ESTIMATING CREDIBILITY IN COLOMBIA'S EXCHANGE RATE TARGET ZONE

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Abstract

This paper analyzes credibility in the Colombian exchange rate target zone. A model of imperfect credibility, in the spirit of Bertola and Svensson (1993) is derived, and used to explain why the exchange rate in Colombia has remained for prolonged periods near the borders of the zone. The main feature of the model is that central parity depreciation expectations are endogenous with respect to the position of the exchange rate inside the target zone. The Krugman (1991) perfectly credible setup is a particular case of this general model. A key implication of the model is that depreciation expectations rise rather than fall as the exchange rate approaches the upper band. It is also shown, that this model shares other empirical features with the basic target zone model. Using the method of simulated moments to estimate the structural parameters underlying the model, I find evidence suggesting that Colombia's exchange rate target zone is properly described by a model of this nature.

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1 INTRODUCTION

This paper develops an extension of a target zone model with imperfect credibility based on Bertola and Svensson [1993]. Bertola and Svensson modify the basic target zone model due to Krugman [1991] by allowing the possibility that the target zone is not perfectly credible as in the basic setup[2]. They model imperfect credibility as a random process which is possibly correlated with the stochastic shocks that drive the fundamentals that determine the nominal exchange rate.

The model presented in this paper, incorporates the position of the exchange rate with respect to the upper and lower bands in the formation of expectations. Agents read information on fundamentals in the exchange rate, and based on their perception about policy makers and on what they think policy makers know about fundamentals, form expectations on future realignments of the central parity. If agents perceive that the policy makers that run the target zone are not dependable, in the sense that they may not be willing to undertake the costs of defending the target zone, and rather would prefer to realign the central parity when the exchange rate reaches a band, the expectation of a realignment will increase as the exchange rate approaches a band [3]. It is shown that when central parity realignment expectations are modeled in such a way, the exchange rate's variance stabilizing properties attributed to target zones with perfect credibility, are diminished. Moreover, it can be the case that the resulting exchange rate is even more volatile than that of a free float regime.

[2]Imperfect credibility in this context means that a non negligible probability of a realignment of the central parity of the target zone exists.

[3]If agents believe that the central bank will incur in costs by defending the band, and further they also assume that a non dependable policy maker runs the central bank and would prefer to abandon the band commitment rather than paying such costs, it is natural to expect a higher realignment expectation as the exchange rate approaches the bands. Cukierman et al. [1993] develop a political economy model of imperfectly credible target zones, following these lines.

Unfortunately, in practice, identifying if the regime is credible or not is a difficult task. As shown in this paper the behavior of the exchange rate inside the target zone can be similar in either model, and hence, it is highly possible that, based on observed market exchange rate information, policy makers draw improper inferences about how agents perceive credibility.

The Krugman model predicts that the exchange rate should remain most of the time near the bands than in the center of the target zone. That is, it predicts that the unconditional distribution of the exchange rate must be bimodal (U-shaped). It is shown that when we allow for imperfect credibility in the way modeled here, the solution also yields a U shaped distribution similar to the Krugman one. However, in the model with imperfect credibility even more mass is concentrated in the bands and less in the center when compared to the Krugman setup. To identify what kind of regime is in place, an econometric estimation of the structural parameters of the model is needed.

Alternatives to structural estimation to identify realignment expectations are tests based on interest rate differentials. In fact, the most popular credibility tests are based on them directly or on information on expected realignments derived from them [4]. However, as noted by Frankel and Phillips [1991] these procedures fail to take into account risk premia which can be a significant source of interest rate differential fluctuation (specially in developing countries). Another drawback of these approaches is that they rely on forecasts of the exchange rate inside the target zone which are used to derive the expected realignment of the central parity [5]. The problem faced by such procedures is the low performance of monetary models in predicting short run movements of the exchange rate. This unfortunate empirical feature has been amply

[4] Examples of these type of tests are found in Svensson [1991a], Chen and Giovannini [1992] and Rose and Svensson [1995]

[5] The expected realignment is estimated as $(i - i^*) - E(dx)/dt$ with i, i^* and x the domestic and foreign interest rates, and the exchange rate inside the target zone, respectively.

documented in literature [6]. Erroneous forecasts of the exchange rate imply erroneous estimations of expected realignments.

In this paper, the method of simulated moments is used to estimate the parameters of the model. Based on these, inference about the credibility of the Colombian target zone is made. The rest of the paper is organized as follows. Section 2 presents a brief description and some basic stylized facts of Colombia's target zone. Section 3 presents the model and derives its implications. Section 4 focuses on estimation issues and section 5 on results. Concluding remarks are included at the end.

2 THE COLOMBIAN TARGET ZONE

Colombia formally adopted a target zone regime in January 1994 [7]. The prevailing market exchange rate at that date was set as the central parity. The width was set at 7.5% from the parity, and the band was allowed to

[6] Examples are found in Meese and Rogoff [1983], and more recently Chinn and Meese [1995] who show that no monetary model has a better forecasting performance than a random walk.

[7] Before this date, and as a monetary response of the Colombian central bank to massive foreign capital inflows, a dollar denominated certificate called the *certificado de cambio* (from here on CC) was constantly issued to exporters and other sellers of foreign exchange to sterilize these flows. In short the exchange rate system worked as follows. Holders of CCs had three alternatives: keep them until maturity and receive the pesos after at maturity, sell them to the central bank at a 12.5% discount rate with respect to the day's official exchange rate(the official exchange rate in this context is the exchange rate at which the central bank redeemed CCs at maturity), or sell them to third parties in the foreign exchange market at a rate lower than the official exchange rate, but higher than that received at the central bank. Carrasquilla [1995a] argues that in practice, a target zone with ceiling equal to the official daily exchange rate and floor equal to 87.5% of that value was developed. However rules directly related with the behavior and defense of the bands were not explicit. These rules, which determine the institutional framework of the target zone are the essence of much of the common results attributed to target zone models. Given this, in this paper only the period of formal target zones is analyzed.

crawl at a rate of 11% per annum. Several authors [8] have argued that the January-December 1994 band was not credible because it was not consistent with a real appreciation trend that fundamentals where exhibiting. Given this, in December 1994 the target zone was realigned downward by 7.5%, making the central parity equal the previous lower bound. In August 1998, the target zone was realigned again. At that time the central parity was devalued 9%.

An interesting feature of the Colombian target zone is that the exchange rate tends to remain very close to either the upper or the lower band for prolonged periods. Figure 1 shows this behavior. This pattern is apparently consistent with the implications of basic target zone models [9]. Figure 2 plots the estimate of the unconditional distribution of the exchange rate in Colombia since 1994 , following the Chen and Giovannini [1992] procedure detailed in the appendix. As can be seen the bimodal distribution is a stylized fact in Colombian data. This, in itself is a very important finding since, even if the basic model predicts such a behavior, international empirical evidence[10] has shown that this pattern does not hold, and that the distribution in most countries is hump shaped (density concentrates in the middle) instead.

As stated above, even if the distribution implied by Krugman's model is apparently satisfied in Colombia, there is no evidence that it is the *correct* bimodal distribution, that is, if it is the empirical U-shaped distribution forecasted by the Krugman model of perfect credibility. International evidence raises doubts about this. A comparison of the average time period that the exchange rate spends near a band suggests that Colombia's behavior is atyp-

[8] e.g. Carrasquilla [1995a] and Urrutia [1995]

[9] See Krugman [1991].

[10] See Chen and Giovannini [1992], Flood et al. [1991] , Garber and Svensson [1994] and Lindberg and Soderlind [1994] for findings on hump shaped distributions in European currencies, and Feliz and J.Welch [1994] for a discussion of Mexico and Chile. This empirical result has been linked to the fact that in practice there is a significant amount of intervention inside the bands, and that central banks usually act conservatively and prefer to offset departures form the central parity.

ical. In Colombia, an average visit to the upper band lasts 3.5 weeks, and an average visit to the lower one lasts 7 weeks. In France an average visit to the upper and lower bands lasted only 1.4 and 3.5 weeks respectively. In Italy an average visit to the lower band lasted in average 2 weeks; and in Mexico, a visit to the upper one lasted only 2.5 weeks. In other words, in Colombia the exchange rate seems to *stick* to the bands for longer periods, compared to other target zones. As shown below this is a salient feature of target zones with certain patterns of imperfect credibility.

3 A MODEL WITH IMPERFECT CREDIBILITY

Based on Bertola and Svensson [1993] a model which describes the behavior of the exchange rate subject to realignment expectations is derived. Realignment expectations are not exclusively modeled as a function of fundamentals, but additionally, a component which relates the position of the exchange rate inside the band with realignments is included . As will be shown one of the mayor implications of a model with these characteristics is that the exchange rate remains near the bands for prolonged periods. That is, the distribution of the exchange rate inside the bands is bimodal with very high concentration of mass at the tails.

As in the basic Krugman [1991] set up, assume that the exchange rate is determined in a purely monetary model, such that:

$$s = f + \alpha \frac{E[ds]}{dt} \tag{1}$$

where s is the logarithm of the exchange rate, α the semi elasticity of money demand with respect to the interest rate, and f a set of fundamentals [11] that are driven by a Brownian motion of the form:

$$df = \mu_f dt + \sigma_f dW_f \tag{2}$$

with W_f a standard Wiener process , i.e. $E[dW_f] = 0$ and $E[(dW_f)^2] = dt$.

As in Bertola and Svensson, the variables are defined in terms of their deviation from the central parity. Specifically $x = s - c$ and $\tilde{f} = f - c$ are defined to be the deviation of the exchange rate with respect to the central parity and the deviation of fundamentals respectively.

Additionally, the expectation in the right hand side of (1) can be rewritten as:

$$\frac{E_t[ds]}{dt} = \frac{E_t[dx]}{dt} + \frac{E_t[dc]}{dt} \tag{3}$$

where the first term is the expected depreciation of the exchange rate inside the target zone and the second one is the expected depreciation of the central parity. The second term captures the effects of imperfect credibility issues in the model. Typically $\frac{E_t[dc]}{dt}$ is modeled as:

[11] In the simplest setup $f = m + v$ with m money supply and v random shocks. Equation 1 is a standard monetary equation of the exchange rate. To derive it, assume that money demand is given by $m - p = y - \alpha i$ where m is money supply, p the price level, i the interest rate and y is income. All variables except the interest rate are in logs. Assume also that there is a foreign economy with the same money demand. We denote foreign variables with a $*$. Subtracting domestic and foreign money demands yields: $(p - p^*) = (m - m^*) - (y - y^*) + \alpha(i - i^*)$. Under the PPP assumption the exchange rate, e, equals $(p - p^*)$, hence: $s = (m - m^*) - (y - y^*) + \alpha(i - i^*)$. Moreover, assuming uncovered interest rate parity such that $(i - i^*) = \frac{E[ds]}{dt}$ then: $s = (m - m^*) - (y - y^*) + \alpha\frac{E[ds]}{dt}$ or: $s = f + \alpha\frac{E[ds]}{dt}$ which is equation (1). A risk premium could also be included and the result would remain similar as above.

$$\frac{E_t[dc]}{dt} = g \tag{4}$$

where g follows a Brownian motion:

$$dg = \mu_g dt + \sigma_g dW_g \tag{5}$$

The stochastic processes driving g and $\tilde{f}$ can be positively correlated, which means that expectations of realignments are allowed to increase with fundamental shocks. This paper extends the Bertola and Svensson model, and additionally to a random g, the expectations of depreciation of the central parity are also determined by the position of the exchange rate inside the target zone. This allows us to endogenize the expected realignment process. For simplicity the relationship between the position of the exchange rate inside the band and expectations is assumed linear. Realignment expectations increase when the exchange rate is near the bands, and decreases as it approaches the central parity. Formally, assume that:

$$\begin{aligned} \frac{E_t[dc]}{dt} &= g_1 + g_2 \\ g_2 &= \beta x \; ; \; \beta \geq 0 \end{aligned} \tag{6}$$

and g_1 follows the Brownian motion above. Replacing (6) and (3) in (1), and defining variables in terms of deviations from central parity, yields

$$x = \tilde{f} + \alpha g_1 + \alpha\beta x + \alpha\frac{E[dx]}{dt} \tag{7}$$

Solving for x :

$$x = \frac{\tilde{f} + \alpha g_1}{1 - \alpha\beta} + \frac{\alpha}{1 - \alpha\beta}\frac{E[dx]}{dt} \tag{8}$$

As usual in this type of literature, a composite fundamental h can be defined such that $h = \tilde{f} + \alpha g_1$ and

$$x = \frac{h}{1 - \alpha\beta} + \frac{\alpha}{1 - \alpha\beta}\frac{E[dx]}{dt} \tag{9}$$

where h is a Brownian motion that evolves according to

$$\begin{aligned} dh &= \mu dt + \sigma dW \\ \mu &= \mu_f + \alpha\mu_g \\ \sigma &= \sqrt{\sigma_f^2 + \alpha\sigma_g^2 + 2\rho\alpha\sigma_f\sigma_g} \end{aligned} \tag{10}$$

where ρ is the coefficient of correlation between dW_f and dW_g. As in Krugman, the model is solved by deriving $\frac{E[dx]}{dt}$, solving the resulting differential equation, and imposing marginal conditions to identify the unique solution. To do so we start by stating a solution $X(h)$[12] to the exchange rate determination problem. Using equation (9) and invoking Itô's lemma[13] we get:

$$dx = [X_h(h)\mu + \frac{1}{2}X_{hh}(h)\sigma^2]dt + X_h(h)dW \tag{11}$$

Taking expectations and plugging the result in (9) yields the following second order differential equation:

[12] X(h) is assumed to be continuous and two times differentiable in h.

[13] Itô's Lemma states that if $y = f(x)$ and x follows a Brownian motion of the form $dx = \mu dt + \sigma dw$ then$dy = [f_x(x)\mu + \frac{1}{2}f_{xx}(x)\sigma^2]dt + f_x(x)\sigma dw$.

$$X(h) = \frac{h}{1-\alpha\beta} + \frac{\alpha}{1-\alpha\beta}[X_h(h)\mu + \frac{1}{2}X_{hh}(h)\sigma^2] \tag{12}$$

with general solution:

$$X(h) = \frac{h}{1-\alpha\beta} + \frac{\alpha\mu}{(\alpha\beta-1)^2} + C_1 e^{\lambda_1 h} + C_2 e^{\lambda_2 h} \tag{13}$$

where

$$\begin{aligned} \lambda_1 &= \frac{-\sqrt{\alpha}\mu - \sqrt{\alpha\mu^2 + 2\sigma^2(1-\alpha\beta)}}{\sqrt{\alpha}\sigma^2} \\ \lambda_2 &= \frac{-\sqrt{\alpha}\mu + \sqrt{\alpha\mu^2 + 2\sigma^2(1-\alpha\beta)}}{\sqrt{\alpha}\sigma^2} \end{aligned} \tag{14}$$

The model assumes that the central bank intervenes when fundamentals reach certain implicit limits ,$\underline{h}$ and $\bar{h}$, that are consistent with the maximum and minimum values of the exchange rate:$\underline{x}, \bar{x}$[14]. The central bank buys foreign currency when $h = \underline{h}$, and sells when $h = \bar{h}$.

C_1 and C_2, as well as the implicit limits on the fundamentals $\underline{h}$ and $\bar{h}$, are determined by solving the following system:

$$X(\underline{h}) = \frac{\underline{h}}{1-\alpha\beta} + \frac{\alpha\mu}{(\alpha\beta-1)^2} + C_1 e^{\lambda_1 \underline{h}} + C_2 e^{\lambda_2 \underline{h}} = \underline{x} \tag{15}$$

$$X(\bar{h}) = \frac{\bar{h}}{1-\alpha\beta} + \frac{\alpha\mu}{(\alpha\beta-1)^2} + C_1 e^{\lambda_1 \bar{h}} + C_2 e^{\lambda_2 \bar{h}} = \bar{x} \tag{16}$$

[14] $\bar{x}$ and $\underline{x}$ are the upper and lower bands of the normalized exchange rate.

$$\left.\frac{\partial X}{\partial h}\right|_{h=\underline{h}} = \frac{1}{1-\alpha\beta} + C_1\lambda_1 e^{\lambda_1 \underline{h}} + C_2\lambda_2 e^{\lambda_2 \underline{h}} = 0 \tag{17}$$

$$\left.\frac{\partial X}{\partial h}\right|_{h=\bar{h}} = \frac{1}{1-\alpha\beta} + C_1\lambda_1 e^{\lambda_1 \bar{h}} + C_2\lambda_2 e^{\lambda_2 \bar{h}} = 0 \tag{18}$$

Equations (15) and (16) determine the levels of the composite fundamentals at which the exchange rate inside the target zone reaches the upper and lower bounds. Equations (17) and (18) are known as the smooth pasting conditions. These imply that at the boundaries the exchange rate is completely insensitive to fundamentals. The intuition behind these conditions is as follows. Inside the band, since fundamentals follow a Brownian motion, their expected rate of change is constant (zero, if μ=0). But at the bands the expected rate of change of fundamentals is different from zero; there is a discontinuity. At the bands the fundamental can either remain there or return to the interior. Hence, at the upper band the expected change is negative, and is positive at the lower one. However, the exchange rate can not exhibit these discontinuities, that is it can not jump, since this would allow for arbitrage opportunities. If the smooth pasting conditions do not hold, the discontinuity in the expected rate of change of fundamentals would imply a discontinuity in the expected change of the exchange rate, and hence, a discontinuity in the exchange rate itself. The only way that at the bands the exchange rate follows a continuous solution given that fundamentals follow a discontinuous path, is if it is completely insensitive to the fundamentals (or horizontal).

For particular values of the parameters the system can be solved numerically. Note that if there is full credibility , that is if $\beta = 0$ and $g_1 = 0$, the solution is identical to that of the basic Krugman model. If the expectation of a realignment is not a function of the position of the exchange rate inside the target zone ($\beta = 0$ only) then the solution is Bertola and Svensson's.

Figure 3 plots this solution for two alternative values of β: $\beta = 0$, the Krugman case, and $\beta = 0.7$ a case where there is imperfect credibility [15]. As can be seen when there is imperfect credibility the solution lies over the Krugman solution for positive values of the exchange rate, and under it (in absolute values) for negative ones. This happens because when there is imperfect credibility there is an additional depreciating (appreciating) expectation when the exchange rate is in the upper (lower) half of the band , caused by the "nervousness" inspired by the position of the exchange rate.

The main difference between these models is how expected depreciation is modeled. Expected depreciation can be derived from equation (1):

$$\delta^e = \frac{x - \bar{f}}{\alpha} \tag{19}$$

Replacing x by its solution yields:

$$\delta^e = \frac{h\beta}{1 - \alpha\beta} + g_1 + \frac{\mu}{(\alpha\beta - 1)^2} + \frac{C_1 e^{\lambda_1 h} + C_2 e^{\lambda_2 h}}{\alpha} \tag{20}$$

Figures 4 and 5 plot δ^e when $\beta = 0.7$ and $\beta = 0.0$ [16]. The basic target zone model ($\beta = 0.0$) assumes that expected depreciation decreases as the exchange rate departs from the lower bound and approaches the upper one. That is, expected depreciation is positive at the upper band and negative at the lower one. This happens because agents expect that the central bank will intervene at the margins and with that, they will revert the path of the exchange rate. In the model presented here, exactly the opposite happens. Agents expect that authorities will not intervene to defend the target zone, but that instead the central parity is going to be realigned. Hence, expected depreciation will be higher at the upper band, and an appreciation will be expected at the lower one.

[15]The rest of the parameters used are: $\alpha = 0.88$, $\sigma = 0.03$ and $\mu = 0.0$.

[16]In the graph $g_1 = 0.0$.

A relevant feature of the model above is concerned with the distribution of the exchange rate inside the target zone. Krugman shows that in his model the exchange rate follows an unconditional U-shaped (bimodal) distribution. That is, more mass is concentrated in the borders of the target zone than in the center. The model with imperfect credibility exhibits the same property, however, it concentrates even more mass near the borders and less in the center than in the Krugman model. Following Harrison [1985] and Svensson [1991] we know that the unconditional density function of the composite fundamental, given that it follows a Brownian motion, is given by:

$$\begin{aligned} \phi^h(h) &= \frac{1}{\bar{h}-\underline{h}}, \quad if\ \mu = 0 \\ \phi^h(h) &= \frac{\theta e^{\theta h}}{e^{\theta \bar{h}} - e^{\theta \underline{h}}} \quad if\ \mu \neq 0 \end{aligned} \tag{21}$$

where $\theta = \frac{2\mu}{\sigma^2}$. Using the change of variable rule we can find the unconditional distribution of the exchange rate:

$$\phi^x(x) = \frac{\phi^h(x^{-1}(x))}{|x_h(x^{-1}(x))|} \tag{22}$$

with $x^{-1}[x]$ the inverse of (13). Given that $x^{-1}[x]$ is implicitly defined, $\phi^x(x)$ can only be numerically simulated. Figure 6 plots the distributions for $\beta = 0$ and $\beta = 0.7$. The Krugman model concentrates more observations at the borders because the exchange rate moves slower at the borders than at the center, given that when it departs from the central parity two effects operate. A depreciating effect lead by fundamentals, and an appreciating one lead by the fact that agents expect that the central bank will intervene (sell foreign reserves) to defend the band. In the model with imperfect credibility the exchange rate will move even faster towards the border of the band given the additional depreciating effect provoked by the way expectations are defined, but there still is some appreciating effect given that the central bank is partially credible. This is tied to the assumptions that a smooth pasting equilibrium is still reached, even when there is low credibility.

Figure 7 plots the conditional variance of the exchange rate for the same cases above. The conditional variance of the exchange rate inside the target zone is given by:

$$\sigma_x^2 = (x_h(h)\sigma)^2 = \left[\left(\frac{1}{1-\alpha\beta} + C_1\lambda_1 e^{\lambda_1 h} + C_2\lambda_2 e^{\lambda_2 h}\right)\sigma\right]^2 \tag{23}$$

In the Krugman case the exchange rate's variance is always less than the variance of the fundamentals (since $x_h(h) < 1$), which would be the variance of the exchange rate in absence of bands. This is one of the most important properties of the Krugman model. When the target zones are credible there is a stabilizing mechanism in the exchange rate. When there is imperfect credibility this stabilizing mechanism is reduced, and vanishes in areas surrounding the center of the band. This is clearly shown in the graph. When $\beta \neq 0$ the instantaneous variance of the exchange rate exceeds that of the free float solution, that is exceeds the variance of the fundamental process.

In the following section, the model is estimated using Colombian data and empirical regularities are tested.

4 ESTIMATION ISSUES

In order to test if Colombian data satisfy the model above, the system (13)-(18) is estimated using the method of simulated moments(MSM) [17]. The es-

[17]This method has been used by Smith and Spencer [1991] to estimate if German Mark / Italian Lira exchange rate data satisfy the Krugman model, De Jong [1994] who tests the same model for the Belgian franc, Dutch guilder, Danish krone, French franc, Irish punt and Italian lira exchange rates, and by Lindberg and P.Soderlind [1994a] who test if Swedish data satisfy a model with intervention inside the bands.

timation procedure is based on minimizing the distance between a vector of sample moments of the exchange rate and a vector of simulated moments[18]. Recall that GMM minimizes the distance between a vector of sample moments and one of theoretical (analytically derived) moments. The rationale for MSM is that analytical expressions for the theoretical moments of particular models can be very difficult to derive, and even when they are derivable they do not necessarily identify the whole set of parameters. In the exchange rate target zone models there is an additional incentive to use such technique: fundamentals are unobservable and because of this, alternative methods such as maximum likelihood estimation are precluded. Given this, MSM appears as a convenient empirical alternative since fundamentals can be simulated instead.

The empirical objective is to estimate a vector of parameters $\gamma' = (\mu, \sigma, \alpha, \beta)$. For such a purpose I define loss function composed of a weighted sum of the difference between emprical and simulated moments, of the form:

$$L(S,\gamma) = \left[M_T(x) - \frac{1}{S} \sum_{s=1}^{S} M_{N,s}(\gamma, h_{s,\gamma}) \right]' W_T \left[M_T(x) - \frac{1}{S} \sum_{s=1}^{S} M_{N,s}(\gamma, h_{s,\gamma}) \right] \tag{24}$$

where $M_T(x)$ are empirical moments and $M_{N,s}(\gamma, h_s(\gamma))$ are the moments generated by the $s - th$ simulation of the exchange rate. The weigth matrix, W_T is defined as $W_T = (1 + \frac{1}{n})^{-1}\Omega^{-1}$, with $n = N/T$, and Ω is a consistent covariance matrix of the empirical moments estimated following Newey and West [1987]. In the expression above T is the number of observations in the sample, N the number of observations of the simulated series and S the number of times the experiment is replicated. The moments selected are determined according to what are considered salient features of the model. Following literature, the first four moments of the exchange rate and its first difference are considered.

[18]For details on the methodology see Lee and Ingram [1991] and Gourieroux and Monfort [1996].

The fundamentals h are simulated according to :

$$h_n = \begin{cases} \bar{h} \quad if\ h_{n-1} + \mu\Delta n + \sigma\sqrt{\Delta n} \cdot w_n > \bar{h} \\ \underline{h} \quad if\ h_{n-1} + \mu\Delta n + \sigma\sqrt{\Delta n} \cdot w_n < \underline{h} \\ \mu\Delta n + h_{n-1} + \sigma\sqrt{\Delta n} \cdot w_n \quad otherwise \end{cases} \tag{25}$$

where w is a series of N random elements generated from a standard normal density, and $h_0 = 0$.

The sample size T is 250, N is 500 and S is 1000. Once fundamentals are simulated given a vector of parameters , the exchange rate is also simulated using equation (13), and $M_{N,s}(\gamma, h_{s,\gamma})$ for $s = 1..S$ are computed. Over a grid of parameters $L(S,\gamma)$ is minimized. In this case α is allowed to range between 0.5 and 1.0, μ between -0.1 and 0.1, σ between 0.001 and 0.05 and β between 0.1 and 1.1. Twenty values of each parameter separated by a constant distance within their range are considered [19]. The width of the band used, is Colombia's 7.5% value.

The MSM estimator is:

$$\hat{\gamma}_{TN} = \arg\min_{\gamma}\ L(S,\gamma) \tag{26}$$

Lee and Ingram [1991] and Gourieroux and Monfort [1996] show that:

$$\sqrt{T}(\hat{\gamma}_{TN} - \gamma_0) \xrightarrow{D} N\left(0, \left[B'\left(1 + \frac{1}{n}\right)^{-1} \Omega^{-1} B\right]^{-1}\right) \tag{27}$$

as $N, T \to \infty$, and with

$$B = E\left[\frac{\partial M_N}{\partial \gamma}\right] \tag{28}$$

Results are reported in the following section.

[19] Note that in order to simulate the exchange rate process the system (13)-(18) has to be solved (numerically) for each parameter combination, since C_1,C_2,$\bar{h}$, and $\underline{h}$ depend on the values of the parameters.

5 RESULTS

The sample moments where estimated using weekly Colombian peso / US dollar exchange rate data for the December 1994 - August 1998 period. Table 1 reports the results. The first column reports the parameter estimated, the second standard errors [20], and the fourth reports significance levels for tests that the parameters equal zero. The last line reports Lee and Ingram's fitness test which is distributed as a χ^2 with degrees of freedom equal to the difference between the number of moments and the number of parameters of the model. The null hypothesis of this test is that the simulated moments match the empirical ones.

Results suggest that in general the data fit the model appropriately. Moreover the estimates of the parameters, in particular that of β ($\beta = 0.88$ and significant) suggest that there is evidence of imperfect credibility in Colombia's foreign exchange market. Results suggest that agents increase their realignment expectations as the exchange rate approaches either band. Moreover, it is shown that when the exchange rate reaches the upper band a depreciation of the central parity of a magnitude of approximately 6.6%[21] is expected.

Given the estimates above, and the previous explanation on how the volatility of the exchange rate increases with β, it appears that the stabilizing properties of target zones are not fully present in Colombia.

6 CONCLUSIONS

This paper develops a model in which the expected depreciation of the central parity (or the expected realignment of the target zone) is a function of factors linked to fundamental and non fundamental variables. The main implications

[20]For a discussion on the estimation of assymptotic standard errors, see Lee and Ingram [1991] and Gourieroux and Monfort [1996].

[21]$\beta * \bar{x} = 0.066$

of such a model are that depreciation expectations, and hence interest rate differentials, increase with the position of the exchange rate inside the band, the exchange rate sticks to the bands, and the stabilizing properties of the target zone are seriously diminished.

The model is estimated using Colombian data, and significant results are obtained. Results suggest that in Colombia , the lack of credibility in the target zone, can account for approximately 6.6% of the interest rate differential. It is also shown that given the current policy stance in Colombia, a target zone regime is inefficient, given that imperfect credibility induces additional uncertainty in the foreign exchange market, and the monetary independence that target zone regimes usually buy is considerably reduced.

It is important to note that these are average results, and by no means can they explain the evolution of credibility across time. Several techniques have been developed to do so. Examples of such procedures are found in Chen and Giovannini [1992], Bekaert and Gray [1996], Mizrach [1995] and Svensson [1991a]. However this is not the purpose of this paper.

7 FIGURES

Figure 1: Exchange Rate and Target Zone in Colombia

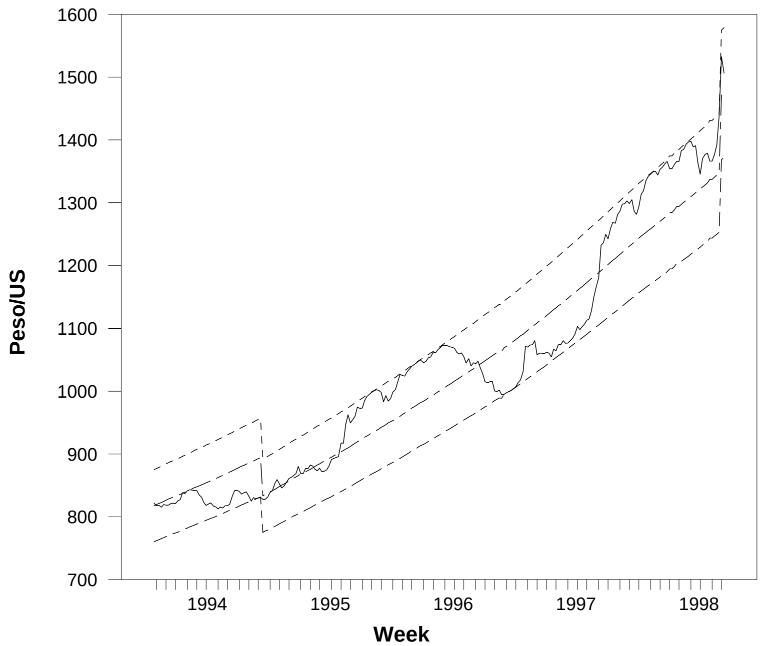


Figure 2: Distribution of Exchange Rate in Target Zone

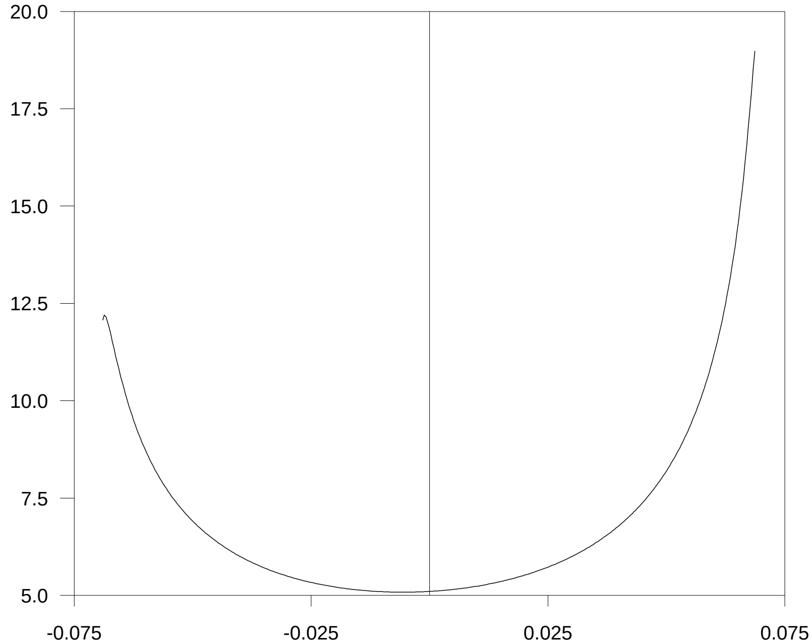


Figure 3: Exchange Rate Solutions

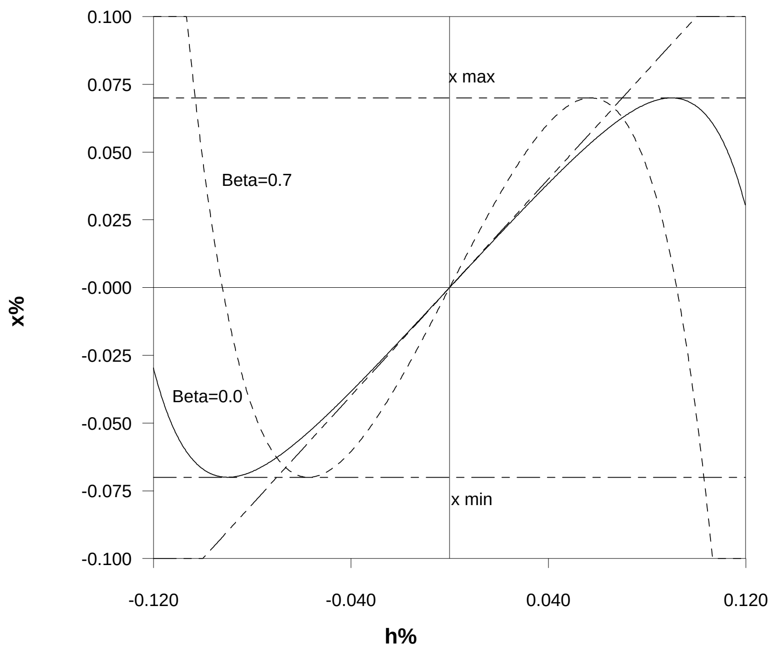


Figure 4: Expected Depreciation

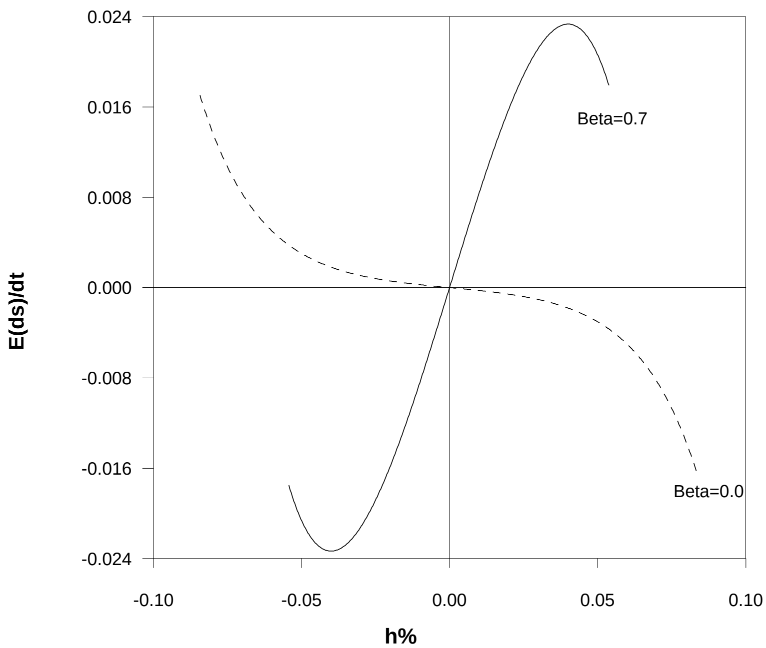


Figure 5: E[ds]/dt, E[dx]/dt and E[dc]/dt

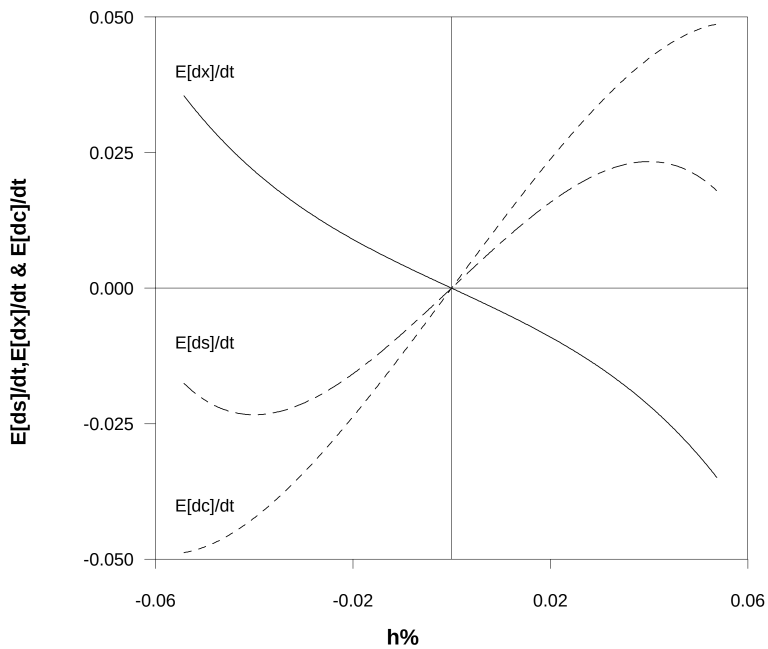


Figure 6: Distribution of Exchange Rate Inside Target Zone

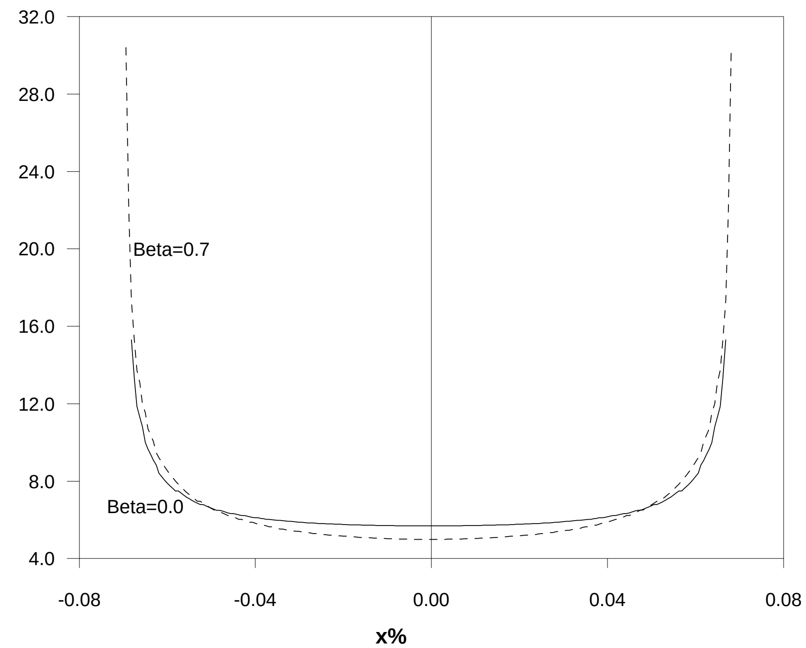
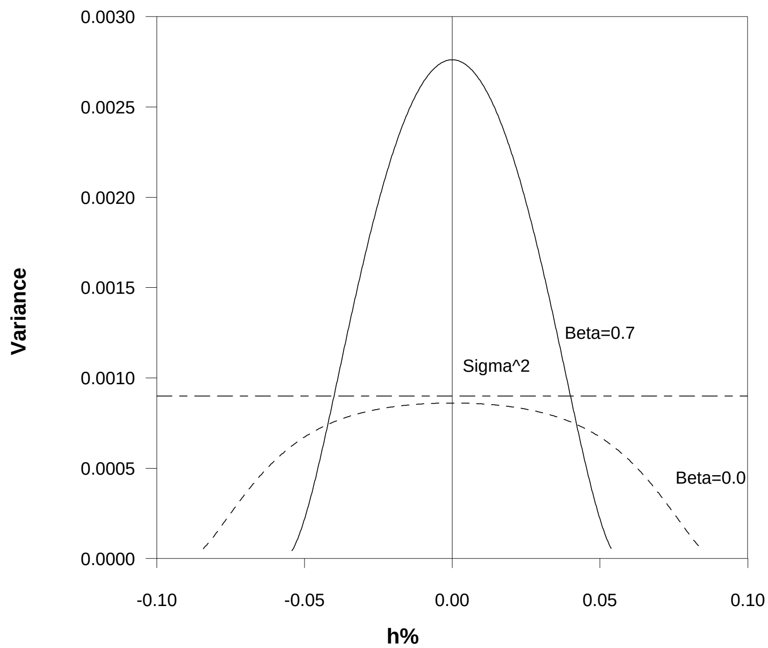


Figure 7: Variance of Exchange Rate Inside Target Zone



8 TABLES

Table 1: Method of Simulated Moments Estimation of Target Zone Model With Imperfect Credibility: Colombia Dec.1994-Aug.1998

	Estimate	Std.Error[a]	Significance[b]
α	0.85	0.127	0.000
μ	0.009	0.003	0.001
σ	0.03	0.016	0.063
β	0.88	0.331	0.008
$\chi^2_{(4)}$	2.0751		0.721

[a] Asymptotic Standard Errors.

[b] Refers to significance level of the test under the null hypothesis that the parameter equals zero.

The moments used for the estimation are the mean, variance, skewness and kurtosis of the log of the exchange rate inside the target zone and its first difference.

APPENDIX
THE CHEN AND GIOVANNINI PROCEDURE

Chen and Giovannini [1992] develop a method to estimate the unconditional distribution of the exchange rate inside the band taking into account the presence of the target zones. They start by considering the following Box-Cox transformation of the exchange rate inside the target zone, x, where $x = s - c$ with s the log of the exchange rate and c the log of the central parity.

$$y = \psi + \delta \left(\frac{B + x}{B - x} \right), \ -B < x < B \tag{29}$$

where B is half the width of the band. The random variable y is assumed to be standard normally distributed. Given this, the distribution of x given B is determined uniquely by ψ and δ. Different combinations of these parameters can resemble most known distributions. To estimate ψ and δ, maximum likelihood methods can be used. Using the change of variable rule, the density of x can be computed:

$$f(x) = J\phi \left(\psi + \delta ln \left(\frac{B + x}{B - x} \right) \right) \tag{30}$$

where ϕ is the standard normal density function, and J, the jacobian is given by:

$$J = \delta \frac{2B}{B^2 - x^2} \tag{31}$$

The log likelihood function can be written as:

Table 2: Maximum Likelihood Estimation of Unconditional Density Parameters

Parameter	Estimate	Std.Error	Significance [a]
ψ	0.0559	0.0711	0.4318
δ	0.4567	0.0255	0.0000

[a] Refers to significance level of the test under the null hypothesis that the parameter equals zero.

$$l = \Sigma\, lnJ + \Sigma\, ln\phi \left(\psi + \delta ln \left(\frac{B+x}{B-x} \right) \right) \tag{32}$$

By maximizing the log likelihood function we obtain estimates of ψ and δ, and the density curve can be numerically simulated.

Results of this maximization for Colombian data are reported in table 2 . The density curve associated with these parameters is plotted in figure 2.

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