Por: Wilmar Cabrera, Jorge Hurtado,
Miguel Morales, Juan Sebastián Rojas

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A Composite Indicator of Systemic Stress (CISS) for Colombia[*]

Wilmar Cabrera[*]   Jorge Hurtado[†]   Miguel Morales[‡]   Juan Sebastián Rojas[§]

Abstract

The most recent global financial crisis (2008-2009) highlighted the importance of systemic risk and promoted academic interest to develop a wide set of warning indicators, which are mechanisms to identify systemically important institutions and global systemic risk indexes. Using the methodology proposed by Holló et al. (2012), along with some considerations from Hakkio & Keeton (2009), this document comprises a Composite Indicator of Systemic Stress (CISS) for Colombia. The index takes into account several dimensions related to financial markets (credit institutions, housing market, external sector, money market and local bond market) and is constructed using portfolio theory, considering the contagion among dimensions. Results suggest the peak of the global financial crisis (September 2008) as the most important episode of systemic risk in Colombia between 2000-2014. Additionally, real activity seems to be adversely affected by an unexpected increase of the systemic risk index.

JEL classification: *G12,G29,C51*
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1. Introduction

Financial markets have shown to have a significant impact on real economic activity, especially in times of stress (Levine (2004), Davig & Hakkio (2010)). Despite this fact, before the latest global financial crisis, economic literature had focused on the effects of real cycles over financial institutions. However, since the most recent international recession originated in the core of the financial sector, the focus has shifted on how to measure the systemic impact of shocks in financial markets on the real sector, and how to quantify the state of systemic stress. This article is mainly related to the latter, since it develops a systemic risk indicator. This approach has become relevant as it represents a tool for policy makers to identify potential vulnerabilities, and respond through prudential mechanisms in order to limit the system's exposure to the sources of risk.

In the literature there is a series of approximations to measuring systemic risk. On one hand, some of these methods intend to rank financial entities according to their systemic importance. In general, these articles use asset size, and measures of connectivity and substitutability as assessments of systemic importance, such as BIS (2013) and León & Machado (2011). On the other hand, some articles seek to measure the risk contribution of each financial institution to the system as a whole. In this line of study, the CoVaR approximation proposed by Adrian & Brunnermeier (2011) is highlighted. The methodology measures the increase in financial risk, defined as the change in the Value at Risk (VaR) of the system, caused by an entity being in distress. Another possible approach within this framework is the systemic and marginal expected shortfall (SES & MES), elaborated by Lehar (2005), which estimates the probability of occurrence of a systemic event, defined either as the scenario in which a given number of financial institutions fail or an arbitrary amount of assets is lost. Additionally, the author calculates the maximum expected loss of the system given the aforementioned events, using the SES, and the contribution of each entity to such loss, through the MES. Using a different methodology, Gauthier et al. (2010) estimates macroprudential capital requirements based on the risk contribution of each entity, the latter being measured using several methods, such as the CoVaR or the incremental Value at Risk (iVaR). Finally, Gray & Jobst (2010) use a Contingent Claims Approach (CCA) to quantify the creditworthiness of financial institutions extending the analysis to the assessment of financial markets' systemic risk.

Despite the existence of different methodologies to measure systemic risk, this research contributes to the policy toolkit in Colombia by constructing an indicator composed of a set of variables relevant in gauging the state of systemic risk in the Colombian financial market, jointly considering the risk levels in different financial dimensions. The implementation of this method goes in line with the interest of central banks to calculate aggregate risk indexes that help to better understand the state of financial markets; some examples are the Federal Reserve (Fed), the European Central Bank (ECB) and the Central Bank of Spain. Moreover, these measures are considered practical tools for policymakers, since they aggregate the main risk factors in an intuitive manner, but also allow to identify the major sources of risk.

To start, an explicit definition of systemic risk is useful. Although there is no consensus on a definition of systemic risk, a widely accepted one is that proposed by the European Central Bank, ECB (2009), which defines it as the risk that financial imbalances become so widespread that they impair the normal functioning of the financial system to the point where the economy and welfare are deteriorated. This definition follows the one proposed by the International Monetary Fund (IMF), the Bank for International Settlements (BIS) and the Financial Stability Board (FSB), which describe systemic risk "as a risk of disruption to financial services that is (i) caused by an impairment of all or parts of the financial system

and (ii) has the potential to have serious negative consequences for the real economy" IMF-BIS-FSB (2009).

Consequently, and in line with the methodology proposed in Holló et al. (2012), the systemic risk index should cover both a "horizontal perspective" related to financial instability and contagion between financial markets, and a "vertical perspective" related to the cost of financial imbalances on the economy. Regarding the "horizontal perspective" of systemic risk, it focuses on the spreading of the crisis, by taking into account correlations across markets. The "vertical perspective" considers the relationship between the financial system and the real economy by measuring the impact that the former has on the latter.

The first challenge is hence to specify how to quantify systemic risk in a single composite index. First of all, the main dimensions (sectors) where systemic risk could emerge and spread were identified, along with the variables in each dimension that are more likely to capture financial stress episodes. The dimensions included were selected from those proposed by Holló et al. (2012), Louzis & Vouldis (2012), Milwood (2013) and Kota & Saqe (2012). Meanwhile, the group of variables used within each dimension was set following Holló et al. (2012) and Hakkio & Keeton (2009), who point out that the set of variables has to be related to the five main symptoms often associated with financial stress events: i) uncertainty about the fundamental value of assets, ii) uncertainty about the behavior of investors, iii) asymmetry of information, iv) decreased willingness to hold risky assets (flight to quality) and v) decreased willingness to hold illiquid assets (flight to liquidity).

These symptoms usually materialize in financial markets as greater asset price volatility, wider spreads between rates of return on different types of assets and higher cost of borrowing for relatively risky borrowers. Hence, variables like interest rate spreads, volatilities and correlations between assets' returns capture the criteria mentioned and may be used to obtain a measure of the system's stress level. Variables allowing the assessment of build-up of risks are also taken into account: credit and housing gaps are relevant for the analysis as the joint imbalance of asset prices and credit provides valuable information of probable future crises (Borio & Drehmann (2009)); additionally, credit risk indicators have proven being helpful since they tend to deteriorate faster before a financial crisis (Inaba et al. (2005)).

The next step is to generate subindices of each of the dimensions using principal component analysis, which help to measure the state of risk within each of the selected sectors. The subsequent aggregation of these subindices into the Composite Indicator of Systemic Stress (CISS) needs to consider the dynamic correlation between the different sources of risk, as contagion between financial markets tends to be higher in periods of financial stress (Bekaert et al. (2005), Chiang et al. (2007)). This is achieved using standard portolio theory and by modelling the change in time of co-movements between dimensions with a DCC-GARCH model, which allows the indicator to gauge the contagion between sectors.

The CISS for Colombia estimates the systemic stress level of the local financial system period by period (on a monthly basis), comprising different financial dimensions (credit institutions, housing market, external sector, money market, and bond market) and the contagion among them. Results show that the CISS reached its highest level on September of 2008, when Lehman Brothers declared bankruptcy at the peak of the financial crisis. However, the CISS also detects some periods of turbulence in the financial system, that given the low correlation of the five dimensions taken into account, are not considered systemic.

This article is organized as follows: Section II describes the data and the methodology used to build the indicator. Section III presents the main results, the evaluation of the index in terms of its ability

to identify financial stress periods and how the index is related with real activity. Finally, Section IV concludes.

2. Systemic Risk Index

As mentioned in the introduction, the purpose of this article is to find an approximation to asses the level of systemic risk in the financial system. Following the main results in the literature, these measures need to consider the major sources of risk that could potentially jeopardize financial stability, which vary depending on the type of economy under analysis. Therefore, to accomplish this task for the Colombian economy, the current work includes the following dimensions: credit institutions, the housing market, the external sector, the money market and the bond market. To construct the index, the main dynamic of each of these dimensions is quantified using the first principal component of a group of variables[1], which generate a subindex of a high relevance in explaining the behaviour of each sector. Later, taking the subindex of each category as an input, a composite indicator of systemic stress (CISS) is calculated, as the aggregation of these variables according to their correlations. As mentioned in the literature, it is important to consider that correlation between risk factors changes in time, and tends to be higher during periods of stress; for this reason, the index would need to reflect this dynamic. Lastly, equally weighted dimensions were assumed to aggregate the subindexes, but it is also an option in the methodology to weight them in the CISS depending on their relation to real activity.

2.1. Data and Variables

Taking into account the aforementioned dimensions, three variables on each category that reflect the build-up risk of the financial system were included. The selection of these variables intends to capture the insights provided by economic and financial theory regarding the accumulation and subsequent realization of financial stress. However, all the variables suggested by the literature could not be considered due to data constraints, and consequently, only variables with a monthly frequency and long time series are considered[2]. It is also important to mention that the chosen variables are stationary in order to use principal component analysis[3]. In what follows, a brief description and explanation of each variable, their importance to systemic risk and their source is presented, organized by market.

I. Credit Institutions

Given that credit institutions hold a considerable share of the assets of the Colombian financial system[4] and their task as financial intermediaries, they play an important role in the correct functioning of the financial system. Therefore, if these institutions face a period of stress, it might have a significant effect on the real and financial sector. For instance, during the most recent financial crisis, US credit institutions were in distress and unable to intermediate resources, which affected market liquidity, credit supply and then, real activity.

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[1] "The principal components decomposition can be viewed as a particular kind of factor model (...) its role in risk measurement is to reduce dimensionality of the problem, that is, to reduce the number of underlying sources of uncertainty or market factors that must be considered" Pearson (2002). The first principal component explains the greatest proportion of the series' variance in each subindex.

[2] For the purpose of this analysis, long time series are those starting at least in 2000.

[3] The unit root tests are presented in the Appendix 4.1, Table 1.

[4] According to the Financial Superintendency of Colombia, as of June, 2013, these institutions held 44.3% of the total assets of the Colombian financial system.

Credit Gap: the credit gap is defined as the difference between the total gross loan portfolio and its long-term trend, estimated through a Hodrick-Prescott filter [5]. It reflects imbalances regarding credit demand and supply; a negative gap may be a sign of aversion to lend resources by credit institutions or a decrease in credit demand, which happens during financial stress periods. On the other hand, when credit is significantly above its long term trend, it could be signalling a risk build-up process, since agents in the economy become more indebted and credit corporations accumulate risky assets. Inasmuch as the relationship between the credit gap and stress in the credit market can be inverse or direct, we let the model to endogenously determine it.

Non-performing Loans (NPL) Gap: the gap of NPL[6] is included as a measure of credit risk. A higher amount of NPL increases its gap and the stress of credit institutions, suggesting the realization of the higher credit risk exposure. Empirical evidence has shown that credit risk indicators are helpful in this purpose since they tend to deteriorate faster before a financial crisis [Inaba et al. (2005)], then a direct relationship is expected.

Interest Margin: profits tend to show a decreasing trend when the financial system is in stress; therefore, its dynamics are a measure of the system's health. In order to capture the dynamics credit institutions' profits, the annual rate of change of the ex-post intermediation margin is included in this subindex. The ex-post intermediation margin is defined as the difference between the implicit rates, being the lending rate the ratio of interest income to performing loans, and the deposit rate the ratio of interest expenditures to interest-generating liabilities.

Despite the fact that the interest margin is an approximation to credit institutions' profitability, it also shows the credit risk faced by those institutions. Hence, a higher interest margin can also be the result of higher risk premiums demanded by credit institutions. Therefore, this variable can have either a direct or inverse relationship with systemic risk, which again will be determined endogenously by the model.

All the variables included in this dimension come from balance sheet information, published by the Financial Superintendency of Colombia, starting in January 1996.

II. Housing Market

The importance of this market rests on the fact that housing constitutes the main asset of most Colombian households, which means that sudden changes in its price might have considerable effects on their wealth[7] and thus on their ability to access credit. Additionally, the construction sector has presented high momentum in recent years in Colombia, which has driven growth in several sectors due to its close linkage to other industries. As expected, housing is the main collateral in the financial system, so changes in its value could result in increases in the risk profile of the financial system.

Housing Credit Gap: the rationale for including this variable is that a negative housing credit gap could be interpreted as a weak demand for credit, or a restriction of credit supply. On the demand

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[5]Gaps were calculated using the smoothing parameter recommended for monthly series, $\lambda = 14400$.

[6]The NPL are defined as those credits which are 30 days past due for all kinds of loans excepting mortgages, which are defined as NPL when they are 60 days past due.

[7]During the period 1996-2006, housing represented more than 70% of total household wealth in Colombia (López & Salamanca 2009).

side, when households have low income, or do not expect a higher income in the future, their demand for loans may decrease, especially for housing loans, because of its nature of longer term debt. On the supply side, when the housing market is under stress, credit institutions may not be willing to lend money at longer terms, and there could be a shortage of housing credit supply which reflects in a negative gap.

As explained for credit gaps, when housing credit is significantly above its trend, it could also reflect a risk build-up process, in which credit demand from risky debtors is matched by risk taking credit institutions. Hence, the relationship between this variable and systemic risk will be determined endogenously by the model.

Non-performing Housing Loans Gap: this variable is included because when non-performing housing loans, considering mortgage backed securities, exceed their long term trend, credit institutions experience a reduction in their income, increasing the stress in the financial system. A rise in the non-performing mortgage loans gap can also reflect previous bad risk-taking decisions made by credit institutions. Consequently, a direct relationship between systemic risk and this variable is expected.

Housing Prices: as mentioned above, an unexpected drop in house prices may have a negative effect on the balance sheet of credit institutions because houses are often used as collateral for loans. Additionally, the wealth of households can be dramatically reduced as a result of lower housing prices, which might contract their credit demand. Hence, house prices and financial risk can be inversely related.

From a different point of view, when house prices reach very high levels, it could be a symptom of the generation of an overvaluation period, which occurs when prices grow significantly above the level that is consistent with its fundamentals. Accordingly, when house prices considerably deviate from their long term trend, whether above or below it, distress on the financial system arise. Consequently, the model will determine endogenously the relationship between housing prices and systemic risk.

Total and non-performing housing loans come from balance sheet information published by the Financial Superintendency of Colombia and the new housing price index was calculated by the National Planning Department [8]. All variables were taken since January 1997.

III. External Sector

Uncertainty in foreign markets can increase systemic stress, as external shocks have significant impacts on both financial and real markets. First of all, foreign counterparties represent a source of funding, especially in the long-term, for real and financial firms; a sudden stop in the flow of these resources might impose important limitations to economic activity. Gonzalez (2012) highlights this point and illustrates the risks that may be posed by the reliance on external financing of some Latin American countries (Brazil, Chile, Colombia, Mexico and Peru) during the period 2000-2011.

Additionally, unexpected variations in the exchange rate will have effects on firms with assets and liabilities denominated in foreign currency. For instance, if a firm is a net exporter of goods or services, sudden changes in the value of the legal tender will generate unwanted volatility in its income, which will difficult financial planning in the medium term. This will also affect firms whose liabilities are denominated in foreign currency, as they will not be able to accurately predict their future outflows.

Finally, growth in emerging economies has been boosted by foreign trade, especially of commodities. Consequently, unexpected movements in the prices of these goods will have a significant and direct impact

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[8]Departamento Nacional de Planeación (DNP in Spanish).

on national income, in particular on firms in the extraction and commerce of commodities business and on the government due to fall in the perceived royalties. The reduction in these inflows might generate difficulties to meet financial obligations, which could eventually have second-round effects on economic activity.

Exchange Rate Volatility: the volatility of the exchange rate, which was estimated using an AR-GARCH(1,1) model for the daily series, captures the underlying uncertainty in the foreign exchange market, which, as mentioned above, imposes harmful volatility to firms' profits and/or liabilities denominated in foreign currency. Higher volatility is a sign of an increase in uncertainty or stress in this market, therefore a positive relationship with systemic risk is expected.

Foreign Debt Gap: on one hand, the gap of the level of indebtedness of local financial intermediaries in the foreign credit markets[9] is an approximation to the exchange rate risk exposure of the local financial system. An increase in the gap reflects a higher dependence on external financial market conditions. On the other hand, a reduction in the levels of foreign debt may indicate a substitution of funding sources or a constraint of external resources; the latter could increase the vulnerability of the financial system. Therefore, the model will endogenously determine the existing relationship between this variable and systemic risk.

Commodity Price Index Volatility: this variable is an estimation of the volatility of the daily commodity index returns through an AR-GARCH(1,1) model. Colombian exports are highly dependent of commodities, which represent 74.9% of total of goods and services sold abroad (December 2013); unexpected changes in commodity prices may affect the real sector, firms, and spread to financial markets through credit risk. Hence, a positive relationship between this volatility and the build-up of risk in the system is expected.

The daily exchange rate (Colombian Peso/US Dollar) and the foreign debt were taken from Banco de la República (central bank of Colombia), while the commodity index returns were obtained from Bloomberg, starting in October 1999.

IV. Money Market

Money markets are the main source of short-term funding and are constituted by short-term, highly marketable, and very low risk debt securities. Money market spreads have caught the attention since the most recent financial crisis, due to the fact that they reflect high levels of counterparty and liquidity risk.

Spread of the 3-month Colombian Sovereign Bond and US Treasury Bill: the monthly difference between the interest rates of the 3-month Colombian Sovereign Bonds (TES) denominated in US dollars and the 3-month US Treasury Bills (considered the benchmark and the risk free interest rate) is an important measure of liquidity and counterparty risk. Since Colombian TES are an important instrument used by Colombian credit institutions, the yield spread contains information on market fundamentals and could also reflect flight to quality and flight to liquidity events. It is expected that during financial stress periods, money markets face higher spreads.

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[9]This variable is taken as a proxy of the total level of indebtedness of the economy, since the series that includes the foreign debt of the corporate sector is only available since 2001. Nonetheless, the foreign debt of financial intermediaries exhibits a high correlation (0.81) with the total.

3-month Colombian Sovereign Bond volatility: this indicator is computed using a monthly AR-GARCH(1,1) model for the US dollar denominated 3-month Colombian TES interest rate. Higher volatility on the 3-month Colombian TES reflects higher uncertainty in the short-term bond market. Uncertainty can be manifested in the disruption or malfunctioning of the money market due to flight to quality, flight to liquidity and/or increasing asymmetric information ((Louzis & Vouldis 2012)), so a positive relationship with systemic risk is expected.

Interbank and the Policy Rate spread: the indicator serves as a general measure of liquidity and credit risk, since it reflects the risk premium associated with lending to commercial banks. The spread is intended to capture liquidity constraints and credit risk premiums in times of financial stress. Therefore, increases in this spread should result in a greater vulnerability of the financial system.

3-month Colombian and US Treasury bonds yields come from Bloomberg, while the source of the interbank and policy rates is Banco de la Republica. All time series in this dimension are taken from April 1998.

V. Bond Market

The bond market in Colombia is mainly composed by sovereign debt securities with longer maturities. Movements in this market reflect the state of economic fundamentals and are related to sovereign risk. Besides the direct effect on the government's expected cash flow, sudden variations in the yield of these assets will have a considerable impact, not only on financial institutions which concentrate a significant portion of their investments in these assets, but also on households that hold a large share of these securities through pension funds and other saving and investment vehicles.

10-year Colombian Sovereign Bond and US Treasury Bond spread: the spread between the 10-year US dollar-denominated Colombian TES and the 10-year US Treasury Bills reflect the behavior of market fundamentals, namely liquidity, sovereign creditworthiness and other risk premia. It is expected that during financial stress periods, this spread widens.

IDXTES-Bond Index volatility: IDXTES is the Colombian bond price index and its volatility reflects the uncertainty regarding the local bond market. The volatility was estimated using an AR-GARCH(1,1) model on the daily change of the index. A positive relationship between this variable and systemic stress is expected.

Correlation between the 10-year Colombian Bond and the Colombian Stock Market Index (IGBC[10]): over the long run, correlation between stock market and sovereign bond returns is positive; however, during periods of financial stress this correlation tends to be negative. The time-varying nature of this correlation is a reflection of the flight to quality phenomenon during periods of financial stress. In fact, during some financial stress episodes in Colombia, a negative correlation between these two assets has indeed appeared.

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[10]Índice General de la Bolsa de Valores de Colombia (IGBC) was the main index of the Colombian stock exchange until November 2013, when it was replaced by COLCAP. However, in this paper the IGBC is used because it has a longer time series.

Spreads and correlations included in this dimension are taken from Bloomberg and the IDXTES from Banco de la República, starting in January 2000.

A summary of the expected and estimated effects of each variable that composes the subindices can be found in Appendix 4.1. Figure 1 graphs the variables that compose each of the aforementioned subindices.

FIGURE 1: Dimensions
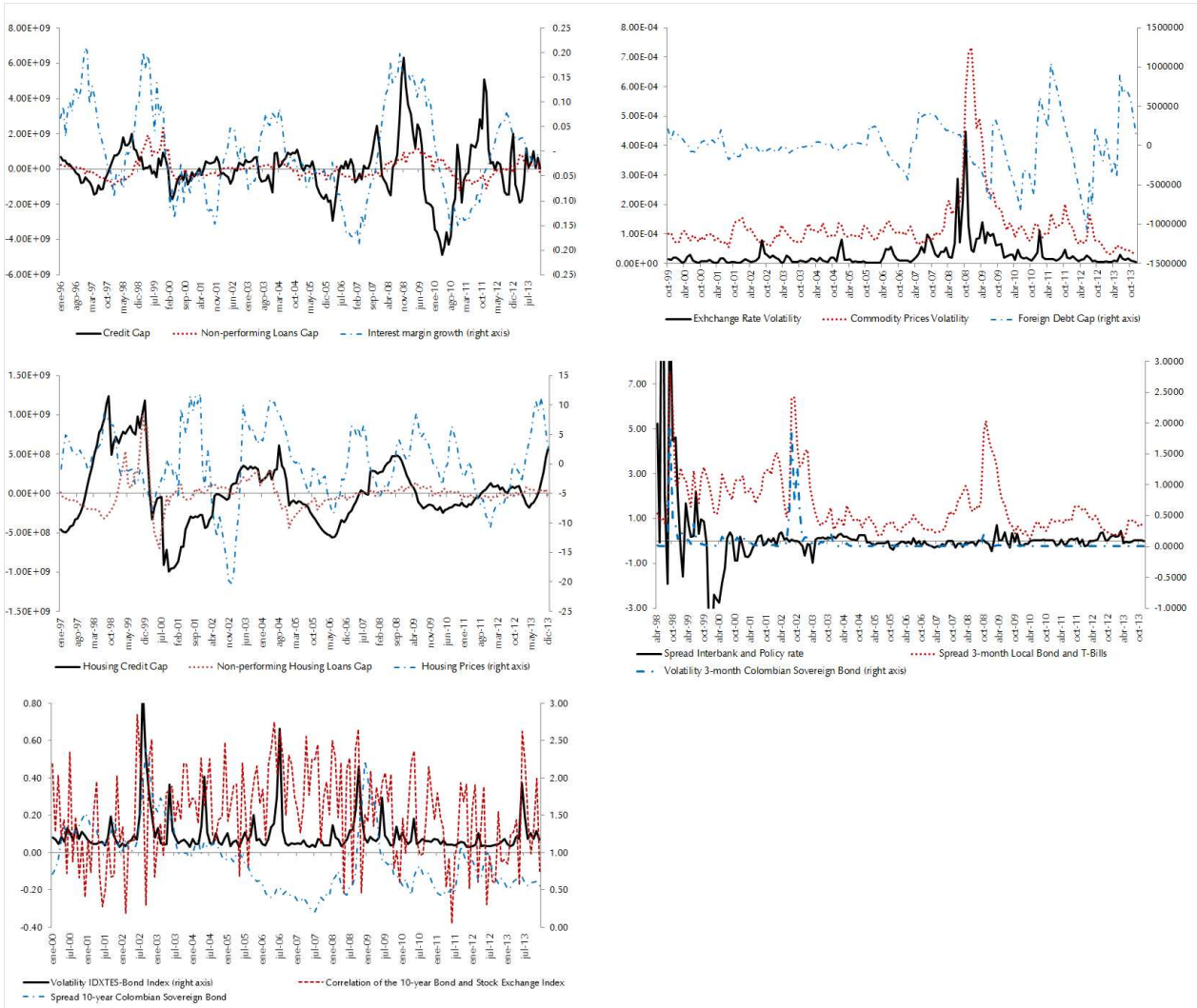
Source: Calculations by the authors. The volatility of the IDXTES was scaled by a factor of 10^4.

2.2. Aggregation Method

Once the variables that compose each category are selected, the next step is to aggregate them into sub-indices that capture the build-up of stress on each of these markets. To do so, a principal component analysis is applied, and the factor explaining the greatest proportion of the series' variance is chosen as the respective subindex. In order to have a comparable set of subindices, they are standardized using the logistic transformation[11].

After obtaining the five different subindices, standard portfolio theory was applied in order to aggregate them into a single a composite indicator of systemic risk, with the advantage that the latter takes into account the correlation among dimensions. The methodology was proposed by Holló et al. (2012) at the European Central Bank, and also implemented by Louzis & Vouldis (2012), Milwood (2013), BDE (2013) and Kota & Saqe (2012), for the design of systemic risk indicators in Greece, Spain, Jamaica and Albania, respectively.

As mentioned above, the subindices are treated as individual risky assets, and aggregated into an overall portfolio, considering the cross correlation among all individual assets' returns[12]. In standard portfolio theory, when highly correlated risky assets are aggregated, the total risk of the portfolio increases, as all assets tend to move simultaneously following market movements. On the other side, when correlation between assets is low, diversifiable risk is reduced, decreasing total portfolio riskiness. According to Holló et al. (2012), this same logic applies to systemic risk; the stronger the correlation of financial stress among subindices, the more widespread is the state of financial instability, which is known as the horizontal view of the definition of systemic risk.

In order to incorporate the correlation between sub-indices, the composite indicator of systemic risk is computed according to:

$$CISS_t = \sqrt{(w \circ s_t) R_t (w \circ s_t)'}, \tag{1}$$

where $w = (w_1, w_2, w_3, w_4, w_5)$ is defined as the vector of subindex weights, $s_t = (s_{1,t}, s_{2,t}, s_{3,t}, s_{4,t}, s_{5,t})$ is the vector of subindices; and $(w \circ s_t)$ is the Hadamard-product (also known as element-wise product)[13] of subindex weights and subindex values vectors in time t. Finally, R_t is the 5×5 time-varying correlation matrix:

$$R_t = \begin{bmatrix} 1 & \rho_{12,t} & \rho_{13,t} & \rho_{14,t} & \rho_{15,t} \\ \rho_{12,t} & 1 & \rho_{23,t} & \rho_{24,t} & \rho_{25,t} \\ \rho_{13,t} & \rho_{23,t} & 1 & \rho_{34,t} & \rho_{35,t} \\ \rho_{14,t} & \rho_{24,t} & \rho_{34,t} & 1 & \rho_{45,t} \\ \rho_{15,t} & \rho_{25,t} & \rho_{35,t} & \rho_{45,t} & 1 \end{bmatrix} \tag{2}$$

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[11]Each standardized subindex, $s_{j,t}$ is calculated using the following formula: $s_{j,t} = \frac{e^{rs_{j,t}}}{1+e^{rs_{j,t}}}$, where $rs_{j,t}$ is the raw subindex (obtained from applying principal components to the variables that compose it) of dimension j, for $j = 1 \ldots 5$. This also guarantees that subindices are defined between zero and one.

[12]There is empirical evidence that these correlations increase during financial turmoil and thereby, increase risk even further; therefore, modeling correlation dynamics is crucial to a risk manager (Engle (2002)).

[13]A binary operation that takes two matrices/vectors of the same dimensions, and produces another matrix/vector where each element ij is the product of elements ij of the original two matrices/vectors. This is computed simply through element-by-element multiplication.

The time-varying cross-correlations $\rho_{ij,t}$ are estimated using a Multivariate GARCH (MGARCH) approach over the five different dimensions, in which the conditional variances and covariances of the errors follow an autoregressive moving average structure. The model allows the conditional covariance matrix of the dependent variables to follow a dynamic structure and allows the conditional mean to follow a vector autoregressive structure (VAR)[14].

After estimating the dynamic correlations, the weights w have to be chosen taking into account the importance of each dimension for the real sector. Although it is possible to define the weights from statistical methods, here has been assumed that each dimension has an equal weight (20%). Kota & Saqe (2012) provide an example of the usage of statistical tools to find the weights, by performing a minimization process for the sum of squared differences between GDP growth and the weighted dimensions $(w \cdot s_t)$, subject to the sum of weights (w) being equal to one and each w_i higher or equal to zero, which is equivalent to the restricted ordinary least squares estimation[15].

Note that when all correlations are equal to one (perfect correlations) and each w_i is the same (equally weighted), the composite indicator is equivalent to the arithmetic average of subindices and would constitute the upper boundary of the CISS. This special case may imply that all five subindices stand either in an extreme stress situation.

2.3. Results

This section identifies financial stress periods in Colombia between 2000 and 2013, presents the estimated dynamic correlations of the five different dimensions and introduces the CISS. Later, the systemic risk indicator is compared with real activity indicators and a multipliers model is applied in order to test their relationship (vertical view of the definition of systemic risk).

In this paper, the financial stress periods considered from 2000 to 2008 were defined as those identified by Gómez et al. (2011), who characterize the main trends observed in capital and credit markets and their relation with periods of stress in the real economy. After 2008, three additional financial stress periods are considered.

Gómez et al. (2011) state that there were three periods of financial stress between 2000 and 2008. The first one was a period of high turbulence in the government bonds market, between July and September of 2002, caused by an increase in local interest rates, as a consequence of adverse changes in country risk. These conditions were reflected in low bond prices, high market volatility and losses experienced by financial institutions. Even though the impact was strong on financial variables, the effects on economic activity were not significant. The second stress period was between February and June of 2006, when the Fed decided to increase the policy rate, changing the future path of monetary policy in that country. As a result, investors expected higher returns in the US and liquidated their positions in emerging countries, including Colombia. This led to valuation losses in Colombian financial institutions given their high exposure to these assets. Once again, the real sector was not highly damaged due to the long-term investment horizon of pension and severance funds' investment portfolio, and the rapid recomposition of the banking sector's balance sheet from government bonds to loans. The last period identified by Gómez et al. (2011) was the Lehman Brothers collapse in September of 2008 during the peak of the

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[14]For further details see Appendix 4.3

[15]This exercise was also performed in this document, but results were not satisfactory since some dimensions got a weight equal to zero. In any case, the CISS constructed using those weights did not differ significantly from the equally weighted index.

US financial crisis. This period was characterized by high uncertainty and an increase in risk aversion, which simultaneously affected the volatility and correlation of local markets (i.e. bonds markets, money markets, exchange markets, etc.). In this period, economic activity also exhibited a slowdown, reaching an annual growth equal to 0.39% in the fourth quarter of 2008.

Additional to the periods mentioned above, three recent episodes which could have caused financial distress were identified: the Greek crisis concerns, the Interbolsa collapse and the tapering announcement by the Fed. Regarding the first period, the Greek government debt was downgraded to junk bond status in April 2010, jeopardizing the long-term viability of the Euro-zone and causing turmoil in global financial markets; those events did not have significant impacts neither on the financial nor on the real sector in Colombia. The next episode took place in November 2012, when the largest brokerage firm in Colombia (Interbolsa) was intervened by the local government due to the default on an intra-day loan with a local commercial bank. The effect of Interbolsa's collapse on local financial and real markets was not noteworthy given the rapid response of local authorities. Finally, the tapering announcement by the Fed, in May 2013, was also identified as a stress period, especially for emerging economies. During that time, the Fed began announced the possibility of tapering its purchases of securities and investors interpreted it as a change in the posture of monetary policy in US, causing a recomposition of investment portfolios in emerging economies, and increased uncertainty in financial markets. Nevertheless, Colombian financial markets did not seem to experience a systemic impact.

Once the stress periods are identified, dynamic correlations and the CISS are contrasted to test if they capture systemic episodes. Figure 2 exhibits the average correlations estimated using the MGARCH[16]. In general, correlations tend to increase rapidly during stress periods, as expected. The highest average correlations have been reached during the peak of the global financial crisis (Lehman Brothers bankruptcy).

The subindices together with their respective dynamic correlations allow the CISS to be constructed using equation (1). The CISS is plotted in Figure 3 (bold line) along with the individual indicators of each dimension and the CISS assuming perfect correlations in every period (dotted line).

The CISS identifies the period of highest systemic stress between September of 2008 and September of 2009. The major peak of systemic risk was observed in September of 2008, when the Lehman Brothers collapsed, and seems to be explained by the simultaneous stress observed in each of the indicators that compose the index. It is worth noting that during the aforementioned stress period, the gap between the CISS and the perfect correlation index is significantly reduced, which is consistent with the empirical findings that markets tend to show greater co-movement during times of increased vulnerability.

Other periods identified as possible times of stress in financial markets did not coincide with increases in systemic risk according to the CISS. This is explained by the fact that these events did not generate a widespread effect on all dimensions (contagion), and hence did not translate into a greater vulnerability for the system as a whole. It is also worth noting, that there are periods in which the gap between CISS and the index assuming perfect correlations closes, but there was not an event that accounts for this synchronization of the different dimensions. For instance, in June 2003, both indices registered similar levels, but systemic risk did not increase significantly. Consequently, when analyzing the CISS it is important to study both the level and the implicit correlations.

Given the link between financial markets and economic activity, it is important to check whether increases in systemic risk can affect the real sector in Colombia. As argued by Cardarelli et al. (2011),

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[16]Only the averages of cross-correlations between sectors are presented for simplicity.

FIGURE 2: Average correlations for each sector and total
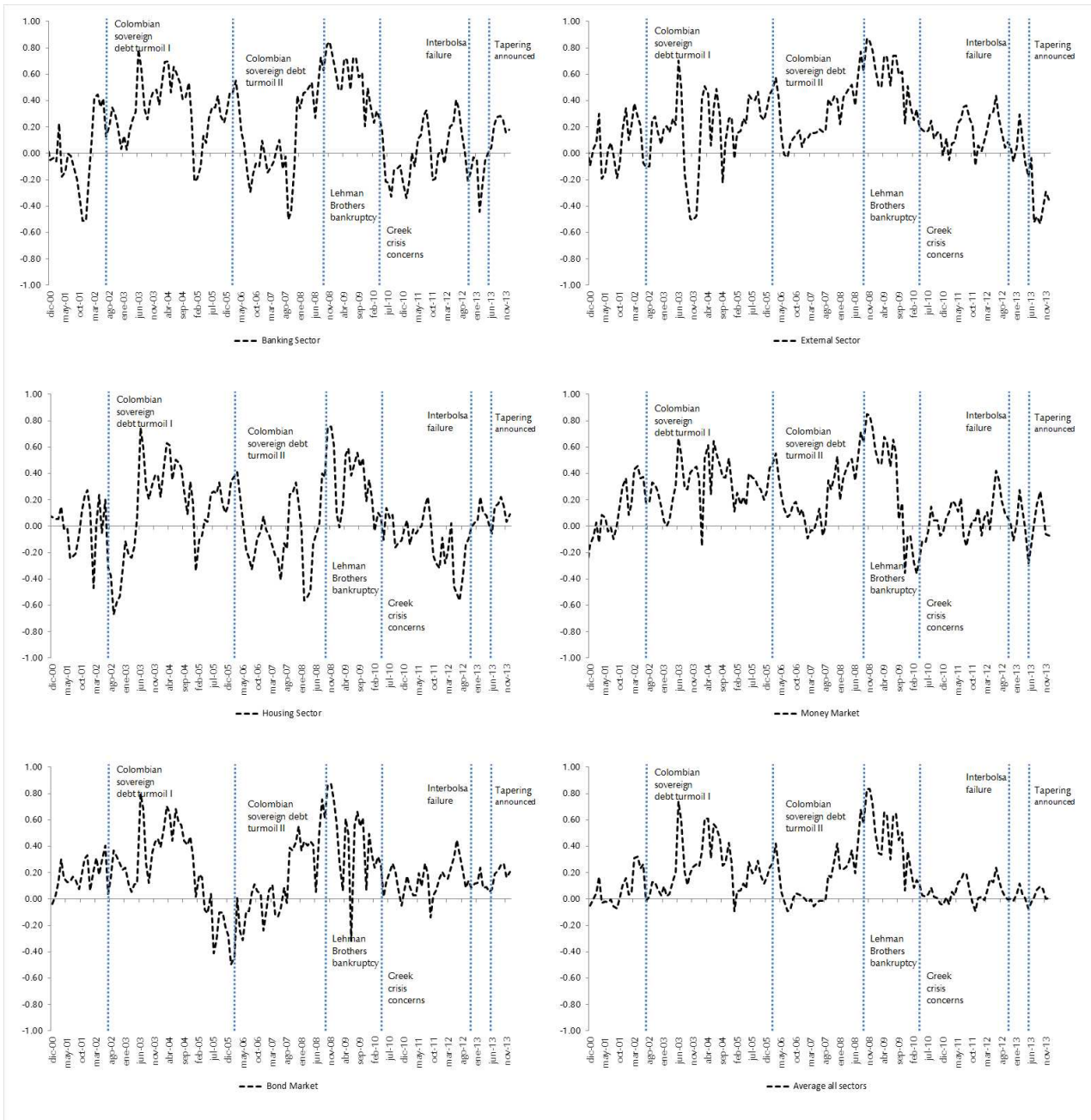
Source: Calculations by the authors.

Davig & Hakkio (2010)and Hakkio & Keeton (2009), data shows that financial stress leads to real activity contractions, and indeed some authors have used systemic risk indicators to test this hypothesis; for instance, Holló et al. (2012) estimate the change in industrial production given an increase in financial distress (CISS) for the Euro area using a TVAR model. The results show that when the CISS exceeds a certain threshold the industrial production declines.

FIGURE 3: Composite Indicator of Systemic Stress
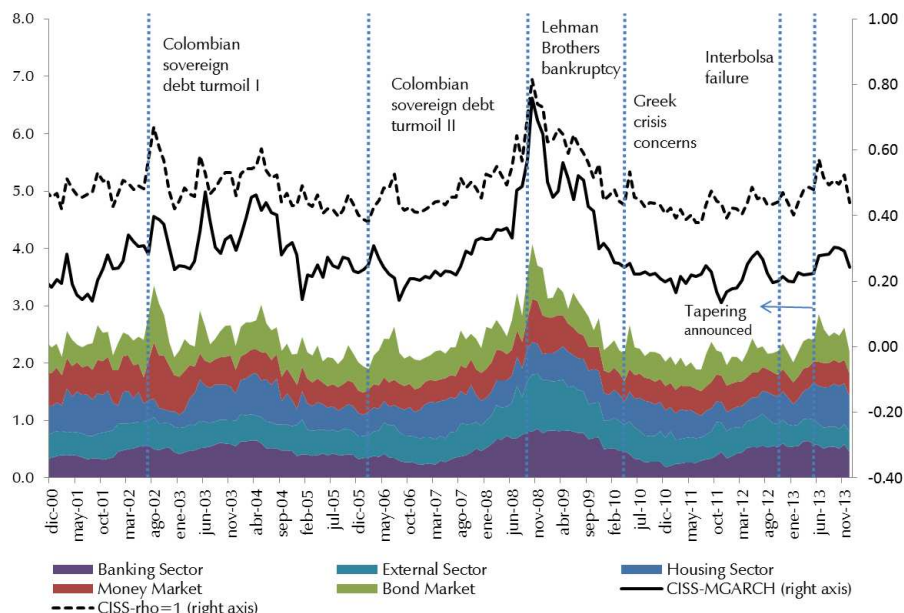
Source: Calculations by the authors.

As a first approximation to analyze the validity of this relation in Colombia, Figure 4 compares the dynamics of the CISS with indicators of real activity, such as the leading indicator of economic activity in Colombia (IMACO)[17] and the annual growth of four-quarter-accumulated GDP.

The graph shows that the CISS appears to lead real activity indicators during periods of higher systemic risk, which suggests that the CISS has a good performance predicting possible real activity downturns caused by disruptions in the financial system.

To statistically test for the relation between the CISS and economic activity, a multiplier analysis is conducted, as proposed by Lütkepohl (2007). This exercise shows the existing relationship between the CISS and quarterly GDP since 2000. The first step was to find the appropriate specification of the autoregressive model by placing GDP as the dependent variable and adding lags of the CISS as additional explicative variables. Using the Akaike information criteria as a benchmark, the best specification is given by:

$$GDP_t = \alpha_0 + \beta_1 CISS_{t-1} + \alpha_1 GDP_{t-1} + \alpha_2 GDP_{t-2} + \alpha_3 GDP_{t-3} + \epsilon_t, \tag{3}$$

where $\epsilon_t \sim N(0, \sigma^2)$ and $t = \{1, \ldots, T\}$. Based on this model, with well behaved residuals in terms of autocorrelation, normality and ARCH effects[18], a multiplier analysis was conducted by applying a 0.99 quantile shock to the CISS, which amounted to an increase of 23.0% in this indicator. The impulse response function up to nine quarters of the GDP (Figure 5) to the CISS shock shows that it has the

---

[17]In spirit, this indicator is similar to the Chicago Fed National Activity Index (CFNAI). For further details see Kamil et al. (2010) & Stock & Watson (1999).

[18]For further details, see Appendix 4.2, Table 2.

Figure 4: Composite Indicator of Systemic Stress (inverted) and Real Activity Indicator
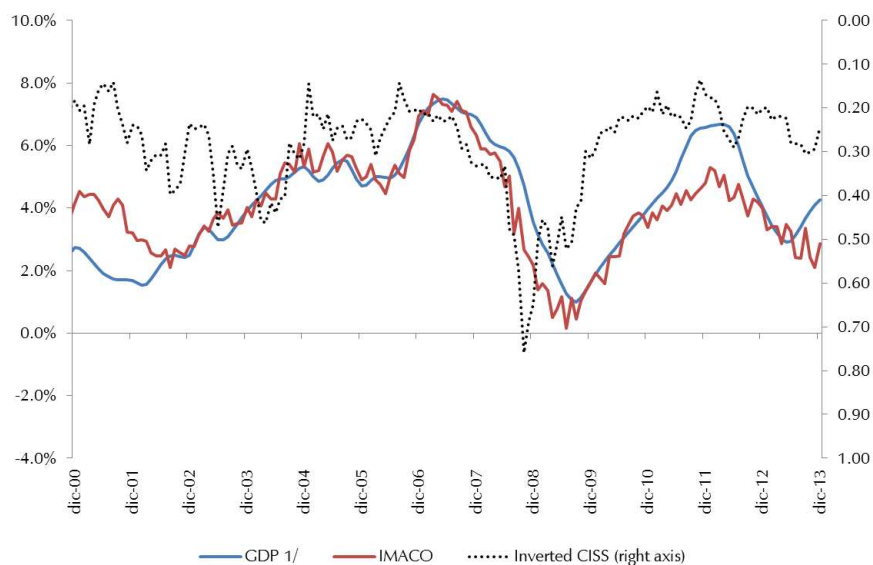
Source: Calculations by the authors. GDP1/: Annual growth of four-quarter-accumulated GDP.

expected effect on the GDP, as in times of financial turmoil it experiences a reduction of -0.5% in its accumulated growth for four periods.

Figure 5: Impulse Response Functions
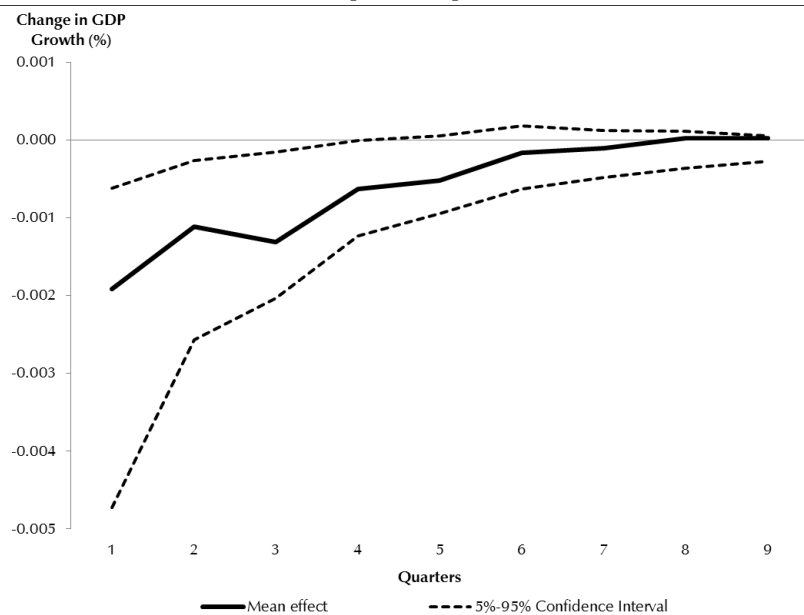
Source: Calculations by the authors.

In summary, the CISS identifies one period of systemic stress in Colombia, which occurred at the end of 2008. At that moment, the different dimensions, except housing sector, showed individually high levels

of stress along with high correlations among them. This is especially valuable for this index as it identifies periods of contagion between dimensions, which is a key element of the horizontal perspective of systemic risk. Moreover, based on the statistical model, it is possible to infer that the CISS can anticipate extreme downturns of GDP at least by one quarter, and the persistence of financial systemic stress on real activity could last until four quarters, which is noteworthy as it regards the vertical perspective of systemic risk.

3. Final Remarks

Following recent developments in the literature of systemic risk, this document builds a Composite Indicator of Systemic Stress (CISS) for Colombia from January of 2000 until December of 2013. The CISS measures the level of systemic distress in the financial system taking into account the contribution of five different sectors of systemic importance: credit institutions, housing market, external sector, money market and bond market. This is the first attempt to build an aggregated measure of systemic risk for Colombia that gauges the state of this risk, as previous studies focused on identifying systemic institutions and on their contribution to the risk of the system.

A key feature of the index is that the state of systemic risk depends, not only on the stress level of the five dimensions, but also on the correlation among them. This aggregation method is based on portfolio theory and tries to capture the notion of higher correlation among sectors when financial markets are in stress (horizontal perspective).

Results show that the CISS reached its highest level on September of 2008, when the Lehman Brothers declared bankruptcy at the peak of the financial crisis. The CISS also detects some periods of turbulence in the financial system, that are not considered systemic given the low correlation of the five dimensions.

In order to check the impact of systemic distress on the real economy (vertical perspective), the CISS was plotted along with two real activity indicators, and it was found that in episodes of high systemic stress, the CISS tends to lead real activity downturns. This hypothesis was tested using multipliers analysis, which revealed that an increase of 23.0% in the CISS has an adverse effect on the accumulated GDP growth of -0.5%.

This aggregate measure contains useful information for policymakers since it allows them to monitor the state of systemic risk and to detect which are the sources of distress. The indicator also permits to identify financial crisis periods and anticipate its possible effects on real activity, which is valuable for the central bank and other macroprudential authorities.

4. Appendix

4.1. Expected and estimated signs of each variable

TABLE 1: Expected and estimated signs

	Expected Sign	Estimated Sign
I. Credit Institutions		
Credit Gap	+ -	+
NPL Gap	+	+
Interest Margin	+ -	+
II. Housing Market		
Housing Credit Gap	+ -	+
NPL Housing Gap	+	+
Housing prices	+ -	+
III. External Sector		
Exchange Rate Volatility	+	+
Foreign Debt Gap	+ -	-
Commodity Price Index Volatility	+	+
IV. Money Market		
Spread 3-month sovereign bond	+	+
Volatility 3-month Colombian Sovereign Bond	+	+
Spread of the Interbank and the Policy Rate	+	+
V. Bond Market		
Spread 10-year Colombian Sovereign Bond and US Treasury Bill	+	+
Volatility of IDXTES-Bond Index	+	+
Correlation of the 10-year Colombian Bond and the IGBC	-	+

Estimated Sign: Using Principal Components Approach (eigenvector sign).

4.2. Unit Root and Residuals Tests

<center>TABLE 2: Unit Root Tests</center>

| | *Series in Levels* | | | |
	ADF	**PP**	**ERS**	**KPSS**
Credit Gap	-5.4290*	-4.2863*	-5.4744*	0.0361
NPL Gap	-4.4215*	-3.7665*	-4.1902*	0.0689
Interest Margin	-2.5635*	-2.8325	-1.4793	0.2616
Housing Credit Gap	-4.4759*	-2.6255	-3.4986*	0.1068
NPL Housing Gap	-6.3541*	-4.6733*	-6.3338*	0.0597
Housing prices	-4.9052*	-3.7320*	-4.8456*	0.0477
Exchange Rate Volatility	-2.681*	-6.9264*	-3.4513*	0.4761†
Foreign Debt Gap	-3.9903*	-4.6788*	-3.2719*	0.0403
Commodity Price Index Volatility	-2.1424*	-2.905*	-3.4573*	0.3149
Spread 3-month sovereign bond and US Treasury Bill	-3.5062*	-4.9750*	-3.5503*	0.1300
Volatility 3-month Colombian Sovereign Bond	-5.9457*	-8.6894*	-5.313*	0.0569
Spread of the Interbank and the Policy Rate	-4.9487*	-11.5485*	-4.1227*	0.1745†
Spread 10-year Colombian Sovereign Bond and US Treasury Bond	-3.6037*	-3.4805*	-3.3129*	0.1932†
Volatility of IDXTES-Bond Index	-6.7913*	-7.5536*	-7.0145*	0.0447
Correlation of the 10-year Colombian Bond and the IGBC	-5.7598*	-10.7761*	-4.4625*	0.4340†

<center>ADF: Augmented Dickey-Fuller Test; PP: Phillips-Perron;</center>
<center>ERS: Elliot-Rothenberg-Stock; KPSS: Kwiatkowski-Phillips-Schmidt-Shin</center>
<center>*If the null hypothesis (unit root) were to be rejected at the 5% significance level.</center>
<center>† If the null hypothesis (stationarity) were to be rejected at 5% significance level. Source: Calculations by the authors</center>

<center>TABLE 3: Autocorrelation, Independence and Normality Tests of the Multiplier Analysis</center>

Test	p-value
Box-Pierce autocorrelation test	0.345§
Engle ARCH effect test	0.936§
Jarque-Bera normality test	0.664§
Shapiro-Wilk normality test	0.483§

<center>§ Implies no rejection of null hypothesis at 10% significance level</center>
<center>Source: Calculations by the authors</center>

4.3. MGARCH-DCC

The Dynamic Conditional Correlation (DCC) proposed by Engle (2002), is one specification of MGARCH, which uses a nonlinear combination of univariate GARCH models with time-varying cross-equation weights to model the conditional covariance matrix. To preserve parsimony, all the conditional quasi correlations are restricted to follow the same dynamics. The DCC representation can be given as follows:

$$
\begin{aligned}
y_t &= C x_t + \epsilon_t \\
\epsilon_t &= H_t^{1/2} \nu_t
\end{aligned}
\tag{4}
$$

$$
\tag{5}
$$

where y_t is a vector of dependent variables, C is a matrix of parameters, x_t a vector of independent variables and ϵ is a normally distributed error term with zero mean and conditional covariance matrix H_t. ν_t is defined as a vector of normal, independent and identically distributed innovations ($\nu \sim N(0, I_n)$) and $H_t^{1/2}$ may be obtained by Cholesky factorization of the conditional covariance matrix H_t.

The covariance matrix (H_t) can be decomposed into the product of dynamic conditional correlations (R_t)[19] and the dynamic conditional standard deviations ($D_t^{1/2}$), where D_t is a diagonal matrix of conditional variances.

$$
H_t = D_t^{1/2} R_t D_t^{1/2}
\tag{6}
$$

equation 6 implies that each element $h_{ij,t}$ in matrix H_t is define as

$$
h_{ij,t} = \rho_{ij,t} \sqrt{h_{ii,t} h_{jj,t}}
\tag{7}
$$

where $h_{ij,t}$ is the covariance between sub-index i and j, $\rho_{ij,t}$ the correlation between sub-index i and j and finally h_{ii} and h_{jj} are variances of sub-indices i and j, respectively.

To compute H_t, the first step consist of estimating matrix D_t using a univariate GARCH processes [20].

$$
D_t = \begin{bmatrix}
\sigma_{1,t}^2 & 0 & 0 & 0 & 0 \\
0 & \sigma_{2,t}^2 & 0 & 0 & 0 \\
0 & 0 & \sigma_{3,t}^2 & 0 & 0 \\
0 & 0 & 0 & \sigma_{4,t}^2 & 0 \\
0 & 0 & 0 & 0 & \sigma_{5,t}^2
\end{bmatrix}
\tag{8}
$$

The second step is estimating the matrix of conditional quasi correlations R_t using the DCC parametrization. In this step, matrix R_t is estimated by considering the dynamics of the conditional variance of the

---

[19] Also known as quasi-correlations.

[20] $\sigma_{i,t}^2$ may be estimated with a univariate GARCH model of the form $\sigma_{1,t}^2 = \omega_i + \sum_{j=1}^{p_i} \alpha_i \epsilon_{i,t-j}^2 + \sum_{j=1}^{q_i} \beta_j \sigma_{i,t-j}^2$. However it is not restricted to this specification.

standardized residuals $\tilde{\epsilon}_t$, which are defined as $D_t^{-1/2}\epsilon_t \sim N(0, R_t)$. From the model, the conditional correlation is the conditional covariance between the standardized disturbances.

$$R_t = \begin{bmatrix} 1 & \rho_{12,t} & \rho_{13,t} & \rho_{14,t} & \rho_{15,t} \\ \rho_{12,t} & 1 & \rho_{23,t} & \rho_{24,t} & \rho_{25,t} \\ \rho_{13,t} & \rho_{23,t} & 1 & \rho_{34,t} & \rho_{35,t} \\ \rho_{14,t} & \rho_{24,t} & \rho_{34,t} & 1 & \rho_{45,t} \\ \rho_{15,t} & \rho_{25,t} & \rho_{35,t} & \rho_{45,t} & 1 \end{bmatrix} \tag{9}$$

Before explaining further how the matrix R_t is obtained, recall that H_t has to be positive definite by the definition of the covariance matrix. Since H_t is a quadratic form based on R_t, it follows that R_t has to be positive definite to ensure that H_t is positive definite. Moreover, by definition, all the elements of the conditional correlation matrix have to be less than or equal to one. Therefore, in order to guarantee both requirements, Engle (2002) proposes to decompose R_t into:

$$R_t = diag(Q_t)^{-1/2} Q_t diag(Q_t)^{-1/2} \tag{10}$$

where Q_t is a positive definite matrix defining the structure of the dynamics and $diag(Q_t)^{-1/2}$ is the inverted diagonal matrix with the square root of the diagonal elements of Q_t. The idea behind the product is to re-scale the elements in Q_t to ensure $q_{ij} \leq 1$.

Additionally, Engle (2002) assumes that Q_t has the following dynamics

$$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha\tilde{\epsilon}_{t-1}\tilde{\epsilon}_{t-1}\prime + \beta Q_{t-1} \tag{11}$$

where $\tilde{\epsilon}_t$ is a vector of standardized residuals $D_t^{-1/2}\epsilon_t$, $\bar{Q}$ is the unconditional covariance of these standardized disturbances and α and β are parameters that lead the dynamics of the conditional quasi correlations. The two scalar parameters satisfy a stability constraint of the form $\alpha + \beta < 1$ and the sequence Q_t should drive the dynamics of the conditional correlations.

In the end, the parameters of the DCC model are estimated by maximizing the Gaussian likelihood function (maximum likelihood-ML) of the multivariate process. Both the ML estimator and the quasi maximum likelihood (QML) estimator, which drops the normality assumption, are assumed to be consistent and normally distributed in large samples. The function based on the multivariate normal distribution for observation t is

$$l_t = -0.5m\log(2\pi) - 0.5\log\{det(R_t)\} - \log\{det(D_t^{1/2})\} - 0.5\tilde{\epsilon}_t R_t^{-1}\tilde{\epsilon}_t\prime \tag{12}$$

where $\tilde{\epsilon}_t = D_t^{-1/2}\epsilon_t$ is a $m \times 1$ vector of standardized residuals, $\epsilon_t = y_t - Cx_t$. Finally, the log-likelihood function is define as $\sum_{t=1}^{T} l_t$[21].

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[21]Dynamic correlations were estimated using Stata (MGARCH-DCC).

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