Sovereign Risk and Real Business Cycles in a Small Open Economy

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Abstract

This paper investigates the effect of sovereign risk on the stochastic rational expectations equilibrium of a real business cycle small open economy. The credit market is imperfect because the sovereign cannot commit to repay its outstanding debt and chooses to default when it is optimal to do so. The possibility of default induces an endogenous sovereign risk premium on foreign debt and an endogenous rationing limit set by foreign creditors. The model is parameterized and solved numerically to explore the determinants of the savings and investment decisions in an economy that can optimally choose to default on its foreign debt and the ability to account for the Sudden Stop of capital inflows.

1 Introduction

The financial crises in emerging markets during the 90's has revealed some empirical regularities in their business cycles. Periods of financial distress in emerging economies are characterized by large current account reversals

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and sudden stops in capital inflows (Calvo, 1998 [3]), soaring sovereign country risk reflected in hikes in the international interest rates faced by these economies and large output contractions (Oviedo, 2001 [13] and Neumeyer and Perri, 2001 [12]), accompanied by collapses in equity prices (Mendoza and Smith 2001, [10]) and prices of nontradable goods relative to tradable goods (Mendoza, 2000 [9]). In several cases, the magnitude of the crisis led countries to default on their outstanding debt (Argentina 2001, Russia 1998, Ecuador 1999 and Indonesia 1998).

In a recent study, Reinhart (2002, [14]) has highlighted the statistical significance of the interaction between default and emerging markets crises. The behavior of the sovereign country risk, reflected on the interest rate that an economy faces in the international credit markets appears to be closely related to the sharp movements in the current account, the collapse in private consumption and the currency crisis. The study finds that 84% of the defaults occurred in emerging markets are associated with currency collapses and about 50% of the currency crises are linked to defaults. This result seems to be a particular characteristic of emerging markets.[1]

Another interesting finding is that sovereign credit ratings (used as an indicator of the likelihood of default) not only have significant impact on sovereign bond yield spreads, but also serve as good predictors of the occurrence of defaults (Larraín et al., 1997 [8]). In consequence, it is not surprising that during periods of financial distress lower ratings are observed and countries face more difficulties borrowing from international credit markets.

The nature of the dynamics of the crises in emerging markets and the empirical findings described above challenges the study of the business cycles in small open economies. The smooth movements in the current account and the level of foreign debt, as well as the neutrality of the business cycle to the external interest rates shocks predicted by conventional models of business cycles in a small open economy[2] are inconsistent with the dynamics of the emerging markets crises and the sudden stops of capital inflows. One

[1] The study finds that there is no statistical significance of the relationship between default and currency crises for developed economies.

[2] See Mendoza (1991, [11]) and Correia et al. (1995, [4]).

important reason of this inconsistency with the facts is the role assigned to international creditors. Since international credit markets are assumed to be perfect, a small open economy can borrow funds at a fixed risk-free rate up to a point limited only by the extent of their wealth.

In a recent survey, Arellano and Mendoza (2002, [2]) point out that the common starting point of much of the literature on emerging markets crises has been to introduce some type of financial-market imperfection that distinguishes emerging economies from industrial countries. However, the majority of studies focuses on partial equilibrium models that qualitatively predict results consistent with the dynamics of the crises. Little is known about the quantitative predictions, not only of these type of models but also of the equilibrium models of business cycles of small open economies.

In this paper, the imperfection arises from the inability of sovereign debtors to commit to repay their their outstanding debt. The small open economy sovereign representative agent can borrow funds at an increasing interest rate up to a point limited by the perception of lenders of the sovereign's willingness to repay the debt. The interest rate that the small open economy faces and the availability of funds are the result of its interaction with foreign lenders. The aim is to explore quantitatively the determinants of sovereign country risk and its role on the dynamics of emerging markets crises in the context of an equilibrium model of business cycles of a small open economy. As the results in Reinhart (2002, [14]) suggest sovereign risk may play an important role on the occurrence of emerging markets crises.

This paper borrows the main elements from the studies of real business cycles in small open economies, as Mendoza (1991), and of sovereign debt, as Eaton and Gersovitz (1981, [5]) and extends the previous work of Hamann (2001, [6]) to allow domestic savings. The aim is to explore two aspects: first, to quantify how different properties of the economy affect the sovereign country risk and the access to international credit. Second, to assess the consistency of the model with the business cycle dynamics of a financial crisis. The rest of the paper proceeds as follows. Section 2 presents the structure of the model and characterizes the equilibrium. Section 3 describes briefly the algorithm for the solution of the model. Section 4 reports the results of different experiments designed to study the determinants of the equilibrium probability of default, the endogenous rationing limit and the average time that it takes for a country to default. Section 5 concludes.

2 The Model

There are two types of agents in the model. A small open economy social planner (country or sovereign hereafter) borrows funds from foreign risk-neutral competitive lenders. The sovereign can default on its debt but at the cost of permanent exclusion from future borrowing. Therefore the borrower's default decision is the result of optimally balancing the cost of exclusion, given by the forgone benefits of consumption smoothing, against the direct costs of repayment, given by the short-run dis-utility of repaying the loan. Lenders are risk-neutral. They are able to assess the probability of default of the country and will restrict the amount of funds available for lending to limit optimally their degree of exposure to default risk. In this fashion, an endogenous borrowing constraint arises.

2.1 The Small Open Economy

2.1.1 Production Technology

Time is discrete, and each period the economy produces an internationally tradable good using the following technology:

$$F(e_t, k_t) = \exp(e_t) k_t^{\alpha}$$

where k_t is the capital stock at the beginning of period t, e_t is the level of productivity at the beginning of period t, and $0 < \alpha < 1$. The productivity shock follows an exogenous stochastic process to be described later. The capital stock evolves according to:

$$k_{t+1} = (1 - \delta) k_t + i_t$$

where i_t is gross investment and $0 \leq \delta \leq 1$ is a constant rate of depreciation.

2.1.2 Preferences and Resource Constraint

There is a representative agent with preferences defined over consumption as:

$$E\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right]$$

where $\beta \in (0,1)$ and $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$. The agent can smooth consumption by holding foreign loans, a_t. It is assumed that the economy can access the international credit market in one-period consumption loans, and is allowed to borrow from a set of loans A, to be determined by international lenders. The determination of this set will be presented later. Typically, the loans level must remain below a borrowing limit $a \leq \underline{a}$. Negative loan levels, $a < 0$, indicate that the country is lending (at the default-risk free rate) while positive loan levels, $a > 0$, indicate that the country is borrowing. Foreign financial loans a_t are charged at the real interest rate r. This rate is to be determined in equilibrium by the interaction of the sovereign and the international lenders. The law of motion of the foreign loan holdings is:

$$a_{t+1} = tb_t - [1 + \exp(n_t)r]a_t$$

where tb_t is the trade balance in period t and n_t is a random disturbance affecting the interest rate.[3]

In this setting, the aggregate resource constraint of the economy states that total absorption and the trade balance cannot exceed the gross domestic product:

$$c_t + i_t + tb_t \leq \exp(e_t)k_t^{\alpha}$$

where c_t is consumption and $0 < \alpha < 1$.

The country can choose to repay or default on its outstanding debt. If the country decides to repay, then the resources constraint is given by:

$$c_t + i_t + tb_t \leq \exp(e_t)k_t^{\alpha} \tag{1}$$

[3] It is implicitly assumed that the type of contracts written are not state contingent. Markets are assumed incomplete and the type of contract assumed here is a standard debt contract.

If the country decides to default, it is permanently excluded from the international credit market and has to consume and invest up to the domestic production:

$$c_t + i_t \leq \exp(e_t) k_t^{\alpha}$$

So, the set of defaulting states is a closed class. Once the country decides to default it cannot return to the foreign asset market and it stays in autarky thereafter. In this setting, the problem of the sovereign can be described recursively as follows. At any point in time t the country's state position is described by a state vector $s_t \in S$, where $s_t = (z_t, k_t, a_t, d_t)$. Let $z_t = (e_t, n_t)$ denote the shocks process with an exogenous transition probability $\pi(z_{t+1}|z_t)$.

The state of the economy at time t, s_t, indicates the exogenous realization of the productivity and interest rate shocks $z_t = (e_t, n_t)$, the country's net foreign asset holdings a_t, the stock of capital k_t and the default state d_t (whether the country is currently in default or not). To simplify the notation, the time subscript is removed and next period's variables are denoted with a prime. The value of following the optimal decisions at any time t is given by:

$$v(s) = \max \left\{ \max_{(c,a',k') \in \Gamma(s)} \{u(c) + \beta E[v(s)|s]\}, w(s) \right\} \tag{2}$$

where s'denotes the next's period's state, the set $\Gamma(s)$ is:

$$\Gamma(s) = \left\{(c, a', k') : c \leq \exp(e)k^{\alpha} - [1 + r(s)]a + a' + (1 - \delta)k - k'; c \geq 0; a' \leq \underline{a}; \right\}$$

and $w(s)$ is the value of switching to autarky:

$$w(s) = \max_{(c,k') \in \Omega(s)} \left\{u(c) + \beta E[w(s')|s]\right\} \tag{3}$$

where:

$$\Omega(s) = \left\{(c, k') : c \leq \exp(e)k^{\alpha} + (1 - \delta)k - k'; c \geq 0\right\}$$

Equation (2) indicates that every period t the country chooses the maximum between the expected discounted value of repaying the loan and the expected discounted value of switching to autarky, given by equation (3). If the former is greater or equal than the latter then the country decides to repay and will have access to the international credit markets for one additional period, by choosing the next period's level of assets, a'. Otherwise the country will default and switch to autarky. If v is the optimal value function of the sovereign's problem, then we can find the optimal decision rules for consumption, loans, capital stock and repayment. [4] These optimal policy rules guarantee that the country will maximize its lifetime expected utility. In the next section the behavior of the international lenders is described.

2.2 International Lenders

The international lenders are assumed risk neutral. They borrow funds from an independent market at the risk free rate, $\widetilde{r}$. They know that the sovereign defaults at t if and only if $w(s) > v(s)$ therefore the probability λ of default at t anticipated at $t-1$ is given by $\lambda(s) = \Pr\left[w(s) > v(s)\right]$. The higher the level of debt the higher the probability of default. Since banks are risk neutral they will lend as long as they are paid a return that is at least as high as the risk free rate:

$$\left[1 - \lambda\left(s\right)\right]\left(1 + r(s)\right) \geq \left(1 + \exp(n)\widetilde{r}\right) \tag{4}$$

where n is the stochastic shock to the default risk free international interest rate. Perfect competition between lenders imply that in equilibrium equation (4) will hold with equality.

To determine the borrowing limit that lenders impose on the sovereign, it is assumed that they will allow the sovereign to borrow up to the annualized value of the lowest possible realization of the stochastic endowment (in every future period) discounted at the interest rate associated with that income stream. Aiyagari (1994,[1]) has shown that as consumption goes to zero its marginal utility goes to infinity, then an agent will never hold an amount of assets that may induce non-positive consumption. Thus a borrowing limit

[4] Implicitly, it is assumed that a bounded measurable solution v to functional equation (2) exists and it is optimal.

is implied by this fact. In his model the interest rate is a constant risk free rate, so the value of the borrowing constraint is the annuity of the worst possible income realization discounted at the risk free rate.

Here the interest rate is stochastic and depends on the probability of default, which in turn depends on the preferences and the stochastic properties of the small open economy. As Zhang (1994,[15]) has pointed out, a natural generalization of Aiyagari's setting is to allow the agent to borrow up to the present value of the worst possible income stream, discounted at the interest rate associated with this income stream, conditional on the constraint binding. More formally, a given probability of default, λ, implies an interest rate r, so the maximum amount of funds available for borrowing is the value of $\underline{a}$ such that

$$\underline{a} = \frac{\min_{S|\underline{k},\underline{a}} F(e, \underline{\underline{k}})}{\max_{S|\underline{k},\underline{a}}(r(s))} \tag{5}$$

where $S|\underline{k}, \underline{a}$ denotes the state space conditioned on the lowest value that the capital stock can take, $\underline{k}$, and the borrowing constraint binding.

Intuitively, the lenders think about the worst case scenario for the repayment of their loans: every period the agent borrow up to the limit and receives the lowest possible endowment. If lenders allow the agent to borrow more than this amount, there will be a positive probability that the present value of the worst income stream is less than the amount of debt, violating the non-negativity of consumption.

The computation of this borrowing limit is as follows. Start with an arbitrary borrowing limit and find the optimal interest rate function, r. The new borrowing limit is the right hand side of (5). Update the borrowing constraint $\underline{a}$ and find a new optimal interest rate function, r. The computation of the optimal interest rate function is described later. In the next section the equilibrium of the model is characterized.

2.3 Equilibrium

The equilibrium concept used is that one of a stochastic, stationary rational expectations equilibrium represented by a time invariant probability of default function, λ, a time invariant interest rate function, r, a time invariant optimal set of loans, A, and a time invariant optimal probability distribution over the state space of the small open economy, ψ. The stationary equilibrium is defined recursively. Given the transition rule $a(s)$, and the transition probabilities of the exogenous process $\pi(z'|z)$, the optimal transition probability $P(s)$ can be computed. $P(s)$ is the probability that a country in state s will reach a state vector that lies in S in the next period.[5] More formally, the stationary stochastic rational expectations equilibrium of the model is characterized by:

1. an optimal interest rate function r and optimal default function λ
2. an optimal set of loans A
3. an optimal foreign asset policy $a'(s)$ and capital accumulation policy $k'(s)$
4. a stationary probability measure $\psi(S) = \int_S P(s)d\psi$

such that:

1. The sovereign solves problem (2)
2. Expected profits for international lenders are zero, i.e. (4) holds with equality
3. The sovereign demand for loans is in the available set of loans, $a'(s) \in A$

The first condition states that the sovereign optimize to find a set of decision rules for consumption, debt allocation and default that depend on the current state of the economy. The second condition states that the profits for international lenders are zero, because the credit market is competitive. The third condition is a market clearing condition.

[5]The optimal transition probability $P(s)$ is very useful to compute the probability of default. Since there are transient and absorbent states, the probability of default is given by the probability of reaching the set of absorbent states.

3 The Solution Technique

The characterization of the equilibrium of the model presented above does not yield a closed form solution. However, the recursive nature of the model allows the computation of the equilibrium and the study of its long-run characteristics using numerical methods. The technique approximates the solution to the functional equation (2) for a discrete version of the state space.[6]

3.1 Discretization of the State-Space

The first step is to discretize the shocks. The stochastic structure of the problem is simplified by assuming that the exogenous disturbances are Markov Chains.[7] Productivity shocks and interest rate shocks assume two possible discrete realizations each, so:

$$z_t \in Z = \{(e^1, n^1), (e^1, n^2), (e^2, n^1), (e^2, n^2)\}$$

The probability of the current state z_t^i moving to state $z_{t+1}^j$is denoted π_{ij} for $i = 1, .., 4$ and $j = 1, ..., 4$. The transition probabilities are given by the "simple persistence" rule:

$$\pi_{ij} = (1 - \theta)\Pi_j + \theta p_{ij}$$

where θ is a parameter governing the persistence of e and n, Π_j is the long-run probability of state z^j and $p_{ij} = 1$ if $i = j$ and zero otherwise.

An additional simplification is to assume that:

$$\Pi(e^h, n^l) = \begin{cases} \Pi \text{ if } h = l \\ 0.5 - \Pi \text{ if } h \neq l \end{cases}$$

[6]The computational method used for the solution of the model is similar to that of Huggett (1993, [7]) and Aiyagari (1994, [1]). The main differences are that here there is an additional discrete state and and action (default-repay) and the market clearing condition.

[7]This simplification follows Mendoza (1991).

and $e^1 = -e^2 = e$, $n^1 = -n^2 = n$. This simplifications imply that the statistical moments of the shocks are $\sigma_e = e$, $\sigma_n = n$, $\rho_{en} = 4\Pi - 1$ and $\rho_n = \rho_e = \theta$.

The second step is to discretize the state space. An evenly spaced grid is chosen to approximate the state space. So, $n_1$values of the capital stock are chosen so that the values that k can take lie in $K = \{k_1, ..., k_{n_1}\}$. Similarly, n_2 values for the loans are chosen so that a lies in $A = \{a_1, ..., a_{n_2}\}$. The economy can be in default or in repayment so that the decision to default, d, lies in $D = \{0, 1\}$. Under these conditions the discretized state space is $S = Z \times A \times K \times D$ and its dimension is $4 \times n_1 \times n_2 \times 2 = n$. Now the implementation of the algorithm to find the solution of the model is described.

3.2 The Algorithm

To simplify notation, let $x = (a', k', d') = (a(s), k(s), d(s))$ be a vector of the optimal decision policies and so we can write them as $x = g(s)$. The solution method is based on the following strategy:

1. Start with an arbitrary probability of default function $\lambda^{(i)}$ and borrowing limit $\underline{a}^{(i)}$, where i denotes the step in the procedure.

2. Given $\lambda^{(i)}$ compute the interest rate $r^{(i)}$ using equation (4) and compute the optimal policy function $g(s)$ and the optimal transition probability matrix $P(s)$ using the contraction equation (2).

3. Update $\lambda^{(i)}$ to $\lambda^{(i+1)}$ using the optimal transition probability matrix $P(s)$ and update the borrowing limit to $\underline{a}^{(i+1)}$ using equation (5).

4. Check whether the zero profit condition is approximately satisfied. If not, use $\lambda^{(i+1)}$ and $\underline{a}^{(i+1)}$ and repeat steps 2 to 4.

5. After convergence of the algorithm, given the optimal policy $g(s)$ and the optimal transition probability matrix $P(s)$ iterate on $\psi^{(i+1)} = \int_S P(s) d\psi^{(i)}$ from an arbitrary initial distribution $\psi^{(0)}$ to obtain the stationary probability distribution.

The first step of the algorithm is solved by policy function iteration on equation (2), for a given $r^{(i)}$ function.

The second step requires to obtain $P(s)$. Notice that the optimal policy $g(s)$ and the Markov chain π on y induce a Markov chain on s, $P(s)$ via the formula:

$$\Pr[s_{t+1} = s' | s_t = s] = \iota(x, s)\pi(z, z') \tag{6}$$

where $\iota(x, s) = 1$ if $x = g(s)$ and 0 otherwise. This indicator function identifies the time t states s that are sent into x at time $t + 1$. Equation (6) defines an $n \times n$ matrix P where n is the number of total possible states. The matrix P is used to compute the ergodic distribution, ψ and the probability of default.

Step 4 updates r , using the zero profit condition of international lenders. If $\left\| \lambda^{(i+1)} - \lambda^{(i)} \right\| < \sqrt{\varepsilon}$ then stop. Otherwise continue iterating on the previous steps. The last step is the standard computation of the ergodic probability distribution.

The computation of the probability of default in the third step deserves special attention. This is the focus of the next section.

3.3 The probability of default

Typically, when there is no default option, all the states of the Markov chain associated with P will be recurrent and its stationary distribution, ψ, can be interpreted as the fraction of time that the country spends in each state. When the possibility of default is introduced, the states of the system are divided in two classes: repayment states and defaulting states. The repayment states are transient and the defaulting states are absorbing by the definition of the problem: once the economy decides optimally to default it cannot return to the international credit markets.

More formally, the optimal transition probability matrix induces a finite Markov chain on the state space S such that it has a single closed class C of states which communicate with one another (the set of defaulting states). The transient states, $T = S - C$, form one or more sets such that members of a set communicate with one another and lead to the set C.

Under these conditions the optimal transition probability matrix P has the following structure:

$$P = \begin{bmatrix} Q & \Lambda_1 \\ 0 & P_1 \end{bmatrix}$$

where P_1 be the transition probability sub-matrix of P of transitions among the members of the set closed class C, Q is a matrix that yield the probability of moving within a set of transient states, T, and Λ_1 is a matrix whose elements express the probability of moving from a repayment state to a defaulting state in the next period. Note that the matrix 0 indicates that once the decision of default has been taken there is no possibility to reach a repayment state in the next period. The optimal transition probability matrix induces an optimal Markov chain $\{X_t, t \geq 0\}$ of the variables in the model.

In period t it is the case that

$$P^t = \begin{bmatrix} Q^t & \Lambda_t \\ 0 & P_1^t \end{bmatrix}$$

where $\Lambda_t = \Lambda_{t-1}P_1 + Q^{t-1}\Lambda_1$, $Q^t$is the $t-$ th power of the matrix Q and $P_1^t$is the $t-$ th power of the matrix P_1. By writing $\Lambda_1 = \Lambda$, the optimal probability of default in period $t+1$ at a given time t can be computed as:

$$\Lambda_{t+1} = \sum_{\tau=0}^{t} Q^{\tau}\Lambda U^{t-\tau} = \sum_{\tau=0}^{t} Q^{t-\tau}\Lambda U^{\tau} \tag{7}$$

The element $\lambda(i,j)$ of the matrix Λ, yields the probability of reaching state $j \in C$ in the period $t+1$ from state $i \in T$ in period t.

The probability of *ever* defaulting can also be computed. Note that the states in C can be lumped together so that the matrix P can be rewritten as:

$$\widehat{P} = \begin{bmatrix} Q & \widehat{\Lambda} \\ 0 & I \end{bmatrix}$$

similarly

$$\widehat{P}^t = \left[\begin{array}{cc} Q^t & \widehat{\Lambda}_t \\ 0 & I \end{array} \right]$$

Let θ_{ik} be the probability that the economy starting in a transient state i eventually defaults and gets absorbed in a state k. The absorption probability matrix will be:

$$\Theta = (\theta_{ik})\,, i \in T, k \in C$$

which can be computed as:

$$\Theta = (I - Q)^{-1}\widehat{\Lambda}$$

3.4 Time until Default

Another indicator of the riskiness of the debtor is the time it would take for the economy to default. Let τ_i be the number of periods, including the starting position at i, in which the economy remains repaying its debts. That is, $\tau_i$is the number steps that a chain starting with $i \in T$ stays in T before entering an absorbing state. The random variable τ_i is denoted as the *time until default* for the economy starting at state i. It is interesting to find the distribution of τ_i.

Start by noting that the events $\{\tau_i > t\}$and $\{X_t \in T\}$are equivalent. So, the event:

$$\{\tau_i > t-1\} = \{\tau_i > t\} \cup \{\tau_i = t\}\,, t \geq 1$$

so that the distribution is given by

$$\Pr\{\tau_i = t\} = \Pr\{\tau_i \geq t-1\} - \Pr\{\tau_i \geq t\} = \Pr\{\tau_i \in T\} - \Pr\{\tau_i \in T\}$$

so that

$$\Pr\{\tau_i = t\} = \sum_{j \in T} \left\{ p_{ij}^{(t-1)} - p_{ij}^{(t)} \right\}$$

Now for $i, j \in T$ the matrix

$$(p_{ij}^{(k)}) = Q^k$$

so that the column vector

$$(\ldots, \sum_{j \in T} p_{ij}^{(k)}, \ldots)_{i \in T} = Q^k \mathbf{e}$$

where $\mathbf{e}$ is a column vector with all elements equal to one. Denote the mean time up to absorption starting at state i by μ_i, so

$$\mu_i = E(\tau_i) = \sum_t t \Pr\{\tau_i = t\}$$

and let

$$\tau(t) = (\ldots, \Pr\{\tau_i = t\}, \ldots)_{i \in T}$$

be the vector whose elements (probability mass functions) correspond to the transient states and

$$\mu(t) = (\ldots, \mu_i, \ldots)_{i \in T}$$

be the vector whose elements (expectations) correspond to the transient states.

Then the vector of time up to absorption can be computed as

$$\tau(t) = (\ldots, \sum_{j \in T} \left\{ p_{ij}^{(t-1)} - p_{ij}^{(t)} \right\}, \ldots)_{i \in T} = \left(Q^{t-1} - Q^t \right) \mathbf{e} = Q^{t-1} (I - Q) \mathbf{e} \text{ for } t \geq 1$$

and $\tau(0) = \mathbf{0}$. Furthermore, the mean time to absorption can be easily computed as

$$\mu = \sum_{t=0}^{\infty} t\tau(t) = \sum_{t=1}^{\infty} tQ^{t-1}(I - Q)\mathbf{e} = (I - Q)^{-2}(I - Q)\mathbf{e} = (I - Q)^{-1}\mathbf{e}$$

The mean time to absorption provides the expected number of periods that the economy will stay in repayment up to the time of default. In this problem, this time will depend not only on the initial state, but also on the parameters of the model and the nature of the stochastic shocks. The next section proceeds to evaluate the determinants of the probability of default and the mean time to default.

4 Parameterization and Results

This section presents the functional forms and the parameters used in the solution of the model. The set of parameters that must be chosen are $\{\beta, \gamma, \delta, \alpha, \widetilde{r}, \sigma_e, \sigma_n, \rho, \rho_{en}\}$. The parameterization of the model follows Mendoza (1991,[11]). The robustness of the solution is tested by varying the values of these parameters. At the same time this exercise permits the exploration of the determinants of the probability of default, the expected time to default and the borrowing limit. The functional form used to represent the preferences is a standard CES utility function, $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ where γ is the coefficient of relative risk aversion.

As it is standard in the literature of RBC for small open economies γ is assumed to be between 1 and 2. For the same reason the depreciation rate, δ, is set at 0.1. The discount factor, β, is set between 0.94 and 0.96. These relatively small discount factors reflect the high degree of impatience that characterizes emerging market economies. The value of the default-risk-free international interest rate, $\widetilde{r}$, is set at 0.04 as in Mendoza (1991,[11]), which is also the value suggested by Kydland and Prescott (1982) and Prescott (1986) for the real interest rate in the US economy. In the production side, the value of the share of income paid to capital, α, is set between 0.3 and 0.4. The values of the parameters of the stochastic shocks, $\{\sigma_e, \sigma_n, \rho, \rho_{en}\}$ are taken from Mendoza (1991) and are $\{0.0118, 0.0118, 0.3, 0.3\}$ respectively. These values are changed to test the sensitivity of the solution of the model to their values.

The solution is found by discretization of the state space by using an equally spaced grid of 1800 points: 15 points in the direction of capital, 15 points in the direction of the loans, 4 points for the exogenous shocks and 2 points for the default-repay state. The grid is refined successively to improve the quality of the approximation.

4.1 Properties of the Solution

Figure 1 shows the value functions under autarky and repayment in the (a, k) space for a given level of the exogenous shocks, z. The values of the parameters are $\beta = 0.95$, $\gamma = 2$, $\alpha = 0.32$, $\delta = 0.1$, $r = 0.04$, $\sigma_e = \sigma_n = 0.0118$ and $\rho = \rho_{en} = 0.3$. This is the benchmark parameterization. As expected, the value function under repayment is strictly concave in both directions, increasing in the capital stock and decreasing in the loans level. The value of continued access to the international credit market is lower when the economy is highly indebted and the capital stock is low. It is optimal to default when the value function under autarky exceeds the value function under repayment. This happens when the debt level is too high.

Figures 2 and 3 show the policy rules for benchmark model of the loan levels, $a(s)$, and the capital stock, $k(s)$, in the (a, k) space at a given level of the exogenous shocks, z. The indebtedness policy, $a(s)$, is increasing in a and decreasing in k. For lower capital levels the economy is essentially borrowing constrained. Note the ceiling in the policy function, $a(s) < \underline{a}$. In the benchmark parameterization $\underline{a} = 1.98$. The economy is also borrowing constrained if the loan levels are too high. For higher capital levels and low debt levels the indebtedness policy exhibits the standard behavior for unconstrained small open economies.

The capital accumulation policy rule, $k(s)$, is increasing in k and decreasing in a. In the borrowing constrained portion of the state space, $k(s)$ varies significantly as the level of debt changes. In the unconstrained portion of the state space the economy smooths consumption by borrowing and lending, while chooses to maintain the capital stock relatively constant. The exhibited shape of the policy functions illustrates the typical behavior of a small open economy subject to borrowing constraints: in good times capital is relatively stable and the consumption smoothing is done by borrowing and lending. In bad times, access to the international credit markets is closed and the adjustment is done by varying the capital stock.

Figures 4 and 5 show the equilibrium probability of default and the expected time to default in number of years in 2 dimensions. The plots deserve some explanation. The horizontal access shows the position of the state in the grid. The grid is ordered as follows z, a, k, d. The exogenous shocks z change more rapidly while the default-repay state, d, changes more slowly. That is, the first 4 points of the grid are the values of z, for given values of the rest of the state variables. The next four points are again the values of z for the given next higher value of a, given the same previous values of k and d. The enumeration continues until the values of a are exhausted. The next step is to increase the value of k until its values are exhausted, and so on.

This layout of the grid produces the shapes in Figures 4 and 5. The graphs are apparently divided in sub-intervals. From the left to the right each sub-interval corresponds to a higher level of capital, given the decision to repay (That is why the graph only have 900 points, instead of 1800). Within the sub-intervals, the level of debt is increasing and the level of the shocks is varying.

For a given point in the grid, i say, Figure 4 shows the probability that an economy would ever default having started in the position of the grid i. If the economy starts with low values of capital the probability that the economy will eventually default is very high. While as capital increases this probability falls. Similarly, if the economy starts with high levels of debt the probability of ever defaulting is very high, but can also be low if the debt level is lower.

More interesting are the results obtained for the expected time to default (or mean time to default, MTTD). For a given point in the grid, i, Figure 5 shows the mean time that an economy takes to default having started in state i. An economy that starts with low levels of capital stock and a high level of indebtedness is expected to default in the shortest period of time, a year. If the economy starts in a better position it could take up to 5 years, in the parameterization of the benchmark model.

In the next section, the sensitivity of the mean time to default (MTTD hereafter) to the characteristics of the economy is evaluated. This exercise permits to determine the relative importance of the different parameters in the model for the determination of the likelihood of default of a given economy.

4.2 Determinants of Default

Before analyzing the sensitivity of the probability of default, the MTTD and the borrowing limit to changes in the parameters of the model it is important to see the impact of them in the value function of default and repayment. Ultimately, the fundamental determinant of the likelihood of default are these value functions. Figures 6 and 7 show the reaction of the value functions of the sovereign to changes in the degree of risk aversion and the discount factor. A higher degree of risk aversion and a lower discount factor reduce the value of having access to the international credit market. However, it also reduces the value of switching to autarky. The relative change in these two forces will determine whether it will be more or less attractive to default. Notice how the impact on the value functions of the value of the time preference parameter is higher than the impact of the risk aversion parameter.

The numerical sensitivity analysis performed shows that it is easier to visualize the impact of the changes in the parameters of the model in the MTTD than in the probability of default. The results are consistent in either two cases. Figure 8shows the impact of increased risk aversion of the MTTD. As γ increases from 1 (solid blue line) to 2 (red dotted line) the impact on the MTTD is negligible for lower values of capital. For higher values, a higher risk aversion reduces the probability of default and can delay the default decision for up to two years. Note that even in the case of a high level of capital, when the level of indebtedness is high, a higher risk aversion reduces the probability of default and does not postpone the expected occurrence of default. A sovereign with a low degree of risk aversion and a relatively high capital to debt ratio doesn't care much about smoothing consumption in the future. Therefore not paying back the debt becomes relatively attractive and so the probability of default is higher, the borrowing limit is tighter and the MTTD is lower. On the contrary a highly risk averse sovereign values more the possibility of consumption smoothing and its probability of default will be lower, unless he holds a large amount of debt relative to capital.

Figure 9 illustrates the effect of a higher time preference parameter.

When β increases from 0.94 (blue solid line) to 0.96 (red dotted line) the effect is very similar to the previous one. The effect depends on the value of debt relative to the capital stock. For higher capital stock levels, a higher time preference parameter reduces the probability of default and increase the MTTD for up to two years. For lower values the effect is negligible. The intuition is also similar to the previous case. As the value of the discount factor increases the sovereign places a higher net value on future consumption smoothing and risk sharing. The value of having access to the international credit markets is increased and so with a lower probability of default the MTTD is delayed and the borrowing limit is relaxed.

Figures 10 to 12 show the impact of the stochastic properties of the shocks on the MTTD. Figure 10 shows the impact of increasing the persistence of the productivity and international real interest rate shocks from a low value to a high value. The lower line (red dotted line) shows the difference between the MTTD that corresponds to a $\rho = 0.3$ and the MTTD that corresponds to a $\rho = 0.9$. Again, this impact depends on the debt to capital ratio. A lower persistence of the shocks reduces the probability of default and delays the expected time to default for up to a year (in some cases it is just a matter of months). So, the magnitude of the impact is lower when compared to the impact of the preference parameters.

The next exercise increases the volatility of the productivity shocks from $\sigma_e = 0.0118$ to $\sigma_e = 0.059$ (i.e. five times). To have an idea of the magnitude of the productivity shocks, recall that $\sigma_e = e$ and the productivity shock assume values between $\exp(e)$ and $\exp(-e)$. The size of a positive shock when $\sigma_e = 0.0118$ is $\exp(e) = 1.0119$ and $\exp(-e) = 0.9883$, while $\exp(e) = 1.0608$ and $\exp(-e) = 0.9427$ when $\sigma_e = 0.059$. In summary, we are considering an increase in the volatility of the productivity shocks of 5 percent points. Figure 11 shows that the impact of increased productivity shocks depends on whether the shock is positive or negative. If the economy starts in a position in which the shock is more favorable, the expected time to default is longer. On the contrary, the MTTD is shortened if the economy starts in a position of more negative shocks.

The final experiment considers an increment in the volatility of the external default-risk-free interest rate. The initial $\sigma_n = 0.0118$, which corresponds to external real interest rates between 2.78% and 5.23%. A threefold increment in the volatility is studied. This corresponds to rates between 0.38% and 7.75%. Despite the fact that the volatility is increased substantially, the results in Figure 12 show that the impact on the MTTD are negligible (of the order of 10^{-5}).[8] This is surprisingly low, even despite the fact that previous models of small open economies with exogenous probabilities of default or endogenous discount factors exhibit this type of neutrality to the external interest rate shocks.

5 Concluding Remarks

This paper builds a model of Real Business Cycle in a Small Open Economy with an endogenous probability of default to understand the effect of sovereign risk on the stochastic rational expectations equilibrium of the model. Following the previous work of Mendoza (1991), the domestic economy is modeled as a RBC model of a small open economy. The international credit market is imperfect because the sovereign cannot commit to repay its outstanding debt and chooses to default when it is optimal to do so, as in Eaton and Gersovitz (1981). The possibility of default induces an endogenous sovereign risk premium on foreign debt and an endogenous rationing limit set by foreign creditors. The sources of the shocks in the model are domestic productivity shocks and external default-risk-free real interest rate shocks. The model is parameterized and solved numerically to explore the determinants of the savings and investment decisions in an economy that can optimally choose to default on its foreign debt and the ability to account for the Sudden Stops of capital inflows.

The results show that the equilibrium probability of default depends not only on the debt to capital ratio, but also on the parameters of the model. Several experiments conducted here show that the importance of the parameters of the model in determining the probability of default and

[8]The same numerical results show also that the impact on the probability of default and the borrowing limit are also negligible (of the order of 10^{-15}).

the dynamics of the model are very important. In particular, the impact on the probability of default, the borrowing limit and the expected time to default of the preference parameters is stronger than the impact of changes in the stochastic properties of the exogenous shocks. Reducing the relative risk aversion or increasing the time preference parameter of the sovereign can lead to postponing the expected occurrence of default for up to 2 years, depending on the initial debt to capital ratio of the economy. Reducing the persistence of the productivity and external interest rate shocks delays the occurrence of default for up to a year. A similar result is obtained in the case of increasing the volatility of the productivity shocks in five percent points. The only difference is the dependence on the positive or negative realization of the initial shock. Finally, a very interesting result is the lack of responsiveness of the model to shocks in the international default-risk-free rate. Enhancing the volatility of external interest rate shocks in two percent points has no impact on the probability of default and the expected time to default. Despite the fact that the risk premium is endogenous, the model still exhibits the type of neutrality that characterizes the family of models of small open economies with exogenous probability of default or endogenous discount factor. The main impact on the sovereign risk premium appears to be coming from the preferences of the sovereign and to a lower degree on the persistence of the productivity and external interest rate shocks and on the volatility of the domestic productivity shocks.

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Figure 1: Value Function under Default and Repayment

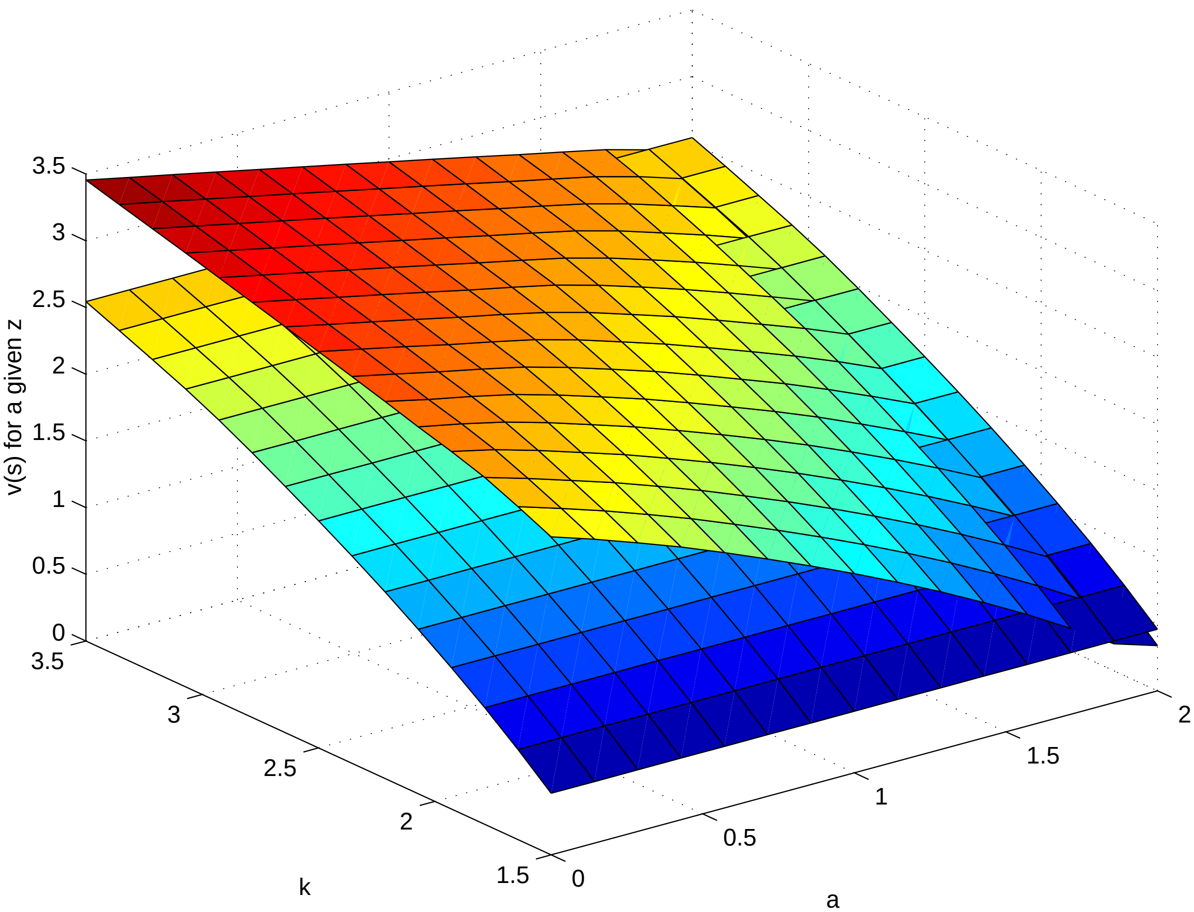

Figure 2: Policy Function for Loans

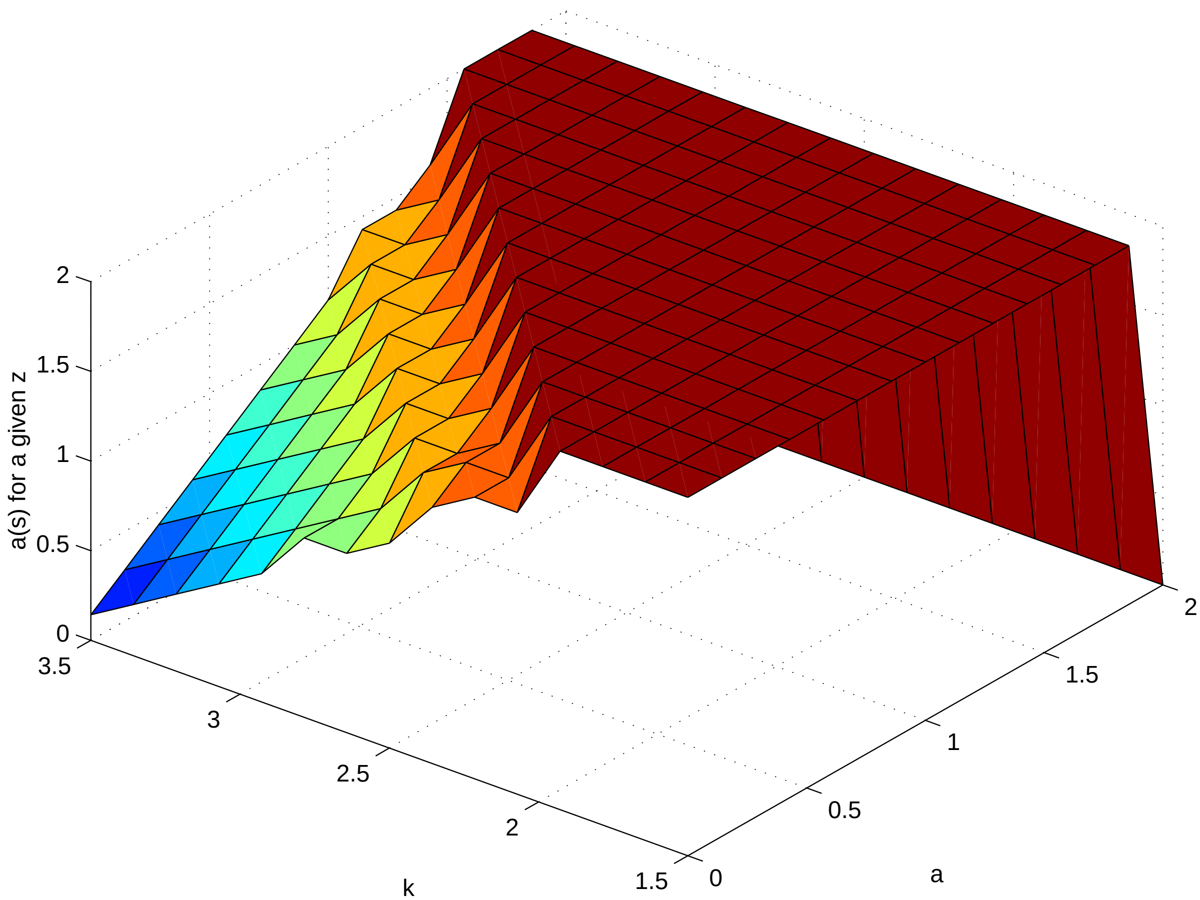

Figure 3: Policy Function for Capital Stock

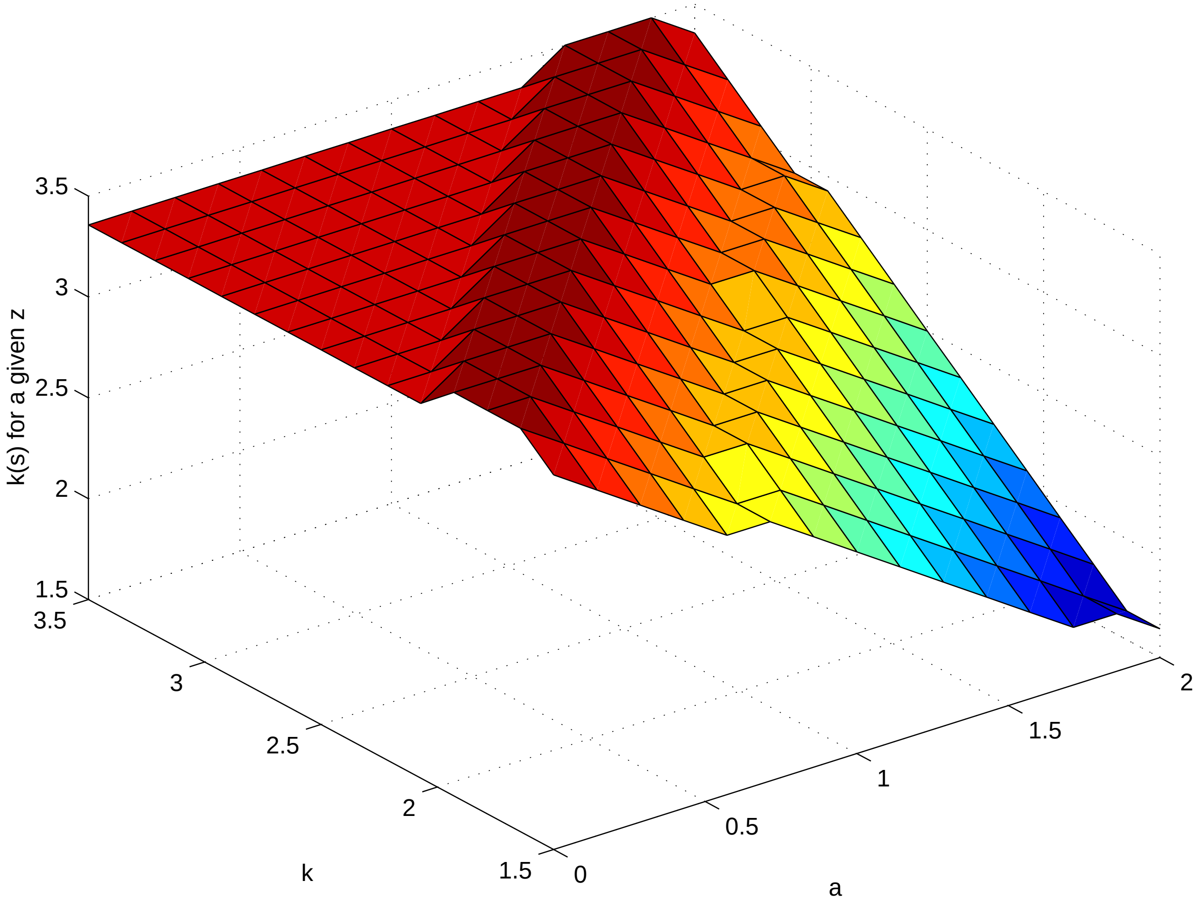

Figure 4: Equilibrium Probability of Default

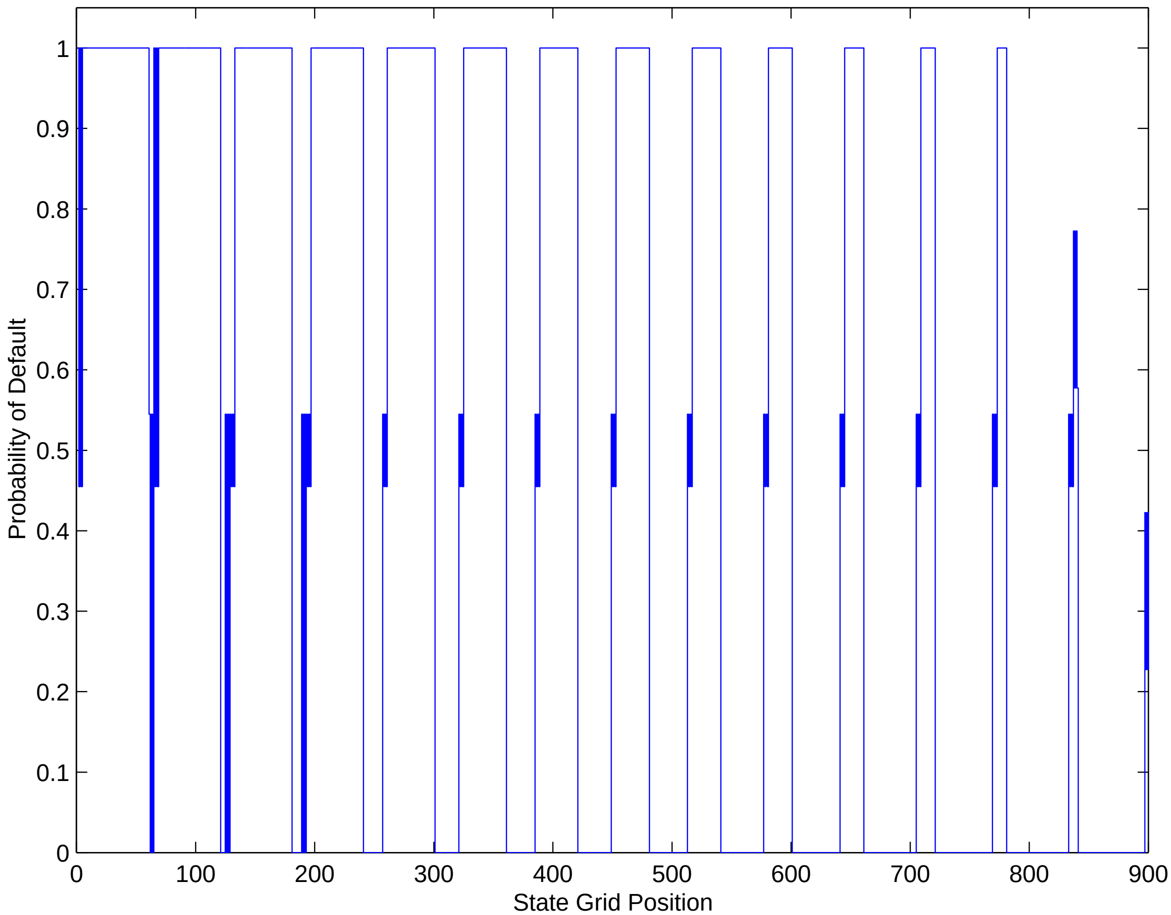

Figure 5: Mean Time to Default (MTTD)

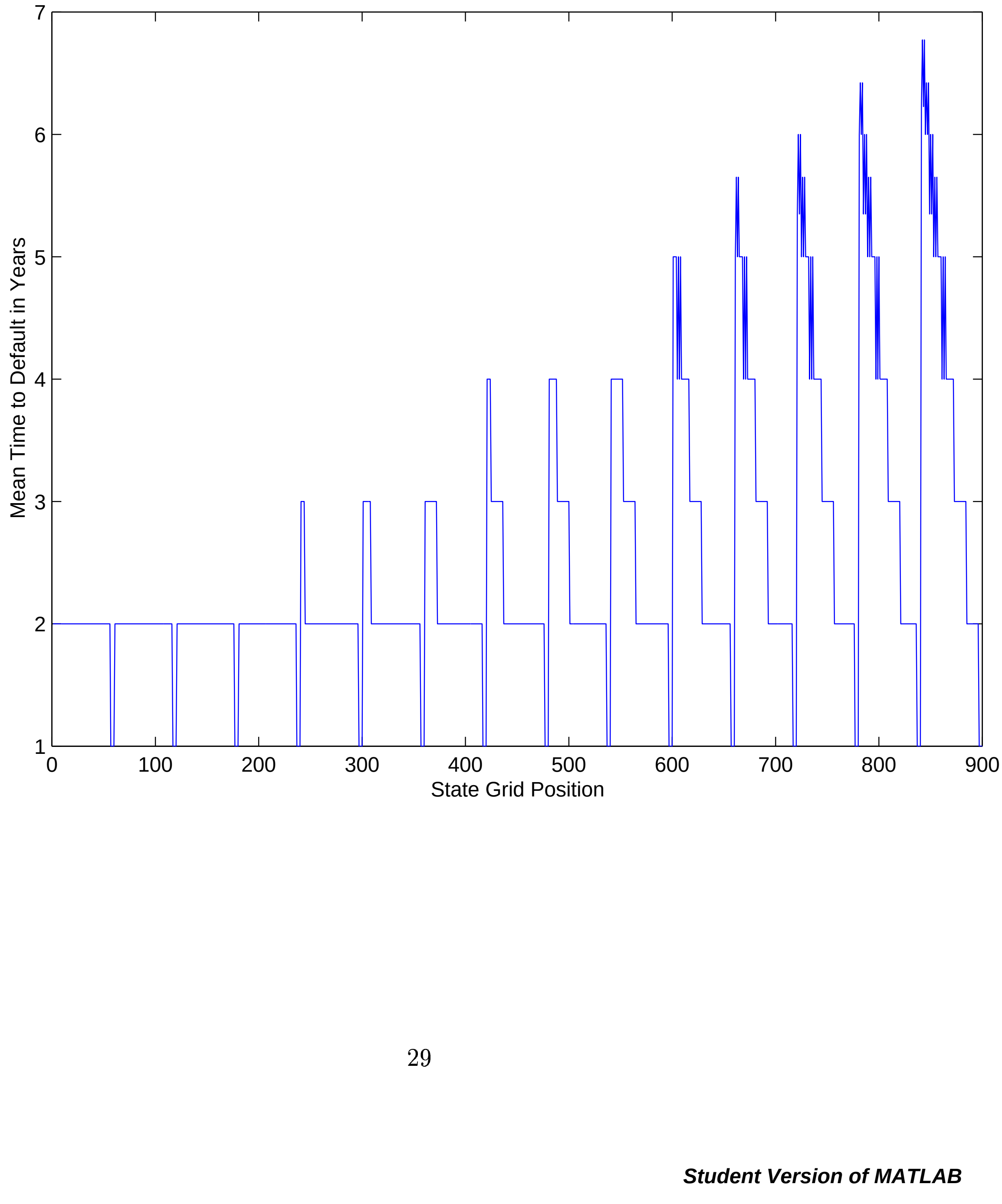

Figure 6: Effect of Increased Relative Risk Aversion on the Value Function

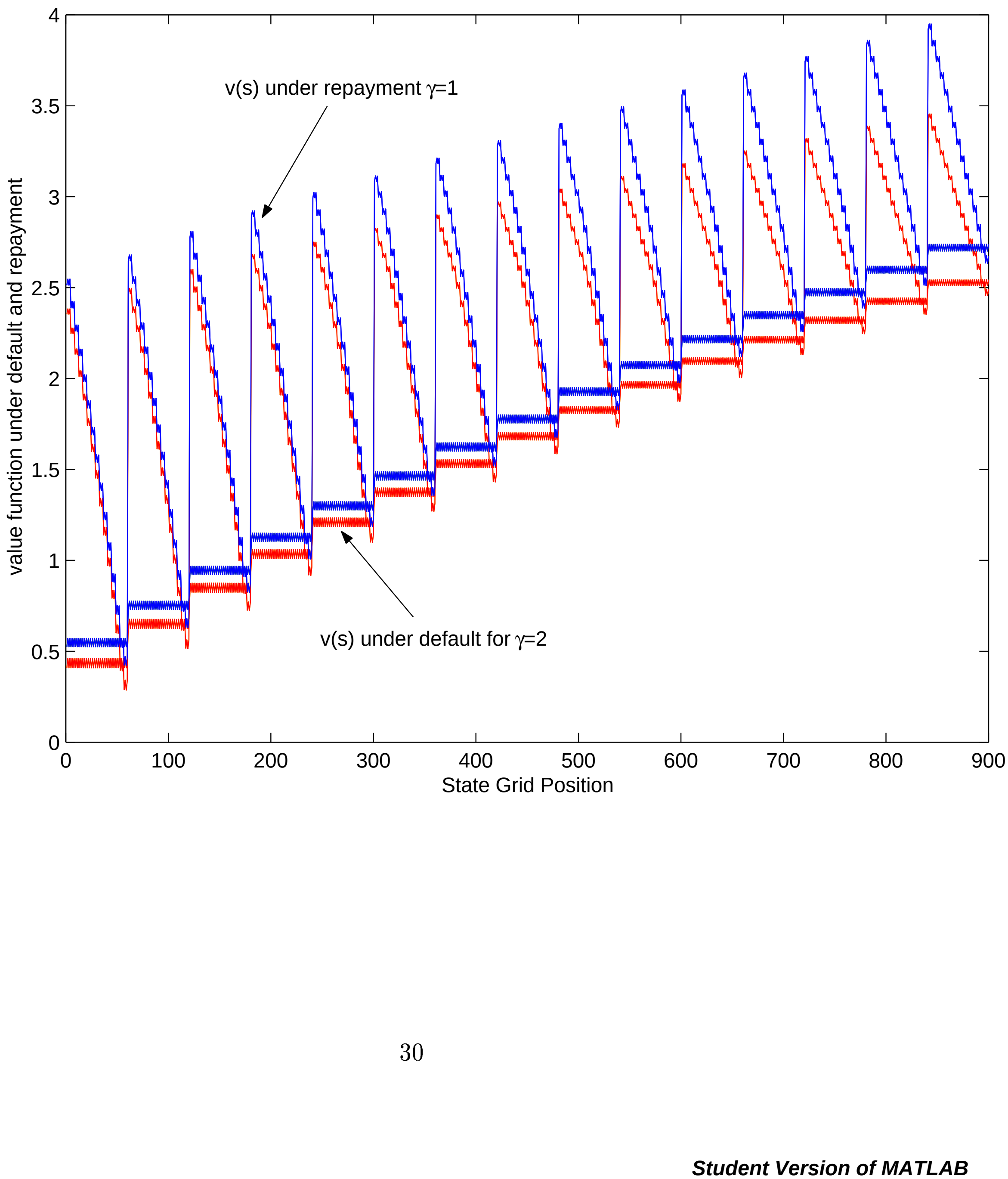

Figure 7: Effect of Increased Discount Factor on the Value Function

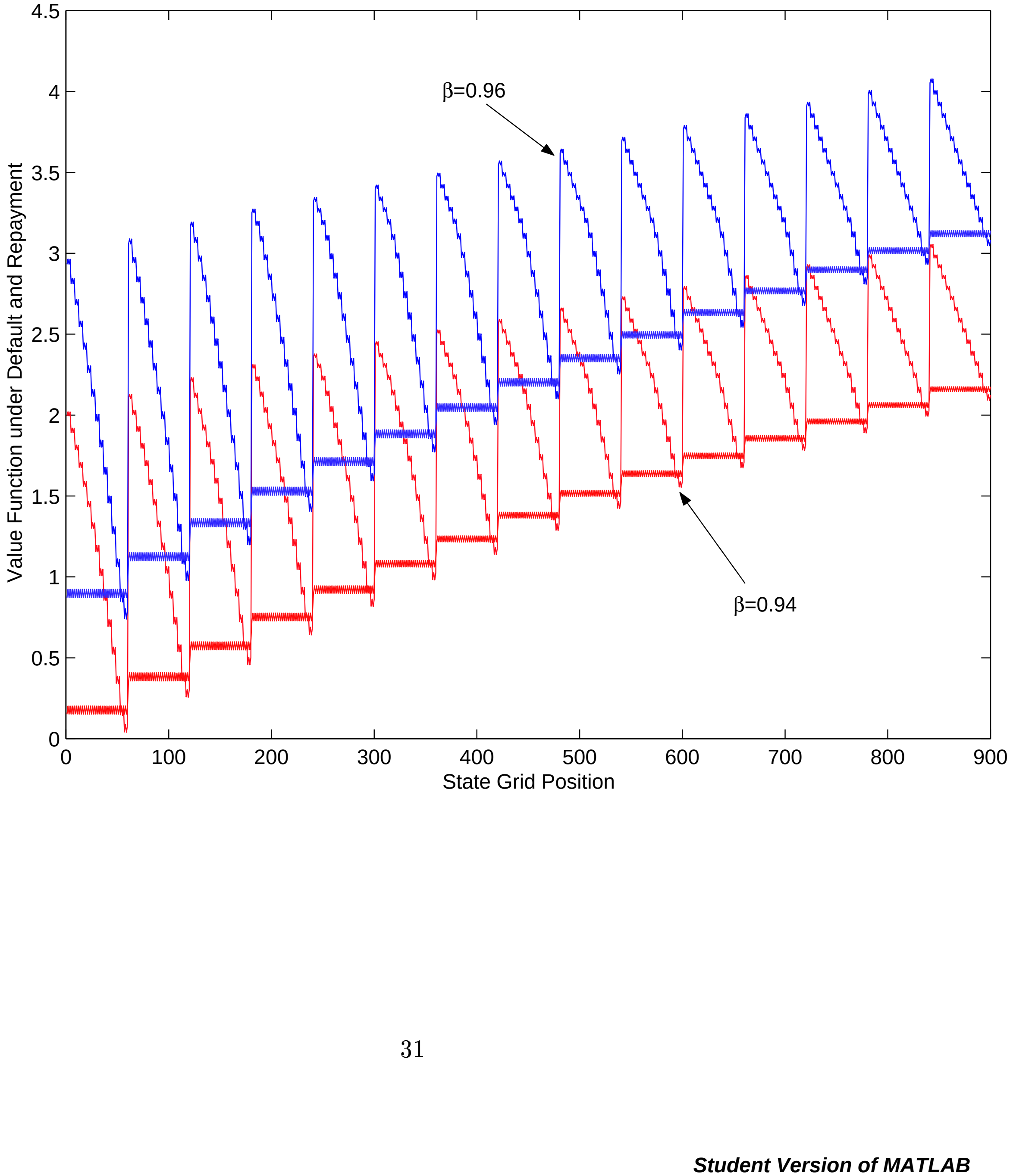

Figure 8: Effect of Increased Relative Risk Aversion on the MTTD

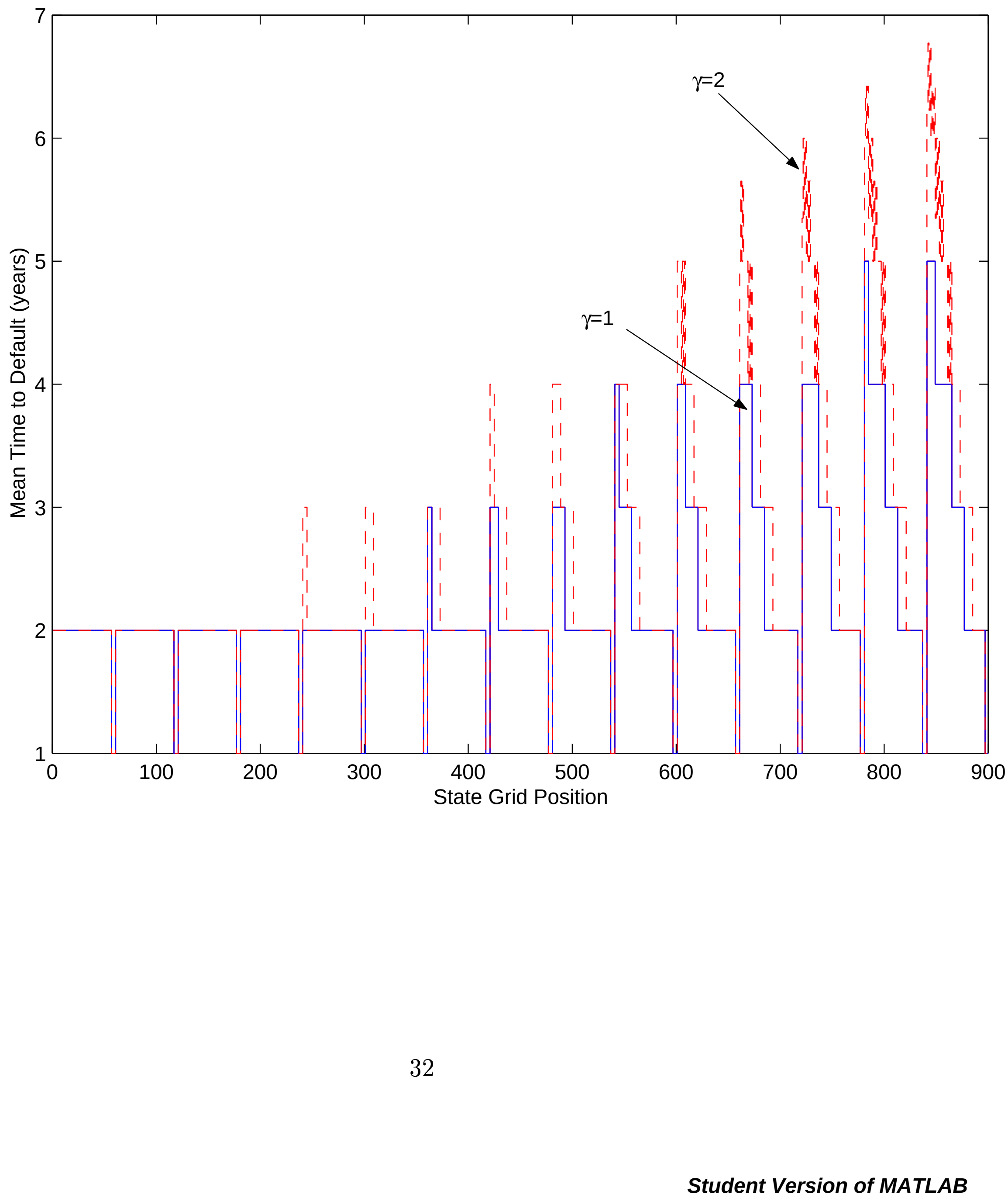

Figure 9: Effect of Increased Discount Factor on the Mean Time to Default

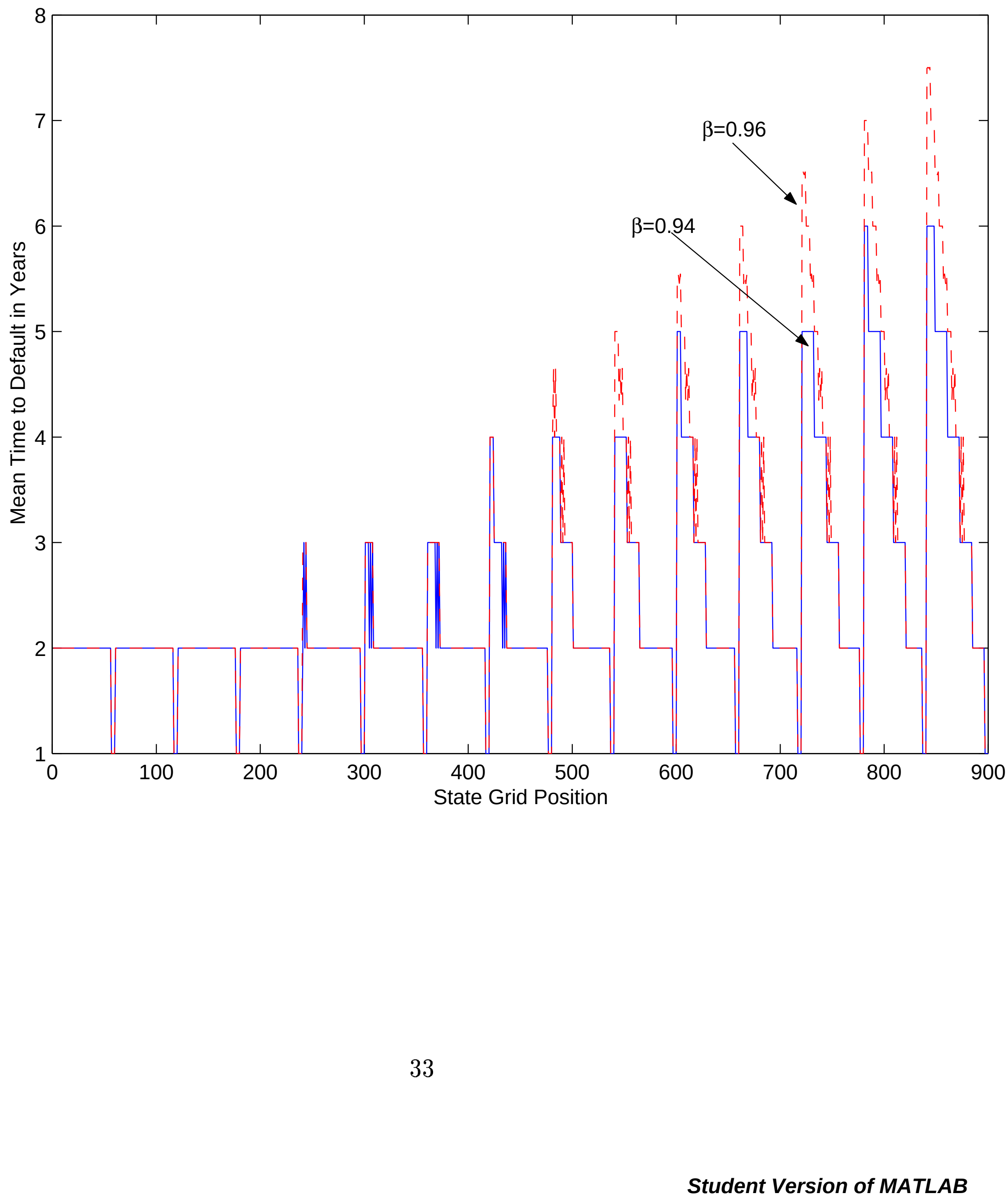

Figure 10: Effect of Increased Persistence of Shocks on the MTTD

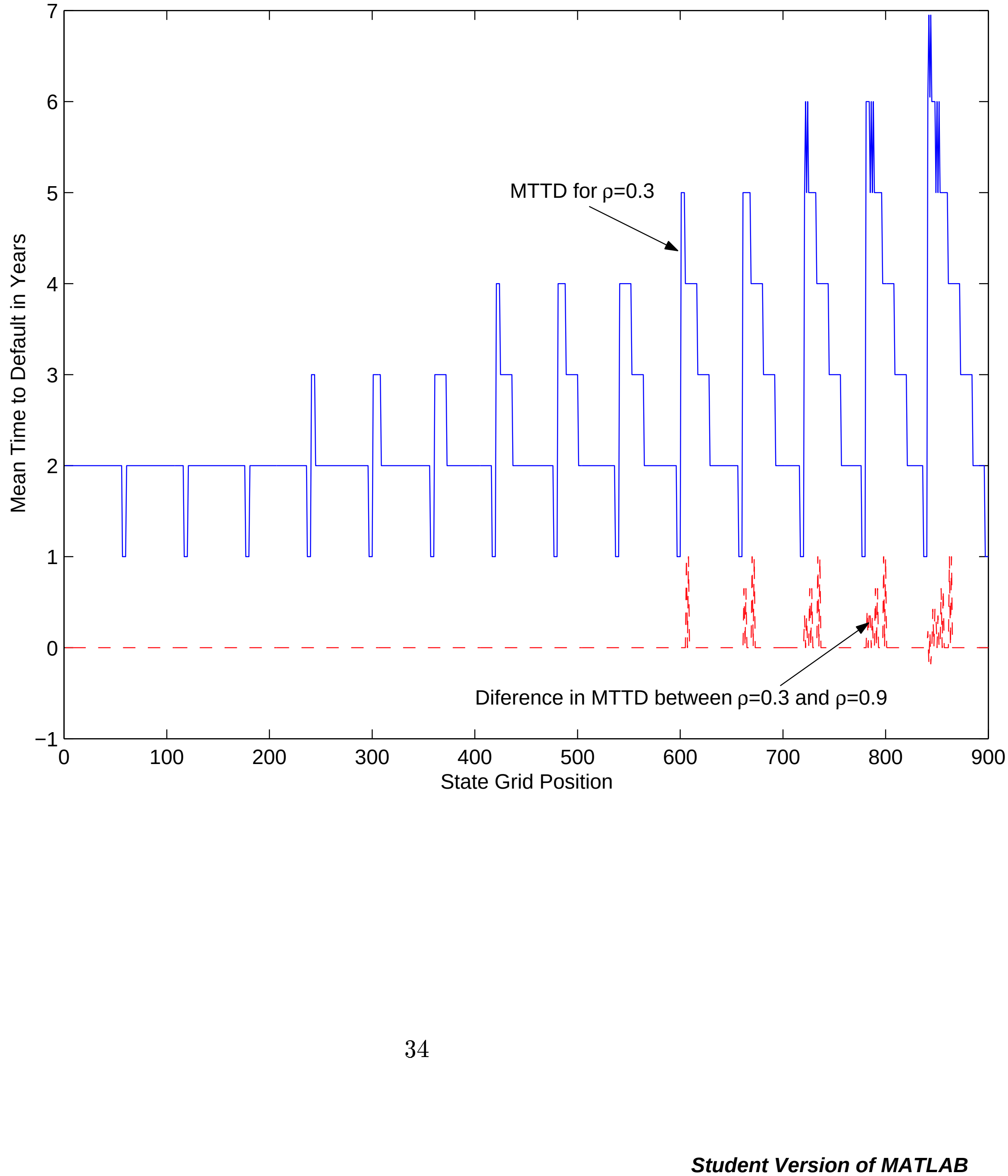

Figure 11: Effect of Increased Volatility of Productivity Shocks on the MTTD

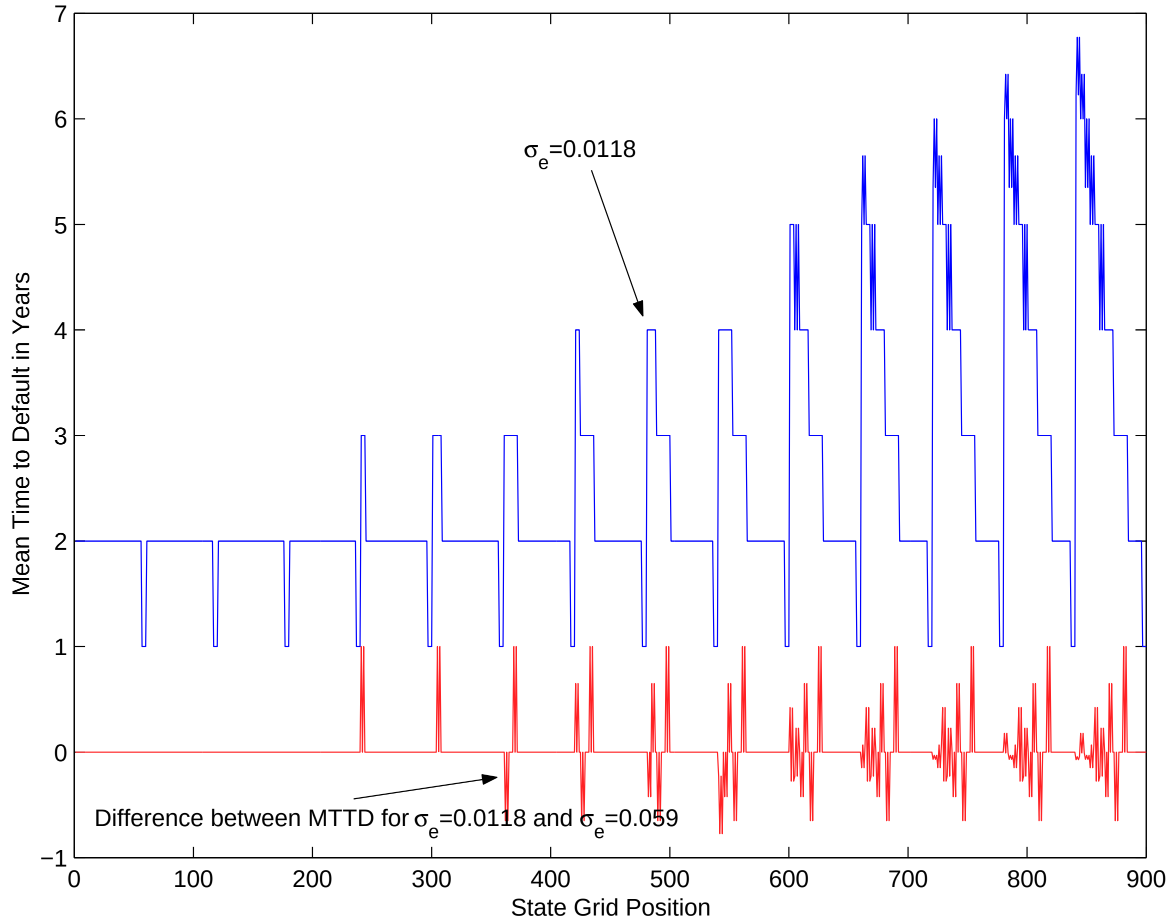

Figure 12: Effect of Increased Volatility of External Interest Rates Shocks on the MTTD