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Borradores de
ECONOMÍA

Núm. 833
2014

Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá

Optimal Policy with Informal Sector and Endogenous Savings

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Abstract

This paper analyzes the effect of social security and lump sum layoff payment in an economy with an informal sector and savings, where the search effort is unobserved. I characterize the optimal consumption/search/non-participant strategy assuming that workers are risk averse and that formal jobs last forever. After including job destruction shocks I solve the model numerically, and focus on the effects of lump sum layoff and social security payments on workers' decision to be formal, informal or non-participant. I find that severance payments protect formal workers against the unemployment risk. With severance payments workers do not over-accumulate to protect themselves agains unemployment, instead they increase the search effort through the *re-entitlement effects*. In this respect my work resembles that of Coles (2006). I find that in the steady state a high severance payment increases the proportion of formal workers while reduces the proportion of informal workers and those who decide not to participate in the labor market. Even though the optimal policy with severance payment is generous, I find that in the steady state the unemployment rate is low and welfare improves.

JEL classification: D91; J32; J64; J65

Keywords: Social security payment, Severance payment, Informal sector, Hidden search effort, Savings

1. Introduction

This paper analyzes an economy with informal sector where workers are risk averse and they are allowed to save. In the first section I characterize the optimal consumption/search/non-participant strategy in an economy where formal jobs last forever. Then I extend the model to the case with job destruction and introduce some policies such as severance payment, income tax and social security payments. I analyze numerically the effect of these policies in the workers' decision of working in the formal or informal sector. Similar to Coles (2006) I find that severance

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payment improves the value of employment, through the *re-entitlement effects*[2], increasing the workers' incentives to search for a formal job. I show that in economies with an informal sector, severance payment policy reduces informality without the moral hazard problem. Furthermore I find that in the steady state the proportion of formal workers increases and the proportion of informal workers and those who decide not to participate in the labour market decreases. The unemployment rate is low and social welfare improves, despite a generous optimal policy with severance payment.

An important number of authors have analyzed the optimal unemployment insurance (UI) when the search effort is hidden and workers are allowed to save. This is the case of Lentz (2009), Shimer and Werning (2003, 2005), Kocherlakota (2004), Lentz and Tranaes (2005), Werning (2002), Acemoglu and Shimer (1999a,b), Hansen and İmrohoroğlu (1992), Joseph and Weitzenblum (2003), among others. Lentz (2009) explored the role of unemployment benefits in a job search model with savings. He found that wealthier individuals experience longer unemployment durations, given the theoretically negative relationship between choice of search intensity and savings, and he found that the higher the moral hazard problem the lower the optimal replacement rate. In this case, workers switch to savings as their main insurance vehicle[3]. Similar results were found by Hansen and İmrohoroğlu (1992). On the other hand, Joseph and Weitzenblum (2003) found that a low replacement rate is not necessarily optimal when we analyze the transition cost of an economy from high to low benefits. They showed that in some cases the adjustment costs are high enough to question the practicability of a cut in benefits. Shimer and Werning (2003, 2005), explored the optimal design of unemployment insurance when there is a hidden financial market and when post-unemployment wages are uncertain. They showed that under these circumstances the optimal policy is a fixed unemployment benefit to the worker in every period that he is unemployed and a lump-sum tax during employment, whose size depends on the unemployment duration. Similar results were found by Lentz and Tranaes (2005) and Kocherlakota (2004) who showed that in an environment with hidden saving and hidden search effort an optimal contract is a constant UI during the unemployment spells. Contrary to the majority of existing literature, Werning (2002) found that the optimal schedule is increasing, although he agrees with other authors that constant benefits are a good approximation to the optimal UI schedule. In general, the majority of existing literature about optimal unemployment insurance with risk averse workers and hidden saving concludes that an optimal benefit is a constant UI.

More recently some authors have started to analyze the optimal unemployment insurance through the life cycle. This is the case of Michelacci and Ruffo (2011) who found that an optimal unemployment insurance implies a high replacement rate for young workers and a low replacement rate for old workers. The reason is that young workers value more the unemployment insurance because they have fewer means to protect themselves against unemployment risk and the moral hazard problem is limited in their case, because young workers want jobs to improve life-time career prospects and to accumulate human capital. The authors show that allowing for unemployment insurance to depend on age brings important welfare gains.

[2]The "re-entitlement effect" refers to the effect on the search effort of a limited unemployment benefit or severance payment. Mortensen (1977) defines the re-entitlement effect as the increase in the search intensity of an unemployed worker when his unemployment benefits are about to exhaust."In the case of a qualified worker who has not yet exhausted his or her unemployment benefits, the escape rate increase realized unemployment duration" p.511. For more details see Mortensen (1977), Fredriksson and Holmlund (2001), Coles and Masters (2007, 2004) among others.

[3]As Deaton (1991) mentioned "assets play the role of a buffer stock, and the consumer saves and dissaves in order to smooth consumption in the face of income uncertainty" p.1223

Other authors analyze the option of severance payments and dismissal delays as an optimal contract offered by firms in a framework without savings. [See Pissarides (2001, 2010), Blanchard and Tirole (2008), among others]. Pissarides (2001, 2010) found that when workers cannot insure against the risk of becoming unemployed, severance compensation and dismissal delays provide a second-best alternative. The author demonstrated that severance payments can provide perfect insurance against the uncertainty of the duration of a job and dismissal delays provide insurance against the uncertainty time that a worker spends unemployed.[4] In the same fashion to Pissarides (2001, 2010), Blanchard and Tirole (2008) argued that unemployment insurance and employment protection are tightly linked. The authors found that in a simple model with risk averse workers a way to achieve the optimal policy is for the government to pay unemployment benefits to insure workers and a layoff tax payed by the firms, so they internalize the cost of unemployment and take an efficient layoff decision. Layoff taxes or severance payments help to internalize the layoff decisions by firms, which is not internalized by the unemployment benefit.

Building upon the above findings, this paper contributes to the analyses of the effect of severance payments in the labour market with a formal and informal sector and savings. This work resembles that of Coles (2006), who introduces severance payments to the optimal mechanism design problem with hidden savings and constant UI. He showed that a constant benefit path combined with a lump sum layoff payment, yields welfare outcomes very close to full information benchmark. Following Coles (2006) I extend these results to an economy with two sectors; formal and informal. My analysis of the informal sector shares some similarities with the model of savings and short-term employment used by Browning et al. (2007). Assuming that workers are unable to borrow beyond a debt ceiling (the natural debt ceiling-NDC), Browning et al. (2007) analyze how job seekers might use short term employment in undesirable jobs as a way to finance consumption during subsequent unemployment search for a good job. As in my model, their study is a partial equilibrium problem, where they study the conditions for which agents move between short term employment and unemployment search. Apart from the NDC, the turnover cost is a key feature in their model. Low turnover costs generate rapid movements between high wage job search and low wage employment. In my model, workers take casual jobs in the informal sector to relieve the cost of the binding liquidity constraint. Under this framework, I focus on the analysis of the optimal policy.

My work corroborates the results of Acemoglu and Shimer (1999a,b), who developed a model with savings and two sectors (low and high productivity), where unemployment insurance increases labour productivity by encouraging workers to seek higher productivity jobs. A moderate level of unemployment insurance encourages workers to take on more risk, including jobs that are harder to get, but probably more productive. Then, an improvement in the composition of jobs, increases the total output and the total welfare. A moderate unemployment insurance not only creates risk-sharing benefits but also increases the general level of output in the economy. In this model I show that social security payment and severance payment increase the number of formal workers (improving the composition of workers type in the economy), therefore the level of output in the economy increases.

This paper is divided into six sections: in the second section I present the model without job destruction shocks and identify the optimal consumption/search/non-participant strategy of

[4]The author argues that rigorous econometric testing has not been able to conclude that employment protection has a big impact on labour market performance. The consensus is that employment protection reduces labour turnover and job reallocation but has no appreciable influence on mean unemployment rates. However it has heterogeneous effects for different groups (marginal benefits for male workers vs. youths, women and older men) on the labour market and for different industries. [OECD (1999), Skedinger (2010), Lazear (1990).

workers when employed and unemployed[5]. In the third section I extend the model with job destruction shocks and include the following policies: a social security payment, b,[6] an income tax, π, and a severance payment, S. In the fourth section I present a numerical solution of this model without the optimal policy. In the fifth section I present the numerical solution of the model solving for the optimal policy that maximizes the objective function of the social planner. Finally, in the last section I conclude the paper summarizing its finding.

2. Model

Time is continuous with $t \in [0, \infty)$. Workers die at rate $\mu > 0$, where μ also describes the inflow of new entrants. For simplicity all have the same subjective discount rate $\rho > 0$. There are two sectors: formal and informal. In the formal sector a worker earns the exogenous wage w. In the informal sector a worker earns the wage w_I. Although there are matching frictions in the formal sector, I assume there are none in the informal sector, following Albrecht et al. (2009) the informal sector is assumed to be unregulated self-employment.

A representative worker has asset $A \geq 0$ and can be in one of two states: unemployed ($s = U$) and employed in the formal sector ($s = E$). Given the state at every point in time, the worker chooses consumption $c \geq 0$ where $u(c)$ describes flow utility of consumption. $u(.)$ is increasing, continuously differentiable and strictly concave with $u(0) = 0$. I assume a perfect annuity market in which the worker enjoys the rate of return $r = \rho + \mu$ to savings and the worker's assets revert to the bank on death. The liquidity constraint $A \geq 0$ implies that banks do not lend to those with no assets.

When unemployed the worker has three possible options. One option is to be non-participant, in which case he/she enjoys additional flow utility $u_B > 0$[7], but the only income stream is asset income rA. A second option is to seek employment in the formal sector in which case he/she enjoys asset income rA plus flow benefit $b > 0$ from the government. The job seeker can, however, split his/her time between job search and employment in the informal sector and this split is unobserved by the government. For simplicity I allow just two possibilities; either (i) search full time for employment and he/she then receives a formal job offer at rate λ, agents who choose this action will be referred as "formal searchers" or (ii) take casual employment which yields additional income w_I in the informal sector but the worker then only receives a formal job offer at rate $\varphi\lambda$ with $\varphi < 1$, agents who choose this action will be referred as "informal searchers". A formal job offer implies the worker becomes employed on wage w where I assume $w > w_I + b$ so that a gain to trade exists. The worker can quit costlessly from employment and so

[5]Following Coles (2006), Browning et al. (2007), Danforth (1979) among others, in this section I assume that jobs last forever. This assumption allows me to obtain analytical results. However, authors like Lentz and Tranaes (2005) introduce a wealth lottery to ensure concavity of the value function and obtain analytical results without further assumptions.

[6]I refer to b as social security payment instead of unemployment insurance, because the government cannot observe between those who are searching full time for a formal job and those who are employed in the informal sector while searching for a formal job, then as Immervoll (2012) reports: the purpose of assistance benefits [or social security payment] is the provision of a minimum level of resources during unemployment rather than insurance against lost earnings p.4.

[7]u_B can be interpreted as the flow utility of being in the beach. I assume u_B strictly positive to be able to differentiate between two types of non-participant workers. Those workers who decide to retire after participating in the labor market and accumulate certain level of assets, which I call retired and those who never participate in the labor market, which are discribed in Florez (2014) as pure informal workers. For an easy exposition in this paper (assuming $w > w_I + b$) I will focus just on those whoare retired.

become unemployed. Switching from unemployment to employment, however, requires search. For simplicity there are no job destruction shocks.

Let $V_s(A)$ denote the worker's expected discounted lifetime payoff in state $s = U, E$ with asset A. The Hamilton/Jacobi/Bellman that describes the value of being unemployed with asset $A \geq 0$ is:

$$rV_U(A) = \max \begin{bmatrix} \max_{c\geq 0}\left[u(c) + u_B + \frac{dV_U}{dA}\left[rA - c\right]\right], \\ \max_{c\geq 0}\left[u(c) + \frac{dV_U}{dA}\left[rA + b + w_I - c\right] + \varphi\lambda \max[V_E(A) - V_U(A), 0]\right], \\ \max_{c\geq 0}\left[u(c) + \frac{dV_U}{dA}\left[rA + b - c\right] + \lambda \max[V_E(A) - V_U(A), 0]\right]. \end{bmatrix}$$

The first line describes the maximized flow payoff by being non-participant: the worker enjoys additional flow utility u_B but only receives income rA, noting that $dA/dt = rA - c$ and so $\frac{dV^U}{dA}\left[rA - c\right]$ describes the worker's capital gain through the optimal savings strategy. The second line describes the maximized payoff by taking casual employment in the informal sector and finding a formal work at rate $\phi\lambda$ where $dA/dt = rA + b + w_I - c$ in this case. The last line describes the flow value of full time job search. For each A, the optimal strategy is the one which yields the highest flow return.

When $V_E(A) > V_U(A)$; i.e. while it is (strictly) suboptimal to quit into unemployment, the Hamilton/Jacobi/Bellman describing the value of being employed with asset $A \geq 0$ is:

$$rV_E(A) = \max_{c\geq 0}\left[u(c) + \frac{dV_E}{dA}\left[rA + w - c\right]\right].$$

If $V_E(A) < V_U(A)$, the worker quits into unemployment. The following part of this section identifies the optimal consumption/search/non-participant strategy by solving the above Bellman equations subject to $A \geq 0$.[8].

2.1. Optimal consumption when employed.

While $V_E(A) > V_U(A)$, the value of being employed is given by the Bellman equation:

$$rV_E(A) = \max_{c\geq 0}\left[u(c) + \frac{dV_E}{dA}\left[rA + w - c\right]\right]. \tag{1}$$

As $u(.)$ is concave, the optimal consumption choice, denoted $c_E(A)$, solves the first order condition

$$u'(c_E) = \frac{dV_E(A)}{dA}.$$

Let

$$\dot{c} = \frac{dc_E}{dA}\left[rA + w - c_E\right]$$

[8]The liquidity constraint assumption is more restrictive than the natural debt ceiling (NDC) assumption, which is used by other authors (see Browning et al. (2007)). The NDC implies that the worker cannot have debt in excess of the maximum that can be serviced in any attainable state. In general the NDC is assumed to be b/r.However assuming that financial markets are incomplete and in particular that borrowing is limited especially for those which low income, it seems ad-hoc to assume that banks are willing to offer a loan to unemployed workers with no collateral. Then the liquidity constraint assumption is more appropriate in this case, especially when we have in mind the case of developing economies with informal sector where the credit is limited.

denote how consumption changes over time while employed. Totally differentiating equation (1) wrt t and the Envelope Theorem establishes $\dot{c} = 0$; i.e. the optimal consumption smoothing strategy implies consumption does not change over time while employed. One possibly optimal strategy is to "work forever" and consume permanent income $c = rA + w$. The expected lifetime payoff to this strategy is

$$\Pi^E(A) = \frac{u(rA + w)}{r}.$$

Alternatively the worker might quit into permanent non-participation. As a permanently retired worker optimally consumes permanent income $c = rA$, this strategy instead yields expected lifetime payoff

$$\Pi^R(A) = \frac{u(rA) + u_B}{r}.$$

The Inada condition $\lim_{c\to\infty} u'(c) = 0$ implies the "retire" strategy dominates the "work forever" strategy for A sufficiently large (e.g. for A satisfying $u'(rA) < u_B/w$).

I identify the solution to the above Bellman equations by adopting a guess and verify approach: I guess the optimal strategy has Property I below, I solve for the corresponding value functions and, in the proofs of Propositions (1) and (2), I verify the solution indeed yields this property.

Property I: if for some A it is optimal that the worker quits into unemployment, the worker retires with payoff $\Pi^R(A)$.

The important restriction implied by Property I is that quitting into unemployment, say, to take a gap year before seeking re-employment, is never an optimal strategy. Of course, establishing property I requires characterizing the optimal strategy of the worker when unemployed. I do that in the next section.

At this stage I take Property I as given and solve for optimal behavior while employed. Consider then the following (possibly optimal) savings plan: suppose an employed worker with asset A consumes $c < rA + w$ and so wealth increases over time. Furthermore, suppose once assets $A = A^R$ the worker retires and thereafter consumes $c^* = rA^R$. In any optimal savings plan, optimal consumption smoothing implies consumption does not change over time: thus the worker always consumes $c = c^*$ during this plan. Note then that $A(.)$ evolves according to

$$\dot{A} = [rA + w - c^*],$$

where $c^* = rA^R$.

Now let $\tau(A, A^R)$ denote time till retirement; i.e. given current asset A, τ describes the time it will take to accumulate wealth A^R. Solving the above linear differential equation implies

$$\tau(A, A^R) = \frac{1}{r} \log \frac{w}{w + r[A - A^R]}.$$

The lifetime payoff yielded by this savings plan is thus:

$$\Pi(A, A^R) = [1 - e^{-r\tau(A,A^R)}]\frac{u(c^*)}{r} + e^{-r\tau(A,A^R)}\frac{u(c^*) + u_B}{r} \tag{2}$$

where the first term describes the discounted utility obtained while saving for retirement, the second describes the discounted payoff by retiring in τ years time. The optimal savings plan in addition chooses A^R. The necessary condition for optimal A^R yields:

$$\frac{\partial \Pi(A, A^R)}{\partial A^R} = u'(c^*) - e^{-r\tau(A,A^R)} u_B \frac{\partial \tau(A, A^R)}{\partial A^R} = 0.$$

Calculating $\partial\tau/\partial A^R$ and simplifying using the above conditions yields the necessary condition for optimality:

$$u'(c^*) = \frac{u_B}{w}. \tag{3}$$

(3) is a transversality condition. At the point along the optimal consumption path where the worker switches into retirement, the optimal strategy requires $w dV_u/dA = u_B$ so that the worker is just indifferent to continuing to accumulate further assets or switch to non-participation.

An optimal savings plan thus consumes $c^* = c^*(w/u_B)$, identified as the solution to equation (3), with retirement asset level $A^R = c^*(w/u_B)/r$. Let $\overline{A}_E = c^*(w/u_B)/r$ denote this asset level. Substituting out optimal c^* and $A^R = \overline{A}_E$ in equation (2) yields the payoff to the optimal savings plan, which I denote $\Pi^P(A)$. Differentiating with respect to A and some algebra further implies

$$\begin{aligned} \frac{d\Pi^P(A)}{dA} &= -\left[\frac{u_B}{r}\right] r e^{-r\tau(A,\overline{A}_E)} \frac{\partial \tau(A, \overline{A}_E)}{\partial A} \\ &= \frac{u_B}{w} = u'(c^*). \end{aligned}$$

Hence $\Pi^P(A)$ is linear with slope $u'(c^*)$. As this savings strategy is not feasible when $A < (c^* - w)/r$ (as assets A then decline over time), define $\underline{A}_E = (c^* - w)/r$. Finally note that $\Pi^P(A) = \Pi^E(A)$ at $A = \underline{A}_E$ (as $\tau = \infty$) and $\Pi^P(A) = \Pi^R(A)$ at $A = \overline{A}_E$ (as $\tau = 0$). I can now describe the optimal consumption and retirement strategy while employed.

Proposition 1. *Conditional on Property I, the solution to the Bellman equation (1) is:*
(i) for $A \le \underline{A}_E$, $V_E(A) = \Pi^E(A)$ with $c^E(A) = w + rA$ (the optimal plan is to work forever);
(ii) for $A \in (\underline{A}_E, \overline{A}_E)$, $V_E(A) = \Pi^P(A)$ and $c^E(A) = c^*(w/u_B)$ (the savings plan is optimal);
(iii) for $A \ge \overline{A}_E$ the worker permanently retires and $V_E(A) = \Pi^R(A)$.

Proof. Given this solution for $V^E(.)$, inspection establishes it is an increasing, concave and continuously differentiable function. Furthermore $c^E(.)$ satisfies the necessary conditions for optimality. As payoffs are bounded below, the Principle of Unimprovability establishes this consumption strategy describes the policy optimum [Kreps (1990)]. This completes the proof of Proposition (1). □

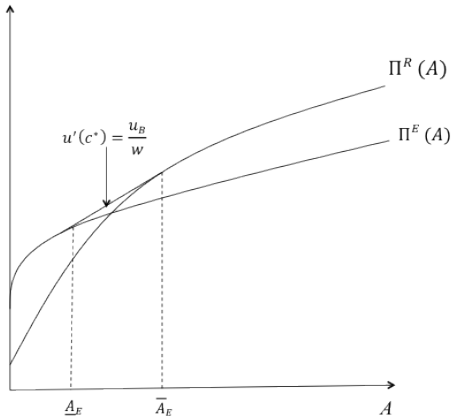
Figure (1) describes $V_E(.)$. Of course this analysis assumes Property I: that if it is ever optimal to quit from employment the worker permanently enters non-participation. I now establish this property by considering optimal behavior when unemployed.

2.2. Optimal search and consumption when unemployed.

Consider now the Bellman equation:

$$rV_U(A) = \max \begin{bmatrix} \max_{c\ge 0}\left[u(c) + u_B + \frac{dV_U}{dA}[rA - c]\right], \\ \max_{c\ge 0}\left[u(c) + \frac{dV_U}{dA}[rA + b + w_I - c] + \varphi\lambda \max[V_E(A) - V_U(A), 0]\right], \\ \max_{c\ge 0}\left[u(c) + \frac{dV_U}{dA}[rA + b - c] + \lambda \max[V_E(A) - V_U(A), 0]\right], \end{bmatrix} \tag{4}$$

Figure 1: $V_E(A)$



subject to the constraint $A \geq 0$, with $V_E(.)$ as described by Proposition (1). As the case $u_B \geq u(w)$ is uninteresting (non-participation is always optimal) the following assumes $u_B < u(w)$. Given that, the assumed properties of $u(.)$ imply $\underline{A}_E > 0$. Hence, by Proposition (1), $V_E(0) = u(w)/r$; i.e. the employed worker with $A = 0$ works forever.

The Bellman equation implies that optimal consumption $c_U(A)$ is the solution to

$$u'(c) = \frac{dV_U(A)}{dA}.$$

To characterize the optimal job search strategy, it is useful to define the flow surplus functions:

$$S^C(A) = [b + w_I]\, u'(c_U) + \varphi\lambda[V_E(A) - V_U(A)]$$

$$S^F(A) = bu'(c_U) + \lambda[V_E(A) - V_U(A)]$$

The Bellman equation implies that optimal behavior is:
i) non-participation while $u_B > \max[S^F(A), S^C(A)]$;
ii) job search with casual employment in the informal sector while $S^C(A) > \max[S^F(A), u_B]$;
ii) full time job search while $S^F(A) > \max[S^C(A), u_B]$.
It is also useful to define $\Delta(A) = S^C(A) - S^F(A)$, where the above implies

$$\Delta(A) = w_I u'(c_U) - \lambda(1 - \phi)[V_E(A) - V_U(A)].$$

$\Delta(A) > 0$ implies casual employment in the informal sector dominates full time job search; i.e. the marginal value of additional casual earnings w_I more than compensates for the fall in job search efficiency.

I characterize the optimal strategy using backward induction from $A = 0$. Note that over each instant of time $dt > 0$, the worker might spend fraction θ of that period in casual employment in the informal sector (earning w_I) and $1 - \theta$ in full time job search. By mixing these two strategies with weight $\theta \in [0, 1]$, the job seeker at $A = 0$ enjoys income $b + \theta w_I$ and job-offer rate

$[\theta\varphi + 1 - \theta]\lambda$. Thus, anticipating the liquidity constraint $A \geq 0$ binds, optimal search behavior at $A = 0$ implies:

$$V_U(0) = \max\left[\frac{u_B}{r}, \max_{\theta\in[0,1]} \frac{u(b + \theta w_I) + \frac{[\theta\varphi+1-\theta]\lambda}{r} u(w)}{r + [\theta\varphi + 1 - \theta]\lambda}\right], \tag{5}$$

where the first term describes the payoff by being non-participant should this strategy be optimal.

I begin with the easiest case, when

$$\frac{u_B}{r} > \max_{\theta\in[0,1]} \frac{u(b + \theta w_I) + \frac{[\theta\varphi+1-\theta]\lambda}{r} u(w)}{r + [\theta\varphi + 1 - \theta]\lambda} \tag{6}$$

and so being non-participant is optimal at $A = 0$. The proof of Proposition (2) now uses backward induction to establish that non-participation is optimal for all $A > 0$. The optimal consumption strategy then follows straightforwardly.

Proposition 2. *Given u_B satisfying (6), the solution to the Bellman equations imply:*

(i) while unemployed, the worker is always non-participant and consumes permanent income $c_U(A) = rA$ with corresponding value $V_U(A) = [u(rA) + u_B]/r$;

(ii) while employed, behavior is as described in Proposition (1).

Proof. While unemployed and for any $A \geq 0$, being non-participant is strictly optimal while

$$u_B > \max[S^F(A), S^C(A)].$$

Given $V_U(A)$, $V_E(A)$ as defined in Proposition (2), noting that $c_U(A) = rA \leq c_E(A)$, differentiation establishes both $S^F(.)$ and $S^C(.)$ are strictly decreasing in A. As equation (6) ensures this inequality holds at $A = 0$, it follows that being non-participant is always optimal. Inspection also establishes that $c^U = rA$ satisfies the necessary condition for optimality. As Property I is satisfied (the worker who quits enters permanent retirement), Proposition (1) also describes the worker's optimal consumption and job search strategy while employed. This concludes the proof of Proposition (2). $\square$

The rest of the section assumes

$$\frac{u_B}{r} < \max_{\theta\in[0,1]} \frac{u(b + \theta w_I) + \frac{[\theta\varphi+1-\theta]\lambda}{r} u(w)}{r + [\theta\varphi + 1 - \theta]\lambda} \tag{7}$$

so that job search is always optimal at $A = 0$. Let θ^* denote the optimal choice of θ; *i.e.*

$$\theta^* = \arg\max_{\theta\in[0,1]} \frac{ru(b + \theta w_I) + [1 - \theta(1 - \varphi)]\lambda u(w)}{r + [1 - \theta(1 - \varphi)]\lambda}.$$

Lemma (3) now describes θ^*.

Lemma 3. *[Initial values for $c_U(0)$ and $V_U(0)$].*

For u_B satisfying (7), there exist two wage levels w^L,$w^H > b + w_I$ such that

(i) for wage $w \geq w^H$ the worker chooses $\theta^* = 0$ [full time job search] and consumes $c_U(0) = b$, where w^H solves:

$$\frac{(1 - \varphi)\lambda}{r + \lambda}\left[u(w^H) - u(b)\right] = w_I u'(b);$$

(ii) for $w \leq w^L$ the worker chooses $\theta^* = 1$ [casual employment in the informal sector] and consumes $c_U(0) = b + w_I$, where w^L solves

$$\frac{(1-\varphi)\lambda}{r+\varphi\lambda}\left[u(w^L) - u(b+w_I)\right] = w_I u'(b+w_I);$$

(iii) for $w \in (w^L, w^H)$ the worker chooses $\theta^* \in (0,1)$ and consumes $c_U(0) = b + \theta^* w_I$ where

$$\frac{(1-\varphi)\lambda}{r+[1-\theta^*(1-\varphi)]\lambda}\left[u(w) - u(b+\theta^* w_I)\right] = w_I u'(b+\theta^* w_I). \tag{8}$$

Given this solution for θ^*,

$$V_U(0) = \frac{u(b+\theta^* w_I) + \frac{[1-\theta^*(1-\varphi)]\lambda}{r} u(w)}{r+[1-\theta^*(1-\varphi)]\lambda}. \tag{9}$$

Proof. Concavity of $u(.)$ implies $w^H > w^L$ as defined in Lemma (3). Equation (8) describes the necessary condition for optimality when θ^* is an interior solution. Wage $w \geq w^H$ implies that the corner solution $\theta^* = 0$ is optimal, while $w \leq w^L$ implies that the corner solution $\theta^* = 1$ is optimal. Given this characterization of θ^*, Equation (5) implies equation (9). □

$w > w^H$ implies the return to search is sufficiently high so that the liquidity constrained worker prefers full time search to taking casual employment in the informal sector. In the intermediate wage range (w^L, w^H), the lower return to search implies that the worker partially substitutes casual employment in the informal sector to relieve the cost of the binding liquidity constraint. Casual employment in the informal sector is optimal only when the wage is sufficiently low; i.e. for $w < w^L$.[9]

Given this characterization of optimal behavior at $A = 0$, and thus optimal consumption, I now use backward induction to characterize the optimal job search and consumption strategy for all $A \geq 0$.

2.2.1. The optimal search and consumption strategy while $S^C(A) > \max[S^F(A), u_B]$.

While $S^C(A) > \max[S^F(A), u_B]$ the worker optimally chooses casual employment in the informal sector. Let

$$\dot{c} = \frac{dc_U}{dA}\left[rA + b + w_I - c_U\right]$$

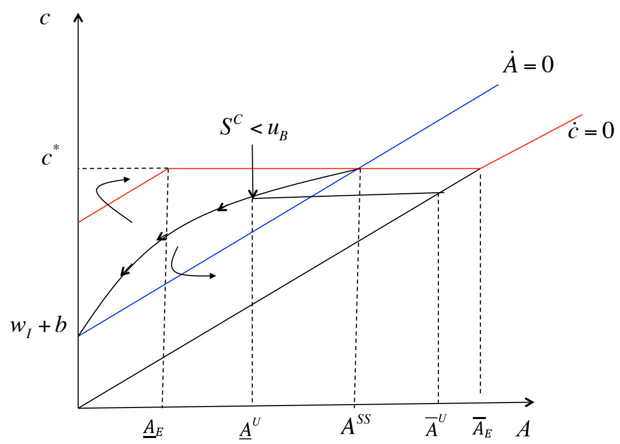
denote how consumption optimally changes over time during this phase. Differentiating equation (4) wrt t and the Envelope Theorem establishes (A, c) evolve according to the pair of differential equations

$$\begin{aligned}
\dot{c} &= \frac{\varphi\lambda[u'(c_E) - u'(c)]}{-u''(c)} \\
\dot{A} &= rA + b + w_I - c,
\end{aligned}$$

where $c_E(.)$ is given by Proposition (1). Figure (2) describes the corresponding phase diagram for the case that $w < w^L$ which, by Lemma (3), implies casual employment in the informal sector is optimal at $A = 0$.

[9]Notice these results are valid for $b > 0$. We assume $b > 0$ to ensure a minimum level of consumption for those who are liquidity constrained. In the case that $b = 0$ full time job search strategy with $\theta^* = 0$ is never optimal.

Figure 2: Phase diagram for the case that $w < w^L$



Note there is a unique steady state at (A^{SS}, c^*) where $rA^{SS} = c^* - b - w_I$ and it is easy to show it is an unstable node (see Appendix A). Thus while $S^C(A) > \max[S^F(A), u_B]$, backward induction implies $c_U(.)$ lies along a path which originates from the steady state (A^{SS}, c^*) as depicted in Figure (2), with initial value $c_U(0) = b + w_I$ at $A = 0$. Note that any such path implies $c_U \in (b + w_I + rA, c^E(A))$ for all $A \in (0, A^{SS})$. Thus during this phase, optimal (c_U, V_U) are jointly determined by

$$\begin{aligned}
\frac{dc_U}{dA} &= \frac{\phi\lambda[u'(c_U) - u'(c_E)]}{[-u''(c_U)]\,[c_U - rA - b - w_I]} \\
\frac{dV_U}{dA} &= u'(c_U(A))
\end{aligned}$$

with initial value $c_U = b + w_I$ and $V_U(0)$ given by (9) at $A = 0$. Given (c_E, V_E) described in Proposition (1), this path for (c_U, V_U) also determines $S^C(.)$ and $S^F(.)$.

Thus while $S^C(A) > \max[S^F(A), u_B]$, I use the above conditions to backward induce (c_U, V_U). This phase ends when either $S^C(A) = S^F(A)$ and the worker switches to full time job search, or when $S^C(A) = u_B$ and the worker switches to non-participation. Whenever any such switch occurs, say at A', the current values $(c_U(A'), V_U(A'))$ yield the start values for the next phase beginning at $A = A'$. I now consider the optimal search and consumption strategy while $S^F(A) > \max[S^C(A), u_B]$.

2.2.2. The optimal search and consumption strategy while $S^F(A) > \max[S^C(A), u_B]$.

While $S^F(A) > \max[S^C(A), u_B]$ the worker optimally chooses full time job search. Let

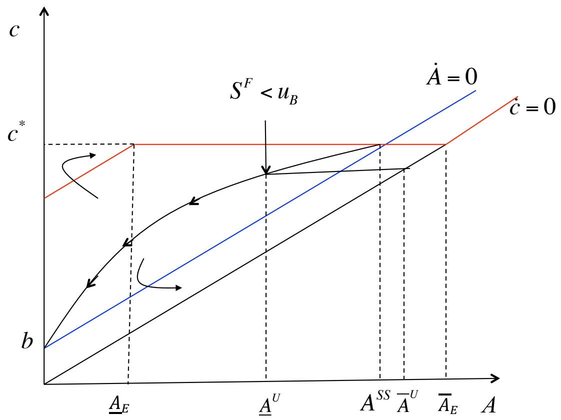
$$\dot{c} = \frac{dc_U}{dA}\,[rA + b - c_U]$$

denote how consumption optimally changes over time during this phase. Differentiating equation (4) wrt t and the Envelope Theorem establishes (A, c) evolve according to the pair of differential equations

$$\begin{aligned} \dot{c} &= \frac{\lambda[u'(c_E) - u'(c)]}{-u''(c)} \\ \dot{A} &= rA + b - c, \end{aligned}$$

where $c_E(.)$ is given by Proposition (1). Figure (3) describes the corresponding phase diagram for the case that $w > w^H$ which, by Lemma (3), implies full time job search is optimal at $A = 0$.

Figure 3: Phase diagram for the case that $w > w^H$



Note there is a unique steady state at (A^{SS}, c^*) where $rA^{SS} = c^* - b$ and it is easy to show it is an unstable node. Thus while $S^F(A) > \max[S^C(A), u_B]$, backward induction implies $c_U(.)$ lies along a path which originates from the steady state (A^{SS}, c^*) as depicted in Figure (3), with initial value $c_U(0) = b$ at $A = 0$. Note that any such path implies $c_U \in (b + rA, c^E(A))$ for all $A \in (0, A^{SS})$. Hence during this phase, optimal (c_U, V_U) are jointly determined by

$$\begin{aligned} \frac{dc_U}{dA} &= \frac{\lambda[u'(c_U) - u'(c_E)]}{[-u''(c_U)]\,[c_U - rA - b]} \\ \frac{dV_U}{dA} &= u'(c_U(A)) \end{aligned}$$

with $c_U(0) = b$ and $V_U(0)$ given by equation (9). Given $V_E(.)$ described in Proposition (1), this path for (c_U, V_U) also determines $S^C(.)$ and $S^F(.)$. While $S^F(A) > \max[S^C(A), u_B]$, I use the above conditions to backward induce (c_U, V_U). This phase ends when either $S^F(A) = S^C(A)$ and the worker switches to causal employment, or when $S^F(A) = u_B$ and the worker switches to non-participation. Whenever any such switch occurs, say at A', the current pair $(c_U(A'), V_U(A'))$ yield the start values for the next regime at $A = A'$.

2.2.3. Optimal regime switching.

Recall that $\Delta(A) = S^C(A) - S^F(A)$. Suppose the optimal job search strategy is full time job search for $A \in [0, A')$ but this switches to casual employment in the informal sector at A'; i.e. $\Delta(A) \le 0$ for $A \in [0, A')$ and $\Delta(A') = 0$. Figure (2) depicts the phase diagram for the continuation dynamics $A \ge A'$ when casual employment in the informal sector is optimal. The initial phase $A \in [0, A')$ yields starting values $(c_U(A'), V_U(A'))$. I now show that whenever any such switch occurs, optimality necessarily implies $c_U \in (b + w_I + rA', c^E(A'))$. Thus as drawn in Figure (2), backward induction implies the continuation path $c_U(.)$ is a path which originates from the steady state (A^{SS}, c^*).

Suppose then a switch occurs at A' where $\Delta(A) \le 0$ for $A = (A')^-$ and $\Delta(A) \ge 0$ for $A = (A')^+$. Differentiation implies

$$\frac{d\Delta}{dA} = w_I u''(c_U)\frac{dc_U}{dA} - \lambda(1-\varphi)[u'(c_E(A)) - u'(c_U(A))].$$

While full-time job search is optimal (for $A < A'$), the above establishes:

$$\frac{dc_U}{dA} = \frac{\lambda[u(c_U) - u'(c_E)]}{[-u''(c_U)]\,[c_U - rA - b]}.$$

Substituting out dc_U/dA in the previous expression and simplifying implies

$$\frac{d\Delta}{dA} = \frac{\lambda(1-\varphi)[u'(c_U) - u'(c_E)]}{c_U - rA - b}\left[c_U - rA - b - \frac{w_I}{1-\varphi}\right]$$

Clearly optimality of the regime switch requires $d\Delta/dA \ge 0$ at $A = A'$. Anticipating the backward induction argument presented in Theorem (4) below, where $c_U \in (b + rA', c^E(A'))$ during any full time search phase, then $d\Delta/dA \ge 0$ at A' holds if and only if:

$$c_U(A') \ge rA' + b + \frac{w_I}{1-\varphi}.$$

As this guarantees $c_U(A') \in (rA' + b + w_I, c_E(A'))$, the continuation path remains a path which originates from the steady state (A^{SS}, c^*) as drawn in Figure (2).

Suppose instead a switch occurs at A' where $\Delta(A) \ge 0$ for $A = (A')^-$ and $\Delta(A) \le 0$ for $A = (A')^+$. Anticipating the backward induction argument presented in Theorem (4) below, where $c_U(A') \in (b + w_I + rA', c^E(A'))$ during any casual employment phase, a switch at A' to full time search phase automatically implies $c_U \in (b + rA', c^E(A'))$. Thus the continuation path remains a path which originates from the steady state (A^{SS}, c^*) as drawn in Figure (3).

I now have enough information to fully characterize the optimal job search and consumption strategies.

Theorem 4. *For u_B satisfying (7), the solution to the Bellman equations imply two asset levels $\underline{A}^U < \overline{A}^U$ such that:*

(i) while unemployed with $A \in [0, \underline{A}^U)$, job search is optimal where $c_U(.), V_U(.)$ solve the differential equations:

$$\begin{aligned}
\frac{dc_U}{dA} &= \begin{bmatrix} \frac{\phi\lambda[u'(c_U)-u'(c_E)]}{[-u''(c_U)][c_U-rA-b-w_I]} \; while\ S^C(A) > S^F(A) \\ \frac{\lambda[u'(c_U)-u'(c_E)]}{[-u''(c_U)][c_U-rA-b]} \; while\ S^F(A) > S^C(A) \end{bmatrix};\\
\frac{dV_U}{dA} &= u'(c_U(A))
\end{aligned}$$

with initial values $c_U(0), V_U(0)$ given by Lemma (3). Furthermore the worker chooses

$$\begin{aligned} \textit{casual employment in the informal sector while } S^C(A) &> S^F(A) \\ \textit{full time job search while } S^F(A) &> S^C(A). \end{aligned}$$

This phase ends at $A = \underline{A}^U < A^{SS}$ where $u_B = \max[S^C(A), S^F(A)]$.

(ii) for $A \in (\underline{A}^U, \overline{A}^U)$ the worker chooses non-participation and consumes $c_U(A) = c^H$ where $c^H = c_U(\underline{A}^U)$.

(iii) for $A \geq \overline{A}^U$ where $\overline{A}^U = c^H/r$, the worker chooses non-participation and permanently retires with consumption $c_U(A) = rA$.

(iv) $V^E(.)$ is as described in Proposition (1).

Proof. The Theorem holds by backward induction from $A = 0$. Equation (7) implies $u_B < \max[S^C(A), S^F(A)]$ at $A = 0$ and so either full time or casual employment in the informal sector is optimal. Lemma (3) describes the initial values for $c_U(0), V_u(0)$. During this phase, the above has established (c_U, V_U) evolve according to Theorem 4(i) where optimal regime switching ensures $c_U \in (b + rA', c^E(A'))$ whenever full time job search is optimal, and $c_U(A') \in (b + w_I + rA', c^E(A'))$ during any casual employment phase. Thus regardless of which job search strategy is currently optimal, the (A, c) dynamics converge to the corresponding steady state (A^{SS}, c^*) as depicted in Figures (2) and (3).

Now along this optimal consumption path, job search only dominates non-participation while $u_B < \max[S^C(A), S^F(A)]$. But both $S^C(A), S^F(A)$ are strictly decreasing functions of A along the optimal consumption path. Furthermore a contradiction argument establishes this condition necessarily fails at either steady state(s) (A^{SS}, c^*). Thus, there exists a unique asset level, denoted $\underline{A}^U < A^{SS}$, where $u_B = \max[S^C(A), S^F(A)]$. Let $c^H = c_U(\underline{A}^U)$ at this asset level and note $c^H < c^*$.

For $A \geq \underline{A}^U$ the worker is non-participant. Define $\overline{A}^U$ where $r\overline{A}^U = c^H$. For $A \in (\underline{A}^U, \overline{A}^U)$, the worker consumes $c_U = c^H$ and assets decline over time. Once assets $A = \underline{A}^U$, the worker switches from non-participation to job search and uses the consumption and job search strategy described above. As $\dot{c} = 0$ over this phase, it describes the optimal consumption plan. For $A \geq \overline{A}^U$, consumption $c = c^H$ would instead imply assets increase over time. For such A, the optimal consumption strategy is instead to permanently retire and consume $c_U(A) = rA$.

All that remains is to show Property I is satisfied: that should the worker ever optimally quit from employment, the worker optimally quits into permanent retirement. It is sufficient to demonstrate the above solution implies $V_E(A) > V_U(A)$ for all $A \leq \overline{A}^E$. Now as $c^H < c^*$, it follows that $\overline{A}^U = c^H/r$ is strictly less than $\overline{A}^E = c^*/r$. Thus $V_E = V_U = \Pi^R(A)$ at $A = \overline{A}^E$. Now by inspection, the above solutions imply $c_U(A) < c_E(A)$ for all $A < \overline{A}^E$. As this implies $dV_E/dA < dV_U/dA$ for $A < \overline{A}^E$, it immediately implies $V_E(A) > V_U(A)$ for all $A < \overline{A}^E$. Thus I have established Property I holds: the employed worker only ever quits into unemployment when $A \geq \overline{A}^E$ in which case permanent retirement is optimal. This completes the proof of Theorem (4). □

3. Model with job destruction shocks

This section consider the results from the previous section and extends the model assuming that employed workers face job destruction shocks, which occur according to an exogenous Poisson process with parameter δ. In this case I extended the insurance program of the government to $B = \{b, \pi, S\}$. As before the government offers the social security payment b, but now formally employed workers are taxed by the lump sum tax $\pi > 0$, and workers receive the lump sum S when laid off. The following describes the representative agent's bellman equation.

As in the previous section the Hamilton/Jacobi/Bellman that describes the value of being unemployed with assets $A \geq 0$ is:

$$rV_U(A) = \max \begin{bmatrix} \max_{c\geq 0}\left[u(c) + u_B + \frac{dV_U}{dA}\left[rA - c\right]\right], \\ \max_{c\geq 0}\left[u(c) + \frac{dV_U}{dA}\left[rA + b + w_I - c\right] + \varphi\lambda \max[V_E(A) - V_U(A), 0]\right], \\ \max_{c\geq 0}\left[u(c) + \frac{dV_U}{dA}\left[rA + b - c\right] + \lambda \max[V_E(A) - V_U(A), 0]\right]. \end{bmatrix}$$

When $V_E(A) > V_U(A)$; i.e while it is (strictly) suboptimal to quit into unemployment, the Hamilton/Jacobi/Bellman that describes the value of being employed with assets $A \geq 0$ changes to:

$$rV_E(A) = \max_{c\geq 0}\left[u(c) + \frac{dV_E}{dA}\left[rA + w - c\right] + \delta[V_U(A+S) - V_E(A)]\right].$$

The last line describes the expected loss of receiving a shock destruction with probability δ. Given the lump sum tax, the net wage is $w = w^G - \pi$, where w^G denotes the gross wage. Moreover, when a worker loses his job the level of assets increases by the severance payment, S.

Using the results in Theorem (4) I solve the pair of bellman equations numerically by the Value Function Iteration Method. I find the pair of optimal consumption rules $c_U(A)$ and $c_E(A)$ and a pair of asset thresholds $\overline{A}^U$, $\underline{A}^U$, $\overline{A}_E > 0$ where property I holds, it means $\overline{A}_E > \overline{A}^U$ [10].

3.1. Model specification

To solve the bellman equations numerically I need to choose some values for the parameters of the model. The choice of parameters is similar to Coles (2006). I assume that the utility function is a constant relative risk aversion function (CRRA)[11], given by:

$$u(c) = \frac{c^{1-\sigma}}{(1-\sigma)},$$

where σ is the risk aversion parameter. Following Coles (2006) I choose $\sigma = 2.2$. I set the wage $w = 100$ in the formal sector. The wage in the informal sector is set to $w_I = 10$, with $\varphi = 0.5$. Using one year as the reference unit of time, I assume that the expected working lifetime is 50 years, which implies $\mu = 0.02$, and the interest rate $r = 6\%$ per annum. I compare two type of economies: the first economy is a low unemployment economy (LU), with $\lambda = 4$ which implies an average duration of unemployment around 3 months and the second economy is a high unemployment economy (HU) with $\lambda = 1.5$ which implies an average duration of

[10]Given that employed workers are facing the risk of unemployment, the optimal strategy when employed is to save until retirement $\overline{A}_E$.

[11]This is a common function used in the literature see for example: Hopenhayn and Nicolini (1997), Hansen and İmrohoroğlu (1992), Lentz (2009) and Coles (2006) among others. Shimer and Werning (2005) compare the results of an optimal policy with saving using the CRRA and CARA functions. They find that in both cases the optimal policy has quantitatively nearly-constant unemployment subsidies.

unemployment of 8 months. Both economies have a job destruction rate of $\delta = 12.5\%$ which implies an average employment spell of 8 years[12].

Given the optimal $\theta^* \in (0,1)$, from Lemma (3) we know that $\Delta(0) = S^C(0) - S^F(0) = 0$, and given the definition of $\frac{d\Delta}{dA}$, it is easy to show that at $A = 0^+$, $\Delta(A) < 0$, which implies that full time job search is optimal at lower level of assets. In this case I define the level of assets A^F where unemployed workers switch from full time job search to causal employment in the informal sector. Then solving the bellman equations numerically I find the following optimal strategy for the unemployed worker:

1. For level of assets $A < A^F$ the unemployed worker search full time for a formal job, I call him "formal searchers".
2. For level of assets $A^F < A < \underline{A}^U$ the unemployed worker take casual employment in the informal sector, I call him "informal searchers".
3. For level of assets $\underline{A}^U < A < \overline{A}^U$ the unemployed worker decide not to participate in the labour market, I call him "on-holidays".
4. For level of assets $A > \overline{A}^U$ the unemployed worker permanent retires, I call him "retired".

3.2. Steady state conditions

Given the numerical solution of the bellman equations in the previous section, I can find the distribution of assets across employed and unemployed workers. Lets $G^E(A)$ and $G^U(A)$ denote the steady state distribution of assets across employed and unemployed, respectively. Given the results in the previous section, unemployed workers are divided in four groups: "informal searchers", "formal searchers", "on-holiday" and "retired". Let $G^U(A^F)$ define the proportion of "formal searchers", $[G^U(\underline{A}^U) - G^U(A^F)]$ the proportion of "informal searchers", $[G^U(\overline{A}^U) - G^U(\underline{A}^U)]$ the proportion of those who are "on-holiday", and finally $[1 - G^U(\overline{A}^U)]$ the proportion of "retired" workers in the economy.

Let u be the steady state unemployment rate. In steady state the outflow of workers into unemployment should be equal to the inflow. Then outflow of unemployment at any interval of time dt, is given by those workers who die or find a job:

$$u[\mu dt + \lambda G^U(A^F)dt + \varphi\lambda(G^U(\underline{A}^U) - G^U(A^F)dt)],$$

and the inflow into unemployment is given by those who being formally employed lose their job and those who are born:

$$(1-u)\delta dt + \mu dt$$

In the steady state the outflow should be equal to the inflow, then the unemployment rate is given by:

$$u = \frac{\delta + \mu}{(\mu + \varphi\lambda(G^U(\underline{A}^U) - G^U(A^F)) + \lambda G^U(A^F) + \delta)}, \tag{10}$$

[12] For the job destruction rate I use the intermedia value used by Coles (2006). Following Lemma (3) and given the chosen parameters I find the optimal $\theta^* = 0.41$ for LU and $\theta^* = 0.48$ for HU, which implies $w^L < w < w^H$. This numerical exercise solves the model including a linear cost of search, as Coles (2006). The inclusion of this assumption does not change the results presented in the first section of this paper.

Proposition 5. *$G^E(A)$ and $G^U(A)$ are jointly determined in the steady state by the following conditions:*

1) $G^U(A)$ satisfies the following differential equations:

$$G^U(A)(\mu+\lambda)+\frac{dG^U}{dA}(rA-c^U(A)+b)=\frac{\mu+(1-u)G^E(A)\delta}{u},\ \ for\ A\le A^F$$

$$[G^U(A)-G^U(A^F)](\varphi\lambda+\mu)=\left\{\begin{array}{c}\frac{dG^U}{dA}(rA-c^U(A)+b+w_I)\\+\frac{(1-u)[G^E(A)-G^E(A^F)]\delta}{u}\end{array}\right\},\ \ for\ A^F<A\le\underline{A}^U$$

$$[G^U(A)-G^U(\underline{A}^U)]\mu=\left\{\begin{array}{c}\frac{dG^U}{dA}(rA-c^U(A))\\+\frac{(1-u)[G^E(A)-G^E(\underline{A}^U)]\delta}{u}\end{array}\right\},\ \ for\ \underline{A}^U<A\le\overline{A}^U$$

$$\frac{(1-u)[1-G^E(\overline{A}^U)]\delta}{u}=\mu[1-G^U(\overline{A}^U)],\ for\ \overline{A}^U<A\le\overline{A}_E$$

2) $G^E(A)$ satisfies the following differential equations:

$$G^E(A)(\mu+\delta)+\frac{dG^E}{dA}(rA-c^E(A)+w)=\frac{u}{1-u}\lambda G^U(A),\ for\ A\le A^F$$

$$\left\{\begin{array}{c}[G^E(A)-G^E(A^F)](\mu+\delta)\\+\frac{dG^E}{dA}(rA-c^E(A)+w)\end{array}\right\}=\left\{\begin{array}{c}\frac{u}{1-u}\lambda\varphi[G^U(A)-G^U(A^F)]\\+\frac{dG^E}{dA^F}(rA^F-c^E(A^F)+w)\end{array}\right\},\ for\ A^F<A\le\underline{A}^U$$

$$\left\{\begin{array}{c}[G^E(A)-G^E(\underline{A}^U)](\mu+\delta)\\+\frac{dG^E}{dA}(rA-c^E(A)+w)\end{array}\right\}=\left\{\frac{dG^E}{d\underline{A}^U}(r\underline{A}^U-c^E(\underline{A}^U)+w)\right\},\ for\ \underline{A}^U<A\le\overline{A}^U$$

$$[1-G^E(\overline{A}^U)](\mu+\delta)=\ \frac{dG^E}{d\bar{A}^U}(r\bar{A}^U-c^E(\bar{A}^U)+w)\ \ ,\ for\ \overline{A}^U<A\le\overline{A}_E$$

subject to the initial value $G^E(0)=0$.

Proof. See proof in Appendix B □

4. Numerical solution without optimal policy

This section presents the results of the optimal search/consumption/non-participation strategy for the two types of economies. First, I assume severance payment $S=0$ and income tax $\pi=0$. Table (1) presents the results for an economy with low unemployment (LU). When social security payment is $b=25$ the level of asset until an unemployed worker search full time for a formal job is $A^F=4.1$ (years of salary), the level of asset until an unemployed worker takes casual employment in the informal sector is $\underline{A}^U=7$ (years of salary), and the level of asset until an unemployed worker decides not to participate (to take a holidays) is $\overline{A}^U=17.5$ (years of salary). Finally, the level of asset until an employed worker retires is $\overline{A}_E=17.9$ [see Figure (4)]. $\bar{A}E$

represents the average level of assets held by an employed worker and $\bar{AU}$ represents the average level of assets held by an unemployed worker. When $b = 25$, the average level of assets held by an employed worker is 1.1 years of his salary, similar to the average level of assets held by an unemployed worker (1.0 years of salary). Notice that when the social security payment increases the average level of assets held by an employed worker decreases. The intuition behind these results is that the higher the social security payment, the lower the amount of income a worker needs to save in order to protect himself against the unemployment risk, then low social security payments are compensated with higher savings. These results are in line with the empirical results found by Engen and Gruber (2001) who found evidence for the US that suggests that individuals do save less when the unemployment insurance-UI is more generous.

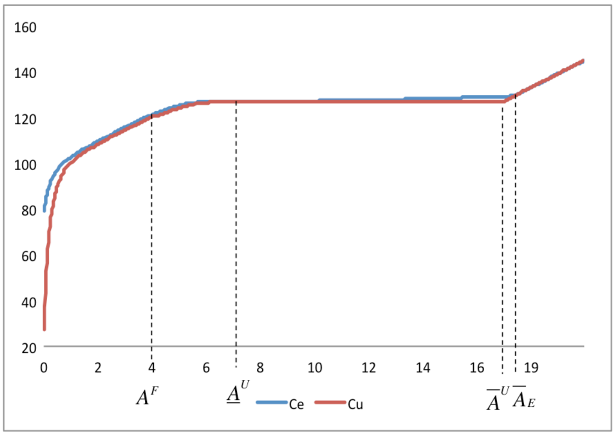
Furthermore, when the social security payment increases the average level of assets held by an unemployed worker increases. Similar results are found by Coles (2006), where the unemployed workers, on average, are wealthier than the employed ones. The reason is because a high social security payment implies a low asset threshold A^F, then as those who are unemployed with $A \leq A^F$ search full time for a formal job and quickly exit unemployment, the pool of unemployed workers is over-represented by relatively wealthy types, those with $A > A^F$ who search for a formal job while working in the informal sector and those who take a "holiday". In this way, when the social security payment increases, the proportion of "formal searchers" given by $G^U(A^F)$, decreases, the proportion of "informal searchers" given by $[G^U(\underline{A}^U) - G^U(A^F)]$ increases, and the proportion of those who are "on-holiday" given by $[G^U(\overline{A}^U) - G^U(\underline{A}^U)]$ increases. This composition effect has implications on the unemployment rate u that increases with the social security payment. Moreover, $G^U(0)$ represents the proportion of unemployed workers who are liquidity constrained, which decreases with a high social security payment. Therefore, a higher social security payment protects workers from being liquidity constrained, but decreases the incentives to search full time for a formal job, decreasing the "formal searchers" and increasing the "informal searchers" and those who are "on-holiday" in the economy (the common well-known moral-hazard problem).

Table 1: Optimal saving and search -LU economy

b	25	30	35	40
LU economy				
A^F	4.1	3.2	2.2	1.2
$\underline{A}^U$	7.0	6.0	5.7	4.7
$\overline{A}^U$	17.5	16	14.4	12.8
$\overline{A}_E$	17.9	16.4	14.8	13.2
$\bar{AE}$	1.1	1.0	0.9	0.9
$\bar{AU}$	1.0	1.5	4.4	7.9
$G^U(A^F)$	0.98	0.93	0.60	0.11
$G^U(\underline{A}^U)$	0.98	0.93	0.62	0.14
$G^U(\overline{A}^U)$	0.99	0.99	0.99	0.99
$G^U(0)$	0.15	0.14	0.09	0.02
u	0.03	0.04	0.05	0.23

Figure (4) describes the optimal consumption for a low unemployment economy with the social security payment $b = 25$. As is expected consumption is increasing with the level of

Figure 4: Optimal consumption



assets, and consumption when employed is higher than consumption when unemployed. When employed, a worker builds up savings to self-insure against job destruction shocks. On the other hand, when unemployed the worker dissaves across time. When his level of assets is between 17.5 and 7 years of salary, the unemployed worker decides to take a "holiday" and not participate in the labor market until his level of assets is lower than 7 years of salary, at which point the worker starts searching for a formal job while working in the informal sector, then when his level of assets is lower than 4.1 years of salary, the unemployed worker stops working in the informal sector and searchs full time for a formal job. Unemployed workers dissave across time until they find a formal job to build up their savings again.

Table (2) on the other hand, represents an economy with high unemployment. As in the previous case, the higher the social security payment b the lower the asset threshold A^F, $\underline{A}^U$ and $\overline{A}^U$. However, in a high unemployment economy for any social security payment b, the proportion of "formal searchers" is lower, while the proportions of "informal searchers" and of those who take a "holiday" are higher compared to the economy with low unemployment. As before the unemployment rate increases with the social security payment b. Notice that these results represent an economy with high unemployment and high informality which is the case of developing economies.

Table (3) presents the results for a high unemployment economy (HU) including the severance payment policy when social security payment is $b = 25$. Notice that in this case the average level of assets held by employed and unemployed workers decreases. Therefore, when the severance payment $S = 30$ (0.3 years of salary), the average level of assets held by an unemployed worker is 5.9 years of salary and the average level of assets held by an employed worker is 1.8 years of salary. Severance payments protect workers against the risk of unemployment, as a result workers do not over-accumulate assets to protect themselves against the unemployment risk. At the same time severance payments increase the incentives to search through the "*re-entitlement effect*" [Mortensen (1977)], therefore with a high severance payment, the proportion of "informal searchers" and those who take a "holiday" in the economy decreases.

In summary, severance payment policy increases the proportion of "formal searchers", which means that it increases the search incentives in the economy. However, as it is presented in table

Table 2: Optimal saving and search -HU economy

b	20	25	30	35	40
HU economy					
A^F	5.1	4.2	3.3	2.3	1.5
$\underline{A}^U$	9.7	8.7	7.2	6.5	5.3
$\overline{A}^U$	19.2	17.7	15.6	14.1	12.5
$\overline{A}_E$	20.3	18.7	16.5	14.9	13.4
$A\bar{E}$	2.3	2.1	2.0	1.9	1.9
$A\bar{U}$	7.7	8.2	8.4	8.6	8.0
$G^U(A^F)$	0.53	0.42	0.29	0.17	0.09
$G^U(\underline{A}^U)$	0.56	0.46	0.34	0.24	0.17
$G^U(\overline{A}^U)$	0.99	0.99	0.98	0.97	0.97
$G^U(0)$	0.09	0.08	0.06	0.04	0.03
u	0.16	0.18	0.23	0.31	0.42

Table 3: Optimal saving and search -HU economy with severance payment

S	15	20	25	30
HU economy $b = 25$				
A^F	4.2	4.2	4.3	4.3
$\underline{A}^U$	9.1	9.2	9.3	9.4
$\overline{A}^U$	17.9	18.0	18.1	18.2
$\overline{A}_E$	19.0	19.1	19.2	19.3
$A\bar{E}$	2.0	1.9	1.8	1.8
$A\bar{U}$	6.8	6.4	6.0	5.9
$G^U(A^F)$	0.55	0.59	0.63	0.66
$G^U(\underline{A}^U)$	0.58	0.62	0.65	0.68
$G^U(\overline{A}^U)$	0.99	0.99	0.99	0.99
$G^U(0)$	0.09	0.10	0.10	0.11
u	0.15	0.14	0.13	0.12

(1) the higher the social security payment b, the lower the incentives for search, (moral hazard problem). Therefore an optimal policy needs to take into account this trade-off. In the next section I discuss the optimal policy for low and high unemployment economies.

5. Optimal policy

This section presents the numerical results solving for the optimal policy. The problem for an optimal insurance scheme is to improve consumption smoothing between spells of employment and unemployment, without inducing less search effort. Following Coles (2006), the social planner's problem is to maximizes the expected payoff of each new labor market entrant when he is employed for the first time, $V_E(0/B)$), subject to the budget balance constraint. Remember that when workers enter for the first time into the labor market they do not have any level of assets, $A = 0$. Thus, the social planner designs the UI program to insure all recently hired new labor market entrants against unemployment risk. In the next section I presents the optimal program with and without a severance payment.

5.1. Optimal policy without severance payment

First I analyze the optimal policy when the severance payment is zero. In this case the social planner's problem is given by:

$$\max_{\{\pi,b\}} V_E(0)$$
$$s.t: \; ub = [1-u]\pi$$

Table (4) presents the results of an optimal policy without the severance payment. Notice that the optimal social security payment is the one which ensure a low proportion of workers taking the non-participation strategy. The first column of table (4) presents the results for a low unemployment economy LU. In this case I find that the optimal social security payment is $b = 25$, and the optimal income tax is $\pi = 1\%$ of the gross formal wage. The optimal level of asset A^F until an unemployed worker searches full time for a formal job is 4.1 years of salary, the optimal level of assets $\underline{A}^U$ until an unemployed worker stops working in the informal sector and decides to take a "holiday" is 7.3 years of salary and the optimal level of assets $\overline{A}^U$ until an unemployed worker decides to retire is 17.4 years of salary. Finally, the optimal level of assets $\overline{A}_E$ until an employed worker decides to retire is 17.8 years of salary, which implies that Property I holds. Moreover, the average level of assets held by an employed worker is 1.1 years of his salary and the average level of assets held by an unemployed worker is 0.9 years of his salary. Notice that these results are similar to those presented in table (1) without the optimal income tax π.

Column two in table (4) presents the results of an optimal policy for a high unemployment economy (HU). In this case the optimal social security payment is $b = 20$ and the optimal income tax is $\pi = 3.43\%$ of the gross formal wage. Where the optimal level of assets $A^F = 5.1$ years of salary, the optimal level of assets $\underline{A}^U = 10.1$ years of salary, the optimal level of assets $\overline{A}^U = 18.8$, and the optimal level of assets $\overline{A}_E = 19.8$. Moreover, the average level of assets held by an employed worker is 2.3 years of his salary and the average level of assets held by an unemployed worker is 7.1 years of his salary. These results are higher compared to a low unemployment economy. The reason is because in a HU economy, the risk of unemployment is higher, thus given that workers are risk averse, they prefer on average to save more than in the case when they are in a LU economy. Moreover, comparing these results with those presented in table (2), I find that the proportion of "formal searchers" increases from 53% to 57% and the

Table 4: Optimal policy with $S = 0$

	LU	HU
b	25	20
π	0.94	3.43
A^F	4.1	5.1
$\underline{A}^U$	7.3	10.1
$\overline{A}^U$	17.4	18.8
$\overline{A}_E$	17.8	19.8
$\bar{AE}$	1.1	2.3
$\bar{AU}$	0.9	7.1
$G^U(A^F)$	0.98	0.57
$G^U(\underline{A}^U)$	0.98	0.60
$G^U(\overline{A}^U)$	0.99	0.98
u	0.03	0.14
$V^E(0)$	-0.0810	-0.0877

proportion of those who take a "holiday" decreases from 43% to 38%. Notice that the income tax π reduces the worker's net income, hence workers need to participate more actively in the labor market in order to maintain the same level of consumption before the income tax. In this way the unemployment rate decreases from 16% to 14%.

5.2. Optimal policy with severance payment

This section presents the results of an optimal policy including the severance payment policy. In this case the social planner's problem is given by:

$$\max_{\{\pi,S,b\}} V_E(0)$$
$$s.t: \; ub + [1-u]\delta S = [1-u]\pi$$

Following Coles (2006) I assume a fully compensating layoff payment. Assuming unemployed workers always search full time for a formal job, a full compensation requires $S = \frac{(w-b)}{(r+\lambda)}$, where $w = w^G - \pi$. Column 1 in table (5) presents the results for a low unemployment economy (LU), where the optimal social security payment is $b = 25$, the optimal lump sum tax is $\pi = 3\%$ of the gross formal wage and the optimal severance payment is of $S = 16$ (which represents 0.2 years of salary). Notice that when we analyze the optimal policy without the severance payment 98% of unemployed workers search full time for a formal job, and when we include the severance payment policy, 99% of unemployed workers search full time for a formal job, hence the severance payment improves the search effort in the economy. However, more important changes are seen when we analyze the case of high unemployment. Column 2 in table (5) presents the results for a high unemployment economy (HU). The optimal social security payment is $b = 20$, the optimal tax is of $\pi = 8.2\%$ of the gross formal wage, with an optimal severance payment of $S = 46$ (which represents 0.5 years of salary). Comparing these results with those in table (4) column two, I find that the optimal assets threshold A^F, $\underline{A}^U$, $\overline{A}^U$ and $\overline{A}_E$ does not change, but the average level of assets held by unemployed workers, $\bar{AU}$, falls to 2.8 years of salary and the average level of assets held by employed workers, $\bar{AE}$, falls to 1.7 years of salary. Furthermore, the proportion of "formal searchers" increases to 89% from 57% and the proportion of "informal searchers" decreases to 1%. Additionally the proportion of workers who are "on-holiday" decreases to 9% from 28%.

The severance payment improves search incentives through the re-entitlement effects. As it is mentioned by Coles (2006): "Re-entitlement to full insurance through becoming re-employed increases the value of becoming re-employed and so improves search incentives".p.31. Comparing the results of the optimal policy without severance payment I find that the unemployment rate decreases to 9% from 14%. Even though this new policy is more generous, the unemployment rate decreases and there is an improvement in welfare. Similar results are found by Coles (2006). There is a welfare improvement through a better consumption smoothing across employment and unemployment spells, and through a better composition of workers, where there are more "formal searchers" than "informal searchers" and those who are "on-holiday"[13].

Table 5: Full compensation policy

	LU	HU
b	25	20
π	3.0	8.2
S	16	46
A^F	4.1	5.1
$\underline{A}^U$	6.8	10.0
$\overline{A}^U$	17.4	18.7
$\overline{A}_E$	17.8	19.7
$\bar{A}E$	0.8	1.7
$\bar{A}U$	0.7	2.8
$G^U(A^F)$	0.99	0.89
$G^U(\underline{A}^U)$	0.99	0.90
$G^U(\overline{A}^U)$	1.00	0.99
u	0.03	0.09
$V^E(0)$	-0.0806	-0.0853

These results are complementary to those of Pissarides (2001, 2010), and Fella (2007) and among others, who find that severance payment is an optimal policy to protect workers against the risk of becoming unemployed, and that it can be offered by the private firms to the workers. Even though my work does not discuss what is the optimal way to finance severance payments, Blanchard and Tirole (2008) suggest that an optimal policy that ensures an efficient layoff decision is a layoff tax to the firms, which means firms should finance the severance payment to workers. Therefore, an optimal program is an unemployment benefit (or social security payment) offered by the government to protect workers during unemployment spells and a severance payment offered by the firms to internalize the layoff decision[14].

[13]This paper ignores other channels to improve welfare, as the consumption smoothing over the life cycle explored by Michelacci and Ruffo (2011).

[14]Even though Michelacci and Ruffo (2011) analyze the optimal policy in a different framework, my results are analogous to theirs when they suggest that an optimal policy that include severance payments will require a combination of high severance payments and low UI benefits to old workers, and high UI benefits and low severance payments to

Furthermore, the results in this paper are in line with those of Acemoglu and Shimer (1999a,b), Albrecht and Axell (1984), and Acemoglu (2001) among others, who argue that unemployment insurance improves the composition of jobs in the economy when workers are risk averse. The reason is that unemployment insurance increases labor productivity by encouraging workers to seek more productive jobs. Therefore, an improvement in the composition of jobs in the economy, increases the total output and the welfare in the economy. In this paper, I show that severance payments improve the composition of workers in the economy, which means that they increase the proportion of "formal searchers" and reduce the proportion of "informal searchers" and those who are "on-holiday" in the economy. As a consequence more workers take formal jobs which are more productive than the informal ones. Hence, on average the total output in the economy increases.

6. Conclusions

This paper extends the analysis of the optimal policy in an economy with two sectors: formal and informal, where workers are risk averse and they are allowed to save. In the first section of this chapter I characterize the optimal consumption/search/non-participant strategy in an economy without job destruction shocks. Then I extend the model to the case with job destruction shocks and introduce some policies such as, lump sum payment and income tax. Analyzing the effect of social security payments without a lump sum layoff, I show given the moral hazard problem, a higher social security payment b, increases the proportion of "informal searchers" and those who are "on-holiday" in the economy, but it reduces the average level of assets held by employed workers. Moreover, the proportion of those who are liquidity constrained decreases with a high social security payment. Therefore, a higher social security payment protects workers from being liquidity constrained, but it decreases the incentives to search. Lentz (2009), Shimer and Werning (2003, 2005), Kocherlakota (2004), Lentz and Tranaes (2005), Werning (2002), Acemoglu and Shimer (1999a,b), Hansen and İmrohoroğlu (1992), Joseph and Weitzenblum (2003), and Coles (2006) among others, find similar results.

Analyzing the effect of severance payments [as did Coles (2006)], on the other hand, I find that severance payments protect workers against the unemployment risk. With a severance payment policy workers do not over-accumulate to protect themselves against the unemployment risk, but have higher incentives to search through the "*re-entitlement effect*". I find that with a high severance payment, the proportion of "formal searchers" in the economy increases and the proportion of "informal searchers" and of those who are "on-holiday" decreases.

Analyzing the optimal policy $\{\pi, S, b\}$, I find that a low social security payment and a positive severance payment are an optimal policy. An optimal severance payment increases the incentives to search, reducing the proportion of those who are "informal searchers" and of those who are "on-holiday". The optimal social security payment protects workers from being liquidity constrained. Even though the optimal policy with a severance payment seems more generous, the unemployment rate is lower and there is welfare improvement. Moreover, as it has been suggested by Acemoglu and Shimer (1999a,b), an optimal UI program improves the composition of workers in the economy, and as a result more workers take formal jobs that are more productive than the informal ones and the average output in the economy increases.

young workers (given that the moral hazard problem is higher in the case of older workers). In my case given that the moral hazard problem is important for formal and informal workers I find that the optimal policy is a low UI benefit for both type of workers and a high severance payment.

Contrary to what the majority of the current literature suggests in the case of developing economies with a high level of informality, I find that severance payment is an optimal policy for these economies to protect workers from unemployment risk and to increase the incentives to search for a formal job, reducing informality. Even though unemployment insurance is difficult to offer because of the impossibility to distinguish between "formal searchers" and "informal searchers", a low social security payment is optimal, because it guarantee a minimum level of resources during unemployment spells and it reduces the proportion of those who are liquidity constrained[15].

Finally further research is needed to analyze the implementation of more active policies in the developing countries with a high levels of informality. Policies that increase the incentives to work in the formal sector and reduce the incentives to work in the informal sector. These types of policies can be done in the form of "in-work benefits" or "back to work" allowances, which are widespread in the OECD countries. As it has been reported by Charlot et al. (2013), workers' decisions are affected substantially by financial work incentives. Low-income groups and lone parents react more strongly, and labor supply for women is more elastic than for men. Hence, policies that focus on these particular groups may have a stronger impact in terms of formal employment and informality.

Appendix A

The steady state (A^{SS}, c^*) is an unstable node

To proof that the steady state (A^{SS}, c^*) is an unstable node, I need to show that $S^C(A^{SS}) < u_B$. At the steady state the value of $V_U(A^{SS})$ is given by:

$$V_U(A^{SS}) = \frac{u(c^*) + \varphi\lambda V_E(A^{SS})}{(r + \varphi\lambda)}, \tag{A.1}$$

and by Proposition (1) the value of $V_U(A^{SS})$ is given by:

$$rV_E(A^{SS}) = u_B + u(c^*) - r[\bar{A}^E - A^{SS}]u(c^*). \tag{A.2}$$

Hence $S^C(A^{SS})$ is defined as:

$$S^C(A^{SS}) = (b + w_I)u(c^*) + \varphi\lambda[V_E(A^{SS}) - V_U(A^{SS})]. \tag{A.3}$$

Using equation (A.1) into (A.3) I get:

$$S^C(A^{SS}) = (b + w_I)u(c^*) + \frac{\varphi\lambda}{(r + \varphi\lambda)}[rV_E(A^{SS}) - u(c^*)], \tag{A.4}$$

and substituting equation (A.2) I can write:

$$S^C(A^{SS}) = (b + w_I)u(c^*) + \frac{\varphi\lambda}{(r + \varphi\lambda)}[u_B - r[\bar{A}^E - A^{SS}]u(c^*)]. \tag{A.5}$$

By definition: $\bar{A}^E = \frac{c^*}{r}$ and $A^{SS} = \frac{c^* - b - w_I}{r}$, then substituting into equation (A.5) I get:

[15] This policy is offered in most of the OECD economies, apart from the unemployment insurance [See Charlot et al. (2013)]

$$S^C(A^{SS}) = (b + w_I)u(c^*) + \frac{\varphi\lambda}{(r + \varphi\lambda)}[u_B - (b + w_I)u(c^*)], \tag{A.6}$$

And given that $u_B = wu(c^*)$, I can write $S^C(A^{SS}) - u_B$ as

$$S^C(A^{SS}) - u_B = -r\frac{u(c^*)}{(r + \varphi\lambda)}[w - (b + w_I)] < 0, \tag{A.7}$$

Then the steady state (A^{SS}, c^*) is an unstable node. I can use the same argument to proof $S^F(A^{SS}) < u_B$.

Appendix B

Proof of proposition 5

1) Consider the pool of the unemployed over an arbitrarily small period of time $dt > 0$. The number who exit this pool with $A \leq A^F$ is given by those unemployed who find a job or die:

$$uG^U(A)(\mu + \lambda)dt \text{ for } A \leq A^F,$$

The inflow towards unemployment is given by those who lose their job, those who are born and those who dissave over time:

$$\mu dt + (1 - u)G^E(A)\delta dt + u\left[G^U(A', x) - G^U(A, x)\right] \text{ for } A \leq A^F,$$

where $A' < A$ and $\dot{A} = rA + b - c^U(A)$. Setting inflow equal to outflow and letting $dt \to 0$ implies:

$$G^U(A)(\mu + \lambda) + \frac{dG^U}{dA}(rA - c^U(A) + b) = \frac{\mu + (1 - u)G^E(A, x)\delta}{u}, \text{ for } A \leq A^F$$

Consider the pool of the unemployed with a level of assets $A^F < A \leq \underline{A}^U$. The outflow into unemployment is given by those unemployed who find a job or die and and those who being in the informal sector dissave over time:

$$u[G^U(A) - G^U(A^F)](\varphi\lambda + \mu)dt + u\left[G^U(A') - G^U(A)\right] \text{ for } A^F < A \leq \underline{A}^U,$$

where $A' < A$ and $\dot{A} = rA + w_I + b - c^U(A)$, while the inflow into unemployment is given by those employed workers who lose their job:

$$(1 - u)[G^E(A) - G^E(A^F)]\delta dt \text{ for } A^F < A \leq \underline{A}^U.$$

Setting inflow equal to outflow and letting $dt \to 0$ implies:

$$[G^U(A) - G^U(A^F)](\varphi\lambda + \mu) = \left\{ \begin{array}{c} \frac{dG^U}{dA}(rA - c^U(A) + b + w_I) \\ + \frac{(1-u)[G^E(A) - G^E(A^F)]\delta}{u} \end{array} \right\}, \text{ for } A^F < A \leq \underline{A}^U$$

Consider the pool of the unemployed with a level of assets $\underline{A}^U < A \leq \bar{A}^U$. The outflow into unemployment is given by those "non-participants" who die and those who dissave over time:

$$u[G^U(A) - G^U(\underline{A}^U)]\mu dt + u\left[G^U(A') - G^U(A)\right] \text{ for } \underline{A}^U < A \leq \bar{A}^U,$$

where $A' < A$ and $\dot{A} = rA - c^U(A)$, while the inflow into "non-participation" is given by those employed workers who lose their job:

$$(1-u)[G^E(A) - G^E(\underline{A}^U)]\delta dt \ \text{ for } \underline{A}^U < A \leq \bar{A}^U.$$

Setting inflow equal to outflow and letting $dt \to 0$ implies:

$$[G^U(A) - G^U(\underline{A}^U)]\mu = \left\{ \begin{array}{c} \frac{dG^U}{dA}(rA - c^U(A)) \\ + \frac{(1-u)[G^E(A) - G^E(\underline{A}^U)]\delta}{u} \end{array} \right\}, \text{ for } \underline{A}^U < A \leq \bar{A}^U$$

Finally consider those retired workers with $\overline{A}^U < A \leq \overline{A}_E$. The outflow into retirement is given by those workers who are retired and die at any period on time dt:

$$u[1 - G^U(\bar{A}^U)]\mu dt \text{ for } \overline{A}^U < A \leq \overline{A}_E,$$

while the inflow into retirement is given by those employed workers who lose their job and given that their assets are $A > \bar{A}^U$ decide not to retire.

$$(1-u)[1 - G^E(A)]\delta dt \text{ for } \overline{A}^U < A \leq \overline{A}_E,$$

Setting inflow equal to outflow I get:

$$\frac{(1-u)[1 - G^E(\bar{A}^U)]\delta}{u} = \mu[1 - G^U(\bar{A}^U)] \text{ for } \overline{A}^U < A \leq \overline{A}_E,$$

2) Consider the pool of employed workers over an arbitrarily small period of time $dt > 0$, the number who exit this pool with $A \leq A^F$ is given by those who being employed lose their job and die, and those who exit through asset accumulation.

$$(1-u)[G^E(A)(\mu + \delta)dt + [G^E(A', x) - G^E(A, x)]] \text{ for } A \leq A^F,$$

where $A' > A$ and $\dot{A} = rA + w - c^E(A)$. The inflow is given by those who being searching full time for a formal offer find a job:

$$\lambda u G^U(A)dt \text{ for } A \leq A^F,$$

Setting inflow equal to outflow and letting $dt \to 0$ implies:

$$G^E(A)(\mu + \delta) + \frac{dG^E}{dA}(rA - c^E(A) + w) = \frac{u}{1-u}\lambda G^U(A), \text{ for } A \leq A^F$$

Consider those employed workers with assets $A^F < A \leq \underline{A}^U$. The outflow of the pool of employed workers is given by those who being employed lose their job or die and those who exit through asset accumulation:

$$(1-u)[(G^E(A) - G^E(A^F))(\mu + \delta)dt + (G^E(A') - G^E(A))] \text{ for } A^F < A \leq \underline{A}^U,$$

where $A' > A$ and $\dot{A} = rA + w - c^E(A)$. The inflow into employment is given by those who being working in the informal sector find a job and those workers with assets A^{F-} who save:

$$\varphi\lambda u[G^U(A) - G^U(A^F)]dt + (1-u)[G^E(A^F) - G^E(A')] \text{ for } A^F < A \leq \underline{A}^U,$$

Setting inflow equal to outflow and letting $dt \to 0$ implies: for $A^F < A \leq \underline{A}^U$:

$$\left\{ \begin{array}{c} [G^E(A) - G^E(A^F)](\mu + \delta) \\ + \frac{dG^E}{dA}(rA - c^E(A) + w) \end{array} \right\} = \left\{ \begin{array}{c} \frac{u}{1-u}\lambda\varphi[G^U(A) - G^U(A^F)] \\ + \frac{dG^E}{dA^F}(rA^F - c^E(A^F) + w) \end{array} \right\},$$

Consider those employed workers with assets $\underline{A}^U < A \leq \bar{A}^U$. The outflow of the pool of employed workers is given by those who being employed lose their job or die and those who exit through asset accumulation:

$$(1 - u)[(G^E(A) - G^E(\underline{A}^U))(\mu + \delta)dt + (G^E(A') - G^E(A))] \text{ for } \underline{A}^U < A \leq \bar{A}^U,$$

where $A' > A$ and $\dot{A} = rA + w - c^E(A)$. The inflow into employment is given by those workers with assets $\underline{A}^{U-}$ who save:

$$(1 - u)[G^E(A') - G^E(\underline{A}^U)] \text{ for } \underline{A}^U < A \leq \bar{A}^U,$$

Setting inflow equal to outflow and letting $dt \to 0$ implies: for $\underline{A}^U < A \leq \bar{A}^U$:

$$\left\{ \begin{array}{c} [G^E(A) - G^E(\underline{A}^U)](\mu + \delta) \\ + \frac{dG^E}{dA}(rA - c^E(A) + w) \end{array} \right\} = \left\{ \frac{dG^E}{d\underline{A}^U}(r\underline{A}^U - c^E(\underline{A}^U) + w) \right\},$$

Finally consider those employed workers with $\overline{A}^U < A \leq \overline{A}_E$. The outflow from the pool of employed workers is given by those who lose their job or die and those who continue saving until they retire and the inflow into employment is given by those workers with assets $\bar{A}^{U-}$ who save. Setting inflow equal to outflow and letting $dt \to 0$ implies: for $\overline{A}^U < A \leq \overline{A}_E$:

$$\left\{ \begin{array}{c} [1 - G^E(\overline{A}^U)](\mu + \delta) \\ + \frac{dG^E}{dA}(rA - c^E(A) + w) \end{array} \right\} = \left\{ \frac{dG^E}{d\bar{A}^U}(r\bar{A}^U - c^E(\bar{A}^U) + w) \right\}$$

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