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Changes in GDP's measurement error volatility and response of the monetary policy rate: two approaches

Julian A. Parra-Polania and Carmiña O. Vargas*

Abstract

Using a stylized model in which output is measured with error, we derive the optimal policy response to the demand shock signal and to changes in the measurement error volatility from two different perspectives: the minimization of the expected loss (from which we derive the 'standard' policy) and the minimization of the maximum possible loss across all potential scenarios (from which we derive the 'prudent' or 'robust' policy). We find that: 1. the prudent policymaker reacts more aggressively to the shock signal than the standard one and 2. while the standard policymaker always mitigates her reaction if the measurement error volatility rises, the prudent one may even increase her response if her risk aversion is very high. When we incorporate forward-looking expectations, the second result is preserved but, in this case, the prudent policymaker is less aggressive than the standard one in responding to the shock signal.

Keywords: *prudence, robustness, measurement error, optimal monetary policy.*

JEL Classification: D81, E52, E58

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1 Introduction

In general, increments in the volatility of an estimated variable could be attributed to an increase in the volatility of the actual variable or to an increase in the volatility of its measurement error.

In practice, increments in the error volatility of estimated macroeconomic variables hinder the process of taking the appropriate economic policy measures. In the decision-making process, the first step is to try to determine what proportion of the observed increase in volatility is the result of measurement error and, once this is established, the second step is to determine what is the optimal policy reaction given these circumstances.

This paper focuses on the analysis of the second step assuming that the increase in volatility corresponds exclusively to measurement error. Specifically, the paper analyzes from two different perspectives how monetary-policy actions should change when there is higher volatility in the measurement error of the aggregate economic activity.

The first perspective corresponds to the standard problem of loss minimization by the monetary policymaker. The second perspective has had good reception recently in economics and corresponds to the concept of "robustness" in which the policymaker seeks to minimize the maximum possible loss across all potential conditional[1] scenarios as a form of *prudence* to avoid huge losses. This criterion has been considered for the design of optimal policies under uncertainty and it is widely discussed in Hansen and Sargent (2007) and Barlevy (2009). A prudent central bank seeks to minimize the social loss in the worst scenario and though its average performance is, by construction, lower than that of the central bank that minimizes the expected loss, following the prudent policy allows the policymaker to avoid scenarios that have low probability but are very costly.

We set up a stylized model which incorporates two features: first, the output gap exhibits some degree of persistence; and second, a lag in the effect of monetary policy such that it affects the output gap more rapidly than inflation. The output gap is measured with error (i.e. there is a noisy signal of the demand shock), and therefore monetary policy faces uncertainty.

We derive both the optimal standard policy and the optimal prudent policy re-

[1] The 'worst' conditional scenario refers to the fact that all scenarios are considered, conditional to the potential policy actions that could be taken. For instance, the policymaker considers not only that, before taking any action, the worst scenario might be the occurrence of a large and negative unanticipated shock, but also cases such as that if she acts as if a large and negative shock were to occur, the worst scenario would instead be that actually a large and positive shock happens.

sponses to the demand shock signal and to an increase in the measurement error volatility. We find that in both cases the central bank reduces (increases) the interest rate when it receives a signal of a negative (positive) demand shock. However, for the same value of the signal, the prudent policymaker reduces (increases) the interest rate more than the standard one.

Furthermore, when there is an increase in the volatility of the measurement error, the standard central bank attenuates its response, that is, when facing a signal of a negative (positive) demand shock but under higher volatility of the measurement error, a standard central bank does not reduce (increase) the interest rate as much. The result is not as straightforward for the case of a prudent central bank. For this type of bank, an increase in the volatility of the measurement error could lead to either an attenuation of its reaction (as in the standard case) or an even stronger response in the interest rate. It depends on which of the following two effects dominates when the volatility increases: the signal's weight in agents' expectations, or the bank's risk aversion. If the first, then the prudent policymaker attenuates its policy.

In the final part of the paper we extend the model to incorporate forward-looking expectations. The model becomes more complex and for the analysis of prudence we derive conclusions from numerical solutions for a specific set of parameters. In this case we find that the second result is preserved i.e. while the standard central bank always mitigates its reaction if the measurement error volatility rises, the prudent one may even increase its response if its risk aversion is very high. However, the first result is different: the prudent policymaker is less aggressive than the standard one in responding to the shock signal.

In the following section, we describe the model and its equilibria under both criteria. In Section 3, we do the same for the model with forward-looking expectations. Section 4 contains our conclusions.

2 The Model and two possible solutions

In a simple model we intend to capture some stylized monetary-policy facts. In particular, it incorporates two features: first, some degree of persistence for the output gap and second, a lag such that monetary policy affects output more rapidly than inflation.

The period loss function for the central bank is

$$L_t = \pi_t^2 + \lambda y_t^2$$

where π is inflation, y is the output gap and $\lambda > 0$ represents the relative weight given to output stabilization. The Phillips curve (PC) and the IS curve are:

$$\pi_t = \eta E_{t-1}\pi_t + \alpha y_{t-1} \tag{PC}$$

$$y_t = \rho y_{t-1} - \delta\left(i_{t-1} - E_{t-1}\pi_t\right) + d_t \tag{IS}$$

where E_{t-1} is the expectations operator, conditional on information available at time $t-1$,[2] $\rho \in (0,1)$ represents the output's degree of persistence, $\eta \in (0,1)$, δ and α are positive constants, i is the nominal interest rate and d is the demand shock, which we assume is uncorrelated over time and normally distributed with zero mean and variance σ_d^2 (i.e. $d_t \sim_{iid} N\left(0, \sigma_d^2\right)$).[3] In this setup, changes in the real interest rate have effects on the output gap with a one-period lag and on inflation with a two-period lag.

This economy also has a statistics office whose purpose is to measure the output gap with the highest precision. In any period t, this office releases a provisional estimation of the output gap for the same period ($\widehat{y}_t$) and the final estimation of the same variable for the previous period (y_{t-1}). The former estimation contains a measurement error (i.e. $\widehat{y}_t \equiv y_t + \varepsilon_t$, $\varepsilon_t \sim_{iid} N\left(0, \sigma_\varepsilon^2\right)$) and the latter estimation contains no error.

The timing is as follows: 1. the statistics office releases $\widehat{y}_t$ and y_{t-1}. 2. Private agents form rational expectations. 3. The central bank picks i_t. 4. Shocks d_t and ε_t are realized but they are unobserved.

2.1 Solutions

2.1.1 Criterion 1: Minimization of the Expected Loss

The central bank picks i_{t-1} so as to minimize the expected loss. This is the standard approach to solve the model described above. In this case the uncertainty problem can be reduced to a signal extraction problem. The central bank can construct a signal of the demand shock using the available information in period $t-1$, $\widehat{d}_{t-1} \equiv \widehat{y}_{t-1} - \rho y_{t-2} + \delta\left(i_{t-2} - E_{t-2}\pi_{t-1}\right)$. Using the IS equation and the definition for $\widehat{y}_{t-1}$:

$$\widehat{d}_{t-1} = d_{t-1} + \varepsilon_{t-1}$$

[2] Section 3 describes some results for the model with forward-looking expectations (i.e. $E_t\pi_{t+1}$).

[3] White-noise supply shocks could be included without affecting the main results of the paper. They would become irrelevant for our analysis due to the monetary policy lag.

Since ε_{t-1} is unobserved, it cannot be separated from the signal and therefore $\widehat{d}_{t-1}$ is a noisy signal of the demand shock. Following Harvey and De Rossi (2006), the demand shock forecast can be expressed as:

$$E_{t-1}\left[d_{t-1} \mid \widehat{d}_{t-1}\right] = \gamma \widehat{d}_{t-1}$$

where $\gamma \equiv \sigma_d^2 / \left(\sigma_d^2 + \sigma_\varepsilon^2\right)$ can be interpreted as a signal-to-noise ratio. The higher the relative amount of noise $\left(\sigma_\varepsilon^2/\sigma_d^2\right)$, the lower the weight given to the signal. In the extreme, when the relative amount of noise is infinite, the signal is completely useless and the best forecast for d_{t-1} becomes its unconditional expected value (zero).

The model is solved by backward induction. The central bank sets i_{t-1} in order to minimize[4]

$$E_{t-1}\left[L_t\right]$$

The solution to this problem yields[5]

$$i_{t-1} = E_{t-1}\pi_t + \frac{\rho}{\delta} E_{t-1} y_{t-1} \tag{1}$$

where $E_{t-1}y_{t-1} = \rho y_{t-2} - \delta\left(i_{t-2} - E_{t-2}\pi_{t-1}\right) + \gamma \widehat{d}_{t-1}$. Since $E_{t-1}y_t = \rho E_{t-1}y_{t-1} - \delta\left(i_{t-1} - E_{t-1}\pi_t\right)$ the central bank is setting i_{t-1} in order to set $E_{t-1}y_t = 0$ (due to the policy lag, $E_{t-1}\pi_t$ is not relevant as it cannot be affected by i_{t-1}).

Using the Phillips curve, we can find the following expression for expectations

$$E_{t-1}\pi_t = \frac{\alpha}{1-\eta} E_{t-1} y_{t-1} \tag{2}$$

Then, from equations (1) and (2) we obtain

$$i_{t-1}^* = \left(\frac{\alpha}{1-\eta} + \frac{\rho}{\delta}\right) E_{t-1} y_{t-1} \tag{3}$$

The change of this policy reaction to changes in the demand shock signal is (taking into

[4] The solution to the one-period problem is equal to that for the multiple-period problem. It can be shown that for infinite periods $E_{t-1}V_{t+1} = A_0 + E_{t-1}\pi_{t+1}^2$ where V_{t+1} is the value function of the problem and A_0 is a constant term. Due to the policy lag, minimizing either $E_{t-1}\left[L_t\right]$ or $E_{t-1}\left[L_t + \beta V_{t+1}\right]$ (where $\beta \in (0,1)$ is the discount factor) is simply equivalent to minimizing $E_{t-1}y_t$ ($E_{t-1}\pi_{t+1}$ is not relevant as it cannot be affected by i_{t-1}).

[5] The Second Order Condition (SOC) of the problem is $\eta\alpha^2 + \lambda\left(1-\eta\right)^2 > 0$, and therefore it is guaranteed that we are finding a global minimum.

account that $\partial E_{t-1} y_{t-1} / \partial \widehat{d}_{t-1} = \gamma$)

$$\frac{\partial i^*_{t-1}}{\partial \widehat{d}_{t-1}} = \left(\frac{\alpha}{1-\eta} + \frac{\rho}{\delta} \right) \gamma > 0 \tag{4}$$

As it is standard, a higher demand-shock signal increases the policy response of the central bank. This response is mitigated by the coefficient γ which acts as a filter of the noisy signal. As explained above, if the amount of noise is infinite the signal is completely useless i.e. $\gamma \to 0$. In this case, the monetary policymaker does not react to such signal.

Since γ depends on the measurement error volatility σ^2_ε, when such volatility changes the effect on the policy response can be expressed as

$$\frac{\partial \left(\partial i^*_{t-1} / \partial \widehat{d}_{t-1} \right)}{\partial \sigma^2_\varepsilon} = \left(\frac{\alpha}{1-\eta} + \frac{\rho}{\delta} \right) \frac{\partial \gamma}{\partial \sigma^2_\varepsilon} < 0 \tag{5}$$

This is also a standard result. An increase in the measurement error volatility implies a higher proportion of noise in the signal, and therefore the central bank's optimal response to changes in the signal is reduced.

2.1.2 Criterion 2: Robustness

In this scenario we assume that the central bank plays a min-max game against a 'cruel' nature. The latter observes what the former does and then acts with the purpose of maximizing the loss. Following van der Ploeg (2009), we construct the stress function which is based on the loss function and the prudence degree of the policymaker:

$$\Gamma_t = \pi_t^2 + \lambda y_t^2 - \frac{\theta}{\sigma^2_\varepsilon} \varepsilon^2_{t-1} \tag{6}$$

where $\theta > 0$ is inversely related to the central bank's risk aversion and the last term in the equation incorporates the fact that there is a finite level of prudence, and therefore nature cannot impose an infinite cost on the central bank.

Since the model is solved by backward induction we start at the last stage in which nature sets ε_{t-1} so as to maximize

$$E_{t-1} \left[\Gamma_t + \beta \Phi_{t+1} \right] \tag{7}$$

where $\beta \in (0,1)$ is the discount factor and Φ_{t+1} corresponds to the value function of the problem. It can be verified that such function takes the following form:

$$\Phi_{t+1} = B_0 + \frac{\theta/\sigma_\varepsilon^2}{\theta/\sigma_\varepsilon^2 - \alpha^2}\left[E_t\pi_{t+1} + \alpha(1-\gamma)\widehat{d}_t\right]^2$$

where B_0 is a constant term. By maximizing (7) we find that nature picks ε_{t-1} following[6]

$$\begin{aligned}\varepsilon_{t-1} &= -\frac{\alpha\eta(1-\eta)^2}{(\theta/\sigma_\varepsilon^2-\alpha^2)(1-\eta)^2-\rho C/\delta}E_{t-1}\pi_t - \frac{\rho C/\delta + \alpha^2(1-\eta)^2}{(\theta/\sigma_\varepsilon^2-\alpha^2)(1-\eta)^2-\rho C/\delta}\widehat{y}_{t-1} \\ &\quad + \frac{C}{(\theta/\sigma_\varepsilon^2-\alpha^2)(1-\eta)^2-\rho C/\delta}(i_{t-1}-E_{t-1}\pi_t) \qquad (8)\end{aligned}$$

where $\widehat{y}_{t-1} = \rho y_{t-2} - \delta(i_{t-2} - E_{t-2}\pi_{t-1}) + \widehat{d}_{t-1}$,

$C \equiv \delta\rho\lambda(1-\eta)^2 + \delta\alpha^2\beta\theta/\sigma_\varepsilon^2\left[\rho/\left(\theta/\sigma_\varepsilon^2-\alpha^2\right)\right] > 0$.

Taking into account equation (8) we proceed to solve the central bank's problem, i.e. to minimize (7) with respect to i_{t-1}, which yields[7]

$$i_{t-1} = D_1E_{t-1}\pi_t + \frac{\rho}{\delta}D_2E_{t-1}y_{t-1} + \frac{\rho}{\delta}D_2(1-\gamma)\widehat{d}_t$$

where $D_1 \equiv \frac{\eta\rho\alpha+\delta\left(\theta/\sigma_\varepsilon^2-\alpha^2\right)}{\delta\left(\theta/\sigma_\varepsilon^2-\alpha^2\right)} > 1$ and $D_2 \equiv \frac{\theta/\sigma_\varepsilon^2}{\theta/\sigma_\varepsilon^2-\alpha^2} > 1$.

Agents expectations are obtained from the Phillips curve and the final expression is equal to that obtained for the standard case (i.e. equation (2)). Then the final expression for the monetary instrument is

$$i_{t-1}^* = \left(\frac{\alpha}{1-\eta}D_1 + \frac{\rho}{\delta}D_2\right)E_{t-1}y_{t-1} + \frac{\rho}{\delta}D_2(1-\gamma)\widehat{d}_t \qquad (9)$$

The change of this policy reaction to changes in the demand shock signal is (taking into account that $\partial E_{t-1}y_{t-1}/\partial\widehat{d}_{t-1} = \gamma$)

$$\frac{\partial i_{t-1}^*}{\partial\widehat{d}_{t-1}} = \frac{\alpha}{1-\eta}D_1\gamma + \frac{\rho}{\delta}D_2 > 0 \qquad (10)$$

As in the standard case, a higher demand-shock signal increases the policy response of

[6] We assume θ is large enough so as to satisfy the SOC of the problem: $\left(\theta/\sigma_\varepsilon^2-\alpha^2\right)(1-\eta)^2 - \rho C/\delta > 0$. A necessary but not sufficient condition is $\theta/\sigma_\varepsilon^2 > \alpha^2 + \lambda\rho^2$.

[7] If the SOC of the nature's problem holds, the SOC of the central bank's problem $\frac{\lambda(1-\eta)^2\left(\theta/\sigma_\varepsilon^2-\alpha^2\right)+\alpha^2\beta\theta/\sigma_\varepsilon^2}{(\theta/\sigma_\varepsilon^2-\alpha^2)(1-\eta)^2-\rho C/\delta} > 0$ is satisfied as well.

the central bank. Since $D_1 > 1$ and $D_2 > 1$, the response of the prudent central bank to changes in the signal is greater than that of the standard one (compare (10) with (4)).

When the measurement error volatility σ_ε^2 changes, the effect on the policy response is

$$\frac{\partial\left(\partial i_{t-1}^*/\partial\widehat{d}_{t-1}\right)}{\partial\sigma_\varepsilon^2} = \frac{\alpha}{1-\eta}D_1\frac{\partial\gamma}{\partial\sigma_\varepsilon^2} + \frac{\rho\alpha^2\theta}{\delta\sigma_\varepsilon^4}\frac{1-\eta\left(1-\gamma\right)}{\left(\theta/\sigma_\varepsilon^2-\alpha^2\right)^2\left(1-\eta\right)} \tag{11}$$

The left-hand side has two opposite effects and, as a result, the total effect can be positive or negative. The first factor corresponds to the impact on the signal's weight in agents' expectations. When the measurement error volatility increases, the signal has less weight when forming expectations ($\partial\gamma/\partial\sigma_\varepsilon^2 < 0$) and, through this channel, it is less relevant for the central bank. The second factor corresponds to the impact on prudence. Increases in σ_ε^2 raise the central bank's relative prudence, as can be seen in its loss function.

It can be shown that the derivative of the left-hand side with respect to θ is negative. For small-enough values of θ ($\theta \to \alpha^2\sigma_\varepsilon^2$), the prudence factor prevails, and the central bank responds more strongly to the signal when there is a perceived increase in the measurement error volatility ($\partial\left(\partial i_{t-1}^*/\partial\widehat{d}_{t-1}\right)/\partial\sigma_\varepsilon^2 > 0$). On the other hand, when θ is big enough ($\theta \to \infty$), the factor associated to the signal's relevance prevails. In this case, a prudent central bank attenuates the response to shocks, similarly to the standard central bank; however, the former attenuates less[8] than the latter.

3 Extension: forward-looking expectations

In this section we provide the model's solution when the relevant expectations are forward looking. In this case, the model becomes more complex and for the analysis of robustness we derive conclusions from numerical solutions for a specific set of parameters.

In particular, with respect to the original model (Section 2) we only change the Phillips curve:

$$\pi_t = \beta E_t\pi_{t+1} + \alpha y_{t-1} \tag{PC'}$$

[8]For the prudent central bank, $\lim_{\theta\to\infty}\frac{\partial\left(\partial i_{t-1}^*/\partial\widehat{d}_{t-1}\right)}{\partial\sigma_\varepsilon^2} = \frac{\alpha}{1-\eta}\frac{\partial\gamma}{\partial\sigma_\varepsilon^2}$. Compare this with the corresponding expression for the standard central bank (equation (5)).

where we have assume that, as in the standard New Keynesian Phillips curve, the coefficient on forward-looking expectations is equal to the discount factor, β. The timing of the model remains the same.

3.1 Solutions

3.1.1 Criterion 1: minimization of the expected loss

The model is solved by backward induction. The central bank sets i_{t-1} in order to minimize:

$$E_{t-1}\left[L_t + \beta V_{t+1}\right]$$

where V_{t+1} corresponds to the value function of the problem. Given the monetary policy lag, L_{t-1} is irrelevant for the decision about i_{t-1}. To obtain a solution for the central bank's problem we postulate the following forms for the value function and expectations:

$$V_{t+1} = A_1 \left(E_t y_t\right)^2 \tag{12}$$

$$E_t \pi_{t+1} = A_2 E_t y_t \tag{13}$$

It should be the case that $A_1 > 0$ as V_{t+1} is the expected discounted value of future losses (all of which must be nonnegative). The optimal interest rate in $t-1$ is[9]

$$i_{t-1} = E_{t-1}\pi_t + \left[\frac{\rho}{\delta} + \frac{\alpha\beta A_2}{\delta\left(\beta^2 A_2^2 + \beta A_1 + \lambda\right)}\right] E_{t-1} y_{t-1} \tag{14}$$

Using the foregoing equation, as well as (PC') and (13), we can find the following expression for expectations

$$E_{t-1}\pi_t = \alpha \frac{\beta A_1 + \lambda}{\beta^2 A_2^2 + \beta A_1 + \lambda} E_{t-1} y_{t-1} \tag{15}$$

which is consistent with the postulated form (13) as long as

$$A_2 = \alpha \frac{\beta A_1 + \lambda}{\beta^2 A_2^2 + \beta A_1 + \lambda} \tag{16}$$

[9]The SOC of the problem is $\beta^2 A_2^2 + \beta A_1 + \lambda > 0$, and therefore it is guaranteed that we are finding a global minimum.

Then, using the obtained results (equations (14) and (15)) we can verify that

$$V_t = E_{t-1}\left[L_t + \beta V_{t+1}\right] = \alpha A_2 \left(E_{t-1} y_{t-1}\right)^2$$

which is consistent with the postulated form (12) as long as $A_1 = \alpha A_2$.[10] From (14), (15) and (16):

$$i^*_{t-1} = \overline{A} E_{t-1} y_{t-1} \tag{17}$$

where $\overline{A} \equiv A_2 + \frac{\rho}{\delta} + \frac{\alpha - A_2}{\delta \beta A_2} > 0$ (from equation (16), $A_2 < \alpha$).

The change of this policy reaction to changes in the demand shock signal is

$$\frac{\partial i^*_{t-1}}{\partial \widehat{d}_{t-1}} = \overline{A}\gamma > 0 \tag{18}$$

Since γ depends on the measurement error volatility σ^2_ε, when such volatility changes the effect on the policy response can be expressed as

$$\frac{\partial \left(\partial i^*_{t-1} / \partial \widehat{d}_{t-1}\right)}{\partial \sigma^2_\varepsilon} = \overline{A} \frac{\partial \gamma}{\partial \sigma^2_\varepsilon} < 0 \tag{19}$$

As in the original model: 1. an increment in the value of the demand shock signal increases the policy response of the central bank and such response is mitigated by the coefficient γ; and 2. an increase in the measurement error volatility implies a higher proportion of noise in the signal, and therefore the central bank's optimal response to changes in the signal is mitigated.

3.1.2 Criterion 2: robustness

The stress function is equal to that of the original model (equation (6)). The model is solved by backward induction. In the last stage nature sets ε_{t-1} so as to maximize

$$E_{t-1}\left[\Gamma_t + \beta \Phi_{t+1}\right] \tag{20}$$

where Φ_{t+1} corresponds to the value function of the problem. To obtain a solution for the central bank's problem we postulate the following forms for the value function and expectations:

$$\Phi_{t+1} = B_0 + B_1 \widehat{y}^2_t \tag{21}$$

[10] Therefore, to determine A_2, we need to solve: $\beta^2 A_2^3 + \beta\alpha A_2^2 + \left(\lambda - \alpha^2\beta\right) A_2 - \alpha\lambda = 0$

$$E_t\pi_{t+1} = B_2 E_t y_t + B_3 (1-\gamma)\widehat{d}_t \tag{22}$$

Nature picks ε_{t-1} following[11]

$$\begin{aligned}\varepsilon_{t-1} &= \frac{\rho(\lambda+\beta B_1)+\beta B_2(\alpha+\rho\beta B_2)}{\theta/\sigma_\varepsilon^2-\rho^2(\lambda+\beta B_1)-(\alpha+\rho\beta B_2)^2}\delta(i_{t-1}-E_{t-1}\pi_t) \\ &\quad -\frac{\rho^2(\lambda+\beta B_1)+(\alpha+\rho\beta B_2)^2}{\theta/\sigma_\varepsilon^2-\rho^2(\lambda+\beta B_1)-(\alpha+\rho\beta B_2)^2}\widehat{y}_{t-1}\end{aligned}$$

Taking into account the foregoing equation we proceed to solve the central bank's problem, i.e. to minimize (20) with respect to i_{t-1}, which yields[12]

$$i_{t-1} = E_{t-1}\pi_t + \frac{1}{\delta}\frac{\rho\left[\lambda+\beta B_1+(\beta B_2)^2\right]+\alpha\beta B_2}{(1-\alpha^2\sigma_\varepsilon^2/\theta)(\lambda+\beta B_1)+(\beta B_2)^2}\widehat{y}_{t-1} \tag{23}$$

Using equations (PC'), (22), (23) and the fact that $\widehat{y}_{t-1} = E_{t-1}y_{t-1} + (1-\gamma)\widehat{d}_{t-1}$ we can find the following expression for expectations

$$\begin{aligned}E_{t-1}\pi_t &= \frac{\alpha(\lambda+\beta B_1)\left[1-\alpha(\sigma_\varepsilon^2/\theta)(\alpha+\rho\beta B_2)\right]}{(1-\alpha^2\sigma_\varepsilon^2/\theta)(\lambda+\beta B_1)+(\beta B_2)^2}E_{t-1}y_{t-1} \\ &\quad -\frac{\rho\left[\lambda+\beta B_1+(\beta B_2)^2\right]+\alpha\beta B_2}{(1-\alpha^2\sigma_\varepsilon^2/\theta)(\lambda+\beta B_1)+(\beta B_2)^2}\beta B_2(1-\gamma)\widehat{d}_{t-1}\end{aligned} \tag{24}$$

which is consistent with the postulated form (22) as long as

$$B_2 = \frac{\alpha(\lambda+\beta B_1)\left[1-\alpha(\sigma_\varepsilon^2/\theta)(\alpha+\rho\beta B_2)\right]}{(1-\alpha^2\sigma_\varepsilon^2/\theta)(\lambda+\beta B_1)+(\beta B_2)^2} \tag{25}$$

and

$$B_3 = -\frac{\rho\left[\lambda+\beta B_1+(\beta B_2)^2\right]+\alpha\beta B_2}{(1-\alpha^2\sigma_\varepsilon^2/\theta)(\lambda+\beta B_1)+(\beta B_2)^2}\beta B_2$$

[11] We assume θ is large enough so as to satisfy the SOC of the problem: $\theta/\sigma_\varepsilon^2-\rho^2(\lambda+\beta B_1)-(\alpha+\rho\beta B_2)^2>0$.This assumption is reasonable because when $\theta\to\infty$, B_1 and B_2 tend to finite values (this can be proved using equations (25) and (26)).

[12] Given the SOC of the nature's problem, the SOC of the central bank's problem is $\left(1-\alpha^2\sigma_\varepsilon^2/\theta\right)(\lambda+\beta B_1)+(\beta B_2)^2>0$. This condition is satisfied as long as θ is large enough. See footnote 11.

Then, using the obtained results we can verify that

$$\Phi_t = \beta \left[B_0 + \left(B_1 (1-\gamma)^2 + \beta B_3^2 \right) \left(\sigma_d^2 + \sigma_\varepsilon^2 \right) \right] + \frac{\alpha^2 (\lambda + \beta B_1)}{(1 - \alpha^2 \sigma_\varepsilon^2 / \theta)(\lambda + \beta B_1) + (\beta B_2)^2} \widehat{y}_{t-1}^2$$

which is consistent with the postulated form (21) as long as

$$B_0 = \frac{\sigma_d^2 + \sigma_\varepsilon^2}{1-\beta} (1-\gamma)^2 \left[\beta B_1 + (\beta B_3)^2 \right]$$

and

$$B_1 = \frac{\alpha^2 (\lambda + \beta B_1)}{(1 - \alpha^2 \sigma_\varepsilon^2 / \theta)(\lambda + \beta B_1) + (\beta B_2)^2} \tag{26}$$

From (23), (24) and $\widehat{y}_{t-1} = E_{t-1} y_{t-1} + (1-\gamma) \widehat{d}_{t-1}$:

$$i_{t-1}^* = \left(B_2 - \frac{B_3}{\delta \beta B_2} \right) E_{t-1} y_{t-1} + \left(1 - \frac{1}{\delta \beta B_2} \right) B_3 (1-\gamma) \widehat{d}_{t-1} \tag{27}$$

The change of this policy reaction to changes in the demand-shock signal is:

$$\frac{\partial i_{t-1}^*}{\partial \widehat{d}_{t-1}} = B_2 \gamma + B_3 (1-\gamma) - \frac{B_3}{\delta \beta B_2} \tag{28}$$

We verify that, for a large set of parameter values,[13] this expression is positive as it was under the criterion of minimization of the expected loss, in Section 3.1.1, and in both cases for the original model in Section 2. However, while in the original model we find that it is always the case that the prudent central bank reacts more aggressively to a higher demand-shock signal, in the present model (with forward-looking expectations) it only happens (for the set of parameters analyzed) when the measurement error volatility is large (and therefore the ratio $\theta / \sigma_\varepsilon^2$ is very small). In general, for the present model, the prudent central bank reacts less aggressively to the demand shock signal. The influence of forward-looking expectations on the transmission mechanism reduces the

[13] We take the baseline ($\beta = 0.99$, $\alpha = 0.024$, $\lambda = 0.003$, $\sigma_d = 2.54$, $\rho = 0.8$, $\delta = 6.25$) from Bodenstein et al. (2012). It must be remarked that while their model corresponds to a standard New Keynesian structure ours is not, due to the incorporation (ad hoc) of lags in equations (IS) and (PC'). We allow for variation of the following parameters (one at a time) within the following intervals $\alpha \in [0.01, 0.06]$, $\lambda \in [0.001, 0.015]$, $\rho \in [0.4, 0.95]$ and $\delta \in [1, 7]$. For each case we set values for θ within the interval $[0.001, 10]$, under the condition that the SOCs be always satisfied and values for σ_ε within the interval $[1/10, 3]\, \sigma_d$, i.e. as a proportion of σ_d. For all combinations $\partial i_{t-1}^* / \partial \widehat{d}_{t-1} > 0$.

damage that nature can cause and hence, as Barlevy (2009) remarks, the robustness criterion does not necessarily imply that policy should be always more aggressive in the face of uncertainty.

Since γ, B_2 and B_3 depend on the measurement error volatility σ_ε^2, when such volatility changes the effect on the policy response can be expressed as

$$\begin{aligned}\frac{\partial\left(\partial i_{t-1}^{*}/\partial\widehat{d}_{t-1}\right)}{\partial\sigma_\varepsilon^2} &= \frac{\beta\delta B_2\left(1-\gamma\right)-1}{\beta\delta B_2}\frac{\partial B_3}{\partial\sigma_\varepsilon^2}+ \\ &\quad \frac{\beta\delta\gamma B_2^2+B_3}{\beta\delta B_2^2}\frac{\partial B_2}{\partial\sigma_\varepsilon^2}+\frac{\partial\gamma}{\partial\sigma_\varepsilon^2}\left(B_2-B_3\right) \qquad (29)\end{aligned}$$

In this case the value of this derivative may be either positive, as in the case under the criterion of minimization of the expected loss, or negative. Whether it is positive or negative, as in the original model and for the set of parameters analyzed, depends on the value of the ratio $\theta/\sigma_\varepsilon^2$. When this ratio is very small, risk aversion prevails in the final effect and a prudent central bank is willing to be more aggressive when it perceives an increase in the measurement error volatility. In contrast, if the ratio $\theta/\sigma_\varepsilon^2$ is not small, the dominant effect is that related to the fact that the signal is becoming noisier, and therefore the central bank's optimal response to changes in the signal is reduced. However, such reduction is lower (i.e. when it is negative, the value of (29) is smaller in absolute value) than the one that occurs under the criterion of minimization of the expected loss.

4 Conclusions

We set up a stylized model which incorporates two features; first, the output gap exhibits some degree of persistence and second, a lag in the effect of monetary policy such that it affects the output gap more rapidly than inflation. The output gap is measured with error and therefore monetary policy faces uncertainty.

We derive the optimal policy response to a noisy signal of the demand shock and to changes in the measurement error volatility from two different perspectives: the minimization of the expected loss (which we refer to as the 'standard' perspective) and the minimization of the maximum possible loss across all potential scenarios (which we refer to as the 'prudent' perspective).

We find that: 1. the prudent policymaker reacts more aggressively to the shock signal than the standard one and 2. while the standard policymaker always mitigates her reaction if the measurement error volatility rises, the prudent one may even increase her response if her risk aversion is very high. The second result is preserved when we incorporate forward-looking expectations but, with regard to the first one, the prudent policymaker is less aggressive than the standard one in responding to the shock signal. The influence of forward-looking expectations on the transmission mechanism reduces the damage that nature can cause and hence, as Barlevy (2009) remarks, the robustness criterion does not necessarily imply that policy should be always more aggressive in the face of uncertainty.

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