches, the purely quantitative PCA-based Systemic Importance Index supports or confirms León and Machado (2011) main findings with their quantitative and qualitative Fuzzy Logic-based Systemic Importance Index. For instance, (i) the three concepts and their metrics are explanatory and non-redundant for differentiating financial institutions' relative systemic importance; (ii) connectedness is the single most important input, followed by size and substitutability, in that order; (iii) Commercial Banks are the most systemically important financial institution in the Colombian case; (iv) some local non-banking institutions (i.e Brokerage Firms), despite being insignificant by the balance sheet exposure they engender, are systemically important because of their role within the local financial system.

Regarding the usage of the PCA-based Systemic Importance Index, financial authorities are to decide whether an important/unimportant threshold is to be defined within the index. Although authors consider that defining such threshold is intricate and outside the scope of this document, they suggest considering (i) the purpose of defining the threshold (e.g. for defining a capital charge or deciding which institutions to follow closely); (ii) the degree of clustering of the Index (i.e. its skewness); and (iii) a detailed knowledge of the functioning of the financial system.

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<sup>20</sup> As aforementioned, each principal component corresponds to a linear combination of the inputs or metrics. Thus, non-linear effects arising from progressively concentrating systemic importance factors within a single institution (akin to an anti-diversification effect, as described in León and Machado (2011)) may not be captured by this PCA-based proposal.

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## 7 Exhibit A: Assessing *connectedness* and *substitutability* with network analysis basics

As explained throughout the document, literature recognizes three criteria as key to measuring and identifying systemic importance: *size*, *connectedness* and *substitutability* (IMF et al., 2009; Manning et al., 2009). Because of their novelty and intricacy, the first two sections of this exhibit briefly describe the methodologies imported from network analysis in order to measure *connectedness* and *substitutability*<sup>22</sup>, where the third section displays some results obtained with the described methodologies.

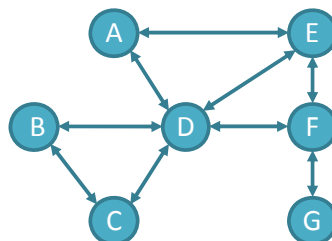
In order to make this exhibit comprehensible please acknowledge the following concepts (Table A1), which pertain to network analysis terminology.

| Term          | Description  |
|---------------|--|
| Vertex        | The fundamental unit of a network. Also referred as node, actor or participant.  |
| Edge          | The line connecting two vertices. Also referred as bond, link or tie. It may be directed if runs in only one direction or undirected if it runs in both directions.  |
| Degree        | The number of edges connected to a vertex. In-degree (out-degree) refers to the number of incoming (outgoing) edges.   |
| Geodesic path | It is the shortest path through the network from one vertex to another. Note that there may be more than one geodesic path between two vertices, and that in a directed network the geodesic path may be different from one vertex to another and its reverse. |
| Distance      | The number of links that is minimally needed to connect two vertices. Neighbors (directly connected) have distance equal to 1; neighbors of neighbors that are not directly connected are at distance 2, and so forth.   |

Source: authors' design, based on Buechel and Buskens (2008) and Newman (2003).

In Figure A1 the previous concepts could be applied as follows: the graph consists of seven vertices or nodes (i.e. A, B, C, D, E, F, G), where the A node is connected to nodes E and D via two undirected edges<sup>23</sup> (i.e. incoming/outgoing from/to E and D); thus, the in-degree and out-degree of A is two. There are two geodesic paths from nodes A to G, consisting of three edges (i.e. the path A-D-F-G and the path A-E-F-G).

**Figure A1**  
**A simple undirected network**



Source: authors' design

<sup>22</sup> Network analysis provides many other metrics and measures related to centrality. This exhibit focuses on the approach chosen by the authors. For a comprehensive review and explanation of alternative metrics and measures please refer to Newman (2010).

<sup>23</sup> Because network analysis is to be applied to payments, where each payment is necessarily related to an immediate, previous or forthcoming transfer of a financial asset (e.g. a bond, a stock, money, etc.), please note that directed edges (i.e. with only one direction) will not be considered.

## 7.1 Connectedness

The main intuition of the *connectedness* criteria asserts that the larger the number – and volume- of the links an institution maintains with other market participants, the larger the contagion or spillovers it may generate; this is, the systemic importance of a financial institution generally increases with its degree of *connectedness*.

According to Newman (2010) and de Nooy et al. (2005), the simplest centrality measure in a network is the *degree* of a vertex; this is, the number of edges connected to it, or the number of its neighbors. This type of centrality metric assesses how intensely the vertex is connected to the network, which relates in our case to how easily can payments arrive to or spread from that vertex. For example, in Figure A1 the most central vertex (i.e. with most edges) is D, which has five of them.

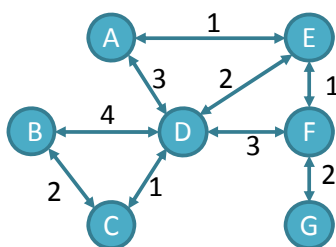
It is common to find that the *degree* of each vertex is normalized with respect to the highest *degree* attainable. Let  $N$  be the number of vertices in a network, and  $E_D$  the *degree* of vertex D, then  $(N - 1)$  is the highest *degree* attainable, then the *degree centrality* of vertex D ( $C_D$ ) may be expressed as in [F1]. Please note that if a vertex is connected to all the other vertices of the network its *degree centrality* will equal 1, whilst an isolated vertex (not connected to the network) will yield a *degree centrality* equal to 0.

$$C_D = \frac{E_D}{N - 1} \quad [F1]$$

Therefore, because of its documented simplicity and usefulness, authors rely on *degree centrality* as a customary metric for *connectedness*. Nevertheless, *degree centrality* of a vertex or node would suffice to assess its centrality only if all edges are judged as equally important; this is the standard case of network analysis applied to social relations or informational networks. In the case in hand, where edges represent payments, it is convenient to recognize the importance of each edge according to the value of the payments it intends to represent.

Consequently, as in other applications of network analysis<sup>24</sup>, it is important to consider each edge's strength, weight or value. Let Figure A2 be a weighted version of Figure A1, where each edge's number represents the weight of the connection between the vertices. It is rather evident that vertex D remains as the most important regarding the weight of the edges it shares with other vertices, with weights adding up to 13.

**Figure A2**  
A weighted network



Source: authors' design

<sup>24</sup> A common case of weighted networks is the bandwidth or the amount of data flowing between nodes within the world wide web (Newman, 2010).



Considering the edges' weights provides new information that was not apparent when using *degree centrality*. Vertices B and C in Figure A1 appear to be equally central, both with two undirected edges, both sharing connections between themselves and to vertex D; moreover, when calculating the *degree centrality* as in [F1], B and C yield the same result ( $E_B/(N-1) = E_C/(N-1) = 2/6 = 0.33$ ). Despite sharing the same *degree centrality*, it is rather clear in Figure B2 that considering weights would signal vertex B as being more intensely connected than vertex C, with weights equal to 6 and 3, respectively.

Analogous to *degree centrality*, authors normalize the total weight of each vertex with respect to the sum of each vertex's edge weight.<sup>25</sup> Let  $W_j$  represent the total weight of all the edges of vertex  $j$ , each vertex's share of the network's weight ( $V_j$ ) may be expressed as in [F2].

$$V_j = \frac{W_j}{\sum_{j=1}^N W_j} \quad [F2]$$

Vertex D, which displays the most weight, yields a 0.342 share of the network's weight. Calculating B's and C's share of the network's weight would yield 0.158 and 0.079, respectively, avoiding considering B and C as equally important within the network because of exhibiting the same number of edges. Unfortunately, comparing B's and F's share of the network's weight (both 0.158) would consider these two vertices as equally important to the network despite F has one more neighbor than B.

Because both approaches to assessing the intensity of the connection are valuable for the case in hand (i.e. payments), where the systemic importance increases with the number of connected institutions and with the share of the total payments, the authors use the product<sup>26</sup> of both metrics as an overall measure of the contribution to the payment system ( $K_j$ ), as in [F3]:

$$K_j = C_j \times V_j \quad [F3]$$

Based on Figure A2, the result of employing such approach is presented in Table A2, where  $K_j$  corresponds to the  $j$ -vertex overall contribution to the network, and  $Kindex_j$  corresponds to the standardized value of  $K_j$  in a 0 to 10 scale.<sup>27</sup>

It is worth noticing that (i) the vertex most contributing to the network is D, receiving the highest score in the 0 to 10 scale, which is intuitive since is the vertex with most connections and with the highest share of the network's weight; (ii) the vertex less contributing to the network is G, receiving the lowest score; (iii) vertices B and C are no longer deemed as equally important as with *degree centrality* alone, where B is more important than C because of the latter's edges weights; and (iv) vertices B and F are no longer deemed as equally important as with the share to the network's total weight,

<sup>25</sup> Please note that the calculation of  $V_j$  as in [F2] counts separately the weights in either direction between each vertex pair, which results in counting each weight twice. As pointed out by Newman (2010), it is possible to compensate for this double-counting by dividing each weight by 2; nevertheless it makes little difference since the analysis focuses on the relative magnitudes and not the absolute values. Moreover, this notation allows for applying [F2] to directed networks as well.

<sup>26</sup> Please note that multiplying both metrics (i.e. the degree centrality and the share of the network's total weight) is analogous to using the AND –conjunction- operator in Fuzzy Logic (Cox, 1994). If a vertex displays both a high (low) level of centrality AND a high (low) level of contribution to the network's total weight, then the product of both levels will be high (low).

<sup>27</sup> As previously introduced (footnote 14), this standardization procedure assigns the maximum index value (10) to the most contributing vertex, and the rest is assigned an index value by means of linear interpolation. Such standardization is straightforward and makes comparisons and calculations easier.

where F is more important than B because of the former's number of edges . Hence, as displayed in Table A2, using the product of both metrics allows for comprehensively and intuitively assessing the importance of the network's vertices.

| <b>Table A2</b>                    |        |        |        |            |
|------------------------------------|--------|--------|--------|------------|
| <b>Contribution to the network</b> |        |        |        |            |
| Vertex                             | $C_j$  | $V_j$  | $K_j$  | $Kindex_j$ |
| A                                  | 0,3333 | 0,1053 | 0,0351 | 1,2        |
| B                                  | 0,3333 | 0,1579 | 0,0526 | 1,8        |
| C                                  | 0,3333 | 0,0789 | 0,0263 | 0,9        |
| D                                  | 0,8333 | 0,3421 | 0,2851 | 10,0       |
| E                                  | 0,5000 | 0,1053 | 0,0526 | 1,8        |
| F                                  | 0,5000 | 0,1579 | 0,0789 | 2,8        |
| G                                  | 0,1667 | 0,0526 | 0,0088 | 0,3        |

Source: authors' calculations

## 7.2 Substitutability

The severity of the impact of a payment system failure, and hence the extent of systemic risk, depends critically on whether substitutes are readily available to allow payment flows to be rerouted via another system (Manning et al. 2009). Consequently, the systemic importance of a financial institution generally decreases with its degree of *substitutability*.

A key concept for assessing *substitutability* comes from network analysis' *betweenness centrality*. *Betweenness centrality* measures the brokerage role of a certain vertex within a network (Buechel and Buskens, 2008) or, as defined by de Nooy et al. (2005), it captures the extent to which a vertex is needed as a link in the chain of contacts that facilitate the spread of information within a network; the more a vertex is a go-between, the more central its position in the network.

The calculation of *betweenness centrality* relies on the *geodesic path* concept, which is the shortest path through the network from one vertex to another. It is calculated for vertex  $j$  as the proportion of all *geodesic paths* between pairs of other vertices that include such vertex (de Nooy et al., 2005). Let  $G_{pq}$  be the number of *geodesic paths* between vertices  $p$  and  $q$ , and  $G_{pq}^j$  the number of *geodesic paths* between  $p$  and  $q$  that go through vertex  $j$ , *betweenness centrality* of vertex  $j$  ( $B_j$ ) is calculated as in [F4]:

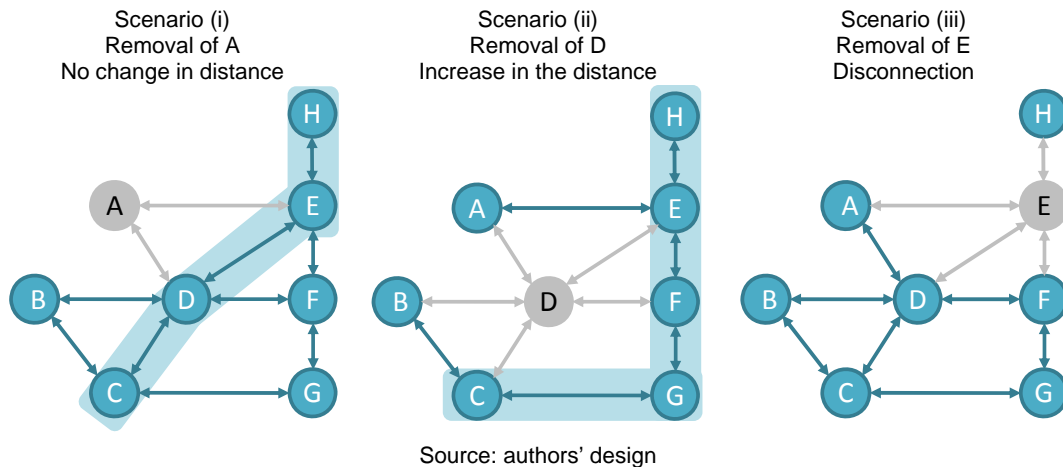
$$B_j = \sum_{pq} \frac{G_{pq}^j}{G_{pq}}, \forall G_{pq} \neq 0, p \neq j, q \neq j, p \neq q \quad [F4]$$

Nevertheless, because *substitutability* is related to the severity of the impact of a vertex being removed, which depends on whether substitutes are readily available to allow preserving the distances between vertices, *betweenness centrality* by itself fails to address this criteria in a proper manner. *Betweenness centrality* captures the importance of a vertex as an intermediary between all the others vertices that compose the network, but it does not capture the severity of the impact resulting from the removal of a vertex.

Because *substitutability* relates to the severity of the impact for the *geodesic path* between vertices  $p$  and  $q$  ( $G_{pq}$ ) resulting from the removal of a  $j$ -vertex, which could be conveniently defined in terms of distance between the remaining vertices, please consider that there are three possible scenarios resulting from removing  $j$  from the network: (i) there is no change in the distance corresponding to the *geodesic path* between vertices  $p$  and  $q$  ( $G_{pq}$ ) because other vertex or vertices could substitute the removed  $j$ -vertex, or because the  $j$ -vertex does not pertain to the *geodesic path* ( $G_{pq}$ ); (ii) other vertex or vertices could substitute the removed  $j$ -vertex, but with an increase in the distance corresponding to the new *geodesic path* between vertices  $p$  and  $q$  ( $G_{pq}$ ); and (iii) there is no other vertex or vertices which may serve the purpose of indirectly connecting vertices  $p$  and  $q$ , hence there would be no *geodesic path* between them ( $G_{pq} = 0$ ), and the distance between vertices  $p$  and  $q$  would be defined as a number larger than any other possible distance in the network.<sup>28</sup>

These three scenarios are depicted in Figure A3. The vertex removed changes in each panel of the figure, and the *geodesic path* to be analyzed is the one corresponding to connecting vertices C and H:

**Figure A3**  
**Scenarios of substitutability**  
**Geodesic path connecting vertices C and H**



In Scenario (i) the impact resulting from the removal of vertex A is nil; the *geodesic path* between C and H ( $G_{CH}$ ) comprises two vertices (D and E, with a total distance of 3), where vertex A is not present. In Scenario (ii) the impact from the removal of vertex D is an increase in the distance between C and H, which increases from 3 to 4 where vertices G, F and E belong to this new *geodesic path*. Scenario (iii) exhibits the third case, where the removal of vertex E results in the impossibility of connecting vertices C and H.

Consequently, for the relation between vertices C and H, vertex E is not *substitutable* (i.e. no other vertex or vertices are able to fulfill its role), vertex A is not relevant (i.e. the *geodesic path* does not include vertex A), and vertex D is *substitutable* but with an increase in the distance between C and H. This result is relevant and emphasizes the importance of considering both *connectedness* and *substitutability*. *degree centrality*

<sup>28</sup> Buechel and Buskens (2008) suggest assigning a number larger than any possible actual distance in the network for those pairs of vertices that cannot reach each other, and choose to use  $N$ ; this is also the authors' choice.

alone would consider Figure A3's vertex E as of lower systemic importance than D (i.e. E has less edges than D), and would not recognize that the removal of D does not result in a major disruption in the network (i.e. it would be substituted by using other – longer- paths), whilst the removal of E would result in a vertex being disconnected from the network.

Accordingly, the approach chosen by the authors to assess the *substitutability* of each vertex comprising the network consists of an iterative procedure of the analysis just described. This is, (i) calculating the average distance of the *geodesic paths* for all the vertices within the network; (ii) removing a vertex from the network and recalculating the average distance of the *geodesic paths* for all the vertices within the network; (iii) calculating the increase in the average distance of all the *geodesic paths* after removing the vertex; (iv) repeating these steps for all the network's vertices.

This procedure will yield each vertex's effect on the average distance of the *geodesic paths* of the network, which is a distance-based metric for assessing the severity of the impact of a vertex being removed, and a useful metric for *substitutability*. It is expected that (i) removing a perfectly *substitutable* vertex (i.e. may be substituted without increasing the distance of the *geodesic paths*) will result in a constant average distance of the *geodesic paths* for all vertices within the network; (ii) removing a non-perfectly *substitutable* vertex will result in an increase in the average distance of the *geodesic paths*, whereas the magnitude of the increase is negatively related to the *substitutability* of the vertex; and (iii) removing a *non-substitutable* vertex will result in an increase in the average distance of the *geodesic paths* and the disconnection of one or more vertices of the network.

## 8 Exhibit B: Principal Components Scoring Factors

In this appendix the values of the first principal components of four systemic characteristics of financial institutions are reported. In addition, the corresponding value of the explained variance by each of the principal components is represented by the value of  $\lambda$ , whereas this information is calculated for the three dates considered in the document.

**Figure B1**  
**Scoring Factors**  
**(May, June, July, 2011)**

| May 2011  | PC1   | PC2   | PC3   | PC4   |
|-----------|-------|-------|-------|-------|
| A         | 0,56  | -0,11 | -0,61 | -0,55 |
| B         | 0,17  | 0,98  | -0,02 | 0,00  |
| C         | 0,77  | -0,12 | 0,62  | 0,13  |
| D         | 0,26  | -0,06 | -0,50 | 0,82  |
| $\lambda$ | 59,2% | 25,6% | 10,2% | 5,0%  |

| June 2011 | PC1   | PC2   | PC3   | PC4   |
|-----------|-------|-------|-------|-------|
| A         | 0,52  | -0,03 | -0,15 | 0,84  |
| B         | 0,15  | 0,88  | 0,45  | 0,02  |
| C         | 0,63  | 0,20  | -0,58 | -0,49 |
| D         | 0,56  | -0,43 | 0,66  | -0,24 |
| $\lambda$ | 56,5% | 21,1% | 18,5% | 3,9%  |

| July 2011 | PC1   | PC2   | PC3   | PC4   |
|-----------|-------|-------|-------|-------|
| A         | 0,65  | -0,18 | -0,07 | 0,74  |
| B         | 0,23  | 0,97  | 0,01  | 0,03  |
| C         | 0,72  | -0,15 | -0,10 | -0,67 |
| D         | 0,11  | -0,04 | 0,99  | -0,01 |
| $\lambda$ | 50,2% | 24,9% | 19,8% | 5,2%  |

Source: authors' calculations

It is worth mentioning that the first principal component explains more than the 50% of the variance for the three dates, and that the four first principal components together (PC1, PC2, PC3 and PC4) explain 100% of the variance of the sample.

The importance of a systemic characteristic such as the volume of financial assets under management (factor B) is increased if the second principal component is considered; in this component that characteristic presents a high and positive scoring factor. In the hypothetic case in which the first principal component would explain a low proportion of the sample variance, the combination of two or more principal components should be used. In that case the characteristic of size represented by the volume of financial assets under management would probably gain more importance into the systemic index. As in the considered observations this is not the case, it is safe to use the first component as an instrument to obtain an appropriate weight of this kind of characteristics.