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The Efficiency of the Informal Sector on the Search and Matching Framework

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Abstract

This paper analyzes efficiency in an economy with an informal sector that consists of unregulated self-employment, and where there are no costs of being informal, (Albrecht et al. (2009)). First, asssuming workers in the formal sector are ex-ante heterogeneous, I show that this type of economy is inefficient. Second, I identify the optimal policies the government can implement, where the informal sector is unobserved (or search effort is unobserved). Allowing the government to use different policies such as social security payment, severance payment, formal tax, and job creation subsidy, I show that the government cannot affect worker's behavior by using severance and social security payments because of the risk neutrality assumption (Lazear (1990)). However, it can achieve an efficient allocation through a tax-credit policy. This result is interesting since it can guide the way in which social security programs can be implemented in developing countries, where in general social protection programs are assumed to subsidize informal activities.

JEL classification: H21; J64; J65

Keywords: Efficiency, Informal Sector, Hidden Search Effort

1. Introduction

This paper analyzes efficiency in a model with an informal sector, that consists of unregulated self-employment, as described in Albrecht et al. (2009).Assuming workers in the formal sector are ex-ante heterogeneous, I show that this economy is characterized by three type of workers: those with high productivity who decide to be "pure formal workers", those with low productivity who decide to be "pure informal workers" and those with medium productivity who stay informal while searching for formal offers "informal searchers". In this paper I show that this type of

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economy is inefficient. There are two facts that explain this inefficiency. First, the hold-up problem which implies that workers do not internalize the firms's cost of posting a vacancy[2]. Second, the "composition externality", which refers to the fact that the search intensity is lower than the efficient one. This is due to the fact that the economy is characterized by two types of workers "formal searchers" and "pure formal workers" who search with different search effort for a formal job.

Using different labor market policies the government can affect the incentives for workers to be formal or informal changing the composition of "formal workers" or "informal searchers" in the economy (composition effect). Therefore, the objective of this paper is to show how the government can achieve efficiency in an economy with an informal sector where the government cannot observe workers' search effort.[3]. Then I allow the government to use different policies such as, social security payment, severance payment, linear formal tax and a job creation subsidy, to get an efficient allocation in the economy. The implementation of a severance payment policy allows me to analyze the "*re-entitlement effect*" mentioned in the literature[4]. Solving the optimal policy when the search effort is unobserved I find that the government cannot affect workers' behavior by using severance and social security payments given the risk neutrality assumption (Lazear (1990)). However a formal linear tax (τ) or tax-credit is an efficient policy in this case. This result is interesting since it can guide the way in which social security programs are implemented in developing countries; especially since there is a lack of labor protection, and where in general social protection programs are assumed to subsidize informal activities (Mazza (2000)).

Most of the existing literature focuses its analysis on the design of an optimal unemployment insurance system when workers are risk averse and the government cannot monitor the agents' search effort. This literature can be divided into to main groups. The first group of researchers focus on a partial equilibrium set-up. This is the case of Shavell and Weiss (1979), Hopenhayn and Nicolini (1997, 2009), and Wang and Williamson (1996), among others. The focus of these authors is the moral hazard problem when workers receive an unemployment benefit and their search effort is not observed. Shavell and Weiss (1979) found that in order to provide the appropriate incentives to search, benefits must decrease monotonically through the unemployment spell. Hopenhayn and Nicolini (1997) reached the same conclusion. They designed an optimal unemployment insurance system using the repeated principal-agent problem, where the principal cannot monitor the agent's search effort. Unlike Shavell and Weiss (1979), Hopenhayn and Nicolini (1997) include in their analysis of the optimal contract a wage tax which depends on the unemployment spell. Similarly, Wang and Williamson (1996) analyzed a model of repeated unemployment spells with endogenous job termination, where a worker's effort affects not only the probability of finding a job but also the probability of keeping it. On the other hand, Cremer et al. (1995) use a search model to identify the optimal level of unemployment insurance assuming risk-averse job seekers that face the possibility of a mismatch between the employment they prefer and the one they are offered. There are a couple of works that analyze unemployment

[2]Similar results are found in Charlot et al. (2013) with homogenous workers.

[3]Florez (2014) presents a solution of the optimal policy when search effort is observed. In this case, if it were possible for the government to observe when a worker is formal or informal, then an unemployment benefit only for those who are formal workers would be an optimal policy. These results are avaible upon request.

[4]The "re-entitlement effect" refers to the effect on the search effort of a limited unemployment benefit or severance payment. Mortensen (1976) defines the "re-entitlement effect" as the increase in the search intensity of an unemployed worker when his unemployment benefits are about to exhaust. "In the case of a qualified worker who has not yet exhausted his or her unemployment benefits, the escape rate increase realized unemployment duration" (p.511). For more details see Mortensen (1976), Fredriksson and Holmlund (2001), Coles and Masters (2006, 2007) among others.

benefits in the presence of informal sector, Alvarez-Parra and Sanchez (2006) and Bardey and Jaramillo (2011). Their work is similar to Hopenhayn and Nicolini (1997). They analyzed the consequences of unemployment benefits on labor markets with informal sector in a partial equilibrium set-up, following the principal-agent framework. The authors showed that an optimal contract is a decreasing unemployment benefit until the period when workers start participating in the informal sector; in which case unemployment benefits jump to zero.

The second group of researchers have analyzed the design of optimal unemployment insurance in a matching equilibrium framework. This is the case of Fredriksson and Holmlund (2001), Cahuc and Lehmann (2000), and Coles (2008). Fredriksson and Holmlund (2001) found that when using an equilibrium model [following Pissarides (2000)], an optimal insurance program implies a declining benefit sequence over the spell of unemployment. The authors focused on a two-tiered UI system[5], which exploits the differential impact of higher benefits on search incentives among insured and noninsured unemployed workers ("re-entitlement effect"). Following Fredriksson and Holmlund (2001), Cahuc and Lehmann (2000) investigated whether unemployment benefits should decrease with the unemployment spell in a model where both job search intensity and wages are endogenous. They found that it is costly to reduce the unemployment rate when wages are endogenous. Finally, Coles (2008) analyzed an optimal unemployment policy in a matching equilibrium model where the social planner chooses unemployment benefits, taxes and job creation subsidies, to maximize a utilitarian welfare function. He found that the optimal unemployment insurance (UI) should combine an initial payment equal to the wage, followed by a decreasing UI payment.

It should be emphasized that the above literature analyzes the optimal policy when workers are risk averse and the search effort is unobserved. In my model, on the other hand, workers are risk neutral, therefore the aim of an unemployment insurance policy is not to smooth consumption across time but to increases the incentive to search during the duration of unemployment (the moral hazard problem), given that the search effort is unobserved. As Shavell and Weiss (1979) affirms:

> "...for the risk-neutral case it is easy to show that it is optimal to give all the benefits in the first period. (This maximizes the incentive to find a job, and the risk that this imposes on those who are unemployed for long periods is of no concern since they are risk neutral)..." p.1357

Shavell and Weiss (1979) suggested that an optimal policy with risk neutral workers may be a severance payment policy, where all benefits are given in the moment that the worker is laid off, as opposed to a decreasing unemployment benefit policy.

Building upon the above findings, this paper contributes to the existing literature extending the analysis of optimal policy in the matching equilibrium framework, with an informal sector and risk neutral workers. Following Shavell and Weiss (1979) I analyze different policies as the severance payment, the social security payment, the formal linear tax, and job creation subsidy, to get an efficient allocation in an economy with informal sector. I show that the severance payment policy and social security payment policy do not play a key role in achieving efficiency. On the other hand, the formal linear tax or tax-credit is a necessary policy for efficiency.

This paper is divided into six sections. In the second section I present the model with an informal sector and solve the market solution. In the third section I present the social planner

[5]This is a program with two benefit levels. Workers who lose their jobs are entitled to UI benefits. But after some period (benefits are not indefinite) some workers lose their benefits and are entitled to "social assistance".

solution and show that the decentralized solution is inefficient. In the fourth section I focus on the case when the search effort is not observed. I show the combination of policies (social security payment, severance payment, formal tax or tax-credit and job creation subsidy) that can be implemented by the government to achieve an efficient allocation. In the fifth section I present a numerical exercise using the previous optimal policies. Finally, in the last section I summarize the main findings of this paper.

2. Model

This analysis considers only the steady state, where time is continuous and workers are risk neutral with finite life. The assumption of risk neutrality implies that workers do not care about smoothing consumption and simply consume all their income in each period. Thus, workers maximize their expected utility by maximizing their income. The rate of death is given by an exogenous Poisson rate μ and at the same rate new workers are born, therefore the labor force is constant and normalized to 1. The future is discounted at the exogenous rate r. The labor market frictions are modeled using a matching function, where search is random and wages are determined by Nash bargaining.

There are two sectors; formal and informal. Workers in the formal sector are assumed to be ex-ante heterogeneous and their productivity x is distributed according to the exogenous cdf $H(x)$ with $0 \leq x \leq 1$. Following Albrecht et al. (2009) the informal sector is assumed as unregulated self-employment, where there are no costs of being informal. When a worker is formally employed he receives the wage $w(x)$, which is a function of his productivity level x. All workers can decide to be informal or unemployed depending on their level of productivity. If a worker decides to be informal he would receive the wage w_I, which is the same for all workers, and when a worker decides to be unemployed he would receive the income flow z (which represents the value of leisure). Once a worker is unemployed he receives opportunities to work in the formal sector at an endogenous Poisson rate λ_1 and when a worker is informal he receives opportunities to work in the formal sector at an endogenous Poisson rate λ_2, where $\lambda_1 > \lambda_2$ (λ_1 and λ_2 are endogenized using the matching function). The job destruction process is exogenous and is given only in the formal sector at the rate δ.

As I show in the following sections this economy is characterized by three types of workers: Those with low productivity $x < x_1$ who only work in the informal sector, which I call "pure informal workers". Those with medium productivity $x_1 \leq x \leq x_2$ who work in the informal sector and accept job offers from the formal sector, which I call "informal searchers", and finally those with high productivity $x > x_2$ who prefer to be unemployed and only accept job offers from the formal sector, which I call "pure formal workers". Let φ be the search effort of employed workers in the informal sector, where φ is $0 < \varphi < 1$. Let us define N_i as the number of workers who are "pure informal workers", N_{is} as the number of workers who are "informal searchers" and N_f as the number of workers who are "pure formal workers", where u_{is} denotes the fraction of "informal searchers" who are employed in the informal sector while searching for a formal job and u_f denotes the fraction of "pure formal workers" who are unemployed. Then the number of effective workers searching for a job is given by $u^e = N_f u_f + \varphi N_{is} u_{is}$ where $N_f u_f$ refers to the total number of workers who are unemployed and searching full time for a formal job, and $\varphi N_{is} u_{is}$ refers to the number of workers who are informally employed searching for a formal job.

The matching process takes place between individual job vacancies and workers who search for a job. The number of job matches is given by a matching function: $m(v, u^e)$. I assume the matching function is increasing in v (number of vacancies) and u^e, and it is concave and homogeneous of degree one. The arrival rate of formal job offers when a worker is unemployed is given by:

$$\lambda_1 = \frac{m(v, u^e)}{u^e} = m(\theta) \tag{1}$$

Let $\theta = \frac{v}{u^e}$ denote the *tightness* of the labor market. The arrival rate of getting a formal job when a worker is informal is given by $\lambda_2 = \varphi\lambda_1$.

However the arrival rate of filling a formal vacancy will depend on the number of workers searching for a formal job and the number of vacancies in the market. Then the arrival rate of filling a formal job offer is[6]:

$$\alpha = \frac{m(v, u^e)}{v} = \frac{m(\theta)}{\theta} \tag{2}$$

Let $U(x)$ denotes the value of being unemployed for a worker type x, $W_f(x)$ the value of being formally employed for a worker type x and $W_i(x)$ the value of being informally employed for a worker type x. The worker's value functions are given by:

$$(r + \mu)U(x) = z + \lambda_1 \left\{max\left[U(x), W_f(x)\right] - U(x)\right\} \tag{3}$$

$$(r + \mu)W_f(x) = w(x) + \delta \left\{max\left[U(x), W_i(x)\right] - W_f(x)\right\} \tag{4}$$

$$(r + \mu)W_i(x) = w_I + \lambda_2 \left\{max\left[W_f(x), W_i(x)\right] - W_i(x)\right\} \tag{5}$$

Equation (3) implies that the opportunity cost of searching for a formal job while unemployed (or the return of being unemployed discounted by the interest rate r and the death rate μ) is equal to income flow z while unemployed, with the addition of the capital gain attributable to searching for an acceptable job, where an acceptable job implies that the value of being formally employed exceeds the value of continuing searching, $W_f(x) > U(x)$. Equation (4) implies that the opportunity cost of being formally employed is equal to the current wage for being formally employed, $w(x)$, plus the capital loss, $\left\{max\left[U(x), W_i(x)\right] - W_f(x)\right\}$, attributable to the exogenous job destruction shock, which arrives at the rate δ. Finally equation (5) shows that the opportunity cost of being informal while searching for a formal job is equal to the income flow w_I while being informal, with the addition of the the capital gain attributable to searching for an acceptable job, $W_f(x) > W_i(x)$.

Let J_u denote the value of an unfilled formal vacancy and $J_f(x)$ the value of a filled formal job with a worker type x, where c represents the cost of holding an unfilled formal vacancy and $w(x)$ the wage, which depends on the worker's productivity.

$$rJ_u = -c + \alpha \left\{max\left[EJ_f(x), J_u\right] - J_u\right\} \tag{6}$$

$$rJ_f(x) = x - w(x) + (\delta + \mu)\left\{J_u - J_f(x)\right\} \tag{7}$$

[6] I assume that $m(\theta)$ and $\alpha(\theta)$ satisfy the standard properties:
i) $m(\theta)$ is increasing in θ,
ii) $\alpha(\theta)$ is decreasing in θ,
iii) $lim_{\theta\to 0} m(\theta) = 0$ and $lim_{\theta\to\infty} m(\theta) = \infty$
iv) $lim_{\theta\to 0} \alpha(\theta) = \infty$ and $lim_{\theta\to\infty} \alpha(\theta) = 0$

Equation (6) implies that the return of holding a vacancy is equal to the capital gain when a firm fills the vacant job with a worker type x minus the cost of posting a vacancy, such that the expected value of the filled vacancy exceeds the value of continuing holding the unfilled vacancy $EJ_f(x) > J_u$. Equation (7) implies that the return of a filled job with a worker type x is equal to the output x minus the wage $w(x)$, plus the capital loss attributable to the exogenous shock destruction δ and worker's death μ. Free entry condition of firms implies $J_u = 0$.

Once workers and firms meet the wage $w(x)$ is determined by Nash bargaining, where β is the worker's bargaining power, and $max\{U(x), W_i(x)\}$ and J_u are the threat points or disagreement's payoff. The Nash bargaining problem is given by:

$$w(x) = arg\,max\left[W_f(x) - max\{U(x), W_i(x)\}\right]^{\beta}\left[J_f(x) - J_u\right]^{1-\beta} \tag{8}$$

The first order condition implies the following sharing rule:

$$(1-\beta)\left[W_f(x) - max\{U(x), W_i(x)\}\right] = \beta\left[J_f(x) - J_u\right], \tag{9}$$

where the total surplus of the match is defined as the worker's surplus plus firm's surplus; i.e., $S(x) = \left[W_f(x) - max\{U(x), W_i(x)\}\right] + \left[J_f(x) - J_u\right]$

2.1. Worker's strategy

First I will describe the workers strategy taking θ as given. To do this I need to solve the bellman equations (3), (4), (5), (7) and the Nash bargaining equation (9). There are three cases I consider. Case A refers to those workers who never participate in the informal sector, I call them "pure formal workers". This case implies that, $W_i(x) \leq U(x) < W_f(x)$. Case B refers to those workers who prefer to stay informal while searching for a formal job offer; I call these workers "informal searchers". In this case $U(x) < W_i(x) < W_f(x)$. Finally, case C refers to those workers who never participate in the formal sector, whom I call "pure informal workers". Case C implies that, $U(x) \leq W_f(x) < W_i(x)$. Solving these three cases and using the Principle of Unimprovability I find the following results.

Proposition 1. *The optimal worker's strategy given θ is:*

i) Workers with productivity $x < x_1$ only work in the informal sector, "pure informal workers", where:

$$x_1 = w_I \tag{10}$$

ii) Workers with productivity $x_1 \leq x \leq x_2(\theta)$ stay working in the informal sector and accept job offers from the formal sector, "informal searchers", where:

$$x_2(\theta) = \frac{w_I(r+\delta+\mu+\beta\lambda_1) - z(r+\delta+\mu+\beta\lambda_2)}{\beta(\lambda_1-\lambda_2)} \tag{11}$$

iii) Workers with productivity $x > x_2(\theta)$ stay unemployed and accept job offers from the formal sector, "pure formal workers"

Proof. See proof in Appendix A. □

2.2. Steady state conditions

Assuming that the productivity distribution of the population is given by the exogenous cdf $H(x)$, with total population normalized at 1, I can define $N_i(\theta) = H(x_1)$ as the number of workers who are "pure informal workers", $N_{is}(\theta) = H(x_2(\theta)) - H(x_1)$ as the number of workers who are "informal searchers" and $N_f(\theta) = 1 - H(x_2(\theta))$ as the number of workers who are "pure formal workers". Using these definitions I can solve for the steady state number of "informal searchers" and "pure formal workers" given θ.

2.2.1. Steady state conditions by type of workers given θ

i) "informal searchers" with productivity $x_1 \leq x \leq x_2(\theta)$

Let u_{is} denote the fraction of "informal searchers" who are employed in the informal sector while searching for a formal job; then the outflow from the informal sector equals the number of those who receive a formal offer: $N_{is}(\theta)u_{is}\varphi\lambda_1$ plus those who die, $N_{is}(\theta)u_{is}\mu$. On the other hand, the inflow into the informal sector is given by those who lose their job in the formal sector: $N_{is}(\theta)(1 - u_{is})\delta$, plus those who are born, $N_{is}(\theta)\mu$. In the steady state the inflow and outflow from the informal sector should be equal, then:

$$u_{is} = \frac{\delta + \mu}{\delta + \varphi\lambda_1 + \mu} \tag{12}$$

ii) "pure formal workers" with productivity $x > x_2(\theta)$

Let u_f denote the fraction of "pure formal workers" who are unemployed; then the outflow from unemployment is given by those who receive a formal offer: $N_f(\theta)u_f\lambda_1$, plus those who die $N_f(\theta)u_f\mu$. On the other hand, the inflow into unemployment is given by those who lose their job in the formal sector: $N_f(\theta)(1 - u_f)\delta$, plus those who are born: $N_f(\theta)\mu$. In the steady state these two flows should be equal, hence:

$$u_f = \frac{\delta + \mu}{\delta + \lambda_1 + \mu} \tag{13}$$

Notice that in the steady state the rate of "informal searchers" who are employed in the informal sector while searching for a formal job, is higher than the rate of "pure formal workers" who are unemployed, $u_{is} > u_f$, given that $\varphi < 1$.

2.2.2. Steady state probability distribution given θ

Let $G_{is}(x)$ define the cumulative probability distribution for those "informal searchers" with $x_1 \leq x \leq x_2(\theta)$ given by:

$$G_{is}(x) = \frac{H(x) - H(x_1)}{H(x_2(\theta)) - H(x_1)} \tag{14}$$

and $G_f(x)$ define the cumulative probability distribution for those "pure formal workers" with $x > x_2(\theta)$ given by:

$$G_f(x) = \frac{H(x) - H(x_2(\theta))}{1 - H(x_2(\theta))} \tag{15}$$

Let $F(x'/\theta)$ define the cumulative probability distribution that a contacted worker has productivity $x \leq x'$ conditional on θ. Using the total number of workers who search effectively for a formal job, given by $u^e = N_f u_f + \varphi N_{is} u_{is}$, and the distribution of workers' type defined in equation (14) and equation (15), I find $F(x'/\theta)$.

Proposition 2. *The cumulative probability that a contacted worker has productivity $x \leq x'$ conditional on θ is given by:*

For $x' \leq x_2(\theta)$

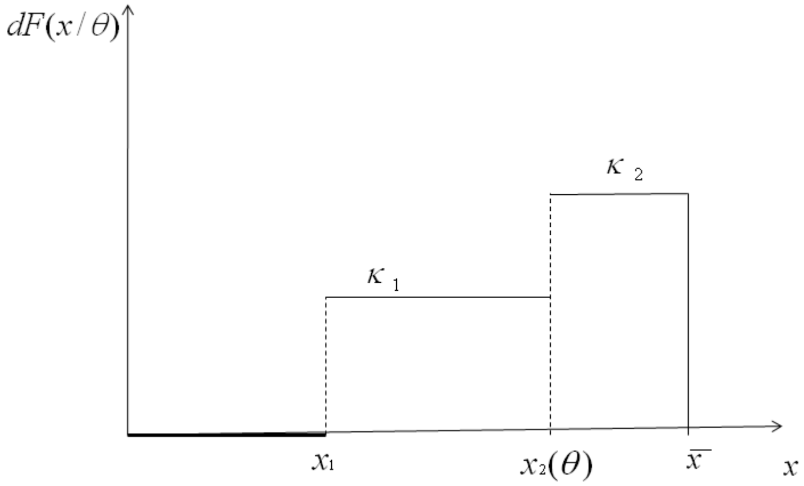
$$F(x'/\theta) = \frac{\frac{\varphi(\delta+\mu)}{\delta+\varphi\lambda_1+\mu}\left[H(x') - H(x_1)\right]}{\frac{\delta+\mu}{\delta+\lambda_1+\mu}\left[1 - H(x_2(\theta)\right] + \frac{\varphi(\delta+\mu)}{\delta+\varphi\lambda_1+\mu}\left[H(x_2(\theta) - H(x_1)\right]}$$

For $x' > x_2(\theta)$

$$F(x'/\theta) = \frac{\frac{\varphi(\delta+\mu)}{\delta+\varphi\lambda_1+\mu}\left[H(x_2(\theta)) - H(x_1)\right] + \frac{\delta+\mu}{\delta+\lambda_1+\mu}\left[H(x') - H(x_2(\theta))\right]}{\frac{\delta+\mu}{\delta+\lambda_1+\mu}\left[1 - H(x_2(\theta)\right] + \frac{\varphi(\delta+\mu)}{\delta+\varphi\lambda_1+\mu}\left[H(x_2(\theta) - H(x_1)\right]}$$

Assuming $H(x)$ has a uniform distribution, I can show that for $x' \leq x_2(\theta)$, $\frac{\partial F(x'/\theta)}{\partial x'} = \kappa_1$ and for $x' > x_2(\theta)$, $\frac{\partial F(x'/\theta)}{\partial x'} = \kappa_2$ are constant probabilities. Moreover assuming $\varphi < 1$, I can show that $\kappa_1 < \kappa_2$ given that the following condition is satisfied: $\varphi(\delta + \lambda_1 + \mu) < (\delta + \varphi\lambda_1 + \mu)$. Figure (1) represents the productivity distribution of a contacted worker.

Figure 1: Productivity distribution of a contacted worker



According to Pissarides (2000) there are two traditional externalities in the search and matching models. There is a negative externality created when firms enter the labor market, since they make it harder for other firms to find workers (congestion externality). There is also a positive externality created when firms enter the labor market, since they increase the probability that workers find employment (thick market externality). In this model I have an additional externality (composition externality). This externality refers to the fact that there are two types of workers in the labor market, the "informal searchers" and the "pure formals", who use different search effort when searching for a formal job. This is reflected in the productivity distribution $F(x'/\theta)$. Thus, given that the economy is characterized by two types of workers "formal searchers" and "pure formal workers" who search with different search effort for a formal job, the search intensity in the economy is lower than the efficient one, as I show in the following section.

2.3. Firm's strategy

The firm's strategy implies that the following three combined conditions should be satisfied: free entry condition, the firm's optimal decision and the worker's optimal decision given θ. Equation (6) below describes the optimal behavior of a firm, and using the free entry condition, $J_u = 0$, the above condition can be re-written as:

$$c = \alpha(\theta) E J_f(x) \tag{16}$$

Equation (16) expresses the optimal condition for a firm to post a vacancy, where the expected value of filling a vacancy, $EJ_f(x)$, depends on the proportion of workers who search for a formal job given θ. However, I already know that given θ, workers with productivity $x \geq x_1$ are willing to search for a formal job. Therefore, equation (16) can be written as: $c = \alpha(\theta) \int_{x_1}^{\bar{x}} J_f(x) dF(x'/\theta)$. Taking into account the productivity distribution of the workers that a firm will contact given by Proposition (2), I find the following result.

Proposition 3. *The optimal strategy for a firm to post a vacancy is given by:*

$$c = \alpha(\theta) \left\{ \int_{x_1}^{x_2(\theta)} \frac{(1-\beta)(x - w_I)}{(r + \delta + \mu + \beta\lambda_2)} dF(x/\theta) + \int_{x_2(\theta)}^{\bar{x}} \frac{(1-\beta)(x - z)}{(r + \delta + \mu + \beta\lambda_1)} dF(x/\theta) \right\} \tag{17}$$

3. Efficiency

Following Hosios (1990), I solve the social planner problem which determines the efficient allocation of this economy. In this case, the social planner chooses $\left\{u_f, u_{is}, x_1^s, x_2^s, \theta^s\right\}$, to maximize the aggregate output in the economy[7], Y, subject to the two steady steady conditions. For simplicity I ignore the discounting rate in order to compare alternative steady states (it means I assume $r = 0$). Then the planner's problem is described as:

$$\max_{\left\{u_f, u_{is}, x_1^s, x_{2,}^s \theta\right\}} \left\{ \begin{array}{l} (1 - u_f) N_f \int_{x_2^s}^{\bar{x}} \frac{x' dH(x')}{1 - H(x_2^s)} + (1 - u_{is}) N_{is} \int_{x_1^s}^{x_2^s} \frac{x' dH(x')}{H(x_2^s) - H(x_1^s)} \\ + z u_f N_f + w_I \left[u_{is} N_{is} + N_i\right] - c\theta \left[u_f N_f + \varphi u_{is} N_{is}\right] \end{array} \right\} \tag{18}$$

$$s.t: \quad N_f u_f (\lambda_1 + \mu) = N_f \left[(1 - u_f)\delta + \mu\right]$$

$$N_{is} u_{is} (\varphi\lambda_1 + \mu) = N_{is} \left[(1 - u_{is})\delta + \mu\right]$$

This is a standard optimization problem solved by the Lagrange Method. The necessary conditions for optimality are described in Appendix B.

[7]Notice that the aggregate output into the economy is divided in four elements: The first one is the average product of those "pure formal" workers who are employed in the formal sector, $(1 - u_f) N_f \int_{x_2^s}^{\bar{x}} \frac{x' dH(x')}{1 - H(x_2^s)}$, the second element is the average product of those "informal searcher" workers who are employed in the formal sector, $(1 - u_{is}) N_{is} \int_{x_1^s}^{x_2^s} \frac{x' dH(x')}{H(x_2^s) - H(x_1^s)}$, the third element is the income received by those who are unemployed and those who are employed informally, $z u_f N_f + w_I \left[u_{is} N_{is} + N_i\right]$ and finally the last element, is the total cost of posting vacancies in the economy, $c\theta \left[u_f N_f + \varphi u_{is} N_{is}\right]$, see Hosios (1990) and Pissarides (2000).

3.1. Socially efficient labor market tightness

Using the first order conditions presented in Appendix B, I solve for the efficient labor market tightness θ.

Proposition 4. *The socially efficient labor market tightness is given by:*

$$c = \frac{\alpha(\theta)(1-\eta(\theta))}{(1-H(x_1^s))}\left\{\int_{x_1^s}^{x_2^s}\frac{(x'-w_I)dH(x')}{(\delta+\varphi\eta(\theta)m(\theta)+\mu)}+\int_{x_2^s}^{\bar{x}}\frac{(x'-z)dH(x')}{(\delta+\eta(\theta)m(\theta)+\mu)}\right\} \tag{19}$$

Proof. See proof in Appendix B.1 □

The absolute elasticity of the matching function with respect to the labor market tightness is defined as: $\eta(\theta) = 1 - \frac{\theta m'(\theta)}{m(\theta)}$, with $m(\theta) = \lambda_1$. Proposition (4) implies that the labor market tightness is determined when the cost of posting a vacancy is equal to the gain of filling a formal job with a worker with productivity $x > x_1^s$. This solution depends on the elasticity of the matching function, $\eta(\theta)$. Proposition (3) presents the labor market tightness in the decentralized case. Comparing the results from Proposition (4) and Proposition (3) I can show that the decentralized solution is inefficient. Even assuming that the Hosios condition holds, [which implies that the worker's bargaining power is equal to the elasticity of the matching function, ($\beta = \eta(\theta)$)], the solution in the decentralized case is inefficient given that $x_1 \neq x_1^s$ and $x_2 \neq x_2^s$, as I will show in the next section.

3.2. Socially efficient productivity level x_1^s and x_2^s

Using the first order conditions presented in Appendix B, I can solve the socially efficient productivity levels x_1^s and x_2^s.

Proposition 5. *The socially efficient productivity levels x_1^s and x_2^s are given by:*

$$x_1^s = w_I + \frac{c(\delta+\mu)}{\alpha(\theta)} \tag{20}$$

$$x_2^s - x_1^s = \frac{(w_I - z)(\delta+\mu+\varphi\lambda_1)}{\lambda_1(1-\varphi)} \tag{21}$$

Proof. See proof in Appendix B.2 □

The value x_1^s is the productivity level at which the social planner is indifferent to allocate workers as "pure informal" or "informal searchers", this implies that the productivity level at which the marginal social productivity gain is equal to the marginal social cost. Then equation (20) implies that the minimum productivity level of a worker to participate in the formal sector should be equal to the income received by a worker while being informal, plus the discounted cost of posting a vacancy in the economy, [taking account of the job destruction shock, the worker's death shock and the average duration of a vacancy, $1/\alpha(\theta)$]. Equation (10) shows the the solution for the decentralized case as $x_1 = w_I$. Then the social productivity level x_1^s is higher compared to the decentralized case. The social planner takes into account the fact that a worker with productivity $x_1 = w_I$ does not overcome the total cost of posting a formal vacancy. Therefore, workers with productivity $x \leq x_1$ are preferred to be "pure informal" workers by the social planner. As a result the social planner increases the average productivity of formally employed workers in the economy, by imposing a higher minimum level of productivity to participate in the formal labor market. This result is related to the findings of Acemoglu and Shimer (1999), which affirm that when firms make ex-ante investments before matching with workers and wages are determined by ex post bargaining, the equilibrium is always inefficient, because workers do not internalize the total investment cost of the firms (hold-up problem).

The value x_2^s is the productivity level at which the social planner is indifferent to allocate a worker as "informal searcher" or "pure formal worker". Equation (21) implies that the marginal productivity gained from allocating an "informal searcher" as "pure formal", $x_2^s - x_1^s$, should be equal to the marginal social cost. This marginal cost is represented as the income loss a worker would get when moving from being an "informal searcher" to a "pure formal searcher" $(w_I - z)$, [taking into account the probability of finding a formal job and some discounting factors]. Notice that $\frac{\partial(x_2^s - x_1^s)}{\partial\varphi} > 0$, then the higher the search effort while informally employed, the higher the gain from allocating an "informal searcher" as "pure formal". Equation (11) shows the solution for the decentralized case as $x_2 = x_1 + \frac{(w_I - z)(\delta+\mu+\beta\lambda_2)}{\beta(\lambda_1-\lambda_2)}$. Then assuming workers have all the bargaining power, $\beta = 1$, I get $x_1^s - x_2^s = x_2 - x_1$ but given that workers do not internalize the full cost of posting a formal vacancy in the economy, $x_1 \neq x_1^s$ and the search intensity in the economy is lower than the efficient one, ("composition externality"), $x_2 \neq x_2^s$. As a result, the solution of the decentralized case is inefficient.

4. Optimal policy

This section analyzes the optimal policy when the worker's search effort is unobserved by the government. In the literature, some authors have studied the design of an optimal unemployment insurance system when the authorities cannot monitor the agents' search effort. This is the case of Shavell and Weiss (1979), Hopenhayn and Nicolini (1997), Fredriksson and Holmlund (2001), Cahuc and Lehmann (2000) and Coles (2008) among others. These authors analyze the case when workers are risk averse. In this section I analyze the optimal policy when workers are risk neutral and the authorities cannot observe the worker's search effort (or the informal sector is unobserved).

In general there are different policy instruments the government can implement as: employment subsidies, hiring subsidies, firing tax, unemployment compensation, wage taxes paid by the worker as lump sum tax or as a proportional tax, among others. Authors such as Mortensen and Pissarides (1999a,b), study the effect of these different policies in the labor market equilibrium without the informal sector. In this paper, I focus on those policy instruments that may be easy to implement. Given that the search effort is unobserved, the social planner has a constrained set of instruments. Notice that if the government wants to offer an unemployment benefit it will have a moral hazard problem, because those formal workers who receive the unemployment benefit may have incentives to work as informal workers (which is unobserved by the government) while receiving the benefit. Then, in this case the unemployment benefit is not an optimal policy. However, given the search effort is unobserved, the government can offer a social security payment (b)[8] to all workers. Furthermore it can offer a severance payment (S) to those who have just been working as formal and have lost their jobs. Moreover, I assume that the government can subsidize job creations (s)[9] and that it can implement a linear tax on workers' productivity

[8]This social security payment refers to any social help the government may offer (for example, any public health and social care program) which is independent of the worker's unemployment status.

[9]According to Coles (2008) this policy can be interpreted as a capital investment subsidy.

$(a+\tau x)$, for those who are formally employed. Where a is a lump sum tax and τ is a proportional tax of the workers' productivity.

Assuming that the social planner can observe the workers' productivity and the worker's wage from the formal sector, τ can be a proportional tax on the productivity level or a proportional tax on the worker's wage. Choosing τ as a proportional tax on the workers' wage affects the worker's bargaining power in the Nash bargaining process. As Pissarides (2000) affirms: "The worker's marginal tax rate reduces labor's share of the surplus, because a unit rise in wages conceded by the firm yields benefit to the worker of one unit less the marginal tax rate"p.210. However, using τ as a proportional tax on the worker's productivity does not affect the worker's bargaining process[10]. Furthermore I can show that the results presented in the next section using τ as a proportional tax on the productivity level are not very different from the results using τ as a proportional tax on the worker's wage.

4.1. Environment

To determine under which conditions there can be an optimal policy when the government cannot observe the worker's search effort, I first need to solve the decentralized solution with the policy instruments. In order to do so, first I describe the model including the different policies offered by the goverment such as: severance payment, social security payment, job creation subsidy and a formal tax.

4.1.1. Worker's and firm's behavior

Workers decide to be unemployed or informal and search randomly for a formal job. This decision, will depend not only on the worker's productivity but also on the different policies implemented by the government, $\{b,\ a,\ \tau,\ s, S\}$, where b is the social security payment, $(a + \tau x)$ is the formal tax payed by those who are formally employed, s is the job creation subsidy offered to the firms and S is the severance payment given to the workers when they are laid off. I assume that firms pay the severance payment S[11]. The worker's value functions are given by:

$$(r+\mu)U(x) = z + b + \lambda_1 \left\{max\left[U(x), W_f(x)\right] - U(x)\right\} \tag{22}$$

$$(r+\mu)W_f(x) = w(x) - (a+\tau x) + \delta\left\{max\left[U(x), W_i(x)\right] - W_f(x)\right\} + \delta S \tag{23}$$

$$(r+\mu)W_i(x) = w_I + b + \lambda_2 \left\{max\left[W_f(x), W_i(x)\right] - W_i(x)\right\} \tag{24}$$

Equation (22) includes the additional income flow given by the social security payment b while unemployed. Equation (23) includes the linear tax for being formally employed, $(a + \tau x)$ and the severance payment S when laid off. Using a linear tax allows me to have some redistribution effects that will be discussed in the next sections. Finally, equation (24) includes the additional income flow given by the social security payment b for those informally employed. Notice that the government cannot observe the worker's search effort, therefore it will offer the same social security payment to all workers.

[10]Using τ as a proportional tax on the worker's wage implies the following Nash bargaining solution: $\beta(1-\tau)\left[J_f(x) - J_u\right] = (1-\beta)\left[W_f(x) - max\{U(x), W_i(x)\}\right]$

[11]Florez (2014) presents the results when severance payment is offered by the government. She shows that the results are similar, so they are independent of whom pays the severance payment.

The value of an unfilled formal vacancy, J_u and the value of a filled formal job with a worker type x, $J_f(x)$, are given by:

$$rJ_u = -c + s + \alpha \left\{ max \left[EJ_f(x), J_u \right] - J_u \right\} \tag{25}$$

$$rJ_f(x) = x - w(x) + (\delta + \mu) \left\{ J_u - J_f(x) \right\} - \delta S, \tag{26}$$

where the wage $w(x)$, depends on the worker's productivity and is negotiated through strategic bargaining. Equation (25) includes the job creation subsidy given to the firms and equation (26) includes the severance payment a firm should pay to the worker when there is a job destruction shock. Free entry condition for firms implies $J_u = 0$.

4.1.2. Strategic bargaining

In this paper, I assume that workers receive a social security payment, b, while they are searching for a job. This benefit is independent of their search effort (or if workers are formal or informal). Moreover, formally employed workers receive a severance payment when they lose their job, S. This implies that this social security program is a special case of an unemployment insurance with duration dependence, where for an unemployment duration $t = 0$, workers receive the severance payment, S and for an unemployment duration $t > 0$ workers receive the social security payment, b.

In this case the Nash bargaining approach is not appropriate. As Coles and Muthoo (2003) point out:

> "The Nash bargaining approach is reasonable in steady-state situations where payoffs do not change over time, however, in situations where agents have time-varying payoffs (for example a worker's unemployment benefit entitlement may be about to expire, or a worker's job skills may decline while unemployed), the strategic bargaining approach determines the equilibrium terms of trade in a way which is consistent with how payoffs are expected to evolve over time"p.71

This particular case of duration-dependent unemployment benefit schemes brings up the well known "re-entitlement effects" defined in the literature by various authors [see Mortensen (1976), Fredriksson and Holmlund (2001), Coles and Masters (2006, 2007), among others]. The "re-entitlement effect" means that when a workers is employed, he qualifies for a severance payment which improves the value of employment, this encourage greater search effort once the worker becomes unemployed. Coles and Masters (2006) show that when workers are risk neutral, the strategic bargaining solution is a special case of the Nash bargaining solution:

> "When agents are risk neutral and have the same discount rate, a simple benchmark case arises, the negotiated wage at duration *t* is equivalent to a Nash bargaining solution when the worker's threat-point is the expected payoff through being indefinitely unemployed" p.111[12].

In this case the value of being indefinitely unemployed is the value of leisure augmented by the additional unemployment insurance payments received from the government. One important assumption of this result is that the government does not observe job offers and so a worker who

[12] See also Coles and Muthoo (2003)

rejects a job offer remains entitled to receive the unemployment benefits. Following Coles and Masters (2006) I can write the strategic bargaining solution as[13]:

$$(1-\beta)\left[W_f(x/a,\tau,S)-\frac{z+b}{(r+\mu)}\right]=\beta\left[J_f(x)-J_u\right] \tag{27}$$

Where $\frac{z+b}{(r+\mu)}$ is the value of being indefinitely unemployed. Once a worker meets a firm, they start bargaining over the wage of the formal job, so the value of being indefinitely unemployed is the lowest offer that a formal worker would accept.

4.2. Worker's and firm's strategies

Following the previous sections, I describe the worker's strategy taking θ as given. In order to do this, I need to solve the Bellman equations (22), (23), (24), and (26) and the strategic bargaining solution (27). As before I consider three cases. Case A refers to those workers who never participate in the informal sector, I will call them "pure formal workers". Case B refers to those workers who prefer to stay informal while searching for a formal job offer, I call these workers "informal searchers" and finally case C refers to those workers who never participate in the formal sector, whom I call "pure informal workers". Solving these three cases I find the optimal worker's strategy.

Proposition 6. *The optimal worker's strategies given θ are:*

i) Workers with productivity $x < x_1$ only work in the informal sector, "pure informal workers", where:

$$x_1=\frac{a(r+\mu+\lambda_2)-\delta(w_I+b)+(r+\mu+\lambda_2+\delta)(z+b)}{(1-\tau)(r+\mu+\lambda_2)} \tag{28}$$

ii)Workers with productivity $x_1 \leq x \leq x_2$ stay working in the informal sector and accept job offers from formal sector, "informal searchers", where:

$$x_2=\left\{\begin{array}{c}\frac{(w_I+b)\left[(r+\delta+\mu)(r+\mu+\lambda_1)-\beta\delta\lambda_1\right]-(z+b)\left[(r+\delta+\mu)(r+\mu+\lambda_2)-\beta\delta\lambda_2\right]}{\beta(r+\mu)(\lambda_1-\lambda_2)(1-\tau)}\\ +\frac{(\lambda_1-\lambda_2)\left[a\beta(r+\mu)-(z+b)(r+\delta+\mu)(1-\beta)\right]}{\beta(r+\mu)(\lambda_1-\lambda_2)(1-\tau)}\end{array}\right\} \tag{29}$$

iii) Workers with $x > x_2$ stay unemployed and accept job offers from formal sector, "pure formal workers"

Equation (28) and equation (29) from Proposition (6), imply that productivity level x_1 and productivity level x_2 are affected positively with the social security payment, $\frac{\partial x_1}{\partial b} > 0$, $\frac{\partial x_2}{\partial b} > 0$, and positively with the linear tax , $\frac{\partial x_1}{\partial \tau} > 0$, $\frac{\partial x_1}{\partial a} > 0$ and $\frac{\partial x_2}{\partial \tau} > 0$, $\frac{\partial x_2}{\partial a} > 0$. The linear tax, disincentivizes the participation in the formal sector of workers with low productivity, given that only those who are productive enough are willing to participate in the formal labor market. The social security payment decreases the participation in the formal sector of those who have low productivity, given that a high social security payment increases the worker's reservation wage, as reported in Pissarides (2000). Notice that since the severance payment does not affect productivity levels x_1 and x_2, we have the following result:

[13]Notice that in this case I do not discuss the two-tier wage structure (as the inside wage and the outside wage), discussed by Pissarides (2000, p.209). This means that once the workers is inside the firm, he cannot force a renegotiation of the wage.

Proposition 7. *When the severance payment is paid by the firms, the negotiated wage decreases in the same amount as the severance payment. This means that in the negotiation process, when firms pay the severance payment, they discount from the worker's wage the full severance payment that a worker would receive if a shock destruction occurred.*

These results are similar to Lazear (1990), who argues that when workers are risk neutral and firms pay the severance payment, the worker would compensate the employer for the expected transfer ex-ante in the form of lower initial wages. Lazear (1990) argues that if workers are risk neutral then a legislated firing cost, which is paid to the worker on layoff, has no real effect on wage, because in the bargaining process the negotiated wage falls one-for-one with the firing cost. Therefore when the severance payment policy is offered by the firms, this policy does not affect the workers decision to participate in the formal labor market, given that the negotiated wage decreases one by one with the severance payment[14]. As a result, offering a severance payment is a waste. Without loss of generality I get that $S = 0$ is an optimal policy.

Using the optimal workers' strategy given θ, described in Proposition (6), the free entry condition, $J_u = 0$, and the firms' optimal condition to post a vacancy given by equation (25); I can get the job creation condition with the policy instruments (linear formal tax, social security payment and job creation subsidy).

Proposition 8. *Job creation condition with the policy instruments is given by:*

$$c - s = \alpha(\theta)(1-\beta)\left\{ \begin{array}{c} \int_{x_1}^{x_2} \frac{\left[(r+\mu+\lambda_2)(x(1-\tau)-a)+\delta(w_I+b)-(r+\mu+\lambda_2+\delta)(z+b)\right]dF(x'/\theta)}{\left[(r+\delta+\mu)(r+\mu+\lambda_2)-\beta\delta\lambda_2\right]} \\ + \int_{x_2}^{\bar{x}} \frac{\left[(r+\mu+\lambda_1)(x(1-\tau)-a-(z+b))\right]dF(x'/\theta)}{\left[(r+\delta+\mu)(r+\mu+\lambda_1)-\beta\delta\lambda_1\right]} \end{array} \right\}$$

Proposition (8) implies that the job creation condition is affected by the policy instruments in different ways. Notice that an increase in the social security payment b, increases the wage at which workers will accept a formal offer, then we can expect that job creation will decrease with b [Mortensen and Pissarides (1999b), and Pissarides (2000)]. Moreover, an increase in the formal tax $(a + \tau x)$ increases the negotiated wage but at the same time, implies that only those with higher productivity will participate in the formal labor market, hence the impact on job creation is ambiguous.

4.3. Efficient policy

Using the previous solutions for $\{x_1, x_2, \theta\}$ from Proposition (6) and Proposition (8) with policy instruments $\{a, \tau, b, s\}$, and the solutions from the social planner problem described by Proposition (4) and Proposition (5) , I get the conditions that allow me to find an efficient policy. To compare these two solutions I assume $r = 0$. Appendix C presents these conditions.

Proposition 9. *The optimal tax τ which targets the efficient productivity level x_2^s is given by:*

$$(1-\tau)^* = \frac{(\lambda_1+\mu)\left[(\mu+\lambda_2)(\delta+\mu)-\lambda_2\beta\delta\right]}{(\mu+\lambda_2)\beta\mu(\delta+\mu+\lambda_2)} \tag{30}$$

Proof. Using conditions C.1 and C.2 from appendix C, I get the optimal proportional tax. □

[14] See full details in Florez (2014)

From Proposition (9) I find that the value of the optimal policy τ depends on the value of the bargaining power β and the parameter φ, which represents the search effort of informal workers (remember that $\lambda_2 = \varphi\lambda_1$). Notice that when the bargaining power $\beta \to 0$, then $\tau \to -\infty$ and when the bargaining power $\beta \to 1$, then $\tau \to 1 - \frac{(\lambda_1+\mu)}{(\mu+\lambda_2)} < 0$. In this case for different values of β and φ, the optimal tax τ is negative which I can interpret as a tax credit [see Brewer and Browne (2006) and Blundell et al. (2000), among others][15]. This policy targets the productivity level x_2^s, then without loss of generality I can asume that those workers with productivity levels between $x_1 < x \leq x_2$ who are employed in the formal sector pay a lump sum tax a and receive a negative tax τ or tax credit proportional to their productivity level, and those whose productivity level is higher $x > x_2$, pay a lump sum tax a (which implies $\tau = 0$). The previous results imply the following optimal tax schedule $T(x)$[16] [see Figure (2)]:

Figure 2: Optimal tax schedule $T(x)$

$$T(x) = \begin{cases} a + \tau x & for\ x_1 < x \leq x_2 \\ a & for\ x > x_2 \end{cases}$$

This optimal tax credit focuses on those who are "informal searchers" and aims to increase their participation in the formal labor market. The higher the productivity of the "informal searchers" when formally employed, the higher the reduction of taxes. Figure (3) shows the tax payed by "informal searchers" when they are formally employed. Then "informal searchers" pay lower taxes (light gray area in the figure) than those who are "pure formal workers" (dark gray area in the figure). The idea is to provide incentives for "informal searchers" to work in the formal sector, because their participation in the formal sector increases the efficiency of the economy (improving the "composition" and search intensity in the economy). Notice that we can interpreted this optimal policy as a progressive taxation, where those workers with lower incomes or lower productivity are subsidized by those with higher incomes or higher productivity [see Diamond (1998, 1980)][17].

Proposition 10. *The optimal policy which targets the efficient productivity level x_1^s is given by:*

$$a + b = \frac{c(\delta+\mu)}{\alpha(\theta)}(1-\tau) + \frac{(w_I - z)(\delta+\mu+\lambda_2) - \tau w_I(\mu+\lambda_2)}{(\mu+\lambda_2)} \tag{31}$$

Proof. Substituting the optimal tax from Proposition (9) in condition C.1 from appendix C and rearranging terms, I find the optimal policy $\{b, a\}$ that ensures an efficient productivity level x_1^s, conditional on $x_2^s = x_2$. □

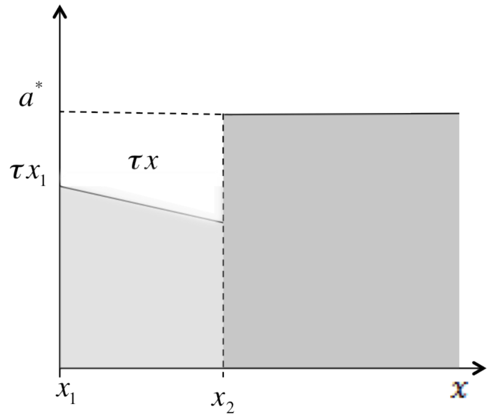
Notice that the previous condition depends on the policy parameters $\{a,\ b\}$. Assuming the social security payment $b = 0$, I can rewrite the optimal taxation condition a^* as:

[15]The work tax credit (WFTC) was introduced in England in 1999. This policy aimed to attract parents without work into the labour market, by directing additional support to those already working but living in families with a low income [Brewer and Browne (2006)].

[16]These results are similar to those found by Diamond (1998), which found an optimal marginal tax rate with a U-shaped pattern.

[17]See more details in Sørensen (1999), who study the degree of tax progressivity which would maximize the welfare of the representative wage earner in four different models of an imperfect labor market.

Figure 3: Optimal tax payment



$$a^* = \frac{c(\delta + \mu)}{\alpha(\theta)}(1 - \tau) + \frac{(w_I - z)(\delta + \mu + \lambda_2) - \tau w_I(\mu + \lambda_2)}{(\mu + \lambda_2)} \tag{32}$$

Equation (32) implies that the optimal lump sum tax a^*, is affected positively with the income flow of being informal w_I and the discounted cost of posting a formal vacancy [taking into account the job destruction shocks, the worker's death shocks and the mean vacancy duration, $\frac{1}{\alpha(\theta)}$], but negatively with the income flow of being formal z. Then the lump sum tax resolves the hold-up problem in the economy. Notice that severance and social security payments are not necessary to determine the efficient productivity level x_1^s.

Proposition 11. *The optimal job creation subsidy s^* that satisfies $\theta^s = \theta$ is given by:*

$$s^* = \alpha(\theta)\left\{\begin{array}{c} \frac{(1-\eta(\theta))}{(1-H(x_1^s))}\left\{\int_{x_1^s}^{x_2^s}\frac{(x'-w_I)dH(x')}{(\delta+\varphi\eta(\theta)m(\theta)+\mu)} + \int_{x_2^s}^{\bar{x}}\frac{(x'-z)dH(x')}{(\delta+\eta(\theta)m(\theta)+\mu)}\right\} \\ -(1-\beta)\left\{\begin{array}{c} \int_{x_1^s}^{x_2^s}\frac{\left[(\mu+\lambda_2)(x(1-\tau)^*-a^*)+\delta w_I-(\mu+\lambda_1+\delta)z\right]dF(x'/\theta)}{\left[(\delta+\mu)(\mu+\lambda_2)-\beta\delta\lambda_2\right]} \\ +\int_{x_2^s}^{\bar{x}}\frac{\left[(\mu+\lambda_1)(x(1-\tau)^*-a^*-z)\right]dF(x'/\theta)}{\left[(\delta+\mu)(\mu+\lambda_1)-\beta\delta\lambda_1\right]} \end{array}\right\} \end{array}\right\} \tag{33}$$

Proof. Using the optimal lump sum tax a^*, which implies $x_1^s = x_1$ from equation (32), and the optimal policy τ^*, which implies $x_2^s = x_2$ from equation (30), into condition C.3 from appendix C, I get the optimal job creation subsidy s^*. □

Proposition (11) implies that the job creation subsidy s^* can be positive or negative. However, given that the search intensity in the decentralized case is lower than the efficient one, we can expect a low job creation in the market solution. In which case the job creation subsidy will be positive. The following numerical exercise confirms these results.

Notice that the severance payment policy S and social security payment b, do not play an important role in the determination of efficiency in the model ($b = 0$ and $S = 0$). The workers' participation in the formal or informal sector is affected mainly by the tax credit $T(x)$, shown in Figure (2). However, these results do not include the Budget Balance Constraint (BBC). An optimal policy with BBC should satisfy the following conditions:

Theorem 12. *Optimal policy would be the values for* $\{\tau^*,\ a^*,\ b^*, s^*\}$ *that satisfy the following conditions:*

$$(1-\tau) = \frac{(\lambda_1+\mu)\left[(\mu+\lambda_2)(\delta+\mu)-\lambda_2\beta\delta\right]}{(\mu+\lambda_2)\beta\mu(\delta+\mu+\lambda_2)} \tag{34}$$

$$a+b = \frac{c(\delta+\mu)}{\alpha(\theta)}(1-\tau) + \frac{(w_I-z)(\delta+\mu+\lambda_2)-\tau w_I(\mu+\lambda_2)}{(\mu+\lambda_2)} \tag{35}$$

$$s = \alpha(\theta)\left\{ \begin{array}{c} \frac{(1-\eta(\theta))}{(1-H(x_1^s))}\left\{\int_{x_1^s}^{x_2^s} \frac{(x'-w_I)dH(x')}{(\delta+\varphi\eta(\theta)m(\theta)+\mu)} + \int_{x_2^s}^{\bar{x}} \frac{(x'-z)dH(x')}{(\delta+\eta(\theta)m(\theta)+\mu)}\right\} \\ -(1-\beta)\left\{ \begin{array}{c} \int_{x_1^s}^{x_2^s} \frac{\left[(\mu+\lambda_2)(x(1-\tau)-(a+b))+\delta w_I-(\mu+\lambda_2+\delta)z\right]dF_1(x'/\theta)}{\left[(\delta+\mu)(\mu+\lambda_2)-\beta\delta\lambda_2\right]} \\ +\int_{x_2^s}^{\bar{x}} \frac{\left[(\mu+\lambda_1)(x(1-\tau)-(a+b+z))\right]dF_2(x'/\theta)}{\left[(\delta+\mu)(\mu+\lambda_1)-\beta\delta\lambda_1\right]} \end{array} \right\} \end{array} \right\} \tag{36}$$

$$s\theta[u_fN_f+\varphi u_{is}N_{is}]+b[u_fN_f+u_{is}N_{is}] = a[(1-u_f)N_f+(1-u_{is})N_{is}]+\tau(1-u_{is})N_{is}\int_{x_1^s}^{x_2^s}\frac{x'dH(x')}{H(x_2^s)-H(x_1^s)} \tag{37}$$

Equation (37) represents the budget balance constraint. This BBC implies that the benefits given through the job creation subsidy and social security payments should be equal to the taxes received by those who are formally employed. Notice that equation (37) takes into account the tax credit received by those "informal searchers" who are formally employed, $\tau(1-u_{is})N_{is}\int_{x_1^s}^{x_2^s}\frac{x'dH(x')}{H(x_2^s)-H(x_1^s)}$. This system of equations can be solved in a recursive way. From equation (34) I can get the optimal tax τ^* which is independent from the optimal policies $\{a,\ b,\ s\}$. Using the value of the optimal policy τ^* into equation (36) I can find the value of the optimal subsidy s^*, for any value of $\{a,\ b\}$ that satisfies equation (35). Then the optimal values for $\{a,\ b\}$ should satisfy equation (35) and equation (37). Substituting the value of $a(S)$ from equation (35) into equation (37), I find the optimal value for b^* and using this value in equation (35) I get the optimal value for a^*. In this case the system has a unique solution.

Notice that with these optimal policies the productivity level of those who participate in the labor market is higher compared to those who participate in the decentralized case without policies. Moreover, the tax credit schedule increases the incentive to participate in the formal labor market for those who are "informal searchers". As Figure (2) shows, the net tax payment decreases with the worker's productivity level for "informal searchers", therefore workers with productivity levels between $x_1 < x \leq x_2$ have more incentives to be formal workers. This policy improves the search intensity in the economy, reducing the "composition externality". The lump sum tax a^* and the job creation subsidy s^* resolve the hold-up problem of the economy. Given that workers do not internalize the cost of posting a vacancy, the social planner subsidize job creation in the economy by taxing the formal workers [similar results are found by Coles (2008)]. Notice that severance payments S and social security payments b do not play any role in determining efficiency. The key role is given by the tax credit policy $T(x)$, which promotes the "informal searchers" to participate in the formal sector. In this case, the social security payment b only helps to balance the budget constraints.

5. Numerical exercise

In this section I present a numerical solution of the optimal policy when the worker's search effort is unobserved by the authorities. The first section presents the numerical solution of a centralized economy and compares these results with those of a decentralized economy. Using different parameters for the cost of posting a vacancy c, the income flow of being informal w_I, and the search effort while being informally employed φ, I find that the "labor market tightness" is always higher in the centralized case compared to the decentralized one, as is suggested in the previous sections. The second section presents the numerical solution of the optimal policy when worker's search effort is unobserved. These policies depend on the income flow of being informal and the cost of posting a vacancy. When the cost of posting a vacancy is high, the job creation subsidy and lump sum tax are high (social security payment does not change). When the income flow of being informal is high the lump sum tax and the social security payment are high.

5.1. Centralized and decentralized solution

To solve the model numerically, I assume a uniform distribution function for the productivity of the population, $H(x)$ with $0 \leq x \leq 1$ and a Cobb-Douglas matching function $m(\theta) = 4\theta^{1/2}$. The parameters are chosen following Albrecht et al. (2009) with a year as the implicit unit of time, where $z = 0$, $\delta = 0.5$, $\beta = 0.5$, and $\varphi = 0.7$. To be able to compare the market solution (MS) with the centralized solution (CS) I assume $r = 0$ with $\mu = 0.02$ as in Coles (2006) [18]. Table (1) presents the market solutions (MS) assuming different values for c and w_I. Table (2) presents the centralized solution (CS) using the same parameters. Notice that as it was expected from the previous sections, the "labor market tightness" is always higher in the centralized case.

Column M1 in Table (1) presents the market solution when the cost of posting a vacancy is $c = 0.2$ and the income flow of being informal is $w_I = 0.2$. In this case the number of "informal searchers" is high ($N_{is} = H(x_2) - H(x_1) = 0.58$), compared to the number of "pure formal workers" ($N_f = 1 - H(x_2) = 0.22$) in the economy. Using the same parameters, column C1 in Table (2) presents the efficient solution. Notice that in the centralized solution the number of "pure formal workers" is higher and the number of "informal searchers" is lower than in the market solution. This happens because, as I mentioned earlier, the social planner increases the productivity level at which workers will participate in the labor market, and as a result the number of "pure informal workers" and "pure formal workers" increases and the number of "informal searchers" decreases. The more productive the workers who are participating in the labor market the higher the level of job creation. Thus, the labor market tightness in the centralized solution is higher than in the market solution ($\theta = 2.37$). Formal employment decreases from $e_f = 0.72$ (72% of labor force) in the market solution, to $e_f = 0.68$ (68% of labor force) in the centralized solution. However, these formally employed workers are on average more productive in the centralized case. On the other hand, the reduction of formal employment is compensated by the reduction of the proportion of those informal workers searching for a job, falling from $e_{is} = 0.24$ (24% of the informal employment) in the market solution to $e_{is} = 0.19$ (19% of the informal employment) in the centralized one. To summarize, in the centralized solution I find more productive workers in the formal labor market and less informally employed workers searching for a job. This increases the total output in the economy from $Y = 0.26$ to $Y = 0.46$.

[18] This implies an expected working lifetime of 50 years.

Even though there are more "pure formal workers" in the centralized solution the unemployment rate does not increase because they are absorbed by the increase in the formal job offers.

Table 1: Market solution with different values for c and w_I

	M1	M2	M3	M4
Parameters				
search effort (φ)	0.7	0.7	0.7	0.7
informal wage (w_I)	0.2	0.1	0.2	0.1
cost vacancy (c)	0.2	0.2	0.3	0.3
Variables				
market tightness (θ)	2.26	2.29	1.44	1.48
productivity level x_1	0.20	0.10	0.20	0.10
productivity level x_2	0.78	0.39	0.81	0.40
unemployment rate (u)	0.02	0.05	0.02	0.06
employment rate (e)	0.98	0.95	0.98	0.94
formal employment rate (e_f)	0.72	0.82	0.70	0.80
informal employment rate (e_i)	0.26	0.13	0.28	0.14
informal searchers (e_{is})	0.24	0.24	0.29	0.29
output (Y)	0.26	0.27	0.26	0.25

Table 2: Centralized solution with different values for c and w_I

	C1	C2	C3	C4
Parameters				
search effort (φ)	0.7	0.7	0.7	0.7
informal wage (w_I)	0.2	0.1	0.2	0.1
cost vacancy (c)	0.2	0.2	0.3	0.3
Variables				
market tightness (θ)	2.37	2.39	1.53	1.55
productivity level x_1	0.24	0.14	0.25	0.15
productivity level x_2	0.76	0.40	0.78	0.42
unemployment rate (u)	0.02	0.05	0.02	0.05
employment rate (e)	0.98	0.95	0.98	0.95
formal employment rate (e_f)	0.68	0.78	0.66	0.76
informal employment rate (e_i)	0.30	0.17	0.32	0.18
informal searchers (e_{is})	0.19	0.17	0.22	0.19
output (Y)	0.46	0.44	0.45	0.42

The second column in Table (2) presents the centralized solution when the income flow w_I is lower than in C1 ($w_I = 0.1$). As one would expect, the lower the income flow w_I, the lower the incentives for a worker to be informal. In this case the number of "informal searches" is lower and the number of "pure formal workers" is higher compared with the solution in C1. As in the previous case the labor market tightness is higher ($\theta = 2.39$) compared to the market solution (M2). Therefore, the proportion of informal workers searching for a formal job is lower ($e_{is} = 0.17$) compared to the market solution in M2 ($e_{is} = 0.24$). Notice that comparing the centralized solution (C2) with the case when the income flow w_I is higher (C1), one observes that, the formal employment is higher ($e_f = 0.78$) and the informal employment is lower ($e_i = 0.17$) compared to column (C1). This happens because the incentives of workers to be informal are low ($w_I = 0.1$).

The third column in Table (2) presents the centralized solution when the cost of posting a vacancy ($c = 0.3$) is higher compared to the solution in column C1. As one would expect, the higher the cost of posting a vacancy the lower the job creation. As in the previous case comparing the centralized solution with the market solution I find a low proportion of informal workers searching for a formal job ($e_{is} = 0.22$) and a high labor market tightness ($\theta = 1.53$). However, notice that if I compare the centralized solution (C3) with the case when the cost of posting a vacancy is lower (C1), I find that when the cost of posting a vacancy is $c = 0.3$, formal employment is lower ($e_f = 0.67$), and the informal employment rate is higher ($e_i = 0.27$) compared to the case when $c = 0.1$ (C1).

Finally, the fourth column in Table (2) presents the centralized solution when the cost of posting a vacancy is higher ($c = 0.3$), but the income flow is lower ($w_I = 0.1$) compared to the solution in column C1. As before, the centralized solution is characterized by a high labor market tightness ($\theta = 1.55$) and a low proportion of informal workers searching for a formal job. However, notice that this case presents the highest unemployment rate and the lower net output in the economy compared to the other scenarios.

In summary, using different parameters for the cost of posting a vacancy c, and the income flow of being informal w_I, I find that the labor market tightness in the centralized case (or social planner case) is always higher than the labor market tightness in the market solution (as it was expected from the previous sectios). Moreover, the proportion of informal workers searching for a formal job is always lower and the proportion of pure formal workers is higher compared to the market solution, witch implies an efficient search intensity in the economy.

5.2. Optimal policy with unobserved worker's search effort

This section presents the numerical results of an optimal policy when the worker's search effort is unobserved by the government. Table (3) presents these results. For this numerical exercise I assume that the worker's bargaining power $\beta = 1$ and an informal search effort $\varphi = 0.7$, which guarantees values of τ between $-1 < \tau < 0$.

The first column in Table (3) (UP1) presents the optimal policy when the cost of posting a vacancy is $c = 0.2$, and the income flow of being informal is $w_I = 0.2$. As it is shown in the previous sections, the linear tax (τ) for those who are formally employed is negative. This means that those who are "informal searchers" and work in the formal sector receive a tax credit of $\tau = 0.43$ (proportional to their productivity). This increases the incentives for workers to be formal. Moreover, the optimal lump sum tax that all formally employed workers should pay, is $a = 0.19$. As mentioned above, the idea of this lump sum tax is to reduces the incentive of workers with low productivity to participate in the formal labor market and the aim of the tax credit is to increase the incentive for those informal searchers to be formal. Finally, the optimal job creation subsidy is $s = 0.12$ and the social security payment for all workers is $b = 0.18$.

The second column UP2 presents the optimal policy when the income flow of being informal is lower compared to the solution in column UP1. In this case, the optimal policy implies a tax credit of $\tau = 0.43$, a lump sum tax of $a = 0.07$, a job creation subsidy of $s = 0.15$ and a social security payment of $b = 0.13$. Notice that compared to column UP1 the number of "informal searches" in the economy is higher, thus the optimal policy implies a lower lump sum tax and a lower social security payment.

Table 3: Optimal policy when search effort is unobserved

	UP1	UP2	UP3	UP4
Parameters				
search effort (φ)	0.7	0.7	0.7	0.7
informal wage (w_I)	0.2	0.1	0.2	0.1
cost vacancy (c)	0.2	0.2	0.3	0.3
Optimal policy with BBC				
market tightness (θ)	2.37	2.39	1.53	1.55
productivity level x_1	0.24	0.14	0.25	0.15
productivity level x_2	0.76	0.40	0.78	0.42
tax credit (τ)	-0.43	-0.43	-0.43	-0.43
lump sum tax (a)	0.19	0.07	0.21	0.09
job creation subsidy (s)	0.12	0.15	0.17	0.22
social security payment (b)	0.18	0.13	0.18	0.14

The third column UP3 in Table (3) presents the optimal policy when the cost of posting a vacancy is higher $c = 0.3$ compared to column UP1. In this case an increase in the cost of posting a vacancy implies a low labor market tightness and a high unemployment rate compared to column UP1. As a consequence, the lump sum tax ($a = 0.21$) and job creation subsidy ($s = 0.17$) are higher compared to column UP1. Notice that in this case the social security payment does not change.

Finally the fourth column in Table (3) presents the optimal policy when the cost of posting a vacancy is $c = 0.3$ and the income flow of being informal is $w_I = 0.1$. As in the previous case, the tax credit is $\tau = 0.43$. However, there is an important change in the lump sum tax and social security payment, which decreases to $a = 0.09$, and to $b = 0.41$, respectively. Moreover, the job creation subsidy increases to $s = 0.22$.

In summary if the authorities want to increase the incentive for workers to be formal, then, an optimal policy is a tax credit policy τ, a lump sum tax a and a job creation subsidy s. These policies depend on the income flow of being informal and the cost of posting a vacancy. When the cost of posting a vacancy is high, the job creation subsidy and the lump sum tax are high (social security payment does not change). When the income flow of being informal is high, the lump sum tax and the social security payment are high.

6. Conclusions

This paper analyzed efficiency in a model, where the informal sector was assumed to be unregulated self-employment. I have shown that the market solution is not efficient for two reasons. The first is the traditional hold-up problem, given that workers do not internalize the firm's cost of posting a vacancy. The second one is given by the "composition externality", since the labor market is composed by two types of workers, "pure formal workers" and "informal searchers", who search for a formal job with different search efforts. Thus, the labor market is overcrowded with those "informal searchers", who search with less effort for a formal job. This "composition externality" implies an inefficient search intensity in the economy. Then, solving the social planner problem I find that the productivity at which workers should participate in the labor market should be higher than the solution found in the decentralized case. Furthermore the efficient number of "informal searchers" should be lower compared to the decentralized case, and the efficient number of "formal workers" should be higher compared to the decentralized case. As a consequence the efficient labor market tightness is higher compared to the market solution.

Since the market solution is not efficient, an optimal policy is required. Assuming the government cannot observe the workers's search effort, I show that the optimal policies to get an efficient allocation are: the social security benefit (b), which is offered to all workers independent of their search effort, the lump sum tax (a), the tax credit (τ), and the job creation subsidy (s). The social security benefit (b), does not play an important role in getting the efficient solution but helps to balance the budget constraint. The key policy to increase a worker's incentive to be formal is the tax credit (τ) policy. This tax credit policy improves the search intensity in the economy by improving the composition of workers in the economy (number of "informal searchers" vs number of "formal workers"). The lump sum tax a^* and the job creation subsidy s^* resolve the hold-up problem of the economy. Given that workers do not internalize the cost of posting a vacancy, the social planner subsidies the job creation in the economy by taxing the formal workers. Finally, the numerical exercise presented in the last section, shows how these policies depend on the income flow of being informal and the cost of posting a vacancy.

The above findings are important because in general there is not a strong support for social security programs in Latin American economies, since these programs are thought to increase informality (moral hazard problem). Mazza (2000) mention that the introduction of UI benefits in developing countries characterized by a high level of informality can subsidize informal activities. In other words, while receiving UI benefits, an unemployed worker may work in the informal sector. However, this paper shows that in the case when workers are risk neutral, there are policies such as a tax credit (τ) policy, that increase a worker's incentive to be formal, even though the government cannot observe the worker's search effort. This result could guide the way in which a social security program can be implemented in Latin American economies, where there is a marked lack of labor protection. This lack of labor protection is one of the main reasons why levels of informality are so high in Latin American economies. According to Maloney (1999, 2004), the Latin American economies have low labor protection, without a proper balance between the taxes and benefits received by workers in the formal sector. This is one of the main reasons why workers prefer to stay informal. Therefore a tax credit policy may be a good instrument to reduce informality in Latin American economies without the moral hazard problem.

Appendix A

A.1 Optimal worker's strategy

Case A: In this case I refer to "pure formal workers" as those with a productivity level x satisfying the condition: $W_i(x) \leq U(x) < W_f(x)$. Notice that if $W_i(x) \leq U(x)$ then workers would never be informal. In this case the only possible option would be to stay unemployed while searching for a formal job; $U(x) < W_f(x)$. The Bellman equations for $U(x)$ and $W_f(x)$ are given by: $(r + \mu)U(x) = z + \lambda_1 \left\{W_f(x) - U(x)\right\}$ and $(r + \mu)W_f(x) = w(x) + \delta \left\{U(x) - W_f(x)\right\}$. Given the free entry condition, and using equation (9) and equation (7) I can find the solution for $w(x)$, $U(x)$, $W_f(x)$, and $J_f(x)$:

$$w(x) = \frac{(r + \delta + \mu + \lambda_1)\beta x + z(1 - \beta)(r + \delta + \mu)}{(r + \delta + \mu + \beta\lambda_1)} \tag{A.1}$$

$$U(x) = \frac{z(r+\delta+\mu)+\lambda_1\beta x}{(r+\mu)(r+\delta+\mu+\beta\lambda_1)} \tag{A.2}$$

$$W_f(x) = \frac{(r+\mu+\lambda_1)\beta x + z(\delta+(r+\mu)(1-\beta))}{(r+\mu)(r+\delta+\mu+\beta\lambda_1)} \tag{A.3}$$

$$J_f(x) = \frac{(1-\beta)(x-z)}{(r+\delta+\mu+\beta\lambda_1)} \tag{A.4}$$

The condition for a worker to accept a formal offer is given by $J_f(x) \geqslant 0$, which implies that the surplus exists if $x \geq z$.

Case B: In this case I refer to "informal searchers" as those workers who prefer to be informal rather than unemployed, hence $U(x) < W_i(x)$ but are able to accept a formal job offer, $W_i(x) < W_f(x)$. The Bellman equations for $W_i(x)$ and $W_f(x)$ are given by: $(r+\mu)W_i(x) = w_I + \lambda_2 \{W(x) - W_i(x)\}$ and $(r+\mu)W_f(x) = w(x) + \delta \left\{W_i(x) - W_f(x)\right\}$. Given the free entry condition, and using equation (9) and equation (7), I can find the solution for $w(x)$, $W_i(x)$, $W_f(x)$, and $J_f(x)$:

$$w(x) = \frac{(r+\delta+\mu+\lambda_2)\beta x + w_I(1-\beta)(r+\delta+\mu)}{(r+\delta+\mu+\beta\lambda_2)} \tag{A.5}$$

$$W_i(x) = \frac{w_I(r+\delta+\mu)+\lambda_2\beta x}{(r+\mu)(r+\delta+\mu+\beta\lambda_2)} \tag{A.6}$$

$$W_f(x) = \frac{(r+\mu+\lambda_2)\beta x + w_I(\delta+(r+\mu)(1-\beta))}{(r+\mu)(r+\delta+\mu+\beta\lambda_2)} \tag{A.7}$$

$$J_f(x) = \frac{(1-\beta)(x-w_I)}{(r+\delta+\mu+\beta\lambda_2)} \tag{A.8}$$

The condition for a worker to accept a formal offer is given by $J_f(x) \geqslant 0$, which implies that surplus exists if $x \geq w_I$.

Case C: In this case I refer to "pure informal workers" as those workers with productivity x such that they prefer to be informal rather than unemployed $U(x) < W_i(x)$ and at the same time they will never accept a formal job offer, which means that $W_f(x) < W_i(x)$. In this case they do not search for a formal job. The Bellman equation for this type of workers is:

$$W_i(x) = \frac{w_I}{(r+\mu)}, \tag{A.9}$$

Given the solution in case A and B, and assuming $w_I > z$, this condition is satisfied when $x < w_I$.

A.2 Principle of Unimprovability

According to (Kreps (1990), p. 812), a strategy is said to be unimprovable if it is satisfied for all initial state θ_0 the following condition:

$$v(\theta_0,\sigma) = \sup_{\alpha \epsilon A(\theta_0)} \left[r(\theta_0,\alpha) + \gamma \sum_{\theta} v(\theta_0,\sigma)\pi(\theta/\theta_0,\alpha) \right] \tag{A.10}$$

$r(\theta_0, \alpha)$ is the immediate reward of one shot deviation α from the initial strategy σ and $\gamma \sum_{\theta} v(\theta_0, \sigma)\pi(\theta/\theta_0, \alpha)$ the discounted expected value of all future rewards from using strategy σ.

Using the *Principle of Unimprovability* I can verify if a candidate's strategy is optimal. To verify if a candidate strategy is the best response, I just need to check one-shot deviation from the candidate strategy. Let σ_f denotes the strategy (analyzed in *case A*) where a worker is unemployed searching for a formal job and does not take any informal job; and consider a period $\triangle$ where the worker deviates from his strategy and takes an informal job. Then σ_f is unimprovable strategy if:

$$w_I \triangle + \frac{1}{1 + r\triangle + \mu\triangle} \left\{ \lambda_2 \triangle W_f(x/\sigma_f) + (1 - \lambda_2 \triangle) U(x/\sigma_f) \right\} \leq U(x/\sigma_f)$$

The first term in the LHS of the inequality, expresses the reward of being informal for a period $\triangle$. The second term expresses the discounted value of future rewards of using strategy σ_f. Where $U(x/\sigma_f)$ is the value function of being unemployed conditional to strategy σ_f while $W_f(x/\sigma_f)$ is the value function of being formally employed conditional to σ_f. Rearranging the terms I can rewrite the above equations as: $w_I \triangle(1 + r\triangle + \mu\triangle) + \lambda_2 \triangle W_f(x/\sigma_f) \leq \triangle U(x/\sigma_f)(r + \lambda_2 + \mu)$; then dividing by $\triangle$, I get $w_I(1 + r\triangle + \mu\triangle) + \lambda_2 W_f(x/\sigma_f) \leq U(x/\sigma_f)(r + \lambda_2 + \mu)$ and letting $\triangle \to 0$, I get: $w_I + \lambda_2 W_f(x/\sigma_f) \leq U(x/\sigma_f)(r + \lambda_2 + \mu)$. Using the value function of being unemployed and formal employed (given by equation (A.2) and (A.3), conditional to strategy σ_f) I find:

$$w_I + \lambda_2 \left[\frac{(r + \mu + \lambda_1)\beta x + z(\delta + (r + \mu)(1 - \beta))}{(r + \mu)(r + \delta + \mu + \beta\lambda_1)} \right] \leq \left[\frac{z(r + \delta + \mu) + \lambda_1 \beta x}{(r + \mu)(r + \delta + \mu + \beta\lambda_1)} \right] (r + \lambda_2 + \mu)$$

This condition is satisfied when:

$$x > \frac{w_I(r + \delta + \mu + \beta\lambda_1) - z(r + \delta + \mu + \beta\lambda_2)}{\beta(\lambda_1 - \lambda_2)}$$

Let σ_i denote the strategy (analyzed in *case B*) where a worker is informal while searching for a formal job, but he is never unemployed. Then consider a period $\triangle$ where the worker deviates from his strategy and decides to be unemployed, σ_i is unimprovable strategy if:

$$z\triangle + \frac{1}{1 + r\triangle + \mu\triangle} \left\{ \lambda_1 \triangle W_f(x/\sigma_i) + (1 - \lambda_1 \triangle) W_i(x/\sigma_i) \right\} \leq W_i(x/\sigma_i)$$

Where $W_i(x/\sigma_i)$ is the value function of being informal conditional to strategy σ_i. $W_f(x/\sigma_f)$ is the value function of being formally employed conditional to σ_i. Using some algebra and the value function of being informally and formally employed conditional to strategy σ_i, (given by equation (A.6) and (A.7)), I find that this condition is satisfied when:

$$x \leq \frac{w_I(r + \delta + \mu + \beta\lambda_1) - z(r + \delta + \mu + \beta\lambda_2)}{\beta(\lambda_1 - \lambda_2)} = x_2$$

Then workers with productivity $x_1 \leq x < x_2$ stay working in the informal sector and accept job offers from formal sector, and workers with productivity $x > x_2$, stay unemployed searching for a formal job and never take an informal job.

Appendix B

Using the Lagrange Method I can solve this standard optimization problem as:

$$L(u_f, u_{is}, x_1^s, x_2^s, \theta, \omega, \gamma) =$$

$$\left\{ \begin{array}{c} (1-u_f)(1-H(x_2^s)) \int_{x_2^s}^{\bar{x}} \frac{x' dH(x')}{1-H(x_2^s)} + (1-u_{is})(H(x_2^s)-H(x_1^s)) \int_{x_1^s}^{x_2^s} \frac{x' dH(x')}{H(x_2^s)-H(x_1^s)} \\ +z u_f (1-H(x_2^s)) + w_I \left[u_{is}(H(x_2^s)-H(x_1^s)) + H(x_1^s) \right] - c\theta \left[u_f (1-H(x_2^s)) + \varphi u_{is}(H(x_2^s)-H(x_1^s)) \right] \\ +\omega \left[(H(x_2^s)-H(x_1^s)) u_{is}(\varphi\lambda_1+\mu) - (H(x_2^s)-H(x_1^s)) \left[(1-u_{is})\delta+\mu \right] \right] \\ +\gamma \left[(1-H(x_2^s)) u_f (\lambda_1+\mu) - (1-H(x_2^s)) \left[(1-u_f)\delta+\mu \right) \right] \end{array} \right\} \tag{B.1}$$

This problem satisfies the following first order conditions:

$$\frac{\partial L}{\partial \omega} = 0 \Rightarrow u_{is} = \frac{\delta+\mu}{\delta+\varphi\lambda_1+\mu} \tag{B.2}$$

$$\frac{\partial L}{\partial \gamma} = 0 \Rightarrow u_f = \frac{\delta+\mu}{\delta+\lambda_1+\mu} \tag{B.3}$$

$$\frac{\partial L}{\partial u_{is}} = 0 \Rightarrow \omega = \frac{\int_{x_1^s}^{x_2^s} \frac{x' dH(x')}{H(x_2^s)-H(x_1^s)} - w_I + c\theta\varphi}{\delta+\varphi\lambda_1+\mu} \tag{B.4}$$

$$\frac{\partial L}{\partial u_f} = 0 \Rightarrow \gamma = \frac{\int_{x_2^s}^{\bar{x}} \frac{x' dH(x')}{1-H(x_2^s)} - z + c\theta}{\delta+\lambda_1+\mu} \tag{B.5}$$

$$\frac{\partial L}{\partial \theta} = 0 \Rightarrow \varphi u_{is}(H(x_2^s)-H(x_1^s)) \left[\omega m'(\theta) - c \right] + u_f (1-H(x_2^s)) \left[\gamma m'(\theta) - c \right] = 0, \tag{B.6}$$

Which imply that the Lagrange multiplier is: $\gamma = \omega = \frac{c}{m'(\theta)}$. Remember that the Lagrange multiplier tells us how the payoff at the optimum varies when we vary the constraint. I can interpret this as the rate of increase on the aggregate output when there is an additional "*informal searcher*" or "*formal worker*" in the economy.

$$\frac{\partial L}{\partial x_1^s} = 0 \Rightarrow \left\{ \begin{array}{c} (1-u_{is}) \frac{\partial}{\partial x_1^s} \left[\int_{x_1^s}^{x_2^s} x' dH(x') \right] + \frac{\partial H(x_1^s)}{\partial x_1^s} \left[w_I - w_I u_{is} + c\theta\varphi u_{is} \right] \\ +\omega \frac{\partial H(x_1^s)}{\partial x_1^s} \left[-u_{is}(\delta+\varphi\lambda_1+\mu) + (\delta+\mu) \right] \end{array} \right\} = 0$$

Imposing condition (B.4) I can rewrite this condition as:

$$\frac{\partial L}{\partial x_1^s} = 0 \Rightarrow (1-u_{is}) \frac{\partial}{\partial x_1^s} \left[\int_{x_1^s}^{x_2^s} x' dH(x') \right] + \frac{\partial H(x_1^s)}{\partial x_1^s} \left[w_I(1-u_{is}) + c\theta\varphi u_{is} \right] = 0, \tag{B.7}$$

by Leibniz's rule I get : $\frac{d}{dx_1^s} \left[\int_{x_1^s}^{x_2^s} x' dH(x') \right] = -x_1^s \frac{\partial H(x_1^s)}{\partial x_1^s}$

$$\frac{\partial L}{\partial x_1^s} = 0 \Rightarrow -(1-u_{is}) x_1^s \frac{\partial H(x_1^s)}{\partial x_1^s} + \frac{\partial H(x_1^s)}{\partial x_1^s} \left[w_I(1-u_{is}) + c\theta\varphi u_{is} \right] = 0, \tag{B.8}$$

$$\frac{\partial L}{\partial x_2^s}=0 \Rightarrow \left\{ \begin{array}{c} (1-u_f)\frac{\partial}{\partial x_2^s}\left[\int_{x_2^s}^{\bar{x}} x' dH(x')\right] + (1-u_{is})\frac{\partial}{\partial x_2^s}\left[\int_{x_1^s}^{x_2^s} x' dH(x')\right] \\ +\frac{\partial H(x_2^s)}{\partial x_2^s}\left[w_I u_{is} - z u_f + c\theta(u_f - \varphi u_{is})\right] \\ +\omega\frac{\partial H(x_2^s)}{\partial x_2^s}\left[u_{is}(\delta+\varphi\lambda_1+\mu)-(\delta+\mu)\right] \\ +\gamma\frac{\partial H(x_2^s)}{\partial x_2^s}\left[-u_f(\delta+\lambda_1+\mu)+(\delta+\mu)\right] \end{array} \right\} = 0$$

Imposing condition (B.4) and (B.5) I can rewrite this condition as:

$$\frac{\partial L}{\partial x_2^s}=0 \Rightarrow \left\{ \begin{array}{c} (1-u_f)\frac{\partial}{\partial x_2^s}\left[\int_{x_2^s}^{\bar{x}} x' dH(x')\right] + (1-u_{is})\frac{\partial}{\partial x_2^s}\left[\int_{x_1^s}^{x_2^s} x' dH(x')\right] \\ +\frac{\partial H(x_2^s)}{\partial x_2^s}\left[w_I u_{is} - z u_f + c\theta(u_f - \varphi u_{is})\right] \end{array} \right\} = 0, \tag{B.9}$$

Remember that by Leibniz's rule I get:
$\frac{d}{dx_2^s}\left[\int_{x_1^s}^{x_2^s} x' dH(x')\right] = x_2^s \frac{\partial H(x_2^s)}{\partial x_2^s}$ and $\frac{d}{dx_2^s}\left[\int_{x_2^s}^{\bar{x}} x' dH(x')\right] = -x_2^s \frac{\partial H(x_2^s)}{\partial x_2^s}$ then:

$$\frac{\partial L}{\partial x_2^s}=0 \Rightarrow \left\{ \begin{array}{c} -(1-u_f)x_2^s\frac{\partial H(x_2^s)}{\partial x_2^s} + (1-u_{is})x_2^s\frac{\partial H(x_2^s)}{\partial x_2^s} \\ +\frac{\partial H(x_2^s)}{\partial x_2^s}\left[w_I u_{is} - z u_f + c\theta(u_f - \varphi u_{is})\right] \end{array} \right\} = 0, \tag{B.10}$$

B.1 Socially efficient labour market tightness

Imposing the optimal condition for θ, given by equation (B.6), into the first order condition u_{is}, given by equation (B.5), I find:

$$\frac{c}{m'(\theta)} = \frac{\int_{x_1^s}^{x_2^s} \frac{x' dH(x')}{H(x_2^s)-H(x_1^s)} - w_I + c\theta\varphi}{\delta+\varphi\lambda_1+\mu} \tag{B.11}$$

Defining the absolute elasticity of the matching function with respect to the *labour market tightness* as:
$\eta(\theta) = 1 - \frac{\theta m'(\theta)}{m(\theta)}$, where $m(\theta) = \lambda_1$
And using the previous definition, I can write:

$$\frac{c\theta}{(1-\eta(\theta))m(\theta)} = \frac{\int_{x_1^s}^{x_2^s} \frac{x' dH(x')}{H(x_2^s)-H(x_1^s)} - w_I + c\theta\varphi}{\delta+\varphi m(\theta)+\mu}$$

Rearranging terms I get:

$$c(H(x_2^s)-H(x_1^s)) = (1-\eta(\theta))\frac{m(\theta)}{\theta}\left\{\int_{x_1^s}^{x_2^s} \frac{(x'-w_I)dH(x')}{\delta+\varphi\eta(\theta)m(\theta)+\mu}\right\} \tag{B.12}$$

In the same way, imposing the optimal condition for θ (B.6) into the first order condition for u_f (B.4) I have:

$$c(1-H(x_2^s)) = (1-\eta(\theta))\frac{m(\theta)}{\theta}\left\{\int_{x_2^s}^{\bar{x}} \frac{(x'-z)dH(x')}{\delta+\eta(\theta)m(\theta)+\mu}\right\} \tag{B.13}$$

Adding equation (B.12) and equation (B.13), I find the condition which determines the socially *efficient labour market tightness* as:

$$c = \frac{m(\theta)}{\theta}\frac{(1-\eta(\theta))}{(1-H(x_1^s))}\left\{\int_{x_1^s}^{x_2^s} \frac{(x'-w_I)dH(x')}{\delta+\varphi\eta(\theta)m(\theta)+\mu} + \int_{x_2^s}^{\bar{x}} \frac{(x'-z)dH(x')}{\delta+\eta(\theta)m(\theta)+\mu}\right\} \tag{B.14}$$

Given $m'(\theta) = \frac{m(\theta)\left[1-\eta(\theta)\right]}{\theta}$ I can write:

$$c = \frac{m'(\theta)}{(1 - H(x_1^s))} \left\{ \int_{x_1^s}^{x_2^s} \frac{(x' - w_I)dH(x')}{\delta + \varphi\eta(\theta)m(\theta) + \mu} + \int_{x_2^s}^{\bar{x}} \frac{(x' - z)dH(x')}{\delta + \eta(\theta)m(\theta) + \mu} \right\} \tag{B.15}$$

B.2 Socially efficient productivity level and x_1^s and x_2^s

Notice that from equation (B.8) I have that the socially efficient productivity level x_1^s is: $x_1^s = w_I + \frac{c\theta\varphi u_{is}}{1-u_{is}}$, then, substituting the steady state value for u_{is} given by condition (B.4) I can write:

$$x_1^s = w_I + \frac{c\theta(\delta + \mu)}{\lambda_1} \tag{B.16}$$

From condition (B.10), I have that the efficient productivity level x_2^s given by: $u_f \left(x_2^s - z + c\theta\right) = u_{is} \left(x_2^s - w_I + c\theta\varphi\right)$, then substituting the steady state value for u_{is} and u_f I can write:

$$x_2^s = \frac{w_I(\delta + \mu + \lambda_1) - z(\delta + \mu + \varphi\lambda_1)}{\lambda_1(1 - \varphi)} + \frac{c\theta(\delta + \mu)}{\lambda_1} \tag{B.17}$$

Using the value x_1^s I can rewrite x_2^s as:

$$x_2^s - x_1^s = \frac{(w_I - z)(\delta + \mu + \varphi\lambda_1)}{\lambda_1(1 - \varphi)} \tag{B.18}$$

Appendix C

Conditions for efficiency

The decentralized solution is efficient if the following conditions are satisfied (with $r = 0$):

C.1 $x_1^s = x_1$, implies:

$$w_I + \frac{c\theta(\delta + \mu)}{\lambda_1} = \frac{a(\mu + \lambda_2) - \delta(w_I + b) + (\mu + \lambda_2 + \delta)(z + b)}{(1 - \tau)(\mu + \lambda_2)}$$

C.2 $x_2^s = x_2$, implies:

$$x_1^s + \frac{(w_I - z)(\delta + \mu + \varphi\lambda_1)}{\lambda_1(1 - \varphi)} = \left\{ \begin{array}{c} \frac{(w_I+b)\left[(\delta+\mu)(\mu+\lambda_1)-\beta\delta\lambda_1\right]-(z+b)\left[(\delta+\mu)(\mu+\lambda_2)-\beta\delta\lambda_2\right]}{\beta\mu(\lambda_1-\lambda_2)(1-\tau)} \\ +\frac{(\lambda_1-\lambda_2)\left[\beta\mu a-(z+b)(\delta+\mu)(1-\beta)\right]}{\beta\mu(\lambda_1-\lambda_2)(1-\tau)} \end{array} \right\}$$

C.3 $JC^s = JC$, implies:

$$c-s-\alpha(\theta)(1-\beta) \left\{ \begin{array}{c} \int_{x_1}^{x_2} \frac{\left[(\mu+\lambda_2)(x-(a+\tau x))+\delta(w_I+b)-(\mu+\lambda_2+\delta)(z+b)\right]dF(x'/\theta)}{\left[(\delta+\mu)(\mu+\lambda_2)-\beta\delta\lambda_2\right]} \\ +\int_{x_2}^{\bar{x}} \frac{\left[(\mu+\lambda_1)(x-(a+\tau x)-(z+b))\right]dF(x'/\theta)}{\left[(\delta+\mu)(\mu+\lambda_1)-\beta\delta\lambda_1\right]} \end{array} \right\} = c-\alpha(\theta)\frac{(1 - \eta(\theta))}{(1 - H(x_1^s))} \left\{ \begin{array}{c} \int_{x_1^s}^{x_2^s} \frac{(x'-w_I)dH(x')}{(\delta+\varphi\eta(\theta)m(\theta)+\mu)} \\ \int_{x_2^s}^{\bar{x}} \frac{(x'-z)dH(x')}{(\delta+\eta(\theta)m(\theta)+\mu)} \end{array} \right\}$$

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