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Exchange Rate Predictability

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Núm. 857  
2014

# Borradores de ECONOMÍA



Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia



# A CONSUMPTION-BASED APPROACH TO EXCHANGE RATE PREDICTABILITY

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This paper provides evidence of short-run predictability for the real exchange rate by performing out-of-sample tests of a forecasting equation which is derived from a consumption-based asset pricing model. In this model, the real exchange rate is predictable as a result of the implications of preferences with habit persistence on the pricing of international assets. The implied predictors are: domestic, US and world consumption growth. Empirical exercises show evidence of short-term predictability on the bilateral rates of 15 out of 17 countries vis-à-vis the US over the post Bretton-Woods float. A GMM estimation of the parameters of the model also finds evidence of the presence of habits in consumers' preferences.

JEL C5, F31, F47, G15

Keywords: exchange rates, out-of-sample, predictability, asset pricing, habits

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# 1. INTRODUCTION

This paper provides a new framework to study real exchange rate (RER) predictability by developing a consumption-based asset-pricing model which includes internal and external habit formation<sup>2</sup>. The econometric methods follow recent literature on out-of-sample predictability tests. Our asset-pricing model shows that the presence of habits in consumers' preferences implies that the RER has a predictable component which depends on past consumption growth.

The econometric methods are applied to RER series for 17 industrialized economies over the Post-Bretton-Woods float. Evidence of short-run out-of-sample predictability is found in 15 countries. This evidence is obtained by computing tests which compare the forecasting power of the model with a random-walk forecast. Additionally, this evidence can be compared with recent papers in the literature which study similar countries, tests and data spans, but use different structural models.

The empirical results are interpreted in the context of a consumption-based asset-pricing model with  $N$  countries, complete markets, imperfect international risk sharing and representative consumers whose preferences include habit persistence through a benchmark consumption level. The economic reason for RER predictability in this framework is the effect of past consumption growth on current marginal utility and therefore on the stochastic discount factors (SDFs) that domestic and foreign investors use to value financial assets. In this theoretical framework, RER variations are driven by changes of SDFs in both countries.

As a robustness check for the predictability results, we measure the degree of habit persistence and its relative importance across countries by estimating the relevant parameters of the utility function using non-linear GMM methods. Results from this estimation show significant and fairly strong habit effects in most of the countries under study.

This paper is related to the empirical literature on exchange rate determination models. In particular, it addresses the puzzle originally described by Meese and Rogoff (1983) about the poor out-of-sample forecasting power of the monetary approach to

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<sup>2</sup> In this paper, external habits are very similar to the definition of catching up with the Joneses in Abel (1990) but within an open-economy interpretation. Another related concept of external habits is the one in Campbell and Cochrane (1999) who use a different specification of preferences.

exchange rate determination.<sup>3</sup> Several papers have shown that alternative specifications of the monetary model have out-of-sample predictability power at long-run horizons (one year or more). Mark (1995), Mark and Sul (2001), Groen (2005), Engel et al (2007), and Cerra and Saxena (2010) find positive results for the standard monetary model on these kinds of horizons.

Additionally, several papers show out-of-sample predictability evidence with alternative exchange rate models. Gourinchas and Rey (2007) study an international financial adjustment model in which real exchange rate changes are the result of disequilibria of the country's external accounts. Molodtsova and Papell (2009) estimate a forecasting equation which is derived from the Taylor rule for monetary policy in each country. Rogoff and Stavrakeva (2008) perform robustness exercises using these alternative models and conclude that the out-of sample predictability evidence is still weak on horizons shorter than one year. One possible reason for this result is that the intensity of the relation between exchange rates and alternative fundamentals is time-varying, as shown by Sarno and Valente (2009). Rossi (2013) surveys this literature and confirms that the most promising models are those based on Taylor rules or external accounts.

This paper is organized in the following way. A consumption-based asset-pricing framework and its implied forecasting equation for the real exchange rate are described in Section 2. The econometrics methods for out-of-sample predictability evaluation are presented in Section 3. Country-by-country results are presented in Section 4. Results for alternative forecasting windows are described in Section 5. Section 6 presents the in-sample estimation of the parameters of the model. Finally, Section 7 concludes.

## 2. A CONSUMPTION-BASED ASSET-PRICING MODEL

### 2.1. Basic Framework

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<sup>3</sup> An early reference for the monetary model of exchange rates is Frenkel (1976). Frankel and Rose (1995) present a survey of the literature on empirical research on nominal exchange rates.

Consider a consumption-based asset-pricing framework which is based on Abel (1990, 2006) but it is extended to include  $N$  countries ( $i = 1, 2, \dots, N$ ). The representative consumer in each country  $i$  maximizes:

$$U_{i,t} = E_t \left[ \sum_{j=0}^{\infty} \beta^j \left( \frac{1}{1-\alpha} \right) \left( \frac{C_{i,t+j}}{V_{i,t+j}^{\gamma_i}} \right)^{1-\alpha} \right]. \quad (1)$$

In Equation (1),  $\alpha$  denotes the risk aversion coefficient,  $\beta$  is the time discount factor,  $C_{i,t}$  is the level of household consumption in each country<sup>4</sup> and  $V_{i,t}^{\gamma_i}$  is the benchmark level of consumption where the parameter  $\gamma_i$  measures the degree of habit persistence.<sup>5</sup> Benchmark consumption includes past domestic consumption as well as past world consumption:

$$V_{i,t} = \left[ (C_{i,t-1})^D (C_{w,t-1})^{1-D} \right]. \quad (2)$$

In Equation (2),  $C_w$  denotes world consumption and  $D$  is a weight that measures the importance of domestic consumption relative to world consumption in the composition of the benchmark level of consumption. World consumption is defined as the geometric weighted average of consumption across countries. The weights  $\omega_i$  in Equation (3) are determined by the relative size of country  $i$ .

$$C_w = \prod_{i=1}^N C_i^{\omega_i}. \quad (3)$$

The utility framework in Equations (1) to (3) nests the standard CRRA case when  $\gamma = 0$ , because in this case the benchmark consumption does not have any influence in utility. When  $\gamma > 0$  instead, utility depends on the ratio between domestic and benchmark consumptions. The presence of  $V_i^{\gamma}$  in the utility function captures two effects: internal and external habit formation. In this paper, the latter effect is interpreted as the satisfaction from consuming as much as the average world level of consumption or more.

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<sup>4</sup>  $C_{i,t}$  corresponds to the level of real consumption per capita. It includes consumption of non-durable goods and services by households.

<sup>5</sup> The parameter  $\gamma_i$  is allowed to be different across countries.

From Equation (1), it is possible to compute the marginal utility of consumption in each country.

$$\frac{\partial U_{i,t}}{\partial C_{i,t}} = \frac{1}{C_{i,t}} E_t \left[ \left( \frac{C_{i,t}}{V_{i,t}} \right)^{1-\alpha} - \gamma_i D \beta \left( \frac{C_{i,t+1}}{V_{i,t+1}} \right)^{1-\alpha} \right]. \quad (4)$$

Note that marginal utility in (4) when  $\gamma_i = 0$ , is exactly equal to the case of a standard CRRA utility function ( $C_{i,t}^{-\alpha}$ ). Therefore, it is possible to partition Equation (4) into three components: standard CRRA, benchmark consumption and habits. These three components are specified in Equation (5).

$$\frac{\partial U_{i,t}}{\partial C_{i,t}} = C_{i,t}^{-\alpha} V_{i,t}^{\gamma_i(\alpha-1)} H_{i,t}. \quad (5)$$

The component  $V_{i,t}^{\gamma_i(\alpha-1)}$  measures the effect of benchmark consumption on marginal utility. This effect has a negative as well as a positive constituent. The negative constituent is the instantaneous drop in utility which happens when  $V_{i,t}$  increases. The positive part is related to the higher marginal utility which is possible to obtain with lower ratios  $C_{i,t}/V_{i,t}$  as a result of the concavity of the utility function. The parameter  $\alpha$  determines the extent of this concavity. Therefore, when  $\alpha > 1$ , the positive effect dominates so that the net effect of  $V_{i,t}$  on marginal utility is positive. In the log-utility case,  $\alpha = 1$  so that both components cancel each other and the net effect is zero. Finally, when the utility function is less concave,  $\alpha < 1$ , the net effect of the benchmark consumption on marginal utility is negative.

The component  $H_{i,t}$  measures the effect of internal habits on marginal utility. It is a number between 0 and 1, which takes into account the fact that a higher consumption today increases the benchmark level of consumption and thus decreases tomorrow's utility.

$$H_{i,t} = 1 - D \gamma_i \beta E_t \left( X_{i,t+1}^{1-\alpha} \right) X_{i,t}^{D \gamma_i (\alpha-1)} X_{w,t}^{(1-D) \gamma_i (\alpha-1)}. \quad (6)$$

In (6),  $X_{i,t}$  corresponds to the gross rate of consumption. Therefore, we define:

$$X_{i,t+1} \equiv C_{i,t+1}/C_{i,t} \quad \text{and} \quad X_{w,t+1} \equiv C_{w,t+1}/C_{w,t}.$$

Equation (5) and the definition of benchmark consumption allow us to easily compute the Stochastic Discount Factor (SDF) or pricing kernel, as the product of the time discount factor and marginal utility growth.

$$M_{i,t+1} = \beta X_{i,t+1}^{-\alpha} X_{i,t}^{D\gamma_i(\alpha-1)} X_{w,t}^{(1-D)\gamma_i(\alpha-1)} \left( \frac{H_{i,t+1}}{H_{i,t}} \right). \quad (7)$$

## 2.2. Implications for the Real Exchange Rate

We describe the relation between exchange rates and SDFs following the asset-pricing framework of Lustig and Verdelhan (2007) who reinterpret, in the context of exchange-rate markets, the result that under free portfolio formation and the law of one price, there exists a unique SDF in the space of traded assets. A similar result was originally derived by Backus et al (2001).

Let  $M_{us,t+1}$  denote the SDF of the US investors.  $Q_t$  is the real exchange rate in US good per foreign good, therefore, when  $Q_t$  goes down the US dollar appreciates in real terms. Both foreign and US investors have access to a foreign-currency return  $R_{i,t+1}$ . The following are the Euler conditions of both investors:

$$E_t(M_{i,t+1}R_{i,t+1}) = 1. \quad (8)$$

$$E_t(M_{us,t+1}R_{i,t+1}Q_{t+1}/Q_t) = 1. \quad (9)$$

The uniqueness of the SDF in the space of traded assets and Equations (8) and (9) imply the following relationship:

$$M_{i,t+1} = M_{us,t+1}Q_{t+1}/Q_t. \quad (10)$$

Computing natural logarithms on both sides of Equation (10) we obtain:

$$q_{t+1} - q_t = m_{i,t+1} - m_{us,t+1}. \quad (11)$$

Throughout this paper, lower case letters are logs of the original variables. In Equation (11),  $m_{us,t+1}$ ,  $m_{i,t+1}$  are the US and country  $i$ 's log SDFs, respectively. This equation says that the log variation in the real exchange rate is equal to the difference between the log SDF in country  $i$  and in the US. Computing logs on both sides of (7) and inserting this result

in (11), we obtain the following expression for the real exchange rate as a function of consumption growth and habit persistence in both countries:

$$\begin{aligned} \Delta q_{i,t+1} = & -\alpha(x_{i,t+1} - x_{us,t+1}) + D\gamma_i(\alpha - 1)x_{i,t} - D\gamma_{us}(\alpha - 1)x_{us,t} + \\ & (1 - D)(\alpha - 1)(\gamma_i - \gamma_{us})x_{w,t} + \Delta b_{i,t+1} + \Delta b_{us,t+1} \end{aligned} \quad (12)$$

In Equation (12), growth rates for the real exchange rate and the habit effect are denoted  $\Delta q_{i,t+1}$  and  $\Delta b_{i,t+1}$ , respectively. Note that (12) can be interpreted as a forecasting equation in which changes in the real exchange rate are determined by lagged values of domestic, US and world consumption growth. The channel for this effect is the presence of habit persistence and its implications on the marginal utility of consumption and thus on SDFs which are the basis for asset pricing.

Some necessary conditions for predictability are observed in Equation (12). First, the risk aversion coefficient  $\alpha$  should be different from one, otherwise, the real exchange rate becomes neutral to the presence of habit persistence. Second, each country's habit persistence degree should be different from the one in the US. This latter condition is necessary for the exchange rate to be predictable with world consumption growth.

### 2.3. Computing a Linear Forecasting Equation

In order to estimate the expected value of (12) using a linear regression framework, it is necessary to use a first-order Taylor approximation to  $b_{i,t}$  and  $b_{us,t}$  since both expressions are nonlinear functions of consumption growth. In order to perform this approximation it is necessary to define the following:

$$z_{i,t} \equiv D\gamma_i(\alpha - 1)x_{i,t} + (1 - D)\gamma_i(\alpha - 1)x_{w,t}. \quad (13)$$

Therefore, using (13), we can write  $b_i$  in the following simplified way:

$$b_{i,t} \equiv \log(H_{i,t}) = \log(1 - D\gamma_i\beta E(X_{i,t+1}^{1-\alpha})e^{z_{i,t}}). \quad (14)$$

Once the derivative of (14) is computed, it is possible to express the first-order Taylor approximation to  $b_{i,t}$  around  $E(z_{i,t}) \equiv \bar{z}_i$  in the following way:

$$b_{i,t} \approx \log(1 - D\gamma_i\beta E(X_{i,t+1}^{1-\alpha})e^{\bar{z}_i}) - \frac{D\gamma_i\beta E(X_{i,t+1}^{1-\alpha})e^{\bar{z}_i}}{1 - D\gamma_i\beta E(X_{i,t+1}^{1-\alpha})e^{\bar{z}_i}}(z_{i,t} - \bar{z}_i). \quad (15)$$



From (15), we can compute  $\Delta b_{i,t+1}$  which consists of a constant multiplied by  $\Delta z_{i,t+1}$ . Therefore, using (13) and (15), we can express the expected value of  $\Delta b_{i,t+1}$ , conditional on information through  $t$ , in the following way:

$$E_t(\Delta b_{i,t+1}) = -\theta_i \gamma_i (\alpha - 1) g + \theta_i D \gamma_i (\alpha - 1) x_{i,t} + \theta_i (1 - D) \gamma_i (\alpha - 1) x_{w,t}, \quad (16)$$

where  $\theta_i$  is a constant parameter:

$$\theta_i \equiv \frac{D \gamma_i \beta E(X_i^{1-\alpha}) e^{\bar{z}_i}}{1 - D \gamma_i \beta E(X_i^{1-\alpha}) e^{\bar{z}_i}}. \quad (17)$$

Equation (16) also assumes a log normal distribution for consumption growth in all countries such that in each period  $t$ :

$$\log(X_{i,t}) \equiv x_{i,t} \sim N(g, \sigma^2). \quad (18)$$

Using (12), (16) and (18), it is possible to derive a linear forecasting equation for the expected variation of the real exchange rate as a function of past consumption growth in the domestic country, the US and the World:

$$E_t(\Delta q_{i,t+1}) = \psi_{i,0} + \psi_{i,1} \Delta c_{i,t} + \psi_{i,2} \Delta c_{us,t} + \psi_{i,3} \Delta c_{w,t}. \quad (19)$$

The parameters to estimate in Equation (19) are functions of the deep parameters of the model:

$$\psi_{i,0} = (\alpha - 1) g (\theta_{us} \gamma_{us} - \theta_i \gamma_i), \quad (20)$$

$$\psi_{i,1} = (1 + \theta_i) D (\alpha - 1) \gamma_i, \quad (21)$$

$$\psi_{i,2} = -(1 + \theta_{us}) D (\alpha - 1) \gamma_{us}, \quad (22)$$

$$\psi_{i,3} = (1 - D) (\alpha - 1) (\gamma_i (1 + \theta_i) - \gamma_{us} (1 + \theta_{us})). \quad (23)$$

Note that the sign of the coefficients  $\psi_{i,0}$  and  $\psi_{i,3}$  is determined by the relative size of the parameters  $\gamma_i$  and  $\gamma_{us}$ . Furthermore, it is necessary that both countries have some internal habit effects ( $D > 0$ ) so that parameters  $\psi_{i,1}$  and  $\psi_{i,2}$  remain different from zero. Additionally, assuming that  $\alpha > 1$ , we should expect a positive sign for  $\psi_{i,1}$  and a negative sign for  $\psi_{i,2}$ .

### 3. ECONOMETRIC METHODS: OUT-OF-SAMPLE PREDICTABILITY TESTS

#### 3.1. Three Alternative Tests

Following Rogoff and Stavrakeva (2008), we compute three alternative tests for out-of-sample predictability power: Theil's U (TU), Diebold-Mariano-West (DMW) and Clark-West (CW). When the mean-square forecasting error is significantly smaller than that from a random-walk model, we regard it as a good forecast. This criterion has been widely used in the exchange rate predictability literature since Meese and Rogoff (1983).

The first step on the out-of-sample predictability exercise consists of choosing a forecasting window. We initially use a 40-observation window to estimate Equation (19) with quarterly data. Since the total sample spans 1973 through 2007, (140 observations), the forecasting window has approximately 100 observations. The second step consists of using rolling regressions, with 40 observations each, to estimate the parameters in Equation (19). Then we use these estimations to perform forecasts of exchange rates one-quarter ahead. The final step is comparing the resulting 100 forecasts with actual real exchange rate data and using these forecast errors to compute predictability tests.

Assume that  $y_t = q_t - q_{t-1}$ , where  $q_t$  is the natural log of the exchange rate for period  $t$ <sup>6</sup>. Let  $X_t$  be the matrix that includes the explanatory variables defined in Equation (19) and let  $\psi$  be the corresponding vector of constant coefficients. We are interested in comparing the forecasting power of the model in Equation (19) with a driftless random-walk model. Under the random-walk model we have:  $y_t = e_{1,t}$ . We can rewrite the model in (19) as:  $y_t = X_{t-1}\psi + e_{2,t}$ . Innovations terms  $e_{1,t}$  and  $e_{2,t}$  are assumed to be unobservable.

The estimated forecasts for the random walk and the structural model are  $\hat{y}_{1,t+1} = 0$ , and  $\hat{y}_{2,t+1} = X_t\hat{\psi}_t$  respectively, where  $\hat{\psi}_t$  is the least-squares estimator of  $\psi_t$ . The

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<sup>6</sup> In this part, I follow the notation in Rogoff and Stavrakeva's (2008) Appendix.

corresponding forecast errors are  $\hat{e}_{1,t}$  and  $\hat{e}_{2,t}$ , respectively. The Mean Squared Forecast Error (MSFE) for either of the forecasting models is:

$$MSFE_i = P^{-1} \sum_{t=R+1}^{t=T} \hat{e}_{i,t+1}^2, i = 1, 2. \quad (24)$$

In Equation (24),  $P$  is the number of forecasts,  $T$  is the sample length and  $R$  is the number of observations used to estimate  $\psi_t$  on the first forecast. The TU test is defined as the ratio between the square root of the MSFE of the structural model and the square root of the MSFE of the random-walk model. Therefore, if  $TU$  is significantly lower than 1, the structural model outperforms the random-walk model.

$$TU = \sqrt{MSFE_2 / MSFE_1}. \quad (25)$$

The DMW test measures the difference between the MSFE of the random walk model and that of the structural model. Therefore a significant and positive DMW test implies that the structural model outperforms the random walk. The formal definition of the DMW test follows:

$$DMW = MSFE_1 - MSFE_2. \quad (26)$$

The literature on forecasting has identified that both statistics, TU and DMW, tend to over-reject the structural model when they are used to compare nested models like those in the current exercise.<sup>7</sup> In view of this problem, Clark and West (2006, 2007) propose a test statistic (CW) which builds on the DMW but takes into account that both models are nested by assuming that, under the null hypothesis, the exchange rate follows a random walk. Therefore, the null hypothesis in the CW test is computed under the assumption that the population parameter vector is  $\psi = 0$ , and that the forecast innovation terms are equal across models:  $e_{1,t} = e_{2,t}$ .

$$\hat{d} = 2P^{-1} \sum_{t=R+1}^{t=T} (y_{t+1} X_t \hat{\psi}_t). \quad (27)$$

Clark and West (2006) show that if  $\hat{d}$ , the quantity defined in (27), is significantly greater than zero, then the structural model outperforms the random walk. Therefore, the

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<sup>7</sup> These models are nested because a random-walk model for the real exchange rate holds as a special case of (19) when all the parameters are equal to zero.

CW test is defined in (28) as a significance test for  $\hat{d}$  where  $\Omega^{\hat{d}}$  is the estimated variance of  $\hat{d}$ .

$$CW = \frac{P^{0.5} \hat{d}}{\sqrt{\Omega^{\hat{d}}}}. \quad (28)$$

We follow Rogoff and Stavrakeva (2008) by computing all three tests, (TU, DMW and CW), when performing out-of-sample predictability exercises, and by using bootstrapped critical values in order to correct for the size distortion which results from working with nested models.

### 3.2. Bootstrap Procedure

We follow Mark and Sul (2001) on the bootstrap procedure used to calculate the p-values of both the TU and DMW tests. Under the non-predictability hypothesis, the exchange rate follows a random-walk model so that its variation is  $y_t = \varepsilon_t^s$ , where  $\varepsilon_t^s$  is an i.i.d. residual. For each right hand side variable in Equation (19) and for each country, we estimate the following OLS regression in order to estimate its autoregressive structure and its correlation with the real exchange rate:

$$\Delta c_t^i = \delta_0 + \sum_{k=1}^d \delta_k y_{t-k} + \sum_{k=1}^l \zeta_k \Delta c_{t-k}^i + \varepsilon_t^i. \quad (29)$$

In Equation (29), the number of lags,  $d$  and  $l$  as well as the appropriate trend (constant or linear), are selected by minimizing a Bayesian information criterion. The estimated residuals for all variables are resampled 1000 times; these resampled residuals are used to recursively simulate the exchange rate and the fundamentals. The first 100 simulated observations are discarded in order to attenuate potential bias related to the choice of starting values for the recursion. Finally, the model is re-estimated and all the test statistics are calculated again for each resampling.

## 4. OUT-OF-SAMPLE PREDICTABILITY RESULTS

We estimate the forecasting equation country by country using least squares and quarterly data for 17 OECD countries<sup>8</sup>. This set of countries is the same one analyzed by Engel et al (2007) and by Rogoff and Stavrakeva (2008). Bilateral Real Exchange Rates (RER) with respect to the US, for all countries, are used to perform out-of-sample predictability tests. These quarterly data span the post Bretton-Woods period through 2007Q4<sup>9</sup>; the starting date of the sample is determined by the availability of consumption data in each country. Most series were retrieved from International Financial Statistics (IFS). Consumption series correspond to nondurable goods and services purchased by households.

Table 1 shows the results from the estimation of the out-of-sample predictability tests described in Section 3.1. The null hypothesis for the TU and DMW tests is that both the model in Equation (19) and a driftless random walk have the same Mean Squared Forecast Error (MSFE); the alternative hypothesis is that the model has lower MSFE than a random walk. In the case of the CW test, the null hypothesis is that the real exchange rate follows a random walk and the alternative hypothesis is that the structural model has a better fit. All p-values in Table 1 are computed with the bootstrap procedure described in Section 3.2 in which, for each series, i.i.d. innovations are estimated with Equation (29), and then 1000 resamplings are used to construct the consumption series and reestimate all the predictability tests.

Results from the TU and DMW tests are very similar to each other in Table 1. There is out-of sample predictability evidence in 12 out of 17 countries according to the TU test. The DMW test shows positive evidence in one additional country, Denmark, with a 90% confidence level. The countries with no predictability evidence are: Belgium, Netherlands, Japan and Korea.

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<sup>8</sup> All implied time series of observable variables are found to be stationary according to unit-root tests. Results are available upon request.

<sup>9</sup> The span of the data allows comparing our results with those in Engel et al (2007).

TABLE 1  
 Out-of-Sample Exchange Rate Predictability Tests  
 Based on One-Quarter Ahead Forecasts

Country	TU	P-value	DMW	P-value	CW	P-value
UK	0.95	0.00	10.16	0.00	3.15	0.01
Austria	0.98	0.01	4.96	0.01	2.71	0.01
Belgium	1.07	0.48	-14.87	0.18	1.10	0.12
Denmark	1.04	0.14	-7.81	0.08	3.02	0.00
France	1.01	0.01	-2.34	0.01	2.86	0.00
Germany	0.97	0.00	7.31	0.00	3.25	0.00
Netherlands	1.11	0.88	-23.27	0.49	2.42	0.01
Canada	1.02	0.01	-1.08	0.02	2.88	0.00
Japan	1.08	0.73	-24.77	0.66	1.75	0.06
Finland	0.95	0.00	15.83	0.00	3.62	0.00
Spain	0.80	0.00	55.37	0.00	6.52	0.00
Australia	1.02	0.05	-4.65	0.06	2.18	0.02
Italy	0.94	0.00	17.27	0.00	3.62	0.00
Switzerland	0.99	0.01	1.83	0.01	3.15	0.00
Korea	1.17	0.99	-43.89	0.99	1.25	0.13
Norway	0.98	0.00	3.78	0.00	2.79	0.01
Sweden	0.96	0.00	11.12	0.00	3.66	0.00

This table presents country-by-country out-of-sample predictability tests estimated from Equation (19) using rolling 40-observation samples. The tests TU , DMW and CW are described in equations (25), (26) and (28) respectively. P-values are computed with the bootstrap procedure described in Section 3.

The evidence on predictability improves when the CW test is examined. In this case, the null hypothesis is not rejected in only two countries: Belgium and Korea. This result implies that for The Netherlands and Japan there is predictability evidence under the slightly different null hypothesis explained on the previous section. For the remaining 13 countries, the consumption based model is able to beat a random walk when forecasting real exchange rates variations one quarter ahead. Figures of predicted versus observed real exchange rate variations are shown in the Appendix.

Engel et al (2007) perform similar tests based on panel data regressions, for the same set of 17 countries, using the monetary model of the exchange rate. Although their long-horizon predictability results are positive for most countries, their short-horizon results work well only in 4 countries. The failure of the monetary model in predicting exchange rate

variations on short-run horizons can be explained by the model's central assumptions. Namely, Purchasing Power Parity (PPP) and Uncovered Interest Parity (UIP) fail to hold in the short run according to the literature on international finance<sup>10</sup>. An alternative explanation for the failure of the monetary model is that fundamentals have a unit root and in addition, the discount factor is near one; in this case, exchange rates have a near-random-walk behavior as shown by Engel and West (2005).

The consumption-based model presented in Section 2 is based on an arbitrage condition for international asset markets and its relation with consumers' stochastic discount factors (Equation 11). Therefore, this approach does not need to assume PPP nor UIP in order to derive the forecasting equation. Additionally, since we use domestic and international consumption growth as fundamentals, we do not have to deal with I(1) fundamentals. Finally, predictability power in this framework is an implication of the presence of consumption habits. Namely, it comes from the effect of past consumption growth on current marginal utility and thus on stochastic discount factors which domestic and foreign investors use to price international financial assets.

## 5. ALTERNATIVE FORECASTING WINDOWS

Rogoff and Stavrageva (2008) argue that it is very important to check for robustness of the results to alternative sizes of the rolling windows in order to make sure that the estimated relationship remains stable.<sup>11</sup> They perform this kind of robustness check to the results of Molodtsova and Papell (2009), Engel et al (2007) and Gourinchas and Rey (2007). Their results show that the out-of-sample predictability evidence weakens when tests are computed with narrower forecast windows or, equivalently, when longer samples are used to compute the parameters. The only exception is Gourinchas and Rey's model since its predictability evidence is stable across all forecast rolling windows.

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<sup>10</sup> See the papers by Rogoff (1996) and Taylor and Taylor (2004) on the failure of the PPP hypothesis, and Fama (1984) on the UIP hypothesis.

<sup>11</sup> The size of the forecasting rolling window is the number of out-of-sample forecasts used to compute the predictability tests. It is defined as  $P$  in Equation (24), i.e. it is the difference between the sample length and the number of observations used to compute the parameters of the regression.

TABLE 2  
CW Test for Alternative Rolling Forecasting Windows

Country	Number of Observations Used for Parameter Estimation					
	30	40	50	60	70	80
UK	1.68**	3.15***	1.68**	1.6**	0.83	0.13
Austria	2.38***	2.71***	1.61*	-0.76	-0.16	-0.51
Belgium	0.44	1.10	2.06**	1.29*	-2.09	-0.83
Denmark	4.09***	3.02***	2.07**	1.23	-0.87	-1.67
France	3.43***	2.86***	1.62*	0.78	0.27	0.42
Germany	3.62***	3.25***	1.98**	-0.33	0.25	-0.23
Netherlands	1.37*	2.42***	2.28**	1.91**	1.62*	-1.68
Canada	4.58***	2.88***	1.68**	1.46*	1.16	0.46
Japan	2.84***	1.75**	1.95**	0.98	1.45*	1.05
Finland	3.85***	3.62***	2.4***	1.78**	0.85	0.68
Spain	6.17***	6.52***	4.99***	4.36***	4.19***	4.28***
Australia	2.78***	2.18***	0.82	1.46*	0.99	0.18
Italy	2.87***	3.62***	2.05**	1.44*	0.96	-0.53
Switzerland	2.50***	3.15***	1.7**	-0.50	0.57	-1.12
Korea	1.58*	1.25	1.51*	1.45*	1.20	1.15
Norway	2.98***	2.79***	1.22	0.76	0.36	0.06
Sweden	2.78***	3.66***	2.24**	1.38*	1.27	1.36*
Overall	16/17	15/17	15/17	10/17	3/17	2/17

\* denotes significance at 10% level; \*\* denotes significance at 5% level; \*\*\* denotes significance at 1% level.  
This table presents the Clark-West predictability test for alternative forecasting windows. The significance of the tests is evaluated according to their asymptotic critical values.

We perform a similar procedure in order to evaluate the robustness of our results, presented in Section 4, to alternative sizes of the forecasting window. Therefore, the Clark-West test is computed for each country and for six alternative sizes which range from 60 to 110 observations, or equivalently, from 80 to 30 observations used to compute the regression parameters. Results are presented in Table 2.

Our results are similar to those in Rogoff and Stavrakeva (2008) for the monetary and Taylor-rule models. Table 2 shows that the predictability results from the consumption-based model are only robust when 30 to 60 observations are used to estimate the parameters of Equation (19). When 70 or more observations are employed, this evidence weakens



notoriously across countries. When the estimations are performed with a sample size of 80 observations, positive evidence only holds for two countries, Spain and Sweden. These results show the possible time-varying nature of the parameters on the consumption-based model, especially, those related to habit persistence.

## 6. IN-SAMPLE ESTIMATION OF THE PARAMETERS OF THE MODEL

The goal of this section is to perform a direct, country-by-country estimation of the structural parameters related to habits and to risk aversion, namely:  $\gamma_i$ ,  $\gamma_{us}$  and  $\alpha$ , in Equation (12). We perform this estimation with a non-linear GMM approach following Hansen (1982). The remaining parameters ( $\beta$ , and  $D$ ), are assumed to take values previously calibrated in order to make this model compatible with several international markets moments<sup>12</sup>. Namely, we assume  $D = 0.18$  and  $\beta = 0.95$ .

This method consists of estimating the sample equivalent of the conditional expectation of Equation (12) by using country-by-country data on real exchange rates and consumption growth. We use three contemporaneous consumption-growth measures (domestic, US and world) as instruments for the GMM estimation. As a result, this set-up gives 4 moment conditions for each country which allows estimating three parameters. This specification assumes that consumption growth data is independent and identically distributed every period. Therefore, the errors from the forecasting equation remain orthogonal to contemporaneous consumption innovations.

A continuously updating GMM estimation method is applied where the initial weighting matrix is proportional to  $Z$ , the matrix of instruments. Namely, the initial matrix is the following:  $W_0 = (Z'Z)^{-1}$ . In the second step, the optimal weighting matrix, which is the inverse of the spectral density matrix, is applied to the estimation. This optimal matrix is re-estimated in the following iterations until an appropriate convergence criterion is reached. Standard errors are computed following Hansen's (1982) GMM asymptotic theory<sup>13</sup>.

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<sup>12</sup> Ojeda-Joya (2010) performs a calibration of a very similar model in order to match the first and second moments of equity returns and exchange rate returns in G7 economies.

<sup>13</sup> Cochrane (2005) and Cliff (2003) are also used as references and guides for this GMM estimation.

TABLE 3

## In-Sample Non-Linear GMM Estimation of Parameters

Country	A.Gamma $i$	B. Gamma US	C. Alpha	F-Test	J-Test
UK	4.89**	4.85**	1.22**	33.53***	2.44
Austria	3.94	2.28	1.16	13.65***	3.49*
Belgium	4.34***	4.26**	1.50*	14.52***	0.88
Denmark	1.12	1.44	5.63	7.09*	0.07
France	5.65*	5.55	1.03*	151***	3.09*
Germany	5.95**	6.07	0.97*	268***	0.05
Netherlands	3.07	4.41	1.14	17.06***	0.00
Canada	1.32	5.52	0.99***	234***	1.23
Japan	5.21	-1.14	1.07	77.4***	2.41
Finland	7.69***	15.17***	0.76***	27.3***	0.58
Spain	7.19***	9.76	0.7***	49.4***	3.25*
Australia	5.43***	5.3***	1.11**	105***	0.49
Italy	6.34***	24.20	0.96***	55.9***	1.38
Switzerland	5.48	1.70	1.04**	89***	5.07**
Korea	5.56***	5.42**	1.03**	157***	0.03
Norway	5.83***	5.83**	0.99***	356***	2.99*
Sweden	5.08*	4.84	1.14*	39.45***	2.50

\* denotes significance at 10% level; \*\* denotes significance at 5% level; \*\*\* denotes significance at 1% level.

This table presents country-by-country estimations of habit-related parameters from Equation (12) using the total sample. The method of estimation is non-linear GMM with 4 instrumental variables. The F-test corresponds to the test for the joint significance of these three parameters. The J-test corresponds to the test for over-identifying restrictions.

The results from the non-linear GMM estimation show that the habit persistence parameter for country  $i$  ( $\gamma_i$ ) is significantly different from zero in 11 out of 17 countries. The average value of this parameter, across all the significant cases, is 5.81. Table 3 also shows that a country-by-country estimation of  $\gamma_{us}$  gives out a heterogeneous set of estimated values. This estimated parameter for the US is, however, only significant in 6 countries and its average value is 6.81.

The estimated value of the risk aversion parameter ( $\alpha$ ) is significantly different from zero in 13 out of 17 cases. Surprisingly, it is very close to 1 in many occasions. In fact, the average estimated value across countries is 1.03. This relatively low degree of risk aversion means that the most important determinant of the volatility of exchange rates is the extent of

habit persistence. Therefore, this estimation helps to understand our predictability results by showing that the presence of habits in the utility function is consistent with data for most countries. Thus, the presence of consumption habits is a missing link which should be incorporated in exchange rate modeling.

Table 3 also shows that the joint significance test for all three parameters imply that they are jointly significant in all countries. However, according to the test for over-identifying restrictions there are five countries (Austria, France, Spain, Switzerland and Norway) for which the selection of instruments may not be the most appropriate and can be further improved.

## 7. CONCLUSIONS

Engel et al (2007) and Rogoff and Stavrageva (2008), among others, describe that it is very difficult to obtain good out-of-sample predictability evidence for the exchange rate in short-run horizons with the existing models in the literature. Therefore, the puzzle described by Meese and Rogoff (1983) still seems to hold in the case of horizons shorter than one year. Recently, positive evidence in short-run horizons has been found by Molodtsova and Papell (2009), using the Taylor-rule approach, and by Gourinchas and Rey (2007) using an external-balance model.

This paper provides an alternative model to study short-run real exchange rate predictability using out-of-sample tests. This framework is an open-economy extension of the model studied by Abel (1990, 2006). It can be described as a consumption-based asset-pricing model with  $N$  countries and complete markets, such that real exchange rate variations are determined by fluctuations in the difference between Stochastic Discount Factors (SDF) across countries.

We show that when preferences include internal and external habit persistence, SDFs are driven by past consumption growth and therefore real exchange rate variations are predictable with consumption data. In other words, habits imply that current consumption growth allows predicting the valuation of financial assets through the effects of consumption on future SDFs. Furthermore, the functional form of the utility function allows deriving an

empirical specification for the real exchange rate as function of the following predictors: domestic consumption growth, US consumption growth and world consumption growth.

Predictability tests with data for 17 developed economies, show good out-of-sample evidence in 15 countries. Additionally, the relevance of this habit-based approach is confirmed through a direct estimation of the key parameters of the utility function using non-linear GMM methods. The estimated habit-related parameters are statistically significant for most countries.

This consumption-based framework to study exchange rates is an alternative approach to long-run risk as in Colacito and Croce (2011). Therefore, it can be useful to perform future studies on macro-financial linkages in open-economy environments. Furthermore, this kind of habit-based utility functions can also be potentially incorporated to asset pricing models with disaster risk (Gourio et al, 2013) in order to address further stylized facts in international environments.

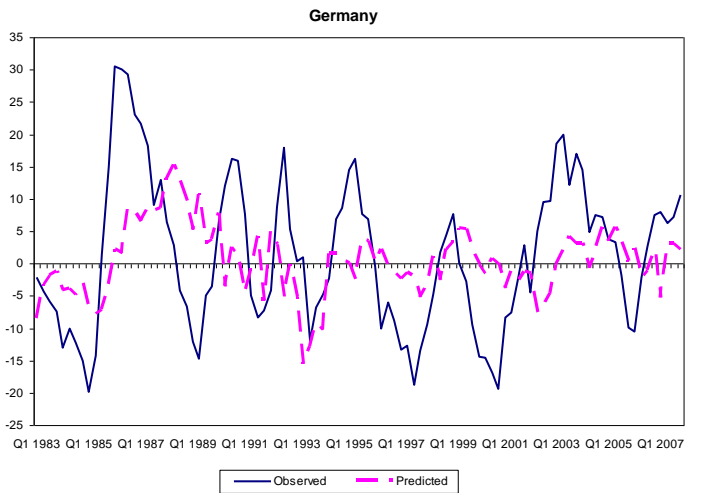
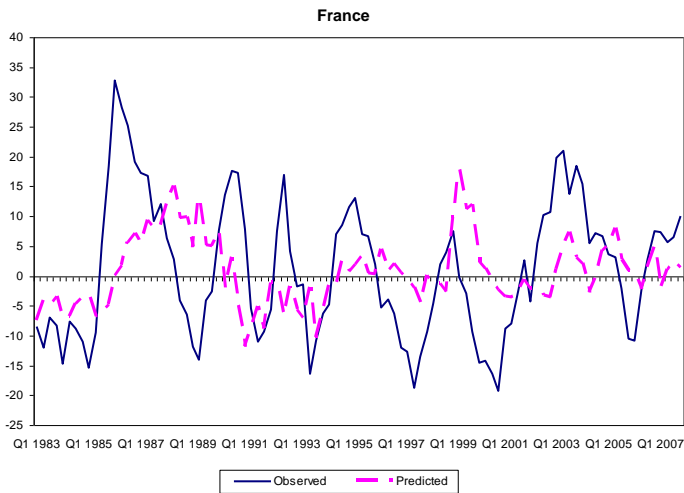
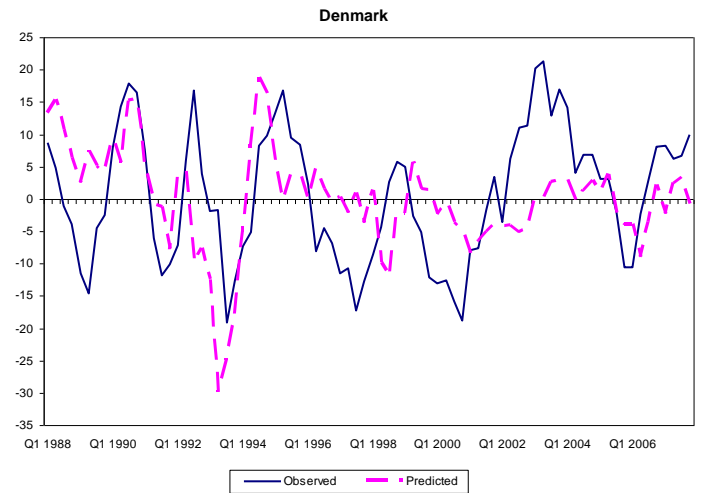
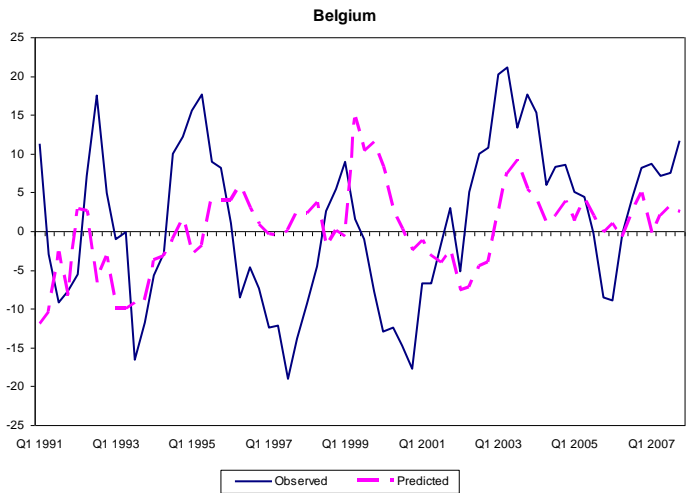
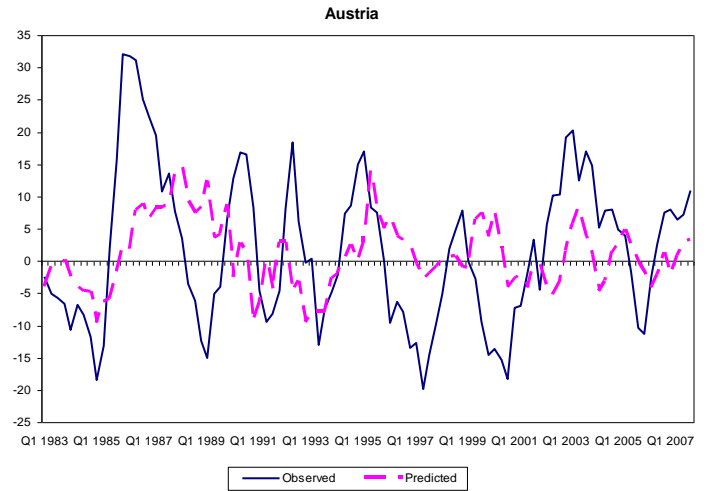
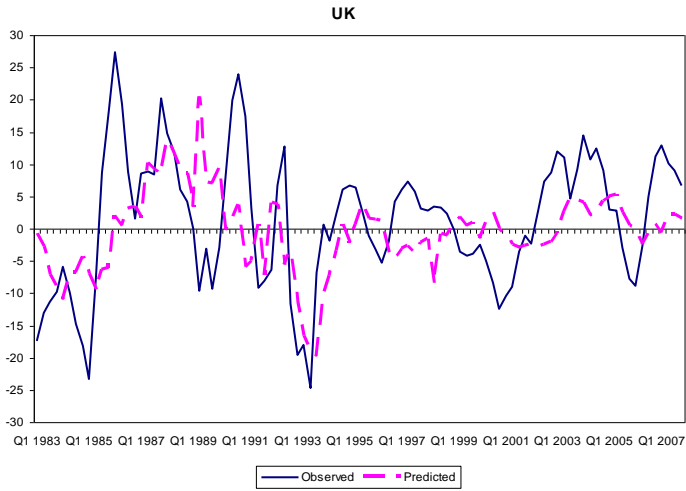
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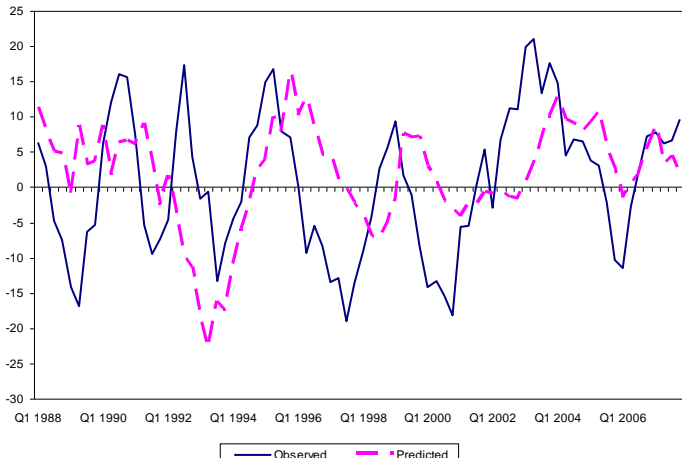
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# Appendix A: Figures for Annual Variations of the Real Exchange Rate: Observed Versus Predicted.

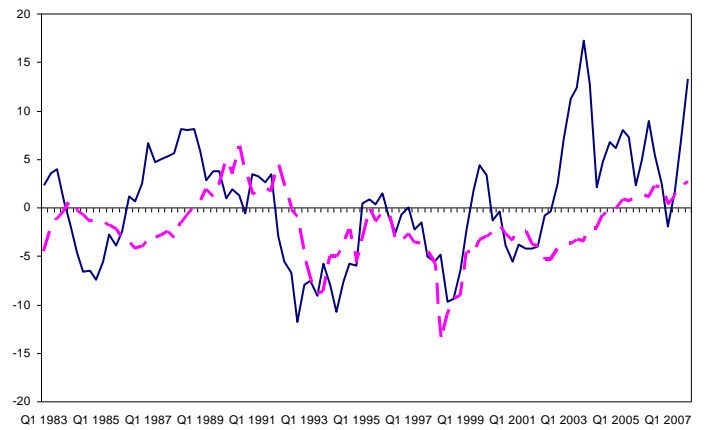




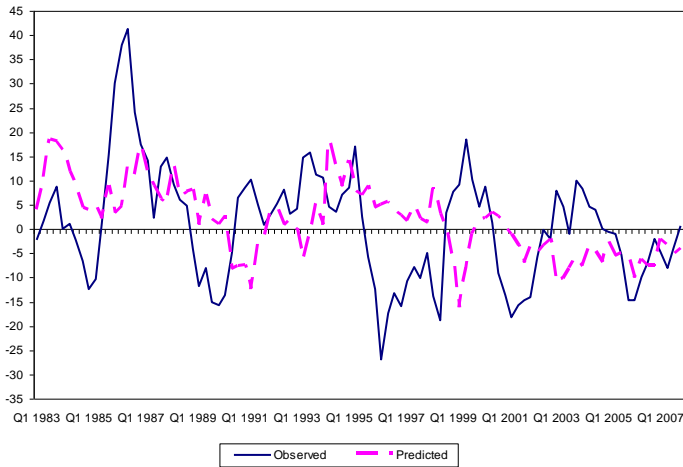
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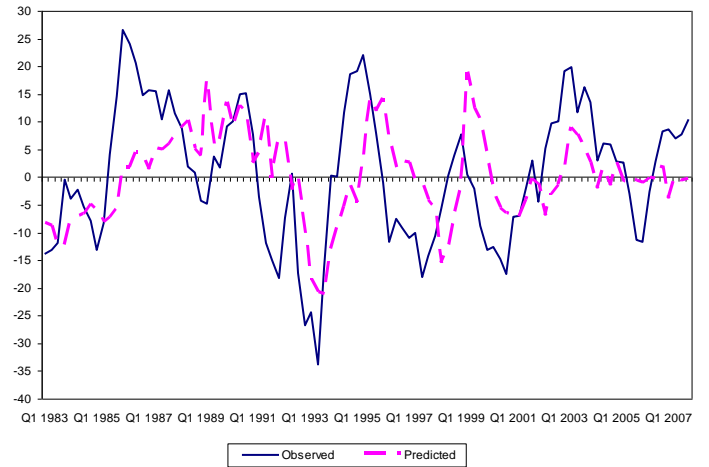
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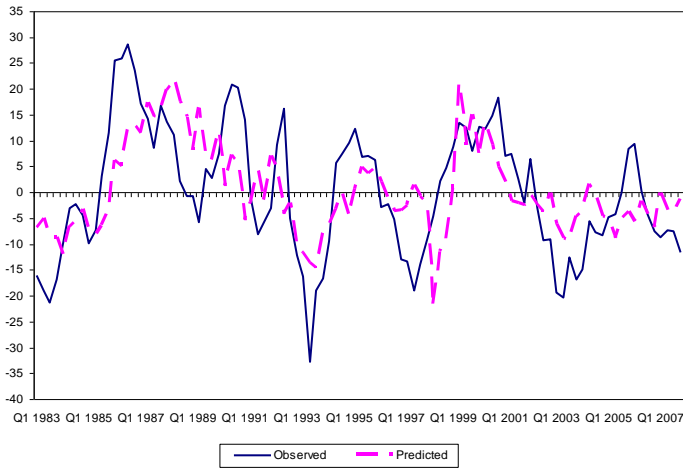
Japan



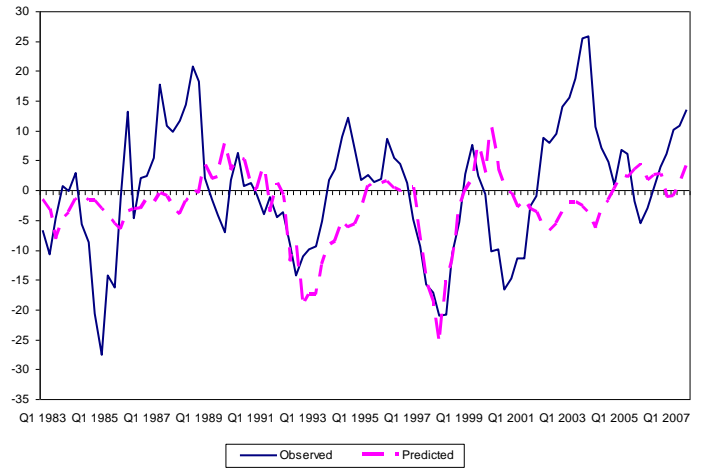
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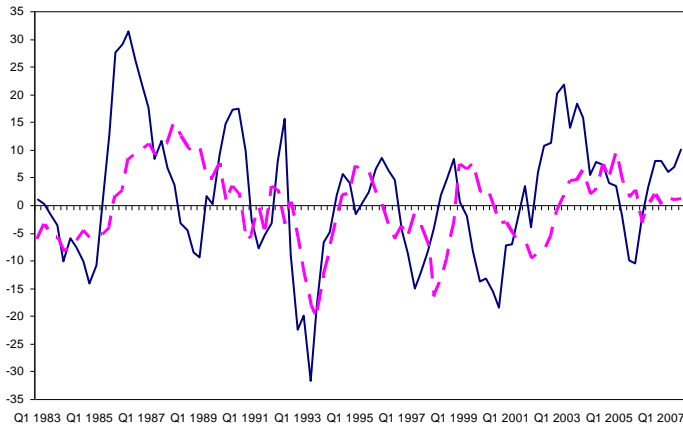
Spain



Australia

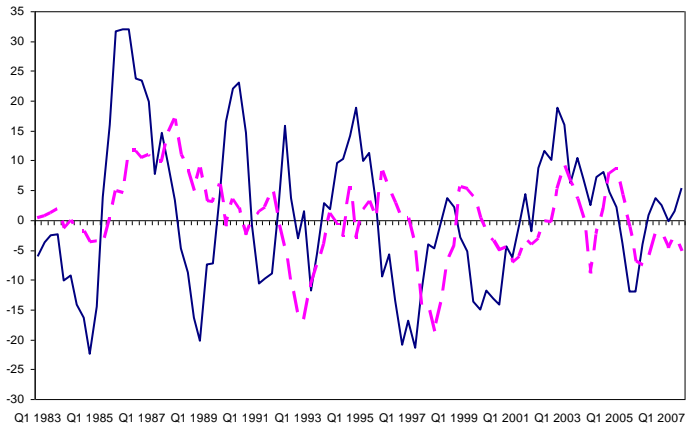


Italy



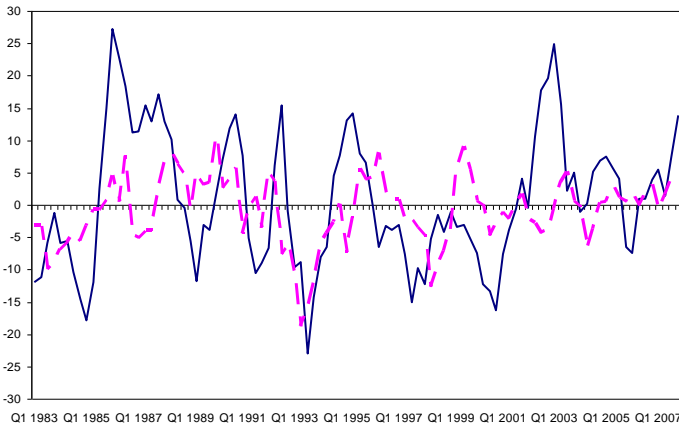
— Observed    - - - Predicted

Switzerland



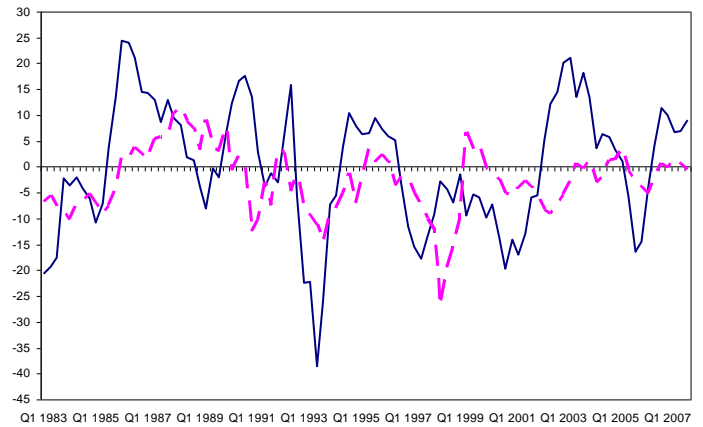
— Observed    - - - Predicted

Norway



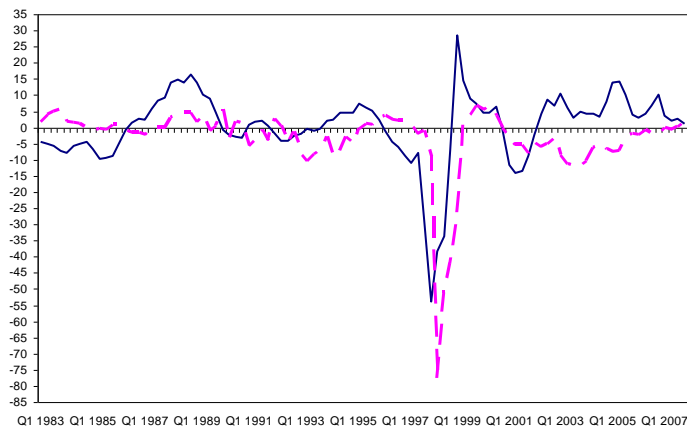
— Observed    - - - Predicted

Sweden



— Observed    - - - Predicted

Korea



— Observed    - - - Predicted

## Appendix B: Data Description

Data consists of quarterly real exchange rates series (RERs) and real per-capita consumption for 18 countries including the United States (US). In order to smooth the seasonality of these data, we use annual variations of the natural logarithm of these variables to perform all the estimations.

RERs are constructed with the consumer price index (CPI) and the average official exchange rate with respect to the US dollar for each country. These data were retrieved from the International Monetary Fund's (IMF) International Financial Statistics (IFS). Log-RERs are computed according to the following formula:

$$q_{i,t} = e_{i,t} - p_{us,t} + p_{i,t} . \quad (A1)$$

In Equation (A1),  $q_{i,t}$  is the log-RER,  $e_{i,t}$  is the log-nominal exchange rate and  $p_{i,t}$  corresponds to the log-CPI for country  $i$ . An increase of the RER, according to this definition, corresponds to a real appreciation of country  $i$ 's currency. For those countries in the European Monetary Union, RERs are computed using the Euro/US-Dollar nominal exchange rate after 1999.

Real per-capita consumption is constructed with the nominal series on households' consumption of non-durable goods and services for each country. These series are deflated with CPI data and turned per-capita with total-population data. The computation of world consumption series is performed as described in Equation (3) and using the weights described in Table A1.

TABLE A1

Weights Used for the Computation of World Consumption

Country	GDP 2007 Billion of US Dollars	Weight %
UK	2148	6.6%
Austria	289	0.9%
Belgium	336	1.0%
Denmark	182	0.6%
France	2059	6.3%
Germany	2623	8.0%
Netherlands	567	1.7%
Canada	1127	3.5%
Japan	4229	12.9%
Finland	185	0.6%
Spain	1221	3.7%
Australia	699	2.1%
Italy	1789	5.5%
Switzerland	305	0.9%
Korea	1152	3.5%
Norway	203	0.6%
Sweden	317	1.0%
US	13233	40.5%
Overall	32664	100.0%

This table describes the weights which are used to compute world consumption in Equation (3). These weights correspond to the relative size of each country's GDP. GDP data is retrieved from the World Bank's World Development Indicators. These GDP data are adjusted by PPP using the World Bank Methodology.