Returns and interest rate: A nonlinear relationship in the Bogotá stock market

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Summary

This work presents some evidence of the nonlinear and inverse relationship between the share prices on the Bogotá stock market and the interest rate as measured by the interbank loan interest rate, which is to some extent affected by monetary policy. The model captures the stylised fact on this market of high dependence of returns in short periods of time. These findings do not support any efficiency on the main stock market in Colombia. Evidence of a non constant equity premium is also found. The work uses daily data from January 1994 up to February 2000.

JEL classification: C22, C52.

Key words: nonlinearities, stock returns, interest rate, smooth transition regression, GARCH models.

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I. Introduction

Stock prices is one of the most active areas of economic research. Some models aimed to the study of asset prices have been developed. Thus, we have the exact present-value formula, which considers the market value of an asset as being determined by the discount present value of the expected dividend payments. According to this view, the rational price of an asset will be less the higher the discount rate. The Gordon growth model, which is perhaps the simplest fundamental-based mechanism to predicting stock prices, is an especial case where dividends are expected to grow at a constant rate (see Heaton and Lucas, 1999, for a recent application of this model).

The intertemporal equilibrium models of asset pricing build on the assumption that people trade assets with the purpose of smoothing consumption over time. The result of this is contained by the *Euler equation*, where the marginal benefit of not consuming one more real dollar today is equated to the marginal benefit from investing the dollar in any asset and selling it in order to consume the proceeds in the future (see Campbell, 1999). Accordingly, this condition describes the optimum. When the *Euler equation* contains an stochastic discount factor, the consumption-based capital asset pricing model arises. These kind of intertemporal models can deal with questions about the forces that determine the rate of return of the zero-beta asset (the riskless interest rate) and the rewards that investors require for bearing risk (for a discussion at length of these models see Campbell, 1999). The Lucas asset-pricing model, as another example, considers consumption as an exogenous variable and equal to output in equilibrium. In this case, the first order conditions can be used to price assets so that these are function of consumption (Lucas, 1978).

In this connection, however, there are some points of debate. Such points focus on the driving forces or fundamentals that determine share prices[1], the channels through which asset prices affect economic activity, the information contained in these prices about future economic activity, the distinction between large fundamental and speculative swings in asset prices and the response of policymakers to face them.

In this paper we explore the empirical link between the asset prices on the Bogotá stock market and the Colombian interbank loan interest rate (TIB).

The objective of this work is to describe the dynamics of the returns and the way in which the TIB affects the behaviour of the Bogotá stock index (IBB). Having a description of the dynamics is a good starting point which could be easily improved to forecast returns and elaborate a trading strategy. The work is carried out by using daily data from January 1994 up to February 2000. This frequency is justified given the velocity of both the money and the stock markets to adjust.

The paper is organised as follows. Section II shows some stylised facts and properties of IBB and TIB during the sample period. Section III explains the empirical

[1] Some relationship between fundamentals and stock prices are associated with changes in corporate earnings growth, in consumer preferences, and in stock-market participation patterns (Heaton and Lucas, 1999). As a source of non fundamental variations of stock prices are the actions of agents with privileged information, poor regulation mechanisms and imperfect rationality of investors (Bernanke and Gertler, 1999).

approach and shows the main findings and results. Finally, section IV provides some conclusions.

II. Properties of IBB

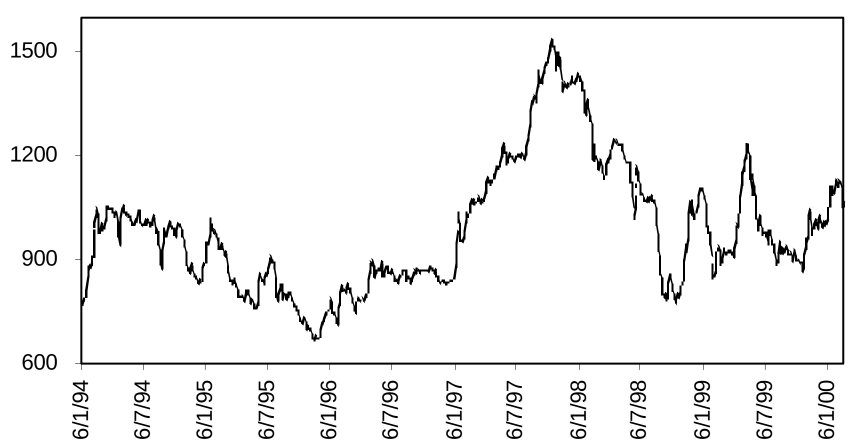
Following Campbell *et al*. (1997), the simple net rate of return, R_t, on the asset between dates t - 1 and t is defined as:

$$R_t = \frac{P_t}{P_{t-1}} - 1$$

where P_t denotes the IBB at date *t*. Figure 1 shows the IBB during the sample period. From this picture we observe that the slope of any linear trend of the IBB, while positive, would look rather small[2].

Figure 2 presents the behaviour of the interbank loan interest rate (TIB) accompanied by the band of no intervention of the central bank. When the TIB is on risk of crossing the band borders, central bank intervenes by counteracting the movement dictated by the market, thus avoiding an inconvenient level of the interest rate[3]. This intervention occurs through the REPO market, in such a way that TIB reflects monetary policy somehow.

Figure 1. Behaviour of the Bogotá stock index (IBB)

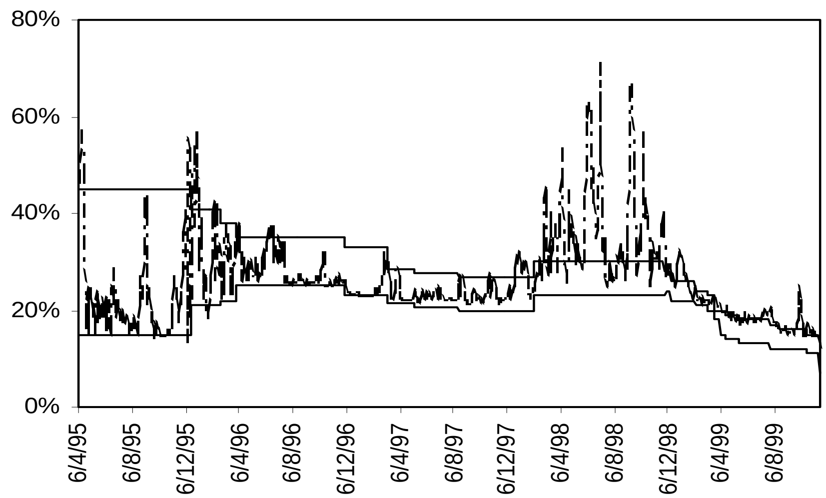


Source: Bogotá Stock Market

[2] It is clear that a moving trend would be more sensible.

[3] The higher levels of the TIB, however, occurred when the very central bank was acting in defence of the previous exchange rate band.

Figure 2. Interbank loan interest rate (TIB) and the intervention band



Source: Banco de la República

Figure 3 shows the joint behaviour of returns, R_t, and the first difference of the TIB, ΔTIB. The mean of the daily rate of return during the sample period is about 0.029% while the standard deviation is 1.16%. Volatility of returns has been increasing, since between January 1994 and December 1996 the deviation was 0.99%, and 1.29% between January 1997 and February 2000.

Figure 3. Returns from the Bogotá stock index
and the first difference of the interbank loan interest rate (TIB)

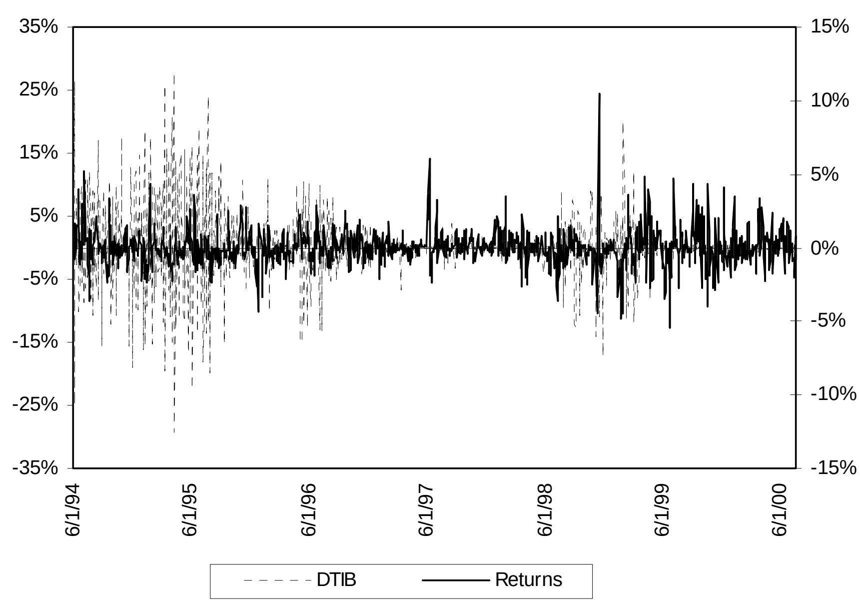


Table 1 shows the participation of the upwards and downwards daily movements of the IBB, when the variation one, two and three days before had the same direction. Most of the index changes, although slightly, go downwards. However, on the mean-of-return basis, the upwards changes are higher than the downwards ones[4]. A striking property is the autocorrelation exhibited by the simple net rate of return on the Bogotá stock market, which may also be interpreted as cuasi-probabilities. For example, 67.7% (=31.2%+36.5%) of the movements of the index repeat the direction of the change happened the day before. When we consider the changes of two days before, the direction of the change repeats 47.8% (=21.4%+25.9%) of the time. Finally, when the changes of three days before are considered, the direction of the change repeats 33.6% of the time[5]. It follows that financial asset returns cannot be considered independently distributed over time.

Table 1. Percentage of the upwards and downwards variations on the Bogotá stock index (IBB) having into account the variation of the one, two, and three days before

Movement → ↓		Today´s change		
		Upwards	Downwards	No change or any other combination of changes
		47.2%	52.4%	0.4%
Last day Change	Upwards	31.2%		32.3%
	Downwards		36.5%	
Change of one and two days before	Upwards	21.4%		52.7%
	Downwards		25.9%	
Change of one, two and three days before	Upwards	15.3%		66.4%
	Downwards		18.3%	

The statistical property of positive autocorrelation of simple net rate of return suggests that there is a high proportion of investors that behave as chartists rather than fundamentalists (for a recent application of this see De Grauwe *et al*., 1993), so that the return process could be either a martingale or a fads process, something that we explore next.

III. Empirical issues

Under conventional procedures, we find evidence that suggests that both IBB and TIB are nonstationary processes. Given this result, we next move to check for any common

[4] No especial pattern is present on a day-by-day basis.

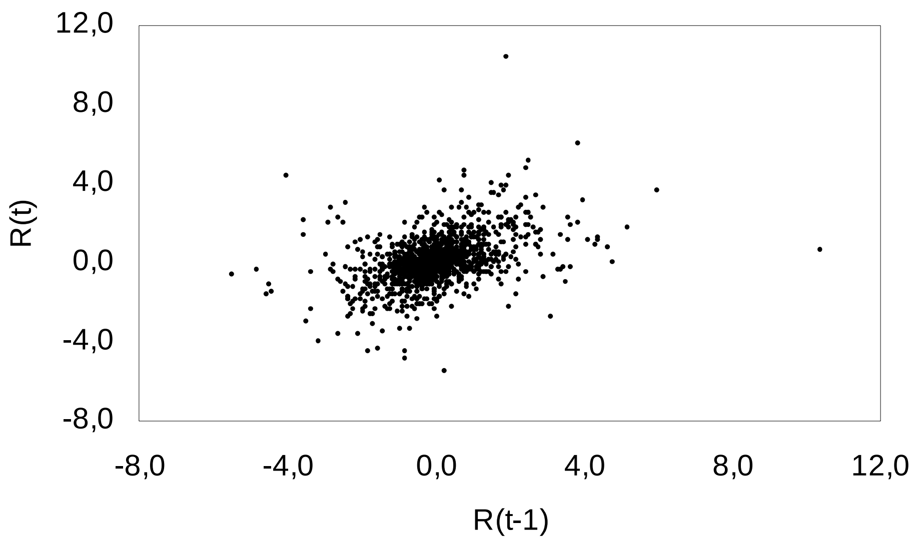
[5] As discussed by Fama (1970) any test for market efficiency involves a joint hypothesis about the rationality of markets and the equilibrium expected rate of return. The efficient market view of the stock markets states that the return on stocks is unforecastable and that all information about future prices is efficiently incorporated into the current price (see MacDonald and Taylor, 1991). Thus, if markets are efficient, the realised returns are expected to be serially uncorrelated, a property no exhibited by the returns on the Bogotá stock market.

trend between the variables. However, by using the Johansen´s procedure we do not fail to reject the hypothesis of a common trend between these two variables[6].

With these preliminary results, apart from the nonlinearity between share prices and interest rate suggested by the theory we also check the hypothesis of a negative (inverse) relationship between these two variables (Appendix 1 sets out the theoretical model used to illustrate the relationship between the variables).

On the other hand, considering both Figure 3 and the scatterplot in Figure 4, where the return of day *t* appears against the return of day *t* –1, the Bogotá stock market seems to conform the stylised fact that relatively volatile periods characterised by large returns alternate with quiet periods of small returns. In other words, in the Bogotá stock market large price changes occur in clusters of outliers which have been associated to the nonlinear fashion in which volatility series evolves over time (Cao and Tsay, 1992)[7]. This gives us another argument to look for nonlinearities in the series of returns itself.

Figure 4. Scatterplot of the return oF the Bogotá stock index (IBB) on day *t* against the return on day *t* – 1



[6] Only when a small number of lags is considered, the variables are cointegrated, but the residuals are autocorrelated and non normal, different from what is required by MLE. In addition, the exclusion test suggests that with a constant in the cointegrating equation TIB is not required. Given this result, weak exogeneity was checked by using the Granger-causality test, where we found some evidence on that ΔTIB does not Granger-cause *R*, while the reverse is different.

[7] Any evidence of nonlinearity in financial time series would suggest that some forecasting improving could be obtained in the short term by changing linear strategy for the nonlinear one. However, as noted by Brooks (1996), evidence of nonlinear dependence in market returns could cast some doubts on the informational efficiency of financial markets, since it may possible to obtain a trading rule to generate positive returns with a probability higher the 0.5. Nonetheless, we recall that our work focuses on the return dynamics and the way in which interest rate affects stock returns.

In this work we use the smooth transition regression (STR) approach to model the behaviour of the stock prices as the alternative to the linear strategy (Granger and Teräsvirta, 1993; Teräsvirta, 1998; Sarno, 1999; Franses and van Dijk, 1999; van Dijk, *et al*., 2000). A version of this type of models can be written as:

$$x_t = \beta' w_t + (\pi_1 + \pi_2 F(s_{i,t};\alpha))' z_t + u_t \qquad (4.1)$$

where x_t is the dependent variable, the return on the Bogotá stock market in this case; $w_t = (w_{1t},.....,w_{Kt})$ is a vector of *K* regressors which enter linearly with constant parameter vector β, $z_t = (z_{1t},.....,z_{Mt})$ is a vector of *M* regressors, $s_t = (s_{1t},.....,s_{Lt})$ is a $(L \times 1)$ vector of regressors whose elements may include those of w_t and z_t, and u_t is an *iid* error process, with $E(u_t) = 0$, and $Var(u_t) = \sigma^2$. *F* is a transition function bounded by 0 and 1 whose parameters are denoted by α. It is assumed that $E(w_t u_t) = 0$, $E(z_t u_t) = 0$, $E(s_t u_t) = 0$.

Some lagged elements of x_t may be included in w_t and z_t although weak exogeneity of the remaining elements of them with respect to the parameters of interest in (4.1) is required. Weak stationarity of x_t, w_t, and z_t processes is assumed.

Notice that when the transition function *F* = 0, the *STR* model (4.1) will be linear. However, the vector of regressor coefficients, $\pi_1 + \pi_2 F(s_{i,t};\alpha)$, will depend, in general, on the values of the transition variable s_t. The transition function can take the logistic form, in which case we have a logistic STR (LSTR) model:

$$F(s_t;\alpha) = (1 + \exp\{-\gamma(s_t - c)\})^{-1}, \;\; \gamma > 0 \qquad (4.2)$$

or as an exponential function, where we have an exponential STR (ESTR) model as:

$$F(s_t;\alpha) = 1 - \exp\{-\gamma(s_t - c)^2\}, \;\; \gamma > 0 \qquad (4.3)$$

where the slope parameter γ stands for the speed of the transition and c for the threshold, that is where the transition occurs.

The null hypothesis to check is that of linearity $(H_0 : \gamma = 0)$. However, (4.1) either with (4.2) or (4.3) is only identified under the alternative $(H_1 : \gamma > 0)$, a fact that invalidates the asymptotic distribution theory. This problem has been overcome through the auxiliary regression (Granger and Teräsvirta, 1993; Teräsvirta, 1994; Teräsvirta, 1998):

$$x_t = \beta' w_t + \lambda_0' z_t + \lambda_1' z_t s_t + \lambda_2' z_t s_t^2 + \lambda_3' z_t s_t^3 + v_t \quad (4.4)$$

where the null hypothesis of linearity becomes $(H_0 : \lambda_1 = \lambda_2 = \lambda_3 = 0)$, with power against both the LSTR and ESTR. The choice between LSTR and ESTR can also be done using equation (4.4), following the sequence proposed by Teräsvirta (1998).

The approach requires first the estimation of a linear model (Table 2). By looking at the results, it is remarkable the significance of the coefficient of ΔTIB as well as its sign. Thus, at this stage and given the way in which we have calculated R_t and ΔTIB_t we have evidence of a negative and nonlinear relationship between P_t and interest rate given by: $P_t = P_{t-1} - \phi P_{t-1}(TIB_{t-11} - TIB_{t-12})$.

Table 2. Benchmark linear model

Dependent variable: R_t				
Variable	Coefficient	Standard error	*t*- statistic	*p* – Value
Constant	0.01045	0.02680	0.38996	0.6966
R_{t-1}	0.43453	0.02381	18.2496	0.0000
R_{t-3}	0.05545	0.02627	2.11084	0.0350
R_{t-4}	-0.06395	0.02592	-2.46723	0.0137
ΔTIB_{t-11}	-0.01039	0.00568	-1.82971	0.0675
R^2	0.19823	Mean of dependent variable		0.01957
Standard deviation of dependent variable	1.14997	Standard error of regression		1.03110
		N		1482

The procedure continues with the selection of the transition variable for which the lags of both *R* and ΔTIB were checked. According to the results, we do not fail to reject the null hypothesis of linearity when the transition variable is R_{t-1} since the marginal level of significance is the lowest. Given this result, the selection process allows us to choose an exponential smooth transition regression model (ESTR).

As noted by Lundbergh (1999) and elsewhere, given that we are dealing with high-frequency financial series, the assumption that the error sequence generated by the STR model for the conditional mean has a constant conditional variance is not realistic and is something to be tested (see also Engle, 1982; Ballie and Bollerslev, 1989). We first test the null hypothesis of a constant conditional variance against the alternative that $\{u_t\}$ follows an ARCH(s) or a GARCH(p, q) process. For that we use the tests suggested by Teräsvirta (1998).

Since we do not fail to reject the null, we fit a ESTR-GARCH model to the data, following a two step estimation[8]. The first step consists of estimating the conditional mean, while the second step consists of the estimation of the conditional variance using the residuals of the ESTR model. On this basis, a GARCH(1,1) model for the innovations was adjusted in such a way that the gain of the model as a whole (ESTR-GARCH), measured in terms of the ratio of the variances, is 0.946 according to the estimates included in Table 3.

Table 3. Smooth transition regression (ESTR) – GARCH model

Dependent variable: R_t				
Variable	Coefficient	Standard error	*t*- statistic	*p*- value
Linear part				
Constant	-0.64709	0.20304	-3.18705	0.00073
R_{t-1}	0.11854	0.09842	1.20445	0.11431
ΔTIB_{t-11}	-0.01070	0.00553	-1.93409	0.02665
Non linear part				
Gamma	0.18724	0.08566	2.18595	0.01449
C	-1.72734	0.35903	-4.81112	0.1×10^{-5}
Constant	1.81977	0.61627	2.95286	0.00160
R_{t-4}	-0.12166	0.05020	-2.42369	0.00774
Dependent variable: (Orthogonalised) Residual				
Constant	0.106284	0.010771	9.867611	0.0×10^{-6}
ARCH (1)	0.228474	0.021687	10.53497	0.0×10^{-7}
GARCH (1)	0.684090	0.024109	28.37500	0.0×10^{-6}
Standard deviation of the regression	1.004793	Mean of dependent variable		-1.17×10^{-17}
Durbin-Watson	2.046558	Standard deviation of dependent variable		1.004114

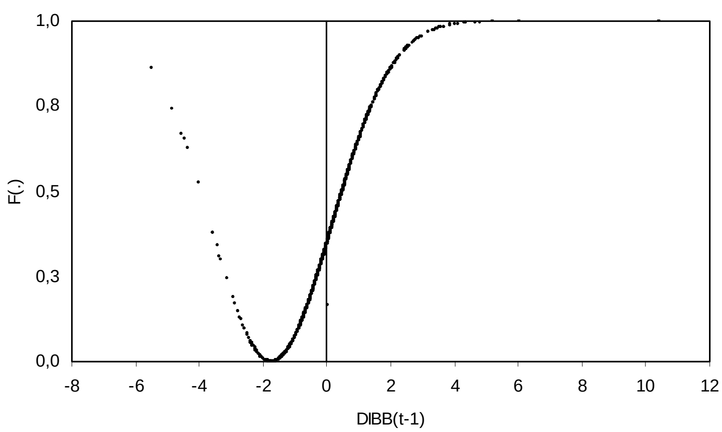
Some comments on these results are in order. First, it is remarkable the presence of ΔTIB_{t-11} in the linear part of the model with the right sign which evidences a nonlinear and negative relationship between IBB and TIB. Second, the model renders evidence of a time-variant equity premium, depending upon if the nonlinearity is active or not. When it is not active, and the model is in essence linear, the premium is negative. However, when by virtue of the value of the transition variable the nonlinearity is active the equity premium will be positive (or less negative). Third, the ESTR-GARCH model suggests that the adjustments of the returns towards the premium are symmetric, without concern on whether the changes are downwards or upwards. Fourth, the autoregressive part (linear and nonlinear) of the model ESTR-GARCH could be associated to the fads component which allows for share returns to deviate from the ones dictated by the fundamentals. Finally, based on the estimated value of the coefficient corresponding to R_{t-1} and its sign, the value of R_{t-1} helps to

[8] This procedure generates consistent estimates, although a tendency to yield over-parameterisation. This is because some effects of the non constant conditional variance may at first be captured by the estimated conditional mean (see Lundbergh, 1999). A joint or simultaneous estimation leads to more parsimonious models.

forecasting the contemporaneous return, which is in conformity with the facts of Table 1. Nonetheless, the value of the interest rate coefficient seems, numerically speaking, very low, regardless of being significant. In addition, the lag of ΔTIB, 11, seems too long for R_t to receive the impact of a ΔTIB, unless, assuming high transaction costs, the agents might wait a few days to make sure that the new level of the TIB is temporary or permanent before modifying their positions.

In Figure 5 we observe the transition function generated by the ESTR model. A negative variation of 1.73% in R_{t-1} (value of coefficient *c* in the regression of Table 3) at the end of the day before trading activates the nonlinearity of the process.

Figure 5. Transition function



Under conventional tests applied to these STR-type of models[9], we do not find evidence of remaining nonlinearities, autocorrelation, instability of parameters, and additional ARCH-GARH phenomena[10]. Under the BDS test (see Table 4), we do no reject the null hypothesis of having *iid* residuals (see the Appendix 2 for some detail on BDS test).

Table 4. Results of BDS test on the standardised and orthogonalised residuals of the *RTSE-GARCH* model

m-dimension	**2**	**3**	**4**	**5**	**6**	**7**
ε						
0.1	0.815	1.664	2.414	2.879	3.074	3.142
0.2	0.187	0.303	0.161	0.954	1.280	1.591
0.3	0.005	0.016	0.034	0.063	0.092	0.121
0.4	0.000	0.000	0.001	0.003	0.004	0.006
0.5	-0.000	-0.000	-0.000	0.000	-0.001	-0.001

[9] Not presented here but available from the authors on request.
[10] However, according to the Jarque-Bera statistic we do not fail to reject the null hypothesis of normality.

IV. Some conclusions

This work was aimed to describe the dynamics of the returns on the Bogotá stock market and to find any empirical link, if at all, between this variable and the short term interest rate, as measured by interbank loan interest rate.

The nonlinear econometric model for the returns is suggested by the stylised fact of relatively volatile periods, characterised by large returns, alternating with quiet periods of small returns on the Bogotá stock market.

Different from what the basic theoretical model predicts, the share prices index does not behave as a martingale. Rather, the model for returns seems to have a component (linear and nonlinear) of fads given its mean-reverting property. In other words, we find that, in the first days of increase (decrease) of the IBB, an agent can think that if today the index moved upwards (downwards), then it could be expected, with a high probability, that tomorrow it will also move upwards (downwards). This type of predictability of the movement of the index or trading rule, raised by the positive autocorrelation, would not support any argument in favour of the market efficiency.

The model gives some evidence about the existence of a non constant equity premium, depending upon if the nonlinearity is active or not. When it is not active, and the model is in essence linear, the prime is negative. However, when by virtue of the value of the transition variable the nonlinearity is active the premium will be positive.

At the same time, we find evidence of the nonlinear and negative effect that, according to the theory, the interest rate exerts on the stock prices. However, that impact is rather small and too lagged somehow. This lag length could be explained if the agents wait some days before acting in order to distinguish temporary movements of interest rate from the permanent ones, because of supposed high transaction costs.

Appendix 1: The interest rate, the stock prices, and the random walk view

Let us assume that the representative agent, of an economy populated by a continuum of them, maximises the lifetime utility function (see Campbell *et al*., 1997, chapter 8, and Sargent 1987, chapter 3, for further details):

$$E_t \sum_{j=t}^{\infty} \beta^{j-t} u(c_{j-t}) \qquad\qquad 0 < \beta < 1$$

where c_t is consumption at time *t*. The consumer is constrained by[11]:

[11] All variables are measured in units of consumption goods.

$$(p_{j+1}+d_{j+1})s_{j+1}=\frac{(p_{j+1}+d_{j+1})}{p_j}[(p_j+d_j)s_j+y_j-c_j]$$

where p_j is the price of a share of an enterprise at time *j*, d_j are nonnegative random dividends paid to the owner of the stock at the beginning of time *j*, s_j is the number of shares held by the agent at the beginning of *j*[12], and y_t is a random process associated with the labour income. E_t is the mathematical expectation conditional on the information available at the beginning of time *t*. Here, we consider $(p+d)_{j+1}/p_j$ as the gross yield on shares between dates *j* and *j* +1. The Euler equation that describes the optimal consumption and portfolio plan is given by:

$$1=\beta E_t\left(\frac{p_{j+1}+d_{j+1}}{p_j}\right)\frac{u'(c_{j+1})}{u'(c_j)}$$

If we have into account that the expected value of a product of stochastic variables is equal to the product of the expected value of each variable plus the covariance of the variables, we can rewrite the later expression as:

$$1=\beta\, E_t\left(\frac{p_{j+1}+d_{j+1}}{p_j}\right)E_t\,\frac{u'(c_{j+1})}{u'(c_j)}+\beta\,\mathrm{cov}_t\left(\frac{p_{j+1}+d_{j+1}}{p_j},\frac{u'(c_{j+1})}{u'(c_j)}\right)$$

assuming that $\mathrm{cov}_t\big((p_{j+1}+d_{j+1})/p_j\,,u'(c_{j+1})/u'(c_j)\big)$=0, which is the case when $u(c_t)$ is linear, and that $E_t u'(c_{j+1})/u'(c_j)$ is constant (equal to 1 for convenience), the random walk model of stock prices obtains, so that the above *Euler equation* reduces to:

$$\beta^{-1}p_j=E_j(p_{j+1}+d_{j+1})$$

which is a first order univariate process for the stock prices. The solution for this expression is given by:

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[12] In this case, $(p_0+d_0)s_0$ is given.

$$p_t = E_t \sum_{j=1}^{\infty} \beta^j d_{t-j} + \left(\frac{1}{\beta}\right)^t \omega_t \qquad \text{(A1.1)}$$

where ω_t is a martingale process.

A martingale is a variable that follows any random process such that: $E_t \omega_{t+1} = \omega_t / \omega_{t-1}, \omega_{t-2}, \ldots$, where ω_t could be defined to be uncorrelated but not necessarily stationary. The implication of this is that dependence between higher conditional moments (variances) could be present (Mills, 1993, pg. 91). An alternative for the martingale is, for example, the AR(1) process put forth by Shiller (1981) as a model of stock markets fads. In general, the fads term added to the random walk also allows for prices different from the those dictated by the fundamentals. The fads term is assumed to be mean-reverting, so that the price will have a tendency to return to fundamentals.

The present-value involved in (A1.1) renders a nonlinear relationship between p_t and the interest rate, r , even making the steady-state assumption that β=1/(1+*r*).

Appendix 2: The BDS test

Brock, Dechert and Scheinkman (1987) developed the BDS test as a set of tests based on the correlation integral. The BDS distinguishes between the null hypothesis of having random iid variables and the alternative hypothesis of deterministic chaos.

The method builds on the concept of correlation dimension of Grassberger and Procaccia (1983). The integral correlation calculated for a time series is defined as:

$$C(T,m,\varepsilon) = \lim_{T\to\infty} \frac{1}{T^2} \sum_i \sum_{j>i} H\left(\varepsilon - \left\| x_i^m - x_j^m \right\|\right) \qquad \text{(A2.1)}$$

where *T=N-m+1*, *N* is the length of the series, the vectors of sequences $\{x^m\}$ are the *m*-histories build from the time series itself, *H* is the Heaviside function which is zero when the equality takes place and one otherwise, $\|.\|$ is the supreme norm and ε is a small but positive number. The two points x_i^m and x_j^m will be spatially correlated if the Euclidian distance is less than a give radius ε of an *m*-dimensional ball centred at one of the two points, i.e., $\left\| x_i^m - x_j^m \right\| < \varepsilon$. The spatial correlation between all points on the attractor for a given ε is determined by counting the number of these pairs located in a ball around every point.

The correlation dimension is defined as:

$$D^{C}(m) = \lim_{\varepsilon \to 0} \frac{\ln C(T, \varepsilon, m)}{\ln \varepsilon} \qquad \text{(A2.2)}$$

The BDS test transforms the correlation integral into one that, asymptotically, is distributed as a normal variable under the null hypothesis of *iid* against a non specified alternative hypothesis. The BDS test observes the dispersion of the points in a number of spaces which dimension goes from 2 up to *m*. The correlation integral of a white noise process with a embedding dimension *m* can be written as the *m*-power of the correlation dimension with embedding dimension equal to 1: $C(T,1,\varepsilon)^{m}$. The BDS test can then be written as:

$$BDS = \frac{C(T, m, \varepsilon) - C(T,1,\varepsilon)^{m}}{\sigma^{2}\left[C(T, m, \varepsilon) - C(T,1,\varepsilon)^{m}\right]} \to N(0,1) \qquad \text{(A2.3)}$$

The null hypothesis of white noise (*iid*) is rejected when BDS excess in absolute value the selected critical values. This will happen if the dispersion of the points in the consecutive spaces is not in line with the expected dispersion for the case of white noise.

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