A nonlinear specification of demand for cash in Colombia

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Summary

A nonlinear smooth transition regression (STR) model of the demand for narrow money in Colombia is specified using monthly data for cash, prices, the scale variable (industrial GDP), the interest rate and the rate of depreciation, within the single equation framework allowed by the data. In comparison with the linear error correction model, the nonlinear specification is highly superior according to the statistics. The dynamics described by this model matches both the magnitudes and the behavior of the aggregate demand for cash in Colombia during the sample period (1982.2-1998.11).

JEL classification: E41, C22, C52

Key words: error correction, demand for money, target bounds, buffers stock models, nonlinearities, smooth transition regression.

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1. INTRODUCTION

A significant part of the empirical research agenda in economics on the last years has been devoted to the search for robust econometric specifications of empirical models of the demand for money with the desired property of theoretical coherence. This tendency was motivated and supported by the appearance of some complementary techniques during the last twenty years. These techniques are the error correction model, the equilibrium long-run relationships of the variables involved in a model, and the concept of Granger causality, which have been used to investigate the issues termed weak, strong and super exogeneity, each with different implications on inference, forecasting, and policy, and related to the Lucas critique, parameter constancy, invertibility of the money demand function and to measurement problems of the monetary aggregate used in the specification. On the theoretical side, precautionary demand, risk aversion, asset demands, adjustment costs, target-bounds, buffer stocks, expectations, learning and financial innovation, have played a central role.

The works of Hendry and Ericson (1991a, b) on the US and the UK demand for money have been of extreme importance to this area as well as the works done on the German case, before and after the unification (e.g. Wolters et al., 1998; Beyer, 1998; Lütkepohl et al., 1999). Some investigations on money demand dealing with the above points have been done in Colombia using quarterly data, monetary base, M1 or M3 as monetary aggregates (an exception is Steiner (1988), who also dealt with cash), the GDP as the scale variable and linear specifications (see Carrasquilla and Renteria, 1991; Misas and Oliveros, 1997; and Gómez, 1998, among others). However, the evidence of having an equation of money demand is rather mixed.

In this work we propose a different approach to arrive at the demand for money in Colombia. First, instead of using monetary base or some other broader aggregate, our definition of the monetary aggregate is cash. We assume that this variable reflects to a greater extent the preferences of agents to hold real balances while some other definitions could be affected by decisions of the Central Bank (e.g. changes in reserve requirements that would affect the measurement of monetary base), which could introduce an identification problem. Second, monthly data (1982:2-1998:11) are used to capture the short run dynamics of the demand for money on the belief that this is the most appropriate frequency to study the demand for real balances. Third, industrial GDP, measured by the Industrial Production Index, is used as a scale variable instead of the (quarterly) GDP. The latter variable is not entirely acceptable in Colombia because of measurement problems. Finally, we specify a nonlinear dynamic model of the demand for money using a smooth transition regression (STR) (see Granger and Teräsvirta, 1993; Teräsvirta, 1998) in which the scale variable, the short term interest rate and the rate of depreciation appear as the explanatory variables. In this paper, we show that the demand for cash in Colombia can be represented by a, noninvertible, nonlinear logistic specification of the STR-type. Moreover, our nonlinear error correction representation is consistent with the theory.

Findings of nonlinearities in money demand functions have been reported recently. They have been addressed, for example, by Hendry and Ericsson (1991b), Muscatelli and Spinelli (1996), Teräsvirta and Eliasson (1998), Ericsson et al. (1998), and Sarno (1999). The nonlinear approach has been used to estimate models of constant parameters in samples of long span and low frequency (annual data for about one century or so). However, we also consider this approach more appropriate to approximate the DGP of a higher frequency series.

Nonlinear models of demand for money may be rationalized as either target-threshold or buffer stock models. The former type of models put forth by Miller and Orr (1966) and developed by Akerlof (1973, 1979) and Milbourne (1983), provide microfoundations for the presence of a close to unity coefficient of the lagged dependent variable in the equation of money demand. Under these target-bound models, the agents define a target zone (bounded above and below) for their real-money balances. The upper and lower long-run thresholds are defined on the basis of expenditure plans and precautionary anticipations. Consequently, nominal balances are forced by the agents to stay near to the mean of the target zone (translated into nominal terms) when facing short-run deviations from the band. One type of the second class of models (see Laidler, 1984) give a buffer stock role to money in the sense that it acts as an asset that absorbs temporary shocks for which agents cannot postpone adjustments which are assumed very costly. As we shall see below other different types of buffer stock models have been developed to account for the fact that, given the existence of adjustment costs, relatively small deviations from the long-run real-money holdings are allowed to persist in the short-run while relatively large are not.

The outline of the paper is as follows. The second section describes the data, shows some preliminary results and discusses the equilibrium long-run relationship and the exogeneity of the system. The third section deals with the linear error correction model of demand for money. The fourth section, extends the target-bound and buffer stock alternatives to rationalize nonlinearities of demand for money. The fifth section introduces the STR models, discusses some estimation and testing issues and shows the results. The sixth section makes some final remarks.

2. DATA, COINTEGRATION SYSTEM AND EXOGENEITY ANALYSIS

From the theoretical point of view, agents may hold money as an inventory to reduce differences between the streams of income and expenditure. However, agents may also hold money as an asset in a multi-asset portfolio. Consequently, we could have a customary long-run specification for nominal money demand (M^d) such as:

$$M^d = f\ (P,Y,I) \qquad (1)$$

where P is the price level, Y is the scale variable and I is a set of rates of returns on assets. The empirical model we consider imposes long-run price homogeneity, takes the industrial GDP as the scale variable, and regards the interest rate from period t to $t+1$ and the depreciation rate from $t-1$ to t as the opportunity cost of holding money. The behavior of these variables during the sample period is shown in Figure 1.

The long-run dynamic model is expressed as:

$$m - p = \lambda_0 + \lambda_1 y + \lambda_2 i + \lambda_3 e \qquad (2)$$

where m is the log of cash, p is the log of CPI, y is the log of industrial GDP, i is the interest rate, e is the annual rate of depreciation and λ_j (j = 0, 1, 2, 3) are parameters[2].

Following Wolters et al. (1998) we use seasonally unadjusted variables on the assumption that seasonal fluctuations are an important source of variation in economic time series and it seems sensible to model them instead of smoothing them out. The set of variables used in this work is

[2] The data corresponding to m, i and e are month average instead of end-of-month.

monthly dated[3] (1982:2-1998:11). The objective of using this frequency is to trace the dynamics of demand for cash since it is more accurate for such an aim than annual or even quarterly data[4]. Given that (*m-p*), *y*, *i*, and *e* are *I*(*1*) under conventional tests, we use Johansen techniques to investigate the cointegrating properties of the stochastic VAR system. The number of cointegrating vectors was tested by using a lag length of twelve[5], an intercept in the cointegration space without allowing for linear trends in the data[6], and including centered seasonal dummies out of the long-run relationship[7]. The results of the cointegration analysis (Table 1) show that according to the trace statistic there is only one cointegrating vector among the stochastic variables of the system. The equation of the demand for cash is correctly signed after normalizing by the coefficient of (*m-p*). On the assumption that there is only a single long-run relationship between the variables of the VAR system, we found evidence of joint weak exogeneity of *y*, *i*, and *e*. This means that we are able to condition *y*, *i*, and *e* on (*m-p*) without losing information relevant for the estimation of the parameters of interest and, consequently, the system of the demand for cash can be reduced to a single equation[8]. Furthermore, according to the results of

[3] To give a rough idea of the Colombian environment, during the sample period there have been at least three remarkable events. First, the financial crisis occurred in the first part of the eighties. Second, the opening up of the economy undertaken between the end of the eighties and the beginning of the nineties. Among the measures adopted within this framework, the change of the exchange rate determination regime from a crawling peg to a target zone is of particular interest for this study. Third, the new order brought to the monetary policy setting given the institutional changes, which allowed a more independent Central Bank.

[4] To our knowledge, no investigation on demand for money using high frequency data has been done yet to the Colombian case.

[5] The lag was chosen on the basis of the Likelihood ratio tests (see Lütkepohl, 1991).

[6] A trend in the cointegration space happened to be statistically insignificant.

[7] As the system contains an unrestricted constant, the asymptotic values do not need any correction because of the seasonal dummies.

[8] If we have a model such as $\boldsymbol{x}_t = \mathbf{f}(\boldsymbol{z}_t)$, $\boldsymbol{z}_t$ will be weakly exogenous if the joint distribution of $\boldsymbol{w}_t=(\boldsymbol{x}_t , \boldsymbol{z}_t)$ conditional on the past, can be factorised as the conditional distribution of $\boldsymbol{x}_t$ given $\boldsymbol{z}_t$ times the marginal distribution of $\boldsymbol{z}_t$. As a result, the parameters of the conditional and marginal distributions are not subject to cross-restrictions and the parameters of interest can be uniquely determined from the parameters of the conditional model.

Table 1 there is evidence of strong exogeneity, given the fact that the null hypothesis of block non Granger-causality cannot be rejected at conventional significance levels. This outcome implies no feedback from (*m-p*) to the stochastic subsystem composed by *y*, *i*, and *e*.

Table 1. Cointegration system analysis

Trace test for the cointegration rank

Eigenvalue	Null hypothesis	Alternative hypothesis	Trace	T. Critical value (90%)
0.1089	$r = 0$	$r \geq 0$	52.23	49.92
0.0727	$r \leq 1$	$r \geq 2$	30.31	31.88
0.0669	$r \leq 2$	$r \geq 3$	15.98	17.79
0.0147	$r \leq 3$	$r \geq 3$	2.82	7.50

Standardized long-run coefficients

(*m-p*)	*Y*	*i*	*e*
1.0000	-0.370	2.619	0.291

P-values of testing

Joint weak exogeneity	*Block non causality*
0.3234	0.1149

In summary we have one equilibrium long-run relation among the variables of the demand for money, in which the variables appear rightly signed. The scale variable, the interest rate and the rate of depreciation are joint weak exogenous and there is no feedback from (*m-p*) to the block composed by *y*, *i*, and *e* so that strong exogeneity obtains.

3. THE ERROR CORRECTION REPRESENTATION OF THE DEMAND FOR MONEY

The results of section 2 permit us to conduct the analysis in a single equation framework. Consequently, the variations of (*m-p*) can be expressed in terms of an error correction mechanism, lags of the dependent variable, lags of the changes of *y*, *i*, and *e* and, possibly, deterministic components. OLS estimation can be used under the assumptions that the coefficients of money demand (*m-p*) are zero on the equations of *y*, *i*, and *e* and the structural shocks of (*m-p*) and *y*, *i*, and *e* are orthogonal.

Starting with twelve lags and seasonal centered dummies for each month, the error correction model we find, after excluding the variables found not statistically significant, is:

$$\Delta(m-p)_t = \underset{(0.056)}{0.2133} - \underset{(0.014)}{0.0549}ecm_{t-1} - \underset{(0.007)}{0.1000}d_{1,t} - \underset{(0.006)}{0.0242}d_{8,t} + \underset{(0.005)}{0.0352}d_{11,t} + \underset{(0.006)}{0.2385}d_{12,t}$$
$$- \underset{(0.033)}{0.1475}\Delta(m-p)_{t-2} - \underset{(0.035)}{0.0907}\Delta(m-p)_{t-3} - \underset{(0.026)}{0.1147}\Delta(m-p)_{t-4} - \underset{(0.020)}{0.1444}\Delta(m-p)_{t-5}$$
$$- \underset{(0.023)}{0.0585}\Delta(m-p)_{t-7} + \underset{(0.041)}{0.1405}\Delta y_{t-1} + \underset{(0.045)}{0.0892}\Delta y_{t-2} + \underset{(0.038)}{0.1111}\Delta y_{t-3} + \underset{(0.137)}{0.3145}\Delta i_{t-7} + \underset{(0.135)}{0.2517}\Delta i_{t-11}$$

where, $ecm_t = (m-p)_t - 0.370\, y_t + 2.619 i_t + 0.291 e_t$. (3)

T = 1982:2-1998:11 = 202; R^2 = 0.948; *SEl* = 0.020; *DW* = 2.1685; *P*-value *LB*(36) = 0.628; *ARCH*(*1*) = 1.5910(0.208); *ARCH*(*4*) = 0.8235(0.5116); *JB* = 11.8679(0.0042); *RESET* = 0.5994(2.36×10^{-10}).

The linear error correction model in (3) exhibits correct signs for the error correction term and the seasonal dummies corresponding to January (d_1), August (d_8), November (d_{11}) and December (d_{12}). Also the coefficients have low standard errors (in parenthesis). The goodness of fit as measured by R^2 is very high and the standard error (*SEl*) seems of adequate magnitude. There is evidence of neither serial autocorrelation at one lag measured by the Durbin-Watson (*DW*) and 36 lags according to the *P*-value of the Ljung-Box coefficient nor *ARCH*-type nonlinearity at one and four lags. However, notice that, according to the Jarque-Bera statistic, the null of normality of the residuals is rejected. This result together with the rejection of the null of no misspecification of the RESET test suggests that an alternative dynamic model should be considered. This suggestion is in line with that of Hendry and Ericsson (1991), Ericsson, Hendry and Prestwich (1998), Teräsvirta and Eliasson (1998), Lütkepohl et al., (1999), and Sarno (1999). The alternative we consider here is an error correction STR model of the demand for money.

4. RATIONALIZING NONLINEARITIES WITHIN THE MONEY DEMAND FRAMEWORK

As mentioned in the introduction, nonlinear models of demand for money can be interpreted at least from two points of view: as the result of target-bounds and as a buffer stock, both micro-founded. With the target-bound models, first proposed by Miller-Orr (1966) and developed by Akerlof (1973, 1979), and Milbourne (1983), the agents, based on expenditure plans and precautionary anticipations define a band for their holdings of real money.

The fact that any target band introduces nonlinearities to the behavior of the targeted process, has been well documented in economics (e.g. Blatt, 1983, chapter 10, for the case of investment; or target zone models for exchange rates as in Krugman, 1991). It is also the case within the money demand framework (Akerlof and Milbourne, 1980) since nominal balances are forced by the agents to keep near to the mean of the target-bound when facing short-run deviations from it or even when the nominal balances are close to the upper and lower bounds[9].

The buffer stock models (see Laidler, 1984) account for the fact that, given the existence of adjustment costs, relatively small deviations from the long-run real-money holdings are allowed to persist in the short-run while relatively large are not. Different approaches can be distinguished within the buffer stock models. First, there exist the disequilibrium models which can be of two different types: *single equation disequilibrium estimates* of money demand that have an autoregressive component which has been associated to slow adjustment of short-run to long-run desired money holdings; and, *complete disequilibrium models* in which a number of real and nominal variables are introduced[10]. These models require that, if the parameters of interest are the coefficients of the long-run money demand, then their estimates are conditional on the full specification of the entire model. Second, the shock absorber approach directly estimates the demand for money function although money supply is assumed to be held in transactions balances. Rational expectations have been assumed and unexpected money supply shocks are voluntarily held in money balances. Fully anticipated money supply changes are reflected in price expectations and if prices are perfectly flexible, then real money balances remain unchanged (Carr et al., 1985). Carr and Darby (1981) found evidence in favor of the shock absorber approach whereas Cuthbertson (1986) rejected it by using a different estimation techniques. Third, the forward-looking approach of the buffer stock models, allow agents to hold temporarily cash due to unanticipated increases in income.

5. A SMOOTH TRANSITION REGRESSIVE MODEL FOR MONEY DEMAND

We consider the smooth transition regressive models (see Granger and Teräsvirta, 1993; Teräsvirta, 1998) to represent the error correction model of the demand for money as the alternative to the linear error correction mechanism estimated in (3); see also Teräsvirta and Eliasson (1998), Lütkepohl et al. (1998), Sarno (1999), for applications of this approach.

The model can be written as:

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[9] Akerlof and Milbourne´s model captures the stylised fact that the short run income elasticity is small and could even be negative.

[10] These models consider an equation for a set of real and nominal variables (output, prices, interest rates, exchange rate, etc.) explained by predetermined variables and a lag polynomial of the differences between money supply and money demand; and other equations for the long-run money demand.

$$x_t = \beta' w_t + (\pi_1 + \pi_2 F(s_{i,t};\alpha))' z_t + u_t \qquad (4)$$

where x_t is the dependent variable, $w_t = (w_{1t},.......,w_{Mt})$ is a vector of K regressors which enter linearly with constant parameter vector β, z_t =($z_{1t},.......,z_{Mt}$) is a vector of M regressors, s_t = ($s_{1t},.......,s_{Lt}$) is a $(L \times 1)$ vector of regressors whose elements may include those of w_t and z_t, and u_t is an *iid* error process, $E(u_t) = 0$, $Var(u_t) = \sigma^2$. F is a transition function bounded by 0 and 1 whose parameters are denoted by α. It is assumed that $E(w_t\, u_t) = 0$, $E(z_t\, u_t) = 0$, $E(s_t\, u_t) = 0$. The x_t, w_t , and z_t processes are assumed to be weakly stationary for the theory of linearity tests to work[11]. Some lagged elements of x_t may be included in w_t and z_t although weak exogeneity of the remaining elements of them, with respect to the parameters of interest in (4) is required.

Notice that when the transition function $F \equiv 0$ the *STR* model (4) will be linear but, in general, the vector of regressor coefficients, $\pi_1 + \pi_2 F(s_{i,t};\alpha)$, will depend on the values of the transition variable s_t. The transition function can be parameterized either as a logistic function, in whose case we have a logistic STR (LSTR) model:

$$F(s_t;\alpha) = (1 + \exp\{-\gamma(s_t - c)\})^{-1}\ ,\ \gamma > 0 \qquad (5)$$

or as an exponential function, in whose case we have an exponential STR (ESTR) model:

$$F(s_t;\alpha) = 1 - \exp\{-\gamma(s_t - c)^2\}\ ,\ \gamma > 0 \qquad (6)$$

The null hypothesis is that of linearity ($H_0 : \gamma = 0$). However, (4) either with (5) or (6) is only identified under the alternative (H_1: $\gamma > 0$) which invalidates the asymptotic distribution theory. It has been shown that the problem can be solved by using the auxiliary regression obtained

[11] However, the theory still applies when s_t is non stationary dominated by a polynomial in t (Lin and Teräsvirta, 1994).

approximating $F(\bullet)$ by a third order Taylor series expansion (Granger and Teräsvirta, 1993; Teräsvirta, 1994; Teräsvirta, 1998):

$$x_t = \beta' w_t + \lambda_0' z_t + \lambda_1' z_t s_t + \lambda_2' z_t s_t^2 + \lambda_3' z_t s_t^3 + v_t \qquad (7)$$

where the null of linearity becomes $H_0 : \lambda_1 = \lambda_2 = \lambda_3 = 0$, with power against both the LSTR and ESTR. The choice between LSTR and ESTR can also be done using equation (7), following the sequence proposed by Teräsvirta (1998). When t takes the place of the transition variable ($s_t \equiv t$), the transition function is either:

$$\begin{aligned} F(t;\alpha) &= (1 + \exp\{-\gamma(t - c_1)(t - c_2)(t - c_3)\})^{-1}; \\ &\gamma > 0, \quad c_1 \le c_2 \le c_3 \end{aligned} \qquad (8)$$

or

$$\begin{aligned} F(t;\alpha) &= (1 + \exp\{-\gamma(t - c_1)(t - c_2)\})^{-1}; \\ &\gamma > 0, \quad c_1 \le c_2 \end{aligned} \qquad (9)$$

or

$$\begin{aligned} F(t;\alpha) &= (1 + \exp\{-\gamma(t - c_1)\})^{-1}; \\ &\gamma > 0 \end{aligned} \qquad (10)$$

Equation (7) can be used for testing parameter constancy in the linear case (Lin and Teräsvirta, 1994; Jansen and Teräsvirta, 1996). For testing parameter constancy in the nonlinear case we use equation (4.6) of Eitrheim and Teräsvirta, (1996). If the test is carried out with either (8), (9), or (10), they will be termed LM3, LM2, or LM1, respectively. The LM1 is a test of H_0: $\lambda_1 = 0 | \lambda_2 = \lambda_3 = 0$, which has good power against a smooth change in the parameters and, depending on the value of gamma, also against a single structural brake. The LM2 is a test of H_0: $\lambda_1 = \lambda_2 =$

$0|\lambda_3 = 0$, and the transition function is a LSTR2 (a logistic transition function with a rest path in the middle regime).

All variables of the linear error correction model were used as possible transition variables. The results do not indicate any nonlinearity at 5% of significance when the possible transition variable is one of the variables used in the linear model (3). The null hypothesis of linearity is rejected when the transition variable is $\Delta_{12}(y)_{t-10}$ given that the *P*-value is the lowest (0.0071). With this result the selection procedure described in detail in Teräsvirta (1998) allowed us to choose a logistic STR model whose final specification is[12]:

$$
\begin{aligned}
\Delta(m-p)_t =\ & \underset{(0.488)}{1.6552} - \underset{(0.131)}{0.44793}ecm_{t-1} - \underset{(0.007)}{0.1168}d_{1,t} - \underset{(0.0502)}{0.2183}d_{2,t} - \underset{(0.3313)}{1.2146}d_{4,t} - \underset{(0.0847)}{0.2730}d_{7,t} \\
& - \underset{(0.048)}{0.1865}d_{8,t} - \underset{(0.062)}{0.2222}d_{9,t} - \underset{(0.095)}{0.3359}d_{10,t} - \underset{(0.061)}{0.1701}d_{11,t} + \underset{(0.046)}{0.0797}d_{12,t} - \underset{(0.302)}{1.1743}\Delta(m-p)_{t-3} \\
& + \underset{(0.691)}{2.498}\Delta(m-p)_{t-4} + \underset{(0.038)}{0.1223}\Delta y_{t-1} + \underset{(0.117)}{0.182}\Delta i_{t-7} + \{\underset{(0.490)}{-1.464} + \underset{(0.132)}{0.3989}ecm_{t-1} + \underset{(0.050)}{0.1672}d_{2,t} \\
& + \underset{(0.331)}{1.1847}d_{4,t} + \underset{(0.084)}{0.2730}d_{7,t} + \underset{(0.049)}{0.1352}d_{8,t} + \underset{(0.063)}{0.1877}d_{9,t} + \underset{(0.095)}{0.3359}d_{10,t} + \underset{(0.061)}{0.1932}d_{11,t} + \underset{(0.046)}{0.1427}d_{12,t} \\
& + \underset{(0.037)}{0.1031}\Delta y_{t-2} + \underset{(0.302)}{1.0630}\Delta(m-p)_{t-3} - \underset{(0.693)}{2.5971}\Delta(m-p)_{t-4} - \underset{(0.018)}{0.1367}\Delta(m-p)_{t-5} - \underset{(0.023)}{0.1481}\Delta(m-p)_{t-7}\} \\
& * [1+\exp\{-\underset{(38.289)}{36.9871}[(\Delta_{12}y)_{t-10} + \underset{(0.002)}{0.0535}]/\sigma_{\Delta_{12}(y)}\}]^{-1}
\end{aligned}
$$

where,. $ecm_t = (m-p)_t - 0.370y_t + 2.619i_t + 0.291e_t$ (11)

T = 1982:2-1998:11= 202; $R^2 = 0.959$; *Senl* = 0.0187; *Varnl/Varl* = 0.852; *P*-value *LB*(4) = 0.418; *ARCH*(*1*) = 0.3178(0.573); *ARCH*(*4*) = 2.7362(0.067); *JB* = 21.3642(0.00).

From the statistics of the error correction *STR* regression in (11), it can be seen that this model outperforms the linear error correction model of equation (3). Thus, the standard error (*SEnl*) is smaller to the extent that the ratio of the residual variances is 0.852, the residuals are white noise but normality is not accomplished according to Jarque-Bera statistic, regardless of some points are correctly captured for our specification. The nonlinear model seems more accurate in capturing the behavior of the demand for cash in Colombia than the linear one, according to the Figures 2 and 3, a fact that can also be observed by looking at the residuals of the respective models (Figures 4 and 5), and the linear and nonlinear error correction mechanisms (Figures 6 and 7).

In addition to these specification tests we also implement the LM-type tests of no remaining autocorrelation, no remaining nonlinearity and parameter constancy developed by Eitrheim and Teräsvirta (1996) in order to check the adequacy of the estimated model in the nonlinear framework. The test of no remaining autocorrelation has under the alternative hypothesis a nonlinear model with autocorrelation of order q. Under the null the test is asymptotically distributed as a $\chi^2_{(q)}$, but with the purpose of having the size under control in small samples we use the F-version. The results (Table 2) indicate that the test fails to reject the null hypothesis of no error autocorrelation of order 1, 4, and 20 at any usual levels of significance.

Table 2. P-values of the LM test of no error autocorrelation against an AR(q) and MA (q) error processes

	Maximum lag q						
Test	1	2	4	6	12	18	20
No error autocorrelation	0.898	0.483	0.419	0.426	0.539	0.683	0.796

[12] We thank Timo Terasvirta for providing us with some of the Gauss code used in this application and also Munir Jalil for further developments of it.

Following Eitrheim and Teräsvirta (1996) we also check whether equation (11) is an adequate characterization of the nonlinear features rendered by the data by looking at remaining nonlinearity. Under the alternative the model is an additive STR model, and the test statistic has an asymptotic $\chi^2_{(q)}$ distribution under the null. However, the model is not identified under the null but this problem is solved in the same way in which the linearity test is solved by Granger and Teräsvirta (1993) and Teräsvirta (1994). In this case, all variables in the linear error correction model of equation (3) were tried as potential transition variables in the additive nonlinear model. Nonetheless the results do not give any evidence in favor of remaining nonlinerity in the model of equation (11).

Finally, we have applied the test of parameter constancy under the alternative hypothesis of a smooth change in the parameters of the model which also includes the test of an abrupt change. The test under the alternative is parameterized using three different functions in which the transition variable is the time (equations 8, 9, and 10 above). These possible parameterizations allow a wide range of non constancy possibilities. Here, the constancy test is carried out over four different sets of parameters (seasonal, linear and nonlinear) of equation (11). The results in Table 3 show that the constancy is not rejected at any level of significance for the parameters in the linear and the nonlinear part of the model. However, the null can be rejected for the parameters of the seasonal dummies. This result is generated by the non constancy of the dummy corresponding to December (H_0: (5) and H_0: (6) in Table 3). The other parameters are constant over time and hence maintain our final specification of equation (11).

Table 3. *P*-values of the LM test of parameters constancy of the LSTR model (11) against STR-type non constancy

	Null hypothesis					
	(1)	(2)	(3)	(4)	(5)	(6)
LM1	0.7042	0.5465	0.0000	0.0000	0.3038	0.4480
LM2	0.9570	0.4672	$3*10^{-6}$	$3*10^{-6}$	0.6397	0.7147
LM3	0.9861	0.6464	$17*10^{-4}$	$15*10^{-4}$	0.7958	0.8302

(1) H_0: "All parameters of the linear part of the model except the constant are constant"
(2) H_0: "All parameters of the nonlinear part of the model except the nonlinear intercept and the dummies are constant".
(3) H_0: "All parameters of the nonlinear seasonal part of the model are constant"
(4) H_0: "All parameters of the nonlinear seasonal part of the model are constant"
(5) H_0: "All parameters of the linear seasonal part of the model without the seasonal parameter d_{12} are constant"
(6) H_0: "All parameters of the nonlinear seasonal part of the model without the seasonal parameter d_{12} are constant"
Note: The parameters not under test are assumed constant also under the alternative

6. FINAL REMARKS

The final specification we have obtained could be analyzed as follows. There is an equilibrium long-run demand for cash in which price homogeneity has been imposed and the normalized coefficients are correctly signed. There is evidence of weak and strong exogeneity and, according to the statistics, our nonlinear model outperforms the linear error correction of equation (3).

The nonlinear dynamics works depending on the value of $(\Delta_{12} y)_{t-10}$, a result highly intuitive, not for the lag but for the variable, in the sense that agents can observe the evolution of the economic activity to decide, based on expenditure plans and precautionary anticipations a band for their holdings of real money. Nominal balances are forced by the agents to keep near to the mean of the target-bound when facing short-run deviations from it or even when the nominal balances are close to the upper and lower bounds.

The logistic STR model (11) contains a nonlinear error correction adjustment (*necm*) which we reproduce here as:

$$\Delta(m-p)_t = 1.6552 - 0.44793 ecm_{t-1} + \{-1.464 + 0.3989 ecm_{t-1}\}$$
$$* [1 + \exp\{-36.9871[(\Delta_{12} y)_{t-10} + 0.0535] / \sigma_{\Delta_{12}(y)}\}]^{-1}$$

To analyze the local dynamics, notice that in the extreme regimes of the transition function, (*F* = 0 and *F* = 1 of Figures 8 and 9), the *necm* becomes:

$$necm_{F=0} = 1.6552 - 0.4479 ecm_{t-1}$$

$$necm_{F=1} = (1.6552 - 1.4642) - (0.4479 - 0.3989) ecm_{t-1}$$

thus, when $(\Delta_{12} y)_{t-10}$ is close or less than the threshold value (*c* = -5.35%), where *F* = 0, the error correction is 9.14 [=0.4479/(0.4479-0.3989)] times the error correction when $(\Delta_{12} y)_{t-10}$ is greater than the threshold value, where *F* = 1.

These different adjustment processes towards the equilibrium are indicative of the asymmetric dynamics rendered by the logistic STR which could match the target-threshold models, in the sense that the agents adjust their holdings of real money to the desired level in different magnitudes and speeds depending on the value (and the sign) of $(\Delta_{12} y)_{t-10}$.

According to our nonlinear specification, the demand for cash in Colombia has remained, most of the time, in the upper regime given the values of the threshold and the historical value of the transition variable. Notice that, according to the value of gamma, the speed to move from regime the other is very high. Finally, the seasonal dummies of July and October tell us nothing about the demand for cash in Colombia since the corresponding linear and nonlinear parameters exactly compensates each other.

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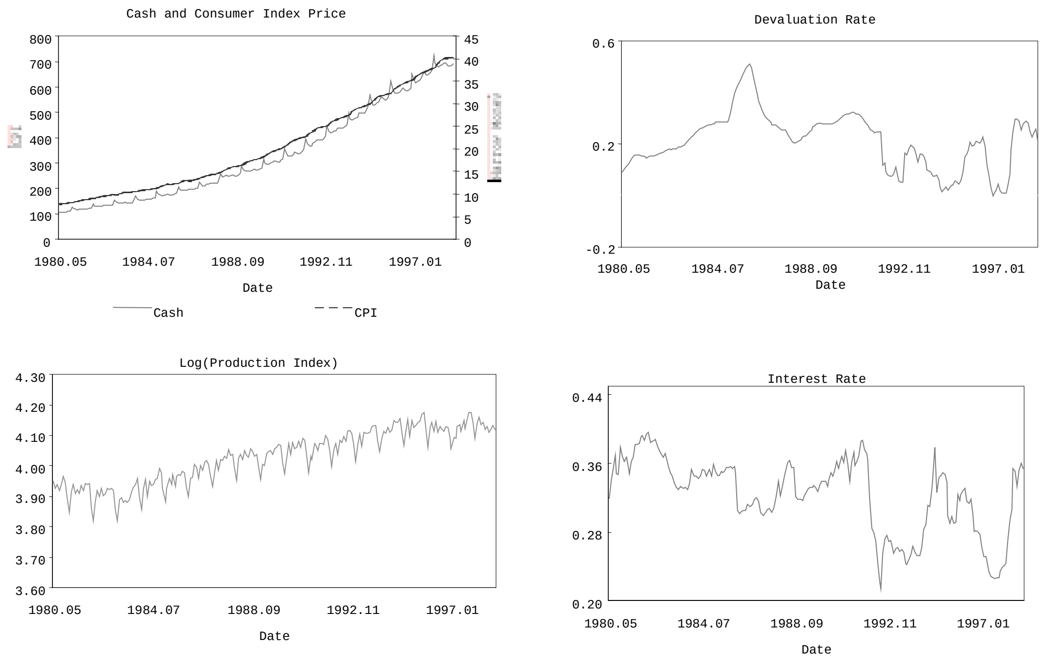
Figure 1. Set of variables (monthly data, 1980:5 – 1998:11)



Sources: cash, interest rate and depreciation rate from the Banco de la República.
CPI and Industrial Production Index from DANE.

Figure 2. Money demand: observed vs. linear estimation

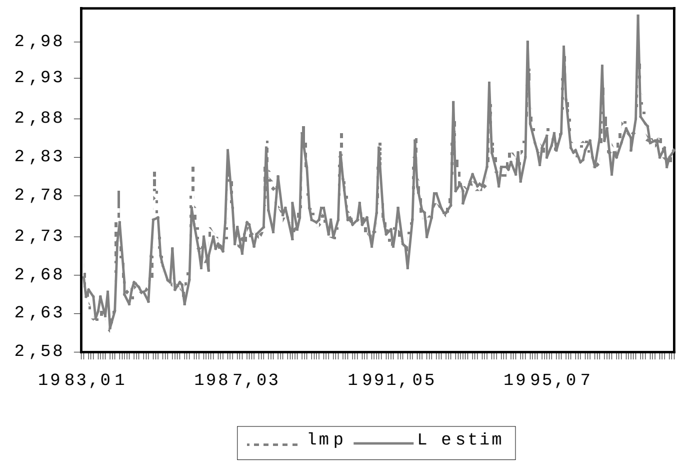


Figure 3. Money demand: observed vs. nonlinear estimation

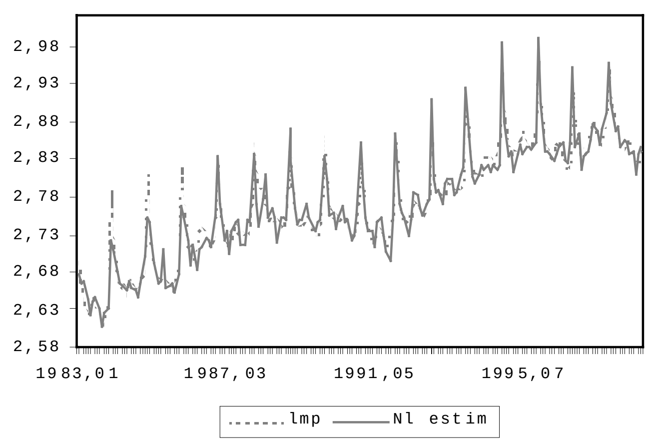


Figure 4. Residual from the linear model

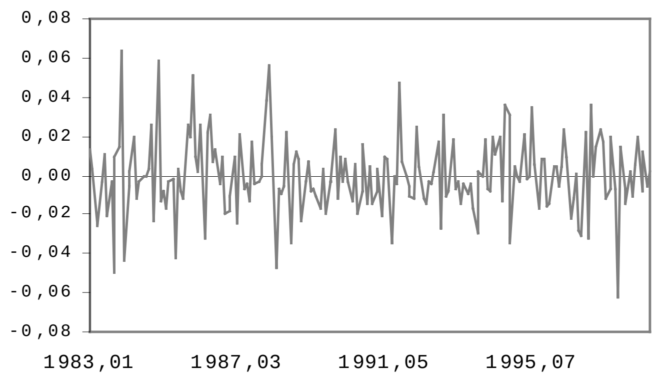


Figure 5. Residual from the nonlinear model

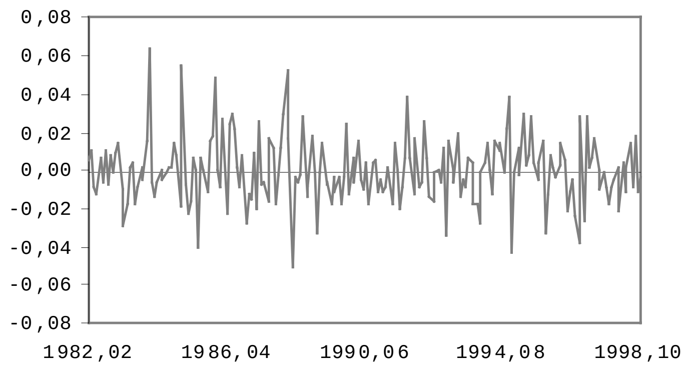


Figure 6. Linear error correction

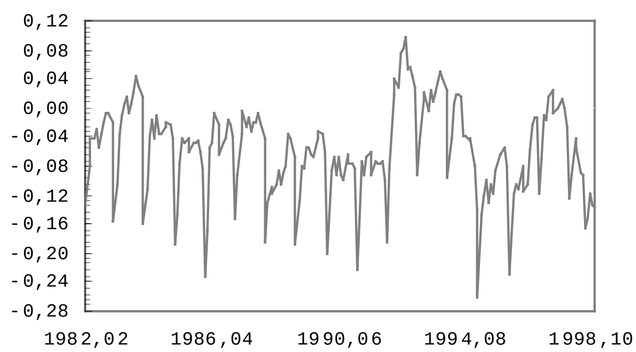


Figure 7. Nonlinear error correction

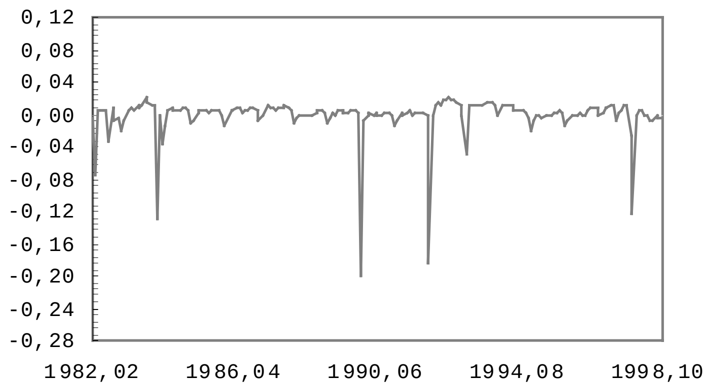


Figure 8. Transition function over time

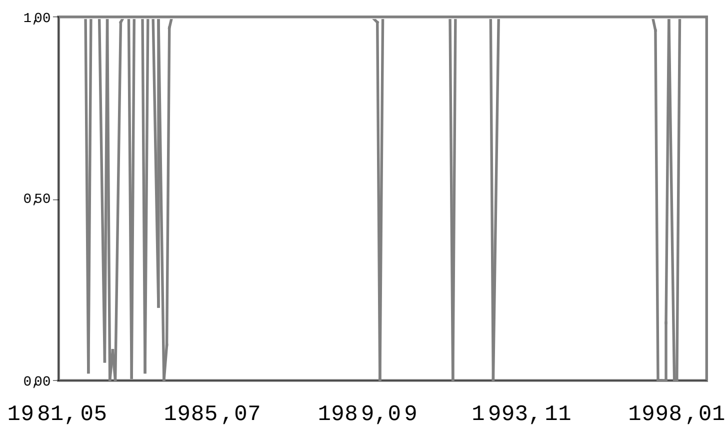


Figure 9. Transition function