How Uncertain are NAIRU Estimates in Colombia?

by

Juan Manuel Julio*

Banco de la República

*The author is a member of the econometrics unit at the Gerencia Técnica in the Colombian Central Bank. The author indebts very much the useful comments made by the attendants of the CEDE seminar at Universidad de los Andes. However, the views and conclussions contained in this paper are the sole responsability of the author and do not compromise the Colombian central bank or its Board of Governors. jjulioro@banrep.gov.co

Abstract

Most of the proposed macro models and Phillips curves for policy design and analysis in Colombia depend on estimates of the potential output. However, it is widely known that this estimates are highly unreliable because of its level of estimation uncertainty. Following Staiger et al.(1996), we explore some common and not very common "fully structural" estimates of the NAIRU, the Non Accelerating Rate of Unemployment, provide confidence bands, and formally test the constancy hypothesis on the NAIRU. We also study the robustness of this results to the specification of the Phillips Curve. We find more reliable estimates of the NAIRU than previous estimates of the output gap, and find evidence in favor of a non constant NAIRU. Our results indicate that it has increased about 4 percentage points along the sample span. However uncertainty results are not robust to specification. There a single policy implication: An increasing NAIRU along with a policy of reduction and stabilization of inflation may imply increasing unemployment levels. Since the NAIRU is the component of unemployment that do not respond to monetary policy, it is up to the goverment to design policies for its reduction.

Key Words and Phrases: NAIRU, Phillips Curve, Estimation Uncertainty, Fieller's Method

1 Introduction

Most of the proposed macro models and Phillips curves for policy design and analysis in Colombia depend on estimates of the unobserved potential output or equivalently the output gap. These estimates are derived from a great variety of methods ranging from the purely automatic filters (that depend on the information available in the time series only) to the fully structural ones (consistent with stable structural relationships). However, recent results have shown that for the case of Colombia these estimates are highly unreliable because of their level of estimation uncertainty. In fact, we can not be sure about the sign or value of the true unobserved output gap except when the economy faces a very deep recession or an unusual recovery. See Julio & Gomez(1998).

On the other hand, after three years of historical record unemployment levels, policy discussants refer to the unemployment as the worst problem facing the Colombian economy at present. This fact have pushed this variable to the center stage of policy analysis and is certainly influencing monetary policy.

These two facts, the lack of reliability on current estimates of the potential output and the increasing importance of the unemployment rate, calls for the study of alternative measures of economic activity to the potential output, and more precisely those based on the unemployment rate.

There are already several estimates of the NAIRU for Colombia. Earlier estimates assumed that the NAIRU was constant and found implausibly low estimations, which are consistent with permanent deflationary pressures never observed in the data. Newer results obtained under the assumption of a constant and time varying NAIRUs, have found more plausible results. However, the issues of estimation uncertainty and formal tests of constancy were not addressed in those papers except by one which draws conclusions only for the nineties (See Gomez and Julio(2000)). Moreover, none of the previous works addressed the issue of robustness to specification for either the NAIRU or the potential output. See Farné et al(1995), Clavijo (1994), Cárdenas and Gutierrez (1997), Henao and Rojas(1998), and Nuñez and Bernal (1997).

In this paper we study the behavior of some estimates of the NAIRU, the Non Accelerating Inflation Rate of Unemployment, following the methodology proposed by Staiger et al(1996). Our estimates fall into the class of "fully structural", that is, those consistent with the existence of a well formulated and stable Phillips curve. We assume that the NAIRU behaves according to alternative deterministic functions of time. In addition, given the absence of enough sample information on inflation expectations, we consider model consistent adaptive expectations according to alternative specifications. By comparing the results under these specifications we asses the robustness of our results, a matter not yet studied either for the NAIRU or the potential output in Colombia.

The estimated NAIRU is found to be the ratio of two correlated gaussian non centered random variables whose distribution is known to have high tails when the variable at the denominator is located close to zero. As a consequence the confidence intervals for the NAIRU may take unexpected shapes like the whole real line or an open interval towards plus or minus infinity. Moreover, the fact that the NAIRU is estimated as a non linear function introduces some complication in computing confidence intervals. We obtain confidence intervals by inverting the non rejection region of a suitable statistical test of hypothesis.

Our estimated NAIRUs are more reliable than previous estimated potential outputs for some particular specifications. These finding suggest that further research on unemployment based Phillips curves may provide better tools for macro modelling and policy analysis in Colombia. Although our uncertainty results depend on the specification of the Phillips curve, we found robust evidence in favor of an increasing NAIRU which is estimated to have risen 4 percentage points during the sample span.

There is a single policy implication of this results: Since the goal of monetary policy during this decade is to reduce and stabilize inflation, an increasing NAIRU may imply increasing levels of unemployment. Moreover, since the NAIRU is the component of unemployment that do not respond to monetary policy, it is up to the government to pull up policies to shift its actual trend.

The paper is divided into five sections including this introduction. The second describes the data and model used, the third summarizes some important statistical issues, the fourth contains the results, and the last presents some concussions and discussion.

2 Model and Data

Our workhorse is a structural equation that represents a dynamically homogeneous expectations augmented Phillips curve of the form

$$\pi_t - \pi_t^e = \alpha_p(B)\left(\pi_{t-1} - \pi_{t-1}^e\right) + \delta_q(B)\,\pi_t^F + \eta_r(B)\,\pi_t^M + \beta_s(B)\,(u_t - \overline{u}_t) + \varepsilon_t \quad (1)$$

where α, δ, η, and β are polynomials in the lag operator and its subscript is the polynomial order, π_t^e is the expected inflation with information up to time $t-1$, π_t^F is the relative price of food inflation as a proxy for supply shocks, and π_t^M is the relative price of imports inflation as a proxy for exchange rate shocks, u_t and $\overline{u}_t$ are the observed rate of unemployment and the unobserved NAIRU respectively, and ε_t is a sequence of (normal) uncorrelated zero mean homoskedastic random variables.

Evidence on the existence of a Phillips curve for Colombia may be found in figure 1. For this figure we have assumed $\pi_t^e = \pi_{t-1}$, and no effect of supply or exchange rate shocks, and omitted the lagged effects of all variables[2].

[2]Our database consists of quarterly measures and the sample used in the estimation process runs from 1978:2 to 2000:4. Price and relative indexes are computed from the quarterly geometric mean of price levels. Before 1983:4 the unemployment rate corresponds to that of the four biggest cities, and after corresponds to the seven biggest cities in Colombia.

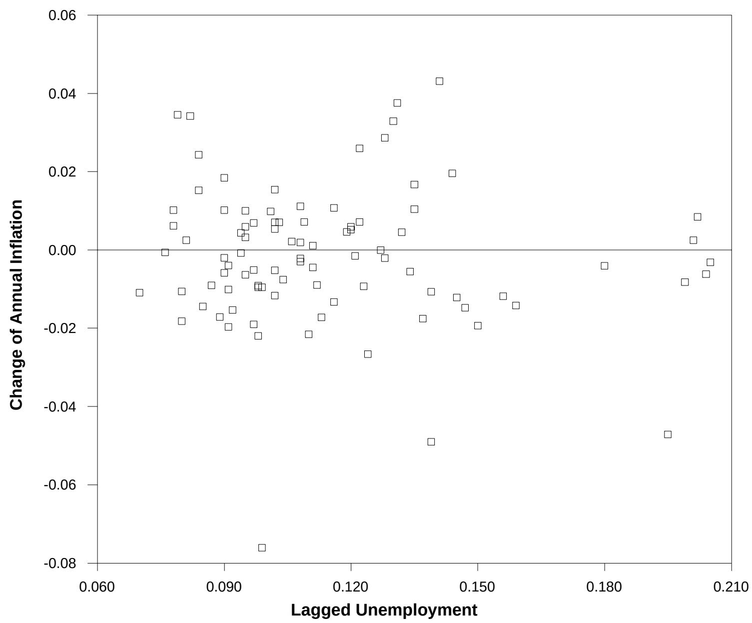

Figure 1: Lagged Unmeployment Rate and Change in Annual Inflation

The data seems to suggest a not very clear negative relationship between lagged unemployment and inflation, but its nature is hardly clear. [3]

In this figure the NAIRU corresponds to the point at which the Phillips curve crosses the x axis, that is, anything between 8 and 15 percent. However unclear the figure may be, it certainly suggests that inflation expectations, supply and exchange rate shocks play an important role in the econometric formulation of the Colombian Phillips curve, and then, depending on the strength of their effect, the Colombian Phillips curve as well as the NAIRU may be difficult to estimate.

[3] For a shorter sample Gomez and Julio(2000) found statistical evidence of non linearity.

Figure 2: Infation Rate and Relative Price of Food Inflation

2.1 Supply Shocks

As argued by Gordon(1988),(1990) there is a set of variables that may shift the intercept of the Phillips curve. One of them is the so called supply shocks series that is assumed to shift inflation but not to affect unemployment. Our measure of supply shocks is the centered relative price of food inflation, see King & Watson(1994, footnote 18). Since almost 30% of the Colombian CPI inflation corresponds to food, this measure of supply shocks clearly has some power in explaining the total variation of inflation.

Figure 2 presents the annual inflation rate and the relative price of food inflation for the active sample used in estimation. Besides the expected and coinciding peaks in the two series, it is interesting to notice that since 1993 the relative price of food inflation seems to have a lower level than ever before, when it was centered around zero. According to our model, this permanent reduction in the level of the relative price of food inflation may partially explain the declining path of total inflation during the nineties.

2.2 Exchange Rate Shocks

Figure 3 depicts the relationship between total annual inflation and the relative price of imports inflation as a measure of exchange rate shocks. It is remarkable that all along the declining path of inflation in the nineties, the relative price of imports inflation remains at a level below zero, and when it comes back to positive the inflation rate decline slows down. The relative price of imports inflation seems to have a very important role in explaining the declining path of inflation during the nineties. This coincides with lower levels of devaluation of the peso during this period as can be seen in figure 4.

2.3 Expectations

In the absence of enough sample information on inflation expectations, we consider model consistent adaptive expectations. We study three alternative specifications

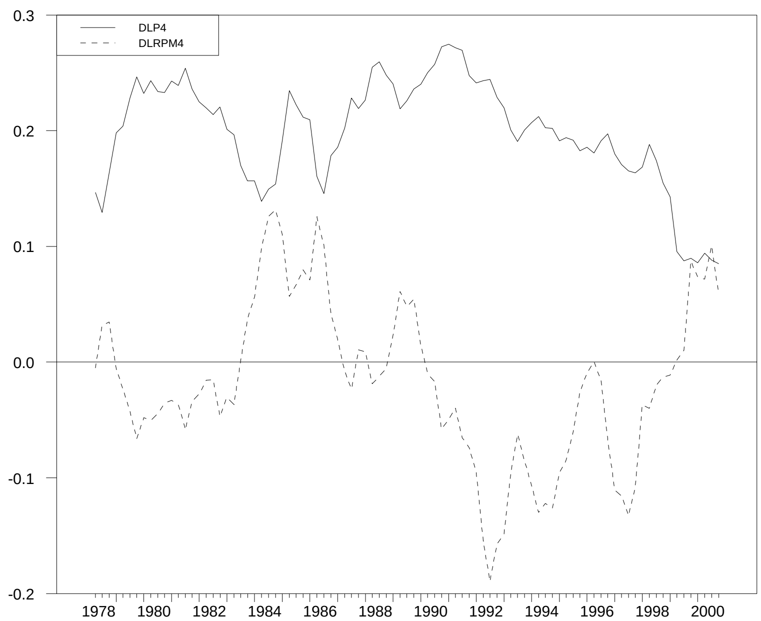

Figure 3: Annual Inflation and Relative Price of Imports Inflation

for expectations; the last observed inflation, the forecast of an AR(p) model with constant parameters, and the forecast of the same AR(p) model with parameters estimated sequentially to simulate real time expectations formation.

$$\pi_t^e = \begin{cases} \pi_{t-1} & \text{Random Walk} \\ \phi_0 + \sum_{i=1}^{p} \phi_i \pi_{t-i} & \text{Full Sample } AR(P) \\ \phi_0 + \sum_{i=1}^{p} \phi_i \pi_{t-i} & \text{Sequential } AR(P) \end{cases} \tag{2}$$

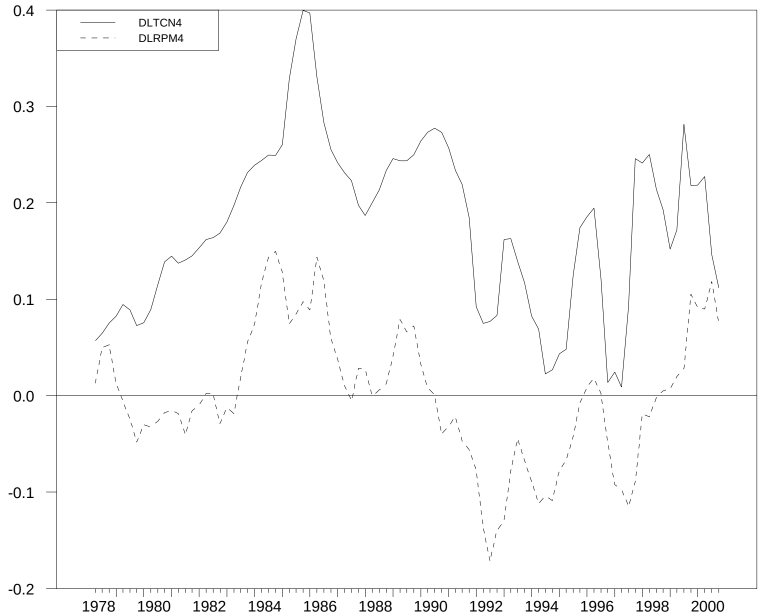

Figure 4: Annual Rate of Nominal Devaluation and Relative Price of Imports Inflation

2.4 Dynamic Homogeneity

Dynamic homogeneity has to do with two properties of the Phillips curve. First, it relates to the possibility of obtaining different steady states of inflation, a necessary restriction when modelling a series with changing mean levels as the Colombian inflation during the nineties. And second, with the long run neutrality from nominal to real variables.

Since equation 1 is equivalent to a specification in levels of inflation under the restriction that the sum of the lag parameters add to one, and since there is not an autonomous deterministic trend in the model, equation 1 represents a dynamically homogeneous Phillips curve.

2.5 Nairu Specification

We will approximate the unobserved NAIRU by using four deterministic specifications for it. The first is a constant, the second is constant with jumps at equispaced points of time, the third jumps at **estimated** points of time, and the fourth is a cubic spline[4] with equidistant knots

$$\overline{u}_t = \begin{cases} \overline{u} & \text{Constant NAIRU} \\ \theta_1 I_t\left[t_0, t_1\right) + .. + \theta_k I_t\left[t_{k-1}, t_k\right) & \text{Equidistant Breaking NAIRU} \\ \theta_1 I_t\left[t_0, \widehat{t}_1\right) + .. + \theta_k I_t\left[\widehat{t}_{k-1}, t_k\right) & \text{Estimated Breaking NAIRU} \\ \mathbf{\Phi}_t^T \boldsymbol{\theta} & \text{Spline NAIRU} \end{cases} \tag{3}$$

The first assumption represents the common belief that the NAIRU is very smooth and does not change very often. Since our sample is small to moderate, this assumption seems to have some ground. However, in a lapse of 30 years it is also likely that the determinants of the NAIRU, hence the NAIRU itself, has registered some slight changes. The second and third assumptions approximate these smooth changes by means of discrete jumps. Particularly useful is the third assumption since it provides estimated jumping times for the NAIRU. The fourth assumption comes from the fact that the last two specifications produce discrete rather than smooth changes, a desirable property of an estimated NAIRU.

[4] A cubic spline is the continuous joining of cubic polynomials at dates known as knots.

3 Statistical Issues

In this section we will present some not very known statistical aspects in estimating and computing confidence intervals for the alternative specifications of the NAIRU. We will also present some results for the assumption of a constant NAIRU with unit root expectations.

3.1 Constant Nairu

3.1.1 Estimation

Under the assumption of a constant NAIRU and random walk expectations, model 1 becomes

$$\Delta\pi_t = \mu + \alpha_p(B)\Delta\pi_{t-1} + \delta_q(B)\,\pi_t^F + \eta_r(B)\,\pi_t^M + \beta_s(B)u_t + \varepsilon_t \tag{4}$$

where $\mu = -\beta_s(1)\overline{u}$. Once the unrestricted model 4 is estimated, by the invariance property of the maximum likelihood estimators, the estimated constant NAIRU is given by

$$\widehat{\overline{u}} = -\frac{\widehat{\mu}}{\widehat{\beta}_s(1)} \tag{5}$$

where $\widehat{\beta}_s(1) = \widehat{\beta}_0 + \widehat{\beta}_1 + ... + \widehat{\beta}_s$. Under the assumptions of the General Linear Model, this estimator is distributed as the ratio of two non-centered and correlated normal variables, that is a doubly non central Cauchy distribution. The significance of the term in the denominator, $\widehat{\beta}_s(1)$, is critical to study the properties of this estimator and to evaluate confidence intervals.

Parameter		Estimate	Std. Error	T Stat	P-Value
Constant		0,013	0,004	2,908	0,005
DDLP4{1}		0,270	0,089	3,042	0,003
DDLP4{4}		-0,207	0,057	-3,615	0,001
DLRPF4		0,551	0,061	8,968	0,000
DLRPF4{1}		-0,710	0,104	-6,799	0,000
DLRPF4{2}		0,206	0,083	2,472	0,015
DLRPM4{1}		0,062	0,018	3,471	0,001
U{0}		-0,121	0,037	-3,258	0,002
Durbin-Watson Statistic				1.94	
Q(22-0)				20.30	0.56
$\overline{R}^2$				0.70	

Table 1: Estimated Unrestricted Model Unit Root Expectations

Table 1 contains the estimation results for equation 4. In this case the polynomial $\beta_s(B)$ has just the term corresponding to the present unemployment, so that $\widehat{\beta}_s(1) = -0.121$, which is highly significant. Our estimated NAIRU is in this case is $\widehat{\overline{u}} = -\frac{\widehat{\mu}}{\widehat{\beta}_s(1)} = -\frac{0.013}{-0.121} = 0.107$, that is 10.7 percent.

3.1.2 Constructing Confidence Sets by Inverting Test Statistics

The fact that the estimated NAIRU is a nonlinear function of the estimated regression coefficients introduces some complication into the computation of confidence intervals for the unobserved NAIRU. However, we can consider the related problem of testing the hypothesis that the NAIRU takes a particular value $\overline{u} = \overline{u}_0$.

In general let us assume that we want a confidence interval for a parameter λ, and $\Psi(\lambda_0)$ is a test statistic for the null $H_0 : \lambda = \lambda_0$ at the significant level α. Let $\Psi(\lambda_0) \leq C_{1-\alpha}$ be the rejection region for this test. Then its complement, the non rejection region, is given by $\Psi(\lambda_0) \geqq C_{1-\alpha}$, and the $1-\alpha$ confidence interval for λ is given by inverting this non rejection region

$$C_\Psi(\lambda, 1-\alpha) = \{\lambda_0 : \Psi(\lambda_0) \geqq C_{1-\alpha}\} \tag{6}$$

that is the set of values λ_0 that are not rejected by the test of hypothesis.

Since under the null $H_0 : \overline{u} = \overline{u}_0$, the restricted model is

$$\Delta\pi_t = \alpha_p(B)\Delta\pi_{t-1} + \delta_q(B)\,\pi_t^F + \eta_r(B)\,\pi_t^M + \beta_s(B)(u_t - \overline{u}_0) + \varepsilon_t \tag{7}$$

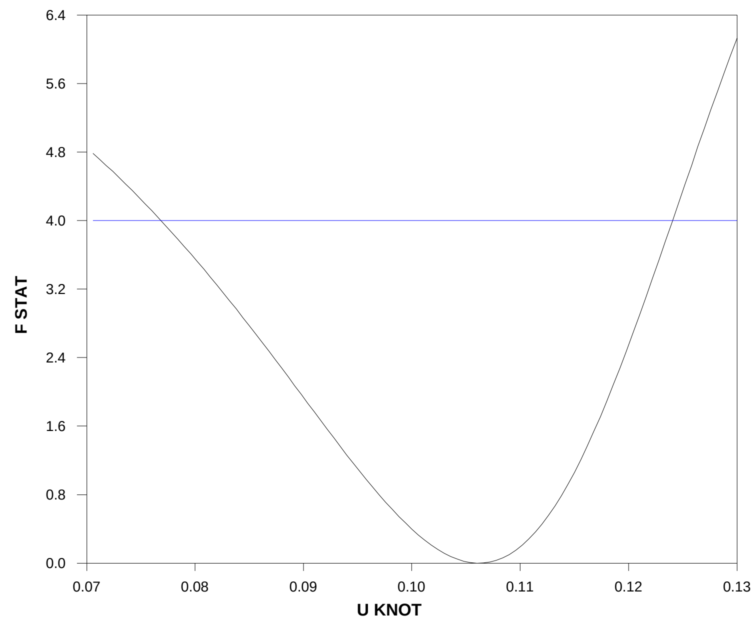

Figure 5: F Statistics and Critical Value for the Null $\overline{u} = \overline{u}_0$

an exact test for the null against the two sided alternative can be obtained by comparing the Sum of Squared Residuals, SSR($\overline{u}_0$), computed from 7 to the unrestricted SSR from 4, SSR($\widehat{\overline{u}}$), using the F statistics

$$F_{\overline{u}_0} = \left[SSR\left(\overline{u}_0\right) - SSR\left(\widehat{\overline{u}}\right)\right] / \left[SSR\left(\widehat{\overline{u}}\right)/d.f.\right] \tag{8}$$

where $d.f.$ are the unrestricted regression degrees of freedom. Under the usual assumptions in gaussian linear regression, this statistics has an exact $F_{1,d.f}$ distribution.

Figure 5 depicts the F statistics 8 for different values $\overline{u}_0$. The critical value for

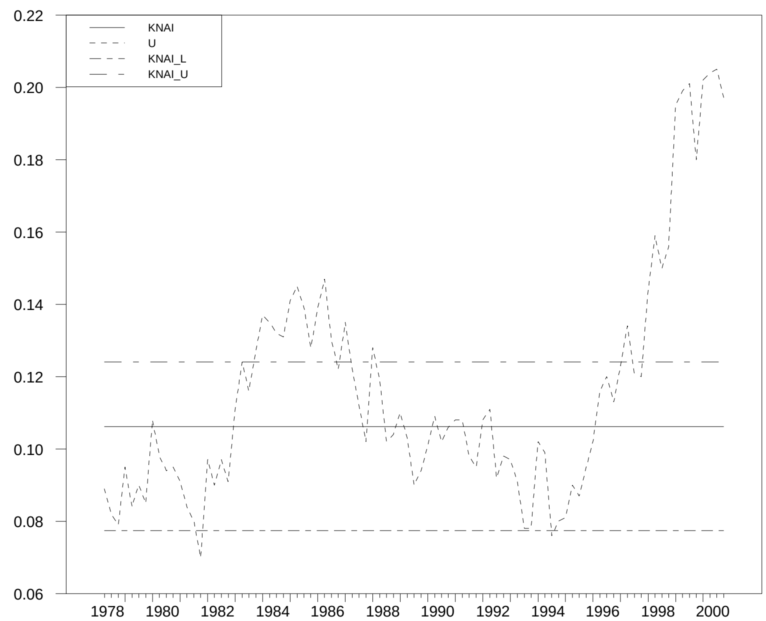

Figure 6: Observed Unemployment, Estimated NAIRU and Confidence Interval

this test is 4.0 so that, for example, the null of $\overline{u}_0 = 0.125$ is rejected, but the null of 0.11 is not rejected.

By drawing a horizontal line at height 4.0, we can find the non rejection region, that is all the points $\overline{u}_0$ (U knot) with F statistics below 4.0 . This yields a 90% confidence interval for the unobserved NAIRU between 7.3% and 12.4%, thinner than originally estimated by just graphical analysis, and showing that in expectations, supply and exchange rate shocks, and lagged effects of them should be considered when estimating the Phillips curve.

Figure 6 shows the estimated NAIRU, its confidence interval and the observed unemployment rate. Even though the uncertainty in estimating the NAIRU is quite high, the unemployment gap is statistically different from zero for several periods of time. This result contrasts with previous results on the output gap which showed that the gap is statistically different from zero in just four periods of time for a similar sample. Moreover, it is important to notice that due to the lack of symmetry, recessions are more easily identified than recoveries.

3.2 Breaking Nairu

In this subsection we will study the case for a NAIRU with a few discrete jumps, at most four. That is, we will approximate the behavior of the NAIRU by means of a step function. The timing of the jumps will be either fixed beforehand at equispaced periods of time or estimated from the sample itself.

3.2.1 Estimation Regardless of how we obtain the jump times, the specification of the NAIRU with $1 \leq j \leq 4$ jumps at times $t_1^*, t_2^*, ..., t_j^*$ can be written as

$$\overline{u}_t = \sum_{i=1}^{j+1} \gamma_i D_t \left[t_{i-1}^*, t_i^*\right)$$

with $t_0^* = 1$, $t_{j+1}^* = T + 1$, and

$$D_t \left[t_{i-1}^*, t_i^*\right) = \begin{cases} 1 & \text{for } t_{i-1}^* \leq t < t_i^* \\ 0 & \text{At any other time} \end{cases}$$

hence model 1 can be written as

$$= -\beta_s(1)\sum_{i=1}^{j+1}\gamma_i D_t\left[t_{i-1}^*, t_i^*\right) + \alpha_p(B)\Delta\pi_{t-1} + \delta_q(B)\pi_t^F + \tag{9}$$
$$\eta_r(B)\pi_t^M + \beta_s(B)u_t + \widetilde{\beta}_s(B)\sum_{i=1}^{j+1}\gamma_i\Delta D_t\left[t_{i-1}^*, t_i^*\right) + \varepsilon_t$$

where $\widetilde{\beta}_s(B)\sum_{i=1}^{j+1}\gamma_i\Delta D_t\left[t_{i-1}^*, t_i^*\right)$ has non null values just at the times of breaks. If we assume that this value is small, we can approximate the model as

$$\Delta\pi_t = \sum_{i=1}^{j+1}\gamma_i^* D_t\left[t_{i-1}^*, t_i^*\right) + \alpha_p(B)\Delta\pi_{t-1} + \delta_q(B)\pi_t^F + \tag{10}$$
$$\eta_r(B)\pi_t^M + \beta_s(B)u_t + \varepsilon_t$$

where $\gamma_i^* = -\beta_s(1)\gamma_i$. Once 10 has been estimated, we can readily estimate the unobserved NAIRU as

$$\widehat{\overline{u}}_t = -\frac{1}{\widehat{\beta}_s(1)}\sum_{i=1}^{j+1}\widehat{\gamma}_i^* D_t\left[t_{i-1}^*, t_i^*\right) \tag{11}$$

which is again distributed as a doubly non central Cauchy variable.

3.2.2 Confidence Intervals

In this case we must consider the related problem of testing the null hypothesis for each value $\overline{u}_0$, for each of the steps of the function. For instance, if we want the confidence interval for the NAIRU in the first step, $t_0^* = 1 \leq t < t_1^*$, we must test the null $H_0 : \overline{u}_t = \overline{u}_0$; $1 \leq t < t_1^*$.

Under the null the restricted regression becomes

$$\Delta\pi_t = \sum_{i=2}^{j+1}\gamma_i^* D_t\left[t_{i-1}^*, t_i^*\right) + \alpha_p(B)\Delta\pi_{t-1} + \delta_q(B)\pi_t^F \tag{12}$$
$$+\eta_r(B)\pi_t^M + \beta_s(B)\left(u_t - \overline{u}_0 D_t\left[t_0^*, t_1^*\right)\right) + \varepsilon_t$$

and the test statistic is given again by 8. By letting $\overline{u}_0$ vary over a suitable interval, we can find in the same way the non rejection region, which gives us the required interval for the NAIRU in the first interval.

3.2.3 Break Times

For the present paper we will consider from one to four breaks. First, we will consider the breaks to occur at equidistant points of time in the sample, but later we will consider the time breaks as endogenous. The breaking times are estimated sequentially according to Bai(1995), that is, finding the time of break that minimizes the residual sum of squares in the unrestricted regression 10 conditional on the existence of previous breaks. This procedure according to Bai(1995) gives consistent estimates of the breaking times.

3.3 Spline Nairu

3.3.1 Estimation

Under this specification we will consider that the NAIRU changes smoothly according to the path of a cubic spline, that is, the continuous joining of cubic polynomials at several breaks or knots known beforehand

$$\overline{u}_t = \mathbf{S}_t^T \overline{\mathbf{\Phi}}$$

where $\mathbf{S}_t^T$ are the values of the spline variables at time t and $\overline{\mathbf{\Phi}}$ is a set of unknown parameters. Again model 1 becomes in this case

$$\Delta\pi_t \quad = \quad -\beta_s(1)\,\mathbf{S}_t^T\overline{\mathbf{\Phi}} + \alpha_p(B)\Delta\pi_{t-1} + \delta_q(B)\,\pi_t^F + \eta_r(B)\,\pi_t^M \tag{13}$$

$$+\beta_s(B)u_t + \widetilde{\beta}_s\,(B)\,\Delta\mathbf{S}_t^T\overline{\mathbf{\Phi}} + \varepsilon_t$$

where $\widetilde{\beta}_s\,(B) = \sum_{i=1}^{p} \widetilde{\beta}_i B^i$ and $\widetilde{\beta}_i = -\sum_{j=i+1}^{p} \beta_j$. If the NAIRU changes slowly then $\Delta\mathbf{S}_t^T$ will be small, and its overall effect on $\Delta\pi_t$ will be negligible. If we drop this term we get the equation

$$\begin{aligned}\Delta\pi_t &= -\beta_s\,(1)\,\mathbf{S}_t^T\overline{\mathbf{\Phi}} + \alpha_p(B)\Delta\pi_{t-1} + \delta_q\,(B)\,\pi_t^F + \\ &\quad \eta_r\,(B)\,\pi_t^M + \beta_s(B)u_t + \varepsilon_t \\ &= \mathbf{S}_t^T\mathbf{\Phi} + \alpha_p(B)\Delta\pi_{t-1} + \delta_q\,(B)\,\pi_t^F + \\ &\quad \eta_r\,(B)\,\pi_t^M + \beta_s(B)u_t + \varepsilon_t\end{aligned} \tag{14}$$

where $\mathbf{\Phi} = -\beta_s\,(1)\,\overline{\mathbf{\Phi}}$. This equation may be estimated by least squares and the estimated NAIRU becomes

$$\overline{u}_t = -\frac{\mathbf{S}_t^T\widehat{\mathbf{\Phi}}}{\widehat{\beta}_s\,(1)}$$

again, the distribution of this estimator corresponds to that of the ratio of two non centered correlated random variables, and the significance of $\widehat{\beta}_s\,(1)$ is of great importance to study the statistical properties of the estimated NAIRU and to compute confidence intervals.

3.3.2 Confidence Intervals

In order to construct confidence interval we can invert the non rejection region for the null $H_0 : \overline{u}_\tau = \overline{u}_{\tau 0}$ for suitable values of $\overline{u}_{\tau 0}$, and for every $1 \leq t \leq T$. In this case the spline regressors have to be changed for each of the assumed values $\overline{u}_{t0}$ and for each period of time. To see that, let us write model 13 as

$$\begin{aligned}\Delta\pi_t &= \beta_s(1)\left(u_t - \mathbf{S}_t^T\overline{\mathbf{\Phi}}\right) + \alpha_p(B)\Delta\pi_{t-1} + \delta_q(B)\pi_t^F + \eta_r(B)\pi_t^M \\ &\quad + \beta_s(B)u_t + \widetilde{\beta}_s(B)\Delta u_t + \varepsilon_t\end{aligned} \tag{15}$$

and assume without loss of generality that the first regressor in $\overline{\mathbf{\Phi}}$ is one, so that $\mathbf{S}_t^T = \left[1, \mathbf{S}_{2,t}^T\right]$. Then, since the space generated by $\mathbf{S}_t^T$, that is $\left\langle \mathbf{S}_t^T \right\rangle = \left\langle \widetilde{\mathbf{S}}_t^T \right\rangle = \left\langle 1, \mathbf{S}_{2,t}^T - \mathbf{S}_{2,\tau}^T \right\rangle$, there is a unique $\widetilde{\mathbf{\Phi}}$ such that $\mathbf{S}_t^T\overline{\mathbf{\Phi}} = \widetilde{\mathbf{S}}_t^T\widetilde{\mathbf{\Phi}}$. Now, if we partition $\widetilde{\mathbf{\Phi}} = \begin{bmatrix} \widetilde{\phi}_1 \\ \widetilde{\mathbf{\Phi}}_2 \end{bmatrix}$ according to the partition in $\mathbf{S}_t^T$ and notice that $\widetilde{\mathbf{S}}_\tau^T = [1, \mathbf{0}]$, then we find that $\overline{u}_\tau = \widetilde{\phi}_1$. Then we can write 15 as

$$\begin{aligned}\Delta\pi_t &= \beta_s(1)(u_t - \overline{u}_t) + \mathbf{S}_{2,t}^T\overline{\mathbf{\Phi}}_2 + \alpha_p(B)\Delta\pi_{t-1} + \delta_q(B)\pi_t^F + \eta_r(B)\pi_t^M + \\ &\quad \beta_s(B)u_t + \widetilde{\beta}_s(B)\Delta u_t + \varepsilon_t\end{aligned} \tag{16}$$

where $\mathbf{\Phi}_2 = -\beta_s(1)\widetilde{\mathbf{\Phi}}_2$.

Since the null $H_0 : \overline{u}_\tau = \overline{u}_{\tau 0}$ does not affect $\widetilde{\mathbf{\Phi}}_2$,$\beta_s(1)$ or the other coefficients, we can use 16 to test the null by comparing the restricted sum of squared residuals from 16 to the unrestricted obtained estimating 16 including an intercept. The confidence interval is obtained by inverting the non rejection region at each point of time.

3.4 Further Econometric Considerations

Under the assumption of fixed regressors and normally distributed residuals, it is clear from statistical theory that the tests statistics we derived our confidence intervals from are Uniformly More Powerful Invariant, and hence our confidence intervals are optimal. However, given that the regressors include lagged endogenous variables and the residuals may not be normally distributed, there is some concern about the robustness of our results to the violation of those assumptions.

In order to asses this issues Staiger et al. (1996) performed a simulation study from a simplified model. The residuals were either sampled from the estimated ones from the model or generated as gaussian random variables. Their results show that average coverage rates closely matched the nominal rates regardless of how the residuals were sampled and the parameter values. This result suggests that this procedure is robust to the violation of these assumptions.

4 Results

In this section we present the most relevant results found for this paper. There is a large appendix containing the remaining results that may be requested from the author.

Tables 2 and 3 presents the specifications used on each of the different assumptions on inflation expectations. These specifications were maintained for all assumptions on the NAIRU behavior. From this tables as well as from table 1 we can observe highly significant effects of the present unemployment rate on the deviation of inflation with respect to its expectation, $\pi_t - \pi_t^e$. We also find a highly significant and strong effect of supply and exchange rate shocks.

A first view of the figures presented in the last section and the appendix reveals that our uncertainty measures depend very much on the specification of the Phillips curve, the assumptions on expectations and the treatment of breaks (estimated versus equidistant). However, with this information we can still draw some basic conclusions about the behavior of the NAIRU.

4.1 Is the NAIRU a Constant?

A question one would like to answer is that of the constancy of the NAIRU. Table 4 contains the estimation results under the null hypothesis of constant NAIRU. Levels of the estimated constant NAIRU do not differ statistically across alternative assumptions on inflation expectations. However, estimated uncertainty does vary as shown in table 4.

The alternatives of breaking, spline and time varying NAIRU give information on the validity of the constancy assumption. In this case we will not look at the breaking NAIRU with equispaced breaks since these arbitrary jumping times might not provide an appropriate approximation to the unknown NAIRU. We will first examine models

Parameter		Estimate	Std. Error	T Stat	P-Value
Constant		0,009	0,004	2,005	0,048
DDLP4E1{4}		0,205	0,083	2,481	0,015
DLRPF4		0,434	0,066	6,582	0,000
DLRPF4{1}		-0,524	0,096	-5,488	0,000
DLRPF4{2}		0,176	0,063	2,804	0,006
DLRPM4		-0,170	0,043	-3,956	0,000
DLRPFM{1}		0,340	0,065	5,201	0,000
DLRPM4{2}		-0,148	0,043	-3,429	0,001
U{0}		-0,079	0,037	-2,146	0,035
Durbin-Watson Statistic				1.80	
Q(22-0)				23.68	0.308
$\overline{R}^2$				0.538	

Table 2: Estimated Unrestricted Model Full Sample AR(p) Expectations

Parameter		Estimate	Std. Error	T Stat	P-Value
Constant		0,015	0,006	2,386	0,020
DDLP4E1{4}		0,221	0,091	2,428	0,018
DLRPF4		0,431	0,083	5,222	0,000
DLRPF4{1}		-0,496	0,119	-4,165	0,000
DLRPF4{2}		0,162	0,080	2,020	0,047
DLRPM4		-0,190	0,055	-3,478	0,001
DLRPFM{1}		0,343	0,083	4,129	0,000
DLRPM4{2}		-0,127	0,055	-2,292	0,025
U{0}		-0,131	0,051	-2,580	0,012
Durbin-Watson Statistic				1,340	
Q(22-0)				28,000	0,140
$\overline{R}^2$				0,467	

Table 3: Estimated Unrestricted Model Sequential AR(p) Expectations

Expectations		Unit Root	AR(p)	Seq. AR(p)
Estimated		10.6	11.29	11.27
L90		7.7	0.7	7.0
U90		12.4	15.8	13.7
Width		4.7	15.1	6.7

Table 4: Estimated Constant NAIRUs and Confidence Intervals Under Alternative Expectations

for breaking NAIRU with estimated jumping times, the models for spline NAIRUs, and finally Time varying NAIRUs.

A first look at the estimated breaking NAIRUs with estimated jumping times reveals unexpectedly high estimates for the first step in the case of sequential AR(p) expectations. As can be observed from table 5 and the corresponding figures in the appendix, the NAIRU for the first step (up to 1980:4), is estimated to be above 20%, a value that is clearly unreasonable. Hence, we will not take into account the results of breaking NAIRUs with sequential AR(p) expectations.

For the remaining two assumptions on inflation expectations, unit root and full sample AR(p), we can observe that there is little evidence to support the existence of more than one jump. Under the assumption of a single jump estimated NAIRUs at the two different steps are significantly different from each other. However, when

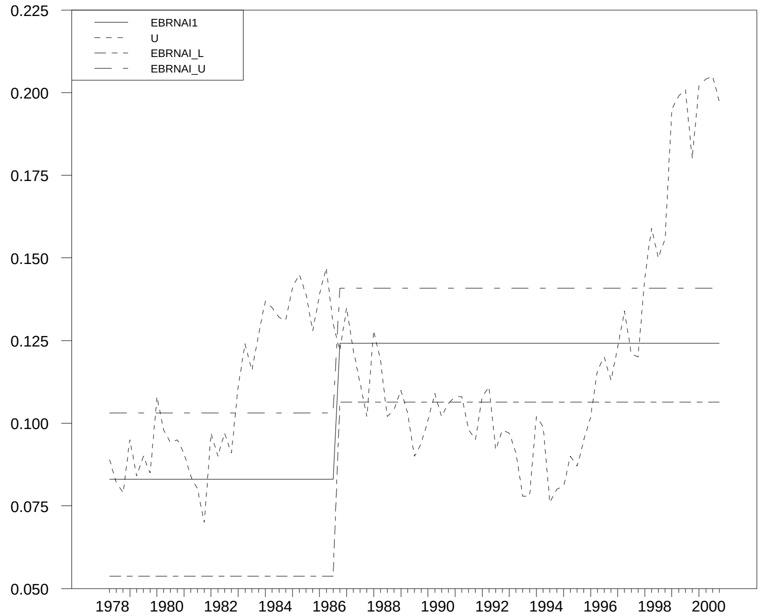

Figure 7: Breaking NAIRU With Estimated Jump Time and Unit Root Expectations.

including further jumps, the estimated NAIRUs at contiguous steps tend not to differ statistically from each other, but the first difference remains. As can be observed from table 5, for unit root and full sample AR(p) expectations, estimated NAIRUs before and after the jump are very close, 8.3 and 8.4% before the break and 12.41 and 13 % after it. See figure 7.

The fact that only one break has significant effect on the change of inflation have important implications on the behavior of the unknown NAIRU. This result implies that there is a significant increase in the average level of the unknown NAIRU, and this increase is estimated to be about 4% along the sample span.

		Inflation Expectations		
		Unit Root	AR(p)	Seq. AR(p)
Est. NAIRU		8.30	8.40	27.17
L90		5.37	5.69	15.20
U90		10.31	10.30	68.35
Width		4. 94	4. 69	53. 15
Break Time		1986:04	1986:04	1980:04
Est. NAIRU		12.41	13.00	10.16
L90		10.64	10.43	5.13
U90		14.09	13.70	11.95
Width		3. 45	3.27	6. 82

Table 5: Estimated Breaking NAIRUs and Confidence Intervals Under Alternative Expectations

A note of caution should be taken when interpreting the results of this exercise. We assume that the unobserved NAIRU is smooth and may be approximated by means of a step function. The number of breaks necessary for this approximation and the times of jump uncover many interesting features of the unobserved NAIRU. The fact that only one jump is required for a "good" approximation suggests that the NAIRU is monotonically increasing, and the fact that the time of jump is estimated to be located before the middle of the sample may indicate that its growth may be slightly higher in the first half of the sample.

Although the approximation of an unknown (and fairly smooth) NAIRU with step functions has given us some important information regarding the behavior of the NAIRU, there is still some valuable information that we can obtain with smoother assumptions on the NAIRU. Figure 8 displays the estimated spline NAIRU and confidence band under the assumption of unit root expectations.

From figure 8 we can observe that at both ends of the sample the estimation uncertainty is (as expected) quite high in comparison with the rest of the sample. Strikingly, we can observe that the true but unknown NAIRU can not be higher that 10 percent from 1981 to 1987, and from 1994 to 1997 it can not be lower than 11 percent, and the lower limit clearly increases with time. This information provides us with enough evidence in favor of a smoothly increasing NAIRU. See figure 9 also.

Figure 8: Spline NAIRU With Zero Knots and Unit Root Expectations.

5 Conclusions

In this paper we studied the behavior of some estimates of the NAIRU in Colombia, its estimation uncertainty, the constancy of the NAIRU, and assessed the robustness of this results to alternative specifications of the Phillips curve. We found that for unit root inflation expectations the resulting estimation uncertainty was lower than found in studies on the output gap. That is, gaps are statistically different from zero for a lot more periods of time in the case of the NAIRU than in the case of the potential output, thus providing more reliable estimates of the gap and the Colombian Phillips

Figure 9: Spline NAIRU with Zero Knots and Sequential AR(p) Expectations.

curve. This same result is found sometimes under full sample AR(p) expectations, and seldom under sequential AR(p) expectations.

We found evidence to say that the NAIRU is not constant, and we estimate that it has increased 4 percentage points over the sample span. Since the goal of monetary policy during this decade is to reduce and stabilize inflation, an increasing NAIRU may imply increasing levels of unemployment, a variable that0 is gaining importance in policy discussions and will clearly influence monetary policy. Moreover, since the NAIRU is the component of unemployment that do not respond to monetary policy, government policies are then required.

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