Expansions and contractions in Brazil, Colombia and Mexico: a view through non-linear models

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Abstract

The study of the asymmetric behavior of macroeconomic variables over the business cycles phases has had a long tradition in economics. In this work we find evidence in favor of the hypothesis of having a STAR-type nonlinear asymmetric behavior of the economic activity, over the last two decades, in three Latin American countries: Brazil, Colombia, and Mexico. For Chile and Venezuela the null hypothesis of a linear process could not be rejected under the method placed by Granger and Teräsvirta (1993). Economic activity is proxied by monthly based industrial production indexes. Evidence of asymmetric behavior is also found according to the generalized impulse response functions analysis for the three countries.

JEL classification: C22, C52.
Key words: real industrial production index, nonlinearities, STAR models, generalized impulse response function, high-density regions.

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I. Introduction

The behavior of macroeconomics variables associated to the business cycles has long been of interest to researchers. It has also been of interest the linearity or nonlinearity of the macroeconomic variables movements over phases of the business cycle. The discussion has also covered the symmetric or asymmetric[1] fashion in which such movements take place[2]. Symmetric fluctuations occur when the time distance from peak to trough is similar to that from trough to peak so that contractions are as short and steep as expansions. By contrast, we can think of asymmetries as fluctuations that have different time distance from peak to trough than from trough to peak so that contractions are much shorter and steeper than expansions. This dynamics clearly suggests that the motion of economic activity is different for booming and for slow down phases (Teräsvirta and Anderson, 1992; Zarnowitz, 1992; Granger, Teräsvirta, and Anderson, 1993; Peel and Speight, 2000). Sichel (1993) distinguished two different properties associated to the size of the asymmetry: *deepness* and *steepness*. The former identifies situations in which troughs are further below trend than peaks are above while the latter refers to situations in which contractions are steeper than expansions.

Asymmetric phases of the business cycle might appear under some circumstances both economic and dynamic. Following the motivation of Kontolemis (1997) based on industrial organization literature it could be the case that exit from an industry is less costly than entry and as a result production could fall rapidly and expand slowly[3]. In addition, the asymmetric property might also be associated to the relative easy in which a firm may reduce production below full capacity when orders decrease compared with the difficulty of increasing production when capacity constraints are present[4,5].

From the point of view of dynamics, cyclical asymmetries might arise when the propagation mechanism is based on the intertemporal substitution of the labor supply when an

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1 References on asymmetries of the macroeconomic variables over the cycles are dated as early as Mitchell (1927, pp. 330-34) and Keynes (1936, p. 314).

2 Boldin (1999) has showed how the effects of monetary policy are stronger during turning points and outright recessions than in expansions.

3 Chetty and Heckman (1985) and Baldwin and Krugman (1986) present models with this characteristic. Sichel (1993) suggests that this kind of asymmetric costs of upward and downward adjustment can generate steepness in the cycles.

4 This view is different from that of Acemoglu and Scott (1994) who, independent from the starting point with respect to the potential output, suggest that adverse supply shocks might correspond to recessions while beneficial demand shocks might correspond to expansions.

5 Sichel (1993) points to this as a potential cause of deepness. For models with this property on prices see, for example, De Long and Summers (1988).

adverse technological shock shifts the economy as in the real business cycle models. Given the existence of a reservation wage, possibly endogenous, when the real wage is below such a reservation wage the labor supply collapses to zero. By contrast, when the shock is positive and the real wage is greater than the reservation wage the supply is positive but it is not possible to say whether the income effect will dominate or not the substitution effect.

Given that some evidence (Boldin, 1999) suggests that most econometric models cannot capture empirically important asymmetries and that linear models are incapable of capturing fluctuation asymmetries (Simpsom et al., 1999), we use the method proposed by Granger and Teräsvirta (1993) to study the nonlinear business cycle properties of the industrial production index of five economies[6]: Brazil, Chile, Colombia, Mexico, and Venezuela over the last two decades. The dynamics is also analyzed with the generalized impulse response functions, *GIRF*, (Potter, 1995; Koop *et al*., 1996) derived from our preferred smooth transition specification.

This work is aimed to obtain some evidence about the regularity associated to asymmetric fluctuations. However, other goals have been previously reached by focusing on the total output. These are the cases of Fernández and Gonzalez (2000) and Torres (1999). The first work showed that the fluctuations of Colombia, Brazil and Costa Rica are highly correlated through coffee. In addition, this work emphasizes on the role of the terms of trade for generating the cycle comovements of the output of some Latin American economies. Torres (1999), found a similarity in the characteristics of the cycles of a set of Latin American countries[7]. This coherence of the movements over the phases of the cycle is explained by external factors such as the capital inflow occurred between 1991 and 1994 (see also Banco de la República, 2001). However, as we have said above, our paper is aimed to check the hypothesis of having asymmetric fluctuations in some Latin American countries.

We characterize the movements of the industrial production by using smooth transition regression models. Armed with a description of the dynamics of each index of the countries where we found evidence of nonlinearities we next estimate *GIRF*´s for the extreme regimes of the cycle to observe the persistence of positive and negative shocks both in expansion and recessions. At the end, we obtain evidence of nonlinear behavior for three out five countries.

[6] Other nonlinear methods used to capture the business cycle features are threshold models (Tsay, 1989; Tiao and Tsay, 1944) and Markov-switching regime models (Hamilton, 1989).
[7] Argentina, Brazil, Chile, Colombia, Peru and Venezuela.

Through the *GIRF′*s we show asymmetric responses of the economic activity depending on the regime they receive the shock and the sign of the shock.

The paper is organized as follows. Section two shows the behavior of the industrial production index for each country included in the sample. Section three describes aspects related to the nonlinear approach we follow. Section four presents some results and discusses the dynamics we find. Finally, section five draws some conclusions.

II. Behavior of the industrial production indexes

The countries included in the study are Brazil, Chile, Colombia, Mexico and Venezuela. The industrial production index, as proxy for economic activity, as well as the countries were chosen on the basis of the availability of monthly data (Figure 1). Appendix 1 to this work includes details about the sample period, the variables and the sources.

The evolution of the industrial activity matches some aggregate behavior of the economies at hand. For example, the slow growth rate of Brazil over the last four years of the sample period (1975-2000); the almost steady growth of Chile within the sample period; the recessions of Colombia at the beginning of the eighties and the end of the nineties; the down turns suffered by Mexican economy about 1983, 1985 and 1995; and, finally, the irregular behavior of the industrial activity in Venezuela with sharp contraction at the end of the eighties. It is important to notice that at glance no common pattern, among the variables, arises.

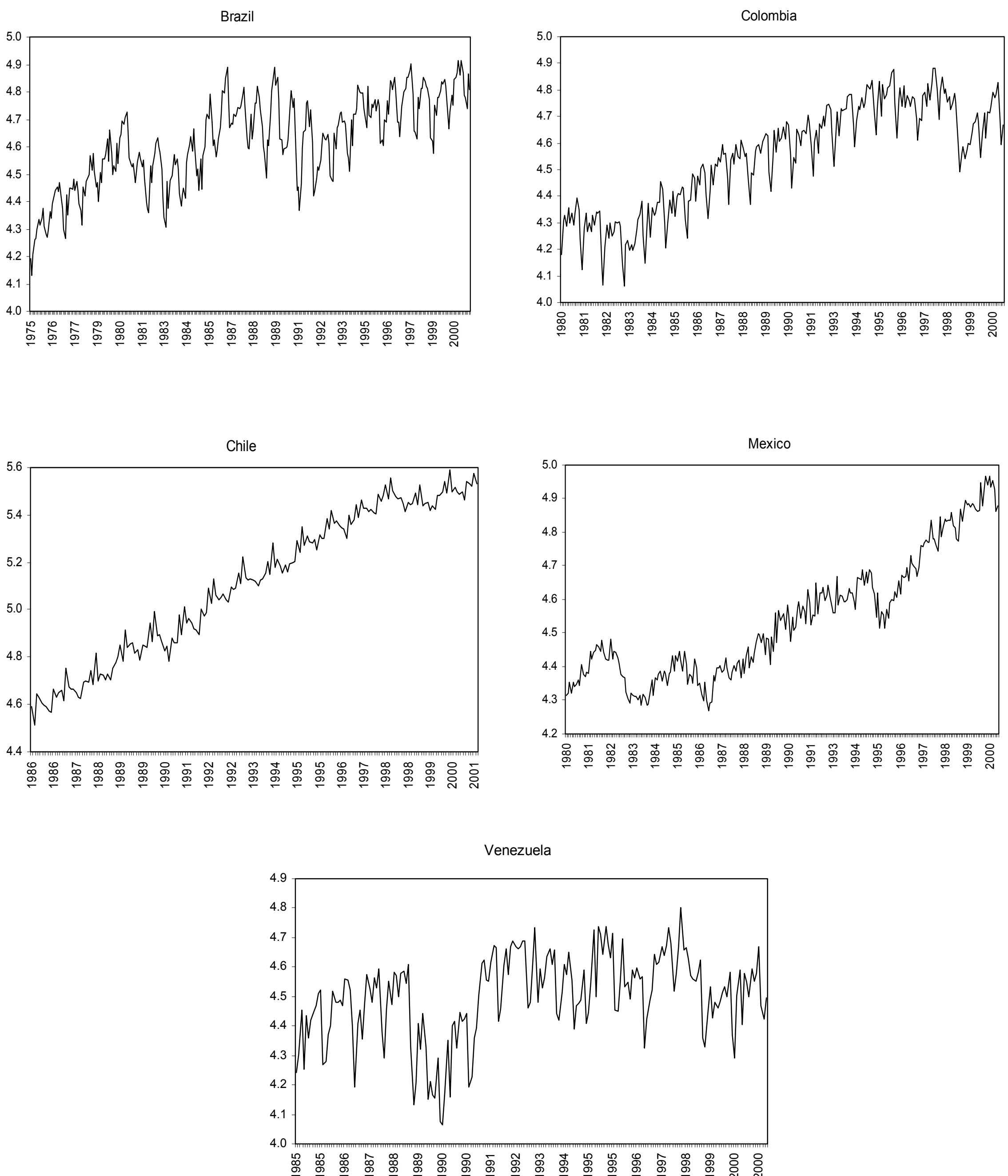
III. Modeling approach

The nonlinear approach we follow, belongs to the smooth transition autoregressive models put forth by Granger and Teräsvirta (1993), Teräsvirta (1994, 1998), and surveyed by van Dijk, *et al*. (2000). In brief, these class of models assume that a (stationary and ergodic) process moves smoothly between the two extreme regimes instead of abruptly from one regime to the other as it is assumed in the threshold autoregressive (TAR) models (Tong, 1990; Priestly, 1988; Tsay, 1989)[8].

[8] For the case of Colombia Arango (1998) applied the same approach to the PIB annually dated between 1925 and 1992.

According to this approach, it could be the case that the DGP of a variable can be represented by a smooth transition autoregressive model of order *p* [STAR(*p*)], which can be written as:

Figure 1. Real industrial production index of selected Latin American countries



$$y_t = \beta_0 + \sum_{j=1}^{p} \beta_j y_{t-j} + (\beta_0^* + \sum_{j=1}^{p} \beta_j^* y_{t-j}) F(y_{t-d}) + \varepsilon_t \tag{1}$$

where y_t is the variable of which we are interested in the dynamics, F is a transition function bounded by zero and one and ε_t is an *i.i.d*. process with zero mean and finite variance.

Following Teräsvirta (1994), the testing strategy is carried out on two transition functions: the *logistic* function:

$$F(y_{t-d}) = (1 + \exp\{-\gamma(y_{t-d} - c)\})^{-1}, \qquad \gamma > 0 \tag{2}$$

which replaced in (1) yields the logistic STAR(p) model [LSTAR(p)], and the U-shaped *exponential* transition function:

$$F(y_{t-d}) = 1 - \exp(-\gamma(y_{t-d} - c)^2), \qquad \gamma > 0 \tag{3}$$

which replaced in (1) gives the exponential STAR(p) model [ESTAR(p)]. The parameter γ represents the speed of the transition process. As we shall see below, the selection between LSTAR and ESTAR models is done by using the data, even in those cases where the economic theory makes some predictions for that.

The "heaviside" properties of the transition function F can be seen as follows. In (2) we note that when $\gamma \to \infty$ and $y_{t-d} > c$ then $F = 1$, but when $c \geq y_{t-d}$, $F = 0$, so that (1) becomes a TAR(p) model. When $\gamma = 0$, (1) becomes an AR(p) model. In (3) we observe that the ESTAR model becomes linear [AR(p)] both when $\gamma \to 0$ and when $\gamma \to \infty$. In either transition function, the variable y_{t-d} can generate monotonic changes in the parameters of (1) rather than discrete movements between regimes[9].

The LSTAR model can describe asymmetric realizations. That is, in our particular case, this model can generate one type of dynamics for increasing growth rate of the industrial index and another for reductions of such a variable. Hence, with the transition function (2) either in the upper ($F = 1$) or the lower regime ($F = 0$), expression (1) becomes a different linear AR(p) model.

The ESTAR model implies that increases and reductions of the transition variable have similar dynamics. For this model, the outer regime ($F = 1$) corresponds to $y_{t-d} = \pm\infty$ and (3) is

[9] Acemoglu and Scott (1994, p. 1305) view this particular transition function, based on past values of the variable at hand, as a potential weakness of this specification.

replaced in (1) to obtain a linear AR(*p*) model; the middle regime ($F = 0$) results when $y_{t-d} = c$, and (3) replaced into (1) yields a linear AR(*p*) model.

The strategy for building a STAR model requires the estimation the artificial regression [see Teräsvirta (1994, 1998) for details]:

$$y_t = \pi_{00} + \sum_{j=1}^{p}(\pi_{0j}y_{t-j} + \pi_{1j}y_{t-j}y_{t-d} + \pi_{2j}y_{t-j}y_{t-d}^2 + \pi_{3j}y_{t-j}y_{t-d}^3) + \varepsilon_t \qquad (4)$$

and test the null H_0: $\pi_{1j} = \pi_{2j} = \pi_{3j} = 0$, (*j =1,...,p*), against a two-tails alternative. In practice, the Lagrange multiplier-type test of linearity is replaced by an *F*-test in order to improve the size and power of the test for small samples. Third, consider the value of *d* as given and use a sequence of tests specified in (5)-(7) to choose between ESTAR and LSTAR models. Such a sequence is:

$$H_{03} : \pi_{3j} = 0, \qquad j = 1,..., p. \qquad (5)$$

$$H_{02} : \pi_{2j} = 0 | \ \pi_{3j} = 0, \qquad j = 1,..., p. \qquad (6)$$

$$H_{01} : \pi_{1j} = 0 | \ \pi_{2j} = \pi_{3j} = 0, \ j = 1,..., p. \qquad (7)$$

and it is based on the relationship between the parameters in (4) and (1) with either (2) or (3). For the ESTAR model $\pi_{3j} = 0$, j = 1,...., *p*, but $\pi_{2j} \neq 0$ for at least one *j* if $\beta_j^* \neq 0$. For the LSTAR model $\pi_{1j} \neq 0$ for at least one *j* if $\beta_j^* \neq 0$. If H_{03} is rejected, a LSTAR model is selected. If H_{03} is not rejected and H_{02} is rejected then an ESTAR model is selected. If H_{03} and H_{02} are not rejected but H_{01} is, then a LSTAR model is selected. No clear-cut conclusion is obtained when H_{02} and H_{01} are rejected. In this case we test:

$$H'_{02} : \pi_{2j} = 0 \ | \ \pi_{1j} = \pi_{3j} = 0, \qquad j = 1,..., p \qquad (8)$$

however, if H_{02} is rejected, then H'_{02} should be rejected even more strongly. In any case, the decision is based on whether H_{03}, H_{02} or H_{01} is rejected more strongly.

IV. Empirical issues

To arrive to an appropriate form of the variables, we first take logs of the five industrial production indexes and, when necessary, eliminate seasonal effects by running a regression on a constant and seasonal dummies for monthly data. Finally, first differences of the resulting variables were used to undertake the estimation process given the evidence of non stationarity of the series in levels. According to the results, evidence of non-linearity, in the sense considered here, is found for three out of five countries when using the lags of $(1\text{-}L^{12})$ times the log of the real industrial production index as transition variable. The models fitted happened to be LSTAR which is an evidence of the asymmetry of the business cycle in these countries (Table 1). No evidence of misspecification of the models is found on the basis of the Ljung-Box, MacLeod-Li, LM-ARCH, and Jarque-Bera tests. Furthermore, non-remaining nonlinearity, and parameter constancy (Teräsvirta, 1998), are highly satisfactory[10].

Table 1A. LSTAR model for Brazil

	Coefficient	**S. D.**	***t*-value**	***p*-value**
Linear part				
Constant	0.003	0.002	1.241	0.215
y_{t-1}	-0.243	0.048	-5.031	0.1×10^{-5}
y_{t-3}	0.121	0.048	2.511	0.0125
y_{t-7}	-0.126	0.048	-2.606	0.009
y_{t-9}	0.128	0.047	2.663	0.0081
Dummy 914	0.140	0.034	4.097	0.5×10^{-4}
Non linear part (Transition variable: $\Delta_{12} y_{t-10}$)				
Constant	-0.010	0.005	-2.070	0.039
$\hat{\gamma}$	183.494	659.702	0.278	0.781
$\hat{c}$	0.066	0.002	24.356	0.1×10^{-10}
y_{t-12}	0.403	0.117	3.452	0.001

With respect to the results, we can pay attention on the estimated values of gamma's ($\hat{\gamma}$) and the thresholds ($\hat{c}$) of each model. As we said before, gamma represents the speed of the transition process while $\hat{c}$, the threshold, represents the value that triggers the change of one regime to the other (see Figure 2).

[10] Also available from the authors on request.

In the cases of Brazil, Colombia and Mexico we observe the same pattern: sudden and abrupt, rather than smooth, movements from one regime to the other, being the corresponding to Brazil the strongest as it has the highest value of the estimated gamma (183.5). This sharp transition observed for Brazil is compatible with the dynamic of the series (Figure 1) where, apart from the seasonal component, we can observe clear fluctuations of the economy.

Table 1B. LSTAR model for Colombia

	Coefficient	**S. D.**	***t*-value**	***p*-value**
Linear part				
Constant	-0.034	0.012	-2.738	0.006
y_{t-1}	-0.516	0.060	-8.517	0.1×10^{-10}
y_{t-7}	-1.221	0.319	-3.830	0.1×10^{-3}
y_{t-10}	-0.157	0.053	-2.991	0.003
y_{t-12}	-1.416	0.327	-4.326	0.2×10^{-4}
Non linear part (Transition variable: $\Delta_{12} y_{t-1}$)				
Constant	0.038	0.012	2.960	0.003
$\hat{\gamma}$	69.501	93.210	0.745	0.456
$\hat{c}$	-0.072	0.003	-24.519	0.1×10^{-10}
y_{t-2}	-0.201	0.065	-3.069	0.002
y_{t-7}	1.175	0.323	3.637	0.3×10^{-3}
y_{t-12}	1.484	0.333	4.445	0.1×10^{-4}

The transition function over time presented in Figure 3 help us to identify the biggest contractions for these countries. This is the case of the slumps (associated here to contractions or decelerations of economic activity) of Brazil occurred between 1981-84 (coincident with debt crisis), 1988-92, 1996-97, and 1998-2001. Booms (associated here to expansions, accelerations or recoveries) occurred in 1977, 1985-88, and, to some extent, 1994. The rest of the time this country faced a high variability of economic activity according to this indicator. Thus, according to the transition function, this country has been more in slump environments than in booms during the last 20 years of past century.

As for Colombia the slumps occurred in 1996-1997 and 1998-1999, the sharpest in that country. However, notice that this model fails to identify the important crisis of 1982-83. With respect to booms, this model suggests that most of the time Colombia was in the upper regime, with a few exceptions neatly observable in the transition function.

Table 1C. ESTAR model for Mexico

	Coefficient	**S. D.**	***t*-value**	***p*-value**
Linear part				
Constant	0.002	0.003	0.678	0.498
y_{t-1}	-0.445	0.053	-8.259	0.1×10^{-10}
y_{t-3}	0.495	0.104	4.757	0.4×10^{-5}
y_{t-6}	0.089	0.052	1.698	0.090
y_{t-9}	0.126	0.052	2.389	0.017
y_{t-12}	-0.195	0.140	-1.395	0.164
Non linear part (Transition variable: $\Delta_{12} y_{t-1}$)				
Constant	-0.7×10^{-7}	0.004	-0.181	0.856
$\hat{\gamma}$	73.987	159.618	0.463	0.643
$\hat{c}$	-0.009	0.003	-2.616	0.009
y_{t-3}	-0.413	0.127	-3.239	0.001
y_{t-12}	0.343	0.155	2.215	0.027

Following the results suggested by the model, Mexico had important downturns in 1982-3 (also coincident with the debt crisis), 1986 and 1995, while the regime associated to booms was more frequent for this country. In summary the evidence tells that Colombia and Mexico have been more in the upper regime than in the lower while the converse situation is the case for Brazil.

The difficulty to interpret some of the estimates of a STAR-type model can be overcome by analyzing the limit values that describe the local dynamics and the impulse response function. For LSTAR models, the lowest and highest growth rates of industrial production index are associated to $F = 1$ and $F = 0$, respectively (Figure 2).

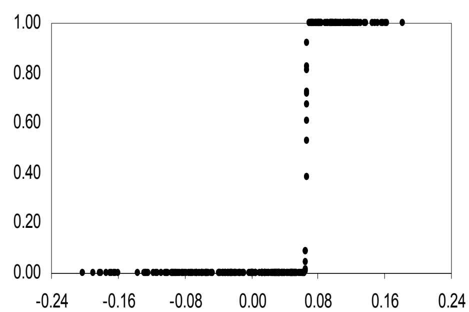
For describing the local dynamics, we use the roots of the models that can be obtained from:

$$z^{p} - \sum_{j=1}^{p} (\hat{\beta}_j + \hat{\beta}_j^{*} F) z^{p-j} = 0 \tag{9}$$

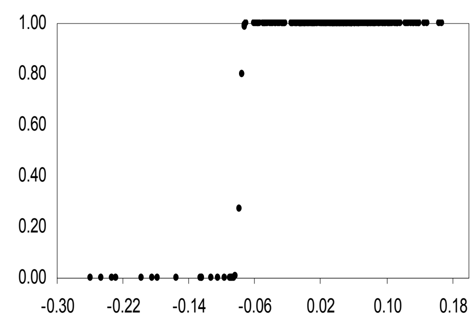
for $F = 0, 1$ (Table 2).

Figure 2. Transition function

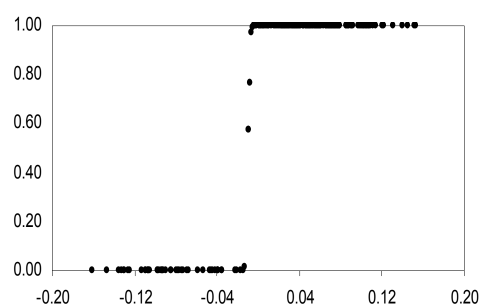
Brazil



Colombia



Mexico



The dominant roots of the regimes of both recession and expansion are locally stable. This is the case for all countries except for the lower regime of Colombia. However, for this country the number of points in the lower regime of the transition function is not high. Such a situation could be interpreted in the following sense: once the industrial activity is in the (extreme) lower regime, any exogenous force arises to reduce the performance of the economy with the aim of moving it out of that regime.

Table 2. Characterization of extreme regimes polynomials and dominant roots

A. Brazil

F=0			F=1		
Root	**Modulus**	**Period**	**Root**	**Modulus**	**Period**
-0.49±0.71i	0.86	2.88	-0.52±0.83i	0.98	2.94
0.12±0.81i	0.82	4.40	-0.93	0.93	.
0.62±0.45i	0.77	10.03	-0.81±0.46i	0.93	2.39
-0.73±0.23i	0.77	2.21	±0.93i	0.93	4.01

B. Colombia

F=0			F=1		
Root	**Modulus**	**Period**	**Root**	**Modulus**	**Period**
-0.80±0.81i	1.14	2.68	-0.53±0.73i	0.90	2.85
0.21±1.06i	1.08	4.57	-0.04±0.88i	0.88	3.87
-1.05±0.15i	1.06	2.10	0.41±0.71i	0.82	6.00
0.97±0.34i	1.03	18.48	-0.76±0.29i	0.82	2.27

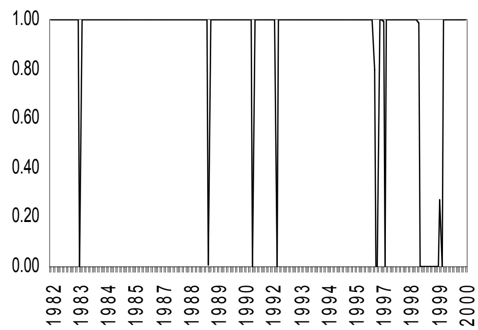
C. Mexico

F=0			F=1		
Root	**Modulus**	**Period**	**Root**	**Modulus**	**Period**
-0.63±0.73i	0.96	2.75	-0.51±0.78i	0.93	2.92
-0.85±0.27i	0.89	2.21	-0.79±0.39i	0.87	2.34
0.84±0.16i	0.85	34.21	0.87	0.87	.
-0.35±0.77i	0.85	3.15	-0.86	0.86	.

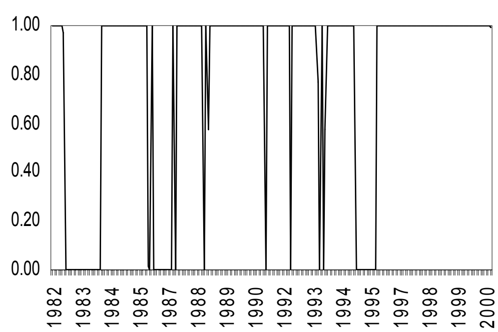
Figure 3. Transition function over time

Brazil

Colombia



Mexico



The dynamics of the variables can also be analyzed by using the impulse response function (*IRF*). This function shows the effect of a shock on a series over time. It can be calculated as the difference between the conditional expected value of the series with and without a shock. That is:

$$\begin{aligned} IRF_Y(h,\delta,t-1) = {} & E\left(Y_{t+h} \ \middle|\varepsilon_t = \delta, \varepsilon_{t+1} = 0, \dots, \varepsilon_{t+h} = 0, Y_{t-1}, Y_{t-2}, \dots\right) \\ & - E\left(Y_{t+h} \ \middle|\varepsilon_t = 0, \varepsilon_{t+1} = 0, \dots, \varepsilon_{t+h} = 0, Y_{t-1}, Y_{t-2}, \dots\right) \end{aligned} \tag{10}$$

for $h = 0, 1, 2, \dots$. In equation (10) the *IRF* indicates the effect of a shock of magnitude δ received by the series $\{Y_t\}$ h periods ago.

The *IRF* for linear models exhibits two main characteristics: symmetry and history independence. The former implies that a shock of magnitude -δ produces, on qualitative grounds, the same effect of a shock of magnitude +δ. The latter implies that the response of a shock does not depend on the time period when the series is shocked.

These properties do not hold for nonlinear models. This is the case of *STAR* models since the effect of a shock depends not only on the sign and size but also on the time period of the shock. In the traditional definition of the *IRF* in (10) the intermediate shocks are assumed to be zero $(\varepsilon_{t+1} = \dots = \varepsilon_{t+h} = 0)$ a fact that could be misleading given the characteristics of this type of models.

The Generalized Impulse Response Function (*GIRF*), set forth by Koop, Pesaran and Potter (1996) provides a generalization of the concept of *IRF*′s for nonlinear models. Let us assume that a specific shock of magnitude δ arises given in time *t* the *GIRF* is defined as[11]:

$$GIRF_Y(h,\delta,\omega_{t-1}) = E\left(Y_{t+h} \ \middle|\varepsilon_t = \delta, \omega_{t-1}\right) - E\left(Y_{t+h} \ \middle|\omega_{t-1}\right) \tag{11}$$

where ω_t represents the history of the process at time *t*. In this definition the expectation is conditional to a shock δ and a particular history. Thus, in contrast to the traditional *IRF*, the intermediate shocks are averaged.

It is natural to regard δ and ω_{t-1} as realizations of the random variables ε_t and Ω_{t-1}. Thus, Koop *et al.* (1996) consider the *GIRF* defined above to be the realization of a random variable defined by

$$GIRF_Y(h,\varepsilon_t,\Omega_{t-1}) = E\left(Y_{t+h} \ \middle|\varepsilon_t, \Omega_{t-1}\right) - E\left(Y_{t+h} \ \middle|\Omega_{t-1}\right) \tag{12}$$

[11] The paragraphs that follow are based on van Dick, Teräsvirta and Franses (2000).

Various conditional versions of the *GIRF* can be defined depending on the subset of shocks and histories included in the analysis. For example, if shocks have asymmetric effects over different regimes, then averaging across all observations will tend to hide the evidence of asymmetry. In this case the *GIRF* of interest is likely to be conditional, such as, negative shocks in the regime corresponding to recessions, say.

The expectations involved in the definition of the *GIRF* in (11) can be interpreted as the optimal forecasts of y_{t+h} at time *t* with and without a shock of magnitude δ at time *t*. Then, the *GIRF* can be estimated by using the point forecast procedures for *STAR* models suggested by Lunderbergh and Terasvirta (2001) which cannot be solved analytically and requires numerical approximations to the expression[12].

Persistence of the shocks is an issue that deserves special attention in the analysis of the *GIRF*. Koop *et al.* (1996) suggest that this can be measured in terms of the dispersion of the distribution of the *GIRF*. To compute confidence regions for the *GIRF* the highest-density regions (*HDR*) are used.

The distributions of the forecasts for nonlinear models can be multimodal, then if we want to compute a forecast region, the use of a symmetric interval around the point forecast might cast some doubt. Hyndman (1995) discusses in detail the construction of forecast confidence regions and argues that the *HDR* are a more effective summary of the forecast distribution than other common forecast regions. Let *x* be a continuous random variable with probability density *f(x)*, the $100\alpha\%$ highest density region $HDR_\alpha(x) = \{x: \ f(x) \geq f_\alpha\}$ where $f_\alpha > 0$ is such that the probability of a given *x* having a density that at least equals f_α is α. Then, the *HDR* is equivalent to the region occupying the smallest possible volume in the sample space. The *HDR*´s can be calculated using standard kernel density estimators for the Monte Carlo or bootstrap simulations used to obtain the point forecasts.

The estimation of *GIRF* that we use here includes all observations in the sample as histories and 60 initial shocks equal to $\delta/\sigma_\varepsilon = \{\pm 3, \pm 2.9, \ldots, \pm 0.2, \pm 0.1\}$, where σ_ε denotes the estimated standard deviation of the residuals from the *STAR* models. For each combination of history and initial shock, we compute *GIRF* for horizons of 60 periods and 1000 replications.

[12] Alternatively, it can be solved through Monte Carlo or bootstrapping techniques.

The dynamic properties of the models fitted for each country is analyzed through the estimated *GIRF′*s (see Figures 4 – 6). Each panel shows the *HDR′*s up to 60 months ahead to illustrate the persistence of a shock, of distinct signs, under different histories. Panel *a* is based on positive shocks in the upper regime, panel *b* is based on positive shocks in the lower regime; panels *c* and *d* show the persistence of negative shocks in the upper and lower regimes, respectively; panels *e* and *f* show the *HDR′*s of the effects of shocks both positive and negative in the upper regime and the lower regime, respectively; panels *g* and *h* show the dynamics generated by a positive shocks in both regimes and the effect of negative shocks in both regimes; finally, panel *i* is based on all shocks in the two regimes. In Figures 4 to 6 we observe two intervals. The dark interval represents the 90% *HDR′*s while the dotted interval represents the 95% *HDR′*s fot the *GIR*.

The results from Brazil (Figure 4) suggest that, in general, shocks are not that persistent. However, they are slightly more persistent in the lower regime than in the upper one (see panels *e* and *f*) since the density function takes longer to shrink. At the same time, positive shocks are more persistent than the negative ones (see panels *g* and *h*). These two results can be observed in panel b where shocks take more than 20 months to contract.

As for Colombia (Figure 5) the results are similar in the sense that persistence is not high. However, what is observed is that shocks are more persistent in the lower regime than in the upper (see panels *e* and *f*) a result consistent with the explosive root described by the lower regime (see panel B of Table 2). For Mexico (Figure 6), as well as for Colombia, no distinction in the dynamics introduced by negative and positive shocks arises but shocks in the lower regime[13] take more time to shrink. Thus, in summary, shocks in the lower regime seem to be more persistent that shocks in the upper regime for the three countries.

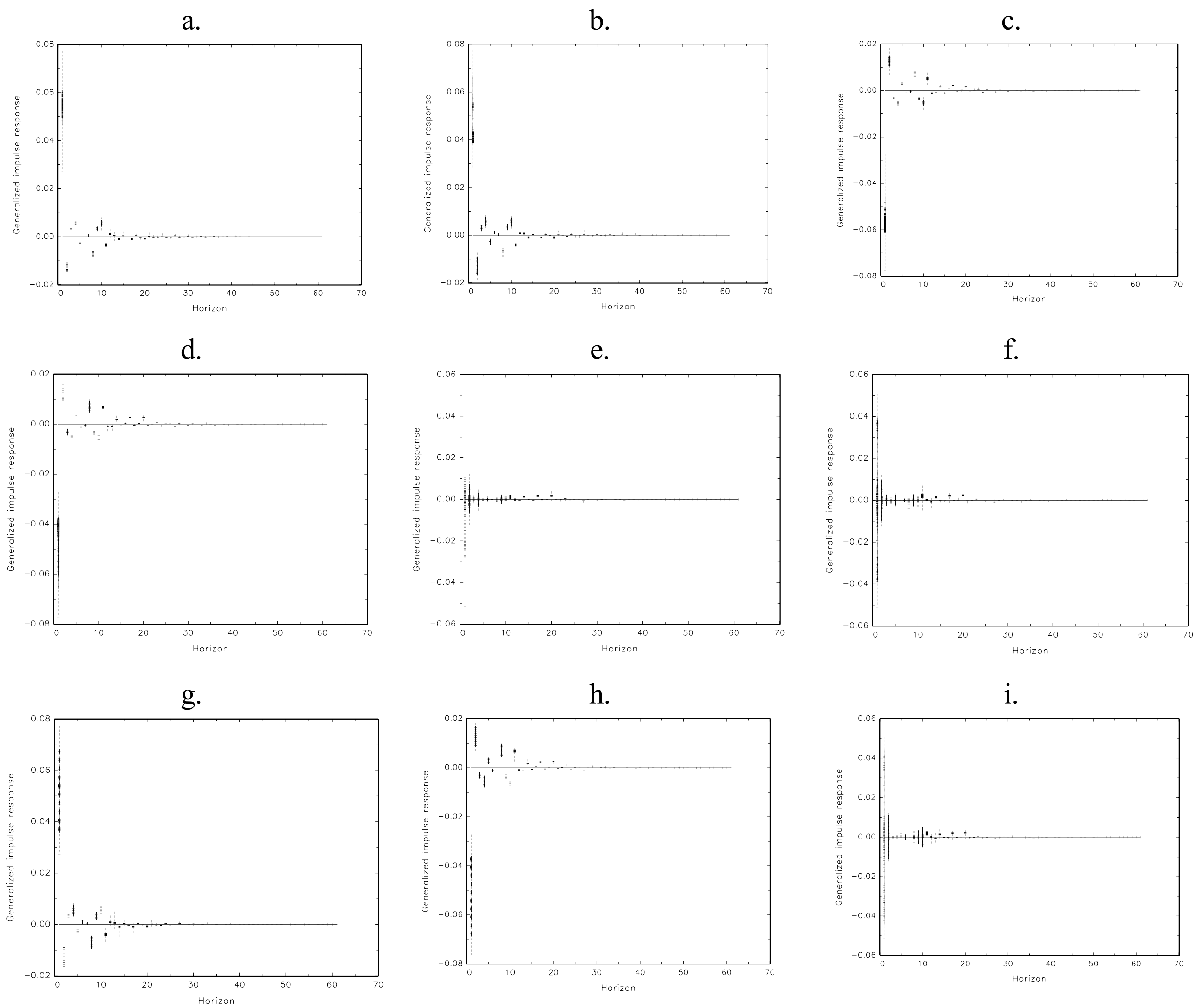
V. Conclusions

In this paper we employ the real industrial production index as the proxy for economic activity and present evidence of having nonlinear business cycles in some of the selected Latin American countries: Brazil, Colombia and Mexico. For Chile and Venezuela, the hypothesis of linearity could not be rejected. The evidence of nonlinearity is supported by the smooth

[13] The definition of the lower regime is not exactly the same since in Figures 4-6 such a state of the economy corresponds to values when $F(X_t)<0.5$ while in Table 2 the lower regime corresponds to $F(X_t)=0$.

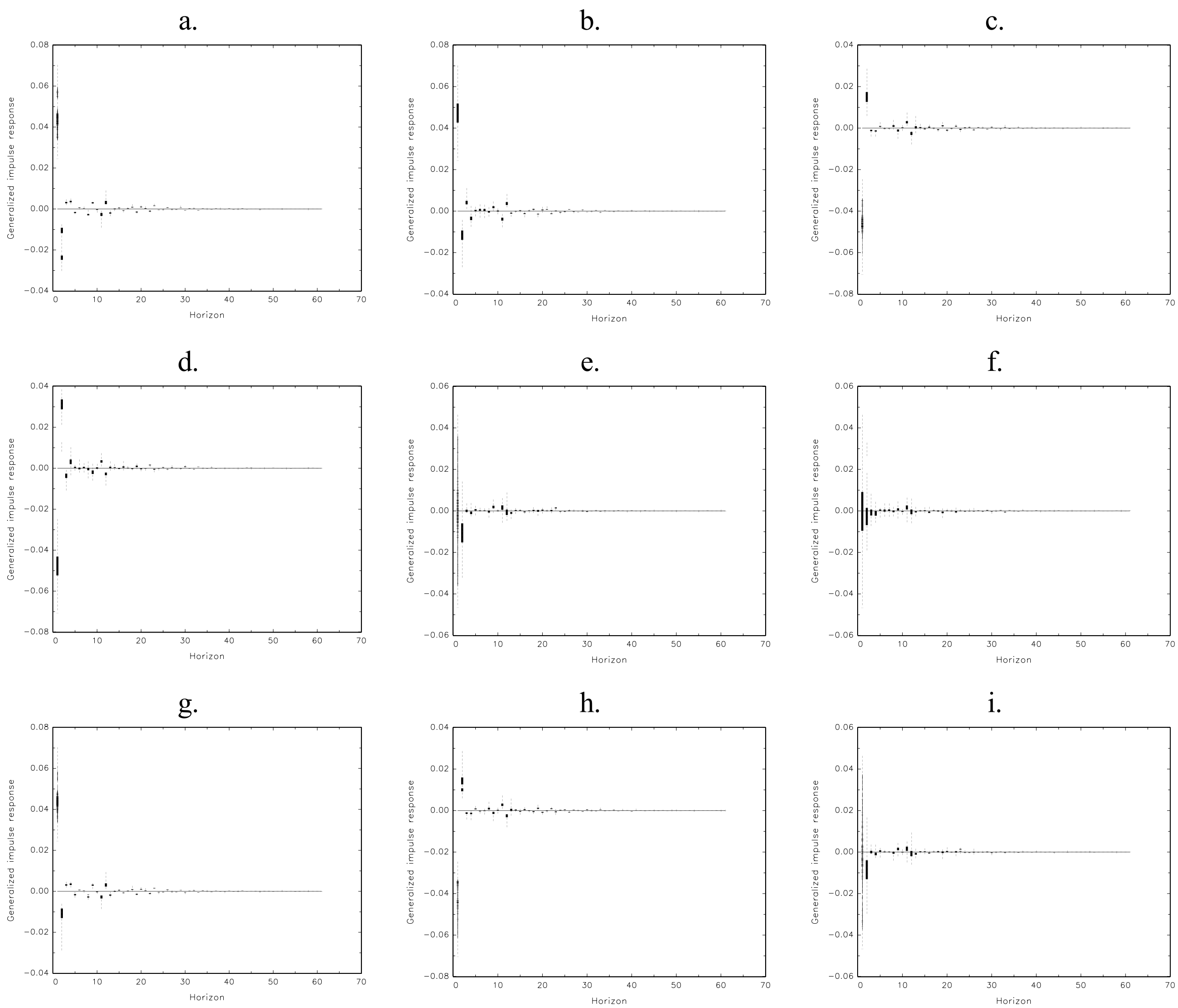
transition autorregresive model adjusted for each country and the asymmetries found in the analysis of the generalized impulse response functions and high density regions.

Figure 4. Generalized impulse response functions for Brazil



The STAR models we have fitted shed some light on the features of the series we have considered. Thus, the nonlinearity characterized for the transition function suggests that the cycles of the three economies are asymmetric. The shape of the estimated transition function of the non linear model meets the dynamics of the data. Its sharp form in the three cases (Brazil, Colombia and Mexico) may be an indication of no clear evidence of transition periods between the extreme regimes. Also, when plotted over time, the transition function can help us to identify the biggest contractions for these countries. This is the case of the 1998-2001 recession of Brazil, the 1999 recession of Colombia and the 1995 recession in Mexico[14].

Figure 5. Generalized impulse response functions for Colombia

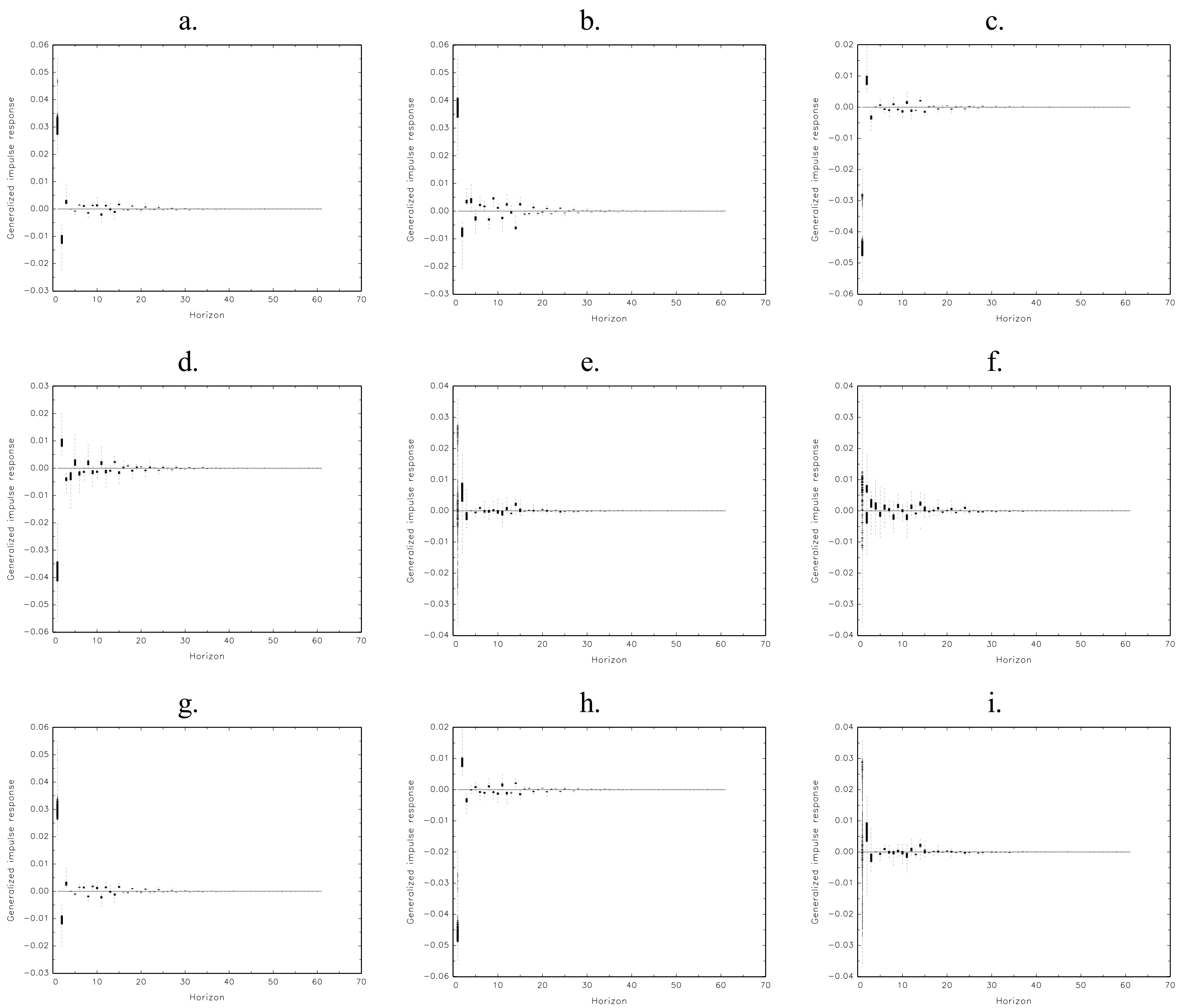


The dynamics suggested by the generalized impulse response analysis is only clear to some extent since there is no evidence on the persistence of the difference of the log of real industrial production index. For the three countries we find responses contingent on the regime

[14] Also remarked by Oliveira (2002).

of the economic activity but for Brazil positive shocks are more persistent in the lower regime than the negative ones.

Figure 6. Generalized impulse response functions for Mexico



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Appendix 1

Data Sources:

Brasil: *Produção industrial – indústria geral - quantum - índice dessaz. – Mensal*". Monthly data from 1975:1 to 2001:1. WEBSITE of the "*Instituto de Pesquisa Econômica Aplicada*".

Chile: Economic Activity Monthly Index (IMACEC). Monthly data from 1986:1 to 2001:2. WEBSITE of the *Banco Central de Chile*.

Colombia: Real Industrial Production Index. Monthly data from 1980:1 to 2001:2. *DANE* Data bases.

Mexico: Physical Volume Industrial Activity Index. Monthly data from 1980:1 to 2001:1. *INEGI* Data bases.

Venezuela: Laspeyres Volume Production Index corresponding to the private manufacturing industry. Monthly data from 1985:1 to 2001:2. *Banco Central de Venezuela* Data bases.

Periods used as references of A: slump (/ contraction / deceleration) and B: boom (/ expansion / acceleration / recovery).
Source: CEPAL, Estudio Económico de América Latina y el Caribe (1999) y (1999-2000)

Brazil:
A: 1981, 1983, 1985, 1989; 1993*; 1994, 1996, 1998, 1999.
B: 1986, 1991, 1997

Chile:
A: 1990*, 1996*, 1998*, 1999.
B: 1989,1992, 1995.

Colombia:
A: 1982*, 1996*, 1998*, 1999.
B: 1986, 1994.

México:
A: 1982*, 1983, 1986, 1995.
B: 1981, 1990, 1997.

Venezuela:
A: 1985, 1989, 1993*, 1994, 1996, 1998, 1999.
B: 1986, 1991, 1997.

* represents a deceleration (qualification from the authors).