Borradores de ECONOMÍA

Forecasting annual inflation with power transformations: the case of inflation targeting countries

Por: Héctor Zárate
Angélica Rengifo

Núm. 756
2013

BANCO DE LA REPÚBLICA COLOMBIA

- Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia

FORECASTING ANNUAL INFLATION WITH POWER TRANSFORMATIONS: THE CASE OF INFLATION TARGETING COUNTRIES[†].

Héctor Zárate*
Angélica Rengifo**

Abstract

This paper investigates whether transforming the Consumer Price Index with a class of power transformations lead to an improvement of inflation forecasting accuracy. We use one of the prototypical models to forecast short run inflation which is known as the univariate time series *ARIMA*. This model is based on past inflation which is traditionally approximated by the difference of logarithms of the underlying consumer price index. The common practice of applying the logarithm could damage the forecast precision if this transformation does not stabilize the variance adequately. In this paper we investigate the benefits of incorporating these transformations using a sample of 28 countries that has adopted the inflation targeting framework. An appropriate transformation reduces problems with estimation, prediction and inference. The choice of the parameter is done by bayesian grounds.

KEYWORDS: ARIMA models, power transformations, seasonality, bayesian analysis.

JEL Classification: C22, C52.

[†] The opinions expressed here are those of the authors and not of the Banco de la República de Colombia nor of its Board.

*Econometrician, Banco de la República. Email: hzaratso@banrep.gov.co.
**Assistant research, Banco de la República and University of ICESI
Email: arengigo@banrep.gov.co

1. Introduction

To date, a total of 28 countries have committed to an inflation target as the anchor of their monetary policy in order to achieve price stability. For this reason, forecasting inflation has become crucial for policy makers to decide on how to conduct the economic policy. From the statistics point of view, the leading approach to forecast short run inflation rely on the traditional Box and Jenkins time series building methodology, which fit seasonal autoregressive integrated moving average models to the first differences of logarithms of the underlying price index. Even though this common practice of applying the logarithm transformation of the Consumer Price Index enjoys advantages related to interpretation issues, the inflation forecasts are not necessarily optimal if this transformation does not stabilize properly the variance. In this paper, we analyze whether or not the incorporation of power transformations into the identification stage of the ARIMA models leads to an improvement of inflation forecasting accuracy. Specifically, we conduct an experiment taking into account four possible transformation strategies for the optimal parameter of the Box-Cox family and we compare the forecast precision through the mean squared error and the mean absolute error for different horizons.

The rest of the paper is organized as follows. In section 2 we review the use of power transformations in models and present the bayesian strategy to choose the optimal transformation for the total CPI for 19 countries. The data models and forecasts are displayed in the next section. Then, in section 4 we present a rolling point forecast comparison with different transformations. Finally, in section 5 we summarize the main contributions of this paper and conclusions.

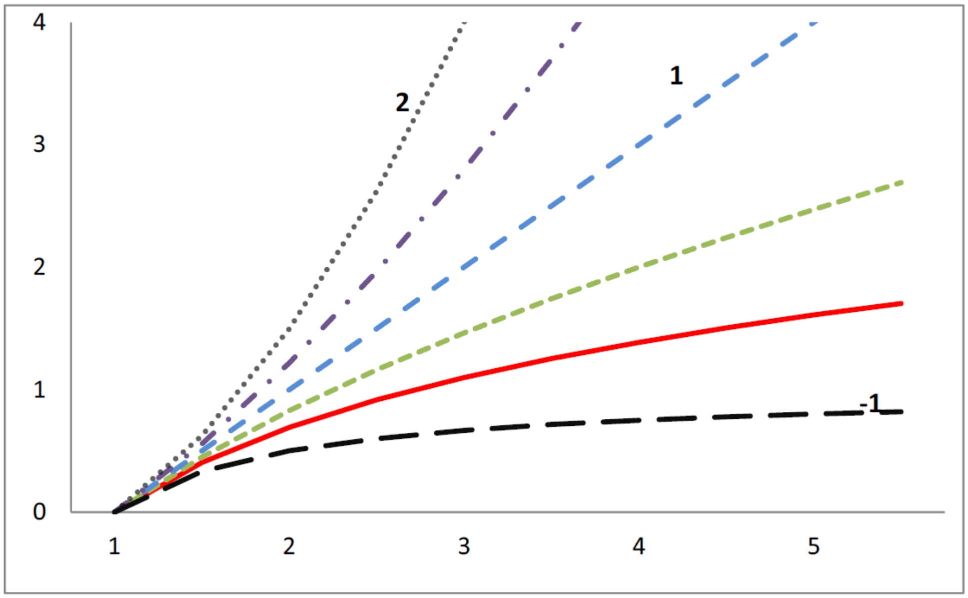
2. Power transformations in ARIMA models

The use of transformations of a variable as a preliminary specification to construct a forecast model has been recommended into the times series model building methodology since the implementation of the ARIMA models by Box and Jenkins (1970). They claim that better forecasts could arise when a model is broadened to include the general class of power transformations. The transformation of the observations known as the Box-Cox (B-C) satisfies certain underlying modeling assumptions on the residuals such as: normality, iid, and a constant variance about a zero mean level and is given by:

$$y_t(\lambda) = \begin{cases} (y_t^{\lambda} - 1)/\lambda & if \ \lambda \neq 0 \\ \ln(y_t) & if \ \lambda = 0 \end{cases}$$

Where $y_t(\lambda)$ denotes the nonstationary variable and λ is the transformation parameter. The role of this transformation is to preserve the ordering of the data and change the shape of the distribution. Thus, when $\lambda = 1$ the series is analyzed in its original scale, and $\lambda = 0$ corresponds to a logarithmic transformation. In Figure 1 different patterns for λ are shown.

Figure 1 - Box-Cox Transformation for values of λ



We consider the following possible transformation strategies: a) use no transformation, b) use the logarithmic transformation, c) use a power transformation with λ estimated by Bayesian MCMC methods.

2.1 Deciding on the parameter λ

The parameter is usually estimated by maximum likelihood under the assumption of a parametric distribution for the transformed series. Constructing the profile likelihood, the parameter λ is chosen taking into account that the change of the scale is corrected by the Jacobian. This approach works well only if the Box Cox transformation converts the distribution to a normal, but there is always uncertainty about the right model. Therefore, we use a Bayesian approach to make inferences about the parameter λ.

Bayesian analysis exploits the combination between the data likelihood and the prior distribution about λ in order to obtain its posterior distribution from which the inferences are based.

$$P(\lambda/y_t) \propto P(y_t/\lambda)P(\lambda)$$

Thus, we use the Markov Chain Montecarlo (MCMC) methods and the Metropolis algorithm to obtain the posterior distribution for λ.

The transformed response $y(\lambda)$ is assumed to follow a normal distribution

$$y_t(\lambda) \sim N(\phi_0 + \phi_1 y_{t-1} + ... + \phi_p y_{t-p}, \sigma^2)$$

The likelihood with respect to the original y_t

$$p(y_t / \lambda, \phi_1, \phi_2, ...\phi_p, \sigma^2) \propto \phi(y_t / \phi_0 + \phi_1 y_{t-1} + ... + \phi_p y_{t-p}, \sigma^2).J(\lambda, y_t)$$

Where $J(\lambda, y_t) = \begin{cases} y_t^{\lambda-1} \; if \; \lambda \neq 0 \\ 1/y_t \;\; if \;\; \lambda = 0 \end{cases}$ is the Jacobian

The parameters of the model are the following: $\{\lambda, \phi_1, ..., \phi_p, \sigma^2\}$

We use a flat prior $\{\phi_1, ..., \phi_p\}$ on and an inverse gamma for σ^2.

3. The data

We use a set of monthly series of CPI, seasonally unadjusted, from 19 countries with inflation target as the anchor of their monetary policy. This set was taken from the IMF database of International Financial Statistics. The sample was chosen to take into account when the inflation targeting scheme began to be implemented in each country. The sample period differs within series and it is available in Table 1 - Sample Period

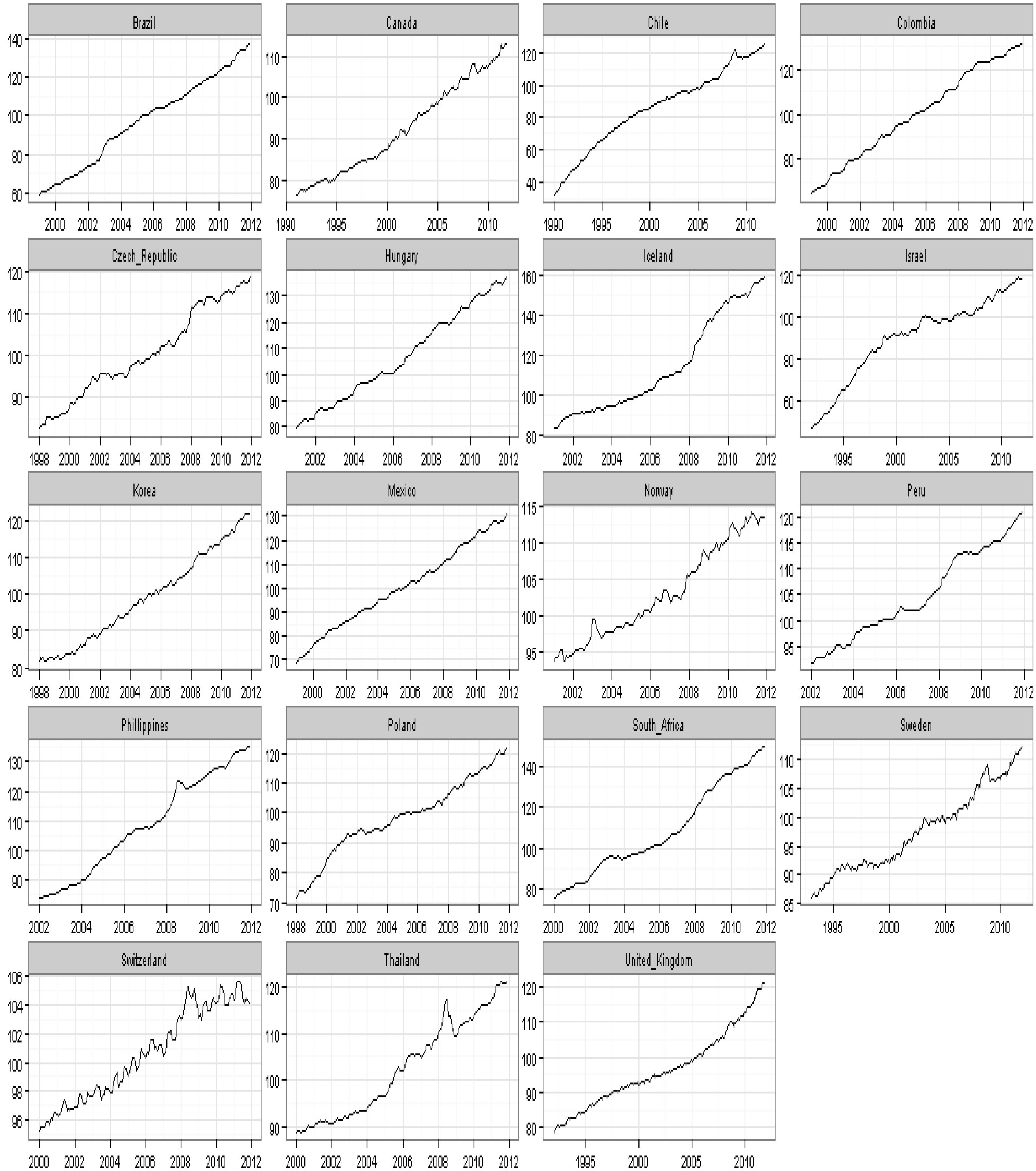
Table 1 - Sample Period

Monthly CPI Series Seasonally Unadjusted	
Country	Sample Period
Brazil	1999 M1 - 2011 M12
Canada	1991 M1 - 2012 M12
Chile	1990 M1 - 2012 M12
Colombia	1999 M1 - 2011 M12
Czech Republic	1998 M1 - 2011 M12
Hungary	2001 M1- 2012 M12
Iceland	2002 M1- 2012 M12
Israel	1992 M1 - 2011 M12
Mexico	1999 M1 - 2011 M12
Norway	2001 M1- 2012 M12
Peru	2002 M1- 2012 M12
Phillippines	2002 M1- 2012 M12
Poland	1998 M1 - 2011 M12
South Africa	2000 M1- 2012 M12
South Korea	1998 M1 - 2011 M12
Sweden	1993 M1 - 2012 M12
Switzerland	2000 M1- 2012 M12
Thailand	2000 M1- 2012 M12
United Kingdom	1992 M1 - 2011 M12

Source: IMF International Financial Statistics Database

Plots of the series are shown on Figure 2. The series have not apparent changes in their seasonal pattern or structural breaks over the period analyzed. It also may seem that some CPI have increases in their seasonal fluctuations in level specification which can be alleviated with transformations of the series. But, in order to test the effectiveness of these transformations on the forecast performance, and following the main goal of this paper, we consider the level specification, the log specification, and the Box-Cox transformation that is recommended for the series in our exercise.

Figure 2 - Monthly CPI Series



We test for zero frequency unit roots and seasonal unit roots, using standard ADF and HEGY approximations, following (Lütkepohl & Xu, 2009). For most of the series, there is evidence for one zero frequency root and some seasonal ones. These results can be observed in Table 2, but we do not present detailed information since these results do not help to decide whether level, log or Box-Cox specifications are better to forecast accurately.

Table 2 - Unit Root Tests

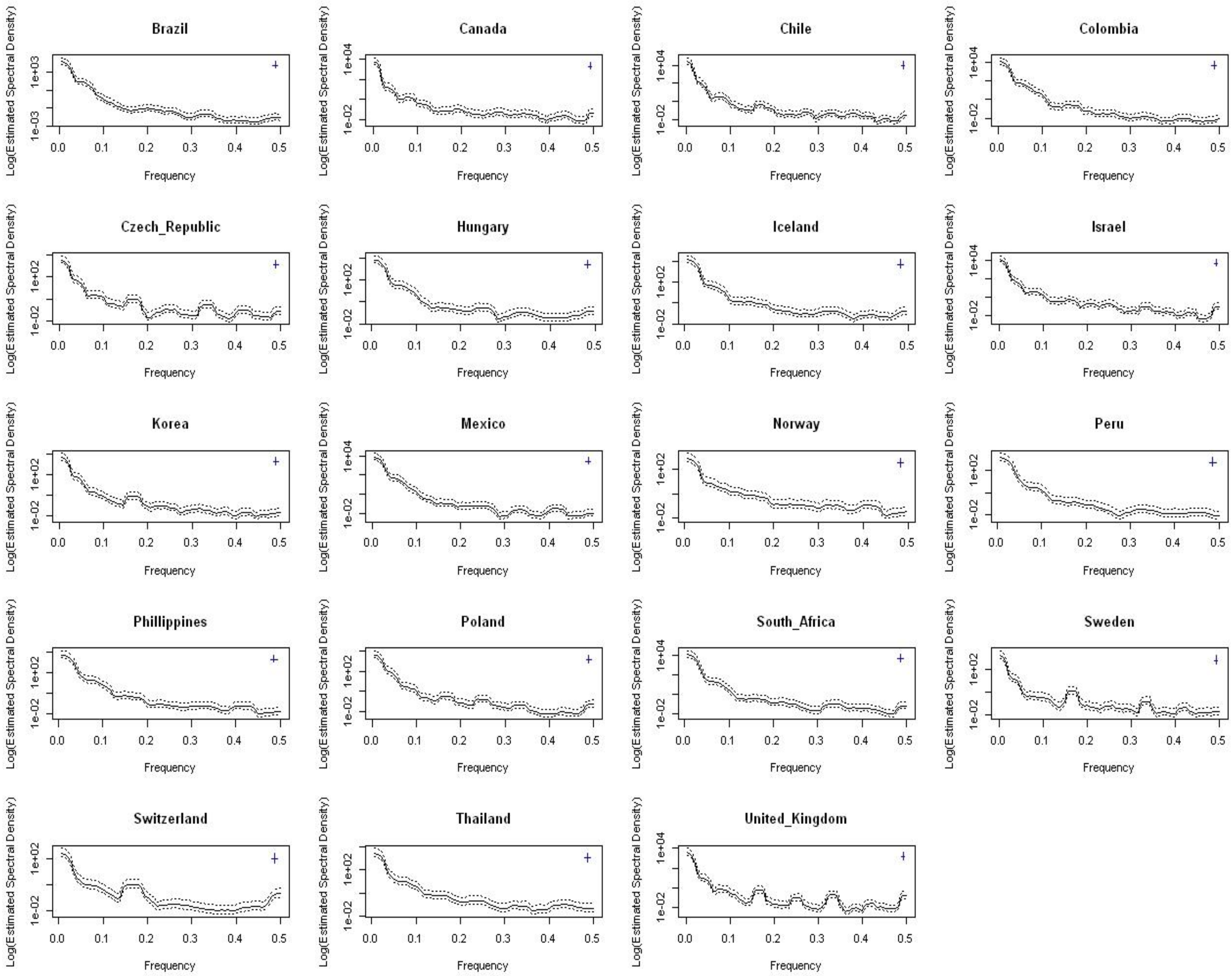
Unit Root Properties of Total CPI Series for Sample Period since Inflation Targeting

Country	ADF with trend and seas. Dummies						ADF with constant						Roots not rejected by HEGY with seas. Dummies					
	level		log		Box-Cox		Δ_{12} level		Δ_{12} log		Δ_{12} Box-Cox		Δ level		Δ log		Δ Box-Cox	
Brazil	-2.1771	(1)	-1.3065	(1)	-1.832	(1)	0.2927	(12)	0.1512	(12)	-0.2480	(13)	1, 2	(0)	1, 2	(0)	1,2	(0)
Canada	-2.3141	(1)	-2.6558	(1)	-2.239	(1)	-2.1931	(12)	-2.3117	(12)	-2.1620	(12)	1,2,3/4	(0)	1,2,3/4	(1)	1,2,3/4	(0)
Chile	-2.9553	(1)	-6.5277 ***	(1)	-2.679	(1)	-1.2720	(13)	-3.9165 ***	(12)	0.5510	(12)	1, 2	(1)	1,2,3/4	(0)	1,2	(0)
Colombia	-1.8581	(1)	-1.3904	(1)	-2.094	(1)	0.3719	(12)	-0.6319	(12)	0.5320	(12)	1,2,3/4	(0)	1,2,3/4,7/8	(0)	1,2,3/4	(0)
Czech Republic	-1.8366	(2)	-1.8366	(2)	-1.747	(2)	0.7869	(12)	0.6362	(12)	0.8200	(12)	1,2,3/4	(0)	1,2,3/4	(0)	1,2,3/4	(0)
Hungary	-1.4465	(7)	-1.9947	(2)	-1.781	(2)	-1.1106	(12)	-0.7893	(12)	-1.1830	(12)	1,2,3/4,7/8	(12)	1,2,3/4,7/8	(14)	All but 11/12	(0)
Iceland	-2.1859	(6)	-2.1859	(6)	-2.183	(6)	0.3062	(12)	0.5551	(12)	0.3030	(12)	1,2,3/4	(0)	1,2,3/4	(0)	1,2,3/4	(0)
Israel	-1.9177	(1)	-2.7132	(1)	-1.783	(1)	-0.4038	(12)	-1.0426	(12)	-1.4670	(13)	1,2,3/4	(0)	1,2,3/4	(0)	1,2,3/4	(0)
Mexico	-2.2926	(1)	-4.4065 ***	(1)	-0.977	(1)	-1.4679	(12)	-3.8352 ***	(12)	-0.6130	(12)	1,2,3/4,7/8	(0)	1,2,3/4	(0)	1,2,3/4	(0)
Norway	-2.9266	(1)	-3.2164 *	(1)	-2.895	(1)	-2.3057	(12)	-2.5042	(12)	-2.2830	(12)	1,2,3/4	(0)	1,2,3/4	(0)	1,2,3/4	(0)
Peru	-1.4105	(1)	-1.7453	(1)	-1.242	(1)	1.0438	(12)	0.7960	(12)	1.1710	(12)	1,2,3/4	(1)	1,2,3/4	(1)	1,2,3/4	(1)
Phillippines	-3.2503	(1)	-2.2290	(1)	-3.361 *	(1)	-0.6620	(12)	-1.7965	(12)	-1.6460	(13)	1,2,3/4	(0)	1,2,3/4	(0)	1,2,3/4	(0)
Poland	-1.4914	(1)	-2.1402	(1)	-1.089	(1)	-0.3744	(12)	-1.2894	(12)	-0.0050	(12)	1,3/4	(0)	1,3/4	(0)	1,3/4	(0)
South Africa	-0.7701	(1)	-1.3150	(1)	-0.978	(1)	1.1096	(12)	1.5773	(12)	0.9630	(12)	1,2,3/4	(0)	1,2,3/4	(0)	1,2,3/4	(0)
South Korea	-2.7675	(1)	-4.0723 ***	(1)	-2.463	(1)	-1.2831	(12)	-1.5956	(12)	-1.1760	(12)	1,2,3/4	(3)	1,2,3/4	(3)	1,2,3/4	(6)
Sweden	-1.0603	(1)	-1.3555	(1)	-0.948	(1)	0.1270	(12)	-0.0062	(12)	0.1270	(12)	1,3/4	(1)	1,3/4	(1)	1,3/4	(1)
Switzerland	-2.1404	(6)	-0.8804	(3)	-2.156	(6)	-1.1944	(12)	-1.2055	(12)	-1.1910	(12)	1,3/4	(0)	1,3/4	(0)	1,3/4	(0)
Thailand	-2.7843	(1)	-2.7515	(1)	-2.782	(1)	-1.6735	(12)	-1.6620	(12)	-1.6730	(12)	1,2,3/4	(0)	1,2,3/4	(0)	1,2,3/4	(0)
United Kingdom	2.9421	(0)	0.9500	(1)	3.494	(0)	-0.2138	(12)	-0.9300	(12)	0.1030	(12)	1,2,3/4	(6)	1,2,3/4	(6)	1,3/4	(0)

Note: Lag selection by AIC with maximum order 14, lag order given in parentheses. 5% critical values for ADF test: -3.41 (with trend), -2.86 (with constant). HEGY test with seasonal dummies, results based on 5% signi¯cance level. Computations of HEGY test were performed with JMulTi (Luetkepohl and Kratzig (2004)). ADF test was performed wit R - Project

Also, we analyze the spectrum of the series in order to identify the importance in total variation of their seasonal component. In Figure 3 we present the logarithm of the smoothed sample spectra. The subtle fluctuations on the spectra may suggest for almost all of the countries that the seasonal part of the series has recently gained importance for explaining their behavior.

Figure 3 - Smoothed Spectrum



Note: we reduce the spectrum variability by using the Daniell spectral window with $m = 6$ and also we consider a cosine bell taper of 10%.

4. Models for forecasting seasonal time series

The leading approach to obtain forecasts is using either models based on stochastic seasonality or deterministic seasonality. The former class strategy relies on the traditional time series building methodology of Box and Jenkins (1970) which fit autoregressive integrated moving average models to the observed consumer prices series. These models are assumed to have seasonal unit roots and to induce stationarity, it is necessary to seasonally differentiate the series. We estimate two versions of these kinds of models for various values of λ. Also, we estimate models with seasonal unit roots and the Airline model. (See Lutketpoll and Xu , 2011).

On the other hand, we consider models with deterministic seasonality which allows the mean to vary with the month in a deterministic way.

$$\phi(L)\pi_t^{\lambda} = \alpha_1 D_{1t} + ... + \alpha_{12} D_{12t} + \varepsilon_t$$

Where D_{it} corresponds to seasonal dummy variables that take the value of one if the observation t is in the month i and zero otherwise. π_t^{λ} is the annual inflation rate calculated with different estimations of λ.

Based on the proposed models, the forecasts and its forecast errors are calculated. To measure the predictive performance of the different models, we use a cross-validation strategy known as rolling-forecasting origin. It is a simple procedure in which we choose a training set and a test set to compare the forecasts results through a measure of the accuracy as the minimum error absolute, MAE. This accuracy measures are obtained as follows:

1. We set $\mathrm{k} = 36$ as the minimum number of observations in the training set.
2. We select the observation k+i+1 for the test set, where $\mathrm{i} = 1, 2, \dots, \mathrm{n} - \mathrm{k} - 1$ and n is the total number of observations and differs for each country.
3. We forecast 12 steps-ahead based on data to k+i and compute MAE
4. Then, we repeat for i observations
5. We compare MAE values for each forecast horizon.

Table 3 contains the results of this evaluation. Considering total inflation for 19 countries, the 1-step forecasts are optimal for the log transformation in 57% of the countries analyzed. On the other hand, in 32% of the countries the best forecast is based under the Box–Cox transformation. Only the 2% of the countries achieve the best forecast with the levels of the consumer price index. Taking into account the 12-step horizon, we found that the use of Box-Cox transformation leads to more accurate forecasts in 53% of the countries analyzed. Models based on levels and logs are optimal in 22% of the cases respectively.

Table 3 - Forecasting Results for total CPI series

Country	Best 1-Step Forecast				Best 12-Step Forecast			
	ds	ss	Airline	Overall	ds	ss	Airline	Overall
Residuals	MAE	MAE	MAE		MAE	MAE	MAE	
Brazil	Log	Log	Box-Cox	ss+ Log	Box-Cox	Level	Level	ds+Box-Cox
Canada	Box-Cox	Log	Log	Airline+Log	Log	Log	Log	ds+Log
Chile	Box-Cox	Box-Cox	Box-Cox	ds+Box-Cox	Box-Cox	Box-Cox	Box-Cox	ds+Box-Cox
Colombia	Log	Box-Cox	Log	Airline+Log	Log	Box-Cox	Box-Cox	ds+Log
Czech Republic	Log	Box-Cox	Log	ss+Box-Cox	Log	Box-Cox	Box-Cox	ds+Log
Hungary	Box-Cox	Log	Log	ss+Log	Box-Cox	Log	Box-Cox	ds+Box-Cox
Iceland	Box-Cox	Log	Level	Airline+Box-Cox	Box-Cox	Box-Cox	Box-Cox	ds+Box-Cox
Israel	Level	Log	Level	Airline+Box-Cox	Box-Cox	Box-Cox	Box-Cox	ds+Box-Cox
Mexico	Level	Level	Box-Cox	ds+Level	Box-Cox	Box-Cox	Level	ds+Box-Cox
Norway	Log	Log	Log	ss+Log	Non Conclusive	Non conclusive	Non Conclusive	ds
Peru	Log	Level	Level	ss+Level	Level	Non conclusive	Non Conclusive	ds+Level
Phillippines	Box-Cox	Level	Level	Airline+Box-Cox	Box-Cox	Log	Level	ds+Box-Cox
Poland	Log	Level	Box-Cox	ss+Log	Level	Box-Cox	Box-Cox	ds+Level
South Africa	Box-Cox	Level	Level	Airline+Box-Cox	Level	Level	Log	ds+Level
South Korea	Box-Cox	Log	Log	ss+Log	Box-Cox	Non conclusive	Log	ds+Box-Cox
Sweden	Log	Log	Box-Cox	ds+Log	Box-Cox	Log	Non Conclusive	ds+Box-Cox
Switzerland	Box-Cox	Log	Box-Cox	ss+Log	Box-Cox	Non conclusive	Non Conclusive	ds+Box-Cox
Thailand	Box-Cox	Box-Cox	Log	Airline+Log	Log	Log	Log	ds+Log
United Kingdom	Log	Log	Box-Cox	ss+Log	Non Conclusive	Log	Log	ds

5. Conclusions

In this paper we investigate the benefits of using a general class of transformations of the underlying consumer price index in order to improve annual inflation forecasts in countries committed with an inflation targeting regime. We found mixed results in evaluating the forecast precision of different horizons. Thus, for the 1-step forecast, the models based on logs leads to optimal forecasts in approximately a half of the sample of countries analyzed. However, for the 12-step forecast the Box-Cox transformation dominates the models based either on logs or levels. Therefore, in a large proportion of countries that adopted the inflation targeting scheme, the tradition of using annual inflation, calculated as first differences of logarithms of the CPI index, produces inaccurate forecasts and the use of power transformations on observations could improve its precision.

Moreover, our results indicate that using deterministic seasonal models lead to precise forecasts than stochastic models for these countries during the period of time analyzed. A possible cause arise because the fall of inflation from two digits to one in the last decade have caused the seasonality component be more explicative and a better job is done through a deterministic modeling strategy.

This is a preliminary, in a future version of the document we will extend the estimation to others type of inflation measurements useful for policy makers. For instance, CPI headline inflation, CPI food and no tradable inflation.

References

Chatfield, C. and D. L. Prothero (1973): “Box Jenkins seasonal forecasting: Problems in a case study” Journal of the Royal Statistics Society A, 136, 295-336.

Franses P. and M. McAleer (1998): “testing for unit roots and non-linear transformations” Journal of time series analysis 19, 147-164.

Granger, C. and P. Newbold (1976): “Forecasting Transformed Series” Journal of the Royal Statistics Society B, vol 38 No 2, 189-203.

Hosoya Y. and T. Terasaka (2009): “Inference on transformed stationary time series” Journal of econometrics, 151, 129-139.

Lee J. , T. Lin, K. Lee, and Y Hsu (2005): “Bayesian analysis of Box-Cox transformed linear models with $ARMA(p,q)$ dependence” Journal of statistical Planning and Inference. 133, 435-451.

Lütkepohl, H. and T. Proietti (2011): “Does the Box cox transformation help in forecasting macroeconomic time series?” International Journal of Forecasting. Vol 29, issue 1, 88-99

Lütkepohl, H. and F. Xu (2011): “Forecasting Annual Inflation with Seasonal Monthly Data: Using Levels versus Logs of the Underlying Price Index” Journal of Time Series Econometrics, vol 3, issue 1.

Nazmi N. and J. Leuthold (1988): “Forecasting Economic Time Series That require a Power Transformation: Case of State Tax Receipts” Journal of Forecasting, vol 7, 173-184.

Stock J. and M. Watson (1999): “Forecasting inflation: Journal of Monetary Economics, 44, 293-335