About a coincident index for the state of the economy

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Abstract

The construction of coincident indexes for the economic activity of a country is a common practice since the fifties. The methodologies vary from heuristic methods to probabilistic or statistical ones. In this paper, we present a new procedure for estimating a coincident index of the state of the economy which is optimum in a statistical sense. This procedure is

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based on state space models that do possess the steady-state property. We apply our methodology for computing a coincident index for the Colombian economy.

Key words: State of the economy, Coincident Index, State Space Models.

1 Introduction

Coincident cyclical indexes have been broadly used since the work of Burns and Mitchell (1946), which was completed in the fifties and sixties by the National Bureau of Economic Research (NBER). These methods are based on the estimation of a weighted average of some observed variables that are supposed to move contemporaneously with the economic cycle or more general with the global state of the economy. There are two problems in these procedures: (1) as it is shown by Stock and Watson (1989, 1991) there is not a precise description or definition of the global state of the economy from a statistical point of view; (2) although this weighted average can be seen as an estimation of a latent variable, there is no way of knowing if this procedure is optimal under some statistical criteria.

In contrast with the traditional NBER methodology, in recent decades several procedures have been developed which use techniques based on econometric and time series analysis[1]. The papers of Stock and Watson (1988,1991, 1992) are some examples of this approach. They develop a probabilistic state space model that can be used to estimate (or predict) a latent process and this estimation is used as a coincident indicator of the economic activity.

Although the Stock and Watson's (1989, 1991, 1992) model involves simultaneously an unobservable process, as the latent state of the economy, and fixed population parameters, in this paper we show that their model does not have the steady-state property in the sense of Harvey (1989). Then, there are two potential problems: (1) divergence of the maximum likelihood esti-

[1] A detailed reference of different approaches used to construct coincident and leading indicators of the economic activity is shown in Lahiri y Moore (1991).

mation algorithm for the hyperparameters of the model and (2) distortions in the dynamic of the estimated non-observable process.

This paper develops an alternative method to compute a coincident index for the economic activity. The proposed methodology is based on a modification to the Stock and Watson's (1989, 1991, 1992) model. The main changes in the model are done in order to have a steady-state model and in this way to formalize the theoretical procedures to be derived from it.

The paper is organized as follows. Section 2 shows the statistic model to be used for the construction of the coincident index; Section 3 presents an empirical application of the methodology to the Colombian economy using monthly data for the sample period 1980:01 - 2001:02. Finally, some conclusions are presented in Section 4.

2 A statistical model

2.1 Specification and basic assumptions

Following the methodology of Stock and Watson (1989, 1991, 1992), from now on SW, we initially define the state of the economy as a latent stochastic processes in the sense of Sargent and Sims (1977) Singleton (1980) and Geweke and Singleton (1981), which is denoted by $\{C_t\}$.

The basic hypothesis for the construction of a coincident index for the economic activity is the following: there are observed variables $X_{1t}, ..., X_{nt}$, integrated of order one, that have a contemporaneous relationship with $\{C_t\}$ given by the equation

$$X_{it} = \beta_{it} + \gamma_i C_t + u_{it},$$

for all $t = 1, ..., N$, N the length of the sample period, and for all $i = 1, ..., n$, where β_{it} is a deterministic component that can include seasonal components, γ_i is a constant that represents the weight of C_t in X_{it} and u_{it} is a stochastic component inherent to X_{it} and independent of C_t which follows the autoregressive process

$$D_i(B)u_{it} = \epsilon_{it} \ ,$$

where $D_i(B) = 1 - d_{i1}B - ... - d_{ik}B^k$, with B as the lag operator and ϵ_{it} a Gaussian zero-mean white noise process with variance σ_i^2. We also assume that the stochastic processes $\{\epsilon_{it}\}$ are mutually independent, which implies the mutual independence of the $\{u_{it}\}$ processes. Another interpretation of the γ_i coefficients is given in the subsection (2.2), in terms of the first differences of X_{it} and C_t.

In contrast with SW methodology, these assumptions imply that the variables $X_{1t}, \cdots, X_{nt}$ are cointegrated. Essentially, the previous equation express that one observed coincident variable is a linear transformation of the state of the economy, plus an intrinsic random noise. Another difference with respect to SW methodology is that the eventual seasonal component in the observed variables is included into the relation between X_{it} and C_t. In this way, we avoid some potential problems due to the seasonal adjustment of the observed variables [see for example Hillmer and Tiao (1982), Harvey and Jaeger (1993) and Harvey and Chung (2000)].

The stochastic dynamic of $\{C_t\}$ is described by the model

$$\phi(B)\Delta C_t = \delta + \eta_t \ ,$$

where $\phi(B)$ is an autoregressive stationary operator of order p, δ is a constant and $\{\eta_t\}$ is a Gaussian zero-mean white noise process with variance σ_η^2. This equation shows another essential assumption of the methodology: $\{C_t\}$ is an integrated process of order 1 [I(1)]. Let $\mathsf{X}_t = (X_{1t}, ..., X_{nt})'$, $\boldsymbol{\beta}_t = (\beta_{1t}, ..., \beta_{nt})'$, $\boldsymbol{\gamma} = (\gamma_1, ..., \gamma_n)'$, $\mathsf{u}_t = (u_{1t}, ..., u_{nt})'$ and $\boldsymbol{\epsilon}_t = (\epsilon_{1t}, ..., \epsilon_{nt})'$, then the previous equations can be rewritten in the following vectorial form:

$$\begin{aligned} \mathsf{X}_t &= \boldsymbol{\beta}_t + \boldsymbol{\gamma} C_t + \mathsf{u}_t & (1) \\ \phi(B)\Delta C_t &= \delta + \eta_t & (2) \\ D(B)\mathsf{u}_t &= \boldsymbol{\epsilon}_t & (3) \end{aligned}$$

where $D(B) = I - D_1 B - ... - D_k B^k$, with I the identity matrix of order n, and $D_i = \text{diag}\{d_{1i}, ..., d_{ni}\}$.

The statistical problem to be solved consists in estimating C_t, for each $t = 1, ..., N$, using the observed information up to time t and taking the estimated process, $\{C_{t|t} : t = 1, ..., N\}$ say, as the coincident index. Technically, it means to compute $C_{t|t} = E(C_t|\mathsf{X}_1, ..., \mathsf{X}_t)$, $t = 1, ..., N$. We can use the Kalman filter to obtain these conditional expected values, therefore equations (1)-(3) must be transformed into a state space model and this is done in Appendix 1.

The state space model developed in Appendix 1 posses the steady-state property, which essentially guarantees that the mean square error matrix (MSE) of

$$\alpha_{t|t} = E(\alpha_t|\mathsf{X}_1, ..., \mathsf{X}_t)$$

with α_t defined in that Appendix, converges to a fixed matrix as $t \to \infty$. The proof of this claim is given in Appendix 2. It is important to note at this point that the state space model used by Stock and Watson (1989, 1992) does not have this property because the sequence of MSE matrices that their model produces does not converge as $t \to \infty$. To show this we present the following counterexample to their methodology. Using a simple simulation where we take one non-seasonal coincident variable ($n = 1$), $\beta_t = 1$, $\gamma = 1$, $p = k = 1$, $\phi = 0.5$, $\delta = 10.0$, $d = 0.7$, $\sigma_\eta^2 = 1.0$, $\sigma^2 = 4.0$ and the Gaussian noises $\{\eta_t\}$ and $\{\epsilon_t\}$ are simulated with seeds 14600 and 12000, respectively; we find that using Stock and Watson's model, the sequence of MSE's of $C_{t|t}$ tends to infinite in a linear form with slope equals to 4.0 (see Figure 2 in Appendix 3). In addition, the process $\{C_{t|t}\}$ does not reflect the stochastic dynamics of the simulated $\{C_t\}$, that is I(1), as it may be deduced from Figures 1 and 3 in Appendix 3. In Figure 3, we have included the 95% prediction band that is calculated with the root-mean-square-error (RMSE) of $\{C_{t|t}\}$. As we can see there, the band does not include the simulated process. It is important to note that under SW model, $C_{t|t} = \Delta C_{t|t} + C_{t-1|t}$ and that consequently

$$\mathrm{MSE}(C_{t|t}) = \mathrm{MSE}(\Delta C_{t|t}) + \mathrm{MSE}(C_{t-1|t}) + 2\mathrm{E}[(\Delta C_t - \Delta C_{t|})(C_{t-1} - C_{t-1|t})]$$

goes to infinity because $\mathrm{MSE}(\Delta C_{t|t})$ and $\mathrm{MSE}(C_{t-1|t})$ do. We carried out another simulations with $n \geq 2$ (the number of coincident variables) and the results were analogous to the previous ones.

Following the NBER methodology, it is important to have an estimation of the weights of each observable variable included into the coincident index.

From the SW approach, we obtain that

$$\alpha_{t|t} = (I - G_t \mathsf{Z})(\delta \boldsymbol{\mu} + T\alpha_{t-1|t-1}) + G_t(\mathsf{X}_t - \beta_t)$$

where Z, T and $\boldsymbol{\mu}$ are defined in Appendix 1 and G_t is the Kalman-filter gain matrix (Harvey, 1989). Since our state space model has the steady-state property, there exists t_0 such that for every $t \geq t_0$,

$$\alpha_{t|t} \approx (I - G\mathsf{Z})(\delta \boldsymbol{\mu} + T\alpha_{t-1|t-1}) + G(\mathsf{X}_t - \beta_t)$$

where G is the limit of the sequence $\{G_t\}$. Therefore,

$$(I - KB)\alpha_{t|t} \approx \boldsymbol{\tau}_0 + G(\mathsf{X}_t - \beta_t)$$

where B denotes the lag operator such that $B\alpha_{t|t} = \alpha_{t-1|t-1}$, $K = (I - G\mathsf{Z})T$ and $\boldsymbol{\tau}_0 = \delta(I - G\mathsf{Z})\boldsymbol{\mu}$. In practice, t_0 is not very large, then the previous approach is valid for almost all the sample period under consideration. Now, if the eigenvalues of K are smaller than one in module, then

$$\alpha_{t|t} \approx \boldsymbol{\tau} + (I - KB)^{-1}G(\mathsf{X}_t - \beta_t)$$

where $\boldsymbol{\tau} = (I - K)^{-1}\boldsymbol{\tau}_0$. Finally, we can write down that

$$\alpha_{t|t} \approx \boldsymbol{\tau} + \sum_{j=0}^{\infty} K^j G(\mathsf{X}_{t-j} - \beta_{t-j}) \ ,$$

thus

$$C_{t|t} = \mathsf{e}_1' \boldsymbol{\tau} + \sum_{j=0}^{\infty} (\mathsf{e}_1' K^j G)(\mathsf{X}_{t-j} - \beta_{t-j}) \ ,$$

where $\mathsf{e}_1' = (1, 0, ..., 0)$ and $K^0 = I$.

We can note the following important facts: (1) in the computation of the index, the observable variables are adjusted by seasonality (similar to the traditional methodology), (2) the calculated coincident index is not only a weighted average of the present values of the seasonally adjusted variables but also of its lagged values (different from the traditional approach that uses just present observations) and (3) since the index depends on the intercept δ, then a local linear trend of the latent process $\{C_t\}$ can be captured.

It is important to note that for each observable variable we have a weights sequence indexed by the lag $j = 0, 1, ...,$ which is similar to an impulse-response function in VAR modelling. This sequence goes towards zero when j tends to infinite since the eigenvalues of K are smaller than one in module. Then, with these weights we can compute the influence of each variable in the coincident index through time.

2.2 Estimation issues

Once a group of coincident variables X_i has been chosen, the first step before computing $C_{t|t}$ is the estimation of the unknown hyperparameters of the state space model (Harvey, 1989). Since we have included seasonal effects in the model, in contrast with SW approach, we do not need to adjust the observed variables by seasonality before they are included in the model. Specifically, we postulated that if $\{X_{it}\}$ is a seasonal process of length 12 (monthly data) then

$$\beta_{it} = b_i + \omega_{1,i}S_{1t} + \cdots + \omega_{11,i}S_{11,t}$$

where $b_i, \omega_{1,i}, ..., \omega_{11,i}$ are fixed parameters for the variable i and $S_{j,t}$, $j = 1, ..., 11$, denotes the jth seasonal dummy variable.

In order to have an identifiable model (Harvey, 1989) we fixed $\sigma_\eta^2 =$

1. The other parameters correspond to the weights γ_i, the coefficients of the operators $\phi(B)$ and $D_i(B)$, and the variances σ_i^2. Altogether, we have $(14+k)n+p+1$ parameters to be considered. For example, with $k=2$, $n=9$ and $p=12$ we obtain 157 unknown parameters.

The relative large number of hyperparameters and the use of the variables in levels cause convergence problems in the numerical algorithms for maximizing the likelihood function and the estimates are very sensible to initial values. This can be observed by simple simulations of the model proposed in (1)-(3). In order to obtain convergence and robustness to initial values, we propose to transform the original model by taking first differences to each member of the equations (1) and (3). For equation (1) we obtain

$$\Delta X_{it} = \sum_{j=1}^{11} \omega_{i,j} \Delta S_{j,t} + \gamma_i \Delta C_t + \Delta u_{it} \ , i=1,...,n, \tag{4}$$

and for equation (3),

$$\mathrm{D(B)} \Delta \mathsf{u}_t \ = \ \Delta \epsilon_t \ . \tag{5}$$

The likelihood function of the model (1)-(3) is equivalent to the one of the transformed model since the proposed transformation is linear and its Jacobian is equal to 1. Consequently, the information that contains the original data about the generating probabilistic model, in particular about the parameters, is equivalent to the one of the differentiated data. The new state space model based on equations (2), (4) and (5) is built up in Appendix 1.

Once the hyperparameters have been estimated, the new estimated state vector $\alpha_{t|t}$ is calculated using the Kalman filter and finally, $C_{t|t}$ is obtained from this estimation. We must observe that $C_t = C_0 + \sum_{j=0}^{t-1} \Delta C_{t-j}$ and that ΔC_t is the first component of the new state vector α_t. Taking $C_{0|t} = 100$ for all t we obtain

$$C_{t|t} = C_{0|t} + \sum_{j=0}^{t-1} \Delta C_{t-j|t}$$

where $\Delta C_{t-j|t} = E(\Delta C_{t-j} | \mathsf{X}_1, ..., \mathsf{X}_t)$, $j = 0, ..., t-1$, is calculated using the fixed interval smoother (Harvey, 1989) for sample sizes varying with t.

In order to be consistent with the identification restriction of the model, that is $\sigma_\eta^2 = 1$, the series $\Delta X's$ are standardized. This is obtained with the transformation

$$x_{it} = \Delta X_{it}/s_i$$

for each $i = 1, ..., n$, where

$$s_i^2 = (1/N) \sum_{t=1}^{N} (\Delta X_{it} - \bar{x}_i)^2$$

and

$$\bar{x}_i = [1/(N-12)] \sum_{t=13}^{N} \Delta X_{it} \ .$$

The motivation behind this transformation is given by the fact that

$$\sum_{t=13}^{N} \Delta X_{it}/(N-12) \ = \ \gamma_i \sum_{t=13}^{N} \Delta C_t/(N-12) + \sum_{t=13}^{N} \Delta u_{it}/(N-12) \rightarrow \mathrm{E}(\Delta C_t)$$

when $N \rightarrow \infty$. Then, we can interpret s_i^2 as a dispersion measure of the data ΔX_{it} around the constant $\mathrm{E}(\Delta C_t) = \delta/(1-\phi(1))$. It is important to note that the process $\{\Delta X_{it}\}$ is not mean-stationary since $\mathrm{E}(\Delta X_{it}) = \beta_{it}$ changes through time, although it is stationary in variance because $\mathrm{Var}(\Delta X_{it}) = \gamma_i^2 \sigma_{\Delta C}^2 + \sigma_{\Delta u_\mathrm{i}}^2$ is a constant.

An interesting interpretation of the parameters γ_i, $i = 1, ..., n$ given in equation (4) is based on the result

$$\mathrm{Corr}(\Delta X_{it}, \Delta C_t) = \sigma_{\Delta C} [\gamma_i / (\gamma_i^2 \sigma_{\Delta C}^2 + \sigma_{\Delta u}^2)^{1/2}]$$

where $\sigma^2_{\Delta C} = \mathrm{Var}(\Delta C_t)$ and $\sigma^2_{\Delta u} = \mathrm{Var}(\Delta u_{it})$. In terms of the differentiated variables, this implies that (1) if $|\gamma_i| \to 0$ then $\mathrm{Corr}(\Delta X_{it}, \Delta C_t) \to 0$ (there is almost no linear association), (2) if $|\gamma_i| \to 1$ then $\mathrm{Corr}(\Delta X_{it}, \Delta C_t) \to \pm\sigma_{\Delta C}/(\sigma^2_{\Delta C} + \sigma^2_{\Delta u})^{1/2}$ (an intermediate case) and (3) if $|\gamma_i| \to \infty$ then $\mathrm{Corr}(\Delta X_{it}, \Delta C_t) \to 1$ (the ideal case). On the other hand, we can see that if γ_i and $\sigma^2_{\Delta C}$ are fixed and $\sigma^2_{\Delta u}$ is very large, $\mathrm{Corr}(\Delta X_{it}, \Delta C_t) \approx 0$. This might be an indication that in $\{\Delta X_{it}\}$ there is no relevant information about $\{\Delta C_t\}$, in terms of linear association.

3 Diagnostics

Since our basic model is in state space form, we can use the standard procedures for validation of this kind of models(Harvey, 1989). This consists on using a Portmanteau test to examine the orthogonality of the one step ahead prediction errors, previously standardized. Additionally, plots of the cumulative sum (CUSUM) and the cumulative sum of squares (CUSUMSQ) are useful to detect structural changes or heteroskedastic behavior of the marginal prediction errors.

Since we might have different groups of coincident variables and different models for the same group of series, following Kitagawa (1987), we propose to use the Akaike information criterion (AIC) as an instrument of selection.

4 An empirical application

In order to illustrate the theoretical results of Section 2, we calculated a coincident index for the Colombian economic activity in the monthly sample period 1980:01-2001:02. The following variables were selected using economic and statistical criteria[2]: current economic conditions (Fp1), number of orders (Fp6)[3], production of cement (Prcem), industrial production index excluding coffee threshing (Ipr), index of employment for unskilled workers (Iemob), currency in circulation in real terms (Efecr), demand of energy and gas (Energa), total imports excluding capital and durable goods (Imp) and loan portfolio of the financial system (Cart). The result of the statistical tests indicated that these series are integrated of order one and that they are cointegrated.

Following Altissimo et al.(2000), we used the index of industrial production as a proxy of the economic activity (C_t) for identifying the autoregressive order p. The initial values for the autoregressive parameters ϕ_i and the intercept δ were taken from the estimation of an ARIMA(13,1,0) model for this variable. The autoregressive order $p = 13$ was obtained using standard methods of time series analysis. Given the results of the subsection (2.2), the initial values of the parameters γ were taken all equal to 1.0. Since we do not have reasonable proxies for the processes $\{u_{it}\}$, we tried several values for the autoregressive order k. Taken into account the large number of

[2]The statistical criteria includes unit root and cointegration tests and analysis of cross-correlation functions between each series and the index of the industrial production.

[3]The variables Fp1 and Fp6 are obtained from the opinion business survey by Fedesarrollo.

hyperparameters and the complexity of the estimation routines, these were $k = 0, 1, 2, 3, 4, 5$. The initial values for the autoregressive parameters d_{ij} were set all equal to 0.1. The identification of k was based on the Akaike information criterion (AIC).

The maximum likelihood estimation of the hyperparameters of the model (1)-(3) is based on the model specified in subsection (2.2). We used the optimization algorithm of Broyden, Fletcher, Goldfarb and Shanno (BFGS) as the numerical method, where the likelihood function is calculated through its decomposition in terms of the one-step prediction errors (Harvey, 1989). These are the results (standard errors in parenthesis):

Equation (1)[4]:

$$\text{Fp1}_t = \widehat{\beta}_1 + \underset{(0.023)}{0.139}\ \widehat{C}_t + \widehat{u}_{1t}$$

$$\text{Fp6}_t = \widehat{\beta}_2 + \underset{(0.022)}{0.106}\ \widehat{C}_t + \widehat{u}_{2t}$$

$$\text{Prcem}_t = \widehat{\beta}_3 + \underset{(0.006)}{0.049}\ \widehat{C}_t + \widehat{u}_{3t}$$

$$\text{Ipr}_t = \widehat{\beta}_4 + \underset{(0.005)}{0.043}\ \widehat{C}_t + \widehat{u}_{4t}$$

$$\text{Iemob}_t = \widehat{\beta}_5 + \underset{(0.014)}{0.071}\ \widehat{C}_t + \widehat{u}_{5t}$$

$$\text{Efecr}_t = \widehat{\beta}_6 + 0.022\ \widehat{C}_t + \widehat{u}_{6t}$$

[4]The results of the unit root and cointegration tests and the estimation of the coefficients $\beta's$, associated with the deterministic part of the equation (1), are not presented in order to mantain simplicity in the results. However, these estimations are available upon request.

$$(0.004)$$

$$\text{Energa}_t = \widehat{\beta}_7 + 0.023\ \widehat{C}_t + \widehat{u}_{7t}$$
$$(0.016)$$

$$\text{Imp}_t = \widehat{\beta}_8 + 0.051\ \widehat{C}_t + \widehat{u}_{8t}$$
$$(0.012)$$

$$\text{Cart}_t = \widehat{\beta}_9 + 0.064\ \widehat{C}_t + \widehat{u}_{9t}$$
$$(0.022)$$

Equation (2):

$$\begin{aligned}
\Delta\widehat{C}_t = &\underset{(0.078)}{0.098} + \underset{(0.258)}{1.581}\ \Delta\widehat{C}_{t-1} - \underset{(0.587)}{0.890}\ \Delta\widehat{C}_{t-2} - \underset{(0.731)}{0.085}\ \Delta\widehat{C}_{t-3} + \underset{(0.638)}{0.378}\ \Delta\widehat{C}_{t-4} \\
&\underset{(0.416)}{-0.065}\ \Delta\widehat{C}_{t-5} + \underset{(0.300)}{0.134}\ \Delta\widehat{C}_{t-6} - \underset{(0.318)}{0.494}\ \Delta\widehat{C}_{t-7} + \underset{(0.290)}{0.568}\ \Delta\widehat{C}_{t-8} \\
&\underset{(0.293)}{-0.292}\ \Delta\widehat{C}_{t-9} + \underset{(0.327)}{0.116}\ \Delta\widehat{C}_{t-10} - \underset{(0.342)}{0.136}\ \Delta\widehat{C}_{t-11} + \underset{(0.234)}{0.279}\ \Delta\widehat{C}_{t-12} \\
&\underset{(0.086)}{-0.182}\ \Delta\widehat{C}_{t-13}
\end{aligned}$$

Equation (3):

$$\widehat{u}_{1t} = \underset{(0.006)}{0.996}\ \widehat{u}_{1t-1}\ ; \qquad \widehat{\sigma}_1^2 = 0.588$$

$$\widehat{u}_{2t} = \underset{(0.007)}{0.992}\ \widehat{u}_{2t-1}\ ; \qquad \widehat{\sigma}_2^2 = 0.594$$

$$\widehat{u}_{3t} = \underset{(0.050)}{0.639}\ \widehat{u}_{3t-1}\ ; \qquad \widehat{\sigma}_3^2 = 0.622$$

$$\widehat{u}_{4t} = \underset{(0.077)}{-0.169}\ \widehat{u}_{4t-1}\ ; \qquad \widehat{\sigma}_4^2 = 0.101$$

$$\widehat{u}_{5t} = 0.997\ \widehat{u}_{5t-1}\ ; \qquad \widehat{\sigma}_5^2 = 0.236$$
$$(0.003)$$
$$\widehat{u}_{6t} = 0.873\ \widehat{u}_{6t-1}\ ; \qquad \widehat{\sigma}_6^2 = 0.128$$
$$(0.037)$$
$$\widehat{u}_{7t} = 0.996\ \widehat{u}_{7t-1}\ ; \qquad \widehat{\sigma}_7^2 = 0.427$$
$$(0.004)$$
$$\widehat{u}_{8t} = 0.904\ \widehat{u}_{8t-1}\ ; \qquad \widehat{\sigma}_8^2 = 0.981$$
$$(0.028)$$
$$\widehat{u}_{9t} = 0.993\ \widehat{u}_{9t-1}\ ; \qquad \widehat{\sigma}_9^2 = 0.641$$
$$(0.002)$$

It is important to note that according to subsection (2.2), the estimates of the parameters γ_i are adjusted by the "standard deviation" s_i.This means that these values must be multiplied by s_i to obtain estimations in terms of the original variables. This also applies for the interpretation of the variances $\widehat{\sigma}_i^2$.

The CUSUM and CUSUMSQ statistics for the marginal prediction errors are plotted in Appendix 4. These graphics show no evidence of miss-specification of the model. Additionally, the plots of the weight sequences for the variables that determine the coincident index $\widehat{\Delta C}_{t|t}$ are presented in Appendix 5. Here, the hat symbol denotes that, in practice, we need to use the estimated hyperparameters for obtaining the index growth $\Delta C_{t|t}$ (and in turn $C_{t|t}$). Computation of these sequences is accomplished in accordance with the results in subsection (2.1), but using the model specified in subsection (2.2) and the interpretation of the weights magnitude is scale free because of the previous standardization of the time series. The results indicate that the contributions of all the coincident variables included in the model have a very similar pattern, where the effects are positive and large in the first lags. The most important contributions to the growth of the coincident index are given by the industrial production index excluding coffee threshing (Ipr), the index of employment for unskilled workers (Iemob), the current economic conditions (Fp1), the number of orders (Fp6) and the currency in circulation in real terms (Efecr), respectively.

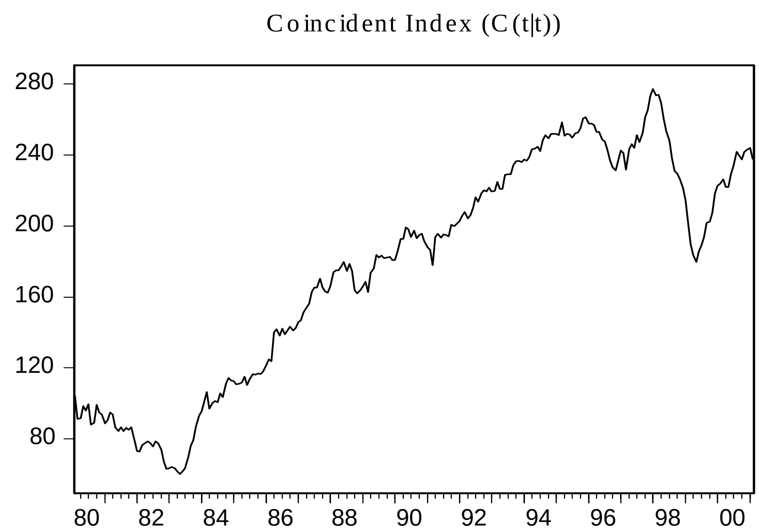


Figure 4

The coincident index $\widehat{C}_{t|t}$ is plotted in Figure 4. The dynamics of the estimated index agrees with the stylized facts of the Colombian economy. For example, the contractions of the index in the 1983 and 1989-1991 periods are also found in the works of Melo et al. (1988) and Ripoll et al. (1995). The slowdown of the economic activity in 1996 is also observed in several economic series including the industrial production index . Finally, the major contraction of the index in the observed sample is presented in the 1998-1999 period.

5 Conclusions and Recommendations

In this work we have developed a new methodology for estimating a coincident index of the aggregate economic activity. The proposed methodology follows the work of Stock and Watson (1989, 1991) including the following modifications: (1) the statistical model requires that the coincident variables are cointegrated, (2) in contrast to the SW model, our proposed state space model has the desirable steady-state property, which permits useful and formal interpretations of the model and the results based on it, (3) since we include seasonal effects in the model we do not need to adjust the observed variables by seasonality prior to be included in the model, (4) a practical strategy is developed for estimating the unknown parameters and providing the necessary initial values for the estimation stage.

We must note that the estimation algorithm for the hyperparameters tends to produce very persistent autoregressive processes for the intrinsic components of the observed variables. This relative problem will be investigated in the future.

APPENDIX 1

In order to put equations (1)-(3) in a state space form, let $c_{t+j|t} = E(C_{t+j}|C_0, C_1, ..., C_t)$; $j = 1, ..., p$; $C_0 = 100$; $\bar{c}_{t|t-1} = C_{t|t-1} - \delta$;

$$\alpha_t = (C_t, \bar{c}_{t+1|t}, C_{t+2|t}, ..., C_{t+p|t}, \mathbf{u}_t', \mathbf{u}_{t-1}', ..., \mathbf{u}_{t-k+1}')' \ ,$$

$$T = \left(\begin{array}{cccc|cccc}
0 & & I & & & & & \\
 & & & & & 0 & & \\
\phi^*_{p+1} & \phi^*_p & \cdots & \phi^*_1 & & & & \\
\hline
 & & & & D_1 & \cdots & D_{k-1} & D_k \\
 & 0 & & & & & & \\
 & & & & & I & & 0
\end{array}\right);$$

where ϕ^*_i denotes the ith coefficient of the polynomial $\phi^*(B) = \phi(B)\Delta = \phi(B)(1-B)$;

$$G = \begin{pmatrix} 1 \\ \psi^*_1 \\ \vdots \\ \psi^*_p \end{pmatrix} ,$$

with ψ^*_j the jth coefficient of the infinite polynomial

$$\psi^*(B) = 1 + \psi^*_1 B + \psi^*_2 B^2 + \cdots$$

such that $\phi^*(B)\psi^*(B) = 1$;

$$R = \begin{pmatrix} G & 0 \\ 0 & I_n \\ 0 & 0 \end{pmatrix} ,$$

$$\mathsf{Z} = \left(\begin{array}{c|c|c|c|c|c} \boldsymbol{\gamma} & \mathsf{0} & I & \mathsf{0} & \cdots & \mathsf{0} \end{array} \right) .$$

and $\boldsymbol{\zeta}_t = (\eta_t, \boldsymbol{\epsilon}_t')'$.

Then, the state space model is specified by the following two equations:

$$\alpha_t = \boldsymbol{\mu}\delta + T\alpha_{t-1} + R\boldsymbol{\zeta}_{\mathsf{t}}$$

as the system equation and

$$\mathsf{X}_t = \boldsymbol{\beta}_t + Z\alpha_t$$

as the observation equation, where additionally

$$\boldsymbol{\mu} = (1, -1, 0, \cdots, 0, \phi_p^*, \mathsf{0}', \cdots, \mathsf{0}')'$$

if $p > 1$ and $\boldsymbol{\mu} = (1, \phi_1^* - 1, \mathsf{0}')'$ if $p = 1$.

We can establish the following reasonable initial conditions for this state space model: as initial state vector

$$\alpha_0 = (100, 100 - \delta_0, 100, \cdots, 100, \mathsf{0}', \cdots, \mathsf{0}')'$$

and as initial variance-covariance matrix

$$P_0 = \kappa I \ ,$$

with κ sufficiently large.

The state space form for the differenced data is obtained by redefining the state vector α_t as follows:

$$\alpha_t = (\Delta C_t, \Delta C_{t-1}, ..., \Delta C_{t-p+1}, \Delta \mathsf{u}_t', \Delta \mathsf{u}_{t+1|t}', ..., \Delta \mathsf{u}_{t+r-1|t}')'$$

with $\Delta \mathsf{u}_{t+j|t} = E(\Delta \mathsf{u}_{t+j} | \mathsf{u}_1, ..., \mathsf{u}_t)$; $j = 1, ..., r$ and $r = \max\{2, k\}$. The new matrix and vectors of the system are given by

$$T = \begin{pmatrix} \phi_1 & \cdots & \phi_{p-1} & & \phi_p & | & & & & \\ & & & & & | & & \mathsf{0} & & \\ & I & & | & \mathsf{0} & | & & & & \\ - & - & - & - & - & - & - & - & - & - \\ & & & & & | & \mathsf{0} & & I & \\ & & \mathsf{0} & & & | & & & & \\ & & & & & | & D_r & D_{r-1} & \cdots & D_1 \end{pmatrix};$$

$$R = \begin{pmatrix} 1 & | & \\ & | & \mathsf{0} \\ \mathsf{0} & | & \\ - & - & - \\ & | & I \\ \mathsf{0} & | & \Psi_1 \\ & | & \vdots \\ & | & \Psi_{r-1} \end{pmatrix},$$

where the matrices Ψ_j are the coefficients of the infinite polynomial matrix $\Psi(B) = I + \Psi_1 B + \Psi_2 B^2 + \cdots$ such that $D(B)\Psi(B) = I - IB$; $\mathsf{Z} = (\boldsymbol{\gamma}, \mathsf{0}, I, \mathsf{0})$ and $\boldsymbol{\mu} = (1, \mathsf{0}^{\prime})^{\prime}$. In fact, this state space model also satisfies the steady-state property. The initial conditions are similar to those of the original model in levels.

APPENDIX 2

In order to prove that the state space model for the data in levels developed in Appendix 1 possess the steady-state property, we shall prove first that the model is controllable and observable. We use the same notation of subsection (2.1).

Proposition 1. The state space model for the data in levels is controllable.

Proof. Initially, we can observe that the transition matrix T is block diagonal, that is,

$$T = \begin{pmatrix} A & \mathbf{0} \\ \mathbf{0} & B \end{pmatrix} ,$$

where

$$A = \begin{pmatrix} \mathbf{0} & | & I_p & & \\ & & & & \\ \phi^*_{p+1} & & \phi^*_p & \cdots & \phi^*_1 \end{pmatrix}$$

and

$$B = \begin{pmatrix} D_1 & \cdots & D_{k-1} & D_k \\ & & & \\ & I_{n(k-1)} & & \mathbf{0} \end{pmatrix}$$

with I_m denoting, in general, the identity matrix of order m. We must note that matrix A has dimension $(p+1) \times (p+1)$, the size of B is $nk \times nk$, the zero matrix in the position $(1, 2)$ of T is $(p+1) \times nk$ and the one in $(2, 1)$ is the transpose of the previous matrix. Since T is a block diagonal matrix, it is not difficult to prove that

$$T^m = \begin{pmatrix} A^m & \mathbf{0} \\ \mathbf{0} & B^m \end{pmatrix} ,$$

for all $m = 1, 2, ...$

In order to verify the controllability of the model, that is, that the block matrix $[R, TR, ..., T^{p+nk}R]$ is of full row rank, we need to find the general form of B^m. Beginning with

$$B^2 = \begin{pmatrix} E_{11}^{(2)} & E_{12}^{(2)} & \cdots & E_{1,k-2}^{(2)} & E_{1,k-1}^{(2)} & E_{1k}^{(2)} \\ D_1 & D_2 & \cdots & D_{k-2} & D_{k-1} & D_k \\ & & & & & \\ & & I_{n(k-2)} & & \mathsf{0}_{n(k-2)\times 2n} & \end{pmatrix},$$

where

$$E_{1,j}^{(m)} = \begin{cases} D_1 D_j + D_{j+1} \ ; & j = 1, ..., k-1 \\ D_1 D_k \ , & j = k \end{cases},$$

and proceeding in a recurrent way from $m = 3$, we find that for $m = k$,

$$B^m = \begin{pmatrix} E_{11}^{(m)} & E_{12}^{(m)} & \cdots & E_{1(k-1)}^{(m)} & E_{1k}^{(m)} \\ & & \vdots & & \\ E_{(k-1)1}^{(m)} & E_{(k-1)2}^{(m)} & \cdots & E_{(k-1)(k-1)}^{(m)} & E_{(k-1)k}^{(m)} \\ D_1 & D_2 & \cdots & D_{k-1} & D_k \end{pmatrix}$$

where, for each $l = 1, ..., k-2$,

$$E_{l,j}^{(m)} = \begin{cases} E_{l,1}^{(m-1)} D_j + E_{l,j+1}^{(m-1)} \ ; & j = 1, ..., k-1 \\ E_{l,1}^{(m-1)} D_k \ , & j = k \end{cases}$$

and

$$E_{k-1,j}^{(m)} = \begin{cases} D_1 D_j + D_{j+1} \ ; & j = 1, ..., k-1 \\ D_1 D_k \ , & j = k \end{cases}.$$

Now, for $j = 1, ..., k-2$,

$$T^j R = \begin{pmatrix} A^j \mathsf{G} & \mathsf{0} & \mathsf{0} \\ \mathsf{0} & E_{11}^{(j)} & \mathsf{0} \\ & \vdots & \\ \mathsf{0} & E_{j-1,1}^{(j)} & \mathsf{0} \\ \mathsf{0} & D_1 & \mathsf{0} \\ \mathsf{0} & I_n & \mathsf{0} \\ \mathsf{0} & \mathsf{0} & \mathsf{0} \\ & \vdots & \\ \mathsf{0} & \mathsf{0} & \mathsf{0} \end{pmatrix} ,$$

for $j = k-1$ the last block of the second column in the previous matrix is I_n and for $j = k$ this last block is D_1.

Considering all the matrix $[R, TR, ..., T^k R]$ we get the following possibilities. If $D_1 = \mathsf{0}$ and $D_2 \neq \mathsf{0}$, we have already full row-rank. If $D_1 = D_2 = \mathsf{0}$ and $D_3 \neq \mathsf{0}$, we aggregate the matrix $T^{k+1}R$ and we obtain full row rank. In the extreme case $D_1 = \cdots = D_{k-1} = \mathsf{0}$ and $D_k \neq \mathsf{0}$ we aggregate $T^{2k-1}R$ and the full row-rank condition is obtained by means of these aggregations because of the following reasons.

Let $E_{1j}^{(1)} = D_j$, $j = 1, ..., k$. Using a recursive procedure from $m = 1$ and the previous expression for $E_{lj}^{(m)}$, which is also valid for all $l = 1, ..., m$ with $2 \leq m \leq k$, we obtain that

$$E_{11}^{(m)} = f(D_1, ..., D_{m-1}) + D_m ,$$

where $f(D_1, ..., D_{m-1})$ is a polynomial of order m in the matrices D_1,...,D_{m-1}, such that each term is a product that involves at least one of these matrix.

As an illustration we have that

$$E_{11}^{(3)} = D_1^3 + 2D_1D_2 + D_3$$

and

$$E_{11}^{(4)} = D_1^4 + 3D_1^2D_2 + 2D_1D_3 + +D_2^2 + D_4 \ .$$

Consequently, in the extreme case $D_1 = ...D_{k-1} = \mathbf{0}$, we obtain that $E_{11}^{(k)} = D_k$, a matrix that is supposed to have non-zero components on its diagonal.

For the model to be controllable it is required that $2k - 1 \leq p + nk$ or, equivalently, $0 \leq (n-2)k + p + 1$. Then, we have to analyze the following cases: (1) if $k = 0$, i.e. the processes $\{u_{it}\}$ are white noise, the previous condition is satisfied for all n and all p. (2) If $k > 0$, $0 \leq (n-2)k + p + 1$ if and only if $[-(p+1)/k] + 2 \leq n$. In this situation if $n = 1$ and $k > p+1$, the condition is not satisfied, but if $n \geq 2$, then for all k and for all p we always obtain $[-(p+1)/k] + 2 \leq n$. In practice, $n \geq 2$, but even if this is not the case, k and p can be restricted in such a way that $k < p+1$. The restriction is plausible since is not reasonable to have an autoregressive order for $\{u_t\}$ larger that the one of $\{C_t\}$.

Proposition 2. The state space model for the data in levels is observable.

Proof. Using a recurrent procedure for the powers of T' ("'" denotes transpose) we find that

$$[Z', T'Z', ..., (T')^p Z'] = \begin{bmatrix} \gamma' & \mathsf{0}' & \mathsf{0}' & & \mathsf{0}' \\ \mathsf{0}' & \gamma' & \mathsf{0}' & & \mathsf{0}' \\ \mathsf{0}' & \mathsf{0}' & \gamma' & & \mathsf{0}' \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \mathsf{0}' & \mathsf{0}' & \mathsf{0}' & & \gamma' \\ - & - & - & - & - \\ I_n & D_1 & E_{11}^{(2)\prime} & & E_{11}^{(p)\prime} \\ \mathsf{0} & D_2 & E_{12}^{(2)\prime} & & E_{12}^{(p)\prime} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \mathsf{0} & D_{k-1} & E_{1k}^{(2)\prime} & & E_{1k}^{(p)\prime} \end{bmatrix},$$

where $\mathsf{0}'$ denotes a row vector of zeros.

Then, the row rank of this matrix is less or equal to $p + 1 + nk$. Using the structure of the matrix $E_{1j}^{(m)}$; $j = 1, ..., k$; $2 \leq m \leq k$ and that $(p + 1 + nk) - 1 \geq p$ for all $n \geq 1$ and $p, k \geq 0$, we conclude that the row rank of $[Z', T'Z', ..., (T')^{p+nk} Z']$ is $p + 1 + nk$, the dimension of the state vector.

Proposition 3. The state space model for the original data has the steady-state property.

Proof. Since the model is controllable and observable, consequently detectable and estabilizable, respectively, and the matrix $P_{1|0} = TP_0T' + R\Sigma R'$ is positive semidefinite, it satisfies the steady-state property (Harvey, 1989, pp. 119); that is, the sequence of minimum-mean-square-error matrices in the Kalman filter, converges to a fixed matrix.

APPENDIX 3

Simulation of C_t using Stock and Watson's model

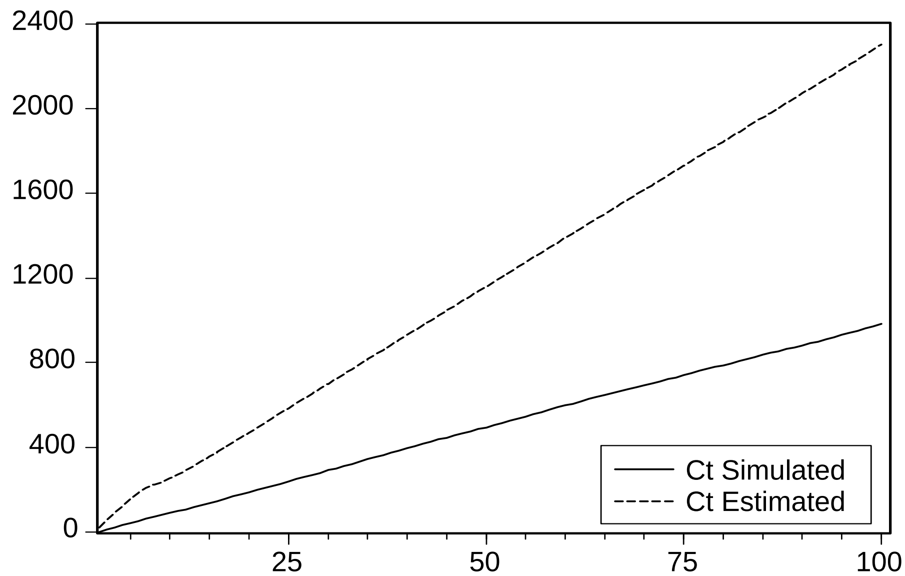


Figure 1: Ct

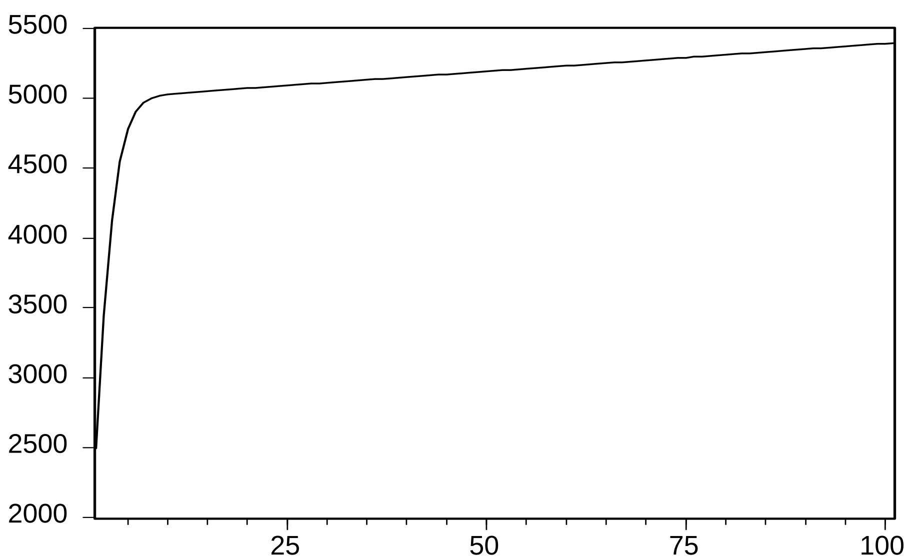


Figure 2: MSE of Ct

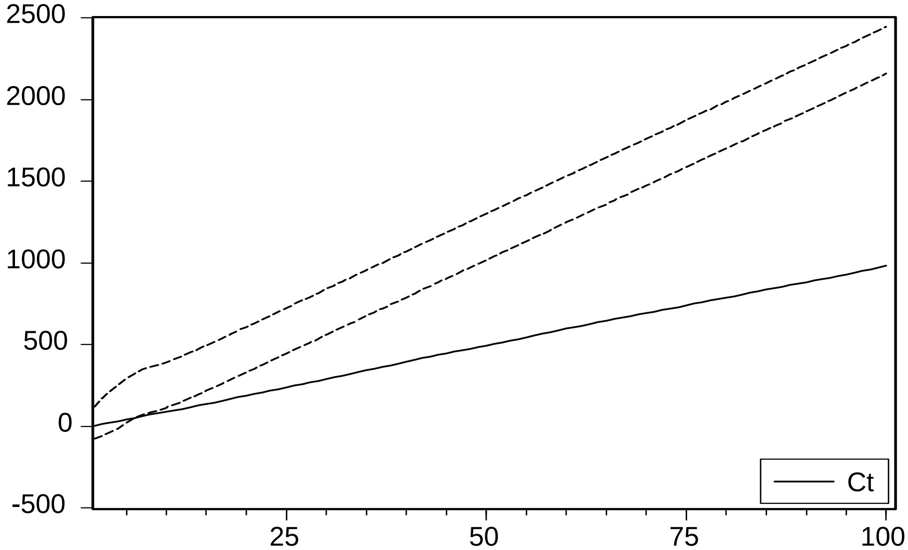
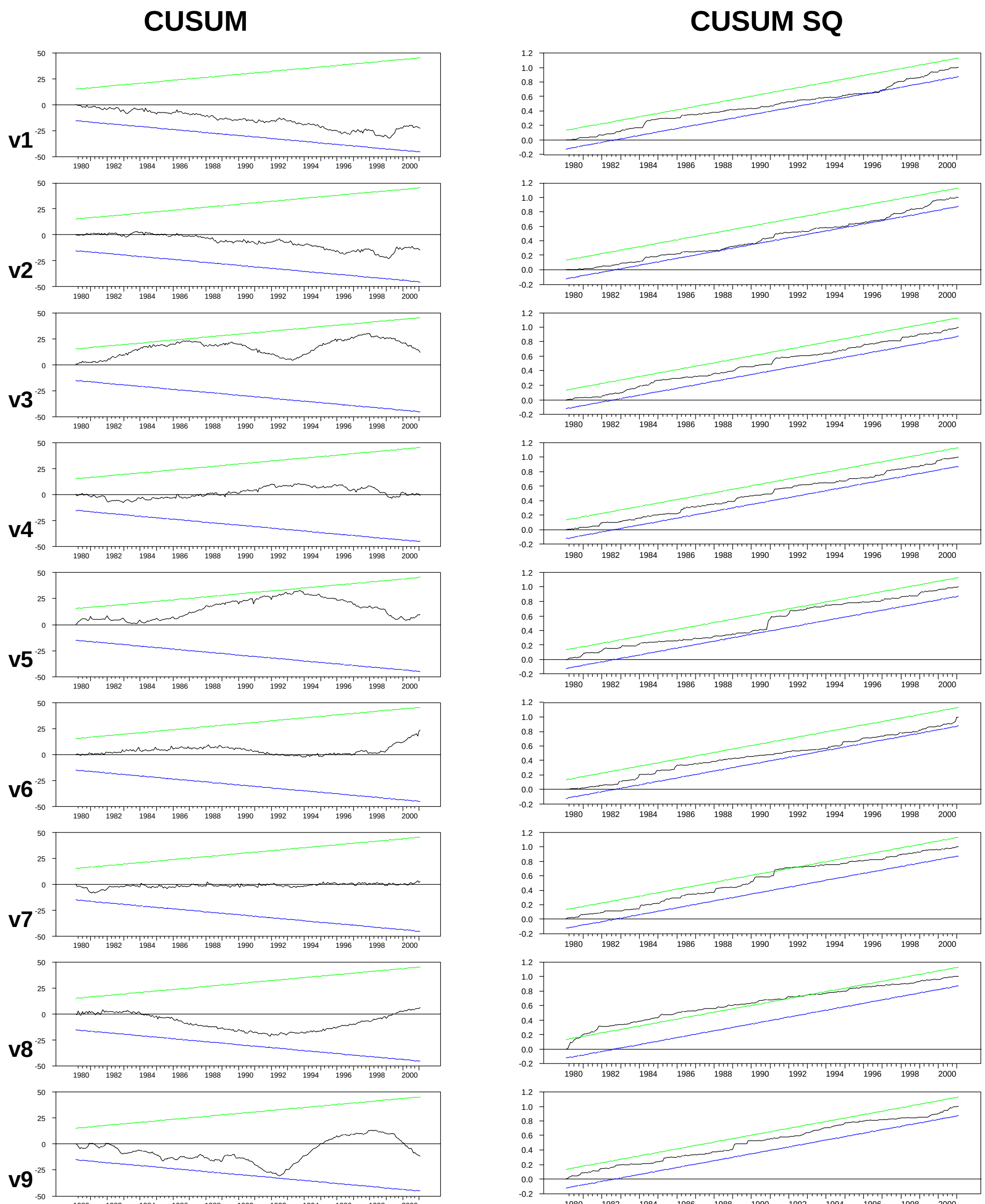


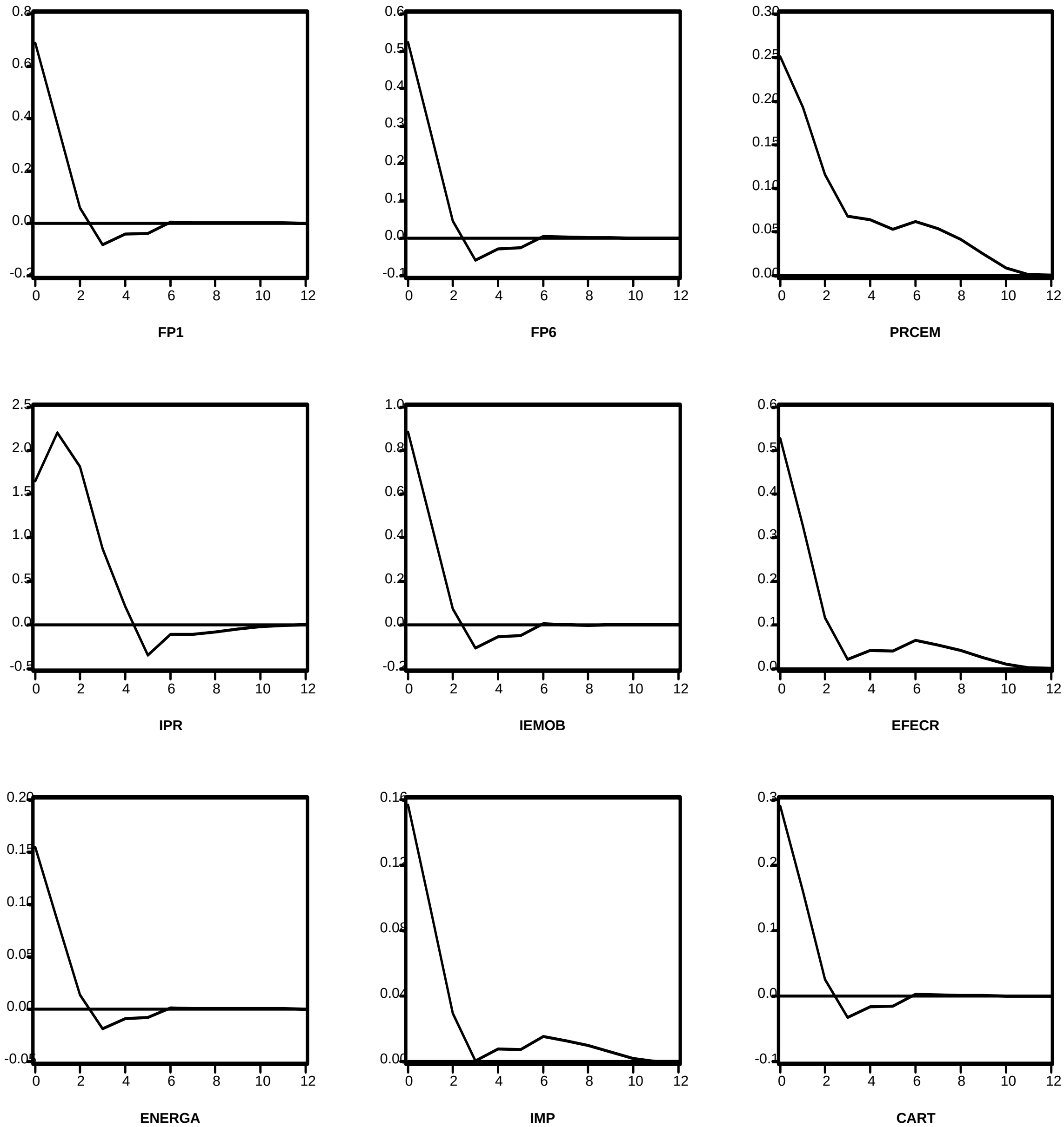
Figure 3: 95% Prediction Band for Ct

APPENDIX 4



APPENDIX 5

Lagged coefficients involved in the coincident index



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