

Macro-prudential assessment of Colombian financial institutions' systemic importance †

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Abstract

This document presents an enhanced and condensed version of preceding proposals for identifying systemically important financial institutions in Colombia. Three systemic importance metrics are implemented: (i) money market net exposures network hub centrality; (ii) large-value payment system network hub centrality; and (iii) an adjusted assets measure. Two complementary aggregation methods for those metrics are implemented: fuzzy logic and principal component analysis.

The two resulting indexes concur in several features: (i) the ranking and remoteness of the top-two most systemically important financial institutions; (ii) the preeminence of credit institutions in the indexes; (iii) the appearance of a brokerage firm in the top-six; (iv) the skewed nature of the indexes, which match the skewed (i.e. inhomogeneous) nature of the three metrics and their approximate scale-free distribution.

The indexes are non-redundant and provide a comprehensive relative assessment of each financial institution's systemic importance, in which the choice of metrics pursues the macro-prudential perspective of financial stability. The indexes may serve financial authorities as quantitative tools for focusing their attention and resources where the severity resulting from an institution failing or near-failing is estimated to be the greatest. They may also serve them for enhanced policy and decision-making.

Keywords: Systemic Importance, Systemic Risk, Fuzzy Logic, Principal Component Analysis, Financial Stability, Macro-prudential.

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† The opinions and statements are the sole responsibility of the authors, and they do not necessarily represent neither those of Banco de la República and the Ministry of Finance and Public Credit, nor of its Board of Directors and Minister, respectively. **Results are illustrative; they may not be used to infer credit quality or to make any type of assessment for any financial institution.** Authors are grateful to the officers and technical staff involved in the design of the expert knowledge base. Comments and suggestions from Joaquín Bernal are greatly appreciated; comments and suggestions to previous research received at seminars at Central Bank of Brazil, Bank of Finland, CEMLA and Banco de la República, and from Journal of Financial Market Infrastructures' anonymous referees are also acknowledged. Data was processed with assistance from Carlos Cadena, Jorge Cely and Santiago Hernández. Any remaining errors are the authors' own.

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1 Introduction

An institution may be considered as systemically important if its failure or malfunction causes widespread distress, either as a direct or indirect impact (i.e. contagion), where the main criterion for assessing systemic importance relates to their potential to have a large negative impact on the financial system and the real economy (IMF et al., 2009).

Defining whether a financial institution⁴ is systemically important (or not) may be intricate but it is key to the oversight, supervision and regulation of financial systems. To be able to identify systemic importance may assist financial authorities in focusing their attention and resources – the intensity of oversight, supervision and regulation - where the systemic severity resulting from a financial institution failing or near-failing is estimated to be the greatest. Identifying systemically important institutions may also help financial authorities in policy-making (e.g. prudential regulation, oversight and supervision) and decision-making (e.g. resolving, restructuring or providing emergency liquidity).

Literature has acknowledged the existence of three main key criteria for assessing and identifying the systemic importance of financial institutions: *size*, *connectedness* and *substitutability* (IMF et al., 2009; Manning et al., 2009).⁵ According to IMF et al. (2009), it is possible to relate *size* and *non-substitutability* to the direct impact of an institution failing to fulfill its role within the financial markets, whilst *connectedness* relates to the indirect impact of such event.

Despite the intuitiveness of these concepts, assessing and identifying systemic important institutions within an *indicator-based approach*⁶ remains a non-trivial task that implies several challenges. Two challenges are particularly demanding. First, designing indicators or metrics for *connectedness* and *substitutability* may require, as acknowledged by recent literature, non-standard data sources and techniques, such as financial infrastructures' data and network analysis, respectively. Second, choosing a methodology capable of robustly aggregating the metrics designed for the three aforementioned concepts into a systemic importance index may be intricate.

⁴ "Financial institution" comprises depository institutions (e.g. banks or savings associations), brokers, dealers, investment companies (e.g. mutual funds), insurance companies, investment advisers and credit unions; that is, those that may not be regarded as a "financial market infrastructures" (i.e. firms providing payment, settlement, clearing, trading platforms or systems).

⁵ (BCBS-BIS, 2013) suggests adding two criteria (i.e. cross-jurisdictional activity and complexity) in order to attain banks' global systemically importance and the difficulty of resolving a systemic event. Because the herein document focuses on non-global banking and non-banking institutions' systemic importance, and since derivatives and other complex instruments are rather scarce in the Colombian market, the criteria is limited to *size*, *connectedness* and *substitutability*, as originally suggested by IMF, BIS and FSB (IMF et al., 2009). However, the proposed aggregation methods are able to consider these two (or other) criteria.

⁶ There is an alternative to indicator-based approaches as the one here proposed; a model-based approach, which uses quantitative models to estimate financial institutions' contributions to systemic risk. However, as highlighted by BCBS-BIS (2013) *models for measuring systemic importance of financial institutions are at a very early stage of development and concerns remain about the robustness of the results; for instance, the models may not capture all the ways that a financial institution is systemically important (both quantitative and qualitative).*

The Basel Committee on Banking Supervision introduced an early approach to both challenges (BCBS-BIS, 2011), which has been recently updated (BCBS-BIS, 2013). Regarding the first challenge, the BCBS-BIS proposal relies mainly on traditional balance sheet data, against growing agreement on the convenience of using other data sources and technical approaches (Uribe, 2011a,b; León et al., 2011; ECB, 2010). About the second challenge, somewhat divergent from IMF et al. (2009) concerns and suggestions, the proposal by BCBS-BIS (2013) employs an equal and fixed weighting scheme for aggregating five metrics' categories (i.e. each one assigned a 20% weight), where the relevance of each metric does not seem to follow any technique –quantitative or qualitative. Furthermore, the equal and fixed weighted scheme proposed by BCBS-BIS may yield undesired results, such as biasing results towards the most volatile categories, and may naively assume that all financial systems are similar to each other.⁷

Unlike the BCBS-BIS (2013) proposal, León and Machado (2013) and León and Murcia (2012) tackle the first challenge by using balance sheet data and an application of network analysis to the large-value payment system's data. They both design four indicators or metrics for assessing *size*, *connectedness* and *substitutability* for the Colombian financial system. *Size* is captured via the volume of deposits and money market borrowing, and the volume of financial assets under management; *connectedness* is captured by measuring the contribution of each institution to the number and volume of the large-value payment system's transactions; whereas *substitutability* is captured by measuring the *betweenness centrality* (i.e. the brokerage role within a network) of each institution within the large-value payment system's network of transactions.

Regarding the second challenge, León and Machado (2013) employ a *fuzzy logic inference system* (henceforth a FLIS), an engineering-type approach based on the deconstruction of expert knowledge into a method that imitates the way experts themselves think about the decision process regarding what a systemically important financial institution is. *Au contraire*, based on the same metrics, León and Murcia (2012) implement *principal component analysis* (PCA), a standard (i.e. purely quantitative) dimension reduction method. According to León and Machado (2013), comparing both methods' results verify their non-redundancy, and suggest that both indexes should be regarded as complementary.

Hindsight, along with discussions and comments received by the authors of both research works⁸, encouraged enhancing and condensing the preceding proposals into the present document. Thus, based on the aggregation methods by León and Machado (2013) and León and Murcia (2012), this document aims to identifying systemically important financial institutions with respect to three new systemic importance metrics: (i) money

⁷ Other drawbacks of the BCBS-BIS (2013) methodological proposal are briefly discussed in León and Machado (2013). Other comments to the consultative document are posted in the BIS' webpage (<http://www.bis.org/publ/bcbs201/cacomments.htm>). Moreover, regarding the existence of biases towards the most volatile categories, BCBS-BIS (2013) implemented a cap to the substitutability category because the weights yielded a greater impact on the assessment of systemic importance than was intended. This issue is explained and addressed by Hurlin and Pérignon (2013).

⁸ Seminars and discussion sessions at Central Bank of Brazil (VII Annual Seminar on Risk, Financial Stability and Banking, 2012), Bank of Finland (XI Payment and Settlement System Simulator, 2013) and CEMLA (II Meeting on Financial Stability, 2012) were particularly useful for designing and introducing some of the methodological changes here proposed.

market net exposures network hub centrality; (ii) large-value payment system network hub centrality; and (iii) an adjusted balance sheet measure of asset size.

Modifying the choice of metrics follows theoretical and practical reasons. Changing from the weighted contribution of each institution to the large value payment system (i.e. a local centrality metric) to *hub centrality* is a qualitative leap towards a comprehensive (i.e. global) metric of the direct and indirect linkages of financial institutions in the two networks considered. Likewise, introducing the money market net exposures as a source of systemic impact conveys a greater explanatory power regarding connectedness as the potential factor behind contagion arising from the failure to fulfill payment commitments between financial institutions. In this sense, this document contributes to the design of metrics that are closer to the macro-prudential perspective of financial stability, in which the proximate objective is to limit financial system-wide distress, and in which prudential standards should be calibrated with respect to the marginal contribution of financial institutions to system-wide risk (Borio, 2003).

Furthermore, estimating the hub centrality on the large-value payment system network not only measures a commonly overlooked source of connectedness (i.e. intraday payments connectedness), but also captures *substitutability* based on BCBS-BIS (2013) definitions, whilst avoiding questions about the accuracy of betweenness centrality for payment networks (as in Soramäki and Cook, 2012). Finally, reducing the number of metrics from four to three makes the deconstruction of expert knowledge simpler and more tractable.

Results confirm that (i) credit institutions (i.e. commercial banks and other banking institutions) are the most systemically important type of financial institution in the local market; (ii) despite being unimportant because of their size, some non-banking institutions are systemically important because of their connectedness within the money market and the large value payment system; (iii) based on principal components analysis, the three metrics are non-redundant and explanatory; (iv) the deconstruction of expert knowledge suggests that linkages among financial institutions are considered as a major systemic impact factor, but size is judged as the main driver of systemic importance; (v) and both aggregating methods (i.e. FLIS and PCA) are non-redundant. Moreover, all in all, results concur with those of León and Machado (2013) and León and Murcia (2012).

This research document is structured as follows: next section introduces the systemic importance concept, focusing on the recommendations and concerns provided by IMF et al. (2009). Third section introduces the design of the systemic importance metrics and the intuition and basics behind the two aggregation methods: FLIS and PCA. Fourth section describes the database, the resulting systemic importance metrics and the aggregation procedure implemented, and presents the resulting assessment of systemic importance (i.e. the indexes) for Colombian financial institutions as of June 2013. The fifth section presents some final remarks.

2 Systemic risk and systemic importance⁹

As presented by IMF et al. (2009), G20 countries use a general definition of systemic risk: *the risk of disruption to financial services that (i) is caused by an impairment of all or parts of the financial system and (ii) has the potential to have serious negative consequences for the real economy.* In respect of payment systems, the Committee on Payment and Settlement Systems and the International Organization of Securities Commissions (CPSS-BIS and IOSCO, 2012) defines it as *the risk that the inability of one or more participants to perform as expected will cause other participants to be unable to meet their obligations when due.*

Irrespective of which of these definitions is embraced, and despite there is not a single definition of risk that can be completely satisfactory in every situation (Dowd, 2005)¹⁰, it is common to think of risk as a function based on two parameters: frequency and severity (Condamin et al., 2006), also referred as likelihood and impact, respectively (Gallati, 2003). Although academic effort has traditionally focused on systemic concerns based on the estimation of systemic risk (i.e. the product of frequency and impact, as in Norman et al. (2009)), there is a recent interest in focusing on systemic severity or importance.¹¹

For example, Paul Tucker, the then Executive Director for Markets and member of the Monetary Policy Committee of the Bank of England, pointed out the following (Tucker, 2005):

[T]he interesting question is not whether or not risk will crystallize, as in one form or another risks crystallize every day. Rather, the important question is whether, in the event of nasty shocks, our capital markets can absorb them or whether they have developed characteristics which may, as some suggest, leave them vulnerable.

More recently, as a result of the global financial crisis after 2007, the “Principles for Financial Market Infrastructures” (CPSS-BIS and IOSCO, 2012) includes several principles that aim to provide a high degree of confidence that a financial market infrastructure will continue operating and serve as a source of financial stability even in extreme market conditions. Principles 4 and 7 emphasize the importance of focusing on the severity of the systemic shocks; the latter addresses liquidity risk for financial market infrastructures:

Principle 7 (Liquidity Risk): A financial market infrastructure should maintain sufficient liquid resources to effect same-day and, where appropriate, intraday settlement of payment obligations with a high degree of confidence under a wide range of potential stress scenarios

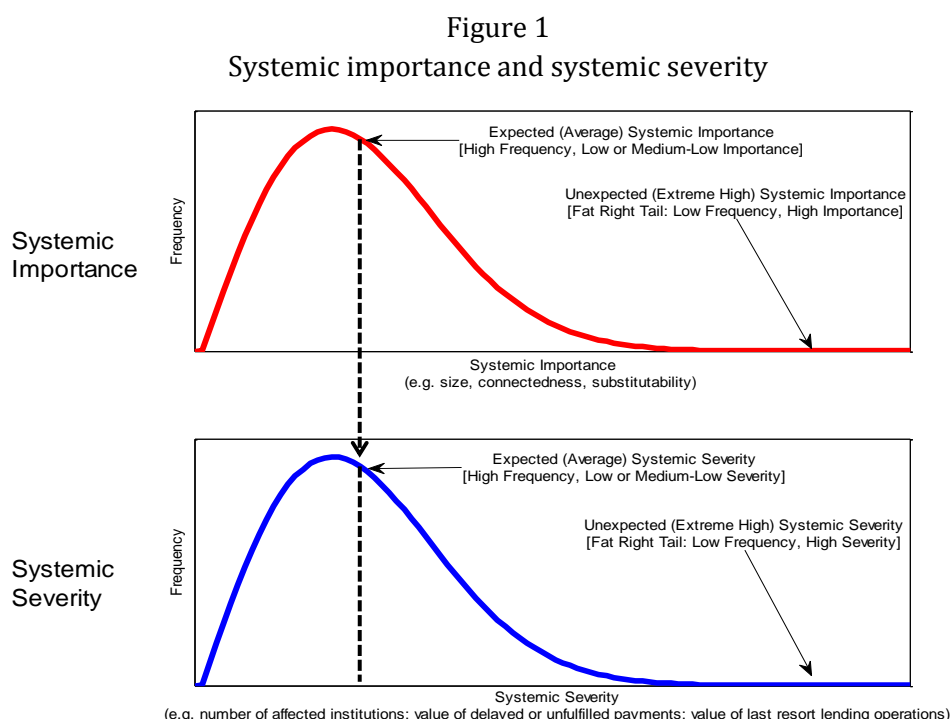
⁹ This section is an updated and augmented partial reproduction of León and Machado (2013).

¹⁰ A proper definition of risk is beyond the aim of this paper. Interesting reviews of risk definitions and their implications can be found in Hubbard (2009).

¹¹ Some authors (Rebonato, 2007; Taleb, 2007) argue that models and techniques for estimating very low probabilities of very disastrous occurrences have demonstrated to yield poor results, and even question the usefulness of those models and techniques for capturing extreme adverse events not found in historical data. Rebonato (2007) also questions the convenience of regulators using VaR-type approaches (i.e. based on estimating low probabilities) to determine prudential capital since even a high percentile (e.g. 99%) would allow a firm to incur losses equal to its regulatory capital rather often (i.e. 2-3 times a year); not to mention if extreme losses cluster, as they do.

that should include, but not be limited to, the default of the participant and its affiliates that would generate the largest aggregate liquidity obligation for the financial market infrastructure in extreme but plausible market conditions.

Such increasing interest in the impact of systemic shocks – beyond the interest in their frequency – results from the intrinsic characteristics of financial and payments systems. As pointed out by Haldane (2009), León et al. (2012) and León and Berndsen (2013), financial and payments networks today may be described as robust to random disturbances, but highly susceptible to targeted attacks.¹² This results from the systemic importance of financial institutions (e.g. *size, connectedness, substitutability*) being distributed with a high degree of asymmetry (right skew) and excess kurtosis, as under a *power-law* distribution, where the average institution is of low systemic importance (Figure 1, upper panel) and –thus– the average default or failure-to-pay results in low systemic severity (Figure 1, lower panel); correspondingly, systemically important institutions and their consequent high systemic severities lurk in the extreme right tail of the distributions.



Source: authors' design.

This means that the traditional focus on estimating risk as the sum of multiplying each participant's estimated frequency of failure (or near failure) by its corresponding estimated impact may be dangerously diverting financial authorities from their aim of preserving financial stability and payment systems safety: on average the financial stability and payments system safety may be "guaranteed", but not when confronted with the failure of a systemically important participant. That is, if financial authorities focus on

¹² As mentioned by Haldane (2009), this explains why there exist long periods of apparent robustness, where peripheral – not systemically important – nodes are subject to random shocks, and short but severe episodes of systemic distress, where systemically important institutions endanger financial stability. Therefore Haldane's characterization of the current international financial network: "robust-yet-fragile".

estimating probabilities of systemic events happening, they would be preparing themselves for a severe systemic shock based only on the impact of a single, systemically modest, average institution.

Moreover, estimating systemic risk as the sum of each participant's estimated frequency of failure (or near failure) multiplied by its corresponding estimated impact assumes that failures or near failures of different participants do not occur together (i.e. they are independent).¹³ As recently shown by the concurrent episodes of distress of AIG, Lehman and Bear Sterns, such independence is by no means guaranteed.

Therefore, as financial authorities should be prepared to confront a non-average but extreme threat to financial stability or payment systems' safety, their activities (i.e. supervision, oversight and regulation) should be designed to cope with one systemically important institution failing or near failing, as suggested by CPSS-BIS and IOSCO (2012). In this sense, financial authorities' policy and decision-making must rely on defining what systemic importance is, and identifying institutions that fall within such a definition.

According to IMF et al. (2009), an institution may be considered as systemically important if its failure or malfunction causes widespread distress, either directly or indirectly through contagion. The main criterion for assessing systemic importance relates to the institutions' potential to have a large negative impact on the financial system and the real economy. This overall criterion may be conveniently explained by three more precise criteria: *size*, *connectedness* and *substitutability* (as in IMF et al., 2009; Manning et al., 2009).

2.1 Size

Some authors regard an institution as systemically important if it exceeds an *asset-size* cutoff (Saunders et al., 2009), whilst others (IMF et al., 2009) prefer to assess the amount of financial services it provides to the system. This is the traditional approach to systemic risk, where the systemic importance of a financial institution generally increases with its *size*, and where systemically important institutions are labeled as *too-big-to-fail*.

There are some intuitive and straightforward metrics for the size of financial services provided to the system. Standard institution-centric accounting data already contains relevant information, such as balance and off-balance sheet exposures (e.g. deposits, money market borrowing and lending) and volume of assets warehoused or managed, among others. Other relevant *size* indicators such as the volume of payments by individual institutions are not publicly disclosed, but are available for financial authorities from their involvement in large-value payment systems or from their oversight and supervision duties.

¹³ The assumption of independence is fair for market risk where only one outcome can occur at a time. But for systemic or credit risk, where simultaneous occurrence of outcomes is feasible (e.g. several firms may enter into default simultaneously or within a short period), the independence assumption may be inappropriate.

2.2 Connectedness

According to the European Central Bank (2010) the properties and behavior of an institution may be affected by institutions that have links to it, and also by other institutions that have no direct links, but are linked to its neighbors. Therefore, the larger the number and volume of the links an institution has with other market participants, the larger the contagion or spillovers it may generate; that is, the systemic importance of a financial institution generally increases with its degree of *connectedness*. Despite its intuitiveness, this is a rather novel approach to systemic risk, where systemically important institutions are labeled as *too-connected-to-fail* (León et al., 2012; Machado et al. 2010; Chan-Lau, 2010; ECB, 2010; Clark, 2010; Zhou, 2009).

Unlike financial institutions' *size*, *connectedness* is more intricate to assess, with regulators and central banks currently lacking the resources to carry out this kind of analysis (Clark, 2010). Network analysis¹⁴ provides some concepts and metrics that may assist the assessment of *connectedness*. The most simple concept is the *in-degree* and *out-degree centrality*, which is related to the number of "neighbors" or "partners" an institution has within the network, where the former (latter) corresponds to incoming (outgoing) flows. Other concepts take into account the global properties of the entire network, where centrality arises from the centrality of the neighbors, as is the case with eigenvector centrality and other related centrality measures (e.g. *PageRank*, *authority* and *hub centrality*, *Katz centrality*).

Traditional application of network analysis for assessing systemic risk relies on balance sheet data such as interbank funding and lending, as in Garrat et al. (2011) or Chan-Lau (2010); however, balance sheet data may be unreliable due to accounting practices by reporting firms, as highlighted by Smith (2011)¹⁵. Alternatively León and Berndsen (2013), León and Pérez (2013b), León et al. (2012) and Machado et al. (2010) use financial transactions from financial market infrastructures (e.g. large value payment systems, and securities/foreign exchange settlement systems), which grants some advantages in granularity, completeness and opportunity; the choice of financial transactions over balance sheet data and other reports by financial institutions is suggested by Kyriakopoulos et al. (2009).

2.3 Substitutability

If the absence of a financial institution distorts the system because it is difficult (or impossible) to find another institution able to provide the same (or similar) type and

¹⁴ As discussed in the authors' prior works (León et al., 2012; Machado et al., 2010), network analysis provides appealing methods and techniques to cope with the need to change from an institution-centric to a systemic approach. The studies by Soramäki et al. (2006) and Bech and Garrat (2006) use network analysis to characterize the United States (Fedwire) payment system, while Inaoka et al. (2004) apply it to the Japan case (BoJ-Net). Cepeda (2008), León and Pérez (2013b) and León and Berndsen (2013) applies network analysis to the Colombian large-value payment system (CUD) and other financial market infrastructures to quantify the impact of failures on its stability.

¹⁵ Smith (2011) reports that Lehman used sale-repurchase (repo) agreements to reduce its recognized debt for dates surrounding quarterly reporting periods; by means of interpreting accounting standards, Lehman removed over \$50 billion from its balance sheet at the end of the fiscal quarter in May 2008, which reduced net leverage from 13.9 to 12.1.

volume of financial services (e.g. settlement, payments, interbank lending, custody, brokerage), such an institution is systemically important. As pointed out by Manning et al. (2009), the severity of the impact of a payment system failure, and hence the extent of systemic risk, depends critically on whether substitutes are readily available to allow payment flows to be rerouted via another system. Consequently, the systemic importance of a financial institution generally decreases with its degree of *substitutability*. *Connectedness* and *substitutability* are therefore both related to the *too-connected-to-fail* criteria.

Unlike financial institutions' *size*, the degree of *substitutability* may be difficult to assess. Despite there are cases in which it is easy to determine that a participant or infrastructure is *non-substitutable* (e.g. if there is only one infrastructure for all the market's clearing), it may be difficult to determine other participants' degree of *non-substitutability*.

For these cases network analysis provides some concepts and metrics that may assist the assessment of *substitutability*. An interesting concept is *betweenness centrality* (Newman, 2010 and 2003; Buechel and Buskens, 2008; de Nooy et al., 2005), which is a measure of a network's resilience based on the assessment of the involvement of a participant in the indirect connection of all other participants. *Betweenness centrality* is implemented in León and Machado (2013) and León and Murcia (2012) for assessing *substitutability*.

Nonetheless, despite the rationale behind capturing substitutability is evident in a traditional network analysis framework, dealing with a financial network has some particularities worth stating. First, as stressed by Inaoka et al. (2004), financial networks may be characterized by their *dynamic stability*, which means that connections between financial institutions may be reconfigured rather promptly given the proper intervention of financial authorities to either bail out the failing institution or supporting the remaining (i.e. non-failing) institutions; this is, unlike –for example– a physical network as the Internet or a power transmission grid, whose hardware may not be reconfigured rapidly or economically, financial institutions' networks may be reconfigured (i.e. rewired) to some extent.¹⁶ In this sense, the connective in-between role of a financial institution within a transactions or exposures network may not reveal whether a financial institution is substitutable (or not); similarly, Soramäki and Cook (2012) highlights that metrics that rely on the length of paths, such as *betweenness centrality*, may not be accurate within payment networks.

As with *connectedness*, network analysis for assessing *substitutability* could rely on data gathered from institutions' balance sheets (e.g. interbank funding and lending) or from financial market infrastructures (e.g. trade repositories, large-value payment systems and securities settlement systems). The choice of *connectedness* and *substitutability* metrics and of data source (i.e. balance sheet or large-value payment system) is next.

¹⁶ However, it is worth highlighting that a network of financial market infrastructures displays legal and operative rigidities that yield a network that may not be reconfigured promptly, as in the case of financial institutions' networks. In this sense, a financial market infrastructures' network may be regarded as similar to a physical network, with substitutability being a major issue.

3 Designing the systemic importance indexes

Based on the basic concepts previously stated (i.e. *size*, *connectedness* and *substitutability*), this section presents the authors' proposal for designing two systemic importance indexes corresponding to two different aggregation methods: FLIS and PCA. The first part introduces the three systemic importance metrics, their rationale and design; the second briefly presents the two aggregation methods.¹⁷

3.1 Designing the systemic importance metrics

According to recent literature on systemic importance for financial institutions and payment systems (IMF et al., 2009; Manning et al., 2009), the most relevant criteria are *size*, *connectedness* and *substitutability*, where the first two relate to their potential to have a large negative impact on the financial system and the real economy, whereas the latter relates to the magnitude of the indirect impact. Hence, the three chosen metrics must –to some extent– capture the three relevant criteria.

3.1.1 Money market net exposures network hub centrality

The first metric consists of the *money market net exposures network hub centrality*. This metric employs a centrality metric (i.e. hub centrality) as a measure of the importance of each financial institution within the sovereign securities collateralized (i.e. sell/buy backs or *simultáneas*¹⁸) borrowing network.

Several authors have addressed the importance of the sovereign sell/buy backs for characterizing the local money market (e.g. Martínez and León, 2013; León, 2012; Cardozo et al., 2011). For instance, after excluding Central Bank's repos, sovereign securities sell/buy backs are the most important source of liquidity among financial institutions, with 2010, 2011 and 2012 daily average value of transactions around 83% of the total, whereas repos between financial institutions account for about 1%, and non-collateralized borrowing around 16%, respectively.¹⁹ Accordingly, "money market" will refer loosely to the sell/buy backs (i.e. *simultáneas*) market.

The collateralized borrowing network among financial institutions used for this metric is constructed with data from DCV (Depósito Centralizado de Valores), which is the financial market infrastructure that serves as the sovereign securities settlement system for the

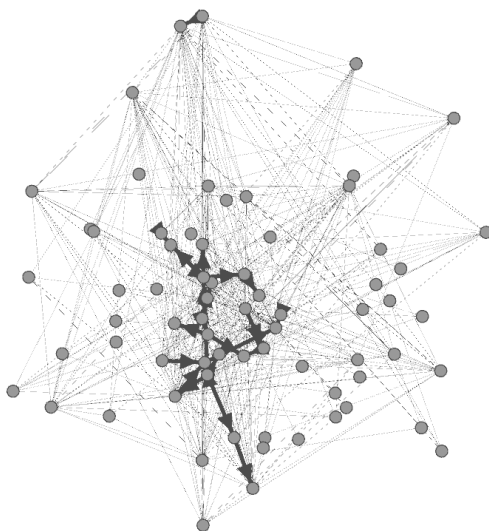
¹⁷ An interested reader may find further details on the fuzzy logic inference system (FLIS) method in León and Machado (2013) or Reveiz and León (2010).

¹⁸ Sell/buy backs consist of two sell and buy transactions simultaneously contracted, with the same principal amount and security, with both parties obliged to take the inverse position at maturity (i.e. the buyer becomes the seller), where the property of the collateral is transferred to its buyer. Under local regulation, unlike repos, haircuts and mobility limitations are not imposed on collateral, which may explain why Colombian financial firms prefer sell/buy backs to other sources, including repos with the Central Bank during some periods (León, 2012).

¹⁹ Not considering non-sovereign securities collateralized markets is due to data limitations. However, based on approximate figures, in 2012 79% of collateralized money market transactions (i.e. sell/buy backs and repos, excluding Central Bank's repos) used sovereign local securities (i.e. TES) as collateral, whereas 17% used other fixed income securities, and 4% used equities; therefore, this limitation is non-negligible, but it is by no means critical.

local market; this infrastructure is owned and operated by the Central Bank. This network consists of the net payment commitments of each financial institution in the local sell/buy backs market; put another way, this network corresponds to the money market net exposures among financial institutions. Figure 2 presents the graph corresponding to the average exposures during June 2013, where the direction and width of the arrow corresponds to the direction and value of the payment to be fulfilled, respectively.

Figure 2
Money market net exposures network



An arrow from i to j represents a commitment to pay from i to j .
The width represents the value of the commitment to pay.

Source: authors' estimations.

Using exposure data from the sovereign securities settlement system has several advantages. First, the sovereign securities market is the most liquid local market, where they serve as the main collateral for borrowing purposes. Second, unlike exposures from reported data (e.g. financial statements or periodic data transmissions by financial institutions), settlement system's information is complete, granular and opportune, and concurs with the main properties reported by Kyriakopoulos et al. (2009) as particularly advantageous from a supervisory and oversight perspective (i.e. transactions are available in real-time and cannot be –easily- falsified). Third, despite failure to pay in the money market does not entail principal risk due to delivery versus payment (DvP) arrangements, it may trigger several problems: (i) cash liquidity pressures due to counterparties of the failing institution not receiving the payment in a timely manner; (ii) cash and securities settlement gridlocks due to counterparties of the failing institutions not being able to fulfill their commitments with their counterparties; (iii) fire-sales (i.e. forced sales at a dislocated price) by the collateral holder, which may lower the ability of other participants to borrow against the same securities, and may have a negative effect on the mark-to-market and volatility of other institutions' portfolios, margin accounts and solvency, which may even yield a vicious reinforcing process (i.e. *liquidity spirals*).²⁰

²⁰ "Liquidity spirals" refers to the internal amplifying process whereby a falling asset leads to more sales (deleveraging), which further drives down asset prices, financial intermediaries' profit and loss statements,

Consequently, this network may be regarded as a depiction of the connectedness among financial institutions, where an institution concentrating future commitments to pay (i.e. to make the reverse sell/buy back or *retrocesión*) is expected to be more important for the money market; under BCBS-BIS (2013) definitions, this metric captures the *interconnectedness vis-à-vis other financial institutions* that arises from *the network of contractual obligations in which these institutions operate*. Moreover, since this network corresponds to the net exposures among financial institutions, the level of the obligations is also indicative of the size of the institution within the money market, and of the potential negative externalities that may arise from exposed institutions being forced to fire-sale the corresponding collaterals.

Despite measuring the relative importance of an institution in the *money market net exposures network* may be easily captured by measuring the number of counterparties (i.e. degree) or the value of the commitments to pay (i.e. strength), those measures are local in nature. Thus, they fall short to take into account the global or network effect of an institution failing to fulfill its obligations. Therefore, since the main interest is to assess financial institutions' contribution to system-wide risk, the metric should be able to acknowledge that systemic importance arises in the money market network from directly and indirectly owing to important counterparties.

Hence, based on the HITS algorithm by Kleinberg (1998), this document estimates the hub centrality of each financial institution in the *money market net exposure network*.²¹ The HITS (Hypertext Induced Topic Search) algorithm is an enhanced version of eigenvector centrality, where the main premise is to identify importance within a network based on a pair of interdependent circular thesis: (i) a participant is a good *hub* if it points to good *authorities*, and (ii) a participant is a good *authority* if it is pointed-to by good *hubs*.

In the case in hand, a financial institution is a good *hub* if (i) it is committed to pay to many counterparties, (ii) it is committed to pay a significant amount of all commitments, (iii) or both; whereas an institution is a good *authority* if (i) it is entitled to receive payments from many counterparties, (ii) it is entitled to receive a significant amount of all payment commitments, or both. Most importantly, due to its mathematical foundations (i.e. eigenvector centrality), the importance of an institution is proportional to the weighted sum of the importance of its counterparties at all possible order adjacencies, which allows for capturing the global role of each institutions within the network; a formal description of the HITS' *authority* and *hub centrality* is presented in Exhibit A.

and balance sheets' net worth (Brunnermeier et al. 2009). The issue regarding the negative externalities arising from fire-sales in collateralized borrowing is addressed by Stein (2012).

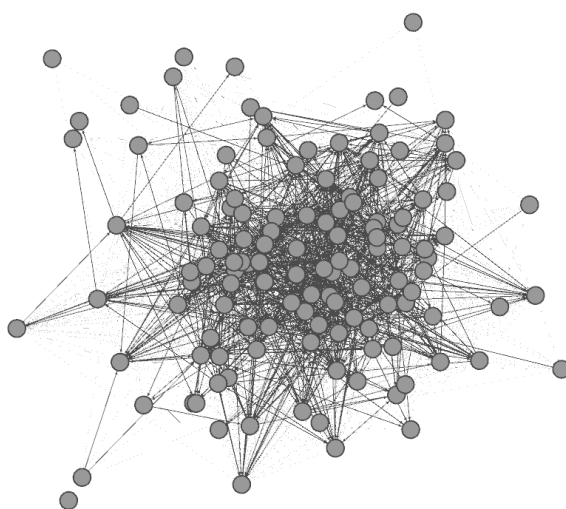
²¹ Other alternatives are available. However, as presented in León and Pérez (2013a), some shortcomings make the HITS authority particularly attractive. For instance, unlike standard eigenvector centrality, HITS may be estimated on directed networks such as the one in hand; unlike PageRank, no randomness is introduced in the model in order to attain convergence; unlike Katz centrality, no arbitrary initial centrality should be allocated to attain convergence. León and Berndsen (2013) provide a brief comparison between the basics behind HITS, PageRank and eigenvector centrality.

3.1.2 Large-value payment system network's hub centrality

The second metric consists of estimating the hub centrality of each financial institution within the large value payment system network. The data is provided by CUD, the only large-value payment system in the Colombian financial market, where all cash leg's settlement (in local currency) takes place; akin to DCV, this financial market infrastructure is owned and operated by the Central Bank.

This network consists of all payments (i.e. cash settlements) among financial institutions in the local market, where all institutions may settle their payments in a direct fashion (i.e. it is non-tiered). Figure 3 presents the graph corresponding to the average payments settled during June 2013, where the direction and width of the arrow corresponds to the direction and value of the payment, correspondingly.

Figure 3
Large-value payments network



An arrow from i to j represents a payment from i to j .
The width represents the value of the payment.

Source: authors' estimations.

Using data from the large value payment system has several advantages: (i) all financial transactions among financial institutions are settled in local currency, where the sole large-value payment system (i.e. CUD) processes and registers all of them; (ii) as emphasized by Kodres (2009), failure or insolvency are not the only sources of systemic shocks, but mere failure-to-pay or non-payment of transactions can gridlock the entire financial system; (iii) as acknowledged by Tumpel-Gugerell (2009), a particular institution might not only be systemically relevant because other institutions are financially exposed to it via balance sheet positions, but also because other market participants rely on the continued provision of its services.

Hence, this network may be regarded as representative of the *connectedness* among financial institutions, where institutions concentrating payments to other counterparties

are expected to be more important for the entire payment system, and for all the markets that settle their transactions in local currency (i.e. fixed income, foreign exchange, equity, derivatives). Furthermore, this network may be indicative of the amount of financial services provided by each financial institution, a non-balance sheet based measure of their relative size within the financial system.

Additionally, estimating the hub centrality on the large-value payment system network not only measures a commonly overlooked source of connectedness (i.e. intraday payments *connectedness*) and serves as a measure of financial institutions' size, but also captures *substitutability* based on BCBS-BIS (2013) definitions. In this sense, the more central a financial institution is within the large-value payments system, the more it may be regarded as a service provider in the underlying infrastructure, and the larger the service and liquidity disruption that would follow its failure.

As with the previous metric, the relative importance of the participating institutions is measured by their *hub centrality*. In this sense, a financial institution is important because it pays to a large number of counterparties, because it pays large volumes to its counterparties, or both, where all orders of adjacent counterparties are considered. Again, most importantly, due to its mathematical foundations (i.e. eigenvector centrality), the importance of an institution is proportional to the weighted sum of the importance of its counterparties at all possible order adjacencies.

3.1.3 Adjusted assets

The third metric corresponds to an adjusted version of the traditional approach to systemic importance: size. Based on the size of financial institutions' assets reported in periodic financial statements (i.e. balance sheets), the chosen metric consists of a broad measure of the value of the assets owned by each financial institution that may be involved in case of its failure, and that may cause some type of negative disruption in financial markets and the real sector. In this sense, this metric seeks to fulfill BCBS-BIS (2013) rationale: *the larger the financial institution, the more difficult it is for its activities to be quickly replaced by other institutions and therefore the greater the chance that its distress or failure would cause disruption to the financial markets in which it operates.*

More precisely, the metric corresponds to the total value of assets net of cash and property, plant and equipment²², where such netting aims at filtering out the size of the core financial services (e.g. money market, corporate and retail lending) provided by each financial firm to the financial market and the real sector, and the size of the proprietary portfolios that may be used in a fire-sale, and may –in turn- cause *liquidity spirals*.

Furthermore, since it is expected that depository institutions do not hold a significant share of deposits and other short-term liabilities in the form of cash or property, plant and equipment, this metric of adjusted assets should serve also as a fair proxy of the size of

²² "Property, plant and equipment" is a term used in the United States (e.g. FASB) and the Colombian accounting standards, and it generally refers to any physical structure or equipment attached to the real estate that cannot be removed and used separately without incurring significant cost.

financial institutions' liabilities that may be at risk for their financial and non-financial (e.g. households, corporations) counterparties.²³

3.1.4 Matching the importance metrics with the criteria

A single systemic importance metric may capture one, two or three systemic importance criteria, where defining the boundaries of each metric with precision is difficult at best. In the case of the three chosen metrics it is possible –and illustrative- to summarize their expected contribution to the assessment of the metrics as follows:

Table 1
How the selected metrics of systemic importance relate to criteria from IMF et al. (2009) and Manning et al. (2009)

Metrics / Criteria	IMF et al. (2009) and Manning et al. (2009) criteria		
	Size	Connectedness	Substitutability
Money market net exposures network hub centrality			
Large value payment system network hub centrality			
Adjusted assets			

Directly captured	
Indirectly captured	
Non-captured	

Source: authors' design

Size is directly captured by the *adjusted assets* metric; it is a forthright measure of the potential disruption due to the volume of financial services provided by each financial institution to the economy as a whole (e.g. lending, servicing deposit withdrawals), and of the potential to disturb the normal functioning of financial markets due to the volume of financial assets it owns –and may sell under critical circumstances. However, the two hub centrality related networks also capture the relative size of each financial institution, where size corresponds to the importance of its net commitments exposures within the money market, and to the importance of its payments within the large value payment system.

Connectedness, which is intended to measure the systemic importance due to the number and volume of linkages a financial institution has, is directly captured by two metrics: *money market net exposures network hub centrality* and *large value payment system network's hub centrality*. Both metrics are based on the *hub centrality* of each financial institution within the corresponding network, in which the higher the number and volume of linkages the higher the *hub centrality*.

²³ For instance, for the period under analysis (i.e. June 2013) the estimated correlation coefficient between the adjusted asset metric and deposits is close to 1. This estimation is restricted to institutions allowed to take deposits from the public (i.e. credit institutions).

Additionally, the *adjusted assets* size metric includes the volume of financial assets each financial institution owns, which may be regarded as a measure of the potential disruption that may be indirectly caused to other agents in the economy (e.g. other financial institutions, households, corporations) if the failing institution is forced to fire-sale its portfolios; this is, since the adjusted assets metric comprises the volume of proprietary outstanding securities, it captures to some extent the potential contagion by means of price mechanisms. In this sense, under the definitions by BCBS-BIS (2013), adjusted assets may be regarded as a measure of interconnectedness as well.

Since substitutability is expected to be positively related to the role of financial institutions as market participants and client service providers (BCBS-BIS, 2013), both hub centrality-based metrics capture such role within the money market and the payment system, where the mathematical nature of the hub centrality (i.e. the importance being proportional to the importance of its counterparties at all possible order adjacencies) necessarily involves the global role of each financial institution for the payment system and the money market. However, it is worth noticing that BCBS-BIS (2013) regards payments activity as a leading indicator for substitutability, whereas the (money market) exposures are considered as leading indicators of interconnectedness only.

Despite the indisputable relevance of substitutability as a systemic importance factor, its measurement from a network perspective is by no means straightforward. As previously stated, Inaoka et al. (2004) points out that financial networks may be reconfigured promptly given the proper intervention of financial authorities to either bail out the failing institution or supporting the remaining (i.e. non-failing) institutions. In this sense, regulation may partially cope with substitutability via supporting the effective reconfiguration (i.e. rewiring) of financial networks.

In the Colombian case, regulation may ease the reconfiguration due to the segregation of customer's assets (i.e. from those of the financial institution and from other customers²⁴) and to the existence of resolution mechanisms (e.g. transferring assets, liabilities and contracts from the failing institution to a sound institution), whereas the robustness of financial market infrastructures and their related payment and settlement systems allows market participants to *continue to trade, knowing that the transactions would almost certainly be settled and cleared without difficulty* (Dudley, 2012).

Moreover, the relative size of each financial institution, measured by the *adjusted assets* metric, also involves their role as participants and service providers within critical markets (e.g. housing, lending, and commerce); thus, this size metric also captures substitutability to some extent.

As presented in Table 2, the three systemic importance metrics are consistent with the 2010 Dodd-Frank Wall Street Reform and Consumer Protection Act.²⁵

²⁴ French et al. (2010) highlight the importance of assets' segregation: *in the event of bankruptcy [...] the customer continues to own the securities in a segregated account. [...] If its assets are not segregated, the customer merely holds a contractual claim [...]*. Moreover, due to assets' segregation within Colombian regulation, assets from third parties (i.e. clients) are not considered in this proposal.

²⁵ The 2010 Dodd-Frank Wall Street Reform and Consumer Protection Act (hereafter Dodd-Frank Act) is United States of America's legislative response to the most recent episode of international financial crisis. Its

Table 2
How the selected metrics of systemic importance relate to criteria from the 2010 Dodd-Frank Wall Street Reform and Consumer Protection Act.

Metrics / Criteria	Dodd-Frank Act criteria			
	Aggregate monetary value of transactions	Aggregate exposure to counterparties	Inter-dependencies and interactions with other participants	Effect on critical markets, institutions and the system
Money market net exposures network hub centrality				
Large value payment system networks hub centrality				
Adjusted assets				

Directly captured	
Indirectly captured	
Non-captured	

Source: authors' design.

It is important to emphasize that the choice of broad metrics follows several considerations. First and most important, broad metrics allow for simultaneously assessing systemic importance of banking and non-banking financial institutions.²⁶ Unlike most models on assessing systemic importance, which are focused on banking institutions (as in BCBS-BIS (2013)), the authors judge imperative to be able to consider non-banking institutions as relevant as banking institutions; as non-banking-related systemic events have demonstrated (e.g. LTCM, Lehman, AIG, Bear Sterns, Freddie Mac and Fannie Mae), and as put forward by Ötoker-Robe et al. (2011), it is essential to improve the understanding of the shadow banking system to prevent non-banking institutions from gaining systemic importance in an unnoticed manner.

Second, broad metrics allow for a parsimonious model, which would allow for continuous (e.g. monthly) monitoring of systemic importance. Third, broad metrics are convenient for comparing results across different financial systems.

main objective is to promote the financial stability of the United States: as Section 804 of the Act addresses the main considerations in designating what systemic importance.

²⁶ However each key indicator may be broken down into other –more specific- metrics, as the document by the BCBS-BIS (2013) suggests. Nevertheless, such decomposition may result in an implicit preference for assessing systemic importance of some types of financial institutions (e.g. commercial banks), whilst overlooking others (e.g. brokerage firms, hedge funds). Hence, authors suggest to use wide-range metrics when initially implementing the proposed model, and subsequently increasing their specificity if necessary.

3.2 Aggregating the systemic importance metrics

Based on the fuzzy logic inference system (FLIS) and the principal component analysis (PCA) methods proposed by León and Machado (2013) and León and Murcia (2012), respectively, this section describes the aggregation procedures that yield the two systemic importance indexes here suggested.

3.2.1 Fuzzy logic inference system (FLIS)²⁷

The fundamental concept of *ordinary sets* is “membership”, which states that an element belongs or not to a set. This type of set, described by an unambiguous definition and boundaries, is known as *ordinary* or *crisp sets*; these sets are characterized by discrete-bivariate membership (*yes* or *no*, 1 or 0, *true* or *false*) and classic, Boolean or Aristotelic logic.

In contrast to *ordinary sets*, Lofti A. Zadeh (1965) acknowledged the fact that in reality there are elements characterized by *membership functions* which are not discrete, but continuous, where different degrees of membership exist between *yes* or *no*, 1 or 0, *true* or *false*; this type of set has unclear boundaries, therefore Zadeh named them as *fuzzy sets*. As stated by Sivanandam et al. (2007), the main contribution of the *fuzzy set* concept is the ability to model uncertain and ambiguous information, the kind of information frequently found in real life.

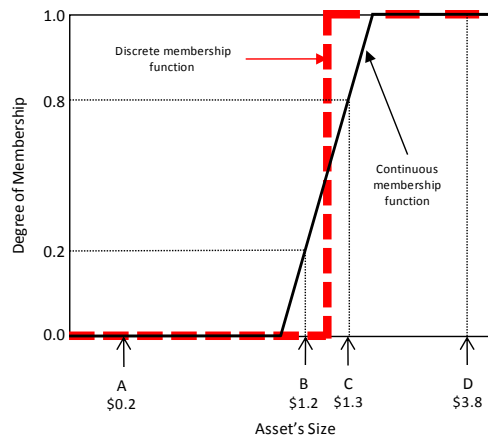
In this study it would be difficult to label a financial institution as *too-big-to-fail* based on a unique threshold related to the *size* of its assets, as suggested by Saunders et al. (2009). Figure 4 compares a discrete *membership function* typical of ordinary sets with a *fuzzy sets'* continuous *membership function*, where the criterion is financial institutions' assets' *size*²⁸.

The discrete *membership function* (dashed line) may yield non-intuitive and impractical results, which could seriously misguide financial authorities' analysis and decision making: (i) despite being clearly different, institution A and B are both regarded as non-systemically important; (ii) institution C's *size*, despite not being significantly different from B's, is considered as resulting in a *too-big-to-fail* institution; (iii) notwithstanding institution D is significantly bigger than C, they are both regarded as equally important because of their *size*.

²⁷ This chapter is based extensively on Reveiz and León (2010), where fuzzy logic theory and the design of a fuzzy logic inference system (FLIS) are briefly explained. Several references were omitted for practical reasons. The familiar reader may skip this section.

²⁸ A *membership function* is the line which defines the transition between sets, thus mapping the degree of membership of the elements of such sets. A continuous *membership function*, typical of *fuzzy sets*, recognizes that elements may belong to different categories in some degree, with this degree varying in a smooth and continuous manner.

Figure 4
Assets' size as an ordinary and a fuzzy variable



Source: authors design

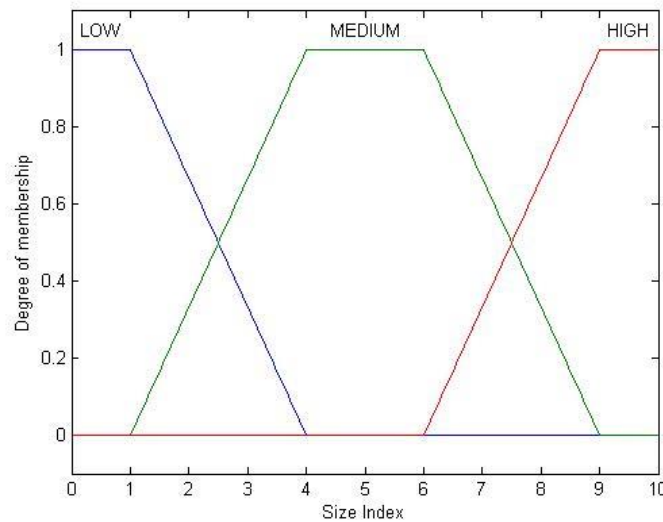
On the other hand, a simple continuous *membership function* that replicates the same *too-big-to-fail* approach to systemic risk yields intuitive and practical results. Institution A is regarded as non-systemically important, with a null degree membership on the *size* criterion; institution B and C are systemically important to some extent, where B's degree of membership to the *too-big-to-fail* criterion (20%) is lower than C's (80%); and where D's *size* corresponds unequivocally (100%) to a systemically important institution due to its *size*.

It can be seen that the membership of the *size* set is not clearly bounded, it is a matter of degree, and it is better described by a *fuzzy set*. It is straightforward to apply the same rationale to other criteria, such as *connectedness* and *substitutability*. This is rather important since, as noted by IMF et al. (2009), assessing the systemic importance of a financial institution does not lend itself to binary outcomes.

Additionally, in order to attain greater generality, higher expressive power, an enhanced ability to model real-world problems and, most importantly, a methodology for exploiting the tolerance for imprecision (Klir and Yuan, 1995), it is possible to use a mixture of continuous *membership functions* to further characterize the degree of membership.²⁹ Such a mixture is presented in Figure 5.

²⁹ The choice of the *membership function* is somewhat arbitrary but should be done with simplicity, convenience, speed and efficiency in view (Mathworks, 2009).

Figure 5
Assets' size as a fuzzy variable



Source: authors' design

Three trapezoidal *membership functions* are used to evaluate the degree of membership with three categories for *size* (i.e. LOW, MEDIUM, HIGH) instead of using a single category (i.e. big), where the nominal value of *size* (e.g. US billion) is replaced by a *size index* from 0 to 10.³⁰ All three trapezoidal *membership functions* have the same mathematical form by means of dividing the x-axis numerical range in three non-symmetrical ranges that yield the overlapping feature required for a continuous global function to exist.³¹

This procedure, consisting of the conversion of a crisp quantity to the appropriate *fuzzy sets* through the use of *membership functions*, is known as *fuzzification* (Sivanandam et al., 2007; Klir and Yuan, 1995; McNeill and Thro, 1994). An important property of this procedure is that *fuzzy logic* models are rarely sensitive to the choice of *membership function* (Cox, 1994), making them quite robust, which is an important property when models are initially prototyped.

Concerning the logic used to evaluate propositions, *ordinary sets* rely on *ordinary logic*; this type of logic, also known as classical, Aristotelic or Boolean logic, conceives the universe in terms of well-structured categories, where an item is either a member of a set or not. Using the logical operators AND, OR and NOT, which correspond to conjunction, disjunction and complement, respectively, propositions are evaluated as follows:

³⁰ The *size index* consists of a typical standardization of the nominal values of assets' *size* for each institution considered; the biggest institution is assigned the maximum index value (10 in this case) and the rest is assigned an index value by means of linear interpolation. As will be explained below, such standardization is straightforward and makes comparisons and calculations easier.

³¹ Cox (1994) emphasize that special attention should be drawn to the overlapping between *membership functions*: the overlapping is a natural result of fuzziness and ambiguity associated with the segmentation and classification of a continuous space.

Figure 6
Ordinary logical operators

Conjunction			Disjunction			Complement	
A	B	A AND B	A	B	A OR B	A	NOT A
0	0	0	0	0	0	0	1
0	1	0	0	1	1	1	0
1	0	0	1	0	1		
1	1	1	1	1	1		

Source: León and Reveiz (2010), based on Mathworks (2009).

Ordinary sets can be regarded as a particular case of *fuzzy sets*, in which degrees of membership are restricted to two extreme alternatives (0 or 1). As a result the choice of the *fuzzy logical operators* should be able to preserve the *ordinary logical operators* for bivariate memberships as in Figure 6 and be capable of evaluating multivariate degrees of membership. This is conveniently and typically attained by using $\min(\cdot)$ instead of AND for conjunction, $\max(\cdot)$ instead of OR for disjunction and $1-(\cdot)$ instead of NOT for complement.

The existence of these fuzzy logical operators allows for developing and evaluating *fuzzy inference rules*, which are rules for deriving truths from stated or proven truths (McNeill and Thro, 1994). The set of *fuzzy inference rules* or *knowledge base* that contains general knowledge pertaining to a problem domain connects antecedents with consequences, premises with conclusions, or conditions with actions (Klir and Yuan, 1995). If *A* and *B* are fuzzy sets, the simplest form of a *fuzzy inference rule* is the following:

if A, then B

For the case in hand, with the three criteria previously considered, the rules may look like the following:

*If SIZE is [LOW / MEDIUM / HIGH] AND ...
SUBSTITUTABILITY is [LOW / MEDIUM / HIGH] AND ...
CONNECTEDNESS is [LOW / MEDIUM / HIGH],
THEN SYSTEMIC IMPORTANCE IS...
[VERY LOW / LOW / MEDIUM LOW / MEDIUM HIGH / HIGH / VERY HIGH]*

Inference rules result from expert knowledge and try to imitate human reasoning capabilities. Cox (1994) claims that the process of building a *knowledge base* via the design of *fuzzy inference rules* forces experts to deconstruct their expertise into fragments of knowledge, which results in a significant benefit from fuzzy system modeling: to gain the ability to encode knowledge directly in a form that is very close to the way experts themselves think about the decision process³²; this is commonly referred as “approximate reasoning” (Serrano and Seraji, 2007).

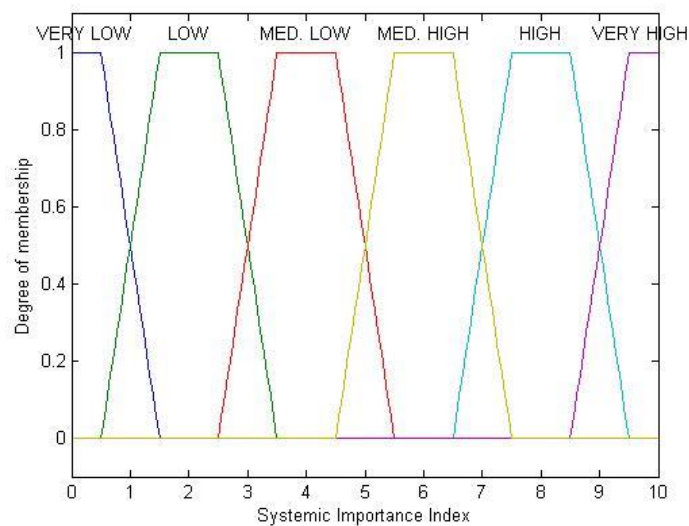
As stressed by Sivanandam et al. (2007), the Achilles’ heel of a fuzzy system is its rules; smart rules give smart systems and other rules give less smart or even dumb systems.

³² Cox (1994) emphasizes that conventional expert and decision systems fail because they force experts to crisply dichotomize rules, resulting in an unnecessary multiplication of rules and the inability to articulate solutions to complex problems.

Bojadziev and Bojadziev (2007) emphasize the important role played by the experience and knowledge of human experts when developing the *knowledge base* because they are appointed to state the objective of the system to be controlled.

The evaluation of the *inference rules* is carried out by a *fuzzy inference processing engine*, which is based on the *fuzzy logical operators* previously introduced. The *fuzzy inference processing engine* is in charge of evaluating input's degree of membership to the fuzzy output sets (Figure 7) according to all the *inference rules*, where such evaluation is done simultaneously.³³ As exhibited in Figure 7 the fuzzy output set consists of a mixture of six trapezoidal *membership functions* for systemic importance [VERY LOW / LOW / MEDIUM LOW / MEDIUM HIGH / HIGH / VERY HIGH].³⁴

Figure 7
Systemic importance as a fuzzy variable



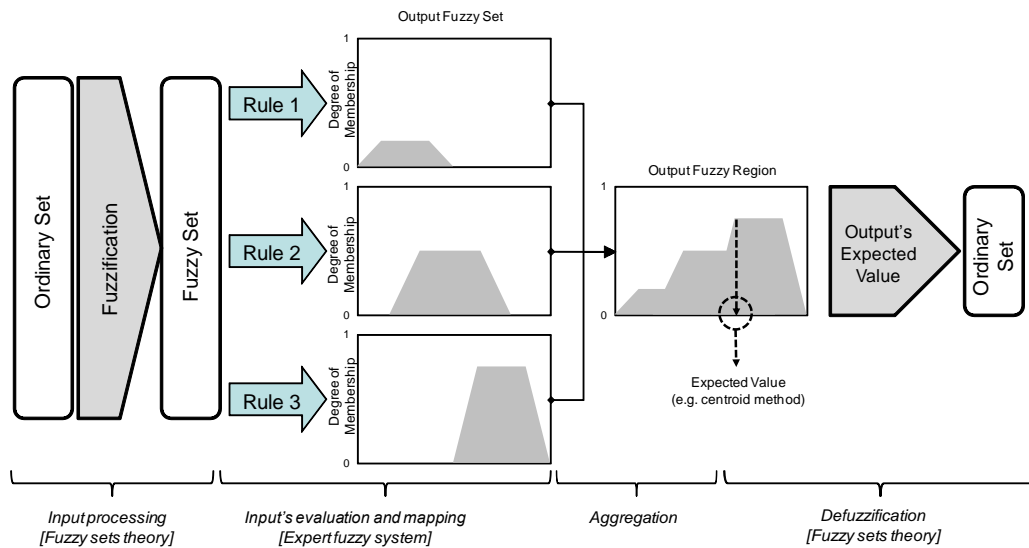
Source: authors' design.

Each time the *fuzzy inference processing engine* evaluates an input's degree of membership to the *inference rules* it maps each solution variable into its corresponding output *fuzzy set*, where the resulting number of output *fuzzy sets* matches the number of *inference rules* used to evaluate the inputs. For example, as in the left part of Figure 8, evaluating and mapping an input with three *inference rules* would result in three output *fuzzy sets*. The aggregation of these three *fuzzy sets* produces the final output fuzzy region, which contains the information of the degree of membership (or truth) of the inputs (or propositions) after the simultaneous evaluation of the *inference rules*.

³³ According to Cox (1994) the main difference between conventional expert systems and a *fuzzy expert system* is the latter's simultaneous evaluation of inference rules, which compared to the serial evaluation of the former has the advantage of being able to examine all the rules and their impact in the output space.

³⁴ The choice of the input's and output's number of *membership functions* follows two criteria. First, the number of *membership functions* should allow a detailed characterization and differentiation of what a systemically important institution is. Second, the number of membership functions should be limited in order to avoid unnecessary complexity for the model, and to facilitate deconstructing experts' knowledge. As in Figure 5, the x-axis range is divided in order to preserve the mathematical form of all the trapezoidal *membership functions* and to obtain the overlapping feature required for a continuous global function to exist.

Figure 8
The Fuzzy Logic Inference System



Source: Reveiz and León (2010).

Afterwards, because a single and crisp quantity is required (i.e. the index), the best representative value (i.e. expected value) of the output fuzzy region has to be calculated; this process is known as *defuzzification*, and corresponds to the calculation of the expected value of the output (Cox, 1994).

According to Sivanandam et al. (2007), Klir and Yuan (1995) and Cox (1994), the most used *defuzzification* method is the *centroid*, also known as the *center of gravity method* or *center of area method*.³⁵ It is calculated as the weighted average of the output fuzzy region, and corresponds to the point in the x-axis which divides the output fuzzy region into two equal subareas (Figure 6).

In this study the result of the *defuzzification* is a systemic importance index level, as presented in Figure 7's x-axis. This index level corresponds to a quantitative relative assessment of the systemic importance of each institution based on its inputs (metrics) and the expert knowledge embedded in the fuzzy rules set.

Finally, according to McNeill and Thro (1994), the combination of *fuzzy inference rules* and the *fuzzy inference processing engine* –based on *fuzzy logical operators*- results in an *expert fuzzy system*. Jointly, as in Figure 8, the use of an *expert fuzzy system* and *fuzzy sets theory* results in a *fuzzy logic inference system* (FLIS).

³⁵ Cox (1994) highlights *centroid's* consistency and well-balanced approach, its sensitiveness to the height and width of the total fuzzy region and the smooth changes in the expected value of the output across observations, behaving similarly to Bayesian estimates; that is, it selects a value that is supported by the knowledge accumulated from each executed proposition. Cox also emphasizes that unless there are reasons to believe that the model requires a more advanced or specialized method of *defuzzification*, the model should be limited to either the *centroid* or the *max-membership-principle* method; therefore, this is the authors' choice. Sivanandam et al. (2007), Klir and Yuan (1995) and Cox (1994) refer to other less common methods.

Fuzzy logic has been extensively employed in the real world, mostly in an engineering context, to control systems where the timing and level of inputs are at least to some extent uncertain (Cruz, 2002). Some of its most important applications include NASA's software design for safe and reliable autonomous landing of spacecrafts and rover navigation (Serrano and Seraji, 2007; Howard and Seraji, 2002 and 2000; Tunstel et al., 2001; Seraji, 2000), along with everyday applications to medicine, automotive industry, water treatment, air and ground traffic control, and home appliances design (Sivanandam et al., 2007; von Altrok, 2002 and 1996; Klir and Yuan, 1995; McNeill and Thro, 1994). Its application to finance and economics is related to insurance, credit card fraud detection, credit risk analysis, bond ratings and operational risk (Reveiz and León, 2010; Bojadziev and Bojadziev, 2007; Bundesbank, 1999; McNeill and Thro, 1994).

To the best knowledge of the authors, León and Machado (2013) is the first attempt to use a FLIS to aggregate systemic importance metrics or factors into a single systemic importance index for financial institutions.

3.2.2 Principal component analysis (PCA)

The aim of principal component analysis (PCA) is to reduce the dimensionality of highly correlated data by finding a small number of uncorrelated linear combinations that account for most of the variability of the original data (McNeil et al., 2005); alternatively, as described by Campbell et al. (1997), PCA is as a technique that permits to reduce the number of variables being analyzed without losing too much information in the covariance matrix.

The case in hand is particularly suitable for PCA since there are three metrics related to systemic importance for a set of financial institutions, and the weighting scheme is unknown. Therefore, the main objective is to construct a consolidated measure of systemic importance taking into account the chosen set of metrics.

Let R_t represent the original set of variables with dimension $(i \times n)$ and Ω its sample covariance matrix with dimension $(n \times n)$, the PCA model uses the *spectral decomposition* of the positively semidefinite and symmetric Ω as in [§1], where Λ corresponds to a diagonal matrix of eigenvalues of Ω , and Γ is an orthogonal matrix satisfying $\Gamma\Gamma' = \Gamma\Gamma = I_n$, whose columns are eigenvectors of Ω .

$$\Omega = \Gamma\Lambda\Gamma' \quad [\text{§1}]$$

If the diagonal matrix of eigenvalues (Λ) is ordered so that $\lambda_1 \geq \lambda_2 \cdots \lambda_n$, the first column in Γ corresponds to the principal eigenvector of Ω . The principal eigenvector (Γ_1) may be considered as the leading vector of the system or the first principal component, the one that is able to explain the most of the underlying system, where the positive n -scaled scores corresponding to each element may be considered as their weights within an

index.³⁶ As in McNeil et al. (2005), the first vector of loadings is positively weighted and can be thought of as describing a kind of index.

Regarding the explanatory power of the principal eigenvector, it is customary to calculate the ratio in [§3], which yields the contribution of the first eigenvalue to the sum of the eigenvalues.

$$\varpi_{\lambda_1} = \frac{\lambda_1}{\sum_{k=1}^n \lambda_k} \quad [\text{§3}]$$

If the linear combination expressed in the first eigenvector (Γ_1) can explain a representative fraction of the information of the covariance matrix (e.g. if $\varpi_{\lambda_1} \gg n^{-1}$), then it is feasible and sound to use the first principal component in order to assign an appropriate weight to the different variables. As a result we can summarize in an effective way the implicit information of different characteristics and individuals in a linear form, where such usage of PCA may be described as *principal components as factors* (as in McNeil et al., 2005).

PCA methodology has been widely applied in many fields. Commonly, the main objective is to construct an aggregate measure combining different characteristics which can be correlated among them. Some PCA-based related applications are listed for illustrative purposes: the construction of an index for the quality of international universities (Steiner, 2006); households' wealth indexes for India (Filmer and Pritchett, 1998); stock market indexes (Feeney and Hester, 1964; McNeil et al., 2005); an index of credit rating history of loans granted by financial institutions to particulars in Colombia (Murcia, 2007); a composite index to measure economic activity (FRB-Dallas, 2003); a Real Sector Business Confidence Index for Turkey (Oral et al., 2005); an index to measure financial markets' stress (Amol, 2010); a financial stability index for Colombia (Morales and Estrada, 2010) and financial conditions indexes for different countries (Hatzius et al., 2010; Gómez et al., 2011), among many others.

PCA approach has already been used in the systemic risk literature as well. For instance, in order to capture the systemic importance of financial institutions in the United States, Billio et al. (2010) used this approach to capture the interconnectedness among the monthly returns of hedge funds, banks, brokers, and insurance.

Rodriguez-Moreno and Peña (2011) study and compare different systemic risk measures for the biggest banks in Europe and the United States. They find that simple measures based on PCA of banks' credit default swaps (CDS) and interbank rates performed better

³⁶ Alternatively, based on Campbell et al. (1997), the first principal component (Γ_1) is the $(1 \times n)$ vector that is the solution to the following maximization problem:

$$\begin{aligned} & \text{Max}_{\Gamma_1} \Gamma_1 \Omega \Gamma_1' && [\text{§2}] \\ \text{subject to} & && \\ & \Gamma_1 \Gamma_1' = 1 && \end{aligned}$$

The solution to this problem corresponds to the eigenvector associated with the largest eigenvalue of Ω .

than more complicated measures based on structural credit risk models (à la Merton, 1974), collateralized debt obligations (CDO) indices and their tranches, multivariate densities and co-risk measures. Additionally, De Cadenas et al. (2010) used PCA in order to identify and evaluate different sources of risk when identifying the systemic nature of an entity. Their analysis shows that all considered institutions contribute to systemic risk, albeit to a different degree, depending on various risk factors such as size, inter-connection, substitutability, balance sheet and risk quality.

Perhaps the most relevant work on systemic risk under PCA approach is Kritzman et al. (2011), which addresses the statistical properties and associations of a large set of financial assets. In this paper they introduce a useful concept called “absorption ratio” as a measure of financial fragility. This ratio is defined as the proportion of variance which is explained by a finite number of eigenvectors. In the words of the authors, *a high value for the absorption ratio corresponds to a high level of systemic risk, because it implies the sources of risk are more unified*, whereas *a low absorption ratio indicates less systemic risk, because it implies the sources of risk are more disparate*. Kritzman et al. (2011) stress the fact that scenarios with high systemic risk do not necessarily lead to asset depreciation or financial turbulence; it could be simply an indication of market fragility since a shock is more likely to propagate quickly and broadly when sources of risk are tightly coupled.

However, to the best knowledge of the authors, this methodology has been used for evaluating the systemic importance of financial institutions by León and Murcia (2012) alone. The basic idea, as in León and Murcia, is to combine appropriately the metrics that are associated to the characteristics that the literature has identified as the determinants of systemic importance (i.e. size, connectedness and substitutability), and then to construct a PCA-based index using the scoring factors of the first principal component. The value of the index would allow for ranking different financial institutions according to their systemic importance.

3.2.3 Balancing the aggregation methods

It is worth emphasizing three advantages resulting from using expert knowledge within the proposed *fuzzy logic inference system* (FLIS). First, deconstructing expert knowledge enables the recognition of the main characteristics of the financial system under analysis. It is most likely to find that two different countries’ financial systems result in two different sets of *inference rules*, even if the panel of experts is the same. Likewise, it is natural to find that the same financial system results in different sets of *inference rules* across time; the evolution of the institutional framework, participants, products, services and regulation would explain such finding. This is why expert knowledge gathered from Colombia’s central bank is relevant for the current Colombian case only.

Second, unlike a weighting approach³⁷, where the aggregated index results from the linear weighted sum of all metrics, deconstructing expert knowledge enables the capture of non-

³⁷ Unlike the suggested FLIS approach, which uses expert knowledge to determine the importance of each key indicator and of all their possible combinations, the document by the BCBS-BIS (2013) suggests an arbitrary, equal and fixed weighting approach (i.e. five major metrics, each one assigned a 20% weight). Besides not being able to capture non-linearities arising from combining metrics, the weighting approach may be

linearities arising from accumulating metrics. This is a convenient feature since it is intuitive that the systemic impact arising from merging two financial institutions is expected to be higher than the mere weighted sum of their systemic importance; different from portfolio theory, in which adding assets results in an equal or lower volatility of the portfolio, aggregating financial institutions (i.e. their metrics) may result in higher systemic importance.

Third, unlike a fixed weighting approach, where weights remain constant across different combinations of metrics, deconstructing expert knowledge allows for capturing non-linearities arising from combining metrics. For instance, it is rather intuitive that the bigger or more connected a financial institution, the more important its degree of *substitutability*; *substitutability* may not be a significant factor when the institution's contribution to the payment system is low, but may become decisive when it is high. This type of non-linearity is ignored in a simple weighting approach, but may be captured by experts' judgment.

All in all, these three advantages allow the proposed model to provide a quantitative framework (through *fuzzification* and *defuzzification*) to intuitively and non-linearly aggregate each financial institution's metrics (i.e. the quantitative input) based on the experts' criteria (i.e. the qualitative input). Therefore, unlike using expert knowledge or standard quantitative methods to determine "the appropriate set of weights" for aggregating each metric, this approach is closer to capturing the *static complexity* (Casti, 1979) emerging from the intricate relation between the interacting metrics and the systemic importance (i.e. the output is a non-simple function of the input).

However, despite FLIS' advantages, the significance of each input or metric is not tested (i.e. it relies on expert knowledge), and the weight assigned to each input or metric is non-observable. The PCA aggregation method overcomes some of these limitations.

First, by construction, the leading principal component (Γ_1) provides a set of scores for each systemic importance metric, which allows for observing the weights that the method assigns to each metric within a linear aggregating framework. This may be important since a particularly low weight in the leading principal component may reveal the redundancy or lack of significance of the corresponding metric; a piece of information that is unavailable in the FLIS aggregation method.

Second, the contribution of the first or leading eigenvalue (ϖ_{λ_1}) provides a measure of the explanatory power of the leading principal component, which allows for quantitatively assessing the overall fitness of the aggregation method; again, the FLIS aggregation method does not reveal this kind of information.

oversimplifying: it is not clear that all criteria are equally important for all markets, or at all times. As suggested by IMF et al. 2009, a qualitative framework could be used to integrate the different components of the assessment and help arrive at judgments of systemic importance, where a high degree of judgment founded in a detailed knowledge of the functioning of the financial system is required in any assessment of systemic importance; this is the rationale behind the presented approach.

Third, following Kritzman et al. (2011), the contribution of the first eigenvalue (ϖ_{λ_1}) may provide additional information regarding the extent to which the different systemic importance metrics are coupled together within considered financial institutions; this is, PCA may point out whether (or not) high levels of individual metrics of systemic importance are concentrated in a few financial institutions. This may be a statistic worth monitoring across time for financial stability purposes, and is a clear advantage of PCA over FLIS.

Both aggregation methods share a common advantage when compared to BCBS-BIS (2013) proposal: they avoid relying on an arbitrary, homogeneous and fixed weighted scheme. Finally, due to their distinct methodological framework and their related (dis)advantages, both methods are expected to provide a complementary view of the systemic importance metrics' aggregation problem, as suggested by León and Machado (2013).

4 Systemic importance assessment³⁸

This section presents the results of implementing the proposed metrics and aggregating methods. The data and the procedures are briefly described first; the main results for each aggregating method are presented afterwards.

4.1 Data and procedures

Data as of June 2013 from Colombia's sovereign securities settlement system (DCV) and large-value payment system (CUD), along with reported financial statements from the Colombian Financial Superintendence, are used. During the chosen period 125 financial institutions were available for analysis, and they were classified as in Table 4.

³⁸ Results are illustrative. They may not be used to infer credit quality or to make any type of assessment for any financial institution. The results do not represent an opinion or statement of Banco de la República and the Ministry of Finance and Public Credit, nor of its Board of Directors and Ministry, respectively. The name of each institution is not revealed due to disclosure restrictions.

Table 4
Main Colombian market's financial institutions
(as of June 2013)^d

Class	Institution type	Main purpose ^c
Credit Institutions (C) ^a		It includes commercial banks (i.e. provision of deposit and loans, including mortgages), financial corporations (i.e. focused on medium term industrial financing; akin to an investment bank) and financing corporations (provision of deposits and loans focused on goods and services commercialization) [50]
Non-Credit Institutions	Investment Fund (F)	Provision of investment vehicles with the purpose of investing in securities and other assets according to the risk profile of the investor. [27]
	Brokerage Firm (K)	Provision of brokerage services with the purpose of buying and selling securities (e.g. stocks, bonds, currencies); allowed to trade for its own account. [25]
	Pension Fund Manager (P)	Provision of investment vehicles with the purpose of investing for retirement. [5]
	Other ^b (O)	Insurance companies, financial cooperatives and other. [18]

^a Financial cooperatives are Credit Institutions (C) but because of their low contribution to the metrics considered they were included in the "Other" class; Cs are the only institutions able to receive last-resort lending liquidity.

^b The "Other" class gathers financial institutions characterized by their particularly low (or nil) relevance for the key systemic importance indicators.

^c Only the main differentiating feature appears; the number of institutions as of June 2013 appears in brackets.

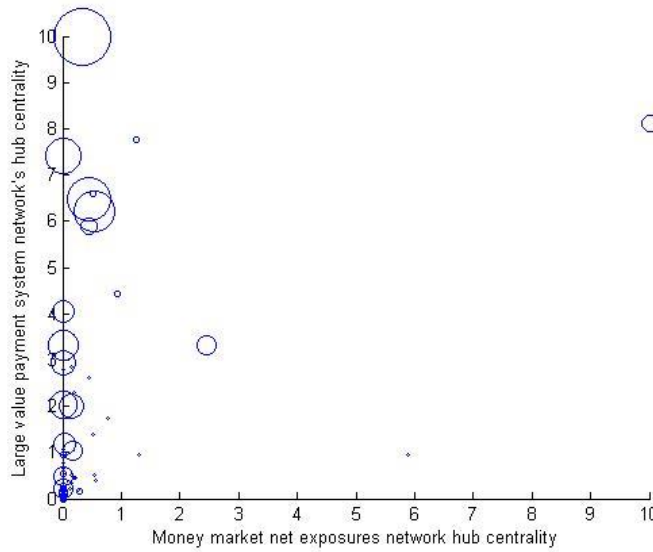
^d The Central Bank, Ministry of Finance and Public Credit, financial infrastructures and official financial institutions are excluded from this Table and were not analyzed.

Source: authors' design.

The three systemic importance metrics were calculated for each of the considered financial institutions. Instead of using the original units (e.g. pesos or percentages), 0 to 10 indexes were designed. For instance, the size index consists of a typical standardization of the nominal values of the adjusted assets for each institution considered, where the biggest institution is assigned the maximum index value (10 in this case) and the rest is assigned an index value by means of linear interpolation. Such standardization is straightforward and makes comparisons and calculations easier. It is important to emphasize that the assessment obtained with these factor indexes and the aggregated systemic importance index are not absolute, but relative to the most systemically important institution.

Figure 9 presents a scatter plot containing the three individual metrics for all the financial institutions considered. The horizontal axis corresponds to the first metric (i.e. *money market net exposures network hub centrality*), standardized as previously stated; the vertical axis corresponds to the second metric (i.e. *large-value payment system network's hub centrality*); whereas the diameter of each circle represents the *adjusted assets* metric.

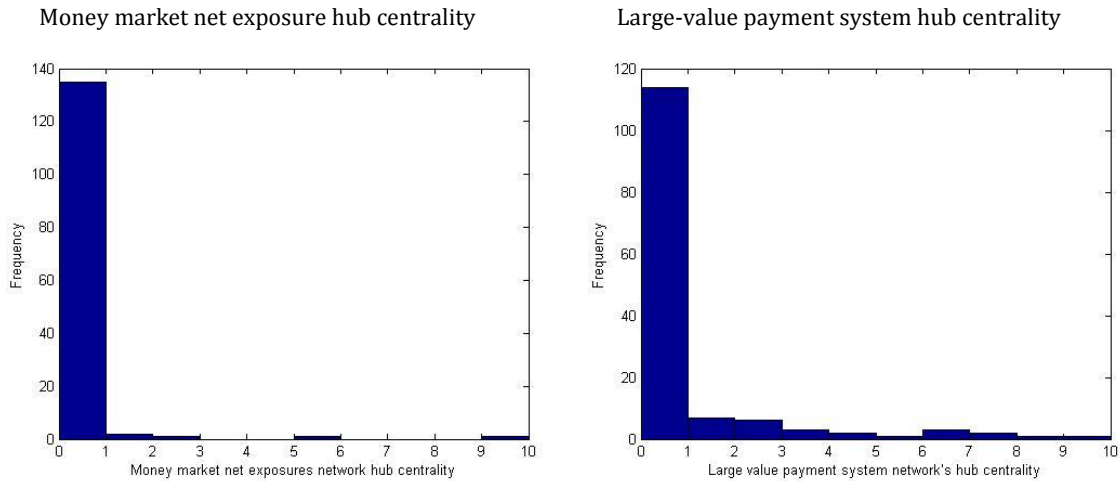
Figure 9
Systemic importance metrics

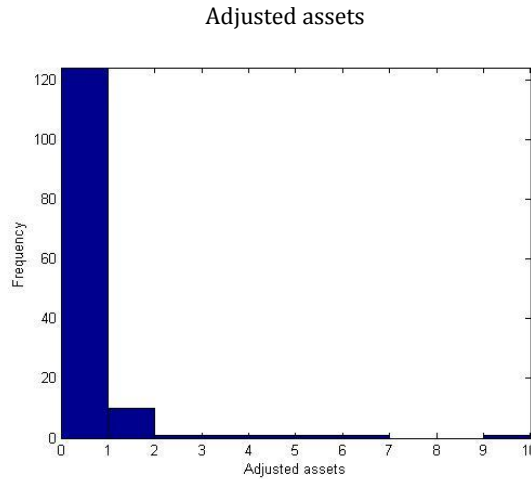


Source: authors' calculations

Interestingly enough, concurrent with financial networks' literature, the first two metrics display skewed distributions where it is evident that a few financial institutions concentrate most of the importance, whilst a large number of financial institutions are of low importance (Figure 10). Likewise, the metric corresponding to the size of financial institutions exhibits such a skewed distribution as well.

Figure 10
Distribution of systemic importance metrics





Source: authors' calculations.

Besides, not only the distribution of the three systemic importance metrics is skewed, but approximates a particular type of distribution: a power-law. For instance, the power-law (or Pareto-law) distribution suggests that the probability of observing a financial institution with size k obeys the potential functional form in [§4], where z is an uninteresting and arbitrary constant, and γ is known as the *exponent* of the power-law.

$$\mathcal{P}_k \propto z k^{-\gamma} \quad [§4]$$

According to relevant literature (Taleb, 2007; Mandelbrot and Hudson, 2004; Peak and Frame, 1994) values in the range $1 \leq \gamma \leq 3$ are typical of scale-free distributions, where an early rule of thumb is $\gamma \sim 3/2$, as in Pareto's findings regarding the distribution of wealth.³⁹ Based on the algorithm designed by Clauset et al. (2009), the exponents resulting from fitting the power-law to the three metrics presented in Figure 10 are 1.90, 1.92 and 2.46, respectively.⁴⁰

Such exponents correspond to distributions where most of the observations are confined to low figures (e.g. low importance, intensity, impact, size), whereas a few observations hold high figures. In the case in hand, for instance, this distribution points out that few (most) financial institutions are big (small). In this sense, following Bak (1996), since there is no typical size of a financial institution that may describe the whole distribution at any internal scale (as if the distribution were a Gaussian), this distribution is also referred as "scale-free", meaning that widely different financial institutions coexist.

³⁹ Other typical exponents are those of Zipf for the use of words in literature ($\gamma \sim 1$) and Mandelbrot for cotton prices ($\gamma \sim 1.7$). Moreover, regarding the distribution of connections in real-world networks, Newman (2010) suggests that exponents in the range $2 \leq \gamma \leq 3$ are typical, although values slightly outside it are possible and are observed occasionally; Barabási (2013) suggests that networks where the distribution of connections yield $\gamma \leq 2$ pertain to an "anomalous regime".

⁴⁰ The simplest method for estimating the exponent of the power-law (γ) consists of an ordinary least squares (OLS) regression. However, as stressed by Clauset et al. (2009), OLS fitting may be inaccurate due to large fluctuations in the most relevant part of the distribution (i.e. the tail), where a Maximum Likelihood may be more appropriate.

Concurrent with a growing volume of literature from distinct sciences (e.g. physics, biology, economics and engineering) finding a scale-free distribution is a patent hallmark of self-organizing systems (León and Berndsen, 2013; Andriani and McKelvey, 2009; Dorogovtsev and Mendes, 2003; Strogatz, 2003; Barabási, 2003; Barabási and Albert, 1999; Bak, 1996; Krugman, 1996). Furthermore, the scale-free nature of the three metrics not only suggests that the financial system displays self-organizing features, but that it has configured itself with a structure that is robust but fragile. This type of self-organization is referred as self-organizing criticality (Bak, 1996), and agrees with Haldane (2009) characterization of the current international financial network: “robust-yet-fragile”.⁴¹

The three resulting standardized metrics or individual indexes served as an input for each of the two aggregating methods. In the case of the FLIS, as previously stated, each standardized index (i.e. from 0 to 10) is used to define each financial institution’s degree of membership to the available categories (i.e. LOW, MEDIUM, HIGH) and, based on the inference rules (i.e. the expert knowledge base) and the fuzzy logical operators, the aggregation method yielded an aggregated systemic importance index that is standardized into a 0 to 10 scale. In this case the aggregated index (iFLIS) corresponds to a quantitative assessment of the expected systemic importance index based on the simultaneous evaluation of each institution’s particular combination of individual metrics and their resulting overall systemic incidence according to the expert input.

Figure 11 presents three intensity plots corresponding to the combination of the three metrics. Each plot may be considered as a representation of the expert knowledge regarding how two metrics (in the axis) result in different degrees of systemic importance (i.e. the intensity) with the third (non-displayed) metric held constant at 5 (left column in Figure 11) and 8 (right column).

The intensity plots displayed in Figure 11 reveal that experts find that few combinations of the *large-value payment system hub centrality* and the *money market net exposure hub centrality* (first row in Figure 11) yield levels of systemic importance that may be considered as critical (e.g. $iFLIS \geq 8$); this is, let adjusted assets be fixed at the two considered levels (i.e. 5 and 8), severe systemic importance arises only with very high levels of *hub centrality* in the money market and the payment systems.

On the other hand, the *adjusted assets* metric are judged by the surveyed experts as particularly critical for determining systemic importance; both plots involving adjusted assets (second and third row in Figure 11) exhibit larger areas (i.e. more combinations) where systemic importance may be considered critical (e.g. $iFLIS \geq 8$) for a given level of the non-displayed metrics.

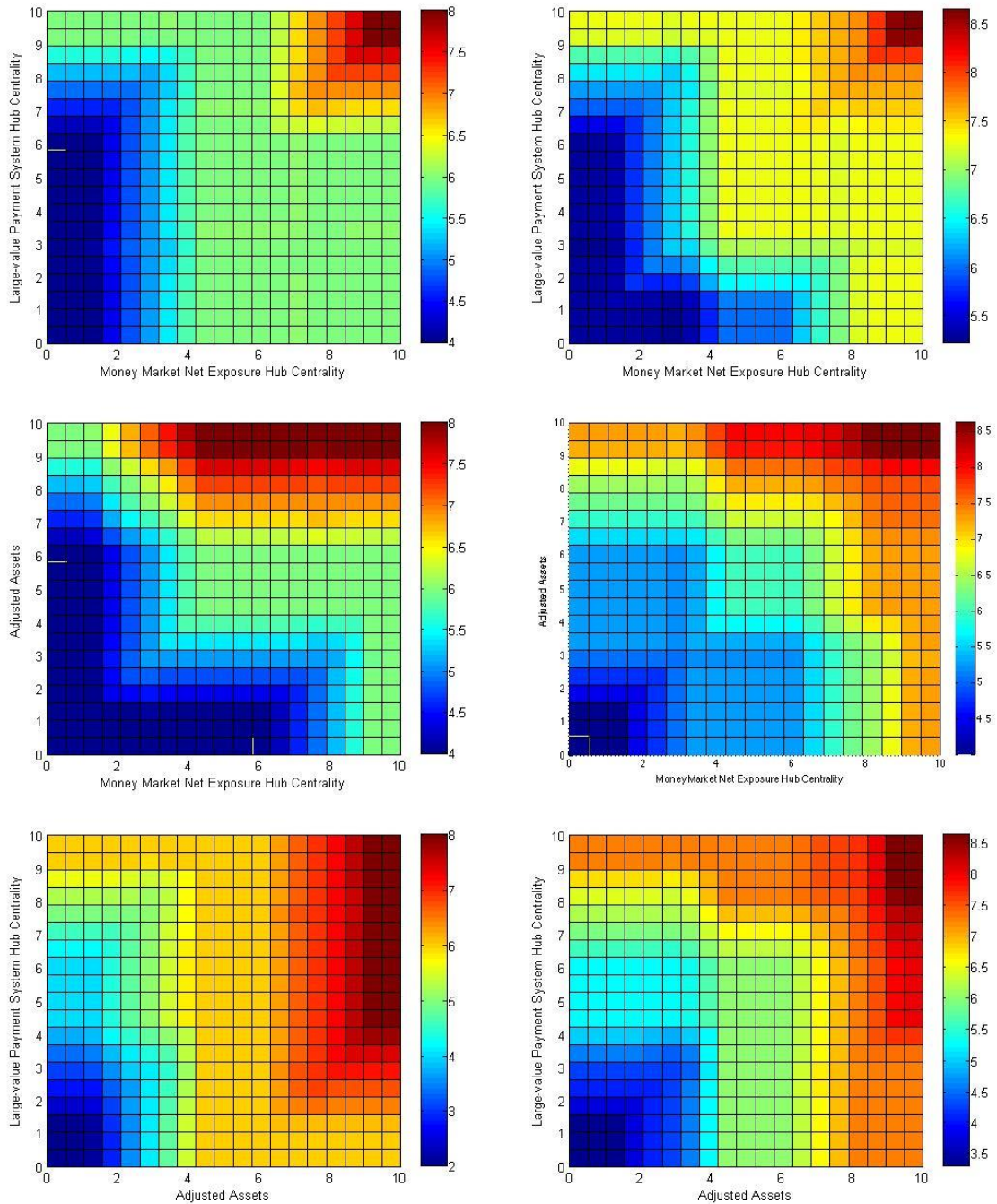
⁴¹ León and Berndsen (2013) presents a comprehensive analysis of the scale-free and self-organizing nature of Colombian financial networks.

Figure 11

Expert knowledge as intensity plots

Non-displayed metric set equal to 5

Non-displayed metric set equal to 8



Source: authors' calculations.

Nevertheless, the intensity plots in Figure 11 provide a limited view of the experts' judgment. Many other combinations are possible since only two levels of the non-displayed metric have been used for illustrative purposes. The existence of many other

levels of the non-displayed metric hinders the way experts judge all possible combinations of the considered metrics.

In the case of PCA, the three standardized indexes were used for estimating the covariance matrix (Ω) and performing the calculations in [§1]. The resulting scores were scaled (i.e. squared) in order to obtain scores that sum up to 1, and thus may be considered as weights within a lineal aggregation index. Let \mathcal{A} , \mathcal{B} and \mathcal{C} represent *the money market net exposure hub centrality*, *the large-value payment system hub centrality* and *the adjusted assets* metrics for a x -financial institution, respectively, the PCA Systemic Importance Indicator (*iPCA*) will be calculated as the standardized value (i.e. in a 0 to 10 scale) of the following linear expression:

$$iPCA_x = 0.1525 \times \mathcal{A}_x + 0.4770 \times \mathcal{B}_x + 0.3704 \times \mathcal{C}_x \quad [§4]$$

where

$$\Gamma = \begin{bmatrix} 0.3906 & 0.8657 & -0.3131 \\ 0.6907 & -0.0507 & 0.7214 \\ 0.6086 & -0.4980 & -0.6177 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} 1.8588 & 0 & 0 \\ 0 & 0.9281 & 0 \\ 0 & 0 & 0.2131 \end{bmatrix}$$

and

$$\varpi_{\lambda_1} = 0.6196$$

Since $\varpi_{\lambda_1} \gg 1/3$, it is feasible and sound to use the PCA model as in [§4]; the designed linear system captures about 2/3 of the information in the covariance matrix. Also, the level attained by ϖ_{λ_1} suggests that a few financial institutions concentrate high levels of individual metrics of systemic importance; as previously stated, this may be a statistic worth monitoring across time for financial stability purposes.

Regarding Γ_1 , as it is evident in the *iPCA* linear expression, the leading metric is the large-value payment system hub centrality, which has a score (weight) of 0.6907 (0.4770), followed by the adjusted assets metric and the money market net exposure hub centrality.

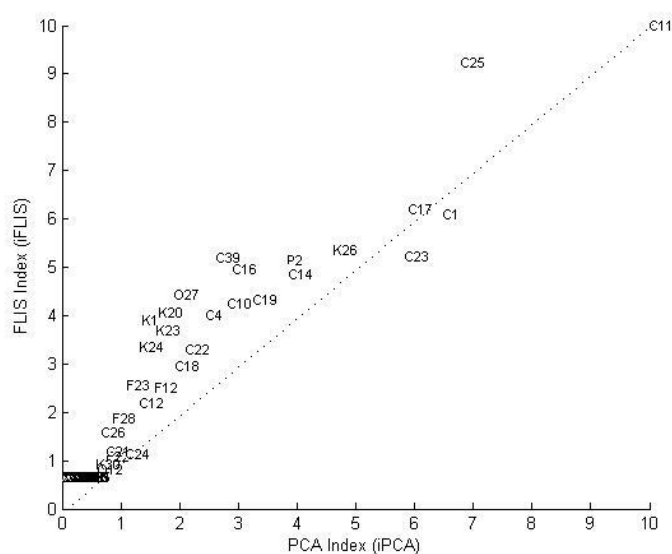
Interestingly, based on *spectral partitioning* basics (Kolaczyk 2009; Straffin, 1980) the signs and magnitudes of the elements in the second eigenvector (Γ_2) may suggest that there is some degree of second order linear connection between financial institution's *adjusted assets* and *large-value payment system hub centrality*; this is, there is some degree of coincidence in big and large-value payment central financial institutions. However, since the corresponding scores and weights are significant (i.e. $\gg 0$), both metrics are non-redundant and remain informative for the linear system in [§4].

4.2 Main results

Figure 12 compares both aggregation methods' results. Each financial institution is identified with a letter corresponding to their type (as in Table 4), along with a tag

number. Each financial institution is mapped in a Cartesian framework, where the horizontal axis corresponds to the PCA Index (iPCA), whereas the vertical corresponds to the FLIS Index (iFLIS); intuitively, financial institutions located on the diagonal received the same systemic importance index from both aggregating methods, whereas those above (below) the diagonal correspond to cases in which the iFLIS (iPCA) index is higher. It is worth emphasizing that both indexes provide a relative assessment of each institution's systemic importance: an index equal to zero does not correspond to the absolute absence of systemic importance for that institution, but a negligible importance with respect to the most important institution.

Figure 12
Systemic Importance Index^a
(as of June 2013)



^a credit institution (C); investment fund (F); brokerage firm (B); pension fund manager (P)
Source: authors' calculations.

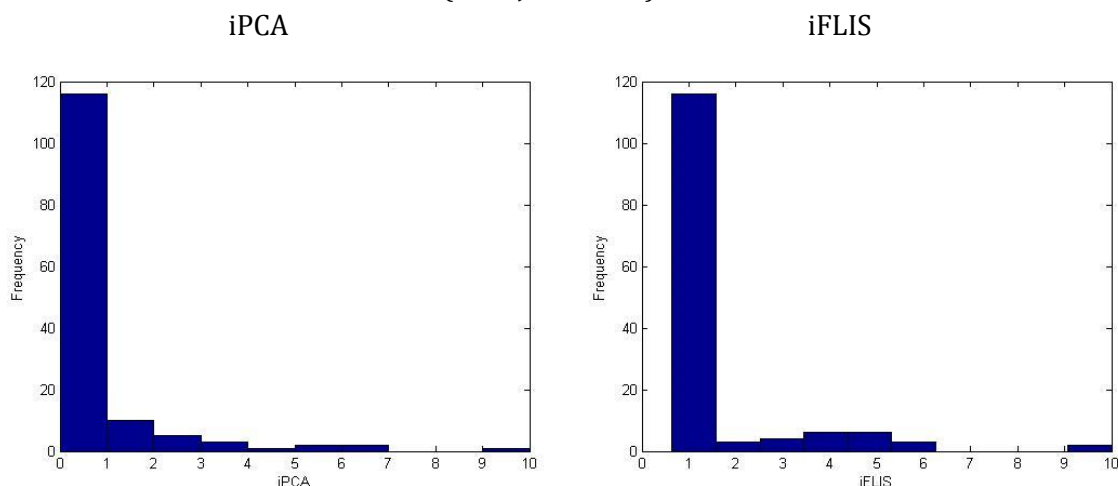
Both indexes agree on C11 and C25 being the two most systemically important financial institutions in the local financial markets. The two indexes also concur in the dominance of credit institutions (Cs) as the type of financial institution most likely to display high levels of systemic importance according to the 0 to 10 scale. Moreover, both indexes agree on the top-six financial institutions (i.e. C11, C25, C1, C17, C23, K26); however, the ranking is dissimilar. Beyond the sixth ranked financial institution differences tend to emerge between indexes.

The first two non-credit institutions appearing in both indexes are K26 and P2, a brokerage firm and a pension fund, respectively. Below the 5 mark both indexes exhibit an heterogeneous mix of financial institutions, with credit institutions (Cs) and brokerage firms (Ks) displaying higher systemic importance figures, and where all investment funds (Fs) have scores below 3.

An important attribute of both indexes is their high level of skewness (Figure 13). Only a handful of financial institutions have indexes above 5; 4 for the iFLIS, and 5 for the iPCA.

About 95% of the financial institutions have indexes below 5, whereas more than 83% have indexes below 2.

Figure 13
Distribution of the Systemic Importance Index
(as of June 2013)



Source: authors' calculations.

Such skewness confirms the intuition regarding the high degree of asymmetry (right skew) of systemic importance, where the average institution is of low systemic importance and the average default or failure-to-pay results in low systemic severity; in this case the average financial iPCA and iFLIS is 0.66 and 1.33, respectively. Thus, concurrent with the scale-free literature, relying on the systemic importance of the average financial institution would divert financial authorities from its aim of ever preserving financial stability and payment systems safety. Likewise, working under the assumption of homogeneous financial institutions (as in Allen and Gale, 2000; Freixas et al., 2000) may mislead financial systems' contagion modeling and analysis.

5 Final remarks

This document presents an enhanced and condensed version of preceding proposals by its authors for identifying systemically important financial institutions in the Colombian case (Leon and Machado, 2013; León and Murcia, 2012). The main enhancements come in the form of (i) improving the network analysis metrics implemented, which consisted of shifting from local to global metrics of centrality; (ii) using network analysis metrics on two different networks, namely large-value payment and money market exposures networks, which allows for assessing transactions and exposures as systemic importance factors, respectively; (iii) filtering out the size of the core financial services and the size of the proprietary portfolios of financial institutions. These enhancements contribute to the design of metrics that are closer to the macro-prudential perspective of financial stability.

However, despite their distinctive methodological backgrounds, both aggregation methods (i.e. FLIS and PCA) yield indexes that concur in several features: (i) the ranking and remoteness of the top-two most systemically important financial institutions, namely C11

and C25; (ii) the preeminence of credit institutions as the type of financial institution most likely to display high levels of systemic importance; (iii) the top-six financial institutions, which are credit institutions and a single brokerage firm; (iv) the skewed nature of the indexes, which match the skewed nature of the three metrics and their approximate scale-free (i.e. Power-law) distribution. The main divergences between both indexes emerge when assessing the systemic importance of non-top-ranked financial institutions.

The two indexes give financial authorities the ability to acquire a comprehensive and methodologically non-redundant relative assessment of each financial institution's systemic importance. They may assist financial authorities in focusing their attention and resources (i.e. the intensity of oversight, supervision and regulation) where the systemic severity resulting from a financial institution failing or near-failing is estimated to be the greatest. Moreover, the two indexes may also help financial authorities in policy and decision-making (e.g. resolving, restructuring or providing emergency liquidity).

Nevertheless, it is important to emphasize that this methodology is by no means a substitute for sound judgment by financial authorities, or the sole metrics to use when deciding the systemic importance of a financial institution. The authors regard the two indexes as providing valuable and novel relative metrics for assessing systemic importance by financial authorities, which conveniently complements existing methods (e.g. BCBS-BIS, 2013; León and Machado, 2013; León and Murcia, 2012).

Some challenges not addressed in this document are worth highlighting. First, for instance, as envisaged by León and Machado (2013), considering ownership links may enhance the assessment of the systemic financial institutions by acknowledging the existence of conglomerates within the financial system. Second, defining an important/unimportant threshold may be interesting for financial authorities, but is by no means simple, and should consider many relevant factors, such as the purpose of the threshold (e.g. for defining a capital charge or deciding which institutions to follow closely) and the skewness of the indexes. Third, if the contribution of the first eigenvalue (ϖ_{λ_1}) is considered as a measure of the extent to which the different systemic importance metrics are coupled together within considered financial institutions, analyzing its dynamics may provide some insights about the evolution of systemic importance throughout time.

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7 Exhibit A (Authority and hub centrality)⁴²

Kleinberg (1998) introduces the HITS (Hyper Text Induced Topic Search) algorithm for estimating authority and hub centrality. The algorithm recognizes that the *authority centrality* of each node is defined to be proportional to the sum of the *hub centrality* of the nodes that point to it, and that the *hub centrality* of each node is defined to be proportional to the sum of the *authority centrality* of the nodes it points-to. In order to make such recognition the algorithm uses *eigenvector centrality* on two modified versions of the original adjacency matrix, where these two matrices correspond to an *authority* matrix (\mathcal{A}) and a *hub* matrix (\mathcal{H}). Let Ω be the adjacency matrix resulting from a network, the *authority* and *hub* matrices (\mathcal{A} and \mathcal{H}) are estimated as follows:

Authority matrix

$$\mathcal{A} = \Omega^T \Omega$$

Hub matrix

$$\mathcal{H} = \Omega \Omega^T$$

Multiplying a symmetrical adjacency matrix with itself allows identifying all nodes that can reach each other in two steps –second order adjacencies (Haining, 2004). However, in the case of non-symmetrical (i.e. directed) adjacency matrices, multiplying with a transposed version of itself allows identifying directed (*in* or *out*) second order adjacencies. Regarding \mathcal{A} , multiplying Ω^T with Ω sends weights backwards –against the arrows, towards the pointing node-, whereas multiplying Ω with Ω^T (as in \mathcal{H}) sends scores forwards –with the arrows, towards the pointed-to node (Bjelland et al., 2008). In this sense, for a non-weighted network off-diagonal elements H_{ii} correspond to the number of nodes that node i points to, whereas elements A_{ii} correspond to the number of nodes pointing to i .

Since \mathcal{A} and \mathcal{H} are symmetrical nonnegative matrices (even if Ω is directed and acyclic), a unique *eigenvector centrality* of \mathcal{A} and \mathcal{H} may be estimated, and the resulting *authority* and *hub centrality* scores will be positive non-zero scores for each node; this contrasts with standard *eigenvector centrality* on a directed and acyclic adjacency matrix, where *eigenvalue centrality* will yield equal –zero- scores for each node.

⁴² This exhibit is extracted from León and Pérez (2013a).