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Implied probabilities of default from Colombian money market spreads: The Merton Model under equity market informational constraints

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Implied probabilities of default from Colombian money market spreads: The Merton Model under equity market informational constraints[1]

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Abstract

Informational constraints may turn the Merton Model for corporate credit risk impractical. Applying this framework to the Colombian financial sector is limited to four stock-market-listed firms; more than a hundred banking and non-banking firms are not listed.

Within the same framework, firms' debt spread over the risk-free rate may be considered as the market value of the sold put option that makes risky debt trade below default-risk-free debt. In this sense, under some supplementary but reasonable assumptions, this paper uses money market spreads implicit in sell/buy backs to infer default probabilities for local financial firms.

Results comprise a richer set of (38) banking and non-banking firms. As expected, default probabilities are non-negligible, where the ratio of default-probability-to-leverage is lower for firms with access to lender-of-last-resort facilities.

The approach is valuable since it allows for inferring forward-looking default probabilities in the absence of stock prices. Yet, two issues may limit the validity of results to serial and cross-section analysis: overvaluation of default probabilities due to (i) spreads containing non-credit risk factors, and (ii) systematic undervaluation of the firm's value. However, cross-section assessments of default probabilities within a wider range of firms are vital for financial authorities' decision making, and represent a major improvement in the implementation of the Merton Model in absence of equity market data.

Key words: Merton model, structural model, credit risk, probability of default, distance to default.

JEL classification: G2, G13, G33, G32

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[1] The opinions and statements are the sole responsibility of the author and do not necessarily represent neither those of Banco de la República nor of its Board of Directors. Results are illustrative; they may not be used to infer credit quality or to make any type of assessment for any financial institution. The author is indebted to Clara Machado for the discussions that supported the model's design and the document's final version. The author is also indebted to various local money market practitioners for sharing their valuable expertise. Comments, suggestions and assistance from Joaquín Bernal, Pamela Cardozo, Jairo Contreras, Agustín Restrepo, Orlando Chipatecua, Fabio Ortega, Jhonatan Pérez, Karen Leiton, Jorge Cely and Peter Crosbie are acknowledged and appreciated. As usual, any remaining errors are the author's own.

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1. Introduction

Credit risk literature recognizes three main sources of information relevant to the estimation of default probabilities (BCBS-BIS, 2000): (i) financial statements, (ii) market prices, and (iii) subjective appraisals of firm's prospects and risk. Regarding the second source, two main approaches for modeling default probabilities exist: (i) structural models and (ii) reduced-form models. Both approaches use market observed data and some ratios from financial statements in order to model the likelihood of a firm or issuer defaulting on its financial obligations, where the former is based on the definition of a default-triggering event, whilst the second is based on the arrival intensity of default.

Both approaches have advantages and disadvantages, as comprehensively documented in Duffie and Singleton (2003). However, structural models have been recognized as useful and practical for regulatory purposes since they encompass and connect ordinary concepts such as leverage and volatility into a default probability model that may be easily interpreted and discussed by practitioners and financial authorities alike; on the other hand, reduced-form models lack of clear economic rationale for defining the nature of default (Arora et al., 2005).

Structural models begun with seminal work by Merton (1974), who used Black and Scholes (1973) option pricing theory to model the relation between the price of equity, leverage and asset's volatility in order to estimate firm's default probability. Such work, commonly known as the Merton Model (MM), provides the most basic framework for structural models of corporate default.

Nevertheless, because structural models rely on market prices, the availability and quality of such prices determine their suitability and validity. In the Colombian case, where firms with liquid market-listed stocks are scarce, structural models are difficult to implement. Furthermore, if structural models are to be applied for liquid market-listed financial firms the informational constraints are even stricter, and do not allow for a wide range of financial firms to work with (i.e. less than a handful of banking firms), as is the case of Capera et al. (2011) and Souto and Abrego (2009); in this sense, applying structural models for Colombian financial firms is impractical.

In order to overcome the informational constraints arising from the absence of stock market prices, under some supplementary but reasonable assumptions, the herein presented approach uses MM's original framework and money market spreads implicit in sell/buy backs (*simultáneas*) as an alternative to estimate market-implied default probabilities for Colombian financial firms. As will be clear below, money market spreads are particularly informative of the credit quality of a financial firm because their peers (counterparties) have clear and predictable incentives to gather and evaluate information about it and incorporate that information into its transactions; this is, because the money market is a source of market discipline in the sense of Flannery and Sorescu (1996).

Results comprise a richer set of (38) banking and non-banking firms to analyze, mainly banks and broker-dealer firms, which have been identified by León and Murcia (2012) and León and Machado (2011) as the types that concentrate systemic importance in Colombian financial markets. As expected, default probabilities are non-negligible for all firms, where the ratio of default-probability-to-leverage is lower for firms with access to Central Bank's lender-of-last-resort facilities.

The approach is valuable since it allows for inferring market-implied forward-looking default probabilities in absence of stock prices. Yet, results may not be interpreted or used as the "actual" probability of default, and thus their validity is limited to cross-section and serial analysis. This drawback arises from the overvaluation of default probabilities due to (i) spreads containing non-credit risk factors, and (ii) systematic undervaluation of the firm's value. Additionally, the proposed approach relies on the public availability of money market spreads, which is typically limited to financial authorities and financial market infrastructures.

However, cross-section assessments of default probabilities within a wider range of firms are vital for financial authorities' decision making. For instance, as presented at the end of the document, the ability to associate financial firms' systemic importance and default probability provides a comprehensive view of systemic risk, where financial authorities may focus the intensity of supervision, oversight and regulation on those firms that combine higher systemic importance and higher probability of default.

The document is structured as follows: next section describes how the MM is used to infer default probabilities from stock prices. Third section briefly discusses the main informational constraints for applying MM in Colombian financial markets. The fourth presents the alternatives undertaken to overcome the informational constraints found in the Colombian market. Fifth section describes the database used, and enumerates and discusses the main assumptions of the model. Sixth section presents the results obtained by the model. Seventh section addresses a particular use of the results: assessing systemic risk by merging systemic importance and default probabilities. Lastly, the last section discusses some advantages and disadvantages of the model and the challenges ahead.

2. Implied default probabilities using the Merton model (MM)[3]

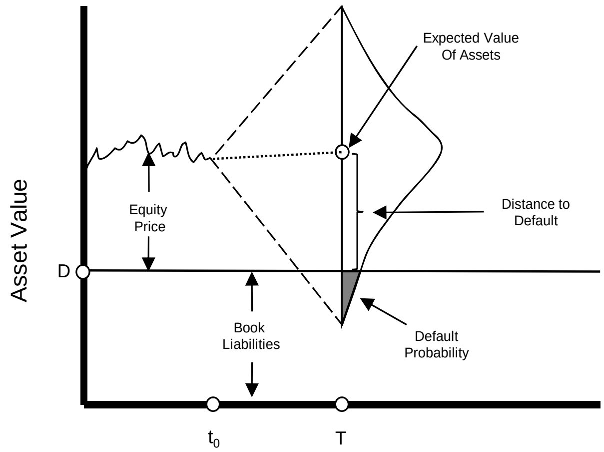
Based on Black and Scholes (1973) option pricing model formulae and assumptions, Merton (1974) infers the probability that the firm's assets will not suffice to satisfy its liabilities (i.e. the probability of default or insolvency). Merton's seminal work, known as the Merton Model (MM), assumes that a diffusion-type stochastic process can describe the dynamics of the value of the firm, where the liabilities of the firm (D) are set at their face or book value (i.e. they are

[3] This section is devoted to describe the Merton model and the usage of spreads as an input for such model. The familiar reader may skip this section without loss of continuity.

time invariant) and, thus, the equity value (E_t) is perfectly correlated to the firm's asset value (A_t).

As exhibited in Figure 1, within MM's framework, and under the –questioned- assumption of stocks and asset values following a geometric Brownian motion[4], the default probability is estimated from the number of standard deviations by which the expected value of assets exceeds liabilities (i.e. distance to default).

Figure 1
The MM's structural model of default



Source: author's design.

Besides the mentioned assumption of the asset's process being described by a geometric Brownian motion or random walk, MM assumes the following:

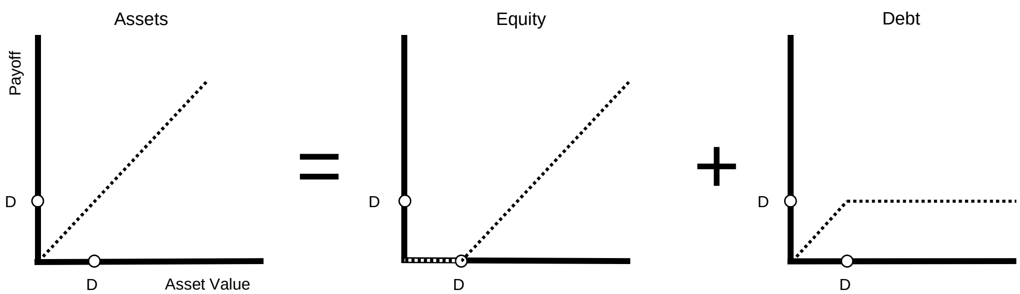
- Markets are frictionless, with no taxes, and there are no bankruptcy costs;
- The typical balance sheet identity holds ($A_t = E_t + D$);
- The capital structure of the firm consists of two claims: an homogeneous senior-class debt involving a risky zero-coupon bond (D) and common equity (E_t);
- The book value of liabilities relative to the market value of assets is the pertinent measure of firm's leverage (Crosbie and Bohn, 2003);
- On the maturity date T the firm must either pay the face value (i.e. book value) of the risky zero-coupon bond (D) to the debt holders or else the equity (E_t) will turn valueless;
- The term structure is "flat" and known with certainty, where r corresponds to the continuously compounded risk-free rate of interest for all maturities.
- Asset's value volatility is constant;

[4] Namely, the geometric Brownian motion or random walk assumption comprises the process being normally distributed, serially independent, continuous (without jumps) and stationary. Since the underlying follows a random walk the standard deviation of the process is proportional to the square-root-of-time rule, which allows for convenient estimation of the distance to default (and the probability of default) at any maturity.

- The assumption of temporal consistency of volatility holds: for instance, daily volatilities contain precisely the same information as five-year volatilities (Holton, 1992), and thus volatility may be scalable to any time horizon by means of the square-root-of-time rule.

Merton (1974) also identified that the payoffs of each constituent of the balance sheet correspond to a simpler exposure or standard financial instrument at maturity T. With the asset value (A_T) as the underlying, the asset side of the balance sheet corresponds to the simple holding of a long position on the underlying; equity corresponds to a call option on the underlying, with strike price D; and debt resembles[5] to a sold put option on the underlying, with strike price D (Figure 2).

Figure 2
Payoffs from balance sheet constituents ($A_t = E_t + D$)



Source: author's design.

Accordingly, the payoffs to the equity holder and debt holder at maturity T may be mathematically expressed as in [F1] and [F2], respectively:

$$max(0, A_T - D) \quad \text{[F1]}$$

$$min(D, A_T) \quad \text{[F2]}$$

Consequently, under the Black & Scholes pricing formulae for European options, the price or market value of the equity (i.e. of the payoffs received by the equity holder) may be calculated at any time t as follows:

$$E_t = A_t \mathcal{N}(d_1) - De^{-r(T-t)} \mathcal{N}(d_2) \quad \text{[F3]}$$

Where $\mathcal{N}(\cdot)$ is the cumulative standard normal distribution, and d_1 and d_2:

$$d_1 = \frac{ln\left(\frac{A_t}{D}\right) + \left(r + \frac{\sigma_A}{2}\right)(T-t)}{\sigma_A \sqrt{(T-t)}} \qquad d_2 = \frac{ln\left(\frac{A_t}{D}\right) + \left(r - \frac{\sigma_A}{2}\right)(T-t)}{\sigma_A \sqrt{(T-t)}}$$

[5] Please note that the debt resembles (it does not correspond) to a sold put option on the underlying, with strike price D. As will be addressed below, debt corresponds to a sold put option plus a spread equal to D.

In this sense, MM traditionally takes advantage of publicly available stock prices to obtain E_t, and the Black & Scholes formulae in order to find two missing –unobservable- variables: the value and volatility of the firm's assets. Obtaining both missing variables is possible by means of two different approaches. The first alternative consists of an *iterative approach* that examines the market value of equity for various dates until attaining a reasonable level of convergence, whilst the second consists of a *numerical solution* by introducing another equation that also contains both missing variables; both approaches are described in Löffler and Posch (2007) and Lando (2004).

After obtaining the asset's implied value and volatility, MM estimates the distance to default, which is the number of standard deviations by which the expected value of assets exceeds liabilities (as in Figure 1). Again, based on Black & Scholes option pricing model, MM estimates such distance ($\widehat{d_2}$) as in [F4].

$$\widehat{d_2} = \frac{ln\left(\frac{\widehat{A_t}}{D}\right) + \left(r - \frac{\widehat{\sigma_A^2}}{2}\right)(T-t)}{\widehat{\sigma_A}\sqrt{(T-t)}} \qquad \text{[F4]}$$

It is worth emphasizing that the equity value used to imply both missing parameters is an equilibrium price, reflecting information known to analysts and investors, and as such it provides their best estimates (Lando, 2004), but they are not the true variables[6]; therefore, MM's distance to default, and the value and volatility of the firm's assets, they carry the traditional "hat" operator ($\widehat{d_2}$, $\widehat{A_t}$ and $\widehat{\sigma_A}$).

The intuition behind $\widehat{d_2}$ being an estimation of the distance to default is now addressed. The first (left) term of the numerator corresponds to the log-difference between the implied value of the firm's assets $\widehat{A_t}$ and the face value of its liabilities (D), whereas the second (right) adjusts this log-difference under the assumption that the expected rate of return of the firm's assets is a drift (deterministic) process dominated by the risk-free rate (r) and proportional to the time-to-maturity of the option ($T-t$).[7] The denominator corresponds to a time-to-maturity adjusted implied standard deviation, where this adjustment results from the application of the square-root-of-time rule for scaling volatility to different maturities; this adjustment is possible since Black & Scholes assumes that no serial memory, either short-term or long-term[8], affects the underlying's process. Thus, the ratio behind $\widehat{d_2}$ may be regarded as the distance to default, or

[6] Lando (2004) explicitly refers to equity value as providing the best estimate for the value of the value of the assets; however, –implicitly- the same rationale applies to the volatility of the assets.

[7] The presence of the volatility term in the right term of the numerator is due to solving the geometric Brownian motion firm's assets process by means of the Itô's lemma. This is addressed in Mikosch (2004).

[8] Evidence demonstrates that long-term dependence is rather frequent in financial time-series (León et al., 2012; León and Reveiz, 2011 and 2012; Peters, 1992; Mandelbrot, 1972). Therefore, this adjustment critically affects the ability of the MM to estimate the distance to default and the corresponding probability of default; for instance, for persistent time-series (i.e. positively correlated serial processes) MM would underestimate the probability of default.

the number of standard deviations that separate the observed leverage (i.e. the book value of liabilities relative to the market value of assets) from reaching 1.

Afterwards, based on the assumption of normally distributed variations of the firm's asset value, MM estimates the probability of default ($\widehat{PD}_t$) as in [F5]:

$$\widehat{PD}_t = \mathcal{N}\left(-\widehat{d_2}\right) \quad \text{[F5]}$$

Furthermore, besides estimating the probability of default, MM allows for estimating the market value of the firm's liabilities (i.e. of the payoffs received by the debt holder) at time t ($\widehat{L_t}$). Based on $\widehat{A_t}$ and $\widehat{\sigma_A}$,

$$\widehat{L_t} = De^{-r(T-t)}\mathcal{N}\left(-\widehat{d_2}\right) - \widehat{A_t}\mathcal{N}\left(-\widehat{d_1}\right) \quad \text{[F6]}$$

$\widehat{L_t}$ has been used for several purposes in the banking industry. For instance, based on the correspondence between loan guarantees and common stock put options, Merton (1978 and 1977) estimates the cost of deposit insurance and its impact on the equilibrium rate of return on deposits in the United States. Capera et al. (2011) assumes that the Colombian sovereign will be responsible for repaying banking firms' outstanding debt net of the market value of their assets, where such contingent liability on the government balance sheet resembles a put option on the banking firm's assets. Flannery and Sorescu (1996) examine whether –observed- bank subordinated debt yields reflect the issuer's estimated risk of default as an evidence of bank market discipline.

3. The equity market: the main informational constraint for applying MM in Colombia

As evident from the previous section, the mainstay of the MM is publicly available stock prices. Under its original framework, without stock prices the model is unable to estimate the market value and volatility of the firm's assets, its distance to default, and the corresponding probability of default.

Such informational constraint should be minor for developed markets, where financial and non-financial firms are listed in their corresponding stock exchanges, whilst emerging or illiquid markets should find this constraint more difficult to overcome. In this sense, some authors choose not to overcome the constraint and decide to preserve the original MM framework. For instance, Souto and Abrego (2009) limit their analysis to five banks within the Colombian financial market, whereas Capera et al. (2011) use information from four banks. In both cases, despite authors claim that their samples account for about half of the banking firm's assets (55% and 46%, respectively), about a hundred financial firms were discarded in the process. Furthermore, non-banking firms (e.g. broker-dealer firms and other credit intermediaries) were absent from the analysis despite some of them are highly leveraged[9] or have been identified as systemically important within Colombian financial markets[10]; for instance, as of June 2012, large–value payments executed by the banking firms used by Souto and Abrego (2009) or Capera et al. (2011) account for less than 25% of the total large-value payments executed by financial firms.

On the other hand, as documented by Saunders and Allen (2002), several authors use asset's book value in order to partially surmount the lack of market information. Gorton and Santomero (1990) and Flannery and Sorescu (1996) follow this alternative approach for the US banking sector, whilst Souto (2008) does for the Uruguayan banking sector. However, without equity prices the volatility of the firm remains unobservable, and some other assumption should be made to be able to estimate volatility based on balance sheet data; for instance, Souto (2008) estimates volatility based on banking sector's assets log-return and deposits log-return.

It is important to realize that the usefulness of stock market prices to solve for the –unobservable- market value and volatility of the firm's assets is not warranted. As highlighted by Souto and Abrego (2009), as any other model that uses market information, the quality of the output depends on how well market information reflects changes in fundamentals. This dependence on the efficiency of the market is especially relevant for illiquid markets, where the efficient market hypothesis is contested by evidence of stock prices time-series' significant long-term persistence.[11]

Other caveats regarding the use of stock market prices within the MM framework exist. According to Löffler and Posch (2007) and Lando (2004), both –previously mentioned- approaches (i.e. *iterative* and *numerical*) yield significantly different estimations of assets' value and volatility, where their divergence increases with the firm's leverage level and variation; this may be important when implementing MM for high-leverage firms, such as some banking firms and other financial institutions. A step further, Lando (2004) even questions how this joint estimation of both unobserved parameters may affect MM's default prediction capabilities. Also, Löffler and Posch (2007) suggest that MM's default probabilities consistently underestimate actual probabilities.

[9] Based on aggregated balance sheet data, some non-banking firms exhibit leverage levels above the average banking firm; because of this leverage level and the absence of lending-of-last-resort facilities according to local regulation, these firms should be of particular interest when applying the MM.

[10] León and Murcia (2012) and León and Machado (2011), based on metrics for size, connectedness and substitutability, conclude that not only large banks are systemically important in the Colombian financial market; heavily interconnected firms (banks, broker-dealers and other credit institutions) are also systemically important financial institutions.

[11] There is an increasing volume of literature that finds significant persistence of financial time-series in emerging markets in the Peters (1996) sense (León et al., 2012; León and Reveiz, 2011; Leiton, 2011; Tabak and Cajueiro, 2008; Jagric et al., 2005).

4. Informational alternatives for estimating default probabilities for Colombian financial institutions

Colombia, as many other emerging markets, suffers from severe informational constraints. As of July 2012, the local stock market lists about 100 common and preferred stocks representing 85 firms, of which 39 stocks are considered as liquid according to local regulatory standards; regarding financial firms, 13 have their stocks listed, of which only 4 firms have common stocks considered as liquid.[12] Hence, in order to successfully and comprehensively apply the MM or some of its variations (e.g. KMV), some informational adjustments to the model have to be designed.

This section presents a proposal to overcome the main informational constraints that limit MM from being fully applicable for a meaningful and diverse sample of Colombian financial firms. The proposal consists of two parts. First, instead of using equity market prices as the key market forward-looking input, to use money market spreads for estimating the probability of default of local financial firms; this choice is supported by compelling evidence regarding the usefulness of debt as a source of information about risk and market discipline. Second, instead of relying on the assumption of market's efficiency to price the value of the firm, and due to the evolution of local and global standards towards fair value accounting, to use balance sheet information as a source of a cross-section unbiased estimate of the firm's value.

4.1. Money market spreads as the market price for credit risk

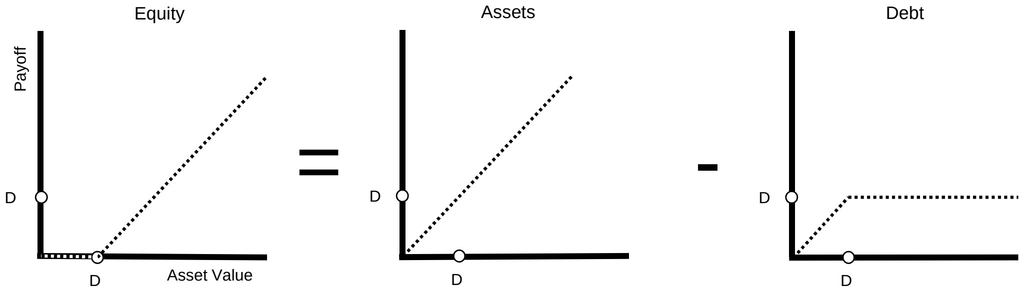
This part addresses two issues. First, how to use debt spreads (i.e. spreads over default-risk free debt) to infer default probabilities within the MM's framework. Second, due to the absence of liquid and homogeneous securities (e.g. bonds, certificates of deposit) issued by a substantial number of financial institutions, how to use local money market spreads as a source of market discipline and informed expectations of credit quality between financial institutions.

4.1.1. Spreads as the market price for credit risk

As previously mentioned, the payoffs for an equity holder equal those of a call option on the assets' value, with strike price D, where the payoffs result from subtracting debt's payoffs from asset's payoffs (Figure 3).

[12] Preferred stock blends the characteristics of a bond and common share, pays dividends and gives the investor a senior claim (over common stock) on a firm's assets in the event of a liquidation or sale (Alexander and Sheedy, 2005). Hence, it may be misleading to compare the probability of default of two firms when they don't issue the same type of stock. Local financial firms' with liquid common or preferred stocks are 6, whereas only four have liquid common stocks.

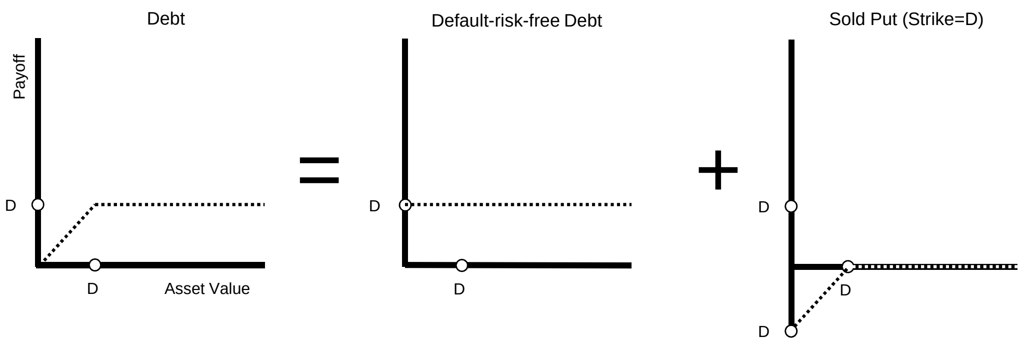
Figure 3
Payoffs from balance sheet constituents ($E_t = A_t - D$)



Source: author's design.

However, debt's payoffs may be broken into two simpler instruments: (i) lending an amount equal to D, and (ii) selling a put option on the firm's assets, with strike price D (Figure 4). In these two instruments the equity holder is the counterparty of the debt holder.

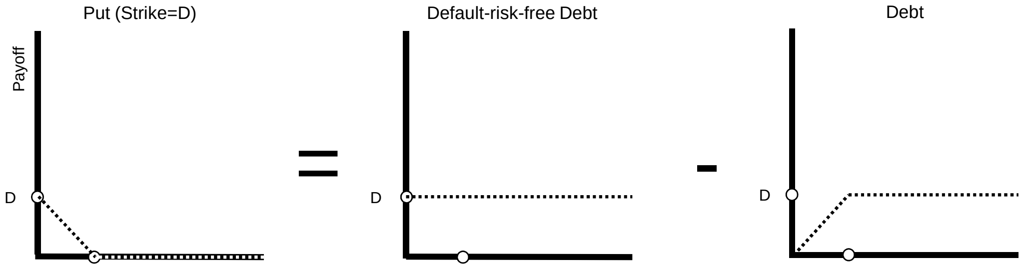
Figure 4
Payoffs from debt



Source: author's design.

Consequently, following Caouette et al. (1998), the debt holders at the same time that they lent to the firm, by recognizing the possibility of default, have also sold a put option to the equity holders that enable the latter to hand the firm's assets in lieu of paying off the debt. Under non-arbitrage conditions, the firm's debt value equals the value of default-risk-free debt plus the value of the sold put option on the firm's assets. Rearranging, a put option on the firm's assets is equal to the difference between the value of default-risk-free debt and the value of (defaultable) debt, where both debts have a face value D (Figure 5).

Figure 5
Put payoffs as a combination of default-risk-free debt and debt



Source: author's design.

Consequently, the value of the put option on the firm's assets with strike D ($Put_{t,K=D}$) is a function of the spread between the risky debt yield (y) and the default-free-risk debt yield (r) [F7]. Hence, by means of the Black & Scholes put option price formulae, it is possible to numerically solve in [F8] for the firm's assets volatility.

$$Put_{t,K=D} = De^{-r(T-t)} - De^{-y(T-t)} \quad \text{[F7]}$$

$$De^{-r(T-t)}\mathcal{N}(-d_2) - A_t\mathcal{N}(-d_1) = De^{-r(T-t)} - De^{-y(T-t)} \quad \text{[F8]}$$

Solving [F8] for the implied volatility of the firm's assets is uncomplicated since it is assumed that the book value of the firm is the market value of the firm under the informational constraints herein considered;[13] a forthcoming section addresses the convenience and suitability of this vital assumption.

4.1.2. Money market spreads as a source of informed expectations of credit quality between financial institutions

As documented by Saunders and Allen (2002), the market values and trading dynamics of risky corporate debt are hard to get all but a few firms, and is generally not available to the public; again, this is the Colombian case. Nevertheless, even if bond prices were publicly available for all firms, it is most likely that they would differ in their financial or legal conditions (e.g. coupon rate, maturity, seniority, collateral, covenants, etc.), making comparisons difficult at best.[14]

Regarding financial institutions, it is tempting to use the rates they pay to their depositors when bond prices are not available. Unfortunately, using deposits' rates may be inconvenient since (i) deposit insurance has the consequence of eliminating incentive for insured depositors to

[13] However, it is possible to simultaneously solve for the firm's assets value and volatility, as in MM's original framework. This would require undertaking the *iterative approach* or the *numerical solution* previously mentioned. Yet, model risk resulting from the joint estimation of both unobserved parameters would arise, as suggested by Lando (2004).

[14] This is why Calomiris (2011, 2009 and 2003) suggests requiring banks to offer credibly uninsured debt instruments as part of their capital structure as a way to obtain market information about risk and market discipline.

monitor their banks (Furfine, 2001); (ii) informational restrictions and costs faced by depositors –especially by the "ordinary depositor"- makes its market discipline content questionable; and (iii) it dismisses firms not authorized to take deposits.[15]

Consequently, an alternative that tackles the three aforementioned inconveniences from using deposit rates must be (i) a subordinated and non-insured liability; (ii) an efficient source of market discipline by means of an efficient monitoring of the financial firms; and (iii) available for a broad base of banking and non-banking firms. In the Colombian case this requisites may be fulfilled by money market transactions.

For the US case Furfine (2001) finds that information from the overnight federal funds market[16] has two main advantages. First, federal funds transactions are large, subordinated and uncollateralized, and thus expose lending institutions to significant credit risk. Second, concurring with Rochet and Tirole (1996) and Calomiris (2003), banks might be particularly effective when monitoring other banks because similar institutions might be expected to identify a peer's risk best.

In the Colombian case, repos, sell/buy backs (*simultáneas*) and interbank lending are the most significant money market operations in the local financial market. Based on daily averages for 2010 and 2011, their relevance as sources of financial firms' liquidity is presented in the following table:

Table 1
Main money market operations as liquidity sources in the Colombian financial market
(as % of the total, based on 2010 and 2011 daily averages)

Operation	Including Central Bank repos		*Excluding Central Bank repos*	
	Number of transactions	Value	*Number of transactions*	*Value*
Sell/buy backs (*simultáneas*) [a]	67.5%	35.2%	*85.0%*	*82.8%*
Repos (between financial firms) [a]	1.7%	0.5%	*2.1%*	*1.2%*
Non-collateralized lending	10.2%	6.8%	*12.8%*	*16.0%*
Repos (with the Central Bank) [b]	20.6%	57.6%		

[a] Operations collateralized with corporate debt or stocks are not considered.
[b] Comprises monetary (open market operations) and intraday repos.
Source: Reporte de Sistemas de Pago - 2011 (2012) – Banco de la República

Non-collateralized[17] lending is usually overnight, and their trades are placed by phone. Unlike other money markets, the contribution of non-collateralized lending as a liquidity source in the Colombian case is rather low (6.8% of money market operations' value), presumably due to the absence of collateral and the corresponding credit risk exposure. Similarly, repos between

[15] The last two inconveniences also apply for Certificate of Deposits (CDs) rates in the US and the Colombian market.
[16] The federal funds market consists of unsecured loans of reserve balances at Federal Reserve Banks that depository institutions make to one another, where the rate at which these transactions occur is called the fed funds rate. These are uncollateralized, have an overnight maturity and are homogenous in their overall conditions.
[17] Please note that the term "non-collateralized lending" is preferred to "interbank lending" in order not to exclude non-banking firms.

financial firms are low contributors to money market liquidity (0.5% of money market operations' value), apparently because –unlike sell/buy backs- the collateral is received with a non-negligible haircut in the Colombian Stock Exchange trading and registering platform (MEC), and because regulation imposes limits on collateral's trading mobility.[18]

As exhibited in Table 1, financial institutions' repos with the Central Bank are their main source of liquidity for years 2010 and 2011, with its contribution varying mainly because of monetary policy objectives. Central bank's repos limit the eligible collateral to central government's securities, mainly local currency denominated TES (*Títulos de Tesorería*), which are also the most commonly accepted collateral for other sources of money market liquidity. Akin to repos between financial firms, repos with the Central Bank encompass a non-negligible haircut[19], but their contribution to the money market liquidity is significant, presumably because of Central Bank's operational advantages and readiness.

Sell/buy backs are the second most important source of money market liquidity, as demonstrated in years 2010 and 2011. Sell/buy backs consist of two sell and buy transactions simultaneously contracted, with the same principal amount and security, with both parties obliged to take the inverse position at maturity (i.e. the buyer becomes the seller), where the property of the collateral is transferred to its buyer. Unlike repos, haircuts and mobility limitations are not imposed on collateral, which may explain why financial firms prefer sell/buy backs to other sources, including repos with the Central Bank during some periods.

Since repos between financial firms and non-collateralized lending are subsidiary sources of liquidity in the money market, and because the cost of the repos with the Central Bank does not follow active credit risk monitoring considerations[20], the most appropriate source of money market information for inferring credit quality is sell/buy back transactions. In this sense, after excluding repos with the Central Bank because their cost does not follow active credit risk monitoring, sell/buy backs are the most important source of credit risk monitoring and market discipline, with their number of transactions and value exceeding 80% of the total (Table 1).

[18] Haircuts applied to local central government's securities (TES) for repos between financial firms in the Colombian Stock Exchange trading platform (MEC) vary between 6% and 15.77% according to the modified duration of the collateral (as of June 2012). As documented by Gorton and Metrick (2010), in the US the haircuts were zero in the pre-crisis period for all asset classes; only after the upheaval of the crisis haircuts eventually rose to 100% for subprime-related asset classes, and to 20% for non-subprime-related asset classes, with investment grade bonds haircuts reaching 2.5%. Local regulation considers repos with (closed) and without (open) limitations to collateral's trading mobility, known as "closed" and "open" repos, respectively; since only the former (closed repos) are effectively traded in the local market, this document refers to this type of repo transactions exclusively.

[19] August 2012 haircuts for eligible collateral for 1-day repos with the Central Bank (i.e. TES) averaged about 2.7% (2.5% for TES with residual maturity of less than one year, 2.7% for one to five years, and 2.9% for longer maturities). According to Hördahl and King (2008), under Basel II the standard supervisory haircut for a repo transaction is a 0.5% for sovereign bonds with a residual maturity of less than one year, 2% for residual maturity from one to five years, and 4% for longer-maturity issues.

[20] There exist limits to the outstanding repos a financial firm may have with the Central Bank. However, those limits do not result from active credit risk monitoring activities from the Central Bank, but from 14-day average accounting figures, and thus the cost of the repo may not be related to credit risk considerations. Hence, Cardozo et al. (2011) highlight money market operations between Colombian financial firms as a source of monitoring.

However, as highlighted by Cardozo et al. (2011) and Hördahl and King (2008), not all sell/buy back or repo transactions result from equal motivations, and thus they convey some degree of "noise". Some sell/buy back operations result from liquidity-demanding financial firms, whilst other result from securities-demanding firms.

In the former case the liquidity-demanding institution uses anonymous trading platforms provided by the Colombian Stock Exchange (MEC) and the Central Bank (SEN), or the over-the-counter (OTC) market to satisfy their liquidity needs. The MEC platform is open to a broad base of financial firms (about 140), where each firm determines a quota or exposure limit for each other potential counterparty, where this limit follows active credit risk assessment from the liquidity-offering firm.[21] Therefore, if a firm is regarded by its peers as being of high credit quality it will easily satisfy its liquidity demands because of extensive loose limits, which will allow it to *cherry-pick* the most convenient –lower- rates; in this sense, a high credit quality firm may take advantage of a "deep market", where its liquidity needs will be satisfied without significant marginal funding costs. Conversely, a low credit quality firm will face tight limits, which will force it to increasing marginal funding costs due to a "shallow market".

Regarding the SEN platform, participation is limited to a select group of 15 financial institutions that are known as "market makers" (*creadores de mercado*), where trades between them are anonymous and counterparty limits or quotas do not exist[22]; hence, as will be clear in a forthcoming section, sell/buy backs in the SEN platform would not be a useful source of market discipline and credit risk assessment by the market. About OTC sell/buy backs, due to their bilateral nature, they convey information concerning the credit quality of the counterparties, and thus it is useful as a source of market discipline.

On the other hand, when a security-demanding firm drives the sell/buy back transaction, this firm is ready to deliver liquidity for a low cost in order to obtain the needed security; as documented by Hördahl and King (2008), cash providers will accept a lower return on their cash when they need to borrow a specific security, for example to be able to cover a short position. The cost the security-demanding firm would be prepared to receive should be below the Central Bank's lowest-cost collateralized money market liquidity facility; otherwise, its potential counterparty would prefer to use its security as collateral in a repo with the Central Bank.

[21] Market practitioners claim that counterparty limits are based on several approaches to credit risk monitoring. They mention the CAMEL rating system (i.e. Capital adequacy, Asset quality, Management quality, Earnings, Liquidity, Sensitivity to market risk), credit rating agencies reports, local and foreign reports on Colombian financial firms, and publicly available market information. Some practitioners do not discard factors such as a counterparty being part of a conglomerate, reputational risk or the importance of long-established business relations. As will be highlighted in the next section, practitioners claim that this counterparty limits are typically valid for collateralized operations from 1-day to 365-day maturities.

[22] Technically the SEN platform allows participants for establishing counterparty limits. However, because participants are limited to a select group of financial firms pertaining to the Ministry of Finance's market maker program, this capability has been disabled in order to promote liquidity.

In this sense, it is possible to distinguish from the two types of sell/buy back transactions (i.e. security-demanding or liquidity-demanding). Under the assumption of the Central Bank's lowest-cost collateralized money market liquidity facility being the aforementioned cost threshold, liquidity-demanding (securities-demanding) driven sell/buy backs correspond to those that exhibit rates above or equal to (below) such threshold.[23] Moreover, this is not only intuitive but also convenient, otherwise negative spreads may result, and the MM would turn unfeasible (i.e. the value of the put option in [F8] would be negative).

Lastly, it is worth discussing the main difference between using the US overnight federal funds market (Furfine, 2001) and using the liquidity-demanding sell/buy back operations: the former is based on non-collateralized lending, whereas the latter is collateralized. Despite Furfine (2001) argues that non-collateralized operations are convenient for assessing market discipline because differential pricing should readily appear in the transaction's interest rate, the Colombian collateralized sell/buy back market exhibits significant differential pricing across firms and types of firms –as will be clear in forthcoming sections-, and are the main source of liquidity within financial firms (Table 1).

Besides numerical evidence of their differential pricing, there are reasons why collateralized transactions, such as repos and sell/buy backs, are not risk-free transactions for the counterparty of the liquidity-demanding firm. As stressed by Gorton and Metrick (2010), the ideal collateral is a security that functions like cash: this is, collaterals must be information-insensitive securities by design, with their price being immune to adverse selection whenever they are traded. Therefore, if collaterals are not information-insensitive securities, concerns arise about the ability to recover the collateral value when sold in the market if the counterparty did default, and the creditor protects against this endogenous adverse selection by requiring overcollateralization (i.e. imposing a haircut).[24]

Following Hördahl and King (2008)[25], collateralized transactions are subject to counterparty credit risk, market risk and operational risk, whereas these risks are mitigated through different management tools. Counterparty credit risk is addressed by posting securities as collateral, where the nature of the collateral determines the cost of the transaction; in this sense, the lower the quality of the collateral, the higher the cost. Market risk is mitigated via the imposition of haircuts on the collateral, which protects the creditor from undercollateralization resulting from price declines; the size of the haircut reflects the market risk of the collateral,

[23] According to consulted local market practitioners there is a rationale for a liquidity-demanding firm to prefer the sell/buy back instead of the repo with the Central Bank when both have the same cost. Besides the sell/buy back advantages previously mentioned (e.g. collateral's mobility and the absence of a haircut), practitioners claim that they prefer to preserve their repo outstanding limit with the Central Bank in case of unexpected liquidity requirements, and thus liquidity-demanding sell/buy backs would correspond to rates above or equal the Central Bank's intervention rate.

[24] Similarly, French et al. (2010) highlights that despite pledged collateral is senior to the claims of other creditors, if failure is a concern the potential cost of having the collateral trapped in a bankruptcy proceeding for even a short period is large relative to the interest due on a one-day loan.

[25] Hördahl and King (2008) address repos only. Still, their rationale is applicable to any collateralized transaction.

where more volatile securities are required higher haircuts. Operational risk results from the holding, transfer and management of the collateral, where delivery versus payment (DvP) is usually used to mitigate the last two. Hence, these three risks are reflected, directly or indirectly, in the interest rate at which a collateralized transaction is agreed.

Under this rationale, the Colombian sell/buy backs displays some interesting features from the risk management perspective. Regarding counterparty risk, due to the information-sensitivity of the most common and highest quality collateral in the local market (i.e. TES), a spread over the risk-free rate (e.g. the overnight policy rate) is to be expected. About market risk, because local sell/buy backs do not involve a haircut, this risk must be priced in the transaction's cost too. Operational risks are mitigated amid the DvP implemented in the Central Bank's Large-Value Payment System (CUD) and the Central Securities Depositary (DCV).

Therefore, it is likely that local sell/buy backs, which are collateralized with information-sensitive securities (e.g. TES[26]) that are received without any haircut, exhibit spreads that combine credit and market risk. Consequently, it is intuitive to assume that (i) creditors mitigate credit risk by imposing counterparty limits in MEC based trades, or by charging higher rates in OTC trades; and (ii) creditors mitigate market risk by quoting their rates with an imbedded market risk premium that is assumed "flat" across counterparties.[27] This is, sell/buy back rates and spreads extracted from MEC and OTC traded market transactions will include credit and market risk, but the analysis –and usage- of the spreads is still valid in cross-section since market risk results in a common premium for the only collateral herein considered: central government's securities (TES). As will be mentioned in a forthcoming section, other common sources of a premium in the cost of the sell/buy back transactions are acknowledged (e.g. taxation, overall market liquidity), but –as is the case with market risk- they are assumed to homogenously affect all financial firms, preserving the cross-section properties of the model.

[26] It is rather straightforward to confirm that central government's securities, mainly local currency denominated TES, are not particularly information-insensitive. During the two most recent local turmoil periods (2002 and 2006) the price of the TES decreased significantly; TES are not necessarily regarded as a "safe haven" for the local market, where there is evidence that cash and US dollars are preferred under market uncertainty. On the other hand, in the latest US financial crisis the US Treasuries acted as safe haven, with repos that used them as collateral exhibiting rates close to zero (Hördahl and King, 2008), as is expected from an ideal collateral in the sense of Gorton and Metrick (2010). Hördahl and King (2008) also report a significant widening of the spread between repos with sovereign German and Greek collaterals during the crisis, which validates the importance of the properties of the collateral in the cost of the transaction.

[27] According to market participants the market premium should not be very disperse across different TES references, with a premium in the 10-15bps range for an off-the-run illiquid TES; therefore, the market premium will be taken as "flat" across counterparties. On the other hand, market practitioners point out that collateral different from TES (not considered in this paper) may exhibit significant premiums, or even be considered as non-eligible collaterals. The degree of "flatness" of market risk premium may be approximated by measuring the dispersion of the haircut required by the Central Bank for extending TES-collateralized liquidity to financial firms: as of August 2012, the mean haircut is 2.7%, with a standard deviation of about 80bps.

4.2. Balance sheet data as cross-section unbiased estimate of the value of the firm

As previously emphasized, the equity value used to imply both missing parameters is an equilibrium price, reflecting information known to analysts and investors, and as such it provides their best estimates of the market value and volatility of the firm's value. In this sense, as pointed out by Damodaran (2001), even if markets are efficient, stock prices tend to fluctuate around the true value and markets sometimes do make mistakes, where a firm may see its stock price increase at the same time it destroys value. Additionally, nowadays there is a growing concern about the informational content of stock market prices with the rise of algorithmic or high-frequency trading, which has replaced investors' fundamental analysis with computers' high-speed arbitrage-driven trading; since it has been reported that the majority of equity trading in the US stock market is done by algorithmic trading[28], this issue is far from being negligible, and is expected to escalate.

Consequently, even if the model risk arising from the equity-based joint estimation of both unobserved parameters (Lando, 2004) is ignored, these estimates may not be considered as the true –observed- variables. The assumption of the equity value being a fair estimate of unobservable parameters is valid as long as the market is efficient, and is suitable as long as market prices exist.

In the Colombian market, as is the case in other emerging economies, the validity and usefulness of this assumption is questionable. First, as demonstrated by some authors (León and Vivas, 2011; Leiton, 2011; Pérez and Mendoza, 2010), the local stock market is not efficient in the classical sense (i.e. efficient market hypothesis). Second, due to the absence of a liquid and broad base of stock prices to apply MM's original framework to local financial institutions, the assumption is unsuitable. The assumption being invalid does not make the customary use of the MM impossible for the herein addressed case, but its lack of suitability does.

The most obvious second-best assumption is to use observable balance sheet information as the market value of the firm. Since it is easily available it may be regarded as a suitable and useful assumption. Yet, the main concern about this assumption is its validity.

It is important to realize that the MM framework was designed back in the seventies, when accounting practices differed significantly from nowadays. As documented by Holthausen and Watts (2001), from 1940 until the 1970s the Securities Exchange Commission (SEC) effectively banned upward asset revaluation in the financial statements and even disclosures of current values. Hence, neglecting book value as a source of information seemed appropriate at that time, when historic cost was the basis of accounting standards.

[28] Several authors claim that the majority of US equity trading results from algorithmic trading. Adler (2012) reports 55%; Lowenstein (2012) reports a range from 50% to 70%; Clark and Ranjan (2011) report a range from 25% to at least 50% for two major US stock exchanges; Clark (2010) reports that algorithmic trading grew from 30 percent of total volume in 2005 to about 70 percent in 2009.

Nevertheless, since mid-1980 accounting standards have changed considerably, where the US Financial Accounting Standards Board (FASB) and the International Accounting Standards Board (IASB) have systematically shifted from historical cost-based accounting measures to market-based measures (Matis and Bonaci, 2008). Back in year 2000 Barth et al. (2000) document that FASB was considering extending fair value (i.e. a price at which two parties would agree to an exchange transaction)[29] accounting to all financial instruments and some related non-financial assets, including core deposits intangibles and credit card relationships. According to Holthausen and Watts (2001), the reintroduction of market value accounting for individual assets is consistent with the balance sheet providing an estimate of the market value of net assets; this is, market value accounting will provide an input to equity and debt valuation, in particular the value of the abandonment option, but will not provide a direct valuation of the equity or the firm.[30]

In this sense, *value relevance research* studies the statistical association between financial information and stock prices or returns (Agostino et al., 2010). Barth et al. (2008), Barth et al. (1996) and Barth (1994) have documented that fair value accounting standards by the IASB and FASB has added *value relevance* to accounting amounts such as firm's book value and net income. Regarding financial firms, Landsman (2006) reports that several authors support the incremental *value relevance* of fair value accounting of assets and liabilities in explaining bank's share prices.[31]

Therefore, unlike 40 years ago, nowadays' balance sheet information may be a reasonable second-best to what Merton originally envisaged. In fact, it is important to realize that Merton (1977) highlighted that the firm's value *"is something like the 'fair' or 'market' value of the assets and not the book value"*, where his reluctance to use historical cost-based book values seems vindicated.

Moreover, besides the evidence behind the *value relevance* argument and its evolution amid the fair value accounting standard, literature also identifies balance sheet information as convenient despite its evident shortcomings. For instance, Gorton and Santomero (1990) regard book values of balance sheet items as unbiased cross-section estimates of their market value, whereas Ohlson (1995) find that that book values are unbiased estimators of market values over long periods.

[29] Fair value is defined as "the amount at which that asset (or liability) could be bought (or incurred) or sold (or settled) in a current transaction between willing parties, that is, other than in a forced or liquidation sale". FASB provides a hierarchy of fair value estimates: (i) quoted market prices; (ii) quoted market prices for similar assets and liabilities; (iii) valuation techniques (e.g. present value models, option-pricing models, fundamental analysis) (Foster and Upton, 2001).

[30] The market value of the firm's net assets will diverge from the firm's value since (i) not all assets and liabilities are marked-to-market, and (ii) the market value of the separable assets will not capture the value of the firm's rents from combining the firm's assets. In this sense, following Holthausen and Watts (2001), particular regulatory and litigation concerns also play a role in determining the balance sheet's nature.

[31] A comprehensive examination of the value relevance research in financial industry is outside the scope of this paper. Other authors discussing the subject are Agostino et al. (2010), Barth (1996), Nelson (1996) and Bernard et al. (1995). Early work by Ohlson (1995) analytically supports value relevance of accounting figures in equity valuation.

However, even if book value provides unbiased estimates of firm's value, it is important to identify the source of the potential discrepancy between the estimated and the true –observed- value. According to Holthausen and Watts (2001), the *goodwill (i.e.* the sum of non-separable economic assets[32]) of the firm corresponds to the difference between the market value of the firm and its net assets. In other words, fair value measurement leads to systematic undervaluation of a firm since market value of individual assets do not incorporate competitive advantages of the firm (Matis and Bonaci, 2008). This suggests that the proposed usage of book –fair- value will underestimate the market value of the firm, and will result in underestimated distances to default and overestimated default probabilities.

Lastly, following Dwyer and Eggleton (2009), using balance sheet data is particularly convenient in the banking industry since regulators will typically use accounting metrics (i.e. book value of assets, liabilities and equity) to shut a bank down unless they find an investor willing to inject equity into the bank[33]. In this sense, it is not likely that a financial supervisor will make decisions based on implied market values of financial firm's assets, but on accounting figures.

5. Data and assumptions

The original database corresponds to sell/buy back transactions registered by the Central Bank's Central Securities Depositary (DCV), where both anonymous trading platforms (Central Bank's SEN and Colombian Stock Exchange's MEC) and OTC register platforms (e.g. MEC-Register and Icap) report all central government's TES-related transactions; corporate-related transactions are not considered. Some particularities of the database used to estimate the spread between the risky debt yield and the default-free-risk debt yield are enumerated next.

- The original database corresponds to short-term (1 to 3 days) sell/buy back transactions taking place from June 2011 to May 2012; only sell/buy backs with central government's TES as collateral are considered. All calculations are based on a subset of transactions of six consecutive months within the original database, where the true names of the financial firms and their spreads are not revealed when presenting the results; this follows disclosure reasons.
- As discussed before, the information resulting from the SEN trading platform is not related to market discipline within the money market participants (i.e. it is anonymous and without counterparty limits), and thus it is not informative about the credit risk of the (15) financial firms that are allowed to trade in this platform. Therefore, sell/buy backs from SEN are excluded, which results in the remaining information accounting for

[32] Goodwill is an intangible asset, as patents or licenses. However, unlike a patent, the Goodwill may not be sold separately from the firm, and can only be realized by the sale of the entire firm. Thus, goodwill is not a separable economic asset of the firm. (Holthausen and Watts, 2001)

[33] On the other hand, other corporate (non-banking) may be viable even if book liabilities exceed book assets. (Dwyer and Eggleton, 2009)

merely 25% of the original database by value; still more significant than the non-collateralized lending as a source of liquidity in the local money market.

- Sell/buy backs where financial firms were intermediaries of other firms were excluded; this avoids the issue documented by Furfine (2001), who was unable to discard transactions in which financial firms were not the actual parties to the transaction.
- Transactions regarded as securities-demanding sell/buy backs (i.e. absent of market discipline information) were discarded; as already mentioned, the Central Bank's intervention rate is selected as the threshold for such procedure.[34]
- In order to avoid non-significant transactions by size, the 5-percentile of lowest size transactions was discarded.
- In order to discard non-significant counterparties, the 5-percentile of lowest contributors to the total number of transactions was discarded.
- After all the aforementioned transformation of the original database, the subset consists of 38 financial firms: 12 commercial banking firms (CBFs), 16 broker-dealer firms (BDFs), 5 investment funds (IFs), and 4 non-banking credit firms (FCs)).[35] This sample accounts for about two thirds of Colombian banking and non-banking firm's assets, and about 70% of the payments executed by financial institutions in the local large-value payment system.
- The spread for each firm was calculated as the transaction value weighted average of the margin over the Central Bank's intervention rate corresponding to each date.

Regarding the use of –fair value- balance sheet data as a proxy of the value of the firm, it corresponds to information available from the Colombian Financial Superintendence. Since the money market spread is calculated based on a six-month sample, the matching six-month average of the balance sheet data was used in the calculations.

The assumptions are a mixture from MM's original framework and the proposed informational alternatives herein designed. Some assumptions are worth highlighting and commenting:

- ***MM's default threshold ($A_T \leq D$) is preserved.*** After Merton (1974) seminal work several authors have proposed other thresholds. Despite being straightforward to modify this threshold, the original choice is preferred in this first attempt to overcome the informational constraints previously mentioned.

- ***The Black & Scholes framework is a valid model for option pricing.*** This results in assuming the validity of (i) the random walk theory; (ii) the square-root-of-time rule;

[34] Using the local overnight Interbank Rate (IBR) as the threshold yielded similar results.

[35] On average each firm participated with approximately 98 transactions during the six-month period. By type of firm, CBFs averaged 64, with a minimum (maximum) of 5 (379); BDFs averaged 150, with a minimum (maximum) of 4 (646); IFs averaged 40, with a minimum (maximum) of 4 (77); FCs averaged 88, with a minimum (maximum) of 5 (212). CBFs accounted for 26.61% (20.40%) of the value (number) of the sell/buy backs; BDFs 36.53% (57.78%); IFs for 2.71% (4.58%); FCs for 19.03% (9.93%); other types (e.g. insurance firms) accounted for the rest.

(iii) the volatility and the default-free risk interest rate being constant. If this assumption is invalid, the whole MM framework is invalid.

- ***The money market spreads resulting from sell/buy backs are the financial market price for credit risk.*** This is one of the two main assumptions of the herein proposed alternative to the informational constraints of the Colombian market. This assumption is based on the existence of market discipline between financial firms, as suggested or confirmed by several authors (Cardozo et al., 2011; Furfine, 2001; Calomiris, 2003; Rochet and Tirole, 1996). If this assumption is wrong the spreads between financial firms should be close to zero, or the spread and its cross-section differences should be caused by other underlying risks (e.g. market, liquidity), taxation, etc. However, as in the original MM framework, this other factors may not be directly accounted for under this model[36], and they are assumed to be affecting all firms in a homogeneous manner, preserving the cross-section content of the results and analysis.

- ***The money market spread is assumed to be constant at different maturities.*** Sell/buy backs are short-term transactions, and the observed spreads are short-term. However, since the risk-free rate is assumed to be constant, and the spread is used to solve for the assets' short-term volatility within the Black & Scholes framework, this volatility may be scaled by means of the square-root-of-time rule to long-term maturities (e.g. 1-year).[37] This assumption is not necessary because probabilities may be estimated for the same (short) term of the sell/buy backs; however, it is convenient to express probabilities as is customary in this type of exercises: the probability of a firm defaulting in the following year.

- ***The balance sheet data is cross-section unbiased estimate of the value of the firm.*** Based on several authors (Landsman, 2006; Barth et al., 1996; Ohlson, 1995; Gorton and Santomero, 1990), this is one of the two main assumptions of the herein proposed alternative to the informational constraints of the Colombian market. This assumption is based on the evolution of accounting practices, from Merton's era historical cost accounting to nowadays' fair value accounting; for the Colombian case it is important to highlight that the adoption of fair value accounting practices by the financial sector is well ahead other sectors within the country, as documented by Arias-Bello (2011).[38] If this assumption is invalid the accounting ratios are non-informative in cross section and

[36] It has been documented that the higher the credit solvency of a firm, the higher the non-credit component of its spread. For instance, BCBS-BIS (2000) reports that a US market AAA credit (55bps spread over the US Treasury) has a spread with a non-credit component of 90%, whereas a BB credit (200bps spread) has a non-credit component of 35%. However, MM model explicitly states that markets are frictionless, with no taxes or bankruptcy costs.

[37] This assumption may be partially backed by statements from market practitioners regarding that the counterparty exposure limits are typically valid for transactions ranging from 1 to 365 days.

[38] Developing *value relevance research* projects for the Colombian financial markets may validate this assumption. To the best knowledge of the author, such type of research does not exist.

should have no significant statistical relation with the spread, and typical credit risk analysis based on balance sheet data (e.g. CAMEL) would be questionable.

Additionally, using balance sheet data is convenient for the Colombian financial firms' case since the financial regulator (i.e. Financial Superintendence) uses accounting metrics (i.e. book value of assets, liabilities and equity) to make decisions on the viability of a supervised firm. Following Dwyer and Eggleton (2009), this argument is also reasonable for other countries' financial firms.

Yet, despite being assumed that balance sheet data are cross-section unbiased estimates of the value of the firm, it is well documented that book value -even under fair value accounting- underestimates firm's value (Holthausen and Watts, 2001; Matis and Bonaci, 2008). Therefore, distance to default and probability of default will be systematically underestimated and overestimated, respectively.

6. Main results

Let D be the face value of debt; $\ddot{A}_t$ the book –fair- value of assets; y; the default-free-risk debt yield; and $\sigma_{\ddot{A}}^2$ the assets' implied volatility from solving [F9] based on the risky debt yield estimated from sell/buy backs (r), the probability of default ($\widehat{PD}_t$) of a financial firm is estimated as in [F10]:

$$De^{-r(T-t)}\mathcal{N}\left(-\widehat{d_2}\right) - \ddot{A}_t\mathcal{N}\left(-\widehat{d_1}\right) = De^{-r(T-t)} - De^{-y(T-t)} \quad \text{[F9]}$$

$$\widehat{PD}_t = \mathcal{N}\left(-\widehat{d_2}\right) = \mathcal{N}\left(-\frac{ln\left(\frac{\widehat{\ddot{A}_t}}{D}\right) + \left(r - \frac{\widehat{\sigma_{\ddot{A}}^2}}{2}\right)(T-t)}{\widehat{\sigma_{\ddot{A}}}\sqrt{(T-t)}}\right) \quad \text{[F10]}$$

The main results of the approach are presented next (Table 2). It is worth emphasizing that the distance to default and the probability of default are underestimated and overestimated, respectively. This bias result from two issues previously identified and documented: (i) the spreads contain non-credit risk factors, and (ii) the systematic undervaluation of the firm's value. Thus, results may not be interpreted or used as the "actual" probability of default, but should be analyzed as a cross-section or relative inference of the likelihood of default for each firm.

It is important to realize that the non-credit-risk content of the spreads and the systematic undervaluation of the assets may yield odd results; for instance, the implied volatility of some firms exceed 0.80, which results from spreads being high despite these firms' low leverage (i.e. below 0.22, when the average is 0.67). Conversely, high leverage firms in the sample (CBF11, BDF15, BDF16, CBF12) exhibit low levels of asset volatility (i.e. volatilities below 0.16, when the average volatility is 0.39); this concurs with the characterization of asset's volatility in the banking industry by Crosbie and Bohn (2003), who find that banks typically display low asset volatility and high leverage.

Table 2
Implied 1-year probability of default from money market spreads
(Ranked by Probability of Default)

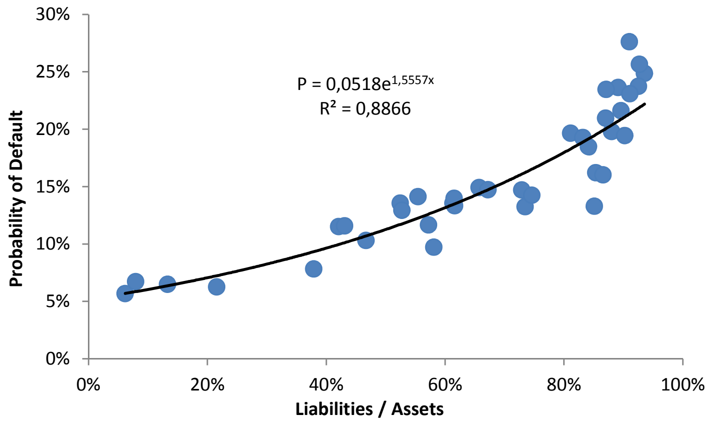
Rank	Firm	Liabilities / Assets	Implied Volatility	Distance to Default	1-y Prob. of Default	Rank	Firm	Liabilities / Assets	Implied Volatility	Distance to Default	1-y Prob. of Default
1	IF1	0.06	1.28	0.74	5.7%	20	BDF10	0.73	0.30	0.90	14.7%
2	BDF1	0.22	0.81	0.96	6.3%	21	BDF11	0.67	0.36	0.91	14.7%
3	BDF2	0.13	1.02	0.85	6.5%	22	BDF12	0.66	0.38	0.91	14.9%
4	FC1	0.08	1.22	0.75	6.7%	23	CBF3	0.87	0.17	0.77	16.0%
5	FC2	0.38	0.59	1.05	7.8%	24	CBF4	0.85	0.19	0.78	16.2%
6	BDF3	0.58	0.39	1.07	9.7%	25	CBF5	0.84	0.21	0.74	18.5%
7	BDF4	0.47	0.53	1.01	10.3%	26	CBF6	0.83	0.23	0.73	19.3%
8	BDF5	0.42	0.60	0.96	11.5%	27	CBF7	0.90	0.16	0.63	19.5%
9	IF2	0.43	0.59	0.96	11.6%	28	CBF8	0.81	0.26	0.74	19.7%
10	FC3	0.57	0.43	1.00	11.7%	29	CBF9	0.88	0.18	0.66	19.8%
11	IF3	0.53	0.50	0.95	12.9%	30	BDF13	0.87	0.20	0.65	21.0%
12	CBF1	0.73	0.28	0.95	13.3%	31	CBF10	0.90	0.17	0.60	21.6%
13	CBF2	0.85	0.17	0.87	13.3%	32	FC5	0.91	0.17	0.54	23.1%
14	BDF6	0.62	0.40	0.95	13.3%	33	IF5	0.87	0.22	0.60	23.5%
15	BDF7	0.61	0.41	0.95	13.6%	34	BDF14	0.89	0.19	0.56	23.6%
16	IF4	0.52	0.51	0.94	13.6%	35	BDF15	0.93	0.15	0.49	23.7%
17	FC4	0.62	0.41	0.94	14.0%	36	CBF11	0.94	0.14	0.45	24.9%
18	BDF8	0.55	0.48	0.93	14.1%	37	BDF16	0.93	0.16	0.45	25.7%
19	BDF9	0.75	0.28	0.91	14.3%	38	CBF12	0.91	0.20	0.46	27.6%

CBF (commercial banking firm); BDF (broker-dealer firm); IF (investment fund); FC (non-banking credit firms)
Source: authors' calculations.

Regarding the relevance of the firms included in Table 2, the top-six and the top-seven most systemically important financial firms of León and Machado (2011) and León and Murcia (2012) are included in the sample, respectively. When considering the top-twenty most systemically important financial firms from both mentioned approaches, Table 2 contains 10 and 13 of those firms, respectively, where most of the missing firms correspond to pension funds, investment funds and banks that are not very active in the liquidity-demanding side of the sell/buy backs in the period herein considered.

As expected, the distance to default is directly related to the leverage of the firm, where most leveraged financial firms tend to exhibit higher probabilities of default (Figure 6); since the normality assumption of the Black & Scholes results in an exponential relation between leverage and probabilities of default, an exponential fit is also provided.

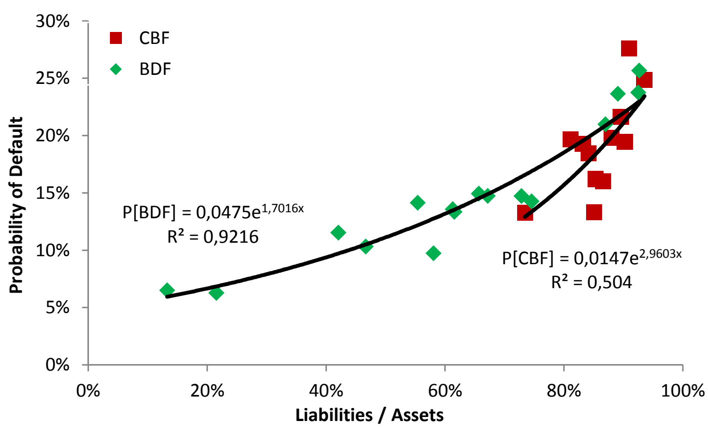
Figure 6
Leverage and 1-year implied probability of default



Source: author's calculations

Despite the fit is rather high across different types of financial firms ($R^2 = 0.89$), it is important to discriminate between different financial businesses. Figure 7 presents the relation between leverage and implied probability of default for banking firms (CBFs) and broker-dealer firms (BDFs), where both preserve a positive and significant relation between both variables.

Figure 7
Leverage and 1-year implied probability of default
(CBFs and BDFs)



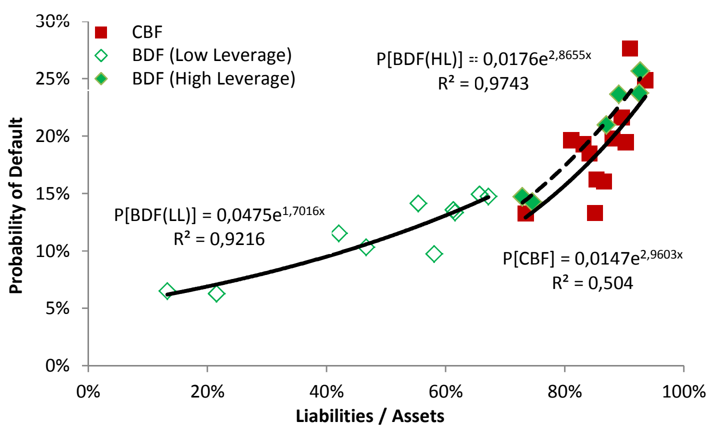
Source: author's calculations

CBFs tend to be highly leveraged, as documented by Crosbie and Bohn (2003); average leverage is about 0.86, where the least (most) leveraged CBF has a 0.73 (0.94) liabilities-to-assets ratio. On the other hand, BDFs show no tendency at all; the average leverage for BDFs is 0.63, where the least (most) leveraged BDF has a 0.13 (0.93) ratio.

If the capital structure is informative of the business line of a firm, this suggests that the banking industry has a greater degree of homogeneity regarding their business line, whilst the broker-dealer industry has different business lines within. This segmentation inside the broker-dealer firms is evident when comparing their asset's book value and leverage (the three most leveraged BDFs account for 68.6% of the BDFs' total assets), where their capital structure is presumably driven by size and efficiency considerations from their owners.[39]

Figure 8 presents the relation between leverage and probability of default for CBFs, and for two classes of BDFs according to their leverage (i.e. high-leverage (HL) and low-leverage (LL)), where the differentiating threshold was chosen intuitively: high-leverage BDFs (BDF(HL)) are those exhibiting a banking-type leverage (i.e. equal or higher than 0.73). According to the exponential fit, it is worth noticing the vertical spread between high-leverage BDFs (dashed line) and CFBs (solid line), which suggests that the market regards banking firms (high-leverage broker-dealer firms) as less (more) likely to default for a given level of leverage.

Figure 8
Leverage and 1-year implied probability of default
(CBFs, Low Leverage BDFs and High Leverage BDFs)



Source: author's calculations.

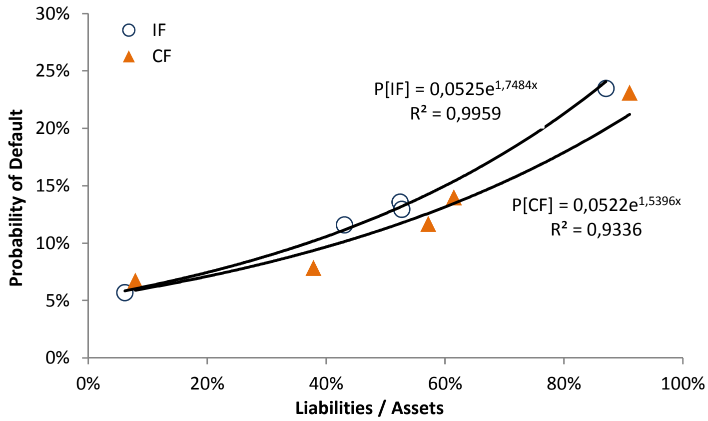
The 1-year implied probability of default spread (about 1.86%) between high-leverage BDFs' and CFBs' exponential fits may be due to factors not considered in the model's framework, such as business risk[40], market participants pricing CBFs' privileged access to Central Bank's

[39] This argument was mentioned by various local market participants, who identify large and leveraged BDFs as the most efficient and dominant.

[40] Business risk relates to the uncertainty arising from business decisions that firms make and to the business environment in which they operate, with factors such as marketing strategies, product development, competitive differentiation strategies, pricing decisions, and sales volume. (Gallati, 2003)

lender-of-last-resort facilities[41], or pricing CBFs' systemic importance and eventual bail out priority; despite these intuitions seem reasonable, the source of this spread must be studied thoroughly in a separate document. Yet, it is interesting that a spread is also present when comparing the two remaining types of financial firms (Figure 9); CFs, which share Central Bank's lender-of-last-resort facilities with CBFs, tend to display lower probabilities of default than IFs.

Figure 9
Leverage and 1-year implied probability of default
(CFs and IFs)



Source: author's calculations.

Table 3 presents the average probability of default for each type of firm; averages were calculated as the asset's book value weighted average of the implied volatilities of default. Low-leverage firms (IFs and CFs) display a lower probability of default, whereas high-leverage (CBFs and BDFs) exhibit higher default probabilities. Results are ranked according to the probability-of-default-to-leverage ratio, where it is confirmed that the two types of firms that have access to the Central Bank's lender-of-last-resort facilities (i.e. CBFs and CFs) receive a lower default probability charge per unit of leverage; however, as before, more analysis and tests should be done to properly confirm the significance of this intuition.

[41] The Constitution of Colombia grants access to lender-of-last-resort facilities to credit institutions only. Credit institutions comprise banks (CBFs), Financial Corporations and Financing Companies (FCs).

Table 3
Probability-of-default-to-leverage ratio
(Ranked by Probability-of-default-to-leverage ratio)

Rank	Type of firm	Liab./Assets	1-y Prob. of Default	(1-y Prob. of Default) / (Liab./Assets)
1	CBFs	0.85	19.20%	0.22
2	CFs	0.76	18.71%	0.25
3	BDFs	0.83	21.35%	0.26
4	IFs	0.71	18.98%	0.27

Source: author's calculations.

It is worth highlighting that the 1-year implied probabilities of default in Table 2 appear to be rather high: the average is 16%, with a maximum of 28% and a minimum of 6%. However, these levels are by no means unprecedented. For instance, Standard & Poor's (2012) bank's observed annual default rates by rating category (Table 4) exhibit non-negligible frequencies for ratings compatible with the expected international ratings of Colombian financial firms (i.e. equal or below BBB-); please note that since the categories used by Standard & Poor's are based on international ratings (i.e. non-local ratings), Colombian financial institutions are expected to be rated equal or below the sovereign[42], which is BBB- as of August 2012 for the mentioned credit rating agency.

Table 4
Bank's Annual Default Rates By Rating Category [a]
(1981-2011)

	AAA	AA	A	BBB	BB	B	CCC/C	*Implied 1-y prob. of Default*[b]
Mean	0,0%	0,0%	0,0%	0,2%	0,7%	4,5%	9,5%	*15.7%*
Minimum	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	*5.7%*
Maximum	0,0%	0,0%	0,9%	3,3%	8,1%	23,1%	60,0%	*27.6%*

[a] Based on Standard & Poor's international rating scales.
[b] Includes banking CBFs, BDFs, IFs and CFs.
Source: author's calculations, with Standard & Poor's (2012).

Again, it is worth emphasizing that the distance to default and the probability of default are underestimated and overestimated, respectively.

[42] Following Fitch (2012), national or local rating scales provide a relative measure of creditworthiness for rated entities only within the country concerned. Therefore, AAA long-term local rating will be assigned to the lowest relative risk within that country, which, in most but not all cases, will be the sovereign state; hence, Colombian financial institutions are expected to be assigned a non-local rating below or equal the sovereign's.

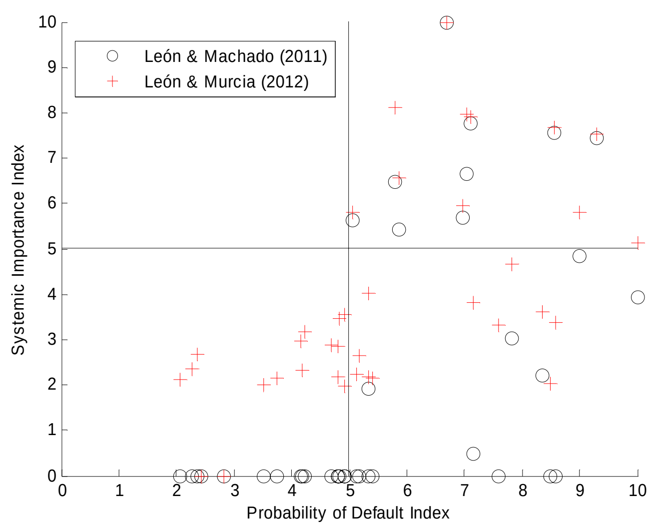
7. Systemic risk assessment by merging financial firms' systemic importance and default probabilities

Despite the resulting default probabilities may not be interpreted or used as the "actual" probabilities of default, the cross-section properties of the results are particularly convenient for financial authorities, especially if jointly used with systemic importance indexes.

For instance, merging the Systemic Importance Index of León and Murcia (2012) and León and Machado (2011) with the herein presented probabilities of default (Table 2) yields a particularly interesting and comprehensive portrait of local financial market's systemic risk, where financial authorities may easily discriminate between firms according to their expected relative contribution to financial instability.

Figure 11 presents such merge, where the default probabilities of Table 2 and systemic importance are presented as indexes from 0 to 10 in *x* and *y*-axis, respectively, with 10 displaying the highest probability and systemic importance; this simple graph may help financial authorities to focus the intensity of supervision, oversight and regulation on those firms that combine higher systemic importance and higher probability of default (e.g. upper-right quadrant of Figure 11).

Figure 11
Probability of default and systemic importance [a]
(Systemic importance as in León and Murcia (2012) and León and Machado (2011))



[a] Systemic importance indexes as of May 2011.
Source: author's calculations, with León and Murcia (2012) and León and Machado (2011)

8. Final remarks

As demonstrated in this paper, under some supplementary but reasonable assumptions, money market spreads may be used in order to infer forward-looking probabilities of default for financial firms within MM's theoretical framework. For the Colombian case the herein presented alternative significantly widens the number and variety of financial firms under analysis when compared with previous attempts to use this useful standard theoretical framework; instead of a handful of banking firms (as in Capera et al., 2011; Souto and Abrego, 2009), this paper obtained default probabilities for 38 financial firms, mostly banks and broker-dealers.

As expected, default probabilities are non-negligible for all firms, where the ratio of default-probability-to-leverage is lower for firms with access to lender-of-last-resort facilities. Moreover, given the broad set of financial firms under analysis, it is possible for financial authorities to merge systemic impact and frequency measures into a convenient and comprehensive map of systemic risk; this provides a simple, yet valuable, tool for focusing the intensity of supervision, oversight and regulation on those firms that combine higher systemic importance and higher probability of default.

As usual, the proposed approach follows and relies on its assumptions. It is important to highlight that traditional assumptions of the MM framework are supplemented by some others that may be debatable since their validity is difficult to test, such as (i) the non-credit component of spreads homogeneously affecting all firms, and (ii) the underestimation of the firm's value due to the use of fair value accounting standards instead of the observed firm's value.

Besides the cross-section and serial boundaries of the approach, some other caveats are worth mentioning. For example, despite covering a significant part of the most systemically important financial firms (as in León and Murcia (2012) and León and Machado (2011)), some relevant firms are not included in the analysis. Most of these missing firms are pension funds, investment funds and banks that are not active in the liquidity-demanding side of sell/buy backs (e.g. they are in the liquidity-offering side), but may not be regarded as of negligible importance for financial authorities.

Moreover, besides assuming that the MM is explanatory of corporate credit risk, this approach assumes that MM is explanatory of financial firms' credit risk. This may be especially dubious since financial firms exhibit particularities such as (i) a continuously varying mix of services and investment business lines, and (ii) a particular ability to adjust their balance sheet in ways no other corporate can, which may both further compromise the ability of MM to model volatility in a consistent manner.

There are several challenges ahead. For instance, it may be interesting (i) to use other thresholds for default; (ii) to relax the assumption of debt maturing at T (i.e. implementing a first-passage model); (iii) to compare results across time, especially with periods of local financial turmoil (e.g. mid-2006) or monetary tightening; (iv) to test the significance of the spread between firms that do and do not have access to last-resort-lending facilities; (v) incorporating dependence between financial firms.; (vi) to estimate the non-credit risk components of sell/buy backs' spreads by using an appropriate benchmark (e.g. SEN sell/buy backs' spreads). These are challenges that the author intends to undertake in forthcoming documents.

There are two challenges outside the author's agenda that are worth addressing: (i) undertaking *value relevance research* of accounting standards for the Colombian financial sector, which may further support the use of fair value accounting as a source of information for analytical and supervisory purposes, and (ii) the convenience of requiring financial firms to offer uninsured debt instruments as part of their capital structure as a way to obtain market information about risk and market discipline, as suggested by Calomiris (2011, 2009, 2003).

Finally, besides contributing to the implementation of the MM under equity market informational constraints, and providing valuable cross-section information for financial authorities, this approach stresses the complementary nature of data in financial market infrastructures (e.g. large-value payment systems, central securities depositaries, trade repositories). As demonstrated in this paper, these infrastructures are relevant sources of granular but comprehensive information on the –opaque- dynamics and structures of financial markets.

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