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Por: Carlos León
Ron J. Berndsen

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Modular scale-free architecture of Colombian financial networks: Evidence and challenges with financial stability in view[1]

Carlos León[2]
Banco de la República

Ron J. Berndsen[3]
Tilburg University and
De Nederlandsche Bank

Abstract

Scale-free (inhomogeneous) connective structures with modular (highly clustered) hierarchies are ubiquitous in real–world networks. Evidence from the main Colombian payment and settlement systems verifies that local financial networks have self-organized into a modular scale-free architecture that favors everyday robustness and performance in exchange for rare episodes of fragility but rapid evolution.

Results provide new elements for understanding and modeling the formation and structure of financial networks, and suggest new insights and challenges for authorities contributing to their stability. For instance, (i) the observed architecture suggests that financial systems are complex adaptive systems; (ii) complex adaptive features invalidate traditional reductionist assumptions for modeling financial systems (e.g. homogeneity, normality, static equilibrium, linearity); (iii) the observed modular scale-free architecture tends to limit cascades and isolate feedbacks; and (iv) with financial stability in view, authorities should understand and take advantage of the existing architecture by means of designing and implementing macro-prudential regulation and system-calibrated requirements. Yet, the quest for discovering, explaining and handling the emerging structure of financial systems is an enduring task.

JEL: D85, E42, C38, D53, G20, L14

Keywords: networks, complex adaptive systems, self-organization, financial system, scale-free, financial stability.

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[2] Financial Infrastructure Oversight Department, Banco de la República, cleonrin@banrep.gov.co / carlosleonr@hotmail.com. [corresponding author]

[3] Department of Economics, Tilburg University; Oversight Department, De Nederlandsche Bank. r.j.berndsen@dnb.nl.

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1. Introduction

Nature is plenty of *complex adaptive systems*. These systems result from the intricate and dynamic interaction between numerous and non-linearly-behaved participants (i.e. the living organisms) and their environment (i.e. the nonliving components). Due to their complexity and non-linear dynamics, understanding and analyzing biological systems (e.g. metabolic, genetic, neural) has demanded appropriate analytical tools, where the fundamental approach consists of analyzing the system as a whole, as a living organism, and not merely as the simple sum of the organisms that compose it.

Understanding and analyzing biological systems has evolved through time. As acknowledged by Von Bertalanffy (1950), science used to *explain phenomena by reducing them to an interplay of elementary units which could be investigated independently of each other*, whereas contemporary modern science, irrespective of the object of study, deal with *what is rather vaguely termed "wholeness"*.

Akin to nature's systems, most human activities involve intricate and dynamic interactions between human beings or their creations (e.g. websites, academic papers, cities, countries, firms). Among many manmade systems, financial networks may be regarded as particularly convoluted, active and critical. As pointed out by Sornette (2003), *financial markets constitute one among many other systems exhibiting a complex organization and dynamics*, where *a large number of mutually interacting parts* [...] *self-organize their internal structure and dynamics with novel and sometimes surprising macroscopic ("emergent") properties*.

Despite the paradigm change from reductionism to wholeness took place nearly a century ago, financial systems' analysis has embraced such change rather recently. Traditional (reductionist) understanding of financial systems has relied on the individual understanding of financial firms, which has been known as the *micro-prudential* dimension of financial stability (De Nicoló et al., 2012; Hanson et al., 2011; Clement, 2010; Borio, 2003; Crocket, 2000;), where, as highlighted by Crockett (2000), *financial stability is ensured as long as each and every institution is sound*. Despite the term *macro-prudential* is not new[4], the analysis of financial systems as a whole appeared recently, mainly after the crisis that begun around 2007 (henceforth referred as "the crisis"), where the perspective of financial authorities is system-calibrated, rather from that of the safety and soundness of individual institutions on a stand-alone basis (Borio, 2010).

In this sense, as stressed by Barabási (2003), economic theory has considered agents as interacting not with each other but rather with "the market", a mythical entity (e.g. Adam Smith's invisible hand) that mediates all economic transactions. However, in reality, *the market is nothing but a* weighted *and directed network*, with economic agents as nodes, and with interactions (i.e. transactions, exposures) as links among them; therefore, *the structure*

[4] Clement (2010) suggests that the term first appeared at a meeting of the BIS' Cooke Committee (the forerunner of the Basel Committee on Banking Supervision) in June 1979.

and evolution of this weighted and directed network determine the outcome of all macroeconomic processes.

Accordingly, a macro-prudential analytical approach to financial systems should begin by recognizing that they are *adaptive nonlinear networks* (Holland, 1998) or *complex adaptive systems*.[5] Anderson (1999) suggests a description of complex adaptive systems by stating their four key elements:

a. There is a cognitive structure (i.e. *schema*) that determines agents' actions (*agents with schemata*);[6]
b. The behavior of an agent depends on the behavior –or state- of some subset of all agents in the system, where the presence of feedback loops among agents result in self-organization by means of nonlinear interactions such as amplification and crowding out of agents' behavior (*self-organizing networks*);
c. Each agent's payoff function depends on choices that the other agents make, where a dynamic equilibrium prevails such that small changes in behavior can have small, medium or large impacts on the system as a whole, according to a *power-law* (*coevolution to the edge of chaos*);[7]
d. Systems evolve based on the entry, exit and transformation of agents or schemata (*recombination and system evolution*).

It is because of these features that the building blocks of traditional economics, such as homogeneity and symmetry assumptions (Miller and Page, 2007); *fixed rational agents that operate in a linear, static, statistically predictable environment* (Holland, 1998); or *the introduction of ceteris paribus conditions, summarizing or ignoring feedback loops, and making assumptions about the order of magnitude of counteracting effects* (Berndsen, 1992), explicitly contradict the true nature of financial systems. Similarly, Anderson (1999) states that *complex systems resist simple reductionist analysis* –such as those typical of mainstream economic analysis- *because interconnections and feedback loops preclude holding some subsystems constant in order to study others in isolation*. This explains some of the limitations of traditional economic models for understanding and analyzing financial systems.

It is useful to synthesize the definition of complex adaptive systems even further by means of etymology. "System" consists of interconnected components that work together (Anderson, 1999); they are "complex" due to the highly nonlinear interactions within (Miller and Page, 2007), which result from optimizing, predicting and anticipating –but potentially confused-

[5] *Complex adaptive systems* are equivalent to *adaptive nonlinear networks* since nonlinearity results in complexity (Miller and Page, 2007), and a network is a representation of the interactions between the parts of a system (Newman, 2010). Under the adaptive nonlinear networks term Holland (1998) states that "the economy" is an example of a complex adaptive system.

[6] Each schema provides, in its own way, some combination of description, prediction, and prescriptions for action (Gell-Mann, 1994); it is a highly compressed description of the identified regularities in the observed system (Gell-Mann, 1992). Schemata (also referred as models, theories or blueprints) may compete with each other, mutate, recombine, and there may be a selective process based on their success. The existence of schemata is what distinguishes adaptive from evolving yet non-adaptive systems (e.g. galaxies), which tend to rely on fixed rules.

[7] In this sense, complex adaptive systems do not reach ordinary equilibrium (i.e. small changes are self-corrective) or chaotic (i.e. small changes are self-reinforcing) states. Complex adaptive systems lying between equilibrium and chaos (i.e. small changes yield power-law distributed impacts) result in a balance between flexibility and stability that allows for continuously evolving systems.

participants; and they are "adaptive" because agents coevolve with one another, adapting to its environment by striving to increase a payoff or fitness function over time (Holland, 1998). Based on a similar synthetized definition, Haldane (2009) concludes that financial systems should be regarded as complex adaptive systems.

Furthermore, not only financial systems may be characterized as complex adaptive systems, but they also share a common feature with the vast majority of real-world systems covered by network theory literature: they tend to be *scale-free networks*. First documented by Barabasi and Albert (1999), the ubiquity of scale-free networks refers to a broad spectrum of networks displaying degree (i.e. connections) distributions approximating a *power-law*, where the number of connections is distributed heterogeneously, with a few heavily connected participants and many poorly connected participants; due to the inhomogeneity there is no typical participant in the network, thus it has no scale (i.e. it is scale-free or scale-invariant).

Such type of distributions contradicts the original *homogeneous* or *exponential networks* models first developed by Solomonoff and Rapoport (1951) and Erdös and Rényi (1960), where those models assumed that connections were homogenously distributed between participants due to the assumption of exponentially decaying tail processes such as the Poisson distribution.[8] Divergence from exponentially decaying tail processes has significant consequences for understanding networks and their underlying systems: inhomogeneity results in the emergence of some key structural features of real-world networks that may significantly govern the efficiency and stability of the network, and may also explain the evolutionary process behind their formation.

Based on the General Theory of Systems (Von Bertalanffy, 1972 & 1950), financial systems being complex adaptive and scale-free is not casual, but may be related to the existence of "isomorphic laws" or "system laws", *where different systems follow laws that are formally identical but pertain to quite different phenomena or even appear in different disciplines*. In the same direction, Bak (1996) points out that *there are a number of ubiquitous empirical observations across the individual sciences that cannot be understood within the set of references developed within the specific scientific domains*, such as power-laws.

Financial networks being scale-free, with connections obeying a power-law, is analogous to the law of *allometric* growth used in biology and demography; to Pareto's law of wealth distribution; to Hurst's law in hydrology; to the Gutenberg-Richter law of earthquakes and to the fractal nature of financial time-series' returns. These and other analogies in the underlying probability mechanisms following a power-law have been extensively documented (e.g. Taleb, 2007; Mandelbrot and Hudson, 2004; Bak, 1996; Peak and Frame, 1994; Von Bertalanffy, 1972 & 1950; Mandelbrot, 1963; Simon, 1955).

Financial networks displaying scale-free structures was documented by the time of the crisis (e.g. Pröpper et al., 2008; May et al., 2008; Cepeda, 2008; Renault et al. (2007) and even well-before its arrival (e.g. Soramäki et al., 2006; Inaoka et al., 2004; Boss et al., 2004). However, it is customary and widespread to –explicitly or implicitly- assume that *homogeneous networks* describe financial systems (e.g. Gai and Kapadia, 2010; Nier et al., 2008; Iori et al., 2006;

[8] Consequently homogeneous networks models are commonly referred as "Poisson random graph".

Cifuentes et al., 2004; Allen and Gale, 2000; Freixas et al., 2000), where most works converge to diversification arguments that suggest a direct relation between connectedness and stability.

After the crisis numerous efforts have aimed at (i) identifying the conditions upon connectedness does not convey stability under homogeneous networks (e.g. Battiston et al., 2012 & 2009; Haldane and May, 2011; May and Arinaminpathy, 2010; Gai and Kapadia, 2010) and (ii) identifying networks' observed connectedness structure and analyzing the resulting impact in the efficiency and stability of the system, as in León and Pérez (2013), Martínez-Jaramillo et al. (2012), Cont et al. (2012), Markose et al. (2012), Markose (2012), Arinaminpathy et al. (2012), Fricke and Lux (2012), Craig and von Peter (2010), Schweitzer et al. (2009), Haldane (2009), Bech and Atalay (2008).

Whenever financial networks' observed connectedness structure is inhomogeneous (e.g. scale-free) the issue of the resiliency of the system arises. In those networks, where most participants have very few connections and very few have most connections, the extraction or failure of a participant will have significantly different outcomes depending on how the participant is selected. When randomly selected, the effect will be negligible, and the network may withstand the removal of several randomly selected participants without significant structural changes; however, if selected because of their high connectivity, the effect of extracting a small number of participants may significantly affect the network's structure. Therefore, a rising amount of financial literature is devoted to encouraging the usage of network theory metrics of importance (e.g. centrality) for identifying "super-spreaders" (Markose et al., 2012; Haldane and May, 2011) or systemically important participants (as in León and Pérez, 2013b; León and Machado, 2013; León and Murcia, 2012; Soramäki and Cook, 2012; Lovin, 2012).

Nevertheless, not all scale-free networks are the same. Networks with degree distributions approximating a power-law may display a community or modular structure as well. According to Newman (2003) a network displays a community structure when groups of vertices have a high density of edges within them, with a lower density of edges between groups. Despite the coincidence of power-law degree distributions and modular hierarchies in networks, the standard scale-free model is unable to reconcile both observed features, which has yielded a new type of network: a *modular scale-free network* (Barabási, 2003). To the best knowledge of the authors, modular scale-free financial networks are not well documented in related literature.

Regarding data sources for building financial networks, two main sources have been used in the literature: (i) financial transactions (i.e. flows), and (ii) financial exposures (i.e. stocks). Networks of financial transactions correspond to payments (i.e. delivery of money), settlements (i.e. delivery of securities or currencies) or trades (i.e. exchange of buy-sell orders) among financial institutions, which are automatically registered and safeguarded by financial market infrastructures (e.g. large-value payment systems, clearing houses, securities settlement systems, central securities depositaries, trading platforms, trade repositories) whenever a transaction occurs. As highlighted by some authors (Kyriakopoulos et al., 2009; Uribe, 2011a,b) the information conveyed in financial transactions is particularly valuable due to (i) its granularity, with informative details such as sender, recipient, amount, type of transaction, underlying asset, etc.[9]; (ii) completeness, since all financial transactions ineludibly involve the delivery of money or securities, or a trade; and (iii) opportunity, with data usually available in real-time (or with a minimal lag).

On the other hand, financial exposures ordinarily emerge from reports prepared and delivered by each financial firm to the corresponding authorities (e.g. financial statements), where the most commonly used for building financial networks are interbank credit and derivatives exposures. This type of information tends to be aggregated (i.e. details of individual exposures, counterparties, instruments, etc. are usually unavailable) and lagged, and its completeness, consistency and validity depends on accounting practices by each financial firm and the corresponding jurisdiction.

Despite interbank credit and derivatives exposures have been the traditional source of information for understanding the financial system, its usefulness has been questioned after the crisis. For example, as documented in the BIS 81st Annual Report (BIS, 2011), the lack of detailed firm-level information (i.e. asset and liability positions broken down by currency, counterparty, instrument type) resulted in market uncertainty that contributed to funding problems for exposed and non-exposed institutions. Moreover, there is strong evidence of non-trivial debt masking in Enron and Lehman Brothers audited financial statements prior to their failures (Smith, 2011)[10], which may verify the lack of completeness, consistency and validity of reported exposures as a rigorous source of information for financial networks' building. These two facts contrast with the arguments of Kyriakopoulos et al. (2009), who states that financial transaction data sets *provides a real-time picture of transactions* and is particularly reliable from a supervisory perspective because payments and settlements *cannot be falsified (or at least at substantially high costs and an increased likelihood of detection)*.

Correspondingly, in order to make a contribution to the understanding of the structure of financial systems by means of network analysis, this document aims at characterizing and analyzing 236 observations of Colombian payment and settlement networks as modular scale-free networks, an isomorphic topology of clusters of dense interaction resulting from financial systems being complex adaptive and self-organizing. Namely, the dataset consists of daily transactions registered during 2012 in the large-value payment system (CUD), the sovereign securities settlement system (DCV) and the currency settlement system (CCDC), which together represent about 88.4% of the value of the payments and deliveries within the local financial market infrastructures during 2012 (Banco de la República, 2013) and correspond to

[9] Contrary to interbank and derivatives exposures, which is commonly extracted from reported financial statements (e.g. balance sheet data), payments and settlements data includes informative details for each transaction, such as the collateral involved (e.g. in a repo); the underlying asset (e.g. in an option); the time to maturity (e.g. in interbank lending); the intermediaries involved (e.g. debt market intermediaries) and the beneficiaries (e.g. financial institutions, households, non-financial companies).

[10] Smith (2011) reports that *Lehman used sale-repurchase (repo) agreements to reduce its recognized debt for dates surrounding quarterly reporting periods*; by means of interpreting accounting standards, Lehman removed over $50 billion from its balance sheet at the end of the fiscal quarter in May 2008, which reduced net leverage from 13.9 to 12.1. Likewise, by means of interpreting accounting standards, Enron reduced its recognized debt about 30%.

the three foremost systemically important financial market infrastructures in Colombia according to León and Pérez (2013).

Due to its aim, the document differs from most financial network literature. First, as already mentioned, to the best knowledge of the authors there is no empirical work regarding financial networks as modular scale-free networks. Second, three different –but interrelated- financial networks are analyzed. Third, unlike most of the literature, the characterization of the three selected networks is not based on a single snapshot (i.e. observation) of the network;[11] all network analysis metrics were applied to a set of 236 consecutive observed networks, which allowed for constructing time-series for the calculated metrics. Fourth, different from most empirical work on financial networks, the type of financial institutions considered is not limited to banking firms, which may reveal some otherwise concealed connective and hierarchical patterns in the networks; this is critical for analytical purposes due to the increasing interest in non-banking financial institutions and the so-called "shadow banking system".

The main quantitative findings confirm the isomorphism of the selected payment and settlement networks with other social networks, where they approximate a modular scale-free architecture in the sense of Barabási (2003). Furthermore, the rationale behind such isomorphism agrees with three interrelated observations: (i) the economy is a complex adaptive system (Holland, 1998); (ii) the economy is a self-organizing system (Krugman, 1996); and (iii) in the sense of Bak (1996), financial systems are complex adaptive systems that have self-organized in order to prevent criticality from arising. Thus, authors consider this document a significant contribution to the existing related literature, where the long-standing call for *mathematics and modeling techniques that emphasize the discovery of building blocks and the emergence of structure through the combination and interaction of these building blocks* (Holland, 1998) is vindicated.

2. Network analysis

Systems and networks are closely related concepts. Trewavas (2006) defines "system" as *a network of mutually dependent and thus interconnected components comprising a unified whole*, whereas Newman (2010) defines network as *a general yet powerful mean of representing patterns of connections or interactions between the parts of a system.* Both statements concur in that a network is a depiction or simplification of the connective structure of a system, where *it seems evident that without a connective structure, there would be no system at all* (Casti, 1979).

Network science is an emerging research area that contrasts, compares and integrates techniques and algorithms developed in disciplines as diverse as mathematics, statistics, physics, social network analysis, information science, and computer science, with the objective of developing theoretical and practical approaches and techniques to increase the understanding of natural and manmade networks (Börner et al., 2007), thus to increase the

[11] Martínez-Jaramillo et al. (2012), Bech and Atalay (2008) and Sorämaki et al. (2006) are other related works with a similar time-series perspective.

understanding of systems. Intuitively, complex systems are the main target of network science.

However, defining what a "complex system" is has proved to be particularly elusive, with standard definitions converging to the existence of a large number of participants that are related in an intricate way, as in May (1973) or Simon (1962). Yet, as emphasized by Casti (1979), different types of complexity coexist, namely *static complexity* and *dynamic complexity*, with the former matching the standard definition (i.e. number of participants and their intricate connections) and the latter relating to system's time behavior (i.e. its motion and predictability). Hence, since the aim of this document is to make a contribution to the understanding of the structure of financial systems, the complexity to be captured and examined is mostly of the *static* type.

The network science research process provides two different paths for understanding the structure of financial systems: *network analysis* and *network modeling.* As in Börner et al. (2007), the first path is dedicated to describing and understanding an underlying system, where the focus is on capturing the system's structure, whereas the second attempts to design processes that reproduce empirical data and also serve the purpose of making predictions, where the focus is on model validation. Consistent with the aim of this document, this document employs the *network analysis* process (i.e. network sampling, measurement and visualization), which includes fitting empirical data to conventional models (i.e. exponential, *scale-free*, *modular scale-free*); however, no –new- models are designed to reproduce empirical data or make predictions.

This section is intended to provide the theoretical and methodological background required to properly examine the static complexity of systems by means of network analysis. Accordingly, this section follows a particular structure, specifically aimed at the two principal aspects of static complexity, namely the system's connective pattern and hierarchical structure (Casti, 1979). Basic concepts and notation are stated first. Afterwards, measures related to network's connective pattern and to network's hierarchical structure are presented, in that order.

As usual, measures serve the purpose of characterizing the observed networks as pertaining to the existing network models. Emphasis will be made on centrality measures as relevant metrics for analyzing inhomogeneous networks.

2.1. Concepts and notation

Due to its interdisciplinary origin and recent use in economics and finance, network science's concepts and notation are worth stating. Most of those concepts and notation is inherited from *graph theory*, the branch of mathematics that deals with networks since the XVII century.[12]

[12] Euler´s solution to the *Königsberg Bridge Problem* in 1735 is documented as the origin of mathematical graph theory.

A network, or graph, represents patterns of connections between the parts of a system. Two concepts arise from this definition: parts and connections. The parts of the system correspond to the participants or elements, and are commonly referred to as *vertices*, whereas the connections correspond to the relations between the elements of the system, and are called *edges*. These concepts tend to have an equivalent when applied to specific networks, such as *nodes* and *links* in computer science, *actors* and *ties* in sociology, *neuron* and *synapse* in neural networks, *web page* and *hyperlink* in the World Wide Web network, or *financial institution* and *payment* (or *exposure*) in financial networks.

The most common mathematical representation of a network is the adjacency matrix. Let n represent the number of vertices, the adjacency matrix A is a square matrix of dimensions $n \times n$ with elements A_{ij} such that

$$A_{ij} = \begin{Bmatrix} 1 \text{ if there is an edge between vertices } i \text{ and } j, \\ 0 \text{ otherwise.} \end{Bmatrix} \quad [§1]$$

A network defined by the adjacency matrix in [§1] is referred as an undirected graph, where the existence of the (i, j) edge makes both vertices i and j adjacent or connected, and where the direction of the edge is unimportant; this may be the case in some social networks (e.g. acquaintances' networks such as Facebook or Linkedin), in which the existence of a relation (e.g. friendship, professional link) implies a reciprocal relation between the vertices (e.g. friends, colleagues).

However, the assumption of a reciprocal relation between vertices is inconvenient for some networks. For instance, the delivery of money between financial institutions (i.e. a payment network) constitutes a graph where the character of sender and recipient of the funds is a particularly sensitive source of information for analytical purposes, where the assumption of a reciprocal relation between both parties is unwarranted; likewise, *co-citation* networks (i.e. citations between academic papers) and the World Wide Web (i.e. hyperlinks between web pages) are directed networks by construction.

Thus, the adjacency matrix of a directed network or *digraph* differs from the undirected case, with elements A_{ij} such that

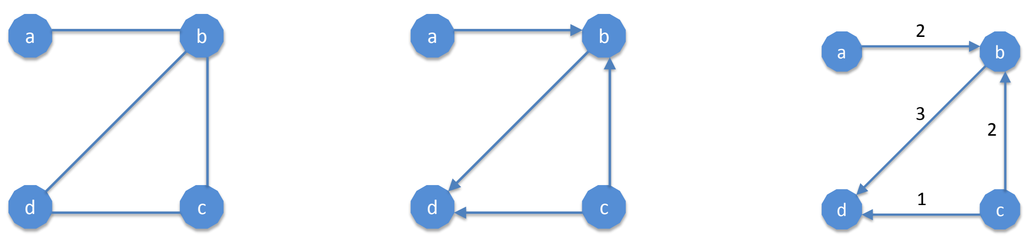
$$A_{ij} = \begin{Bmatrix} 1 \text{ if there is an edge from } i \text{ to } j, \\ 0 \text{ otherwise.} \end{Bmatrix} \quad [§2]$$

Consequently, the undirected adjacency matrix is always symmetrical with respect to the main diagonal, whereas the directed case tends to be non-symmetrical; the direction of the connection is usually displayed with an arrow, and it is common to use the terms *arc* and *directed edge* interchangeably. If self-edges are allowed the main diagonal may have non-zero elements, and the graph may be known as a *multigraph*.[13]

Moreover, networks may require edges to represent more information than that conveyed in a simple binary (0 or 1) relation; in the complex network context, edges are not binary, but are weighted according to the economic interaction under consideration (Schweitzer et al., 2009). Therefore, it may be useful to assign real numbers to the edges, where these numbers may represent distance, frequency or value, in what is called a weighted network and its corresponding weighted adjacency matrix (W_{ij}).

For a financial network the weights could be the monetary value of the transaction or of the exposure. Figure 1 presents samples of undirected (a.), directed (b.) and weighted directed (c.) graphs, along with the corresponding adjacency matrices.

Figure 1
Sample graphs



$$A_{ij} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 0 \\ b & 1 & 0 & 1 & 1 \\ c & 0 & 1 & 0 & 1 \\ d & 0 & 1 & 1 & 0 \end{array}$$

a. Undirected graph

$$A_{ij} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 0 \\ b & 0 & 0 & 0 & 1 \\ c & 0 & 1 & 0 & 1 \\ d & 0 & 0 & 0 & 0 \end{array}$$

b. Directed graph

$$W_{ij} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 2 & 0 & 0 \\ b & 0 & 0 & 0 & 3 \\ c & 0 & 2 & 0 & 1 \\ d & 0 & 0 & 0 & 0 \end{array}$$

c. Weighted directed graph

Source: authors' design

Despite graphs are illustrative about the topology of a network, the dimensionality of the system (e.g. the number of vertices or edges) may obscure the visual inspection of the underlying structure. In such cases it is convenient to use a *tree*. A *tree* may be described as a simplified but informative version of a graph that displays the most relevant edge for each one of the vertices, where such relevance is defined according to the type of network.[14] For

[13] The existence of self-edges in financial networks may be non-trivial. For instance, if two clients use the same securities' broker, their transaction will be registered as occurring within the broker accounts (i.e. as a self-edge). If the brokerage business is to be addressed (e.g. León and Pérez, 2013b), assuming the absence of self-edges may be inconvenient.

[14] Formally, a tree is a graph that is connected (i.e. no vertices are disconnected), acyclic (i.e. no loops) and has $n-1$ edges (Jungnickel, 2008). The construction of a tree usually implies the maximization or minimization of the sum of the network's weights. Several algorithms are available for this purpose, but the most cited in the literature are Kruskal's and Prim´s algorithms; yet, the main features of the tree do not depend on the choice of algorithm (Kim et al., 2005).

instance, for a network of payments it is convenient to construct the tree based on the maximization of the system's weights (i.e. a *maximal spanning tree*), although searching for the most efficient route within a highway system would require minimizing distances or commute times (i.e. a *minimal spanning tree*). In this sense, as highlighted by Braunstein et al. (2007) and Wu et al. (2006), the resulting *tree* may be considered as the "skeleton" of the network.

Regarding the characteristics of the system and its elements, a set of concepts is commonly used. The simplest concept is the vertex degree (k_i), which corresponds to the number of edges connected to it. In directed graphs, where the adjacency matrix is non-symmetrical, in degree (k_i^{in}) and out degree (k_i^{out}) quantifies the number of incoming [§3a] and outgoing [§3b] edges, respectively; for undirected graphs, $k_i = k_i^{in} = k_i^{out}$.

$$k_i^{in} = \sum_{j=1}^{n} A_{ji} \qquad\qquad k_i^{out} = \sum_{j=1}^{n} A_{ij} \qquad [§3]$$

a. In degree (k_i^{in}) b. Out degree (k_i^{out})

In the weighted graph case the degree may be informative, yet inadequate for analyzing the network; financial networks are a good case of degree being limited for analytical purposes. The *strength* (s_i) measures the total weight of connections for a given vertex, which provides an assessment of the intensity of the interaction between participants. Akin to degree, in the directed graph case in strength (s_i^{in}) and out strength (s_i^{out}) sum the weight of incoming [§4a] and outgoing [§4b] edges, respectively; for undirected graphs, $s_i = s_i^{in} = s_i^{out}$.

$$s_i^{in} = \sum_{j=1}^{n} W_{ji} \qquad\qquad s_i^{out} = \sum_{j=1}^{n} W_{ij} \qquad [§4]$$

a. In strength (s_i^{in}) b. Out strength (s_i^{out})

Intuitively, the larger the degree or the strength, the more important the vertex is for the network. Nevertheless, as will be discussed in forthcoming sections, the analytical reach of these two metrics as measures of the relative importance of a vertex is limited because they do not take into account the global properties of the network (i.e. they are local measures of importance by construction).

2.2. Identifying connective patterns

Some metrics allow for determining the connective pattern of the graph, which is one of the main aspects any static complexity measure must address according to Casti (1979). The simplest metric for approximating the connective pattern is *density* or *connectance* (d), which measures the cohesion of the network. The *density* of a directed graph is the ratio of the number of actual edges (m) to the maximum possible number of edges ($n(n-1)$), as in [§5].[15]

$$d = \frac{m}{n(n-1)} \qquad [§5]$$

Density (d)

By construction, density is restricted to the $0 \leq d \leq 1$ range. Formally, Newman (2010) states that a sufficiently large network for which the density d tends to a constant as n tends to infinite is said to be *dense*, whereas if density tends to zero as n tends to infinite the network is said to be *sparse*. However, since it is frequent to work with non-sufficiently large networks, it is common to characterize a network as sparse when the density is much smaller than the upper limit ($d \ll 1$), and to use the term *dense* as the density approximates the upper limit ($d \cong 1$), where the term *complete network* is used when $d = 1$.

A particularly informative alternative to density is to examine the degree probability distribution ($\mathcal{P}_k$); such distribution provides a natural summary of the connectivity in the graph (Kolaczyk, 2009). Akin to density, the first moment of the distribution of degree (μ_k) measures the cohesion of the network, and is restricted to the $0 \leq \mu_k \leq n$ range. According to Börner et al. (2007), a *sparse* graph has an average degree (μ_k) that is much smaller than the size of the graph ($\mu_k \ll n$).

Since the number of edges in a directed network is equal to the number of incoming edges and to the number of outgoing edges, there is a unique *average degree* for the network, as in [§6].

$$\mu_k = \frac{1}{n}\sum_{i=1}^{n} k_i^{in} = \frac{1}{n}\sum_{i=1}^{n} k_i^{out} = \frac{m}{n} \qquad [§6]$$

Average degree (μ_k)

The second moment of the distribution (σ_k) indicates how disperse is the vertices' degree around the average degree. The standard deviation of the in and out degree may not be the same [§7].

[15] Please note that the calculation of the density varies according to the type of graph (e.g. graph, digraph, multigraph). In a digraph without self-edges the maximum possible number of edges is $n(n-1)$; if self-edges are allowed n^2.

$$\sigma_{k_{in}} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (k_i^{in} - \mu_k)^2} \qquad \sigma_{k_{out}} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (k_i^{out} - \mu_k)^2} \qquad [§7]$$

a. In degree standard deviation $(\sigma_{k_{in}})$ b. Out degree standard deviation $(\sigma_{k_{out}})$

The third moment (i.e. skewness or asymmetry) of the degree distribution is particularly informative about the connective pattern of the network. If asymmetry is nil or negligible, the average degree is meaningful, and the majority of the vertices display an average degree, and few vertices are of low or high degree. In this case vertex degree is of a fairly similar order of magnitude across the graph –homogeneous-, the corresponding degree distribution is quite concentrated, and typically decay exponentially fast in k (Kolaczyk, 2009); in the limiting case of a symmetric distribution the degree follows a Poisson process, where the probability of observing a vertex with k edges becomes negligibly small when $k \ll \mu_k$ or $\mu_k \gg k$.

However, most real-world networks display right-skewed distributions, where the majority of vertices are of very low degree, and few vertices are of very high degree; hence inhomogeneous. Such right-skew of real-world network's degree distributions has been found to approximate a power-law distribution (Barabási and Albert, 1999). On the other hand, in homogeneous networks all vertices have approximately the same number of edges.

The power-law (or Pareto-law) distribution suggests that the probability of observing a vertex with k edges obeys the potential functional form in [§8], where z is an uninteresting and arbitrary constant, and γ is known as the *exponent* of the power-law.

$$\mathcal{P}_k \propto z k^{-\gamma} \qquad [§8]$$

Degree distribution as a power-law

Verifying that the degree distribution approximates a power-law (i.e. it is a scale-free network) is interesting for several reasons. For instance, power-law distributions not only appear to be ubiquitous in networks across many areas of sciences (Kolaczyk, 2009), but their fractal nature may be also informative about the evolutionary process of the underlying systems, as suggested by Dooley and Van de Ven (1999), Peak and Frame (1998) and Bak (1996). Furthermore, as addressed below, the scale-free nature of networks points out to systems robust to random changes, but fragile to targeted ones.

Besides degree distributions approximating a power-law, other features have been identified as characteristic of real-world networks, such as low mean geodesic distances, high clustering coefficients, and significant degree correlation.

Let g_{ij} be the *geodesic distance* (i.e. the shortest path) from vertex i to j, the mean geodesic distance for vertex i (ℓ_i) corresponds to the mean of g_{ij}, averaged over all vertices j in the network (Newman, 2010), as in [§9a].[16] Respectively, the mean geodesic distance or average path length of a network (i.e. for all pairs of vertices) is denoted as ℓ (without the subscript), as in [§9b], and corresponds to the mean of ℓ_i over all vertices.

$$\ell_i = \frac{1}{(n-1)} \sum_{j(\neq i)} g_{ij} \qquad\qquad \ell = \frac{1}{n} \sum_{i} \ell_i \qquad [§9]$$

a. Mean geodesic distance of a vertex (ℓ_i) b. Mean geodesic distance of a network (ℓ)

Consequently, the mean geodesic distance or average path length reflects the global structure; it measures how big the network is, it depends on the way the entire network is connected, and cannot be inferred from any local measurement (Strogatz, 2003).

The mean geodesic distance (ℓ) of random networks is small, and increases slowly with the size of the network; therefore, as stressed by (Albert and Barabási, 2002), random graphs are small-worlds because in spite of their often large size, in most networks there is relatively a short path between any two vertices. According to Newman et al. (2006), $\ell \sim \ln n$ for random networks, where such slow logarithmic increase with the size of the network coincides with the small-world effect (i.e. short average path lengths).

However, the mean geodesic distance for scale-free networks has been found to be smaller than $\ell \sim \ln n$. As reported by Cohen and Havlin (2010 & 2003), non-degree-correlated scale-free networks with $2 < \gamma < 3$ have a mean geodesic distance that behaves as $\ell \sim \ln\ln n$; networks with $\gamma = 3$ yield $\ell \sim \ln n / (\ln \ln n)$; and with $\gamma > 3$, the small-world $\ell \sim \ln n$. For that reason, Cohen and Havlin (2010 & 2003) state that scale-free networks can be regarded as a generalization of random networks with respect to the mean average geodesic distance, in which scale-free networks with $2 < \gamma < 3$ are “ultra-small”.

The clustering coefficient (c), corresponding to the property of network transitivity, measures the average probability that two neighbors of a vertex are themselves neighbors; this is, it measures the frequency with which loops of length three (i.e. triangles) appear in the network (Newman, 2010). Let a *triangle* be a graph of three vertices that is fully connected, and a *connected triple* be a graph of three vertices with at least two connections, the calculation of the network’s clustering coefficient is as follows: [17]

[16] Some technical details are worth noting. First, the length of a path (or distance) is in terms of number of edges between vertices, not the number of vertices. Second, as in [§9a], it is convenient to exclude the $i = j$ case from the calculations, where $g_{ii} = 0$. Third, in directed networks the distance from i to j and j to i may differ ($g_{ij} \neq g_{ji}$); thus, both distances should be considered. Fourth, if there is no path between two vertices, the length is infinite; however, for the purpose of calculation of the mean geodesic of a network, only finite paths (i.e. reachable vertices) are considered. Fifth, the inverse of ℓ_i is commonly known as the *closeness centrality* of vertex i.

[17] If three vertices (i.e. a, b, c) exist in a graph, a triangle exists when edges (a,b), (b,c) and (c,a) are present (i.e. the graph is complete), whereas a connected triple exists if at least two of these edges are present. In this sense, a triangle occurs when there is transitivity (i.e. two neighbors of a vertex are themselves neighbors). The factor of three in the numerator arises because each triangle is counted three times when the connected triplets are counted (Newman, 2010).

$$c = \frac{(\text{number of triangles in the network}) \times 3}{\text{number of connected triples}} \qquad [§10]$$

Clustering coefficient (c)

Hence, by construction, clustering reflects the local structure; it depends only on the interconnectedness of a typical neighborhood, the inbreeding among nodes tied to a common center, and thus it measures how incestuous the network is (Strogatz, 2003).

Intuitively, in a random graph the probability of connection of two vertices tends to be the same for all vertices regardless the existence of a common neighbor. Therefore, in the case of random graphs the clustering coefficient is expected to be low, about $c \sim \mu_{k}/(n-1) \ll 1$, and tends to zero in the limit of large random networks.

Contrarily, real-world complex networks tend to exhibit a large degree of clustering. Albert and Barabási (2002) report that in most –if not all- real networks the clustering coefficient is typically much larger then it is in a comparable random network (i.e. with same number of vertices and edges), with this factor slowly increasing with the number of vertices. Accordingly, in inhomogeneous graphs, as those resulting from real-world networks, the probability of two neighbors of a vertex being themselves neighbors is reported to be in the 10% and 60% range in most cases (Newman, 2010). In this sense, scale-free networks combining particularly low mean geodesic distance and high clustering implies that the existence of a few too-connected vertices with very large degrees plays a key role in bringing the other vertices close to each other (Wang and Chen, 2003), indicating that the scale-free topology is more efficient in bringing the vertices close than is the topology of random graphs (Albert and Barabási, 2002).

Besides displaying low mean geodesic distances and clustering, real-world graphs also display non-negligible degree correlation between vertices. They are characterized by either positive correlation, where high-degree (low-degree) vertices tend to be connected to other high-degree (low-degree) vertices, or negative correlation, where high-degree vertices tend to be connected to low-degree vertices. Positive degree correlation, also known as *homophily* or *assortative mixing by degree*, results in the core/periphery structure typical of social networks, whereas negative degree correlation (i.e. *dissortative mixing by degree*) is typical of technological, informational and biological networks, which display star-like features that do not usually have a core/periphery but uniform structures (Newman, 2010). On the other hand, the degree of random (i.e. homogeneous) networks tends to be uncorrelated.

Degree correlation may be measured by means of estimating the assortativity coefficient (Newman, 2010). As before, let m be the number of edges, the degree assortativity coefficient of a network (r_{k}) is estimated as follows [§11]:

$$r_{k} = \frac{\sum_{ij}\left(A_{ij} - k_i k_j/2m\right)k_i k_j}{\sum_{ij}\left(k_i \delta_{ij} - k_i k_j/2m\right)k_i k_j} \qquad [§11]$$

Where

$$\delta_{ij} = \begin{cases} 0 \text{ if } i \neq j \\ 1 \text{ if } i = j \end{cases}$$

Degree assortativity coefficient (r_k)

However, the assortativity coefficient is not limited to the degree correlation. Other types of characteristics may be underlying the formation of correlation (e.g. age, income, gender, ethnics, size), which results in *assortative mixing by scalar characteristics* (Newman, 2010). As stressed before, for payment and settlement networks it is important to assess the intensity of the interaction between participants; as highlighted by Leung and Chau (2007) and Barrat et al. (2004), the inclusion of weights and their correlations might consistently change our view of the hierarchical and structural organization of the network.[18] Based on [§11], it is possible to estimate the *assortative mixing by strength* as in [§12].

$$r_s = \frac{\sum_{ij}\left(A_{ij} - k_i k_j / 2m\right) s_i s_j}{\sum_{ij}\left(k_i \delta_{ij} - k_i k_j / 2m\right) s_i s_j} \quad [§12]$$

Strength assortativity coefficient (r_s)

Differences in degree correlation are relevant for understanding the structure and dynamics of networks. For instance, a disease can persist more easily in an assortative mixing (i.e. positively correlated) network by circulating in the dense core, where there are many opportunities for it to spread; in a negatively correlated network the same disease finds it harder to persist, but if it does persist, then it typically spreads to the whole network (Newman, 2008).

2.3. Assessing centrality

Since the manifestation of a power-law suggests that few vertices are very highly connected and many are poorly connected, assessing the relative importance of those highly connected becomes a relevant issue for network analysis. In this sense, some metrics are informative about the importance of a vertex in the network, where *centrality* is the most common concept.

There are many possible definitions of centrality, and correspondingly many centrality measures for networks (Newman, 2010). The simplest measure of centrality is the degree (k), where importance arises from concentrating edges within the network; in the case of directed networks, two measures coexist: in degree (k_i^{in}) and out degree (k_i^{out}) centrality, calculated

[18] Barrat et al. (2004) highlights that it is possible that a network simultaneously displays disassortative mixing by degree (i.e. high-degree vertices connected to a majority of low-degree vertices) and assortative mixing by strength (i.e. high-degree vertices concentrating the largest fraction of their strength only on high-strength vertices). In this sense, the topological features would point to disassortative properties, whereas the network could be considered assortative in an effective way.

based on [§3]. Likewise, concentrating strength (s_i) in a weighted network may be a signal of importance. However, none of these two measures takes into account the global properties of the network (i.e. they are local measures of centrality); this is, the centrality of the adjacent vertices is not taken into account as a source of centrality.

The simplest global measure of centrality is *eigenvector centrality*, whereby the centrality of a vertex is proportional to the sum of the centrality of its adjacent vertices; accordingly, the centrality of a vertex is the weighted sum of centrality at all possible order adjacencies. Hence, centrality arises from (i) being connected to many vertices; (ii) being connected to central vertices; (iii) or both. Let $\lambda_1$be the largest eigenvalue of the adjacency matrix A, the eigenvector centrality (e) is estimated as in [§13]:

$$e_i = \lambda_1^{-1} \sum_j A_{ij} e_j \qquad [§13]$$

In matrix terms,

$$e^T = Ae$$

where

$$Ae = \lambda_1 \, e$$

Eigenvector centrality (e)

Bonacich (1972) envisaged this global measure of centrality, which results from estimating *popularity scores* based on the eigenvector corresponding to the largest eigenvalue. Bonacich's choice of the largest eigenvalue is consistent with it providing the highest accuracy (i.e. explanatory power) for reproducing the original adjacency matrix.[19]

However, eigenvector centrality has some drawbacks. First, as stated by Bonacich (1972), eigenvector centrality works for symmetric structures only (i.e. undirected graphs). The most severe inconvenience from estimating eigenvector centrality on asymmetric matrices arises from vertices with only outgoing or incoming edges, which will always result in zero eigenvector centrality, and may cause some other non-strongly connected vertices to have zero eigenvector centrality as well (Newman, 2010); in the case of acyclic graphs, such as financial market infrastructures networks (León and Pérez, 2013b), this may turn eigenvector centrality useless.

Some measures try to profit from eigenvector centrality's global approach to importance within a network, whilst surmounting its main drawbacks.[20] The most well-known measure

[19] Straffin (1980) and Boots (1984) verify the convenience of estimating the largest eigenvalue for capturing the main features of networks (e.g. total connectivity, spread potential, equilibrium importance of vertices, degree of differentiation of vertices). Before Bonacich (1972), Gould (1967) and Tinkler (1972) suggested using the largest eigenvalue in Geography and Physics for similar reasons. It is worth noticing that if each entry of the eigenvectors is weighted by the square root of the corresponding eigenvalue, so that the elements of the eigenvectors associated with the smaller eigenvalues are successively reduced in scale, this is the conventional Principal Component Analysis (Gould, 1967).

[20] An alternative to the measures addressed in this document is *Katz centrality*. Katz centrality avoids some of the documented drawbacks by giving each node an initial amount of centrality. However, as documented by Newman

was developed for *internet link analysis*: PageRank, the algorithm behind Google's search engine (www.google.com), developed by Brin and Page (1998). Similar to eigenvector centrality, PageRank was designed based on a thesis: a vertex (e.g. webpage) is important if it is pointed-to by other important vertices. However, PageRank's design includes a *stochastic adjustment* to eigenvector centrality that overcomes the existence of vertices with only outgoing or incoming edges (i.e. dangling nodes).

The estimation of PageRank may be stated as an eigenvector problem (Langville and Meyer, 2012). Let λ_1 be the largest eigenvalue of $\mathcal{G}$; ω a scalar between 0 and 1; x^T a row vector of all 1s; and H the row-normalized original adjacency matrix, then the row vector of PageRank scores results from solving $\mathscr{p}$ in [§14]:

$$\mathcal{G}\mathscr{p} = \lambda_1 \mathscr{p} \qquad [§14]$$

where

$$\mathcal{G} = \omega S + (1-\omega)(1/\mathscr{n})(xx^T)$$

$$S = H + \begin{Bmatrix} \left((1/\mathscr{n})x^T\right) \text{ if vertex } i \text{ has no outgoing edges,} \\ 0 \text{ otherwise.} \end{Bmatrix}$$

PageRank centrality ($\mathscr{p}$)

PageRank avoids the main drawbacks of eigenvector centrality in two steps. First, it suppresses dangling vertices (i.e. without outgoing edges) by forcing a $\mathscr{n}$-dimension row-normalized adjacency matrix (H) into S, a $\mathscr{n}$-dimension right stochastic matrix (i.e. square matrix of non-negative real numbers, with each row summing to 1). Second, $\mathcal{G}$, commonly known as the Google matrix, is the weighted sum of S and a $\mathscr{n}$-dimension matrix with all its elements equal to a homogeneous probability ($1/\mathscr{n}$), where the weight is a scalar (ω) between 0 and 1 that controls the proportion of time that the system follows the network structure in S.[21]

In this sense, as put forward by Soramäki and Cook (2012), PageRank and eigenvector centrality can be thought of as the proportion of time spent visiting each vertex in an infinite random walk through the network, where PageRank allows the measure to be calculated for all types of networks by means of adding a random jump probability for dangling vertices.

(2010), this solution implies some other drawbacks that are conquered by PageRank; therefore, it is not discussed or used in the document. Soramäki and Cook (2012) introduced other alternative, SinkRank.

[21] PageRank originally suggested $\omega = 0.85$; it is important to highlight that increasing the value of ω (as it gets closer to unity) significantly increases the time to convergence of the algorithm, and makes the results more volatile. The mathematical foundations of PageRank are outside the scope of this paper. Langville and Meyer (2012) present a comprehensive analysis of the conceptual and mathematical origins of PageRank.

Internet link analysis provides another enhanced version of eigenvector centrality: HITS (Hypertext Induced Topic Search), the algorithm designed by Kleinberg (1998), which powers Teoma's (www.teoma.com) and Ask's (www.ask.com) search engines. HITS main premise is to recognize that webpages serve two purposes: (i) to provide information on a topic, and (ii) to provide links to other webpages containing information on a topic.

Therefore, Kleinberg's algorithm identifies popularity or importance based on a pair of interdependent circular thesis: (i) a webpage is a good *hub* if it points to good *authorities*, and (ii) a webpage is a good *authority* if it is pointed-to by good *hubs*. This may be conveniently reduced as follows: authority central vertices receive edges from hub central vertices, and hub central vertices send edges to authority central vertices, where each vertex has some authority score and some hub score.

As in the case of PageRank, HITS avoids the issues regarding the estimation of eigenvector centrality on directed networks. Instead of adding a random jump, HITS generates two modified versions of the original adjacency matrix, in which these two matrices correspond to an authority matrix ($\mathcal{A}$) and a hub matrix ($\mathcal{H}$).

$$\mathcal{A} = A^T A \qquad\qquad \mathcal{H} = AA^T \qquad [\S15]$$

a. Authority matrix ($\mathcal{A}$) b. Hub matrix ($\mathcal{H}$)

Both, $\mathcal{A}$ and $\mathcal{H}$ are symmetrical matrices by construction. Moreover, multiplying the adjacency matrix with a transposed version of itself allows identifying directed (*in* or *out*) second order adjacencies. Regarding $\mathcal{A}$, multiplying $A^T$with A sends weights backwards –against the arrows, towards the pointing node-, whereas multiplying A with A^T (as in $\mathcal{H}$) sends scores forwards –with the arrows, towards the pointed-to node (Bjelland et al., 2008).

The estimation of authority and hub centrality results from estimating standard eigenvector centrality (as in [§13]) on $\mathcal{A}$ and $\mathcal{H}$. In this sense, the authority centrality ($\mathcal{a}$) of each node is defined to be proportional to the sum of the hub centrality ($\hbar$) of the nodes that point to it, and that the hub centrality of each node is defined to be proportional to the sum of the authority centrality of the nodes it points-to. Let Ω and ψ be two unknown constants, and $\mathcal{a}$ and $\hbar$ the vector of authority and hub centrality, respectively, the vector of authority and hub scores results from solving $\mathcal{a}$ and $\hbar$ in [§16], which is –as in the case of PageRank- an eigenvector problem with respect to the largest eigenvalue (λ_1) of A:

$$\mathcal{a}_i = \Omega \sum_j A_{ji} \hbar_j \qquad\qquad \hbar_i = \psi \sum_j A_{ij} \mathcal{a}_j \qquad [\S16]$$

In matrix terms,

$$\mathcal{a} = \Omega A^T \hbar \qquad\qquad \hbar = \psi A \mathcal{a}$$

Substituting,

$$a = \Omega\psi A^T A a \qquad\qquad \hbar = \psi\Omega A A^T \hbar$$
$$A^T A a = (\Omega\psi)^{-1} a \qquad\qquad A A^T \hbar = (\Omega\psi)^{-1} \hbar$$

Replacing $(\Omega\psi)^{-1}$ with λ_1

$$A^T A a = \lambda_1 a \qquad\qquad A A^T \hbar = \lambda_1 \hbar$$
$$\mathcal{A} a = \lambda_1 a \qquad\qquad \mathcal{H} \hbar = \lambda_1 \hbar$$

a. Authority centrality (a) b. Hub centrality ($\hbar$)

The HITS algorithm has some practical advantages. It provides two sets of centrality measures, corresponding to the importance as a source and recipient of edges; in the payment and settlement networks' case this may be convenient since it is relevant to differentiate the role of financial institutions as originators or recipients of transactions involving money or securities. Second, HITS yields two symmetric modified versions of the adjacency matrix that share a single set of eigenvalues (λ), a byproduct that will be most useful when assessing the hierarchical structure of the network by spectral analysis. Third, as stressed by León and Pérez (2013b), PageRank's introduction of a stochastic adjustment that randomly allows (i.e. creates) connections between nodes may be undesirable since for some graphs such randomness is implausible; this is the case with financial market infrastructures' networks or with tiered payment systems.

Despite some of the eigencentrality measures were originally designed for non-weighted graphs (e.g. PageRank, HITS), there are no formal restrictions to applying them for weighted graphs. As in Bonacich (1972), non-weighted graphs correspond to a particular case (i.e. a binary or Boolean case), and more general cases may be safely evaluated; moreover, as stressed by Casti (1979), the relative strength of the interactions among system's elements is key for the assessment of static complexity.

2.4. Identifying hierarchies

Simon (1962) suggests a narrow definition of "hierarchical system" o "hierarchy": *a system that is composed of interrelated subsystems, each of the latter being, in turn, hierarchic in structure until we reach some lowest level of elementary subsystem*. Correspondingly, Casti (1979) points out that the number of hierarchical levels in a given system represents a rough measure of its complexity.

Some authors link the hierarchical structure of networks to the existence of communities or modules. For instance, Newman (2003) defines that a network displays community structures when groups of vertices have a high density of edges within them, with a lower density of edges between groups. Likewise, Barabási (2003) describes modularity in real-world networks as an architecture where the more connected a vertex is, the smaller its clustering coefficient, with such low clustering from central vertices contradicting the standard scale-free model.

Hence, in order to quantitatively measure the hierarchical modularity of a network Barabási (2003) suggests assessing whether (or not) the most connected vertices display low local (i.e. single vertex) clustering, as the real-world observed hierarchical modularity suggests. Newman (2010) defines local clustering as in [§17]:

$$c_i = \frac{(\text{number of pairs of neighbors of } i \text{ that are connected})}{(\text{number of pairs of neighbors of } i)} \qquad [§17]$$

Local clustering coefficient (c_i)

If there is no dependence between degree and clustering (i.e. clustering is democratically distributed), then the network has no hierarchical modularity, as expected from both standard random and scale-free networks. However, if degree and clustering display an inverse relation (i.e. the higher the degree, the smaller the clustering coefficient), there is evidence of hierarchical modularity, where central vertices tend to connect to vertices in their module and to other central vertices in other modules.

Barabási (2003) and Dorogovtsev et al. (2002) suggest that hierarchical modularity may be captured by fitting a power-law to the distribution of local clustering as a function of average degree (μ_{k}), as in [§18]:

$$\mathcal{P}_{c_i} \propto z\mu_{k}{}^{-\gamma} \qquad [§18]$$

Local clustering distribution as a power-law

Barabási (2003) highlights that the existence of hierarchical modularity in real-world networks is a defining feature of most complex systems, but it is not caused and may not be explained by the mere presence of scale-free properties. Consequently, because the standard scale-free model presumes the existence of a few central vertices connected to nodes in numerous modules (i.e. against the evidence of modularity in real-world networks), Barabási (2003) introduces a new type of network: a modular scale-free network.

Therefore, based on the standard (i.e. Poisson, small-world, scale-free) and the modular scale-free models of networks, Table 1 presents a summary of the statistical properties of networks, which allows for identifying the type of network under analysis. Following Barabási (2003) and Dorogovtsev et al. (2002), the clustering coefficient of a network and its local distribution by degree may determine the type of hierarchy of the system.

Table 1
Main statistical properties of networks

<table>
<tr><th colspan="2">Network model</th><th>Degree distribution ($\mathcal{P}_{k}$)</th><th>Mean geodesic distance (ℓ)</th><th>Clustering coefficient (c)</th><th>Degree/strength correlation (r)</th></tr>
<tr><td rowspan="2">Homogeneous</td><td>Random (e.g. Poisson)</td><td rowspan="2">Homogeneous, non-skewed, exponentially decaying distributions</td><td rowspan="2">Small, with
$\ell \sim \ln n$</td><td>Low, with
$c \sim \frac{\mu_{k}}{(n-1)} \ll 1$</td><td rowspan="2">Non-significant ($r \sim 0$)</td></tr>
<tr><td>Small-world</td><td rowspan="2">$c \gg \frac{\mu_{k}}{(n-1)}$</td></tr>
<tr><td rowspan="2">Inhomogeneous</td><td>Scale-free</td><td rowspan="2">Inhomogeneous, skewed, with distributions decaying as $\mathcal{P}_{k} \propto z k^{-\gamma}$</td><td rowspan="2">Ultra small, with
$\ln \ln n \leq \ell \leq \ln n$</td><td rowspan="2">Significant ($r \neq 0$)</td></tr>
<tr><td>Modular scale-free</td><td>$c \gg \frac{\mu_{k}}{(n-1)}$, but
distributed as a power-law, where
$\mathcal{P}_{c_i} \propto z k^{-\gamma}$</td></tr>
</table>

Source: authors' design.

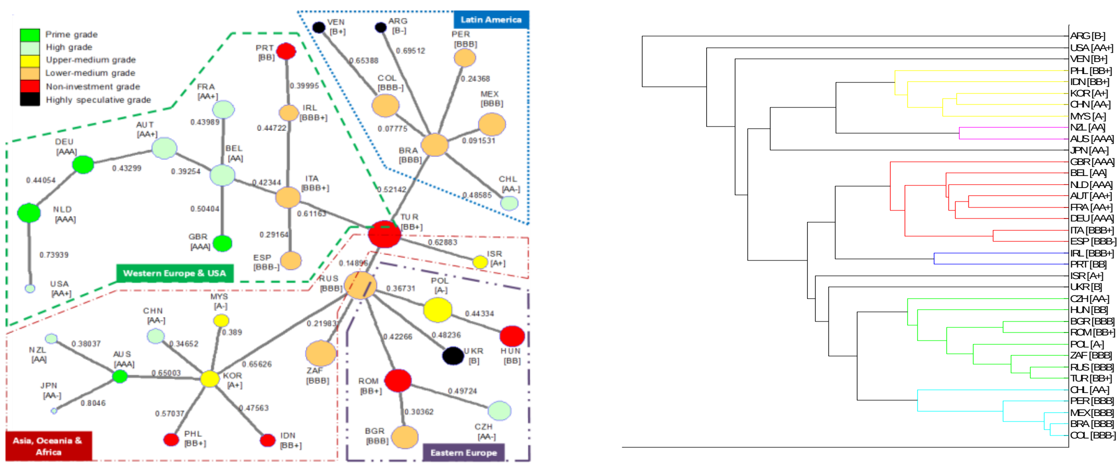
Alternatively, *graph partitioning* is a useful tool for finding subsets of vertices that demonstrate "cohesiveness" with respect to the underlying relational patterns, with two well-established methods: *hierarchical clustering* and *spectral partitioning* (Kolaczyk, 2009). The first method produces a hierarchical representation in which the clusters at each level of the hierarchy are created by merging clusters at the next lower level (Hastie et al., 2009), whereas the second relies on spectral graph theory that associates connectivity with the eigen-analysis of certain matrices (Kolaczyk, 2009).

Hierarchical clustering, also known as cluster analysis, is the traditional method for extracting community or hierarchical structure from a network (Newman, 2003). Hierarchical clustering is a method used in data analysis (i.e. data mining), pertaining to a category commonly referred as unsupervised learning. This method uses similarities of instances to find groups such that instances in a group are more similar to each other than instances in different groups (Alpaydin, 2009), and they usually yield a tree-like graph diagram that represents the hierarchical relations among vertices. Several tree-like graphs are available, such as *spanning trees* and *dendrograms*.

The construction of a tree is based on a measure of cohesiveness, which is a distinctive feature of each system. Simon (1962) suggests defining hierarchies in terms of the intensity of interaction between its elements, which allows for reconciling different types of networks, namely physical, biological and social. Accordingly, based on the measure of cohesiveness, hierarchical clustering optimizes (i.e. maximizes or minimizes) the intensity of interaction for each element. For instance, for a network of payments it is convenient to construct the tree based on the maximization of the system's weights (i.e. a *maximal spanning tree*). In this sense the resulting *spanning tree* may be considered as the "skeleton" of the network (Braunstein et al., 2007; Wu et al., 2006) or the "communication kernel" (Kim et al., 2005), whereas the dendrogram summarizes the process of clustering by displaying similar records joined by lines whose length reflects the distance between the records (Shmueli et al., 2010).

Hierarchical clustering has received a lot of attention lately in economics. One of the most prolific fields has been analyzing the taxonomy of financial markets by means of employing hierarchical clustering on correlation matrices, as suggested by the early work of Mantegna (1999 & 1998).[22] For example, León et al. (2013) describes and analyzes the hierarchical structure behind the sovereigns' CDS market by means of constructing the *minimal spanning tree* that results from transforming CDS correlation matrix into an adjacency matrix, which may be accompanied by the corresponding dendrogram (panel a. and b., Figure 2).

Figure 2
Hierarchical clustering of sovereign CDS correlation matrix
(November 24, 2009 – February 22, 2012)*



a. Minimal spanning tree

b. Dendrogram

(*) The diameter of the vertices in a. corresponds to the eigenvector centrality of the sovereign; each sovereign is reported along with its long-term S&P credit rating as of April 4, 2013.
Source: León et al. (2013) and authors' calculations.

The hierarchical clustering of the sovereigns' CDS market enabled León et al. (2013) to identify the existence of subsystems driven by geographical location and credit rating grade. Moreover, this method allowed for pinpointing the most influential sovereigns in the system, and for detecting the main transmission channels between sovereigns.

[22] Research works on hierarchical clustering for analyzing the taxonomy of financial markets comprise several types of assets, such as stocks (Eryigit and Eryigit, 2009; Bonanno et al., 2004 & 2003; Onnela et al., 2003; Kullmann et al., 2002; Mantegna and Stanley, 2000; Mantegna, 1998 & 1999), fixed income securities (Gilmore et al., 2010), credit default swaps (León et al., 2013), interest rates (Aste and Di Matteo, 2005), currencies (Naylor et al., 2007; Mizuno et al., 2006), commodities (Gilmore et al., 2012).

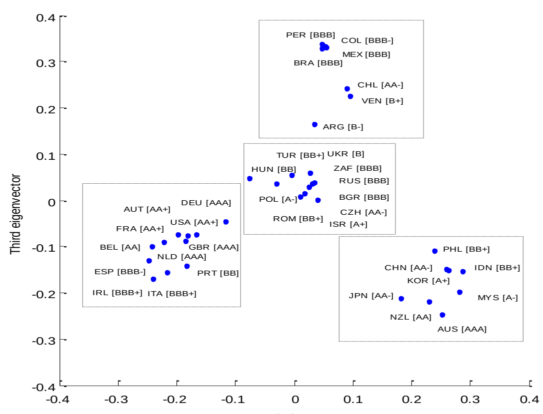
Some other works based on hierarchical clustering have also tried to identify central and peripheral issuers (Marsh et al., 2003) and to overcome the empirical problem of noise in –historical- correlation matrices (Naylor et al., 2007; Bonnano et al., 2003). All in all, most of the authors stress the usefulness of hierarchical clustering for characterizing financial markets by means of identifying their underlying structure, taxonomy or hierarchy (León et al., 2013).

Regarding spectral partitioning, this method relies on spectral graph theory that associates connectivity with the eigen-analysis of certain matrices, such as the adjacency matrix (Kolaczyk, 2009). In that case, (i) the n-eigenvalues of the adjacency matrix A (i.e. the spectrum of the graph) are ordered from the largest to the smallest (i.e. $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$) in absolute terms; (ii) starting with the largest eigenvalues (i.e. the most informative for the system)[23], the entries of the related eigenvectors are sorted; (iii) the vertices corresponding to particularly large positive or negative entries, in conjunction with their immediate neighbors, are declared to be a cluster[24].

Regarding the final step of spectral partitioning, it is worth recalling that the eigenvector corresponding to the principal (i.e. largest) eigenvalue is the *eigenvector centrality*, a global measure of importance (e.g. centrality popularity, accessibility) within a network. As stated by Straffin (1980), since the principal eigenvector has all positive components, all other –orthogonal- eigenvectors have positive and negative components, where such sign and level partition might pick out significant clusters or subsystems of the graph.

For instance, based on the sovereigns' CDS market series used by León et al. (2013), the spectral partitioning confirms the existence of subsystems driven by geographical location (Figure 3).

Figure 3
Spectral partitioning of sovereign CDS correlation matrix
(November 24, 2009 – February 22, 2012)



Source: authors' calculations based on León et al. (2013).

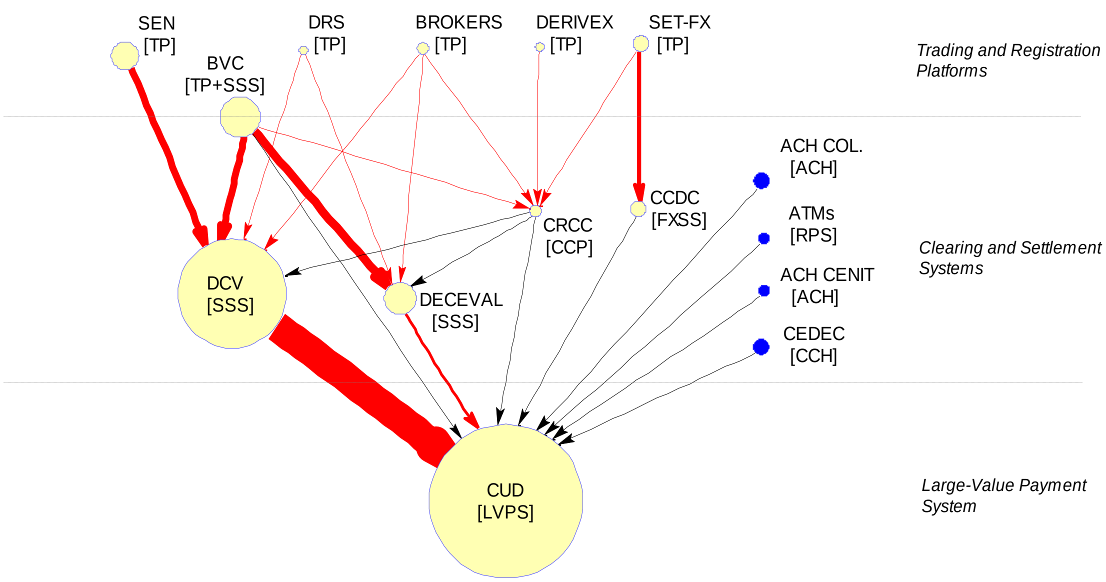
[23] In practice attention is usually restricted to just the q-th largest eigenvalues, where $q \sim \ln n$ (Kolaczyk, 2009).
[24] For instance, Gould (1967) and Straffin (1980) implement this type of spectral analysis to geographical data. The analysis of successive eigenvectors enabled them to break road networks into successive strong nodal regions, and to classify each city in these regions.

In this sense, as highlighted by Albert and Barabási (2002), the interest in spectral properties is related to the fact that spectral density can be directly linked to the graphs topological features.

3. Colombian payment and settlement networks

Each transaction between financial institutions has to complete a sequence of processes, namely (i) the exchange of buy-sell orders; (ii) orders' match and registry; (iii) the calculation of the parties mutual obligations (i.e. clearance); (iv) the transfer of monetary claims by the payer to the payee (i.e. payment); (v) the transfer of securities or financial instruments (i.e. delivery); and, finally, (vi) discharging the obligations between the parties as a consequence of the payment and delivery (i.e. settlement).[25] In this sense, Colombian financial market infrastructures may be classified according to the processes they fulfill: trading and registration platforms, clearing and settlement systems, and large-value payment systems, as presented in Figure 4.

Figure 4
Colombian Financial Market Infrastructures



[a] Vertices' diameter and edges' thickness correspond to the monetary value of transactions. [b] Edges representing net (gross) flows are in black (red). [c] Vertices in blue pertain to the retail payment system.
Source: authors' design.

There are six trading and registration platforms (TPs). Regarding securities' TPs, the Central Bank (Banco de la República - BR) owns and operates SEN (Sistema Electrónico de Negociación), the main sovereign securities' TP. Sovereign securities may also be traded in the

[25] These definitions concur with the standard terms used in payment and settlement systems, as reported by CPSS (2003).

Colombian Stock Exchange (Bolsa de Valores de Colombia - BVC) trading platform (i.e. MEC, Mercado Electrónico Colombiano), which also provides the trading and registration platform for other types of fixed income securities such as corporate, municipal and commercial papers, and for equity and financial futures.

Deceval Registration (DSR) provides registration services for fixed income securities; it is owned and operated by Deceval securities settlement system (SSS). Derivex provides TP services for the energy futures market only. Local branches (subsidiaries) of international inter-dealer brokerage firms (Brokers) allow transactions between participants through hybrid (i.e. voice and data) systems. Regarding Peso/Dollar trading and registration platforms, SET-FX and Brokers provide TP services for foreign exchange market participants.

Regarding clearing and settlement systems, BR owns and operates DCV (Depósito Central de Valores), a FMI that is both the securities settlement system (SSS) and the central securities depository (CSD) for sovereign securities exclusively. DCV and privately owned Deceval (Depósito Centralizado de Valores de Colombia) work under a Real-Time Gross Settlement System (RTGS) and a Delivery-versus-Payment (DvP) mechanism. Deceval provides CSD and SSS services for corporate and non-sovereign public securities, along with CSD services for the equity market. Central counterparty (CCP) services for futures markets are provided by CRCC (Cámara de Riesgo Central de Contraparte de Colombia). BVC provides both TP and SSS services for local equity markets. About foreign exchange, CCDC (Cámara de Compensación de Divisas de Colombia) provides clearing and settlement for the Peso/Dollar spot market, whereas the CRCC offers clearing and settlement services for Peso/Dollar non-delivery forwards.

Four IMFs (vertices in blue) are in charge of the clearing and settlement of retail payments.[26] The Central Bank (BR) owns and operates both CENIT Automated Clearing House (ACH) and Cheques Clearing House (CCH), whereas commercial banks own ACH-Colombia. ATM provides clearing and settlement for transactions made through debit cards and credit cards, via point-of- sale and automated teller machines.

The only large-value payment system (LVPS), where all cash leg's settlement (in local currency) takes place, is owned and operated by Colombia's Central Bank (BR). This IMF is known as CUD (Cuentas de Depósito), and works under a Real-Time Gross Settlement System (RTGS) framework. Unlike many LVPS around the world (e.g. CHAPS in the United Kingdom), the Colombian LVPS works under a non-tiered framework in which all types of financial institutions (i.e. banking and non-banking) are eligible for an account at BR that allows for settling payments directly to other participants of the LVPS; furthermore, central bank's ordinary liquidity facilities (e.g. repos) are not restricted to banking institutions either.

In order to analyze and understand the structure of Colombian financial system three financial market infrastructures were selected as sources of transactions: the large-value payment system (CUD), the sovereign securities settlement system (DCV) and the currency settlement system (CCDC). The rationale behind this selection follows five facts: first, these

[26] Retail payments are those not included in the definition of large-value payments, mainly consumer payments of relatively low value and urgency (CPSS, 2003).

three financial market infrastructures account for 88.4% of the value of the payments and deliveries within the local financial market infrastructure during 2012 (Banco de la República, 2013); second, based on León and Pérez (2013), they are the three most systemically important local financial market infrastructures; third, these three infrastructures provide consolidated and standardized data for most of the existing trading and registering platforms; fourth, the sovereign securities settlement system (DCV) and the foreign exchange settlement system (CCDC) provide detailed data for the two largest local financial markets (i.e. local sovereign securities and foreign exchange); and, fifth, the large-value payment system (CUD) provides aggregated data for all financial transactions occurring in the local market (i.e. from all financial market infrastructures).[27] Therefore, this selection may be considered comprehensive and representative, yet parsimonious.

Consequently, three financial transactions networks will be analyzed: large-value payment, sovereign securities settlement and the foreign exchange settlement systems. The three corresponding datasets consist of daily transactions for year 2012, with each transaction containing the time (date, hour, minute, etc.), sender, receiver and amount. For the large-value payment system the original dataset (i.e. in edge list format) consists of 450.124 transactions during year 2012, whereas for the sovereign securities settlement system (DCV) and the foreign exchange settlement system (CCDC) datasets consist of 169.398 and 115.733 registries, respectively.[28]

Transforming the registries datasets from edge lists to adjacency matrices resulted in three datacubes (i.e. hypermatrices). Each datacube has dimensions $n \times n \times t$, where the first two dimensions correspond to the traditional adjacency matrix of size $n \times n$, and the third dimension corresponds to the number of observations from January 3rd to December 28th 2012 ($t = 236$). Thus, for the CUD, DCV and CCDC, the datacubes have dimensions $144 \times 144 \times 236$, $116 \times 116 \times 236$, $46 \times 46 \times 236$, respectively; differences in the first two dimensions results from not all financial institutions participating in all networks, whereas the coincidence in the third dimension results from choosing the dates in which the three networks concurrently operated.[29]

[27] Data from the three selected financial market infrastructures does not capture the equity, corporate securities markets or retail-value payments directly. Besides not being representative (i.e. less than 12% of all payments and settlements), since all the cash settlement (i.e. payments) of the equity and corporate markets, and retail-value payments is included in the large-value payment system (CUD) data, the loss of detail is by no means critical.

[28] DCV data was filtered out from the CUD database. This is feasible and does not entail any loss of detail for the purpose of this document since CUD works under a real-time gross settlement framework and DCV under delivery-versus-payment, where all related transactions are settled on an individual basis (i.e. one-by-one). Therefore, only transactions that do not result in an exchange of money (e.g. exchanging securities between accounts from a single financial institution or a single institution acting as a broker for different participants) are discarded, about 5.08% and 9.04% of the value and number of transactions, respectively. Moreover, the connective pattern and hierarchies of networks are not affected by such type of self-connecting transactions; only when considering the end-buyer/seller of each transaction this filtering would affect the analysis of the network.

[29] Adjusting the datasets in order to work with the same number of observations ($t = 236$) is convenient for comparative purposes; since the number of observations (i.e. dates) of the original databases is 244, 244 and 236, respectively, the loss of information (i.e. eight non-consecutive days out of 244) due to this adjustment is trivial. Non-financial institutions (e.g. Ministry of Finance, IMFs) and the Central Bank were discarded from the datasets due to their special nature.

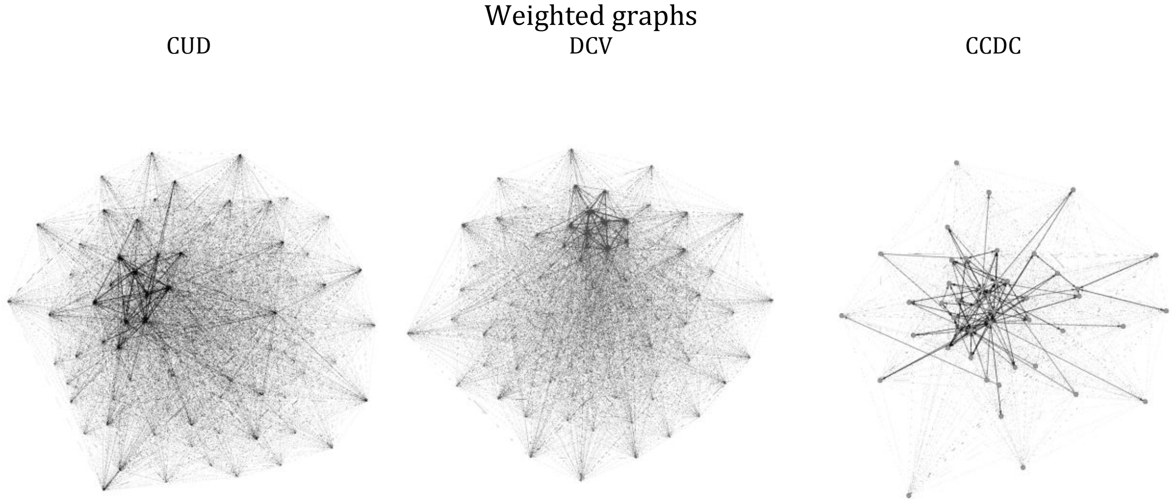
4. Network analysis on Colombian selected payment and settlement systems

The rich scientific literature on networks and graph theory may have some bearing on the management of economic and financial system risk (Kambhu et al., 2007). Therefore, based on the network analysis metrics previously described, this section aims to classify the three selected datasets (i.e. CUD, DCV, CCDC) according to their connective patterns and hierarchical structure. Centrality measures for the three networks will be presented in order to further understand the connective pattern of the networks.

4.1. Identifying connective patterns

Figure 5 presents the graphs corresponding to the selected networks. As expected, due to the dimensionality of each system (i.e. the large number of vertices and edges), visual inspection and analysis of the graphs is rather difficult.

Figure 5
Weighted graphs



Source: authors' calculations.

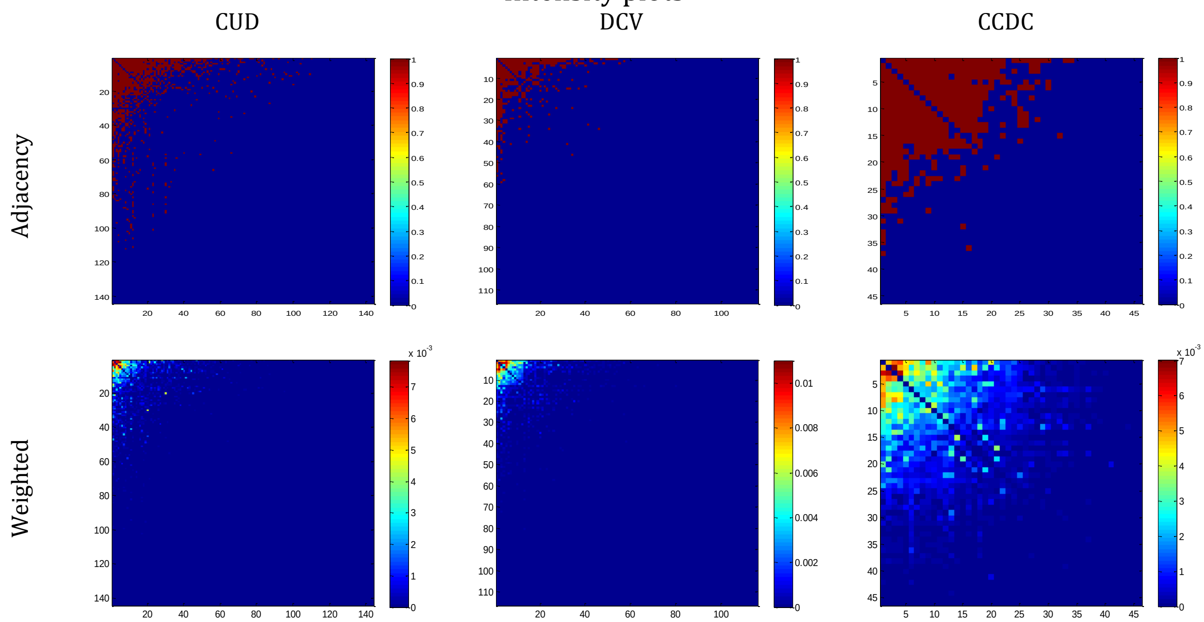
A simpler and more tractable alternative to a graph is an *intensity plot*, a method for displaying three-dimensional data on a two-dimensional plot by using a normalized color scale (i.e. from the lowest to the highest value) to display the values of the third dimension. Figure 6 presents the intensity plots corresponding to the adjacency matrices (first row) and weighted matrices (second row) for each network.

Adjacency matrices result from the mode of the (236) observed networks, whereas weighted matrices correspond to the arithmetic sum of the 236 observed networks;[30] in order to

[30] Using the mode for the adjacency matrix is convenient since aggregating edges across time results in artificially dense networks; this is, since adjacency matrices are binary, the mere existence of a single transaction during the analyzed period would result in a disproportionate bias towards admitting that such edge exists on a regular basis. On the other hand, aggregating weighted matrices is sound since adding monetary values preserves the true intensity of the network.

facilitate visual inspection and analysis, the order of the participating financial institutions in the axis obeys their strength (i.e. high-strength vertices appear in the upper-left corner). Each (i, j) element in the adjacency matrix corresponds to the existence of a local currency payment from i to j on a regular basis, whereas each (i, j) element in the weighted matrix represents the contribution of all i to j payments to all system's payments along the period under analysis.

Figure 6
Intensity plots



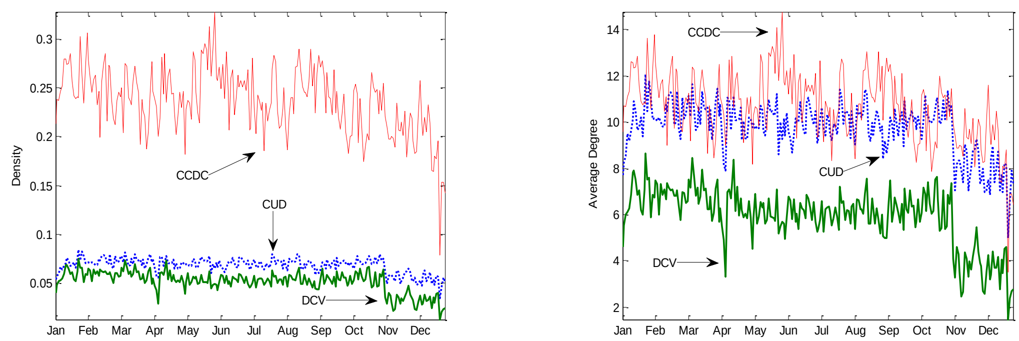
Source: authors' calculations.

All six intensity plots share a common feature: connections and their corresponding intensities are not homogeneous across participants, but heavily and almost symmetrically concentrated in the upper-left corner of the plots, whereas most of the plots are empty. Such concentration provides a preliminary –yet illuminating- indication of the connectedness structure of the networks: they appear to be (i) sparse; (ii) inhomogeneous; and (iii) clustered. Moreover, loosely following the block analysis suggested by Craig and von Peter (2010), there may be evidence of tiered structures, where participants operate in a hierarchical manner in which lower-tier (i.e. peripheral) financial institutions deal with each other through high-tier (i.e. core) institutions; this is an interesting finding since the three systems under analysis are formally non-tiered, where all participating institutions may directly connect to each other.

Regarding the sparseness of the networks, the estimation of density (d) and average degree (μ_{k}) confirms the preceding visual inspection. As is evident in the left panel of Figure 7, CUD and DCV networks are particularly sparse, with densities below 0.10 (i.e. less than 10% of the potential links are observed), whereas CCDC network is sparse but with densities usually in the (0.15, 0.30) range. Likewise, the average degree of each network is much smaller than the number of participants $(\mu_{k} \ll n)$, which verifies the sparse nature of the networks and the particularly high sparseness of CUD and DCV; for instance, CUD's average degree during 2012 was always below 12, much lower than the size of the network ($n_{CUD} = 144$).

Figure 7
Density and average degree

Density | Average degree



Sample mean: CUD 0.07; DCV 0.05; CCDC 0.24 | *Sample mean: CUD 9.75; DCV 5.93; CCDC 10.66*

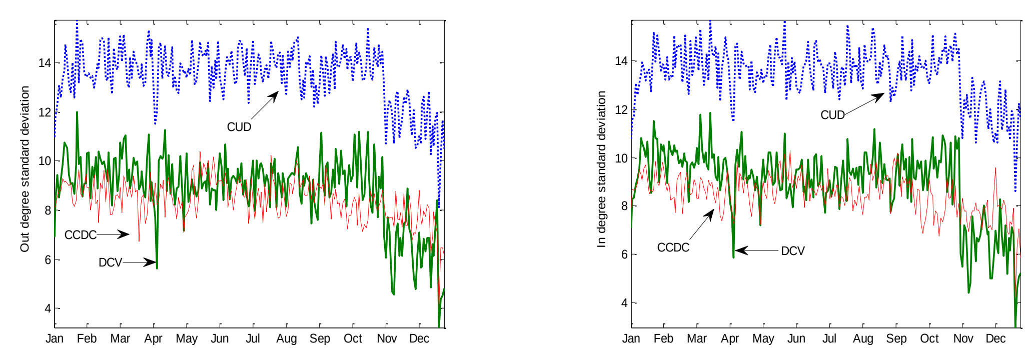
Source: authors' calculations

It is worth noting a sharp drop in the density and average degree levels starting from the first days of November, most evident for CUD and DCV networks. This drop concurs with the failure of a broker-dealer institution (i.e. Interbolsa) on November 2, an institution that had been implicitly or explicitly considered systemically important due to its connectedness.[31]

The second moment of the distribution of the degree, measured by the in and out degree standard deviation, is presented in Figure 8. It is noticeable that the dispersion around the mean is rather high, with the sample mean standard deviation dominating the sample mean average degree for CUD and DCV.

Figure 8
Standard deviation of in and out degree

Out degree | In degree



Sample mean: CUD 13.45; DCV 8.84; CCDC 8.43 | *Sample mean: CUD 13.45; DCV 8.96; CCDC 8.47*

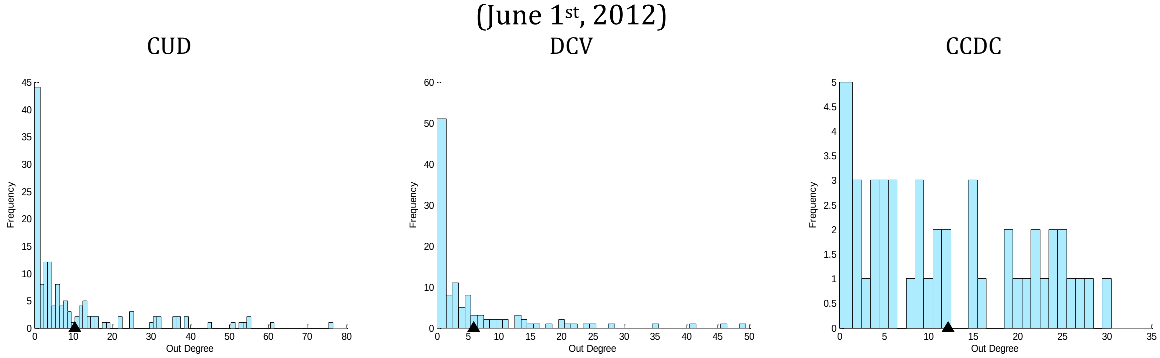
Source: authors' calculations.

[31] Despite the appeal of analyzing the failure of an institution considered systemically important (e.g. IMF, 2013; IMF, 2013b; León and Pérez, 2013; León and Machado, 2013; León and Murcia, 2012; Saade, 2010; Cepeda, 2008), such issue is outside the scope of this document. Notwithstanding the magnitude of the effects of this failure in the other metrics (below), no particular analysis will be provided.

Since the degree is limited to positive numbers, such high dispersion around the mean suggests the presence of skewness and kurtosis. Estimating the third and fourth moments of the degree distribution confirms such suggestion: the sample mean of out (in) degree skewness is 2.05 (1.92), 2.34 (2.38) and 0.46 (0.49) for CUD, DCV and CCDC, respectively, whereas the sample mean of out (in) degree kurtosis is 6.43 (13.45), 9.48 (8.84) and 2.23 (8.43), correspondingly.[32] This concurs with most real-world networks displaying right-skewed distributions.

The histogram of the degree distribution is the customary graphical test for the presence of right-skewed (i.e. heterogeneous) connective patterns. Figure 9 presents three out degree histograms for a single day (i.e. June 1st, 2012). As expected, the distributions are right-skewed, where the average degree (black triangle) does not characterize the distribution of edges among the vertices, especially for the CUD and DCV networks.

Figure 9
Out degree distribution
(June 1st, 2012)



Source: authors' calculations.

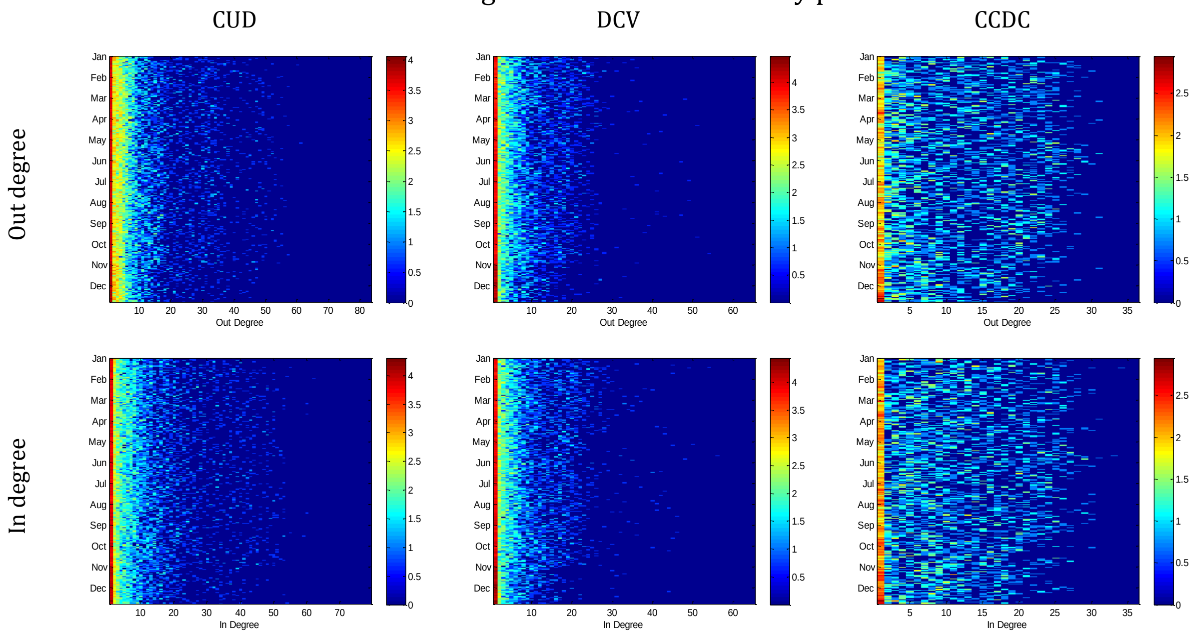
In order to display the in and out degree distribution for the entire sample, Figure 10 presents six intensity plots. In all cases the horizontal axis corresponds to in or out degree, the vertical axis corresponds to the t daily-observations analyzed, and the intensity corresponds to a logarithmic transformation of the frequency; such transformation enhances the visualization of differences across degree levels and observations.

Concurrent with the single-day histograms in Figure 9, all systems consistently display right-skewed in and out degree distributions. Therefore, concurrent with most real-world networks, the majority of vertices are of very low degree and few vertices are of very high degree; hence, the connectedness pattern of the three networks may be characterized as inhomogeneous.

Likewise, Figure 11 confirms that the strength distribution is right-skewed; in this case the horizontal axis corresponds to each participant's contribution to total payments for each observed day, where the intensity still corresponds to a logarithmic transformation of the frequency. This indicates that not only the connections but also the value of the payments are inhomogeneous in nature.

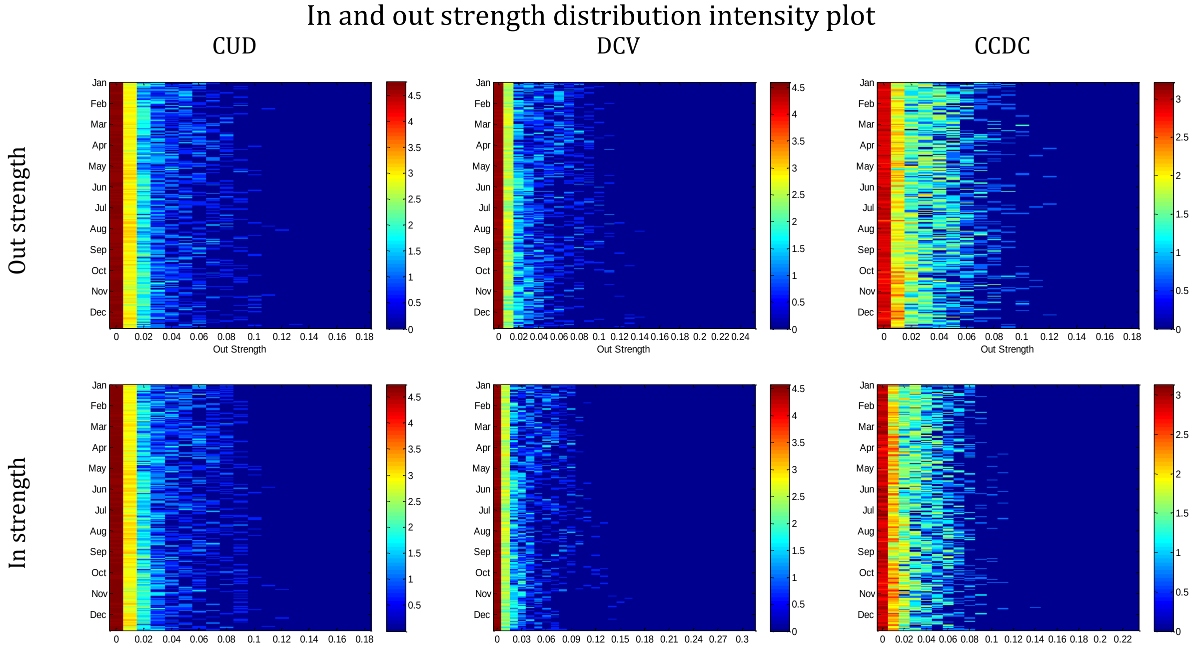
[32] Kolmogorov-Smirnov normality tests were rejected at traditional significance levels.

Figure 10
In and out degree distribution intensity plot



Source: authors' calculations.

Figure 11
In and out strength distribution intensity plot

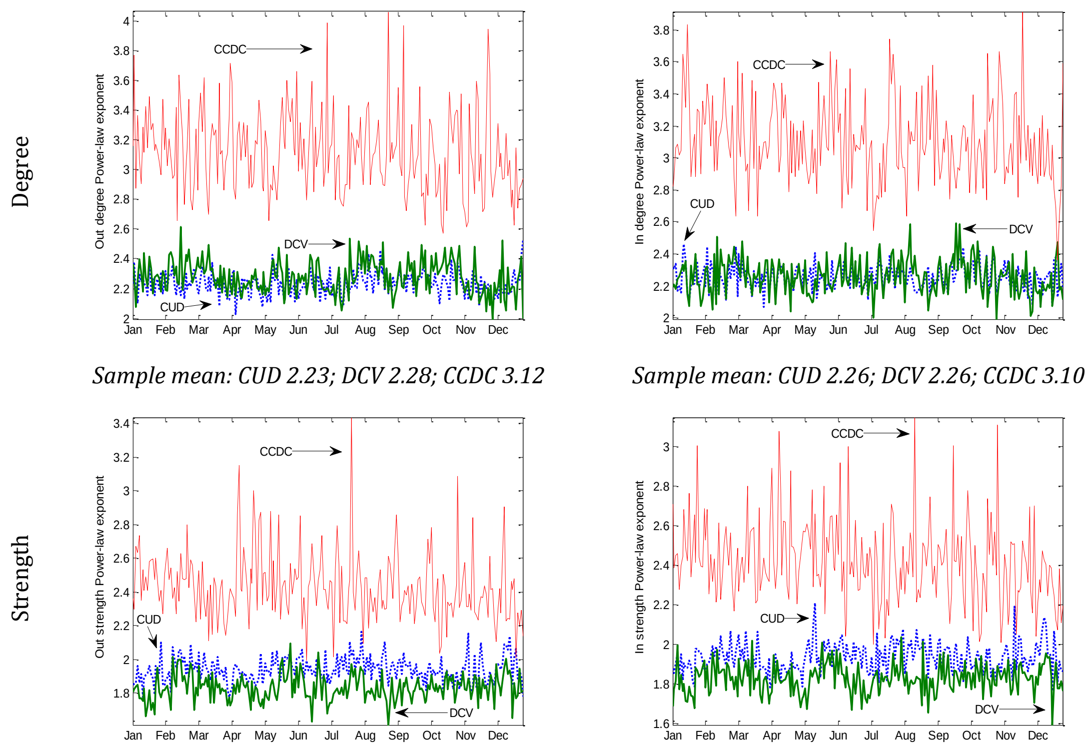


Source: authors' calculations

As usual, agreeing with Barabási and Albert (1999) seminal findings, the right skew in the distribution of degree and strength approximates to a power-law distribution. Figure 12 exhibits the estimated exponent of the power-law for the degree (γ_{k}) and strength (γ_{s}), for each system under analysis, on a daily frequency.[33]

Figure 12
Degree and strength power-law exponent (γ)

Out In



Sample mean: CUD 2.23; DCV 2.28; CCDC 3.12 *Sample mean: CUD 2.26; DCV 2.26; CCDC 3.10*

Sample mean: CUD 1.94; DCV 1.83; CCDC 2.43 *Sample mean: CUD 1.93; DCV 1.83; CCDC 2.42*

Source: authors' calculations

Estimated exponents for the three systems agree with typical values for real-world networks (i.e. $2 \leq \gamma \leq 3$).[34] However, it is evident that CUD and DCV exponents share a common (lower) level, whereas CCDC displays a higher exponent level; such difference suggests that connectedness in the CCDC is less heterogeneous, as manifested in the preceding intensity

[33] The simplest method for estimating the exponent of the power-law (γ) consists of an ordinary least squares (OLS) regression on a logarithmic transformation of [§8]: $\ln(p_k) = \ln(C) - \gamma ln(k)$. However, as stressed by Clauset et al. (2009), OLS fitting may be inaccurate due to large fluctuations in the most relevant part of the distribution (i.e. the tail). Therefore, all estimations of γ employed the maximum-likelihood algorithm developed by Clauset et al. (2009).

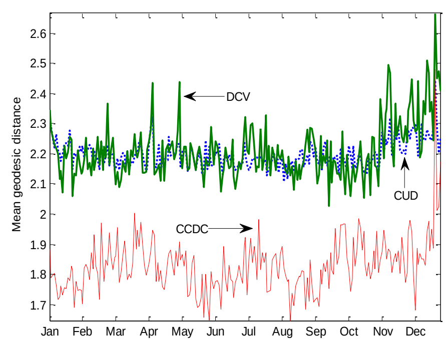
[34] Values in the range $2 \leq \gamma \leq 3$ are typical, although values slightly outside it are possible and are observed occasionally (Newman, 2010). For instance, as reported by Albert and Barabási (2002), different authors converge to a 2.1 exponent for the in degree distribution of the World Wide Web, long-distance telephone calls, internet domains, and of neuroscientists' co-authorship networks. Values close to 2.5 have been reported for networks consisting of mathematicians' co-authors, Internet routers, and the out degree of the World Wide Web. Networks of sexual contacts have been reported to display values above 3, whilst food webs have been reported to display values close to 1.

plots (i.e. figures 6, 9, 10, 11). It is also evident that strength's power-law exponent tends to be lower than degree's; due to the functional form of the power-law distribution, this suggests that the distribution of the payments is more right-skewed (i.e. more heterogeneous) than the distribution of edges.

Based on the graphical and numerical evidence previously reported, it is possible to characterize the networks under analysis as scale-free. Unlike any homogeneous network (e.g. Poisson or small-world), CUD, DCV and CCDC networks lack characteristic vertices, and exhibit structures where most vertices have very few connections and yet a few vertices have many connections.

As formerly stated, other features have been identified as characteristic of real-world networks: low mean geodesic distances, high clustering coefficients, and significant degree correlation. Regarding the first, as presented in Figure 13, the mean geodesic distance is particularly low for the three networks. The CUD, DCV and CCDC networks have sample means about $\ell_{CUD} = 2.20$, $\ell_{DCV} = 2.21$ and $\ell_{CCDC} = 1.83$; this may be interpreted as the average geodesic distance between financial institutions in the three networks being close to 2 edges (i.e. one single institution in-between).

Figure 13
Mean geodesic distance (ℓ)



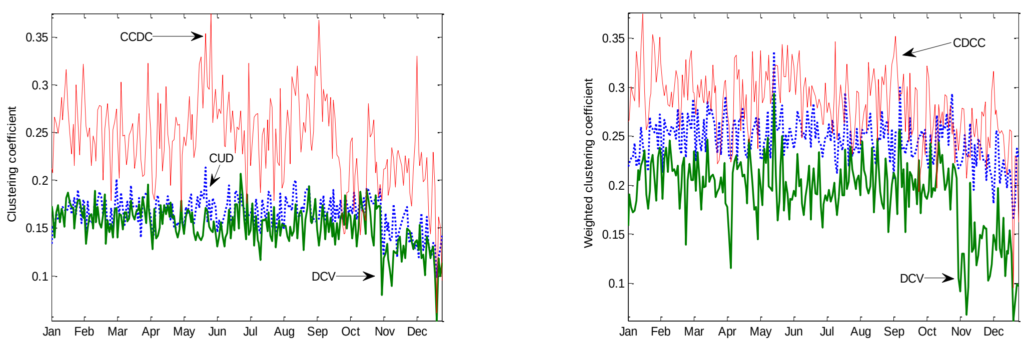
Sample mean: CUD 2.20; DCV 2.21; CCDC 1.83
Source: authors' calculations

The observed mean geodesic distances are much lower than the expected for homogeneous networks of the corresponding size (i.e. $\ln n_{CUD} = 4.97$; $\ln n_{DCV} = 4.75$; $\ln n_{CCDC} = 3.82$). They approximate to the characterization proposed by Cohen and Havlin (2010 & 2003), which states that networks with $2 < \gamma_{k} < 3$ (i.e. CUD and DCV) have a mean geodesic distance that behaves as $\ell \sim \ln\ln n$, whereas networks with $\gamma_{k} = 3$ (i.e. CCDC) yield $\ell \sim \ln n / (\ln\ln n)$. In the first case, the expected mean geodesic distance of CUD and DCV is 1.60 and 1.56, respectively, whereas that of CCDC is 2.85. Therefore, since the mean geodesic distance of the three networks is much lower than the homogeneous case ($\ell \sim \ln n$), and it is closer to those typical of ultra-small networks in the Cohen and Havlin sense, the scale-free characterization is reinforced.[35]

The second additional characteristic of real-world networks is the evidence of clustering. As previously stated, in a random graph the probability of two vertices being connected tends to be the same for all vertices regardless the existence of a common neighbor. Thus, the clustering coefficient of a large random network should be close to zero ($c \sim \mu_{k}/(n-1)$), where the expected clustering coefficient for CUD, DCV and CCDC is about 0.07, 0.05 and 0.24, respectively. As presented in Figure 14, the observed clustering coefficients estimated for CUD and DCV adjacency and weighted matrices are much larger (i.e. more than twice) than those expected for a homogeneous network, and slightly higher in the case of CCDC.

Figure 14
Clustering coefficient (c)

Clustering coefficient — Weighted clustering coefficient



Sample mean: CUD 0.17; DCV 0.15; CCDC 0.24 — *Sample mean: CUD 0.25; DCV 0.19; CCDC 0.28*

Source: authors' calculations

Not only the evidence of clustering reinforces the non-random features of the three networks, but also the sample mean of the weighted clustering coefficient being higher than the non-weighted conveys relevant information about the structure of the networks. According to Barrat et al. (2004), this fact reveals that clusters are more likely formed by edges with larger weights, which further underlines the importance of clusters in the structure of the network; likewise, Leung and Chau (2007) points out that this fact suggests that the topological (i.e. non-weighted) clustering underestimates the cohesiveness of the vertices within their neighborhoods.

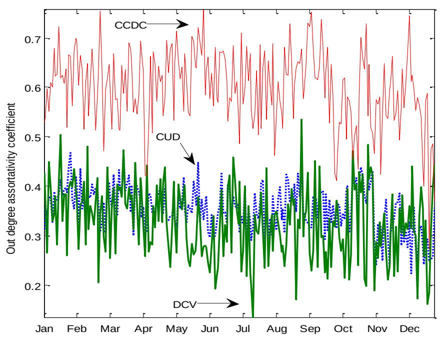
The third additional characteristic of real-world networks is the presence of significant degree correlation (r), as measured by the degree assortativity coefficient in [§11]. Since edges in homogeneous networks are evenly distributed among vertices, where the degree of all vertices does not deviate significantly from the average degree, vertices' degree should display no correlation. Nevertheless, as depicted in Figure 15, degree correlation appears to

[35] The absolute difference of the observed mean geodesic distance for CUD, DCV and CCDC with respect to the mean geodesic distance for a random network is 4.6, 3.9 and 1.95 times the absolute difference with respect to the expected mean geodesic distance according to the "ultra-small" characterization by Cohen and Havlin (2010 & 2003).

be significant for the three networks, where the sample mean out (in) degree correlation for CUD, DCV and CCDC is 0.37 (0.36), 0.31 (0.33) and 0.59 (0.60), respectively.

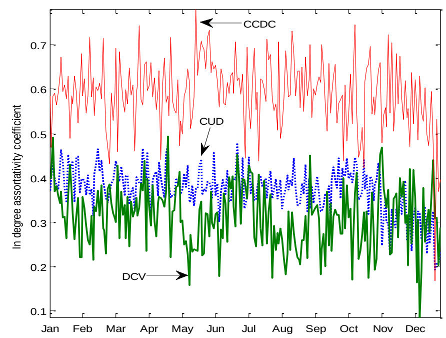
Figure 15
Degree assortativity coefficient (r_k)

Out degree



Sample mean: CUD 0.37; DCV 0.31; CCDC 0.59

In degree



Sample mean: CUD 0.36; DCV 0.36; CCDC 0.60

Source: authors' calculations

The observed positive degree correlation, also known as *assortative mixing by degree*, where high-degree vertices have a larger probability to be connected to other high-degree vertices, is typical of social networks (e.g. co-authoring, film actors), and concurs with the presence of core-periphery structures within a network (Newman, 2010). Consequently, for the three systems under analysis, the level and sign of the degree correlation suggests the existence of a core-periphery structure, as the visualization and interpretation of intensity plots in Figure 6 initially suggested.[36]

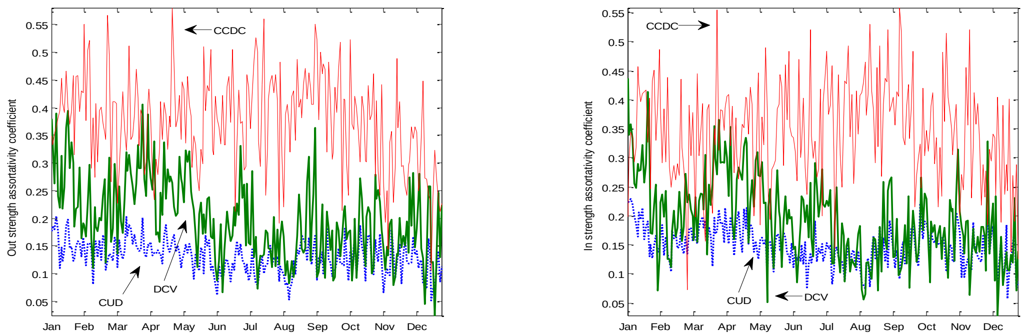
Despite degree correlation is illustrative by itself regarding the topological features of the networks, the intensity of the links among the vertices (i.e. the strength) may reveal additional hierarchical and organizational structures within the systems, verifying (or contradicting) the degree-based correlation analysis. In this sense, based on Newman (2010), Figure 16 displays the strength correlation (i.e. *assortative mixing by strength*).

[36] Evidence of assortative mixing in the selected Colombian payment and settlement networks contradicts the findings of Bech and Atalay (2008) for the Federal Funds Market network and of Soramäki et al. (2006) for Fedwire interbank network. However, those networks being restricted to commercial banks (in Sorämaki et al.) or depositary institutions (in Bech and Atalay) may determine the disassortative nature of those networks; in the selected Colombian networks all types of financial institutions are considered, where such heterogeneity may be leading the results. Alternatively, due to the size of those networks (e.g. Söramaki et al. above 7,000, and Bech and Atalay above 450), results may be biased by the tendency to observe disassortative networks by degree because the number of edges that can fall between high-degree vertices is limited with respect to the total size of the network; correspondingly, Bech and Atalay report that the dissasortative by degree nature of the Federal Funds Market network weakens or vanishes when weights are considered, a result also documented by Leung and Chau (2007) for other networks.

Figure 16

Strength assortativity coefficient (r_s)

Out strength | In strength



Sample mean: CUD 0.13; 0.20; 0.37 | *Sample mean: CUD 0.15; 0.19; 0.34*

Source: authors' calculations

Evidence of significant positive correlation in the three networks confirms the presence of core-periphery structures typical of social networks (Newman, 2010). However, the strength correlation is lower than the degree correlation, which may be interpreted as the degree being more relevant as an explanatory variable than strength for explaining vertices' affinity to connect to others. More importantly, the presence of significant correlation suggests that some *preferential attachment* exists within these networks.

All the statistical properties of the three networks under analysis are presented in Table 2. Expected values for random networks are included –in brackets- when feasible.

Table 2

Basic statistics of the networks*

Statistic	CUD		DCV		CCDC	
n	144		116		46	
d	0.07		0.05		0.24	
μ_k	9.75		5.93		10.66	
$\sigma_{k_{in/out}}$	13.45/13.45		8.96/8.84		8.47/8.43	
$\gamma_{k_{in/out}}$	2.26/2.23		2.26/2.28		3.10/3.12	
$\gamma_{s_{in/out}}$	1.93/1.94		1.83/1.83		2.42/2.43	
ℓ	2.20	[~4.97]	2.21	[~4.75]	1.83	[~3.82]
c	0.17	[~0.00]	0.15	[~0.00]	0.24	[~0.00]
c_w	0.25	[~0.00]	0.19	[~0.00]	0.28	[~0.00]
$r_{k_{in/out}}$	0.36/0.37	[~0.00]	0.33/0.31	[~0.00]	0.60/0.59	[~0.00]
$r_{s_{in/out}}$	0.15/0.13	[~0.00]	0.19/0.20	[~0.00]	0.34/0.37	[~0.00]

(*) Statistics presented are: number of vertices (n); density (d); average degree (μ_k); in/out degree standard deviation ($\sigma_{k_{in/out}}$); in/out degree Power-law exponent ($\gamma_{k_{in/out}}$); in/out strength Power-law exponent ($\gamma_{s_{in/out}}$); mean geodesic distance (ℓ); clustering coefficient (c); degree correlation ($r_{k_{in/out}}$); strength correlation ($r_{s_{in/out}}$). Expected values for random networks are reported in brackets.

Source: authors' calculations.

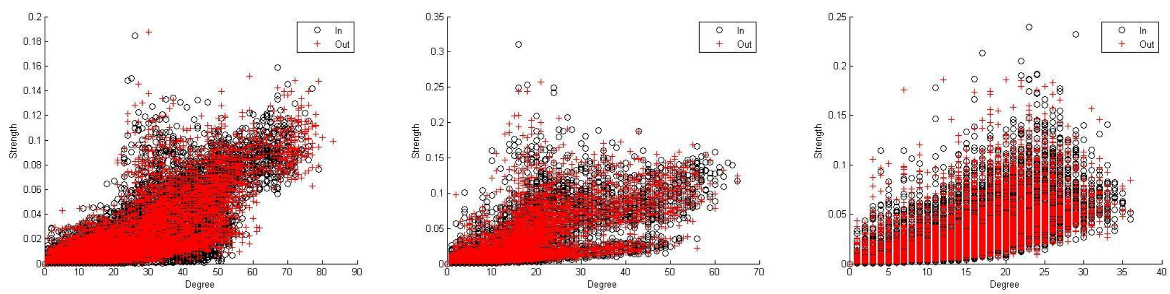
4.2. Assessing centrality

Since all visual and numerical evidence points out to the scale-free nature of the weighted and non-weighted versions of the networks under analysis, assessing the relative importance of those vertices concentrating connections or their related intensities becomes relevant. As previously mentioned, centrality is the most common concept regarding the relative importance of vertices within a network.

The existence of vertices whose degree and strength excel over the rest of vertices is evident in Figure 10 and 11, respectively; those particularly connected and contributing vertices may be identified as central to the system they belong to. Figure 17 displays the relation between degree and strength for the whole sample, where a typical direct relation between both centrality metrics is rather clear; heterogeneity (i.e. skewness) in both metrics is again noticeable.

Figure 17
Degree and strength centrality

CUD DCV CCDC

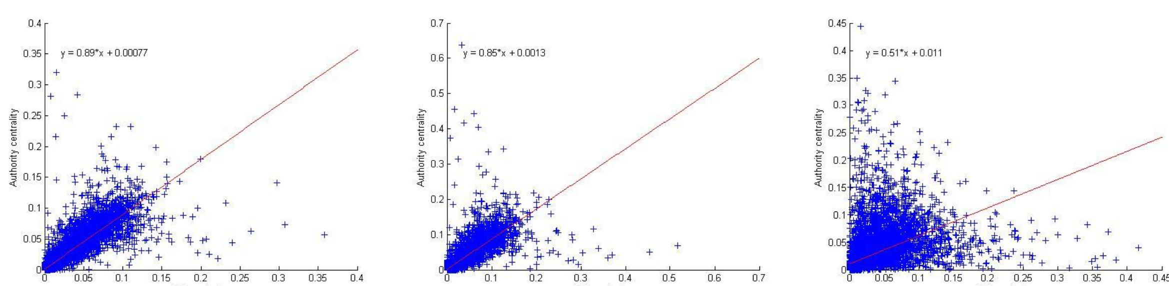


Source: authors' calculations.

Yet, as previously stated, degree and strength are local measures of centrality; these two measures do not take into account the global properties of the network since the centrality of the adjacent vertices is not taken into account as a source of centrality. Figure 18 displays authority and hub centrality for the whole sample, where it is evident the presence of a typical direct relation between both metrics, which reveals that an institution being central usually involves a dual role in the systems: sending and receiving payments.

Figure 18
Authority and hub centrality

CUD DCV CCDC

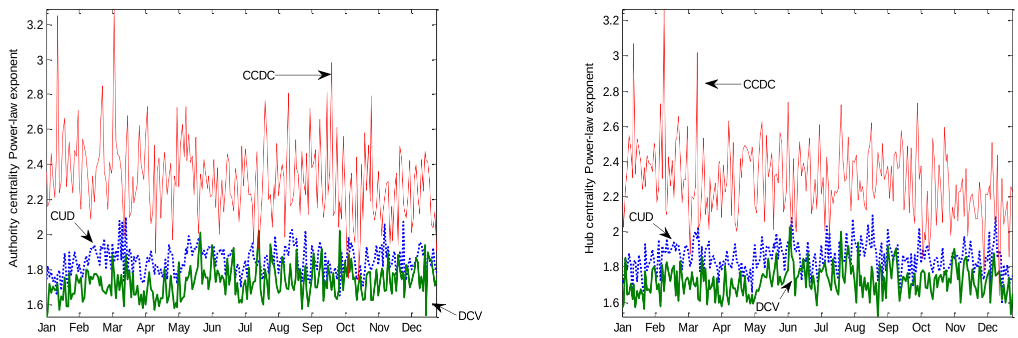


Source: authors' calculations.

Akin to the distribution of degree and strength, authority and hub centrality are concentrated in a few vertices; as suggested by Newman (2010), the distribution of non-degree-related measures, while of lesser importance in the study of networks are nonetheless of some interest. As displayed in Figure 19, authority and hub centrality also follow a power-law distribution, which further emphasizes the skewed (i.e. heterogeneous) nature of the systems under analysis.

Figure 19
Authority and hub centrality power-law exponent (γ)

Authority centrality　　Hub centrality



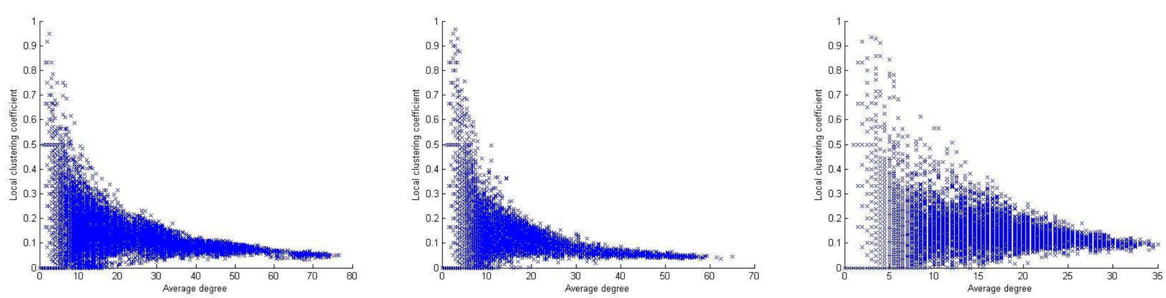
Source: authors' calculations.

4.3. Identifying hierarchies

Following Newman (2010) and Barabási (2003) about the information conveyed in the relation between degree and local clustering for identifying modular hierarchies, Figure 20 exhibits the pair-wise relation between average degree[37] (horizontal axis) and the local clustering coefficient [§17] for the whole sample. It is evident that heavily connected vertices are restricted to low clustering coefficients (i.e. less than 0.10 for CUD and DCV, and less than 0.20 for CCDC), whereas poorly connected vertices may display a broad spectrum of clustering coefficients, including particularly high levels of local clustering (i.e. above 0.30).

Figure 20
Degree and local clustering coefficient

CUD　　DCV　　CCDC

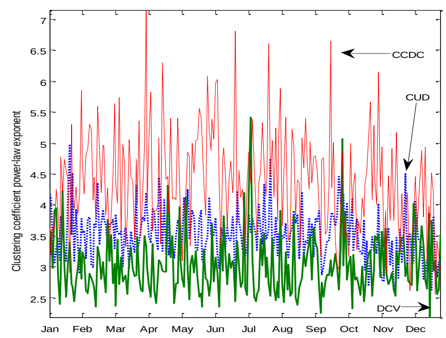


Source: authors´calculations.

[37] This is the simple average of in and out degree.

Correspondingly, the local clustering coefficient as a function of average degree appears to follow a power-law distribution (Figure 21), as suggested by Barabási (2003) and Dorogovtsev et al. (2002) when characterizing modular networks; hence, the low clustering coefficient of central vertices reveals that they are not connected to vertices in numerous modules, as the standard scale-free model suggests, whereas peripheral vertices tend to share neighbors among them. Therefore, the three systems appear to be modular scale-free networks, where such modularity exceeds the framework of standard network models (i.e. Poisson and scale-free).

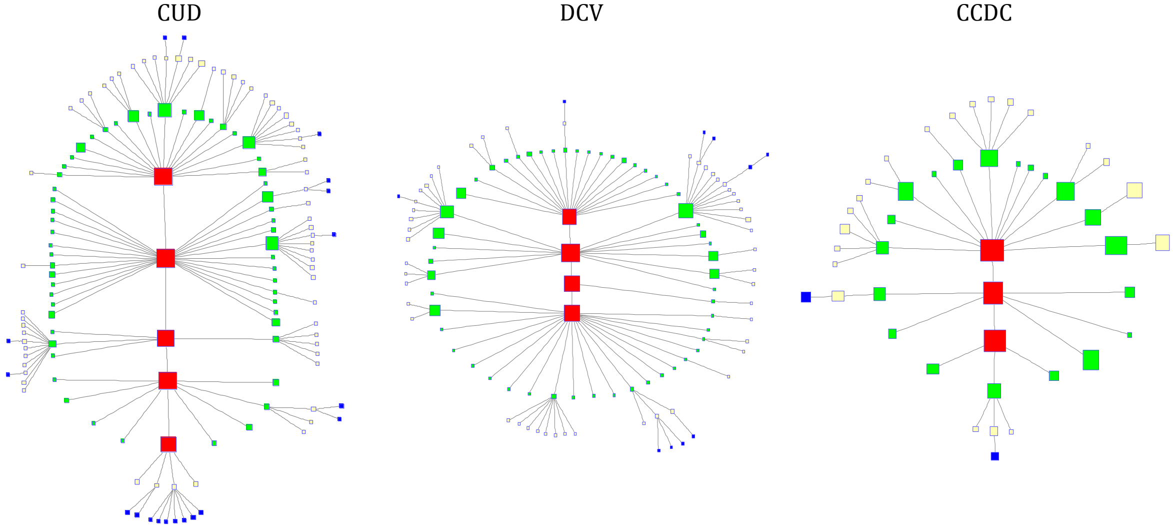
Figure 21
Local clustering power-law exponent (γ)



Sample mean: CUD 3.51; DCV 3.01; CCDC 4.37
Source: authors' calculations

Regarding hierarchical clustering, which typically yields tree-like graphs that represent hierarchical relations among vertices based on a measure of the intensity of their interaction, Figure 22 presents the maximal spanning trees for the weighted aggregated networks under analysis, which results in the "skeleton" or the "communication kernel" of the network (Braunstein et al., 2007; Wu et al., 2006; Kim et al., 2005). In order to assist visual inspection and analysis, the size of each vertex results from its authority and hub centrality [§16], where the former (latter) metric determines the width (height) of the square; also, vertices are assigned different colors according to their remoteness with respect to those vertices that may be identified as pertaining to the core (red vertices) of the maximal spanning tree.

Figure 22
Maximal spanning trees



Source: authors' calculations

The three "skeletons" resulting from the maximal spanning trees have some common features. For instance, a few vertices (in red) are interconnected in the center of the skeleton, generally corresponding to those displaying the largest (i.e. the most hub and authority central) and most connected vertices in the tree; these few central vertices may be considered as the "spine in the skeleton" or the "superhighways" within the system (Braunstein et al., 2007; Wu et al., 2006). As expected from an assortative mixing network, high-degree nodes tend to stick together, and to be surrounded by clusters of other less connected peripheral vertices that tend to be densely connected within each cluster.

It is also worth noticing that the almost perfectly-squared shape of these few central vertices reveals a dual role as hubs and authorities within the network, where they serve as the parent node for less connected vertices, with those less connected vertices making part of a hierarchical module or community around one of these few central vertices. In this sense, despite the resulting spanning tree simplifies the system excessively (Serrano et al., 2009), it retains its salient features (Gilmore et al., 2010), such as its core-periphery and hierarchical structure.

A critical, yet non-observable (due to disclosure reasons) feature in the maximal spanning trees is that some of the largest and most connected vertices (in red) overlap across the three networks. One financial institution concurs in the three networks as a central vertex; three concur in two of them; three appear just in one network. Such overlapping is important for the stability of the financial infrastructure and for the financial system as whole. As stressed by CPSS (2008), financial institutions overlapping across systems may increase the interdependence of domestic systems, with such interdependence raising the potential for disruptions to spread widely and quickly across the financial system.

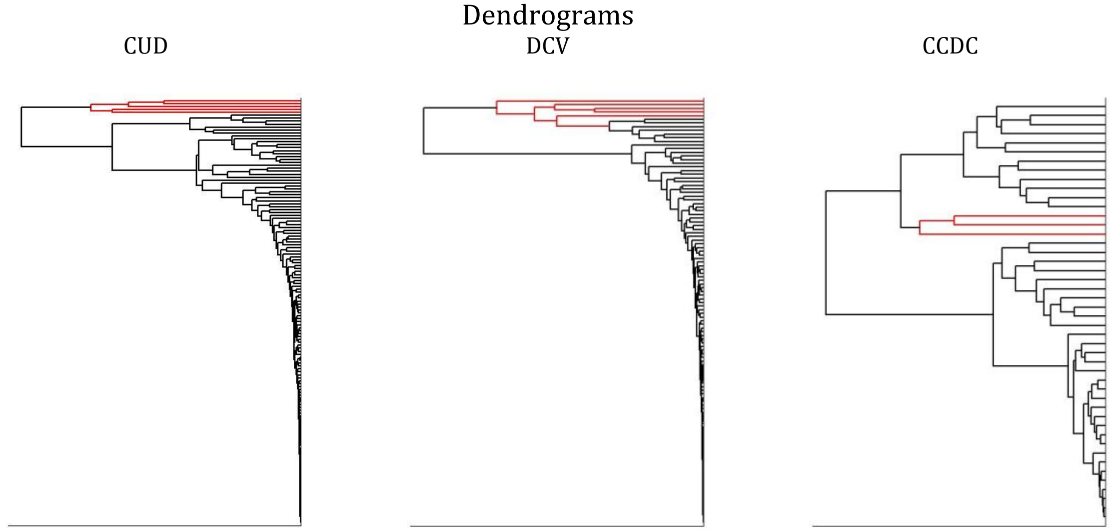
Comparing the three maximal spanning trees reveals that the CCDC network differs from the other two. For example, the size of the vertices (i.e. their authority and hub centrality) located in the core of the graph is not much larger than several others around them, whilst in CUD and DCV networks the differences tend to be manifest. As with preceding metrics and visualizations, CCDC network appears to be less heterogeneous.

Figure 23 presents the dendrogram for each weighted network. Akin to the maximal spanning tree, the dendrogram summarizes the process of clustering among vertices in a tree-like graph. In the CUD dendrogram it is evident the existence of two main partitions: a five-vertex core (in red) that dominates the value of payments and a 139-vertex periphery, where the former (latter) represents about 37% (63%) of the payments, with the five-vertex cortex matching the five central vertices in the maximal spanning tree. Within the periphery other clusters emerge based on the intensity of their connections.

The DCV dendrogram displays a similar structure. However, in the DCV the core and periphery appear to be even more heterogeneous. The core consists of twelve vertices that represent about 73% of the payments, where these twelve vertices correspond to the four red vertices in the maximal spanning tree (in red) plus other five well-connected vertices that are directly linked to these four.

The CCDC dendrogram verifies the distinct features of this system when compared to CUD and DCV. The dendrogram is less dispersed (i.e. it is more homogeneous), with two main clusters, where the most representative (in red) is composed of the three red vertices in the maximal spanning tree plus other twelve vertices; this fifteen-vertex core contributes with 32% of the payments, whereas the remaining 31-vertex periphery contribute with 68%.

Figure 23
Dendrograms



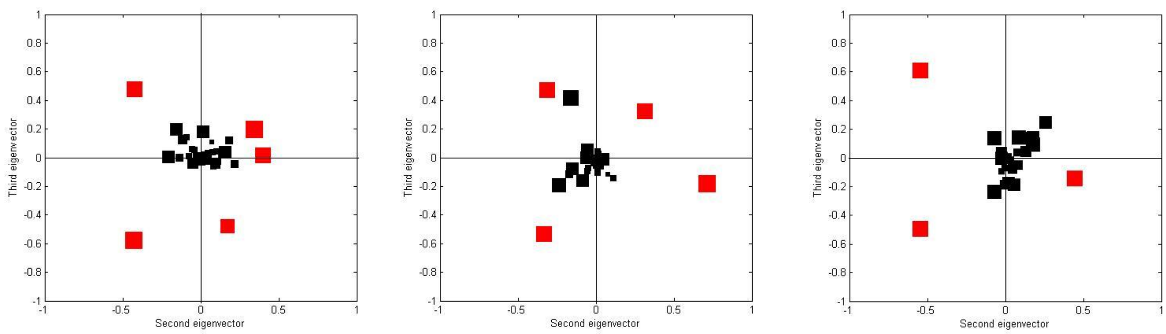
Source: authors' calculations.

Therefore, dendrograms confirm the hierarchical structure of the networks under analysis. The intensity of the linkages among financial institutions reveals communities or groups within the network, in which core (peripheral) institutions are intensely connected among them.

Finally, following Straffin (1980), the spectral partitioning of the weighted matrix uses differences in the sign and level of the first two non-principal eigenvectors to pick out significant clusters or subsystems in the corresponding graph. Figure 24 presents the spectral partitioning of the cumulative weighted matrix for the three systems[38], where the spectral space corresponds to the conjunction of the second and third eigenvector, and where the size of the vertex corresponds to the value of the first eigenvector's (i.e. eigenvector centrality) for each financial institution.

Figure 24
Spectral partitioning

CUD DCV CCDC



Source: authors' calculations

It is evident that there are vertices that display particularly large positive or negative deviations from the origin (i.e. the (0,0) coordinate), where most of these remote vertices (in red) coincide with those considered as central in the maximal spanning tree and the dendrograms. Therefore, as suggested by Kolaczyk (2009), these remote vertices, in conjunction with their most immediate neighbors, may be declared to be a cluster. Very close to the origin of each spectral partitioning plot lie the peripheral vertices, which are of low size due to their low eigenvector centrality.

5. Emergent properties of complex adaptive systems: modular scale-free payment and settlement networks

Disparate networks show the same three tendencies: short chains, high clustering and scale-free link distributions; the coincidences are eerie, and baffling to interpret (Strogatz, 2003). Similarly, Ravasz and Barabási (2003) and Barabási (2003) point out that many real networks in nature and society share two generic properties: they are scale-free and they display hierarchical organizations (i.e. high degree of clustering), where standard models reproducing

[38] Since the weighted matrix is non-symmetrical (i.e. directed), the weighted matrix is no longer guaranteed to have a complete real spectrum. Therefore, spectral analysis commonly uses a *similarity transformation* consisting of multiplying the matrix by its transpose (Kolaczyk, 2009; Gkantsidis et al., 2003). Such transformation is the same used in [§15] for attaining the hub matrix ($\mathcal{H}$).

scale-free (e.g. Barabási and Albert, 1999) or random structures (e.g. Erdös and Rényi, 1960) are blind to the modular hierarchies observed in real-world networks; this is, the Poisson and the scale-free models do not explain the emergence of order from chaos.

Likewise, selected Colombian payment and settlement networks are ultra-small, and exhibit the connective pattern (i.e. scale-free) and the hierarchical structure (i.e. modularity) typical of many real networks in nature and society. Financial and real networks coinciding in their size and connective pattern is by no means new, but their modular hierarchy has not been documented to the best knowledge of the authors.

If isomorphic or "system laws" (Von Bertalanffy, 1972 & 1950) govern financial and real networks, there should be some commonalities that may help explain financial systems' structure and dynamics as non-coincidental outcomes of randomness or chance; in fact, financial networks literature agrees on finding similar structures across different countries and markets[39], which may already signal the existence of some sort of isomorphism among financial systems as well.

The most evident commonality across financial systems is their complex adaptive nature, and it may also be at the origin of their formation and the resulting scale-free and modular architecture. In this sense, *emergence*, the phenomenon whereby well-formulated and robust aggregate behavior arises from individual behavior (Miller and Page, 2007) may explain the connective patterns and hierarchical organizations herein documented, a form of *organized complexity* in the Colombian payment and settlement systems. This concurs with Krugman (1996) view of the economy as a self-organizing system.

Therefore, results provide some elements that financial and economic theory has not been successful at capturing or including in models, and –therefore- may help to understand the formation and structure of financial networks as emergent properties of complex adaptive financial systems.

The proposed analytical approach (i.e. emergence from complex adaptive financial systems) is divided in two parts. The first one address the connective patterns, where the scale-free and ultra-small properties result from the evolution of a network of institutions within the competitive environment of financial systems. The second addresses modularity as the result of financial institutions inadvertently organizing themselves into a self-organized critical state (Bak, 1996), where the particular emergent modular hierarchy favors evolution within a structure that leads the system away from critical regimes.

[39] León and Pérez (2013b) document that some financial networks diverging from the typical structure in the Colombian case result from regulatory issues. For instance, the sovereign securities SEN network has a few participants ($n = 15$), it is dense ($d = 1$) and homogeneous (e.g. all participants share the same degree, $\mathscr{k} = 14$) due to regulatory requirements imposed to the participants, which are considered as a club of market-makers; however, as point out by León and Pérez, such homogeneity reinforces inhomogeneity at an aggregated level (i.e. when adding all sovereign securities' trading and registering platforms).

5.1. Scale-free financial networks: adaptation within a competitive environment

Despite the presence of power-law degree distributions in networks was well documented before Barabási and Albert (1999)[40], their work is particularly influential since they are largely responsible for starting the current wave of interest in scale-free networks, and because they make three important contributions (Newman et al., 2006): first, they verify that a power-law degree distribution is a property of many real-world networks; second, they explain the main properties of such networks by introducing a model in which a network grows dynamically; third, they propose a specific model of a growing network that generates power-law degree distributions.[41]

Regarding the second and third contributions, Barabási and Albert (1999) state the main features of scale-free networks: growth and preferential attachment. About growth, all standard models before Barabási and Albert assumed that networks were static, with an initial fixed number of vertices (n) that are connected under some distributional assumption, without modifying n, whereas Barabási and Albert acknowledged that real-networks continuously expand by the addition of new vertices that are connected to those already present in the system.

Furthermore, Barabási and Albert linked the growing nature of real-world networks to another real-world feature: preferential attachment.[42] Instead of assuming that new vertices connect to the existing vertices in a random (i.e. uniform) manner, the probability of connecting to an existing vertex depends on its present degree; this is, growth and preferential attachment allow for early nodes to have more time to acquire links, and allow for early nodes to be selected more often and to grow faster than their younger and less connected peers (Barabási, 2003), akin to a seniority-based rich-get-richer phenomenon.

However, seniority (i.e. the advantage of older nodes) may be limited for explaining preferential attachment.[43] In a competitive environment each vertex has some ability to get

[40] Barabasi and Albert (1999) introduced the scale-free term for networks with power-law degree distributions, documented its ubiquity in many types of real-world networks, and proposed an explanatory framework and the corresponding genetaring model. However, other authors before them had documented degree distributions approximating a power-law, such as Price (1965) for scientific citation networks; Watts and Strogatz (1998) for networks of film actors; Albert et al. (1999) for the World Wide Web (i.e virtual network of hyperlinked documents); and Faloutsos et al. (1999) for the physical network of computers and routers that compose the Internet.

[41] The standard model introduced by Barabási and Albert (1999) results in a scale-free network with $\gamma=2.9\pm0.1$, about the same order exhibited by some real-world networks such as the collaboration network of movie actors ($\gamma\sim2.9$), the World Wide Web ($\gamma\sim2.1$), the electrical power grid of the western United States ($\gamma\sim4$), and the co-citation of scientific publications ($\gamma = 3$).

[42] Preferential attachment is a systems' feature documented before Barabási and Albert (1999). Simon, Nobel Prize in economics (1978), suggested a growth model producing skewed distributions (Simon, 1955). Similarly, Price (1976) used a "cumulative advantage" rationale for bibliometrics.

[43] For example, the five most central financial institutions in CUD network were established –on average- 57 years ago, whereas the average credit institution was established 44 years ago. A handful of banks that are non-central to the three networks were established about a century ago, whereas three out of the five most central were established less than fifty years ago. Therefore, seniority provides a rather weak rationale for preferential attachment in the Colombian case (with data from Mora et al., (2011) and public information from Financial Superintendence of Colombia).

related relative to its peers, where this ability may be related to some metric of fitness (e.g. seniority, size, geographical location, popularity, etc.); in this sense, the fitness model allows us to describe networks as competitive systems in which nodes fight fiercely for links (Barabási, 2003).

Therefore, the intuition behind the scale-free model may be generalized as follows: together, growth and preferential attachment within a competitive environment allow for fit vertices to progressively attract new edges, and allow for fit edges to be selected more often and to grow faster than their less attractive peers, resulting in a fit-get-rich phenomenon that leads to a scale-free connective pattern.

When choosing the right metric(s) for fitness, this generalization allows for understanding diverse types of social dynamics, such as the formation of agglomerations by humans (e.g. cities) or animals (e.g. termites colonies), the structure of the World Wide Web, the networks of scientific co-citation, etc.; moreover, as will be addressed below, it allows for understanding modular hierarchies within financial systems.

Both features, growth and preferential attachment, are intricately related to the elements that define a complex adaptive system. Together, growth and preferential attachment are behind the *recombination and system evolution* element stated by Anderson (1999), which points out that systems evolve and adapt over time based on the entry, exit and transformation of agents. Likewise, it is also related to the system's adaptive process resulting from those processes where *building blocks are recombined and revised continually as the system accumulates experience* (Holland, 1998).

In financial networks growth and preferential attachment are marked and interrelated features as well. Financial systems are not static; they are the result of a long evolutionary process where new financial institutions, business niches or cognitive structures (i.e. *schemata)* appeared, some old ones disappeared, and where some existing ones recombined (e.g. merged) in a new form, with such evolution modifying the main characteristics (e.g. pattern, intensity, direction) of the connections between financial institutions. As illustrated by Miller and Page (2007), agents in the stock market actively adapt and alter the fundamental behavior of the system and, in so doing, force it into new realms of activity.

Moreover, this evolutionary process is by no means random, but is related to financial systems' competitive environment and to the mutually dependent adaptive efforts of institutions to improve their own fitness, which result in financial institutions acting based on the fitness of their peers, and of financial market's niches and schemata.[44] Accordingly, financial institutions may decide (i) who to make deals with based on its counterparties' efficiency, costs, size, connectedness, seniority, geographical location, market position, access to last-resort lending, reputation, etc.; (ii) to modify its business line based on the rise (or fall) of new (old) market niches; and (iii) to modify its strategies (e.g. trading rules) based on

[44] As highlighted by Anderson (1999), *adaptive efforts of individual agents that attempt to improve their own payoffs* is a distinctive feature of complex adaptive systems, where each agent is to be considered adaptive if its actions can be assigned a value (e.g. payoff, fitness), and the agent behaves so as to increase this value over time dependent on the choices of other agents. Clearly, this is the case of financial institutions.

competitors' success or technological advances.[45] Therefore, as in Barabási and Albert (1999) seminal work, the evolution of financial systems around preferential attachment principles results in a scale-free topology, which also results in financial networks being ultra-small according to Cohen and Havlin (2010 & 2003).

In this sense, financial systems evolutionary process (i.e. growth) implies an adaptation process within the network, where institutions, business niches and schemata evolve based on a preferential attachment principle, with the fittest institutions, niches and schemata dominating the networks' connective pattern; hence, ultra-small and scale-free financial networks appear to be an emergent property of complex adaptive financial systems.

5.2. Self-organizing criticality of payment and settlement systems: order emerging out of chaos

There is a certain agreement regarding a consequence of a system being characterized by power-law distributions: the system may be organizing itself (Andriani and McKelvey, 2009; Dorogovtsev and Mendes, 2003; Strogatz, 2003; Barabási, 2003; Barabási and Albert, 1999; Bak, 1996). In this sense, the origin of the architecture of networks is their self-organization (Dorogovtsev and Mendes, 2003), with power-laws as the patent signature of self-organization in complex systems (Barabási, 2003).

Furthermore, evidence confirms that with adaptive agents a system self-organizes in such a way that frequent small changes yield frequent small consequences for the system, and exceptional critical consequences, as in a power-law distribution of events. In this way, as highlighted by Miller and Page (2007), due to adaptive agents' risk aversion, the system configures itself in a way that mitigates the overall risk by inhibiting criticality from emerging.[46] This phenomenon, commonly referred as self-organizing criticality (Bak, 1996), is consistent with one of the key elements of complex adaptive systems stated by Anderson (1999): *coevolution to the edge of chaos*.[47]

Self-organizing criticality is convenient from an evolutionary perspective. As changes have power-law distributed impacts, systems tend to be robust to everyday behavior, a condition that agrees with agents' search for a stable environment; but, at the same time, the system is exposed to rare massive transformations. Thus, the system adapts to a precipice, a state that

[45] The fitness of niches and schemata may explain the rise and fall business lines and strategies. For instance, in the financial sector case, it may explain the rising importance of the e-trading niche, and of algorithmic trading as a widespread schema (i.e. a cognitive structure that determines institutions' actions).

[46] Adaptive systems tend to be inherently risk averse because, despite the potential gains to be made by taking even a favorable risk, it takes only a single loss to kill of an agent and eliminate it from the system forever (Miller and Page, 2007).

[47] Coevolution to the edge of chaos is a dynamic equilibrium in which small changes in behavior can have small, medium or large impacts on the system as a whole, according to a power-law, where occasional large evolutionary cascades associated with small changes in behavior allows the system to leap to higher fitness peaks than it would likely locate thorough evolutionary refinement (Anderson, 1999). On the other hand, chaotic systems are those in which small changes tend to yield large impacts, whereas static systems are those in which small changes always lead to small impact; therefore, chaotic systems can reach extraordinary fitness peaks but cannot remain on them, whilst static systems can never improve much; loosely speaking, complexity lies somewhere between order and chaos (Miller and Page, 2007).

is optimal and fragile, rich in performance yet rather exposed (Miller and Page, 2007), a state that matches the well-known scale-free networks' property of being robust to random failures, yet fragile to targeted attacks.

From a biological perspective, given that mutations occur at random, natural selection favors designs that can tolerate haphazard insults (Strogatz, 2003). And social and biological networks suggest that modularity offers that kind of robust design.

According to Simon (1962), the existence of clusters of dense interaction in social systems identifies well-defined hierarchical structures that may be defined as *nearly decomposable systems*. In such type of systems each cluster may be regarded as a subsystem composed of subordinates led by a boss, in whom the interactions among subsystems are weak, but not negligible, where intra-component linkages are generally stronger than inter-component linkages.

Hierarchical modularity and the resulting near decomposability of real-world systems are by no means accidental.[48] Hierarchical modularity has significant design advantages, such as making multitasking possible: while the dense connections within each module help the efficient accomplishment of specific tasks, the hubs coordinate the communication between the many parallel functions (Barabási, 2003). Moreover, as highlighted by Anderson (1999), since most components or subsystems receive inputs from only a few of the system's other components, change can be isolated to local neighborhoods; hence, by limiting the potential cascades, modularity protects the systemic resilience of both natural and constructed networks (Haldane and May, 2011).

In the financial systems case, hierarchical modularity and near decomposability may be advantageous for the aforementioned reasons. For instance, different financial industry's niches (e.g. securities trading, derivatives, corporate banking, and asset management) may profit from each niche operating in a specialized subsystem, with fit financial institutions providing the main linkages among subsystems. Additionally, financial institutions may prefer to connect to fit institutions in search of firewalls against risks posed from other less fit ones to prevent criticality; in this sense, as stressed by Schweitzer et al. (2009) and Haldane (2009), heterogeneities of agents can turn out to become a source of stability for financial systems.

Financial institutions' fitness may come in several forms. A preliminary list of sources of fitness may include the following: (i) solvency; (ii) liquidity; (iii) seniority; (iv) access to payment and settlement systems (e.g. formal and *de facto* tiered structures); (v) access to central bank's accounts and ordinary liquidity facilities; (vi) access to last-resort lending; (vii) systemic importance[49]; (viii) fulfilling a holding or parent position within a conglomerate; (ix) privileged access as a counterparty of the central bank in its implementation of monetary policy (e.g. primary dealers); (x) privileged access as underwriter of sovereign securities (e.g.

[48] Despite the dominance of hierarchical modularity in real-world networks, Ravasz and Barabási (2003) acknowledge that hierarchy is absent in networks with strong geographical constraints, as the limitation on the link length strongly constraints the network topology.

[49] As acknowledged by Haldane and May (2011), in financial ecosystems, evolutionary forces have often been survival of the fattest (i.e. too-big, too-connected, too-complex-to-fail) rather than the fittest.

market makers); (xi) geographical location; (xii) long-standing business commitment (e.g. tradition); and (xiii) standard barriers to entry (e.g. economies of scale, sunk costs).

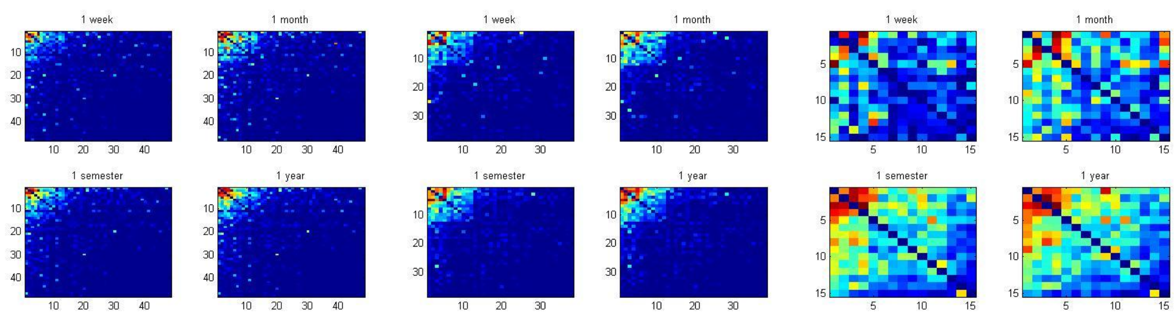
Consequently, as highlighted by Simon (1962), the search for fitness yields an evolutionary explanation of observed hierarchies based on selectivity; this is, direction to the system is provided by the stability of complex forms, but this is nothing more than the survival of the fittest (i.e. of the stable).

In this sense, financial institutions adapt to the feedback information from their search for improving their own fitness, which includes *selective trial and error* and their previous experience.[50] Financial systems' selective evolution yields fit overall configurations in the form of stable path dependent hierarchies, with small (frequent) changes affecting them as in a power-law distribution of events (i.e. again, a self-organizing criticality); thus, complex adaptive systems tend to be particularly path dependent. Concurrently, as pinpointed by Inaoka et al. (2004), financial systems must be based upon a stable network of transactions in order to fulfill its role.

Figure 25 makes a simple –yet informative- test for stable configurations for the three selected Colombian payment and settlement networks. The entire sample of each network was divided in four different subsamples, corresponding to a week, a month, a semester and the whole sample[51]; for legibility reasons, only $\frac{1}{3}$ of the most contributing institutions by their strength are displayed. Since the position of vertices in the axis across subsamples is fixed for each system, if the configuration (i.e. hierarchy) of each network is stable there should be no significant differences among the four intensity plots for each network.

Figure 25
Intensity plots (subsamples)

CUD DCV CCDC



Source: authors' calculations.

Subsamples appear to be self-referential, especially for CUD and DCV. There is a vivid characteristic pattern that not only resembles across subsamples but also resembles the

[50] Thus, since selectivity drives from various rules of thumb, or heuristics, that suggest which paths should be tried first and which leads are promising (Simon, 1962), selectivity is intimately related to the existence of *agents with schemata*, one of the elements that characterize complex adaptive systems according to Anderson (1999).

[51] 1-week corresponds to the first week of February; 1-month corresponds to February; 1-semester corresponds to February-July; 1-year corresponds to February-November. January and December were discarded in order to avoid end-of-the-year seasonality.

original sample (1-year). In the sense of Mandelbrot and Hudson (2004) and Peak and Frame (1994), intensity plots appear to be *self-affine*; that is, they appear to be made of shrunken, but distorted copies of the whole –a characteristic property of fractals.

Yet, the observed similarity and the suggested stable configuration are by no means anticipated. For instance, as in León and Pérez (2013b), the DCV network is the sum of three different sovereign securities networks resulting from two anonymous trading platforms (i.e. SEN and MEC) and an Over the Counter (OTC) registering platform, where 77% of the volume corresponds to anonymous (i.e. blind) interactions among financial institutions. Therefore, since most of the transactions are anonymous, the evolution and emergence of hierarchy by means of selectivity is somewhat unexpected, but not unexplainable: as highlighted by León (2013), MEC is open to a broad base of financial firms (about 140), where each firm determines a quota or exposure limit for each other potential counterparty, where this limit follows active credit risk assessment, with such assessment resulting from financial institutions adapting to the feedback information from their search for improving their own fitness. Additionally, SEN, which contributes with 61% of the volume, is restricted to a select group of 15 financial institutions (i.e. market makers) chosen based on some set of fitness criteria, where trades between them are anonymous and counterparty limits or quotas do not exist.

On the other hand, OTC transactions dominate the local foreign exchange market (i.e. 80% of the volume), where their bilateral nature may explain the emergence of a stable hierarchy in the CCDC network. In this case, it is likely that financial institutions choose their corresponding counterparties in the foreign exchange market by adapting to the feedback information from their search for improving their own fitness. Regarding CUD, which aggregates DCV, CCDC and other non-directly-observable networks, it displays a stable configuration as well, where most of the non-directly-observed transactions correspond to bilateral fund transfers (i.e. non collateralized lending and discretional debits/credits from/to depositary institutions).

Besides, Simon (1962) stresses that the salient concentration of interactions within clusters results in nearly decomposable systems agreeing with the "empty world hypothesis", where most things are only weakly connected with most other things; likewise, systems in which each vertex is connected with almost equal strength with almost all other vertices are rare. As stressed by Anderson (1999), order arises in complex adaptive systems because their components are partially, not fully, connected; on the other hand, full connectedness (e.g. $d \sim 1$) results in decay if feedback dampens out change, or chaos if feedback amplifies changes.[52]

Interestingly enough, the "empty world hypothesis" agrees with the particular sparseness of real-world networks. Furthermore, this hypothesis also matches the sparseness of the

[52] Empirical evidence of the non-linear effects of increasing connectedness in a financial network is provided by Battiston et al. (2012), Gai and Kapadia (2010) and Battiston et al. (2009), who find that financial fragility feedback (i.e. a *financial accelerator* mechanism) may amplify the effect of an initial shock and lead to a widespread systemic crisis; this is, the relation between connectivity and systemic risk is not monotonically decreasing. However, their approach is based on random (i.e. non-real-world) structures.

selected Colombian payment and settlement networks, either measured by density or average degree or by the visual inspection of the intensity plots (Figure 6).

Assenza et al. (2011) propose a specific model of an adaptive network that generates modular scale-free architectures. Their model yields a modular scale-free architecture from two competing feedback mechanisms: *homophily* and *homeostasis*. Homophily is related to increasing the intensity of interactions with other similar institutions, whereas homeostasis consists of an intensity preservation mechanism that weakens prior interactions in favor of new ones; together, these two feedback mechanisms lead to the emergence of real-world (e.g. social an neural systems) features such as scale-free distributions of interaction weights, strong modularity, and local synchronization (Assenza et al., 2011).

The competition between homophily and homeostasis may applied to financial systems with some adjustments. The selective trial and error process of Simon (1962) may be behind increasing the intensity of interactions with other institutions, where such process may be related to financial institutions' fitness, and not similarity or corporate ownership, whereas the intensity preservation mechanism (i.e. homeostasis) may be related to financial institutions being forced to counter-balancing the intensity of their aggregated interactions due to finite resources (e.g. money, securities, risk limits). This new model would explain the main features of real-world adaptive networks, namely the intensity of intra-module edges at the expense of inter-module edges within a scale-free network, and the sparseness of networks. Based on the findings of this document, this model could provide some insights regarding the Colombian payment and settlement networks as well.

6. Final remarks

Evidence verifies that Colombian payment and settlement systems are modular scale-free networks. Furthermore, based on complex adaptive systems theory, evidence also strongly suggests that local payment and settlement systems have self-organized into the observed scale-free and hierarchical structure, a ubiquitous (i.e. isomorphic) structure in social networks that favors everyday robustness and performance in exchange for rare episodes of fragility but rapid evolution.

Therefore, evidence reported here supports three important preceding propositions: (i) the economy is a complex adaptive system (Holland, 1998); (ii) the economy is a self-organizing system (Krugman, 1996); and (iii) in the sense of Bak (1996), financial systems are complex adaptive systems that have self-organized in order to prevent criticality from arising.

Several consequences, both theoretical and practical, arise from these findings. From a theoretical point of view evidence vindicates contemporary calls for a new fundamental understanding of the structure and dynamics of financial networks (e.g. Farmer et al., 2012; Haldane and May, 2011; Schweitzer et al., 2009; Haldane, 2009; Kambhu et al., 2007); this is precisely the type of understanding that was absent before and during the financial crisis that started around 2007.

Attaining such new fundamental understanding requires fundamental shifts in the way economic analysis approaches financial systems as well. Despite the urge for considering the economy as a complex system can be dated back several decades ago (e.g. Holland, 1988; Kauffman, 1988; Krugman, 1996; Bak, 1996)[53], *at present economics stands out as the branch of science where the study of complex systems has had the least impact* (Farmer et al., 2012).

In turn, the required fundamental shifts in economic analysis are by no means new, but correspond to persistent and idealistic assumptions about the functioning of the economy and financial markets. For instance, ignoring that financial time-series do not approximate a random-walk (i.e. Brownian motion) but a power-law (e.g. Taleb, 2007; Sornette, 2003; Stanley et al., 2002; Bak, 1996; Mandelbrot and Van Ness, 1968; Fama, 1965; Mandelbrot, 1963), not only has misled risk management about the likelihood and impact of extreme events, but has also obscured the main evidence of financial markets' self-organized criticality, and has favored self-correcting static equilibrium approaches to financial markets.

Likewise, assumptions of linearity neglect the existence of feedback loops among agents and the resulting self-organization. Under such assumptions of linearity the system's response is proportional to the size of the impact, as in static equilibrium systems, where large fluctuations *can occur only if many random events accidentally pull in the same direction, which is prohibitively unlikely* (Bak, 1996). This type of nonlinearity of financial systems' has appeared recently in related literature, mainly after the crisis (e.g. Haldane, 2009; May et al., 2008; Kambhu et al., 2007), where there is an explicit recognition that feedback mechanisms (e.g. margin calls, fire sales, herding), along with financial networks' connective patterns, derive in the rise of catastrophic events from modest local changes.[54]

Furthermore, the traditional assumption of homogeneity in financial systems (e.g. Allen and Gale, 2000; Freixas et al., 2000) and the resulting emphasis on average or representative behavior, ignores the true –inhomogeneous- nature of financial systems, potentially misleading the discussion about the effects of connectedness on financial contagion. Also, as stressed by Craig and von Peter (2010), in such homogeneous –flat- environments there is no role for financial intermediation, whereas homogeneity spells the inability of a system to adapt (Kambhu et al., 2007). Therefore, *homogeneity is not a feature we often observe in the world but rather a necessity imposed on us by our modeling techniques* (Miller and Page, 2007).

The aforementioned theoretical implications of the quantitative results of this document are particularly challenging for traditional economic analysis. Practical implications are equally defiant.

If the connective pattern and hierarchical structure of financial markets are the result of self-organized criticality, where small changes yield frequent small consequences and exceptional

[53] Interestingly, Adam Smith may be credited for the first view of the economy as a complex adaptive system, where the "invisible hand" is nothing but the self-organization emerging from independently acting economic agents.

[54] As documented in León et al. (2012), the crisis that begun about 2007 display the disproportionate effect of a shock in the overall properties of the system, where there is some degree of consensus about the lack of correspondence between the subprime crisis (i.e. the shock) and the global financial crisis (i.e. the catastrophe), where the former is rather modest when compared to the extent of the whole episode.

critical consequences (i.e. a power-law distribution of events), the traditional aim of financial authorities should be reexamined accordingly. For instance, as highlighted by Bak (1996), since price variations approximating a power-law are scale-free, with no typical size of the variations (as earthquakes, species' extinctions, wars), large price changes have nothing special despite their potentially devastating consequences, and –thus- instability and catastrophes are inevitable in economics, as they are in geology, biology and history.[55] This view certainly poses a major shift in economic thinking.

Even if Bak's (1996) argument is wrong and large economic swings are unique, different from all other observed catastrophes (e.g. earthquakes, species' extinctions, wars), documented financial systems' self-organization based on a modular scale-free architecture highlights the need for a major move towards reinforcing modularity. Since hierarchical modularity protects resilience of systems by means of limiting cascades (Haldane and May, 2011) and isolating feedbacks (Kambhu et al., 2007), financial authorities should enhance modularity (not fight against it) as a firewall against system-wide effects.

In this sense, authorities should understand and embrace hierarchical modularity in order to be able to contribute to financial stability in an effective manner. The most obvious strategy is inherited from biology, more specifically from epidemic theory.[56] As argued after the crisis (e.g. León and Pérez, 2013b; Markose, 2012; Haldane and May, 2011; May et al., 2008; Kambhu et al., 2007), preventive action should be on systemic important financial institutions (i.e. super-spreaders), where prudential regulation should enhance the fitness of financial institutions that have emerged as firewalls or circuit breakers against contagion.

Accordingly, as suggested by Haldane and May (2009), protecting the financial system from future systemic events would require the key super-spreader nodes to run with higher buffers of capital and liquid assets, which are then proportional to the system-wide risk they contribute; this is, prudential regulation has to be system-calibrated rather than institution-calibrated. In this sense, as in forest fire models, compartmentalization (i.e. modularity) by firebreaks or vaccination of super-spreaders allow for countering systemic risk (May et al., 2008).

Nevertheless, not only prudential regulation should adjust to financial markets' hierarchical modularity. Financial authorities' regulation, supervision and oversight efforts should be prioritized accordingly, during tranquil and adverse periods. Moreover, since financial networks are particularly dynamic in the sense of Inaoka et al. (2004), where connections between financial institutions may be reconfigured promptly[57], authorities may decide how to face the failure of a systemically important institution: either the failing institution is bailed

[55] *Large fluctuations observed in economics indicate an economy operating at the self-organized critical state, in which minor shocks can lead to avalanches of all sizes, just like earthquakes. The fluctuations are unavoidable. There is no way that one can stabilize the economy and get rid of the fluctuations through regulations of interest rates or other measures.* (Bak, 1996)

[56] Following Kambhu et al. (2007), since experimental stress testing is not feasible for financial system analysis, examining common structural properties with other systems should be of interest, and may help guide policy making.

[57] Unlike –for example- a physical network as the Internet or a power transmission grid, whose hardware may not be reconfigured rapidly or economically.

out, or the remaining institutions are effectively supported in order to allow for an effective reconfiguration (i.e. rewiring) of the system.

Hence, practical implications for financial authorities are far from simple, but correspond to a better approximation to the true nature of financial systems. Interestingly enough, theoretical and practical implications involve a hard change in the prevailing cognitive structure of economics (i.e. the economic schema).

As suggested by Gell-Mann (1992), the cognitive structure or schema consists of a twin process of compressing the regularities and discarding the randomness found in experience; thus a schema is approximate by construction, and may be wrong. Accordingly, the explanatory power of the prevailing economic models has been proven low to understand financial systems, which suggests that the dominating schema has misidentified regularities and exaggerated mere coincidences.

The prevalence of the dominating economic schema of financial systems in spite of its poor explanatory power may be due to the absence of a clear fitness function for testing its viability (i.e. its ability to account for observed facts). This causes the feedback loop of selective trial and error of economic models to be sympathetic with the existing schema, where evident but potentially problematic regularities tend to be overlooked or denied (e.g. nonlinearities, inhomogeneity, power-laws), and elegant but flawed generalizations (i.e. static equilibrium, homogeneity, rationality, serial independence, normality) tend to be broadly accepted.[58]

The survival of the dominating economic schema may also be explained by frictions within the related community as well. As reported by Kambhu et al. (2007), *an effort to model an entire system, with the aim of learning how to control it better, is a very large-scale project and one that academic economists will not readily take on because of the way the profession is organized and financed*.

Either way, further understanding of financial systems is still needed. This document makes a contribution to the existing literature by identifying and analyzing the connective pattern and hierarchical structure of three Colombian payment and settlement networks, where the main novelties are related to characterizing the selected networks as modular scale-free without relying on a single snapshot of the systems or limiting the type of financial institutions considered.

However, despite numerical results appear to be conclusive about the modular scale-free architecture of the three payments and settlement systems, the understanding of the generating process behind such architecture is still preliminary. As highlighted by Bak (1996), the *problem of explaining the observed statistical features of complex systems can be phrased mathematically as the problem of explaining the underlying power laws, and more specifically the values of the exponents*, whereas Strogatz (2003) stresses that despite years of intense effort, the origin of power-laws remains controversial. Therefore, this document suggests some rationale for the connective patterns and hierarchical architecture within complex adaptive systems' theory, but they are by no means definite, complete or unique.

[58] For instance, as documented by Bak (1996), economists have chosen largely to ignore Mandelbrot's work (e.g. fractional Brownian motion), *mostly because it doesn't fit into the generally accepted picture*.

Additional challenges are worth stating. First, the three selected networks are interconnected (as presented in Figure 4), where financial institutions may overlap across financial market infrastructures (i.e. systems) and –hence- create a transmission channel between different markets (e.g. securities, foreign exchange) and their participants, even in the absence of direct linkages between them; as stressed by CPSS (2008), financial institutions overlapping across systems may increase the interdependence of domestic systems, with such interdependence raising the potential for disruptions to spread widely and quickly across the financial system.

The risk from the connections between systems and the overlapping across systems (i.e. cross system risk) has not been addressed in the literature. It demands a significant –yet realistic- increase in the complexity of the model, where financial institutions are not able to connect to each other directly as is customary assumed in financial networks literature, but require an intermediary in-between: a financial market infrastructure. In other words, existing literature has decided to simplify the network by means of neglecting the existence of what Bernanke (2009) calls the "financial plumbing".[59] In this sense, infrastructure-related systemic risk, *the component of systemic risk that can be brought about the improper functioning of the financial infrastructure, or where the financial infrastructure acts as the conduit for shocks that have arisen elsewhere* (Berndsen, 2011), has been overlooked.

Merging financial institutions and financial market infrastructures into a single network will reveal additional levels of hierarchy in financial systems, concurring with Simon (1962) definition of a hierarchic system: *a system that is composed of interrelated subsystems, each of the latter being in turn, hierarchic in structure until we reach some lowest level of elementary subsystem*.

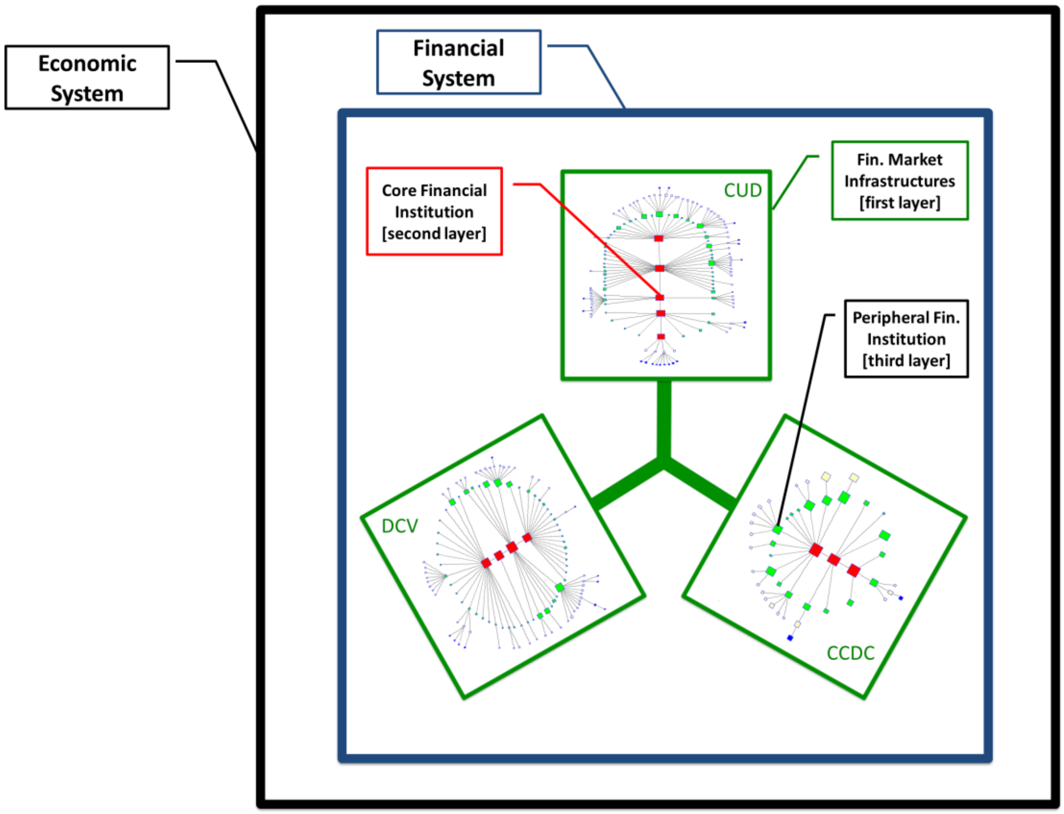
In this sense Figure 26 presents the Colombian financial system as a simplified modular hierarchical subsystem of the economic system. The first layer of the financial system consists of a network comprising the three financial market infrastructures analyzed in this document (i.e. CUD, DCV and CCDC). Each financial market infrastructure nests a second layer or network of financial institutions, which in turns is divided into core (i.e. central) and peripheral financial institutions, the second and third layers of the financial system's hierarchy, respectively.

Three main simplifications in this conceptual figure are worth stating. First, the number of financial market infrastructures is limited to the three analyzed in this document, and their true connective structure (in Figure 4) is overlooked for illustrative purposes; second, since the financial institutions' networks are depicted by their maximal spanning tree, connections between financial institutions not pertaining to the same core module are not presented; third, the figure does not consider financial institutions overlapping across financial market infrastructures, a salient feature of financial systems. However, such simplified architecture is able to enclose the entire hierarchy resulting from merging financial institutions and infrastructures, and from acknowledging the modular scale-free nature of financial

[59] Neglecting the role of financial market infrastructures in financial networks is akin to excluding the role of airports, harbors and roads in the understanding of trade, or analyzing the food chain away from living organisms' environment.

institutions' networks, where the modular and *self-affine* (i.e. fractal) architecture at each layer is apparent.[60]

Figure 26
The Colombian financial system as a (simplified) modular hierarchy



Source: authors' design.

Traditional economic models based on the homogeneity of financial institutions and their linkages (e.g. Allen and Gale, 2000; Freixas et al., 2000) have limited their scope to analyzing financial institutions within a flat environment, where the core and peripheral structure is ignored; this is, traditional models have studied financial institutions without distinguishing the existence of the second and third layers of Figure 26. A step ahead, financial networks' literature has aimed at identifying the true connective patterns among financial institutions, which has allowed acknowledging inhomogeneity and –thus- the existence of the second and third layers; this is the approach of the present document, where three different networks from three different infrastructures have been analyzed.

The existence and systemic role of the first layer, which allows financial markets and their two layers of institutions to link to each other, has not been considered in the literature to the best

[60] Despite the elementary subsystem for analyzing financial systems is the financial institution, it is quite possible that additional levels of modular hierarchy could be found within the financial institutions corporate structure, which would yield another layer of subsystems, and so on. Successive sets of modular subsystems not only match Simon (1962) definition of complexity and near-decomposability, but agree with the fractal or scale-free nature of financial systems, where the different layers are *self-affine*.

knowledge of the authors. Aggregating these three levels of hierarchy is essential for an enhanced and comprehensive understanding of transmission channels within financial markets and for assessing systemic risk under a macro-prudential point of view. The authors of this document will tackle this challenge in a forthcoming research project.

Additional challenges arise from the evidence of modular architecture within financial institutions. If core financial institutions are to be required with higher buffers of capital and liquid assets in order to enhance their firewall capabilities, sound quantitative methods are required. Centrality measures may be an interesting and objective approach to determining systemic-calibrated macro-prudential requirements, as in the eigenvector centrality-based "super-spreader tax" proposal by Markose (2012). An alternative is to regard the role of these super-spreaders institutions as proximate to that of a financial market infrastructure, an analogy that may provide some ideas for their prudential regulation under BIS-IOSCO (2012) "Principles for Financial market Infrastructures". However, due to the originality of the problem of systemic-calibrated requirements, more research is still required.

Furthermore, financial authorities should explore alternatives to requiring systemically important financial institutions higher capital and liquidity buffers. Accordingly, financial market infrastructures' role as sources of stability for the financial system should be emphasized, where their proper design and safe functioning serve as an additional firewall against contagion.

Finally, despite self-organization is often regarded as a good (i.e. desirable) thing, it may be the case that the resulting structure and hierarchy is inconvenient (e.g. inefficient, fragile). As highlighted by Krugman (1996), self-organization is something we observe and try to understand, not necessarily something we want. Thus, reinforcing financial systems' modularity in order to profit from its inherent advantages follows the assumption of financial authorities not being able to design a "better" architecture for the financial system.

However, modifying the architecture by means of regulation may not be discarded; a good example is the current interest in central clearing of OTC derivative contracts through central counterparties[61]. Yet, implementing large-scale changes to the existing architecture should be done under a macro-prudential framework, with financial authorities being able to understand the complexity of the system they are trying to redesign.

[61] Forcing financial contracts (e.g. derivatives) through central counterparties may be creating an additional hierarchical layer to the financial system; it may be reinforcing modularity. Instead of bilateral clearing, the central counterparty would be in charge of the clearing, where the central counterparty would stand between the financial institutions and the other financial market infrastructures. Moreover, the imposition of stringent risk management requirements to central counterparties agrees with the "vaccination" of super-spreaders for countering systemic risk.

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