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An Asset Allocation Framework with Tranches for Foreign Reserves*

Julián David García-Pulgarín†, Javier Gómez-Restrepo‡ & Daniel Vela-Barón§

Abstract

This document explores an alternative strategic asset allocation framework for foreign exchange reserves, whose main purpose is to maximize the risk-adjusted returns maintaining the objectives of liquidity and safety of a foreign reserves' portfolio. The overall portfolio can be fragmented into two tranches. On the one hand the Safety Tranche is comprised of liquid, almost default-free and low volatile assets, where the financial goals of safety and liquidity are met. On the other hand, the Wealth Tranche aims to maximize the return with a broader range in the asset space and a longer investment horizon. It is found that through this framework both the historical and forward looking performance of an aggregate portfolio is improved, while maintaining the safety and liquidity needs of a traditional foreign exchange reserves portfolio.

JEL classification: E58, G11, C61, F30

Keywords: Foreign Exchange Reserves, Reserves Adequacy, Strategic Asset Allocation, Investment Horizon, Mental Accounting

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Un Enfoque para la Asignación Estratégica de Activos Reservas Internacionales mediante Tramos[1]

Abstract

Este documento explora un enfoque de asignación estratégica de activos para las reservas internacionales, cuyo objetivo principal es mejorar su desempeño manteniendo los objetivos de inversión tradicionales de un portafolio de reservas internacionales de liquidez y seguridad. El portafolio global puede ser fragmentado en dos tramos. Por un lado el Tramo de Seguridad que se compone de activos líquidos, casi libres de riesgo de no pago y de baja volatilidad, con el cual los objetivos financieros de seguridad y liquidez son logrados. Por otro lado, el Tramo de Riqueza que tiene como objetivo maximizar el retorno con una gama más amplia de activos admisibles y un horizonte de inversión más largo. Se encontró que a través de este enfoque se mejoran tanto el rendimiento histórico como el retorno esperado a futuro del portafolio global, manteniendo al mismo tiempo las necesidades de seguridad y liquidez de un portafolio de reservas internacionales tradicional.

Clasificación JEL: E58, G11, C61, F30

Palabras Clave: Reservas Internacionales, Nivel Adecuado de Reservas, Asignación Estrategias de Activos, Horizonte de Inversión, Contabilidad Mental

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1. Introduction

The accumulation of foreign exchange reserves has witnessed an unprecedented scale over recent years. The central banks have adopted intervention policies to counter excessive volatility in the foreign exchange market and gathered additional amounts with the purpose of reducing the negative outcomes of balance of payments' crises, such as potential sudden stops or even outflows of capital. Despite the potential benefits that can bring along these high amounts of foreign exchange reserves, authors such as Hviding, Nowak & Ricci (2004) or Chivakul, Llaudes & Salman (2010) have found decreasing advantages of holding high levels of reserves. Among those possible issues is the gap between the lending and the borrowing rates, which can substantially increase the cost of high accumulation policies, moreover the opportunity cost of investing in low return-risk assets. However, some countries may found advantageous to hold similar or higher amounts of foreign reserves compared to peer countries since this may translate into lower borrowing premiums in international markets.

As shown in table 1 the holdings in the central banks have increased substantially during the past fourteen years, particularly in the regions with emerging economies. More noticeable are the increases in the Middle East, Eastern Europe, Africa, Central - Southern Asia and Eastern Asia, with relative changes of 1060%, 747%, 707%, 678% and 663%, respectively, since the year 2000 up to the second quarter of 2014.

Table 1: International Reserves in World Regions (US$ Millions)

	2000	**2005**	**2010**	**2014 (Q2)**
Africa	56,141	180,749	405,137	453,210
America	318,917	478,716	1,193,883	1,395,199
Latin America & The Caribbean	158,091	257,439	647,804	841,319
Northern America	160,827	221,277	546,079	553,880
Asia	1,183,727	3,040,234	6,785,295	8,787,720
Central - Southern Asia	46,781	156,059	344,863	363,908
Eastern Asia	847,879	2,273,417	4,974,103	6,467,736
South-Eastern Asia	187,583	295,268	673,275	778,466
Middle East	101,484	315,490	793,055	1,177,609
Europe	583,368	856,734	1,998,643	2,351,351
Eastern Europe	92,241	339,497	767,189	780,869
Northern Europe	120,619	123,881	241,342	289,572
Southern Europe	117,956	115,193	241,786	254,983
Western Europe	252,552	278,163	748,326	1,025,927
Oceania	19,538	44,329	46,109	62,069

Source: Authors' calculations, IMF (IFS)

Considering the investment objectives among central banks of safety, liquidity and return together with the current high holdings of foreign exchange reserves, this paper proposes a framework in which these financial objectives are maximized, while the implicit costs of reserve holdings are reduced. The framework consists on assessing the adequate level of reserves that allows the central bank to accomplish the safety and liquidity objectives by investing the portfolio in liquid, almost default-free and low volatile securities, as the usually held in a foreign reserve portfolio. These assets are held in what will be denominated as the Safety Tranche. If the current level of reserves is above the adequate level an additional tranche, named the Wealth Tranche, is proposed in which the return guideline can be accomplished, and thus reducing the implicit costs of holding foreign reserves. The procedure by which this tranche is developed follows the same methodological approach of the other tranche, but relaxing the liquidity and volatility constraints using a long term investment horizon. This framework guarantees an appropriate tradeoff between the investment objectives of the international reserves, as mentioned in Fisher & Keeley (2013), without jeopardizing the benefits of holding great amounts of foreign reserves that will decrease the negative outcomes of balance of payments' crises.

This paper is structured in seven sections. The first one is this introduction. The second introduces the methodology to assess the appropriate level of international reserves. Afterwards, the third section describes the methodological framework to estimate the efficient portfolios for each tranche. The subsequent section explains the data used to evaluate the overall framework. The fifth presents the main historical and forward looking results of the framework compared to the usual holdings of a foreign exchange portfolio. On the sixth section the dilemma of having a *mental accounting bias* is discussed. Finally, the seventh section gives some concluding remarks.

2. Appropriate Level of International Reserves

International reserves represent a major resource for emerging economies, as they allow to buffer possible liquidity vulnerabilities within a countries' balance of payments. Consequently, the amount of reserves that a country should hold is very relevant for economic policy. Several authors have approached two different types of methodologies to define the appropriate level of international reserves that a country should accumulate. On one hand, authors like Heller (1966), Ben-Bassat & Gottlieb (1992), Jeanne & Rancire (2011), among others, have pursued an optimal level of reserves, which seeks to maximize a utility function of a central bank that depends on the opportunity cost of holding reserves and on the cost and probability of occurrence of an eventual crisis. However, there is still no consensus in the literature with respect to the accurate measurement of these variables, which makes the results of these models very dependent on the assumptions adopted.

On the other hand, authors like Triffin (1960), Guidotti-Greenspan (1999), Beaufort Wijnholds & Kapteyn (2001) and IMF (2011) have proposed measurements of reserve adequacy, which seek to hedge possible outflows of the balance of payments by accumulating a specific proportion of a macroeconomic variable. In contrast to methods which aim to find an optimal level of reserves, this one is limited by the fact that it disregards the costs of hoarding these assets. Nevertheless, it requires a weaker set of assumptions, which makes it more reliable and robust, and consequently more suitable for policy analysis.

The International Monetary Fund (2011) incorporated the main reserve adequacy indicators into a single measurement:

- M2, which captures possible outflows during a bank-run crisis.
- Short-term debt (STD), which includes possible outflows in the contingency of a balance sheet crisis.

- Other portfolio liabilities (OPL), which captures outflows that occur by the liquidation of foreign short-term portfolio investments in periods of market stress.
- Exports[2] (X), which show possible outflows during a current account crisis.

These authors proposed weights for each variable based on the negative 10^{th} percentile of historical flows, consequently providing some empirical support for this new indicator.

The new metric of the International Monetary Fund (2011) varies according to the exchange rate regime of the country as follows:

$$Fixed: \quad 30\%\ of\ STD + 15\%\ of\ OPL + 10\%\ of\ M2 + 10\%\ of\ X \tag{1}$$

$$Floating: \quad 30\%\ of\ STD + 10\%\ of\ OPL + 5\%\ of\ M2 + 5\%\ of\ X \tag{2}$$

Although the indicator of the IMF (2011) provides a very complete benchmark to measure reserve adequacy, as it considers the major potential risks for an economy in an eventual shock to the balance of payments, its major shortcoming is the fact that it is equally applied to all emerging economies, which tend to have very heterogeneous outflows of capital. Specifically, the IMF (2011) mentions that the level suggested by this metric will be below the adequate levels when the country's exports are dependent on commodity prices, its economic or financial fundamentals are weak, or when the country is highly reliant on remittances, amongst others. Meanwhile, the indicator tends to overestimate actual needs when most of the short term-debt is held by multilateral institutions, when the financial system is highly dollarized and hence its exposure to currency risk is low, or when effective capital controls are in place.

Consequently, it is highly desirable that the reserve adequacy indicator captures the idiosyncratic traits of each country, especially when it is intended for policy guidance. Hence, it is appropriate that the weight of each variable is calibrated based on the historical data of each country or of a sample of economies whose balance of payments is structurally alike and that are exposed to similar external shocks. Gómez & Rojas (2014) studied the Colombian case following the methodology of the IMF, using the variables proposed in IMF (2011) since they represent the greatest short-term potential outflows of the balance of payments for Colombia. However, this might vary between countries and some economies should include other variables such as remittances, repatriation of dividends from multinational companies or even foreign direct investment, among others, given their importance and short-term variability.

After establishing the sample (2003-2012), it is necessary to estimate an index of exchange market pressures (EMP), following Eichengreen, Rose & Wyplosz (1997) [3], in order to identify the periods in which the local currency faces unusual depreciation pressures, as these are the moments in which the international reserves could be required to provide liquidity to the market.

The next step is to calculate the percentage change of each of the selected variables in periods of exchange market pressure with respect to their 12-month average[4]. Then, for each variable Gómez

[2] The authors use exports, instead of the traditionally utilized imports, because they argue that the latter does "not capture risks of external demand collapse and tend to be endogenous to the amount of available financing, and so generally fall during crises, improving the balance of payments" IMF (2011).

[3] Gómez & Rojas (2013) followed this methodology as the IMF (2011).

[4] Unlike the IMF (2011), which uses the percentage change against a 3-year average, Gómez & Rojas (2013) chose a 12-month average because they used monthly data (instead of annual frequency used in IMF (2011)) and needed to compare the variable to a less volatile indicator that captured the medium term behavior of each component.

& Rojas (2014) found the tenth percentile of these changes, which reflects the potential outflows under periods of high stress. Different percentiles can be used according to the risk aversion of the memebers of the board of directors in a central bank. This fact impacts the choice of including a new Wealth tranche as well as its potential size.

For Colombia, this means that the adequate level of international reserves at any period t will be given by:

$$IR_t = 13.3\% \; of \; STD_t + 2.9\% \; of \; OPL_t + 8.7\% \; of \; M2_t + 19.2\% \; of \; X_{12m} \tag{3}$$

Where the sub-index t denotes the aggregate of the variable in month t, and 12m represents the aggregate of the 12 previous months, which is used in the case of exports, since it is the only flow variable in the sample.

This shows that the adequacy indicator estimated in Gómez & Rojas (2014) suggests a higher level of reserves than any of the traditional indexes. This is not surprising as the new indicator is a combined metric that captures more potential outflows of the balance of payments and has a conservative stance as it ignores the existing correlation between these variables.

It is worth mentioning that both the IMF (2011) and Gómez & Rojas (2014) consider very extreme scenarios in which all of the studied components of the balance of payments suffer considerable reversals at the same time, so only a very conservative central bank should guide itself by these exceptional adequacy levels.

Later on, Gómez (2014), focusing in the fact that the metric proposed by the IMF ignores the existing correlation between the variables, develops a new metric to estimate the appropriate reserves level characterized by the use of relative weights of potential outflows of the balance of payments. Also the metric incorporates the calculation of implied correlations among the variables considered. This sheds a less conservative measure, derived on the premise that in a period of pressure in the foreign exchange market the worst case of each variable does not materialize.

In addition, Gómez (2014) discusses some changes that could enhance the calculation of the metric. First, he replaces M2 by M3, since it is a broader monetary aggregate that allows including additional vulnerabilities that could deal with a bank run that might not be captured by M2. Secondly, the author includes foreign direct investment as an additional variable. The relevance of this change is a result of the nature of metric that is perceived as a cover of international reserves to external shocks and vulnerabilities. Third and finally, the author considers the dependence on remittances of some Latin American economies and includes this variable to improve the calculation of the metric.

After calculating the index of exchange market pressure, and understanding the proposed changes to the items to be considered, the next step is to calculate the percentage change of each of the selected variables in periods of exchange market pressure with respect to their 12-month average. Then, for each variable Gómez (2014) found the tenth, fifth and first percentile of these changes, which reflects the potential outflows under periods of high stress. It is worth mentioning that these values are in the farthest extreme of the tail of the distribution, so they show acute potential losses in times of stress in the exchange market. Also, in these extreme scenarios, all of the studied components of the balance of payments suffer considerable reversals at the same time.

Remarking that the variables are highly correlated, which increases when periods of pressure in the foreign exchange market are solely considered, it is confirmed that in times of crisis variables move in the same direction. This means that it would be appropriate to have a parallel coverage for all variables. However, the fact that these correlations are not 100%, indicate that the linear sum between items in the equation of the metric is quite conservative. This leads to develop an alternative metric that takes into account the correlations between the variables in periods of exchange market pressure. This sheds a less conservative measure, derived of the premise that in a period of pressure in the exchange market the worst of each variable does not materialize. The reserves adequacy range results obtained by this new metric are used in the proposal of this paper. Below is depicted a deeper explanation of the calculation of the new metric.

After finding the periods of pressure in the foreign exchange market, the next step is to take the variations in pressure periods for each variable. Meanwhile, the weights of each variable on the aggregate level of the variables are calculated, as indicated in the following equation:

$$\boldsymbol{\omega}_t = [i_{1t}, i_{2t}, i_{3t}, i_{4t}, i_{5t}, i_{6t}] \times \frac{1}{\sum_{j=1}^{6} i_{jt}} \tag{4}$$

where $\boldsymbol{\omega}_t$is the weight vector of each variable at time t and $i_{jt}$is the value of each variable j, where $j = 1$ corresponds to DCP, $j = 2$ to PP, $j = 3$ to X, $j = 4$ to M3, $j = 5$ to IED and $j = 6$ to remittances.

With this, a product of the associated vectors to the weights of each variable and the percentage changes in each variable during periods of market pressure ($\boldsymbol{MP}$) is done (this is done for each period considering the same sample periods of pressure), as shown below:

$$\%NARI_t = \boldsymbol{MP} \times \boldsymbol{\omega}_t^T \tag{5}$$

Where $\%NARI_t$ is the percentage of adequate international reserves level relative to aggregate variables for period t. After this, the percentiles for each period (of the resulting set product vectors) are calculated, and then are multiplied by the aggregate level of the variables for each period:

$$NARI_t = P_{(10,5,1)} \left\{\%NARI_t\right\} \times \sum_{j=1}^{6} i_{jt} \tag{6}$$

The levels found at this point will be defined as adequate reserves levels ($NARI_t$). This new methodology takes into account implicitly the correlations between the variables in periods of pressure, making it less conservative compared to the former methodology (linear summation between items).

The adequate levels of foreign reserves estimated by Gómez (2014), result in a proper guideline for an emerging economy in order to reduce its vulnerability to external shocks. First of all, the proposed indicator is more robust than any of the levels found using the methodology of optimal reserves, as it does not rely on parameters that are difficult to estimate, such as the probability of crisis, the cost of an eventual shock and the opportunity cost of reserves. Secondly, it considers a greater number of potential risks, than any of the traditional indicators, so it provides a more thorough estimation of outflows of the balance of payments than any of the traditional adequacy indicators. Thirdly, it considers the existing interaction between the different variables of the balance of payments that compose the indicator. Lastly, since it is built based only on the historical information of each emerging economy, it captures the idiosyncratic traits of the country, which corrects for the possible inaccuracies of the indicator proposed by the International Monetary Fund (2011).

It is important to note that for Brazil and Peru, Gómez (2014) found that current reserves levels are superior compared to the range of reserve adequacy. This suggests, given the cost of accumulating reserves and the criteria under which are invested, that the amount of reserves that exceeds the range defined as the appropriate level should be invested in a different way, that is, creating an alternative investment segment that pursues higher returns without substantially increasing the risks.

Despite all of the advantages of this indicator, it is not a definite measure, as it is subject to several shortcomings, such as the fact that, given the characteristics of emerging economies, it tends to consider a short sample for the different indicators. It also disregards the costs of reserve accumulation. Consequently, this indicator should be used to complement the existing measures.

As the surplus indicates the amount redirected into an alternative tranche it is important to take into account that the adequate level of reserves should be updated periodically in order to adjust the size of the tranches to the conjuncture of each country.

3. Methodological approach to estimate the efficient portfolios

Once the decision of establishing two separate tranches is made, the next step is defining an appropriate benchmark for both of them under a separate framework. As mentioned by Solnik & McLeavey (2003) three main issues should be considered in this matter: (i) scope of the benchmark; (ii) attitude towards currency risk, and (iii) set of weights chosen.

The first issue involves the selection of asset classes; this point is closely related to the risk aversion defined by the top decision-making body for each of the two different portfolios. The approach suggested in this document is to maintain the same level of risk aversion but under separate time horizons. Therefore, a non-linear constraint is defined which limits the portfolios within the efficient frontier to those not resulting in losses contained by a 95% confidence level in the given time horizon.

Regarding the selection of the time horizon, Bodie (1995) proves that when the time horizon increases, the risk of an asset does not decline. The latter considering that, although the probability of a shortfall decreases, the size of the shortfall increases through time, increasing the overall risk along the time horizon. Then, increasing the time horizon by itself does not guarantee a tranche with a different risk-return profile. Consequently, the liquidity and safety constraints of the long-term invested portfolio (Wealth Tranche) are relaxed, in order to allow investment opportunities usually not considered in foreign reserves' portfolios. Therefore, the Safety Tranche will only be limited to the fixed income universe plus gold, whilst the Wealth Tranche will explore other asset classes, mentioned in detail in the next section, as suggested by Fisher & Keeley (2013). A broad asset space represents most of the actual market, which use provides a better estimate of the Black-Litterman's equilibrium returns. It also aligns with highly diversified portfolios that focus on maximizing returns, such as sovereign wealth funds, foundations or endowments. In an implementation phase a central bank should consider its operational, legal, risk aversion and knowledge constraints before deciding which assets are included in the optimization.

Concerning the second issue for developing an appropriate benchmark, different approaches are used including: minimizing the portfolio's and balance sheet's volatility; or using an asset-liability management framework in order to replicate the country's expected obligations in foreign currency, thus establishing a numeraire close to the currency composition of those liabilities. These methods are consistent for the Safety Tranche, which is the main buffer to cover the required liabilities. Given that the main purpose of the Wealth Tranche is to increase the return of the foreign reserves and the low predictability of the movement of the currencies rates, this paper suggests partially hedging the currency risk in this particular portfolio. This allows an appropriate exposure to currencies in order to maximize the numeraire return, which in this case would vary accordingly to the investment and accounting policies of the central bank, USD is used as a reference.

As explained by Black (1995), investors in different countries can maximize their expected returns by taking some currency risk in their portfolios, this given the effect of the "Siegel's paradox"[5], where the average gain for one currency exceeds the average loss of the other currency. Therefore with the universal hedging formula one can define the appropriate holdings to be hedged (h), taking into account the average across countries of the expected excess return of the world market portfolio (μ_m), the average across countries of the volatility of the world market portfolio (σ_m), and the average across all pairs of countries of the exchange rate volatility (σ_e).

$$h = \frac{\mu_m - \sigma_m^2}{\mu_m - \frac{1}{2}\sigma_e^2} \tag{7}$$

As soon as the first two points are covered one can proceed to determine the weights within the portfolios. The methodological approach applied in this document follows the Black & Litterman (1991) framework with some particular enhancements in the estimation of the covariance matrix. The Black & Litterman (BL) model has been praised for its low turnover and the avoidance of corner solutions, given the use of the global Capital Asset Pricing Model (CAPM) equilibrium as a center of gravity. Additionally, this model allows the investor to incorporate particular views over the asset classes through a Bayesian approach.

The main purpose of the optimization process is to maximize a utility function that considers the first two moments of each portfolio return distribution, as well as the specific risk aversion (λ) of each investor. Let μ_p and σ_p^2 be a portfolio's expected return and variance, respectively, the utility function for selecting the investment portfolio is defined by:

$$U_p = \mu_p - \frac{1}{2}\lambda\sigma_p^2 \tag{8}$$

In order to get the estimates of μ_p and σ_p^2 one should estimate the expected returns of the available assets (μ_i) as well as the covariance matrix among the asset universe ($\mathbf{\Sigma}$). For the first one this paper follows the market equilibrium framework explained by Black & Litterman (1991), which

[5] A straight forward explanation is given by Black (1995). Two countries are defined, with an initial exchange rate of 1:1, but that will change over the next year to either 2:1 or 1:2 with equal probability. Given the paradox, an individual in one of the countries will gain in expected return by trading his domestic consumption goods to the foreign consumption goods. At the end of the year the foreign consumption goods will be worth either 2 or 0.5 domestic consumption goods, which expected value are 1.25 domestic consumption goods, 0.25 higher than the alternative of keeping the domestic consumption goods.

estimates the returns assuming that the current market capitalizations are in equilibrium. Thus through an inverse optimization[6] one can reach the n-dimensional vector ($\mathbf{\Pi}_{mkt}$) of the market-implied expected excess returns over the risk free rate as shown in equation 9.

$$\mathbf{\Pi}_{mkt} = \lambda_{mkt} \mathbf{\Sigma} \boldsymbol{\omega}_{mkt} \tag{9}$$

Hence $\mathbf{\Pi}_{mkt}$ is the set of expected excess returns that would clear the market if all investors had identical views. The market weights ($\boldsymbol{\omega}_{mkt}$) are gathered from the current market capitalization. The market's risk aversion (λ_{mkt}) is approximated by estimating the observed (historical) balance of additional excess return against additional risk[7]. In addition the covariance matrix is estimated through historical weekly data with three particular adjustments:

i) Exponential Decay: the recent data is given more weight following the EWMA model with a decay factor (η) equal to 0.99; which is the value that minimizes the mean square error between the estimated covariance with the EWMA (Exponential Weighted Moving Average) model and an observed variance measure with squared returns in a weekly frequency. Hence, given a total number of observations T and the last observation t, each element of the covariance matrix, given by the covariance between factors i and j, which follows:

$$\sigma_{i,j} = \frac{\sum_{s=1}^{T} \left(r_{j,t+s} - \bar{r}_j\right)\left(r_{i,t+s} - \bar{r}_i\right)\eta^{T-s}}{\sum_{s=1}^{T} \eta^{s-1}} \tag{10}$$

where

$$\bar{r}_i = \frac{1}{T} \sum_{s=1}^{T} r_{i,t+s} \tag{11}$$

The benefit of this adjustment consists on estimating a variance that reflects more recent events in the market, without completely obviating previous economic cycles.

ii) The second enhancement consists on an adjusted version of the Hurst exponent that solves the inappropriate long-term-independence-of-returns assumption. Following León & Reveiz (2010) the

[6] Assuming a quadratic utility function, its maximization is solved by setting the first derivative of the function with respect to portfolio's weights ($\boldsymbol{\omega}$) equal to zero, which results in the optimal weights ($\boldsymbol{\omega}^*$) assigned to each asset within the portfolio.

$$\boldsymbol{\omega}^* = \frac{1}{\lambda} \mathbf{\Pi} \mathbf{\Sigma}^{-1}$$

[7] Following Bodie et al. (2001), let quadratic utility function be expressed in terms of excess returns:

$$U_p = \mu_{rf} + \left(\mu_r - \mu_{rf}\right)\Phi - 0.5\lambda_{mkt}\left(\Phi_{mkt}^2 \boldsymbol{\omega}_r \mathbf{\Sigma}_r \boldsymbol{\omega}_r'\right)$$

where portfolio's utility is a function of the expected return of the market's portfolio of risky assets (μ_r); the expected return of the market's risk-free asset $\left(\mu_{rf}\right)$; the risky asset's covariance matrix ($\mathbf{\Sigma}$); the market's portfolio of risky assets' weights ($\boldsymbol{\omega}_r$); the market's risk aversion (λ_{mkt}), and the markets' preference (weight) for risky assets (Φ_{mkt}). Utility maximization problem is solved by setting the first derivative with respect to Φ_{mkt} equal to zero, which is conveniently solved for λ_{mkt}:

$$\lambda_{mkt} \cong \frac{\mu_r - \mu_{rf}}{\Phi_{mkt} \boldsymbol{\omega}_r \mathbf{\Sigma}_r \boldsymbol{\omega}_r'}$$

Hurst exponent is used in order to estimate the true long-term-serial-dependence of returns and to adjust the estimated covariance matrix according to the investment horizon, by doing this one avoids a concealed risk taking due inadequate risk estimations. As shown by León & Vela (2011) the high-frequency estimated covariance between assets i and j $\left(\hat{\sigma}^2_{\{(i,j),hf\}}\right)$ is scaled to low-frequency estimation $\left(\hat{\sigma}^2_{\{(i,j),lf\}}\right)$ with the respective estimation of the adjusted Hurst exponents for the same assets $\left(\check{H}_i \& \check{H}_j\right)$ as follows:

$$\hat{\sigma}^2_{\{(i,j),lf\}} = \left(m^{\check{H}_i+\check{H}_j}\right)\left(\hat{\sigma}^2_{\{(i,j),hf\}}\right) \tag{12}$$

where m is the number of high-frequency periods which compose a low-frequency period (e.g. 252 days in a year).

iii) The third adjustment follows Ledoit & Wolf (2003) shrinkage estimate of the covariance matrix that consists of an optimally weighted average of two estimators, the sample covariance matrix with the previous adjustments (**S**) and a single-index covariance matrix (**F**). This estimator accounts for extra-market covariance without having to specify an arbitrary multifactor structure, thus a more efficient estimate is withdrawn by combining an unbiased and very variable estimator with a bias and low variable estimator. The estimate of the covariance matrix is:

$$\hat{\boldsymbol{\Sigma}} = \hat{\delta} * \boldsymbol{F} + (1-\delta) * \boldsymbol{S} \tag{13}$$

where δ denotes the shrinkage intensity that results from minimizing a loss function, which consists of a quadratic measure of distance between the true and the estimated covariance matrices based on the Frobenius norm[8]. Conversely, the matrix **F** is given by:

$$f_{ii} = s_{ii} \quad and \quad f_{ij} = \bar{\hat{\varrho}}\sqrt{s_{ii}s_{ij}} \tag{14}$$

where $\bar{\hat{\varrho}}$ stands for the average correlation of the sample:

$$\bar{\hat{\varrho}} = \frac{2}{(N-1)\,N}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\varrho}_{ij} \tag{15}$$

Once the covariance is estimated, the market-implied excess returns $(\boldsymbol{\Pi}_{mkt})$ can be calculated. In terms of the returns used in the quadratic optimizations in the mean-variance space in this paper, the equilibrium returns are the ones selected, considering that, as explained by Zimmerman (2003), they reflect the normal behavior of an average investor and represent a hypothetical passive manager. Hence, the equilibrium- distribution of expected excess returns $(\boldsymbol{\Pi})$ is given by:

$$\boldsymbol{\Pi} \sim N\left(\boldsymbol{\Pi}_{mkt}, \tau\boldsymbol{\Sigma}\right) \tag{16}$$

where τ stands for a scalar measuring the uncertainty of the market equilibrium as a neutral reference.[9]

[8] Matrix norm of an $m \times n$ matrix defined as the square root of the sum of the absolute squares of its elements.

[9] A small value of τ corresponds to a high confidence in the equilibrium return (CAPM) estimates. According to Idzorek (2002) it is customary to use a τ value close to zero (e.g. $0.01 < \tau < 0.5$). This document uses $\tau = 0.025$.

However, the BL model permits blending the equilibrium returns with the investor's views with a self-defined degree of confidence. This is done through a Bayesian approach, where the revised expected excess returns are given by:

$$\bar{\mathbf{\Pi}} = \left[(\tau\mathbf{\Sigma})^{-1} + \boldsymbol{P}'\mathbf{\Omega}^{-1}\boldsymbol{P}\right]^{-1}\left[(\tau\mathbf{\Sigma})^{-1}\mathbf{\Pi}_{mkt} + \boldsymbol{P}'\mathbf{\Omega}^{-1}\boldsymbol{Q}\right] \tag{17}$$

where $\mathbf{\Omega}$ denotes the (kxk) covariance matrix of views' errors;[10] $\boldsymbol{P}$ stands for a (kxn) matrix with each row representing a view portfolio, where an element of $\boldsymbol{P}$ is nonzero if the respective asset is involved in the view and zero otherwise[11]; and $\boldsymbol{Q}$ is a k-vector that contains investor's views on each asset's expected returns.

4. Data Description

The selected weekly data varies accordingly with the tranche in it which it is used. The Safety Tranche's asset universe is limited to gold and fixed income securities, taken from the Merrill Lynch database, of the G10[12] currencies. These assets resemble those usually held in the asset allocation of foreign reserves. Conversely, the Wealth Tranche broadens the asset space and incorporates fixed income securities of emerging markets plus Denmark, Singapore, Hong Kong and Korea, equities (small cap and large cap from US, non US developed and emerging countries), private equity, hedge funds and real estate. These data series are gathered form Bloomberg, among different vendors including S&P, MSCI and Russell.

The length of the data goes back to December of 1998. With the exception of the government bond indices of China and South Korea, which are only available since 2004 and 1999, respectively. These missing values are estimated with the methodology approached by Page (2013). The model combines the maximum likelihood estimation (MLE) approach of Stambaugh (1997) with conditional sampling to backfill missing data. Instead of assuming that the noise term is normally distributed, the model recycles residuals from the short sample.

The vector of m missing returns at time t ($\boldsymbol{Y}_t$) conditioned on the vector of n known returns ($\boldsymbol{X}_t$) is given by the vector of quasi-MLE-estimated mean of the missing returns $\left(\hat{\boldsymbol{\mu}}_{Y,L}\right)$ of the long sample (L), the mean of the known returns over the long sample $\left(\boldsymbol{\mu}_{X,L}\right)$, the estimated covariance for the missing returns over the long sample $\left(\hat{\mathbf{\Sigma}}_{YX,L}\right)$, and the calculated covariances for the known returns over the long sample $(\mathbf{\Sigma}_{XX,L})$ as shown in equation 18.[13] Thus, the expected value of

[10]BL original model assumes that all views are independent from each other, which would result in Ω being a diagonal matrix where non-zero terms correspond to the variance of the errors of each view.

[11]When a relative view is expressed the elements of a row sum up to zero; when an absolute view is expressed, the corresponding row consists of a 1 in the place of the asset under view and zeros everywhere else.

[12]USD, GBP, EUR, AUD, SEK, NZD, CHF, CAD, NOK & JPY.

[13]The estimation of the mentioned coefficients start with the vector of standard regression coefficients over the short sample:

$$\beta = \left[\mathbf{\Sigma}_{XX,S}\right]^{-1} \times \left[\mathbf{\Sigma}_{XY,S}\right]$$

Once β is determined one can estimate the vector of long sample means and the estimated covariances for $\mathbf{Y}$:

$$\hat{\boldsymbol{\mu}}_{Y,L} = \boldsymbol{\mu}_{Y,S} + \beta\left(\boldsymbol{\mu}_{X,L} - \boldsymbol{\mu}_{X,S}\right)$$

$\boldsymbol{Y}_t$ is a beta factor $\left(\hat{\boldsymbol{\Sigma}}_{YX,L}\boldsymbol{\Sigma}^{-1}_{XX,L}\right)$ times $\boldsymbol{X}_t$, adjusted for the difference in means. Additionally, equation 19 shows that the noise around the expected value of $\boldsymbol{Y}_t$ is a function of the volatility of those missing returns, adjusted by beta times the covariance term. The higher the covariance, the lower the noise around the conditional estimate of $\mathbf{Y}$.

$$E[\boldsymbol{Y}_t|\boldsymbol{X}_t] = \hat{\boldsymbol{\mu}}_{Y,L} + \hat{\boldsymbol{\Sigma}}_{YX,L}\boldsymbol{\Sigma}^{-1}_{XX,L}(\boldsymbol{X}_t - \boldsymbol{\mu}_{X,L}) \tag{18}$$

$$\boldsymbol{\Sigma}_{Y|X} = \hat{\boldsymbol{\Sigma}}_{YY,L} - \hat{\boldsymbol{\Sigma}}_{YX,L}\boldsymbol{\Sigma}^{-1}_{XX,L}\hat{\boldsymbol{\Sigma}}_{XY,L} \tag{19}$$

Moreover, an additional adjustment is made over the data series of the returns of alternative investments, to account for the supplementary volatility that is not regularly considered due to performance and the management fees. Thus, the gross returns are estimated, assuming a management fee of 2% and a monthly performance fee of 20%[14], which increases the volatility of the mentioned assets.

As to the numeraire of the Safety Tranche, a representative currency composition of 86% USD, 5% AUD, 4% CAD, 2% SEK and 3% GBP is selected, which resembles that in Colombia[15]. As mentioned in the previous section, the Wealth Tranche is fully accounted in dollars which includes an optimal hedging percentage of 67.47%, which results from an expected excess return of the market of 2.92%, an expected market volatility of 10.13%, and an exchange rate volatility of 4.80%. Thus, the optimization will incorporate this hedging percentage as an input in the data series.

Concerning the time horizon the Safety Tranche follows the usual central bank's practices, which is usually one year. While for the Wealth Tranche the chosen parameter is ten years, which corresponds to the approximately time frame in which a crisis event happens, assuming a time homogeneous Poisson process and a sudden stop probability of 10%. The latter value follows approaches of Calvo, Izquierdo & Mejía (2004), Jeanne (2007) y Ruiz-Arranz & Zavadjil (2008), among others. The independent jump assumption of a Poisson process is soothed by the fact that

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$$\hat{\boldsymbol{\Sigma}}_{YY,L} = \boldsymbol{\Sigma}_{YY,S} + \beta' \left(\boldsymbol{\Sigma}_{XX,L} - \boldsymbol{\Sigma}_{XX,S}\right)\beta$$

The estimated covariance between the vector of known and missing returns is given by:

$$\hat{\boldsymbol{\Sigma}}_{XY,L} = \boldsymbol{\Sigma}_{XY,S} + \beta\left(\boldsymbol{\Sigma}_{XX,L} - \boldsymbol{\Sigma}_{XX,S}\right)$$

Thus, assuming that $\boldsymbol{X}_t$ is normally distributed vector: $\boldsymbol{X}_t \sim N(\mu, \Sigma)$, where µ comprises the means for $\boldsymbol{X}_t$ and $\boldsymbol{Y}_t$:

$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_{X,L} \\ \boldsymbol{\mu}_{Y,L} \end{bmatrix}$$

with dimensions $[n\, m]'$. Σ is partitioned as follows:

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_{XX,L} & \hat{\boldsymbol{\Sigma}}_{XY,L} \\ \hat{\boldsymbol{\Sigma}}_{YX,L} & \hat{\boldsymbol{\Sigma}}_{YY,L} \end{bmatrix}$$

with dimensions $\begin{bmatrix} n \times n & n \times m \\ m \times n & m \times m \end{bmatrix}$. Once the latter is established, conditional sampling generates missing returns ($\boldsymbol{Y}_t$) conditioned on($\boldsymbol{X}_t$) as shown in equation 18.

[14]These management fees are the standard in the Hedge Fund industry as mentioned in Goetzmann, Ingersoll & Ross (2001), Poloner (2010) and Guasoni & Obłój (2011).

[15]The idea behind the currency composition methodology follows Gómez & Hernandez (2011).

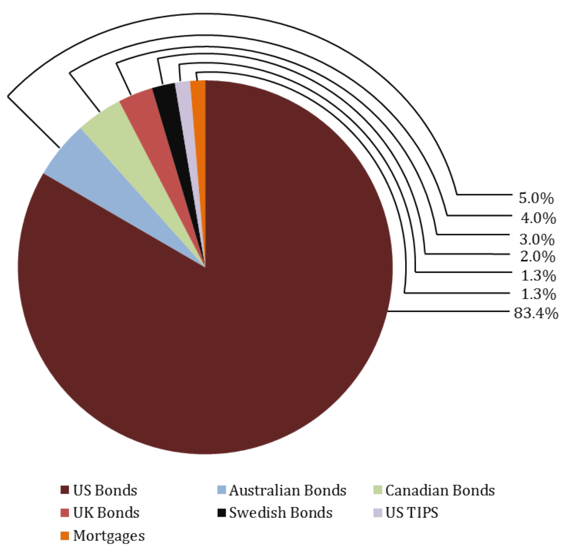
a country under continuous crisis would eventually cease its needs of foreign reserves. Considering that once a sudden stop befalls, the country will cover all its potential outflows under periods of high stress.

Once the available data series are used to optimize the portfolios, the two tranches have to be pooled accordingly with the estimated appropriate level of reserves. This will vary according to the country and the selected percentile of the change in each variable in high pressure periods. For illustrative purposes this paper selects one third of excess reserves that will be allocated into the Wealth Tranche. One of the possible issues that may result as a consequence of distributing the overall portfolio into two tranches is the effect of sub-optimality resulting from a mental accounting behavioral bias. This aspect is analyzed in the sixth section.

5. Results

This section explores the resulting portfolios estimated using the framework mentioned in previous sections, which consists in the two previously defined tranches: a Safety Tranche and a Wealth Tranche. On one hand, the composition of the former is depicted in figure 1; its main objective is to serve as a short-term portfolio investment benchmark for international reserves that covers the needs of safety and liquidity.

Figure 1: Asset allocation of the Safety Tranche

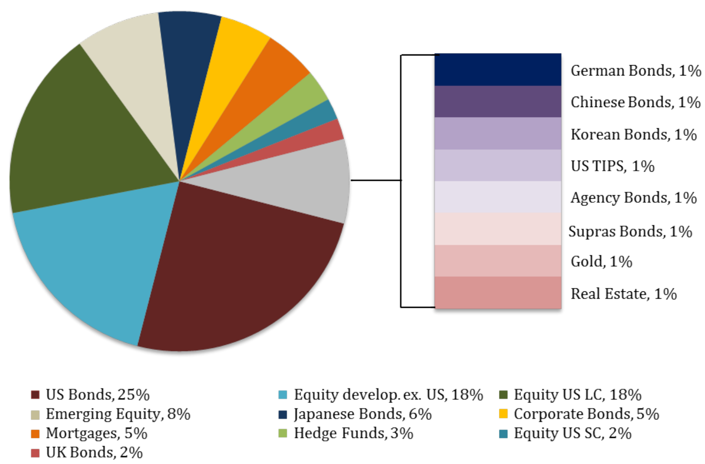


Source: Authors' calculations

The resulting one year investment horizon portfolio is composed entirely of fixed income instruments and concentrates on the short end of the curve (its duration is equal to 0.76) which makes it a low volatility portfolio. The allocation exposed to currency risk is 14% of the portfolio, which is the result of the selected numeraire, as explained in the previous section.

On the other hand, the resulting composition of the Wealth Tranche, by applying the methodology developed in the previous sections with an investment horizon of ten years is shown in Figure 2.

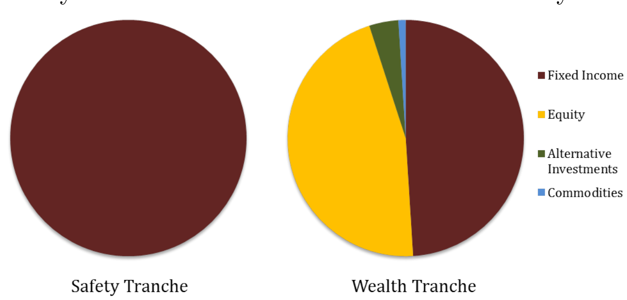
Figure 2: Wealth Tranche asset allocation



Source: Authors' calculations

The resulting portfolio is clearly more diversified in terms of asset classes compared to the short-term portfolio as shown in Figure 3. In addition, as expected, the allocation in fixed income assets has a significantly longer duration (its duration is equal to 5.68). Although, being a portfolio held by a central bank, its composition can vary to that of a long term invested portfolio of an endowment, pension fund, sovereign wealth fund or foundation that may hold a higher percentage of alternative assets.

Figure 3: Safety and Wealth Tranches' asset allocations by asset classes



Source: Authors' calculations

In order to evaluate the historical performance of the Wealth Tranche and the aggregate portfolio against the portfolio with an investment horizon of one year, a backtest exercise is realized in which the methodology described is applied for the available data each month from December 2001 to

September 2014, assuming monthly rebalancing for each portfolio. Table 2 presents the results of the backtesting and Figure 4 shows the evolution of the historical returns for each portfolio.

Table 2: Backtesting main results

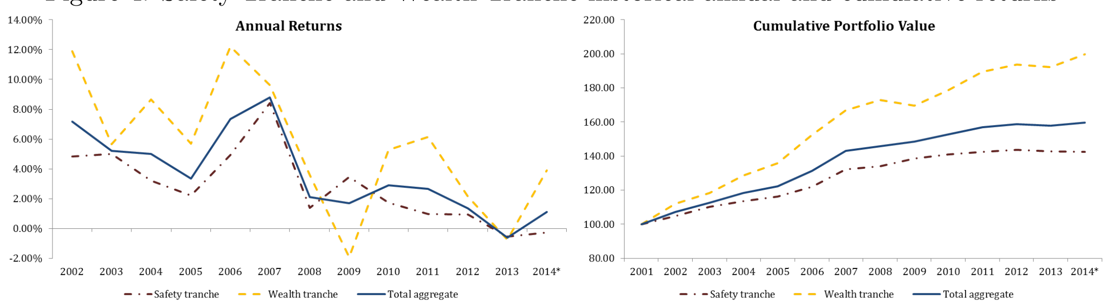
	Safety Tranche	Wealth Tranche	Aggregate portfolio
Mean return (%a.a)	2.79%	5.55%	3.70%
Standard deviation (%a.a)	2.53%	4.33%	2.81%
Safety-First ratio[(1)]	1.10	1.28	1.32
VaR (5%) (%a.a)[(2)]	-0.49%	-0.57%	-0.36%
ES (5%)(%a.a) [(2)]	-0.79%	-1.98%	-0.62%
Max drawdown(%a.a) [(2)]	-1.68%	-6.01%	-1.00%

[(1)]Risk adjusted excess return over a desired profitability threshold, which was assumed as 0%.
[(2)]VaR (Value at Risk) is the 5th percentile of the series of returns and ES (Expected Shortfall) is the average of the lower returns to VaR. The Max Drawdown is the maximum historic drop in portfolio value over a period. To estimate these measures 12-month moving average returns in a monthly periodicity were used.

Source: Authors' calculations

It is observed that the average return and the historical volatility of the long-term tranche are higher than those calculated for the one year investment horizon during the evaluation period (mean: 5.55% vs. 2.79% and volatility: 4.33% vs. 2.53% respectively). This is an effect of increased duration of the fixed income component and the addition of more volatile financial instruments. The average return and volatility of returns of the total aggregate portfolio are 3.70% and 2.81% respectively.

Figure 4: Safety Tranche and Wealth Tranche historical annual and cumulative returns

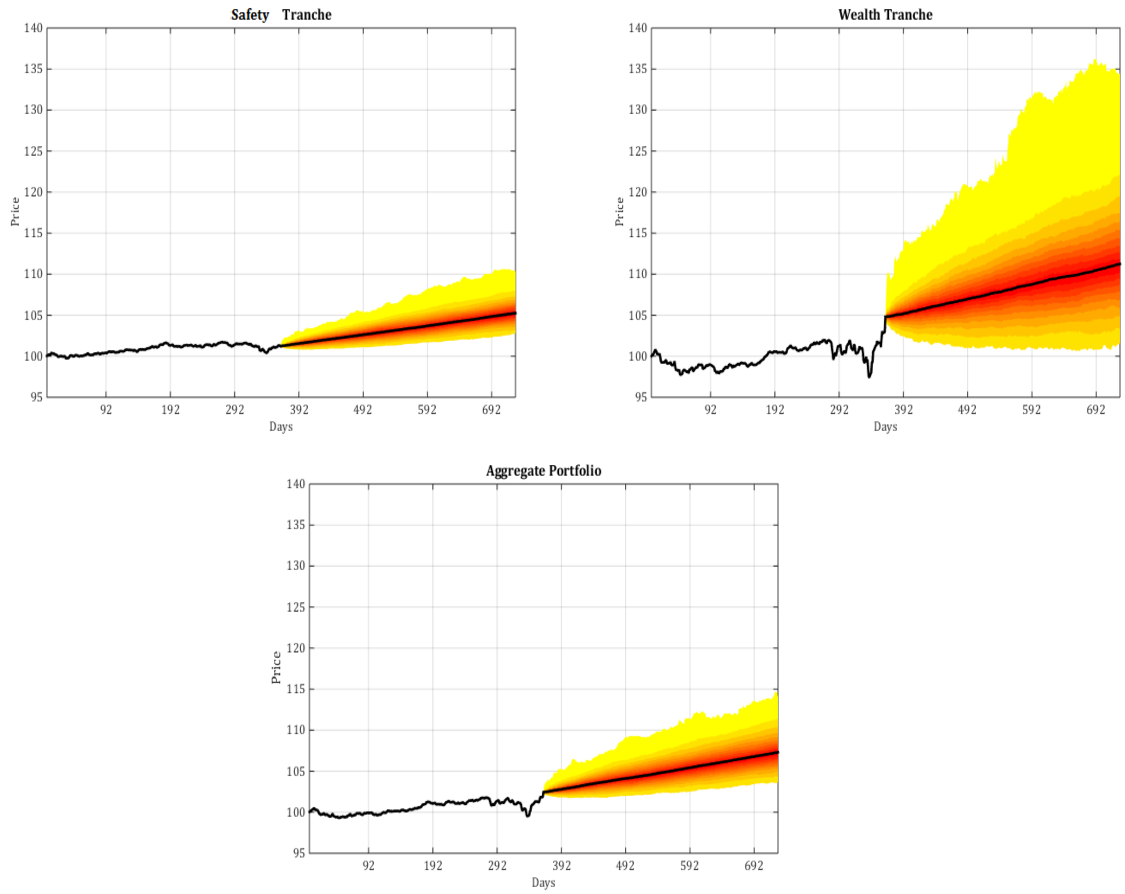


*Data until September 2014

Source: Authors' calculations

In spite of having greater volatility, the aggregate portfolio shows a better risk adjusted performance than the traditional methodology. This is derived from the Safety-first Ratio, which allows an investor to select one portfolio rather than another based on the criterion that the probability of the portfolio's return falling below a minimum desired threshold is minimized. This indicator is equal to 1.32 for total aggregate portfolio versus 1.10 for the Safety Tranche and means that the increased volatility of the total portfolio is more than offset by the increase in the excess returns on the defined threshold. In addition, three measures of tail risk largely favor the pool of the two tranches against the single Safety Tranche, as a consequence of the almost independent relation between the short and long term portfolios, which can be seen at first sight in some years in Figure 4. Thus, according to the evaluated historical data, the proposed strategy of two tranches would have been superior to the traditional foreign reserves portfolio.

Figure 5: Safety Tranche and Wealth Tranche Bootstrap



Source: Authors' calculations

In order to obtain a forward-looking evaluation of the framework, a bootstrapping[16] with time dependency[17] is developed through a one year horizon as shown in the fan plots of figure 5. The

[16]A bootstrap procedure consists in resampling from the existing data set with replacement, the evaluation on these graphs is done using the available data since December of 2001.

[17]The bootstrap time dependence methodology aims to dispose the regular presupposition of time independence among observations, in order to establish a more realistic approach. The technique used in this paper modifies the probabilities with which individual observations are chosen, giving a higher probability to those observations closer to the most recently selected data. To do this the proposed probabilities are related with the estimated Hurst exponent (León & Reveiz (2010)) of the data series. Thus, a Hurst exponent equal or lower than 0.5 gives the

plots start with an index value of 100, and are evaluated through the historical data of the previous year and the simulated returns of the next year. Clearly, the Wealth Tranche shows the higher expected returns, which simultaneously increases those of the Aggregate portfolio proportionally. As expected, given the upside potential of this tranche, a greater risk is implicit which may decrease the overall value of the portfolio, this is depicted with a higher expected shortfall of -9.44% with a confidence level of 99%. This same risk measure for both the Safety Tranche and the aggregate portfolio is 0.02% and -0.55%, respectively. This indicates that regarding the increasing riskier holdings in the Aggregate portfolio, the diversification between both tranches help produce a low expected shortfall.

6. Behavioral Bias Analysis: Mental Accounting

The mental accounting bias results when an investor considers her or his financial objectives separately and develops distinct portfolios to fulfill her or his financial needs, in lieu of grouping all the available resources under the same basket and building an overall optimal portfolio following Markowitz mean-variance theory. Although, this bias might originate critiques over the approach developed in this paper, given that the resulting overall portfolio, that pools the two tranches, might be considered suboptimal; Das, Markowitz, Scheid & Statman (2010) through behavioral portfolio theory show that more imprecisions can be derived for not establishing correctly the financial goals. Thus, this section explores the mental accounting framework developed by these authors in order to verify the effect of selecting a portfolio divided among distinct tranches.

The mentioned framework establishes a lower bound (H) for the returns of each tranche and a maximum probability for non-fulfillment with the threshold (α). Therefore, $Prob(r(p) < H) < \alpha$. For the specific case of both the Safety and the Wealth Tranche the threshold is equal to 0% and the probability of non-fulfillment is equal to 5%, however measure under a different investment horizon and demarcated solely as a constraint. With these parameters is possible to define the efficient portfolios following equation 20.

$$H = \boldsymbol{\omega}(\lambda)^{'}\boldsymbol{\mu} + \Phi^{-1}(\alpha)[\boldsymbol{\omega}(\lambda)^{'}\boldsymbol{\Sigma}\boldsymbol{\omega}(\lambda)]^{1/2} \tag{20}$$

where $\boldsymbol{\omega}$ stands for the assets' weights within the portfolio; $\boldsymbol{\mu}$ is the vector of the expected returns; $\boldsymbol{\Sigma}$ is the covariance matrix; and Φ^{-1} is the inverse normal function. Therefore, one can avoid estimating the risk aversion coefficient (λ), avoiding a possible misspecification, and conversely adopting the parameters H and α that are more intuitive. Thus, the risk aversion coefficient can be defined as follows:

$$\boldsymbol{\omega}(\lambda) = \frac{\mathbf{1}}{\lambda}\boldsymbol{\Sigma}^{-1}\left[\boldsymbol{\mu} - \left(\frac{\mathbf{1}^{'}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} - \lambda}{\mathbf{1}^{'}\boldsymbol{\Sigma}^{-1}\mathbf{1}}\right)\mathbf{1}\right] \tag{21}$$

The aggregate portfolio would be efficient since a combination of portfolios within the frontier is mean-variance efficient. This holds for a portfolio with no short-selling constraints ($\boldsymbol{L}$), given that

same probability to all the observation in spite of the distance to the previously selected data point; whilst a Hurst exponent equal to 1 gives a 50% probability to both the immediately preceding and successive observations. For Hurst exponents in between 0.5 and 1 the selected probabilities of each observation are estimated throughout an approximation of an exponential decay factor. None of the available portfolios presented an anti-persistent behavior.

a boundary of this type increases the optimization problem complexity by adding non-equality constraints that require quadratic programming optimizers to produce a solution. Ultimately this results in a very small loss of efficiency in the aggregate portfolio. Consequently, as shown by the authors the risk aversion coefficient under these conditions can be solved recursively until the investors' preferences are met.

$$Solve_{\lambda}\boldsymbol{\omega}(\lambda)^{'}\boldsymbol{\mu} + \Phi^{-1}(\alpha)[\boldsymbol{\omega}(\lambda)^{'}\boldsymbol{\Sigma}\boldsymbol{\omega}(\lambda)]^{1/2} \tag{22}$$

where $\boldsymbol{\omega}(\lambda)$ solves for:

$$max_{\omega}\boldsymbol{\omega}'\boldsymbol{\mu} - \frac{\lambda}{2}\boldsymbol{\omega}'\boldsymbol{\Sigma}\boldsymbol{\omega} \tag{23}$$

$$\boldsymbol{\omega}'\mathbf{1} = 1 \tag{24}$$

$$\boldsymbol{\omega} \geq \boldsymbol{L} \tag{25}$$

For the specific case analyzed in this paper, additional equality and non-equality constraints will be added to the optimization problem that solves for the risk aversion coefficient, including an upper bound ($\boldsymbol{U}$) and a currency composition as follows:

$$\boldsymbol{\omega} \leq \boldsymbol{U} \tag{26}$$

$$\boldsymbol{\omega}\boldsymbol{P} = \boldsymbol{C} \tag{27}$$

where $\boldsymbol{P}$ is a matrix with rows equal to the number of assets and colums equal to the amount of currencies, in which a one indicates if a determined asset belongs to a currency and a zero otherwise; and $\boldsymbol{C}$ contains columns establishing the percentage assigned to each currency.

This higher level of complexity increases the gap of sub-optimality; however this marginal gap guarantees that the central bank will be able to establish its preferences in a more intuitive way. This facilitates the process of defining the liquidity and currency related financial goals, which ultimately decreases possible liquidity and reputational risks. Overall as explained by Das, Markowitz, Scheid & Statman (2010) this scheme, where mean-variance theory and behavioral portfolio theory are unified, results in a more advantageous framework where investor consumption goals and portfolio production are linked.

7. Concluding Remarks

In this paper the traditional investment objectives of a central bank are maximized through a framework that evaluates the adequate level of reserves in order to divide the overall portfolio between two tranches. On one hand the Safety Tranche comprises liquid, almost default-free and low volatile assets, where the financial goals of safety and liquidity are met. On the other hand, the Wealth Tranche aims to maximize the return, with a broader range in the asset space and a longer time horizon. It is found that through this framework both the historical and forward looking performance of an Aggregate portfolio is improved, while maintaining the safety and liquidity needs of a traditional foreign exchange reserves portfolio.

The reserve adequacy is measured through a metric that includes most potential outflows of the balance of payments with implicitly adding the correlations among the variables in periods of exchange market pressure, which outlines the size of the Safety Tranche. If this measure indicates an excess amount of reserves, the framework proposes the tranche division by calculating both portfolios with the Black-Litterman methodology, adding a series of adjustments in the covariance matrix calculation. Among the main strategic asset allocation decisions that differ between the two tranches are the asset space, the investment horizon, the numeraire and the liquidity constraints. The paper also shows that the decision of adopting a behavioral bias (mental accounting bias) is more aligned with the process of better establishing the financial preferences.

Among other advantages of the framework are: (i) The possibility of decreasing the implicit costs associated with the gap between the borrowing rate and investment return, particularly in emerging countries. (ii) The likelihood of decreasing the borrowing premiums in international markets, as a result of a relatively high amount of foreign exchange reserves that improves the perception of financial robustness amongst market participants and credit rating agencies.

Furthermore, it is important to note that the strategic asset allocation for the Wealth Tranche would diverge among countries as they would have to consider their economic particularities and their own investments constraints. The former include their main produced resources, goods and services and whether they want to establish a counter-cyclical policy with the invested assets or the currency composition. The latter would limit the asset space as the central bank considers their operational, legal, risk aversion and knowledge constraints, this step would also induce further considerations as whether the international reserves are going to be actively or passively managed.

Additionally, if there is full certainty that the entire or a fraction of the amount in the Wealth Tranche will not be needed in future scenarios for complying with the central bank's institutional mission, a separate organization to manage these holding can be considered in order to reduce the effect of the traditional risk averse policies of a central bank. This can in turn give a clear separation among the tranches objectives. However, the introduced framework relies on the fact that both tranches are formed to cover the needs of the foreign reserves. If this is not the case further research can be done to test the viability of alternative options of using the excess reserves, including paying the outstanding foreign debt, developing infrastructure projects or any other social or governmental investments.

Finally, additional improvements and changes can be carried on the methodologies used in the framework. For instance, as the samples of the macroeconomic variables used in the reserve adequacy measure increase one can estimate a more robust indicator. Moreover, additional portfolio optimization methodologies can be tested, as a factor based optimization or increasing the moments under which the optimization is carried on. Additionally, a further analysis can be done on the assets held in the Safety Tranche, considering that in the face of a zero risk-free rate one can discern of the available "safe" assets, whether they are limited to the fixed income space or not.

References

[1] Beaufort-Wijnholds, J. and Kapteyn, A. (2001). Reserve adequacy in emerging market economics. *IMF working paper - International Monetary Fund.*

[2] Ben-Bassat, A. and Gottlieb, D. (1992). On the effect of oportunity cost on international reserve holdings. *The review of economics and statistics.*

[3] Black, F. (1989). Equilibrium exchange rate hedging. *National Bureau of Economic Research.*

[4] Black, F. (1995). Universal hedging: Optimizing currency risk and reward in international equity portfolios. *Financial Analysis Journal*, pages 161–167.

[5] Black, F. and Litterman, R. (1991). Asset allocation: Combining investor views with market equilibrium. *The Journal of Fixed Income.*

[6] Bodie, Z. (1995). On the risk of stocks in the long run. *Financial Analyst Journal*, pages 18–22.

[7] Bodie, Z., Kane, A., and Marcus, A. (2010). *Investments.* Mc Graw-Hill.

[8] Calvo, G. A., Izquierdo, A., and Mejía, L. F. (2004). On the empirics of sudden stops: The relevance of balance-sheet effects. *NBER Working Papers.*

[9] Chivakul, M., Llaudes, R., and Salman, F. (2010). The impact of the great recession on emerging markets. *IMF Working Papers*, No. 10/237.

[10] Das, S., Markowitz, H., Scheid, J., and Statman, M. (2010). Portfolio optimization with mental accounts. *Journal of Financial and Quantitative Analysis*, Vol. 45, No. 2.

[11] Eichengreen, B., Rose, A., and Wyplosz, C. (1996). Contagious currency crises. *NBER working papers - National Bureau of Economic Research.*

[12] Fisher, P. and Keeley, T. (2013). In search of a new official investment paradigm: Rethinking safety, liquidity and return. *BlackRock.*

[13] Goetzmann, W. N., Ingersoll, J., and Ross, S. A. (2001). High-water marks and hedge fund management contracts. *Yale International Center for Finance.*

[14] Gómez, J. (2014). Una nueva métrica para el nivel adecuado de reservas internacionales en economías emergentes. Master's thesis, Universidad de los Andes.

[15] Gómez, J. and Hernández, J. M. (2011). Composición cambiaria y poder adquisitivo de las reservas internacionales. *Borradores de Economía - Banco de la República*, 654.

[16] Gómez, J. and Rojas, J. S. (2014). Assessing reserve adequacy: a country-specific combined metric. *HSBC Reserve Management Trends 2014 (Central Banking Publications)*, pages 71–85.

[17] Greenspan, A. (1999). Currency reserves and debt. In *Before the World Bank Conference on Recent Trends in Reserves Management.*

[18] Guasoni, P. and Oblój, J. (2011). The incentives of hedge fund fees and high-water marks. *Boston U. School of Management Research Paper.*

[19] Heller, R. (1966). Optimal international reserves. *Economic journal*, pages 296–311.

[20] Hviding, K., Nowak, M., and Ricci, L. A. (2004). Can higher reserves help reduce exchange rate volatility? *IMF Working Papers*, No. 04/189.

[21] Idzorek, T. (2004). A step-by-step guide to the black-litterman model. *Zephyr Cove*.

[22] IMF (2011). Assessing reserve adequacy. *International Monetary Fund Discussion paper*.

[23] Jeanne, O. (2007). International reserves in emerging market countries: Too much of a good thing? *Brooking papers on Economic Activity, The Brookings Institution*, pages 1–80.

[24] Jeanne, O. and Rancire, R. (2011). The optimal level of international reserves for emerging market countries: A new formula and some applications. *Economic Journal*, pages 905–930.

[25] Ledoit, O. and Wolf, M. (2003). Improved estimation of the covariance matrix of stock returns with an application to portfolio selection. *Journal of Empirical Finance*, pages 603–621.

[26] León, C. and Reveiz, A. (2011). Portfolio optimization and long-term dependence. *Portfolio and risk management for central banks and sovereign wealth funds - BIS Papers*, 58.

[27] León, C. and Vela, D. (2011). Strategic asset allocation: non-loss constraints and long-term dependence. *RBS Reserve Management Trends 2011 (Central Banking Publications)*, pages 79–106.

[28] Page, S. (2013). How to combine long and short return histories efficiently. *Financial Analysts Journal. CFA Institute*, Vol 69, no 1:45–52.

[29] Poloner, S. H. (2010). Structuring hedge fund manager compensation: Tax and economic considerations. *Journal of Taxation*.

[30] Ruiz-Arranz, M. and Zavadjil, M. (2008). Are emerging asia's reserves really too high? *IMF Working Papers*, 08/192.

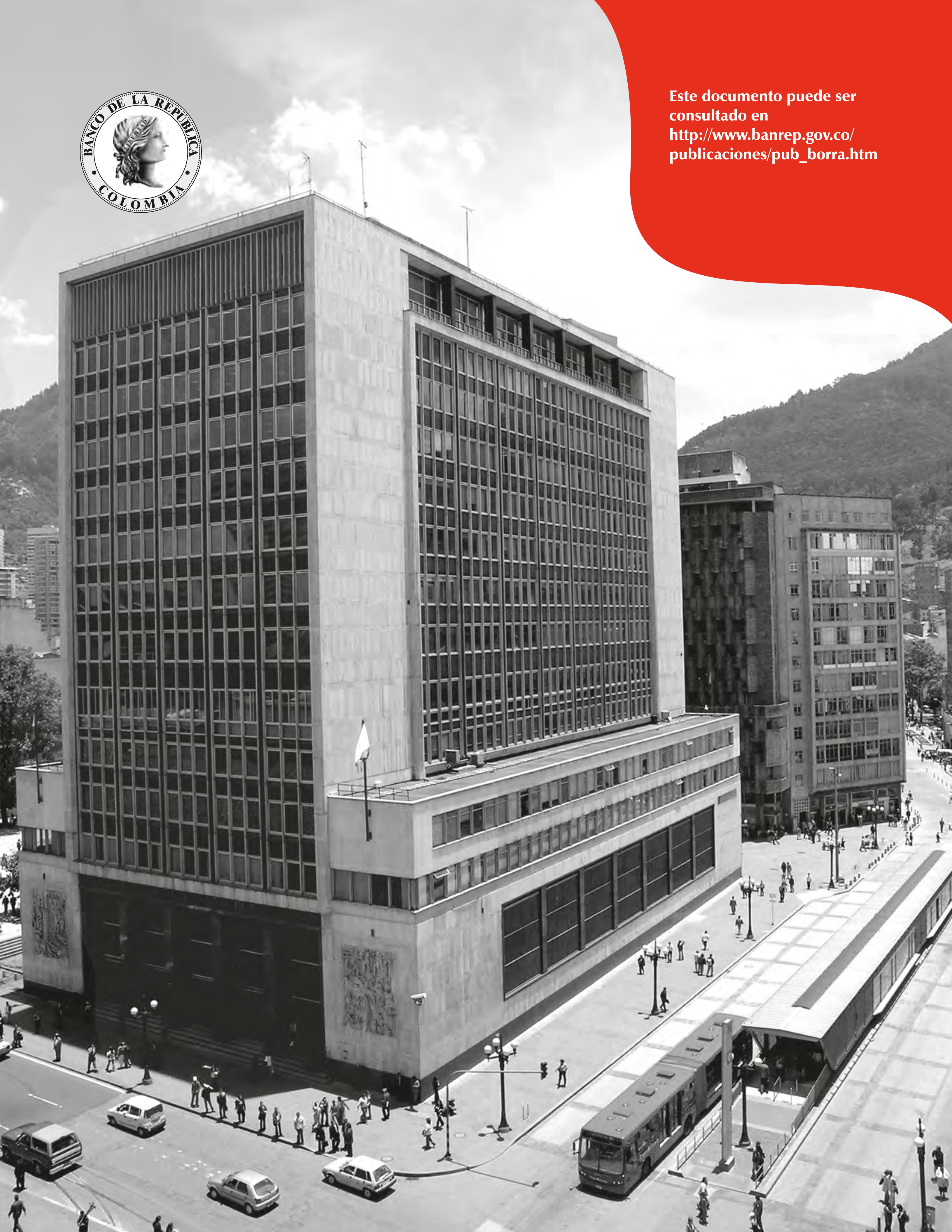
[31] Shefrin, H. and Statman, M. (2000). Behavioral portfolio theory. *Journal of Financial and Quantitative Analysis*, pages 127–151.

[32] Solnik, B. and McLeavey, D. (2003). *International Investments*. Pearson.

[33] Stambaugh, R. F. (1997). Analyzing investments whose histories differ in length. *Journal of Financial Economics*, Vol.45, no. 3:285–331.

[34] Triffin, R. (1960). Gold and the dollar crisis. *Yale University Press*.

[35] Zimmermann, H., Drobetz, W., and Oertmann, P. (2003). *Global Asset Allocation: New Methods and Applications*. Wiley.



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