Inflation Targeting in a Small Open Economy: The Colombian Case*

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DRAFT FOR COMMENTS

12th August 2004

Abstract

This paper presents a dynamic stochastic general equilibrium model of inflation targeting in a small open economy. We calibrate the model to the Colombian economy and present the response of some macroeconomic variables to different types of shocks that are relevant for emerging economies. We also analyze the sensitivity of those responses to some key parameters. Furthermore, using simulated data from the model we study the ability of the model to capture the spectra, the phase and the coherence of observed output and inflation. We follow a frequency domain comparison methodology proposed by Diebold, Ohanian and Berkowitz (1998,[19]). The Colombian data is characterized by: first, cyclical inflation and output gap (as meassured by Hodrick-Prescott filter) are dominated by periodic movements between 2 and 25 quarters with a peak between 10 and 12 quarters. The cross spectrum and coherence show results in the same direction. Second, the coherence does not show any significant dominance of frequencies for the cross movements but the correlation jumps to $0,6$ for periodic movements around 5 quarters. These facts are compared to the data simulated from the model. We conclude that the simulated data spectra and cross spectra do not differ statistically from the respective population quantities for, at least, frequencies beyond $0,05\pi$, which correspond to periodic movements of up to at least 10 quarters. The model spectra presents more persistence than the observed data and the population coherence is captured for most frequencies but

*The authors wish to thank Martín Uribe for helpful conversations and Julián Pérez for helping us to build the database. All the errors are our own. The views expressed here are those of the authors and do not necessarily reflect the position of the Banco de la República de Colombia.

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the ones around the peak of the model's theoretical coherence and very long run periodic movements. Subsequent research will address these issues.

1 Introduction

Nowadays Colombia's Central Bank, uses the so called "Model of Transmission Mechanisms (MTM)" as the main model for monetary policy analysis and forecast[1]. The model, consists of a monetary policy rule, a Phillips curve (augmented by expectations) for "core" inflation, an equation for food inflation, another for imported goods, the output gap and the exchange rate. This setup aims to capture the main channels through which monetary policy is transmitted to the real sector as well as the sluggish response of inflation to monetary policy shocks. The MTM has proven to be a useful tool for monetary policy analysis and forecast. However, it is also recognized that this kind of models have important limitations[2]. The fact that we only use a subset of semi-structural economic relationships, with no specifications about agents, markets and their interaction, limits the scope, the consistency and the interpretability of the model. One consequence is that the language used to support the monetary policy decisions is usually vague and sometimes even inconsistent.

At a theoretical level, it has been argued that nominal rigidities and departures from perfect competition may be an important channel through which monetary policy has real effects. The macroeconomic implications of these types of models are surveyed by Walsh (2003, [24]). Recent explorations by Chari, Kehoe and McGrattan (1996, [4]) show that a high labor-supply elasticity is required in order to explain a significant fraction of price dynamics in the U.S. data. However, Nelson (1998, [18]) has shown that the type of models developed by Chari, Kehoe and McGrattan cannot account for the sluggish response of inflation that seems to be present in the U.S. data. Melo and Riascos (2004, [16]) have found a sluggish response in output, employment and inflation to policy shocks using Colombian data and built a structural model with two types of rigidities: a real rigidity in the labor market and a limited participation constraint in the financial market. Their model can reproduce the sluggish response of inflation, but fails to reproduce the labor market dynamics; the issue of the role of nominal rigidities remains unexplored in Colombia.

This paper evaluates *quantitatively* how much of the observed dynamics of the Colombian macroeconomic data can be explained by the presence of nominal rigidities in the context of a small open economy. It also explores its implications for monetary policy when operating under an Inflation Targeting framework and using the nominal interest rate as the policy instrument[3]. We develop and calibrate a DSGE model for monetary policy analysis in Colombia to study the

[1] See Gomez, Vargas and Uribe (2001, [8]) and Gómez and Julio (2003, [7]).

[2] More precisely, most of the equations of the model are subject to the Lucas critique. For example, the Phillips curve implicitly assumes the presence of some rigidity, but since agents don't optimize and there is not a well defined economic structure in the model, it is impossible to know where the rigidity comes from.

[3] Efforts in this direction are also pursued in other Central Banks. See Smets and Wouters, (2003, [21]), Murchinson, Rennison and Zhu (2003, [17]) , Scott (2003, [22]).

effects of different types of shocks and their transmission mechanisms through the economy. When able to identify the sources of the shocks, one wants to be able to conclude to what types of shocks must the monetary authority react and how to do it. In other words, the model can help to identify the shock and its nature. Then one should be able to define what change should take place on the policy instruments (the interest rate in this case) to take inflation back to its target and the product to its potential[4].

In particular, we introduce a nominal rigidity and a market imperfection to a small open economy model. The model economy has two sectors: a standard competitive sector populated by firms that hire capital and labor from households to produce homogeneous output, and a monopolistically competitive sector in which firms buy the homogeneous product at a competitive price, differentiate their product by putting a tag (a process we call "branding") and sell it to households. The consumption good that enters the agent's utility function is a Dixit-Stiglitz aggregate of all the differentiated goods. The first sector can be thought as a wholesale sector and the second sector as a retail sector. The introduction of monopolistic competition in the retail sector provides a rationale for price-setting behavior. What is important, is to introduce some sort of nominal rigidity. In this respect, we follow Calvo(1983, [3]) assuming that a fraction of retailers adjust their price infrequently. Monetary policy is conducted to target inflation and potential output. The monetary authority adjusts the nominal interest rate to meet these targets. Since this is a small open economy, monetary policy decisions will impact not only domestic savings households decisions but also their external financing.

We test the empirical power of this simple model in explaining the dynamics of output and inflation in Colombia for the period 1980:1-2004:1. Our criteria to evaluate the model is that its calibrated version should be able to reproduce salient and/or interesting features of Colombian data on output gap and inflation. We compare, at the frequency domain, the observed data with the data simulated from the theoretical model. The methodology consists of four steps. First, we estimate the sample data spectrum and compute its uncertainty using bootstrap techniques. Second, from the estimated spectrum and its uncertainity we determine the salient and/or interesting features which we expect the theoretical model to reply. Third, we compute the model's theoretical spectrum. Finally, we compare the theoretical and observed estimated spectrums at the required frequencies.

The rest of the paper proceeds as follows: in the next section we lay out the model. Section 3 shows the calibration procedure. Section 4 shows the response of the model to different shocks and its sensitivity to some key parameters, as the degree of price stickyness and the degree of response of the risk premium to the external debt to output ratio. In section 5 we determine to what extent the model replicates the salient features of Colombian data using the methodology proposed by Diebold et al. (1998). The last section summarizes our findings.

[4]In this framework potential output is understood as the resulting output in an environment characterized by the absence of frictions and/or shocks. See Galí (2002, [6]).

2 The Model

We consider a small open economy with a representative household, two types of firms and a government that makes unproductive expenditure, issues national currency and behaves according to an interest rate policy rule. The first type of firms hire labor and capital from households and produce an homogeneous good. The second type of firms buy the homogeneous good, put a label at no cost, and end up with a differentiated good. One way to think about the second type of firms is as "branding" firms[5]. They buy wheat, pack it and put a label on it. This is just a device to introduce price-stickiness into the model[6]. From now on we will refer to the first type of firms as "producers" and to the second as "retailers". Households consume differentiated consumption goods and pay a liquidity cost, they also supply homogeneous indivisible labor, accumulate capital and supply it to producers. They receive lump sum transfers from the government and hold wealth as cash. Producers hire labor and capital from households as factor inputs and produce homogeneous goods. These homogeneous goods are demanded by retailers, which transform homogeneous goods into differentiated consumption goods and sell these to households. The consolidated monetary and fiscal authority issues money, makes net lump sum transfers to households, makes some unproductive expenditure and collects the liquidity costs from households[7]. All quantities are in per-capita terms if not stated otherwise.

2.1 The Representative Household

Households are the owners of the firms that produce the homogeneous good as well as of the retail sector firms and are consumers. Their income at period t is given by the nominal wage, nominal returns to capital, the benefits from retailers and the net lump sum transfers obtained from the government in this same period. Apart from their income they also count with a real money stock given at the beginning of the period as well as with a stock of real domestic private bonds and foreign assets[8]. Expenditure is determined by consumption, the liquidity costs and investment. At period t, they also decide the level of expected real money holdings, real domestic private bond holdings and foreign asset holdings for period $t+1$. Then the

[5]This type of setup is not new in the literature, to our knowledge it was first implemented by Bernanke, Gertler and Gilchrist (1999, [2]).

[6]See Schmitt-Grohe and Uribe (2004,[20]).

[7]By doing so we intend to eliminate the wealth effect.

[8]Stock variables are given at the beginning of the period and flows are known at the end, i.e. M_t is known at the start of period t, $P_{t-1}$is given at the end of period $t-1$ so it's known at the beginning of period t, as $m_t = \frac{M_t}{P_{t-1}}$, real money holdings are known at the start of period t.

budget constraint is given by:

$$c_t + \Phi + m^d_{t+1} + \frac{P_t x_t}{P^c_t} + b_{t+1} + \frac{e_t F_{t+1}}{P^c_t} = \frac{W_t}{P^c_t} h^s_t + \frac{R_t}{P^c_t} k^s_t + \frac{\Pi^R_t}{P^c_t} + \frac{\Pi_t}{P^c_t} + m^d_t \frac{P^c_{t-1}}{P^c_t} + b_t \frac{P^c_{t-1}}{P^c_t}(1+i_t) + \frac{e_t F_t}{P^c_t}(1+i^f_t) + \tau_t \quad (1)$$

where: c_t is real consumption, m^d_t is real money demand, x_t is real investment, W_t is the nominal wage, h^s_t is the number of hours worked per-capita, R_t is the nominal return to capital, k^s is capital supply, Π_t are the benefits from the homogeneous good producers, Π^R_t are the benefits from the retailers, τ_t are government lump sum transfers to the households, P^c_t is the price index of consumption goods and P_t is the price index of homogeneous goods, b_t are net real private domestic bonds, F_t are net foreign assets (or liabilities depending on the sign) denominated in units of the tradable homogeneous good, e_t is the nominal exchange rate (COP/USD), i_t is the domestic nominal interest rate and i^f_t is the foreign nominal interest rate denominated in dollars. M_0, k_0, b_0 and F_0 are known. As $m_t = \frac{M_t}{P^c_{t-1}}$, hence m_0 is known and the same follows for b_0. Φ is a function which determines the transaction costs, and is given by

$$\Phi(c_t, m_{t+1}, x_t) = \kappa \left(\frac{c_t + \nu \frac{P_t}{P^c_t} x_t}{m_{t+1}} \right)^a \quad (2)$$

where all variables are in real terms (relative to the consumption good) and ν is a parameter that determines the fraction of investment that affects the optimal choice of real money holdings. According to this expression, as the household consumes or invests more, its liquidity costs increase, and they decrease with the real money holdings they save for next period. Money is introduced like this for simplicity.

The external nominal interest rate is defined as

$$\left(1 + i^f_t\right) = (1 + i^*_t)\left(1 + \vartheta\left(\frac{F_t}{y_t}\right)\right) \quad (3)$$

where i^*_t is the international risk free nominal interest rate and ϑ is the risk premium function[9]. Notice that if the net foreign assets (F_t) are negative, then the country is a net debtor and otherwise it is a net lender. It is also assumed that the purchase power parity (PPP) is satisfied, so $P_t = e_t P^*_t$. This means that the price for the homogeneous good equals the foreign price for the homogeneous good times the exchange rate. We set $P^*_t = 1$ for all t, therefore $P_t = e_t$ and so the depreciation rate equals the inflation rate of homogeneous goods, $\pi_t = d_t$. If we define

[9]The risk premium function is defined as $\vartheta\left(\frac{F_t}{y_t}\right) = \omega_{ss} + \omega_1 + \omega_2 * Exp\left[\omega_3\left(\frac{\frac{F_t}{y_t}}{\frac{F_{ss}}{y_{ss}}}\right) * \mu^\vartheta_t\right]$ where the subscript ss stands for the steady state value of the variable and μ^ϑ_t is an exogenous variable which logarithm follows a standard autorregresive process of order one of the form $log(\mu^\vartheta_{t+1}) = \rho_4 log(\mu^\vartheta_t) + (1-\rho_4) log\left(\overline{\mu^\vartheta}\right) + \epsilon_{t+1}$

$q_t = \frac{P_t}{P_t^c}$ as the relative price of homogeneous goods to heterogeneous goods, and $\frac{P_{t-1}^c}{P_t^c} = \frac{1}{(1+\pi_t^c)}$, then the budjet constraint (1) can be rewriten as

$$\begin{aligned} c_t + \Phi + m_{t+1}^d + q_t x_t + b_{t+1} + q_t F_{t+1} &= \frac{W_t}{P_t^c} h_t^s + \frac{R_t}{P_t^c} k_t^s + \frac{\Pi_t^R}{P_t^c} + \frac{\Pi_t}{P_t^c} + \\ & \frac{m_t^d}{(1+\pi_t^c)} + \frac{b_t}{(1+\pi_t^c)}(1+i_t) + F_t q_t (1+i_t^f) + \tau_t \end{aligned} \tag{4}$$

Households accumulate capital according to the following expression:

$$k_{t+1} - (1+\delta)k_t - f\left(\frac{x_t}{k_t}\right) k_t = 0 \tag{5}$$

where f is a twice continously differentiable and concave function, which reflects investment adjustment costs in capital, and δ is the depreciation rate. The specification of the function f, is such that when the economy is in steady state, there are no adjustment costs[10] .

Consumption and leisure generate utility to households, but they have a habit stock which generates dis-utility, this is

$$u(c_t, H_t, h_t, \mu_t^u) = \mu_t^u \log(c_t) - \gamma \log(H_t) - B h_t \tag{6}$$

where H_t is the habit stock, B is a parameter, μ_t^u is an exogenous variable that represents an intertemporal preference shock[11], and

$$c_t = \left[\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} \tag{7}$$

where $c(z)$ is the consumption of a specific good z coming from the retailer z, and θ is the elasticity of consumption of each good z with respect to the whole bundle.

The functional form of the utility function deserves some explanation. First, the linear specification of utility involving h follows Hansen (1985, [10]) where labor is indivisible. Workers can either work some given number of hours or not at all (i.e. they can't work part time). Second, the utility function is separable in consumption and leisure. Third, agents trade employment lotteries instead of hours of work. This implies that hours worked are

[10]We assume that f is a cuadratic function $f\left(\frac{x_t}{k_t}\right) = c_2\left(\frac{x_t}{k_t}\right)^2 + c_1\left(\frac{x_t}{k_t}\right) + c_0$. c_2 determines the concavity of the function, that is, how expenssive it is on the margin to adjust the capital outside the steady state and is fixed in order to replicate investment's volatility. Parameters c_1 and c_2 are determined by the fact that there are no adjustment costs on the steady state.

[11]The log of this exogenous variable follows a standard autorregressive process of order one, $log(\mu_{t+1}^u) = \rho_3 log(\mu_t^u) + (1-\rho_3) log\left(\overline{\mu^u}\right) + \epsilon_{t+1}$.

proportional to employment[12].

On the other hand, H represents the consumption habits of each individual:

$$H_{t+1} - H_t - \rho(c_t - H_t) = 0 \tag{8}$$

where H_0 is given. Consumption habit today depends on last period's consumption and habit[13]. The higher habit is, the more dis-utility it is going to generate. In the present period the individual is going to have to consume more to be as satisfied as last period[14].

Then the representative household's dynamic problem is

$$\max_{\{c,h,x,k,m,H\}} \sum_{t=0}^{\infty} \beta^t u\left(c_t, H_t, h_t, \mu_t^u\right)$$

subject to (2), (3), (4), (5), (7), and (8) .

According to this, the first order conditions of the household's problem are the following:

$$u_{c_t}\left(c_t, H_t, h_t, \mu_t^u\right) + \eta_t \rho = \lambda_t \left(1 + \Phi_{c_t}\left(c_t, m_{t+1}, x_t\right)\right) \tag{9}$$

$$u_{h_t}\left(c_t, H_t, h_t, \mu_t^u\right) + \lambda_t \frac{W_t}{P_t^c} = 0 \tag{10}$$

$$\lambda_t \left(\Phi_{x_t}\left(c_t, m_{t+1}, x_t\right) + q_t\right) = \gamma_t f_{x_t}\left(\frac{x_t}{k_t}\right) k_t \tag{11}$$

$$\beta E_t \left(\lambda_{t+1} \frac{R_{t+1}}{P_{t+1}^c} + \gamma_{t+1}\left(f\left(\frac{x_{t+1}}{k_{t+1}}\right) + \frac{\partial\left(f\left(\frac{x_{t+1}}{k_{t+1}}\right) k_{t+1}\right)}{\partial\left(k_{t+1}\right)}\right) + \gamma_{t+1}(1-\delta)\right) = \gamma_t \tag{12}$$

$$\beta E_t \left(\frac{\lambda_{t+1}}{\left(1+\pi_{t+1}^c\right)\left(1+\Phi_{m_{t+1}}\left(c_t, m_{t+1}, x_t\right)\right)}\right) = \lambda_t \tag{13}$$

[12]Each period instead of choosing manhours households choose a probability of working α. The new commodity being introduced is a contract between the firm and the household that commits to work h_0 hours with a probability α. The contract is what is being traded, so the household gets paid wether it works or not. Since households are identical all are going to choose the same α. So all households are going to offer αh_0 which is a fixed quantity. As the utility function is linear in leissure it implies an infinite elasticity of substitution between leissure in different periods. This follows no matter how small this elasticity is for the individuals in the economy. Therefore the elasticity of substitution between leissure in different periods for the aggregate economy is infinite and independent of the willingness of the individuals to substitute leissure across time.

If α increases then it means that people are willing to work more, this means that a higher portion of people are working. Therefore the sum of hours worked is higher and with the same population (assuming there is no population growth) the number of hours worked per-capita is going to be higher.

[13]Commonly known as inward looking habit.

[14]This friction is introduced in order to obtain the persistence in consumption which is observed in the data.

$$\beta E_t \left(\frac{\lambda_{t+1} \left(1 + i_{t+1}\right)}{\left(1 + \pi^c_{t+1}\right)} \right) = \lambda_t \tag{14}$$

$$\beta E_t \left(\lambda_{t+1} \left(1 + i^f_{t+1}\right) q_{t+1} \right) = \lambda_t q_t \tag{15}$$

$$\beta E_t \left(\eta_{t+1} + U_{H_{t+1}} \left(c_{t+1}, H_{t+1}, h_{t+1}, \mu^u_{t+1}\right) - \eta_{t+1}\rho \right) = \eta_t \tag{16}$$

and equations (4), (5) and (8). Where λ, γ and η are the lagrange multipliers associated with the budget constraint, the evolution of capital and the evolution of the stock of habit, respectively.

2.2 The Producers

This sector is competitive and the producers seek to maximize their profits by choosing the level of capital and labor, given the rental rate of capital, the nominal wage and a technology to produce output, which is sold at price P_t. The technology is assumed to be a standard Cobb-Douglas production function. Hence the problem faced by producers is to solve

$$\max_{\{k,h\}} \Pi_t = P_t A_t (k^d_t)^{\alpha} (h^d_t)^{(1-\alpha)} - R_t k^d_t - W_t h^d_t \tag{17}$$

where A_t is the level of productivity, the subscript d represents the specific input's demand and $log(A_t)$ will follow a standard autorregressive process of order one[15]. The first order conditions for the producers of the homogeneous good are the standard ones.

2.3 The Retailers

The retailers, purchase homogeneous output from producers at a price P_t, and turn it into their specific brand of consumption good at zero additional cost. However, on each period retailers face a constant probability, $1 - \varepsilon$, of receiving a signal, that tells them that they can re-optimize their price, this probability behaves as in Calvo (1983, [3]). The other ε retailers follow a backward indexation rule, see Christiano, Eichenbaum, Evans (2001, [12])[16]. This probability is independent across firms and time. We assume that if a retailer doesn't receive the signal, it fixes his price according to[17]:

$$p^{rule}_t(z) = p^c_{t-1}(z)(1 + \pi^c_{t-1}) \tag{18}$$

[15] $log(A_{t+1}) = \rho_1 log(A_t) + (1 - \rho_1) log(\overline{A}) + \epsilon_{t+1}$ where $\overline{A}$ represents the average value taken by A across time.

[16] This indexation rule makes it possible for the model to have inflation different from zero. It also implies that in the steady state prices are going to have zero dispertion, i.e. the price that follows the backward indexation rule is equal to the optimal price. Other pricing rules are $p^{rule}_t(z) = p^c_{t-1}(z)$ or $p^{rule}_t(z) = p^c_{t-1}(z)(1 + \bar{\pi})$ where $\bar{\pi}$ is the long run inflation. These rules are studied by Dotsey, King and Wolman (1999, [5]).

[17] One way to interpret this pricing rule is to assume that on each period retailers face a constant probability $1 - \varepsilon$, of wanting to gather information about the state of the economy in order to re-optimize their price (see

where p^c_{t-1} is retailer's last periods price and π^c_{t-1} is the period $t-1$ rate of inflation of the aggregate consumption price index.

With probability $1-\varepsilon$ a retailer is going to optimize and set p_t^{opt}. If this is the case the retailer's problem is the following:

Each retailer[18] (z) expected profits at period $t+j$ are given by:

$$E_t\left(\Pi^R(z)_{t+j}\right) = E_t\left(c(z)_{t+j}(p^c(z)_{t+j} - P_{t+j})\right) \tag{19}$$

The real profits of each retailer are $\Pi^R(z)_{t+j}/P^c_{t+j}$ so those firms who are allowed to adjust their price in period t will choose $p^c(z)_{t+j}$ to:

$$\max_{\{p^c(z)_t\}} E_t \sum_{j=0}^{\infty} (1-\varepsilon)\,\varepsilon^j \Delta_{t+j} \frac{\Pi^R(z)_{t+j}}{P^c_{t+j}}$$

where the discount factor $\Delta_{t+j} = \beta^j \frac{u'(C_{t+j},h_{t+j},H_{t+j})}{u'(C_t,h_t,H_t)}$ is an appropriate discount factor according to the market's real interest rate, and households take it as given for their maximization problem. Notice that in period t the firm chooses a price from now on, $p^c(z)_{t+j} = p^c(z)_t$ because of the uncertainty on future price changes, in other words, the firm does the maximization taking into account that today they can re-optimize prices (with probability $(1-\varepsilon)$) and that for j periods they are not going to re-optimize them (with probability ε^j).

From the households problem it can be shown (see appendix 1) that the demand for the consumption good $c(z)_t$ is:

$$c(z)_{t+j} = \left(\frac{p^c(z)_{t+j}}{P^c_{t+j}}\right)^{-\theta} c_{t+j} \tag{20}$$

so the maximization problem ends up being:

$$\max_{\{p^c(z)_t\}} E_t \sum_{j=0}^{\infty}(1-\varepsilon)\varepsilon^j \Delta_{t+j} c_{t+j} \left[\left(\frac{p^c(z)_{t+j}}{P^c_{t+j}}\right)^{1-\theta} - \varphi_{t+j}\left(\frac{p^c(z)_{t+j}}{P^c_{t+j}}\right)^{-\theta}\right]$$

where $\varphi_{t+j} = \frac{P_{t+j}}{P^c_{t+j}}$.

After solving for $p^c(z)_t$, the solution becomes (see appendix 2 for derivation):

$$\frac{p_t^{opt}}{P_t^c} = \frac{\theta}{\theta-1} E_t\left[\frac{\sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \varphi_{t+j} \left(\frac{P^c_{t+j}}{P^c_t}\right)^{\theta}}{\sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \left(\frac{P^c_{t+j}}{P^c_t}\right)^{\theta-1}}\right] \tag{21}$$

Mankiw and Reis, 2002, [15]). So those $1-\varepsilon$ who gather the information, re-optimize their price according to it. In contrast the other ε retailers follow a backward indexation rule, they keep changing their prices according to past information. So in a sense this is not exactly a case of sticky prices, because as one can see everyone is changing prices but not re-optimizing. This is more a case of sticky information.

[18] Retailers are indexed by z.

or what is the same

$$\frac{p_t^{opt}}{P_t^c} = \frac{\theta}{\theta - 1} E_t \left(\frac{\Theta_t}{\Psi_t} \right)$$

where

$$\Theta_t = \Delta_t c_t \varphi_t + \varepsilon E_t \left((1 + \pi_{t+1}^c)^{\theta} \Theta_{t+1} \right)$$

$$\Psi_t = \Delta_t c_t + \varepsilon E_t \left((1 + \pi_{t+1}^c)^{\theta - 1} \Psi_{t+1} \right)$$

and p_t^{opt} denotes the price of the good $c(z)_t$ set by the retailer z in the case in which he decides to optimize. Since (20) implies that the price index is also a CES aggregator, it can also be shown that the price index P_t^c is given by[19]

$$P_t^c = \left[\varepsilon (p_t^{rule})^{1-\theta} + (1 - \varepsilon)(p_t^{opt})^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{22}$$

and then the aggregate inflation dynamics is given by

$$(1 + \pi_t^c) = \left(\varepsilon (1 + \pi_{t-1}^c)^{(1-\theta)} + (1 - \varepsilon) \left(\frac{p_t^{opt}}{P_t^c} \right)^{(1-\theta)} (1 + \pi_t^c)^{(1-\theta)} \right)^{\frac{1}{1-\theta}} \tag{23}$$

2.4 Consolidated Monetary and Fiscal Authority

On each period t, the government issues money, transfers a net lump sum to households and makes unproductive expenditures. It is also assumed that the government collects the liquidity costs payed by households. Seigniorage as well as the liquidity costs represent income for the government so their budget constraint is the following:

$$m_{t+1}^s - \frac{m_t^s}{1 + \pi_t^c} + \Phi \left(c_t, m_{t+1}, I_t \right) = \tau_t + \left(\frac{g_t}{y_t} \right) y_t \tag{24}$$

where the letters with subscript s represent a supply, and g_t is real government expenditure. $log \left(\frac{g_t}{y_t} \right)$ follows a standard autorregressive process of order one[20] .

[19] As we know the consumption index is $c_t = \left[\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$ which implies that the demand for the z-th good is $c(z)_{t+j} = \left(\frac{p^c(z)_{t+j}}{P_{t+j}^c} \right)^{-\theta} c_{t+j}$, where P_{t+j}^c is an index of the cost of buying a unit of $c(z)_t$: $P_t^c = \left[\int_0^1 (p_t^c(z))^{1-\theta} dz \right]^{\frac{1}{1-\theta}}$. This integral can be divided into two. So, retailers can be separated into two groups, a fraction $(1 - \varepsilon)$ that optimizes their price, and a fraction ε that doesn't.

[20] $log \left(\frac{g_{t+1}}{y_{t+1}} \right) = \rho_2 log \left(\frac{g_t}{y_t} \right) + (1 - \rho_2) log \left(\frac{\overline{g}}{y} \right) + \epsilon_{t+1}$ where $\left(\frac{\overline{g}}{y} \right)$ represents the average value taken by $\frac{g}{y}$ across time.

It is also assumed that monetary policy is conducted with an interest rate policy rule, of the form:

$$i_t = i + \zeta \left(\pi_t^c - \overline{\pi}^c\right) + \xi \left(y_t - y^{ss}\right) \tag{25}$$

where i is the steady state nominal interest rate level, $\overline{\pi}^c$ is the inflation target[21], and y^{ss} corresponds to the steady state level of output (this is the level of output in absence of shocks)[22]. y_t is determined by the production technology described in the last subsection. ζ and ξ are parameters that determine the importance that the monetary authority gives to inflation and output respectively when using the nominal interest rate as the policy instrument.

2.5 Competitive Equilibrium

To characterize the competitive equilibrium, the following definitions are used:

Definition: A price system is a positive sequence $\{W_t, R_t, p_t^{rule}, p_t^{opt}, P_t^c, P_t, e_t, i_t, i_t^f\}_{t=0}^{\infty}$.

Definition: $\{A_t, \mu_t^{\vartheta}, \mu_t^u, \frac{g_t}{y_t}, P_t^*\}_{t=0}^{\infty}$are taken as exogenous sequences. m_0, k_0, b_0, F_0, $H_0 > 0$ are also taken as given. An equilibrium is a price system, a sequence of consumption $\{c_t\}_{t=0}^{\infty}$, investment $\{x_t\}_{t=0}^{\infty}$, capital $\{k_t\}_{t=1}^{\infty}$, number of hours worked per-capita $\{h_t\}_{t=0}^{\infty}$, habit stock $\{H_t\}_{t=1}^{\infty}$, domestic real private bonds $\{b_t\}_{t=1}^{\infty}$, net foreign assets $\{F_t\}_{t=1}^{\infty}$ and a positive sequence of real money $\{m_t\}_{t=1}^{\infty}$in order that:

1. Given the price system and net lump sum transfers, household's optimal control problem is solved with $\{m_t^d = m_t^s = m_t\}_{t=1}^{\infty}$, $\{k_t^d = k_t^s = k_t\}_{t=1}^{\infty}$, $\{b_t = 0\}_{t=1}^{\infty}$, $\{h_t^d = h_t^s = h_t\}_{t=0}^{\infty}$, $\{c_t\}_{t=0}^{\infty}$ and a level of $\{F_t\}_{t=1}^{\infty}$ such that $(1 + i_t) = \left(1 + i_t^f\right)(1 + d_t)$ is satisfied.
2. The government's budget constraint (24) and policy rule (25) are satisfied for all $t \geq 0$.
3. $Y_t = C_t + I_t + G_t + F_{t+1} - \left(1 + i_t^f\right) F_t \qquad \text{for all } t.$

This last condition is the standard resource restriction in a small open economy.

3 Calibration

We now proceed to calibrate the model. There are some parameters that are uncontroversial, while others deserve some explanation. Parameter B is calibrated to obtain $h = \frac{1}{3}$ in steady-state. The capital share within the production function is set at $\alpha = \frac{1}{3}$ which approximately corresponds to the capital share in income. The capital stock time series in Colombia is a

[21]Notice that this target is in terms of the inflation of the prices of heterogeneous goods.

[22]It's not the level of output in the absence of frictions because transaction costs are still present in the steady state.

constructed one, which assumes a quarterly depreciation rate of 0.012, so we set $\delta = 0.012$. The parameter θ that determines the degree of competition in the differentiated goods market, is set to 5 in order to obtain a markup of 25% according to the most recent research on market structure available in Colombia[23]. The parameter ε that determines the degree of price stickiness is set to 0.75 in order to have prices changing every one year. β, which in equilibrium is equal to $\frac{1}{1+r}$ is fixed at 0.984 according to Vasquez (2003,[23]) who estimated the anual long term interest rate for Colombia in 6.81% which corresponds to 1.6% quarterly. The inflation target $\overline{\pi}$ is fixed at 5.5% (anual rate) according to the target set for this year by the Central Bank. i was fixed according to $\overline{\pi}$ and r. We set the international interest rate $i_t^* = 0.03$. The parameters ζ and ξ corresponding to the weight given by the monetary authority to the inflation and output gap respectivelly were fixed in 1.7 and 0 according to Melo and Riascos (2004,[16]), although they estimated the rule with a lag on the interest rate.

The parameter ω_{ss} of the risk premium function was calibrated according to the spread between i and i_t^*. We calibrate the rest of parameters of the risk premium function, ϑ, to match the long term total external debt to GDP ratio, which for Colombia is about 30%.

Investment adjustment costs where calibrated so that in the steady state there are no adjustment costs, $f\left(\frac{x}{k}\right) = c_2\left(\frac{x}{k}\right)^2 + c_1\left(\frac{x}{k}\right) + c_0 = \left(\frac{x}{k}\right)$ and $f'\left(\frac{x}{k}\right) = 1$. For a given c_2, this two conditions determine c_1 and c_0. So, c_2 is fixed to replicate investment's volatility which according to the Hodrick and Prescott filter is 18.8% for Colombia.

Since there is no information about the parameters that determine the evolution of habit over time, we calibrate them to replicate some stochastic properties of the consumption time series in Colombia: φ is set to replicate its volatility as close as possible (which is of 1.4% for Colombia according to data filtered with Hodrick and Prescott) and ρ is fixed to obtain the observed persistence of consumption's cyclical component.

We pay special atention to the parameter a in the transaction cost function, which determines the elasticity of the quantity of money demanded to consumption and interest rate. The first order conditions of the model allow us to obtain an approximation to the money demand of this economy. So we decided to estimate the values of a and κ. Using equations (14) and (13) we solve deterministically for m_{t+1} and obtain:

$$m_{t+1}^{1+a} = \frac{a\kappa(c_t + \nu q_t x_t)^a}{\frac{i_{t+1}}{1+i_{t+1}}} \tag{26}$$

Applying logs to equation (26) we obtain:

$$log(m_{t+1}) = \frac{1}{1+a}log(a\kappa) + \frac{a}{1+a}log(c_t + \nu q_t x_t) - \frac{1}{1+a}log(i_{t+1}) + \frac{1}{1+a}log(1 + i_{t+1})$$

and we estimate it in order to solve for the coeficients a, κ and ν. We used non-linear ordinary least squares with the following three restrictions: $a > 1$, $0 < \nu < 1$ and $\kappa > 0.0645$. The restriction on a is to avoid the case of a linear function, the one on ν is straight forward and

[23]See Arango et al. (1991,[1])

in principle κ should be $\kappa > 0$ but 0.0645 is the minimum value for which we were able to obtain the solution. What we found was a corner solution on κ, so our results were $a = 1.858$, $\kappa = 0.0645$ and $\nu = 0.025$. M1 was used for m, for q which can be defined as the real exchange rate (recall that in the model $q = \frac{e_t}{P_t^c} = \frac{P_t}{P_t^c}$) we used the spot's market nominal exchange rate times the U.S. core CPI (CPI minus food and energy) devided by Colombia's CPI, and for i we used the CD's 90 days interest rate.

We finally describe the parameters related to the exogenous shocks. We focus only on the productivity shock since it is the only one used in our simulations. For the productivity shock, A, we performed a standard Solow residual computation to obtain an autocorrelation coefficient of $\rho_1 = 0.83$. The standard deviation is calibrated to reproduce as closely as possible the observed output's volatility (using a Hodrick and Prescott filter it is 1.62%)[24]. Finally, the standard deviation of the forcing variable A is set to reproduce as closely as possible the observed output's volatility which was found to be 1.62% according to Hodrick and Prescott's filter.

The autocorrelation of the remaining shocks, government expenditures, preferences and risk premium were found to be 0.773, 0.8 and 0.69 respectivelly[25]. As we mentioned above this parameters are not considered in our simulation exercise.

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[24]Using labor, capital and product quarterly data from 1984:1 until 2003:4, and expressing the production function in logarithms one can solve for $log(A_t)$ in order to obtain a time series for A. From this new data we found an average value $\overline{A} = 1.19$ (in levels). The parameter ρ_1 was found by runing the follwing regression $log(A_t) = \rho_1 log(A_{t-1}) + (1 - \rho_1) log(\overline{A}) + \epsilon_t$ where ϵ is an error term. We performed a Wald's test to prove the null hypothesis $\rho_1 + (1 - \rho_1) = 1$ and we obtained a F-statistic value of 0.2156 and a P-value of 0.6437, so our null hypothesis is accepted, and A can actually follow a standard autorregressive process of order one as stated before.

[25]The autocorrelation ρ_2 of the variable $\frac{g_t}{y_t}$ was found by doing the following: we took the ratio between real total government expenditure and real GDP, we found the mean of this series and found $\overline{\frac{g}{y}} = 0.15$, then we runned a regression of the autorregressive process and found $\rho_2 = 0.773$ and that the standard deviation of the error is 0.0063.

For the preference shock we took the consumer sentiment survey made by "Fedesarrollo" and specifically used the consumer confidence index. We assumed that by construction the index has media zero, this is because consumers are asked if they feel positive or negative about something and the negative answers are subtracted from the positive ones, so in steady state opinions should be devided in half. As the media of the process was assumed to be zero, then we runned the regression of the autoregressive process without intercept and we found the autocorrelation ρ_3 of the variable μ_t^u to be $\rho_3 = 0.8$ and the standard deviation of the error 0.07.

The autocorrelation ρ_4 of the variable μ_t^ϑ was found by doing the the following: a daily series of the EMBI was used as a proxy of the variable μ_t^ϑ. As our model is quarterly then we found the quarterly geometric average of the series. We know that we are assuming that this variable has $\overline{\mu^\vartheta} = 1$, and so the intercept of the autorregresive process is zero, so we found the logarithm of our quarterly series and subtracted its mean from it. Then we runned a regression of the autorregressive process and found $\rho_4 = 0.69$ and that the standard deviation of the error is 0.0245.

4 Model Dynamics

4.1 Solving the Model

In order to solve the model, we first state the first order nonlinear dynamic system that characterizes the competitive equilibrium. In order to calculate the steady state we transform the system equations into their deterministic steady state representation and solve using numerical methods. Then we log-linearize around the deterministic steady state. At this stage the system is expressed in terms of relative deviations from the steady state.

After solving the model using the method of King, Plosser and Rebelo (2001,[11]) we obtain matrices $\mathbf{M}$ and $\mathbf{H}$ which generate the dynamic solution by iterating on the following two equations:

$$\begin{aligned} \mathbf{Y}_t &= \mathbf{H}\mathbf{x}_t \\ \mathbf{x}_{t+1} &= \mathbf{M}\mathbf{x}_t + \mathbf{R}\eta_{t+1} \end{aligned} \tag{27}$$

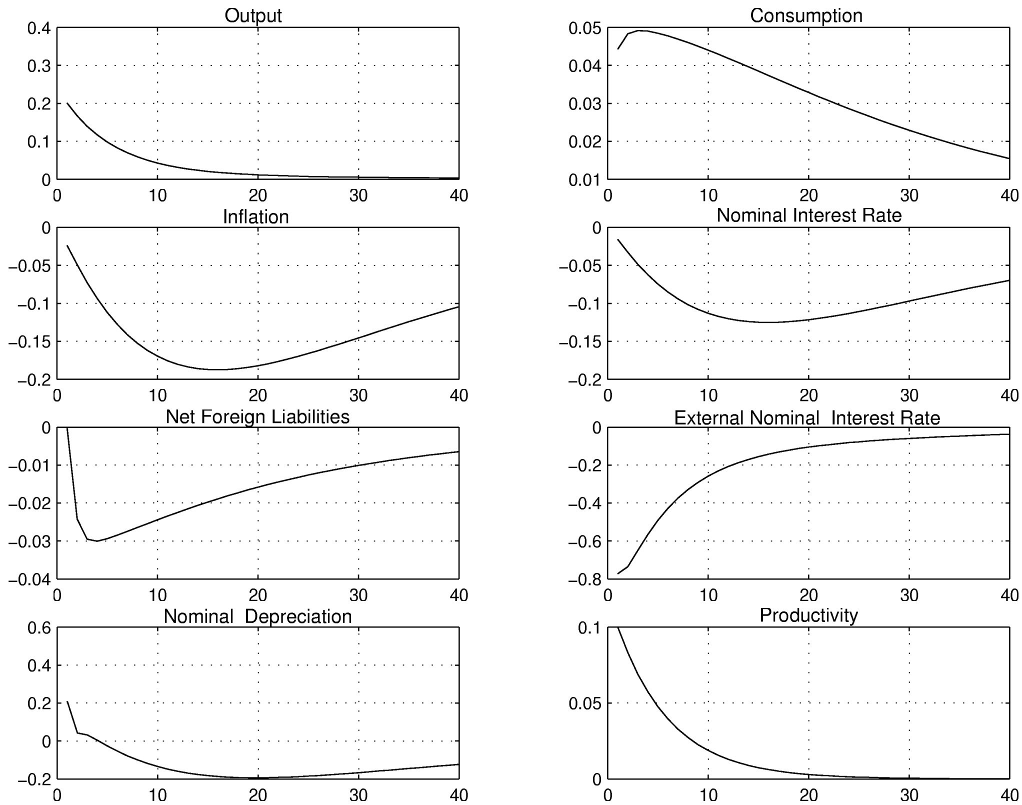
where $\mathbf{Y}$ is a vector composed by control, co-state and flow variables, $\mathbf{x}$ is a vector of endogenous and exogenous states, $\mathbf{H}$ characterizes the policy function and $\mathbf{M}$ the state transition matrix. η_{t+1} is an innovation vector and $\mathbf{R}$ is a matrix composed of zeros, ones or a parameter instead of a one. This matrix determines which variables are hit by the shock and in what magnitude. This state space representation will help us to compute the spectrum of the data.

4.2 Impulse Responses

We report the response of the model to a 10% shock to productivity, preferences, real government expenditures and risk premium. Figure 1 shows the impulse response to a productivity shock. Higher productivity today and in the future increase consumption, investment (not shown) and output. Ouput increases more than absortion and as a result inflation falls below the Central Bank's target[26]. The monetary authority responds reducing the nominal interest rate. For a given level of external debt and due to the increase in output, the risk premium falls and so does the interest rate that the economy faces externally. Capitals flow out of the country, that is the economy accumulates net foreign assets. In the balance of payments, the trade balance improves because output increases more than absorption and the net factor payments abroad fall. This is a standard result in small open economies: during productivity-driven booms the economy prepays external debt and this is reflected in a current account surplus. One interpretation is that the expectations of near-future debt repayments depreciate the nominal exchange rate on impact. Once the economy resumes the debt accumulation, the exchange rate appreciates. Note that between period one and two the exchange rate is

[26] The inflation that is falling is that of heterogeneous goods, and the output that increased was that of homogeneous goods, so the price of homogeneous goods decreased first and as the price of the heterogeneous good depends on the former, then it also falls.

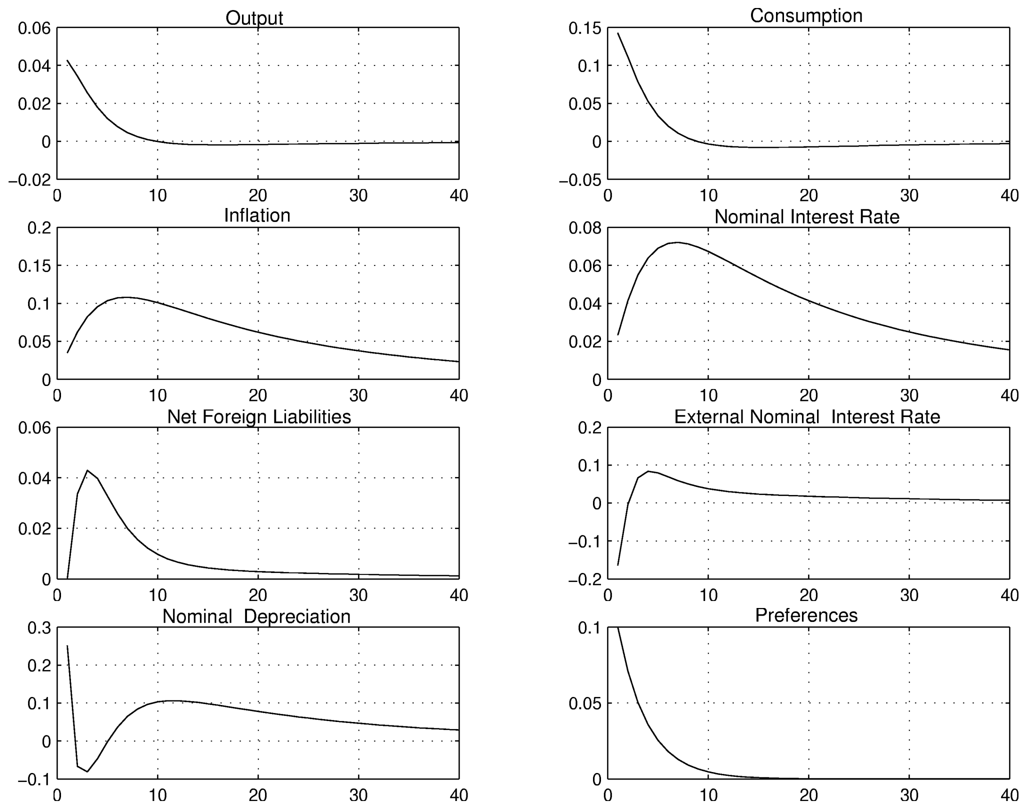
Figure 1: Productivity Shock



apreciating although agents are demanding more foreign assets (in this case one would expect to see the exchange rate depreciating), between this two periods what happens is that there is another transmission mechanism that's acting: as output increased more that absorption, by market clearing conditions the price of homogeneous goods decreaces and this is reflected as a whole in the exhange rate.

What happens to the economy when hit by a preference shock is shown in Figure 2. All of a sudden agents decide to consume more and so output increases. This demand driven expansion generates inflationary pressures, to which the Central Bank responds by increasing the interest rate. The trade balance deteriorates because the increase in consumption is higher than the increase in output and so agents will finance consumption with higher indebtness. Initially, the external interest rate falls, since the external debt to output ratio falls. As indebtness increase so does the external interest rate faced by agents. Inflationary preassures increase price level of homogenous goods, by PPP the nominal exchange rate depreciates on impact, since debt increase next period (net foreign assets decrease), the nominal exchange rate apreciates, from there on the economy starts to pay back debt (increase net foreign assets) making the nominal

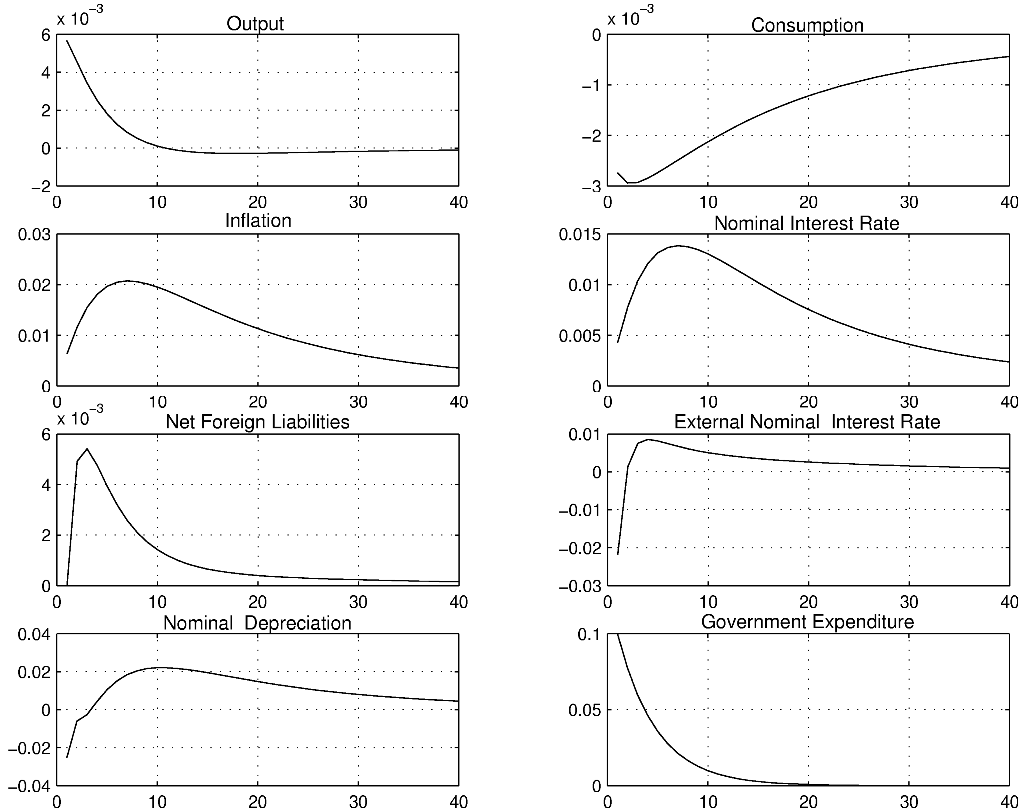
Figure 2: Preference Shock



exchange rate depreciate. The increase in foreign assets drives down the external interest rate.

Another interesting experiment is to analyze the impact of a transitory, but persistent government expenditure shock. Figure 3 shows the results. Recall that in the model, the governement finances governement consumption by using net lump sum taxes. So, net transfers to agents fall (net taxes increase). In order to finance governement expenditures agents take more external debt. The increase in government purchases "crowds out" consumption and investment, but still aggregate demand increases. Although equilibrium employment increases and so does output, this is not enough to compensate the absorption increase, so the trade balance deteriorates. This demand-driven shock increases inflation, calling for an interest rate hike by the Central Bank. Also the external interest rate falls, since the risk premium falls (recall that total external debt is given at the time of the shock and output has increased). Since debt is going to increase next period (net foreign assets decrease), the nominal exchange rate apreciates on impact. Between period one and period three approximately, the PPP mechanism is acting, so the increase in the price of homogeneous goods depreciates the exchange rate. From there on the economy starts to pay back debt (increase net foreign assets) making

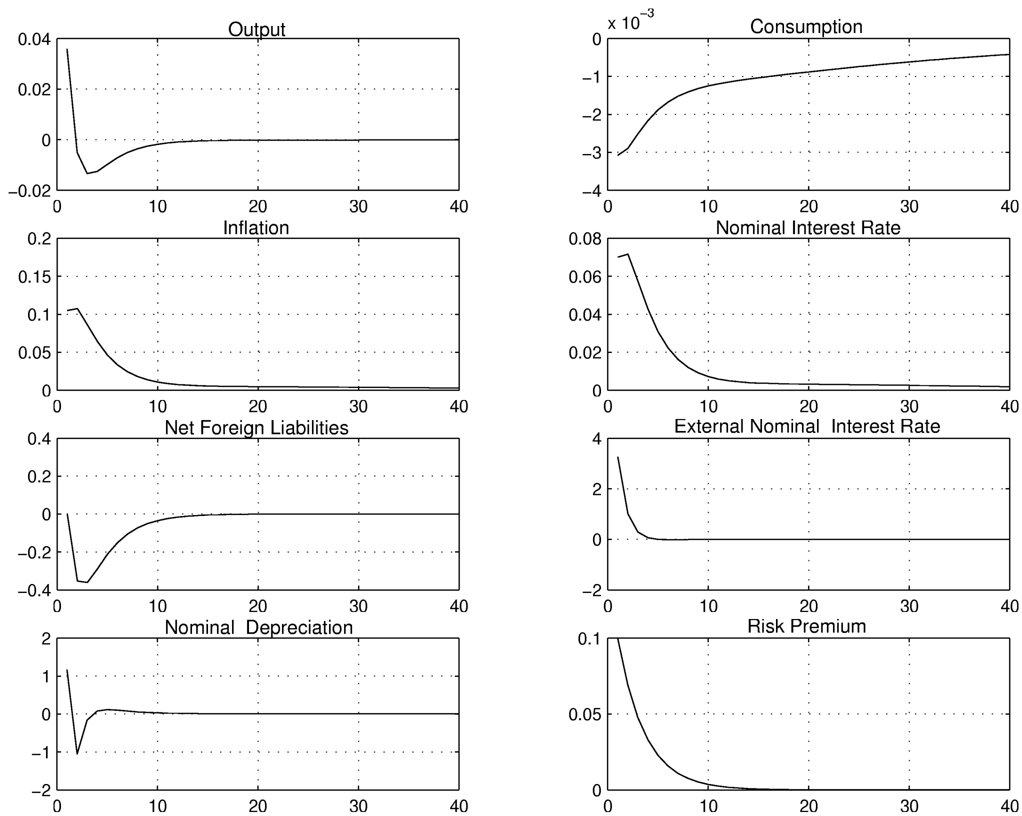
Figure 3: Government Expenditure Shock



the nominal exchange rate depreciate and then slowly go back to its steady state level. The increase in foreign assets drives down the external interest rate.

Finally Figure 4 shows the case of a Risk Premium shock. When the risk premium increases it causes an increase in the external nominal interest rate. As debt becomes more expenssive, agents are going to want to repay debt, this expectations make the nominal depreciation rate to depreciate on impact. In order to be able to pay debt, agents reduce their consumption and investment (not shown), and decide to work more (not shown). The increase in hours worked increases output. An imperfect pass-through is observed from the nominal exchange rate into the prices of heterogeneous goods, which causes inflation to rise. As a response, the monetary authority increases the nominal interest rate. As government expenditure is constant and consumption and investment decreased, the increase in output causes an excess of supply that generates a fall in the prices of the homogeneous good, this is what causes an apreciation and the following behavior of the exchange rate.

Figure 4: Risk Premium Shock



4.3 Sensitivity Analysis

We now study the properties of the dynamic response of the model to two key parameters: the degree of price stickiness and the sensitivity of the risk premium to the external debt to output ratio.

4.3.1 More Flexible Prices

In our benchmark calibration we had set $\epsilon = 0.75$, so that retailers adjust prices every year. Now we show how the dynamics of the model change as retailers adjust prices every 6 months ($\epsilon = 0.5$). Figures 6 to 8 in appendix 4 show the results. By increasing the degree of price flexibility we change the persistence and volatility of most of the nominal variables. Most of the real variables remain unchanged. So, as prices are more flexible:

1. Inflation becomes more responsive to all types of shocks. The response is considerably higher when the sources of the shock comes from the demand side (preferences and public expenditures). However, for more flexible prices the persistence is slightly lower.
2. As a consequence, the Central Bank adjusts the nominal interest rate more but for a shorter period of time.
3. On impact, the nominal depreciation is less responsive to productivity shocks and more responsive to public expenditure shocks. However, the subsequent adjustment process is more agressive when prices are more sticky and the degree of persistence is higher.
4. The persistence of real consumption falls, for the productivity and public expenditure shocks.
5. There is little effect on the response of output, net foreign assets and the external nominal interest rate for all types of shocks.

4.3.2 More Debt-Elastic Risk Premium

In the baseline calibration we had that ($\omega_3 = 0.1$) and we study what happens as the degree of sensitivity to the risk premium is higher ($\omega_3 = 0.5$). Figures 9 to 11 in appendix 4 show the results. By increasing the degree of response of the risk premium to debt, we change the persistence and volatility of the variables related to the external sector. There is little effect on domestic variables. So, as the risk premium becomes more sensitive to debt:

1. The external nominal interest rate becomes more responsive for all types of shocks.
2. The net foreign assets are less responsive for all shocks.
3. There is little or no effect on the rest of the variables for productivity, preferences and government expenditure shocks.

4. In the risk premium shock output, consumption, inflation and the nominal interest rate are less responsive for the higher elasticity.

5 Validating the Model

5.1 The Theoretical Model Vs. the Observed Data

In order to asses the extent to which the calibrated model replicates salient and/or interesting features of the actual economy, we follow a frequency domain methodology proposed by Diebold et al. (1998,[19]). In this subsection we sumarize the methodology and present our results concerning the agreement of the data spectrum with the model spectrum.

The methodology consists of five steps. First, we took a series for the Gross Domestic Product (GDP) and another one for inflation[27], logarithms were applied to the GDP series and then both series (inflation and output) were seasonally adjusted using the X12 filter and then filtered using Hodrick and Prescott, so that frequencies beyond eight years were eliminated. Second, we estimate the sample data spectrum and compute its uncertainty using bootstrap techniques. Third, from the estimated spectrum and its uncertainity we determine the salient and/or interesting features that we expect the theoretical model to reply. Fourth, we compute the model's theoretical spectrum. Finally, we compare the theoretical and observed estimated spectrums at the required frequencies. The methodology proposed by Diebold et al. goes a little further by proposing an spectral maximum likelihood estimation technique to calibrate the model parameters by minimizing the disagreement between sample and theoretical spectrums at pre defined frequencies. This step is left for future work.

5.2 Estimating the Spectra

For an N-variate linearly regular covariance stationary process with population autocovariance matrices $\mathbf{\Gamma}(\tau) = E\left[(\mathbf{Y}_{t+\tau} - \mu)(\mathbf{Y}_t - \mu)^T\right]$, the population spectra at frequency ω is defined as

$$\mathbf{F_Y}(\omega) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} \mathbf{\Gamma}(\tau) \exp(-iw\tau)$$

for $\omega \in [-\pi, \pi]$. An important property of the spectra is that

$$\int_{-\pi}^{\pi} \mathbf{F_Y}(\omega) \exp(iw\tau)\, d\omega = \mathbf{\Gamma}_\tau$$

which is particularly useful when $\tau = 0$. See Hamilton (1994,[9]) Chapters 6 and 10.

[27]The series for the GDP was constructed as follows: For the period 1994-2003, the quarterly data was taken from the national accounts statistics reported by the colombian national department of statistics (DANE). For the period 1977-2003, this series was backward-chained using the quarterly growth rate reported for this period by the national department of planning (DNP). The inflation series is from the Central Bank.

The diagonal elements of $\mathbf{F_Y}(\omega)$, $f_{kk}(\omega)$, are the univariate spectra. According to the spectral representation theorem, areas under this curve are the relative contribution of the frequencies to the total unconditional variance of the k^{th} variable.

Off diagonal elements, $f_{kl}(\omega)$, are the cross spectral densities, and can be expressed in polar form as

$$f_{kl}(\omega) = ga_{kl}(\omega) \times \exp\{i \times ph_{kl}(\omega)\}$$

where $ga_{kl}(\omega) = \sqrt{re^2(f_{kl}(\omega)) + im^2(f_{kl}(\omega))}$ is the gain and $ph_{kl}(\omega) = \arctan\{im(f_{kl}(\omega))/re(f_{kl}(\omega))\}$ is the phase at a frequency ω. The gain tells us by how much the amplitude of y_l has to be multiplied in order to reach the amplitude of y_k at a same frequency ω. The phase measures the lead of y_k over y_l at frequency ω (The phase shift in time units is $ph(\omega)/\omega$). Instead of using the gain it is costumarily to report the coherence defined as $coh_{kl}(\omega) = ga^2(\omega)/(f_{kk}(\omega) \times f_{ll}(\omega))$, which measures the squared correlation between y_k and y_l at a frequency ω (See Hamilton (1994,[9]) Chapter 10).

An obvious non parametric way to estimate the population spectra based on a sample $\{\mathbf{Y}_t\}_{t=1}^{T}$, is to replace the population autocovariances and mean vector μ with sample quantities so that the sample autocovariance at lag τ becomes

$$\widehat{\mathbf{\Gamma}}_\tau = \frac{1}{T}\sum_{t=\tau}^{T-1}\left(\mathbf{Y}_{t-\tau} - \overline{\mathbf{Y}}\right)\left(\mathbf{Y}_t - \overline{\mathbf{Y}}\right) \quad \text{for} \quad -(T-1) \le \tau \le (T-1)$$

and the estimated spectra

$$\widehat{\mathbf{F}}_{\mathbf{Y}}(\omega_j) = \frac{1}{2\pi}\left\{\sum_{\tau=-(T-1)}^{T-1} \widehat{\mathbf{\Gamma}}_\tau \exp(-iw_j\tau)\right\}$$

evaluated at frequencies $\omega_j = 2\pi j/T$ for $j = 1, 2, 3, ..., T/2 - 1$. However, this sample estimate is not consistent. A consistent estimate may be found by windowing the autocovariances sequence using the Blackman-Tuckey approach which gives an estimated spectra of the form,

$$\widehat{\mathbf{F}}^*_{\mathbf{Y}}(\omega_j) = \frac{1}{2\pi}\left\{\sum_{\tau=-(T-1)}^{T-1} \mathbf{\Lambda}(\tau)\,\widehat{\mathbf{\Gamma}}_\tau \exp(-iw_j\tau)\right\}$$

where the window function $\mathbf{\Lambda}(\tau)$ is a matrix of lag windows[28]. By adjusting the lag window according to the sample size we can simultaneously reduce the bias and variance of the spectra estimate and hence obtain a consistent estimator of the population spectra. This approach is the same as smoothing the estimated sample periodogram using an equivalent spectral kernel. From this spectrum estimate we obtain estimates of the population coherence and phase.

[28] A window lag matrix is a generally truncated symetric and positive, weighting function for the lags. The truncation lag defines the window size, and outside this window the weights are zero. By giving small or zero weights to long lagged autocovariance matrices (the poorly estimated ones), the estimated spectra becomes smoother and consistent at the cost of some small sample bias.

5.3 Assesing Sample Variability

In order to asses the sampling variability of this estimator, Diebold et al. propose to use a resampling algorithm called the Cholesky factor bootstrap. If the vector sequence $\varepsilon^{(i)}$ is a random sample of an NT dimensional standard distribution, $(\mathbf{0}_{NT}, \mathrm{I_{NT}})$, then

$$\mathbf{z}^{(i)} = \overline{\mathbf{z}} + \mathbf{P}^{*}\varepsilon^{(i)} \sim \left(\mathbf{1}_T \otimes \mu, \mathbf{\Sigma}^{*} = \mathbf{P}^{*}\mathbf{P}^{*T}\right)$$

where $\overline{\mathbf{z}} = \mathbf{1}_T \otimes \overline{\mathbf{Y}}$ and $\mathbf{\Sigma}^{*}$ is the corresponding variance covariance matrix obtained from the estimated autocovariance matrices multiplied by the corresponding window functions.

For each iteration $(i = 1, 2, 3, ..., R)$ we randomly draw $\mathbf{z}^{(i)}$ and from this we compute $\widehat{\mathbf{F}}^{*(i)}(\omega_j)$ for $\omega_j = 2\pi j/T$ $(j = 1, 2, 3, ..., T/2 - 1)$ and then construct the confidence intervals for the spectra, cross-spectra, coherence and phase of the vector.

5.4 The Theoretical Model Spectra

Given that the model can be written in a State Space Form

$$\begin{aligned} \mathbf{Y}_t &= \mathbf{H}\mathbf{x}_t \\ \mathbf{x}_{t+1} &= \mathbf{M}\mathbf{x}_t + \mathbf{R}\eta_{t+1} \end{aligned} \tag{28}$$

where the innovation vector η_{t+1} is iid$(\mathbf{0}, \mathbf{\Omega})$, it is straightforward to compute the theoretical model spectra by simple spectral density arithmetic (See Hamilton (1994,[9]) Chapter 10). Notice that equation (28) is closely related to (27).

When this is not posible (that is when $\mathbf{\Omega}$ is singular or when observed data and model are assumed to arise from different sets of transformations), it is advisable to generate a very long simulated path of the variables subject to countinuous innovations, and estimate the spectra from this simulation. If the simulation is long enough, the sampling errors are negligible.

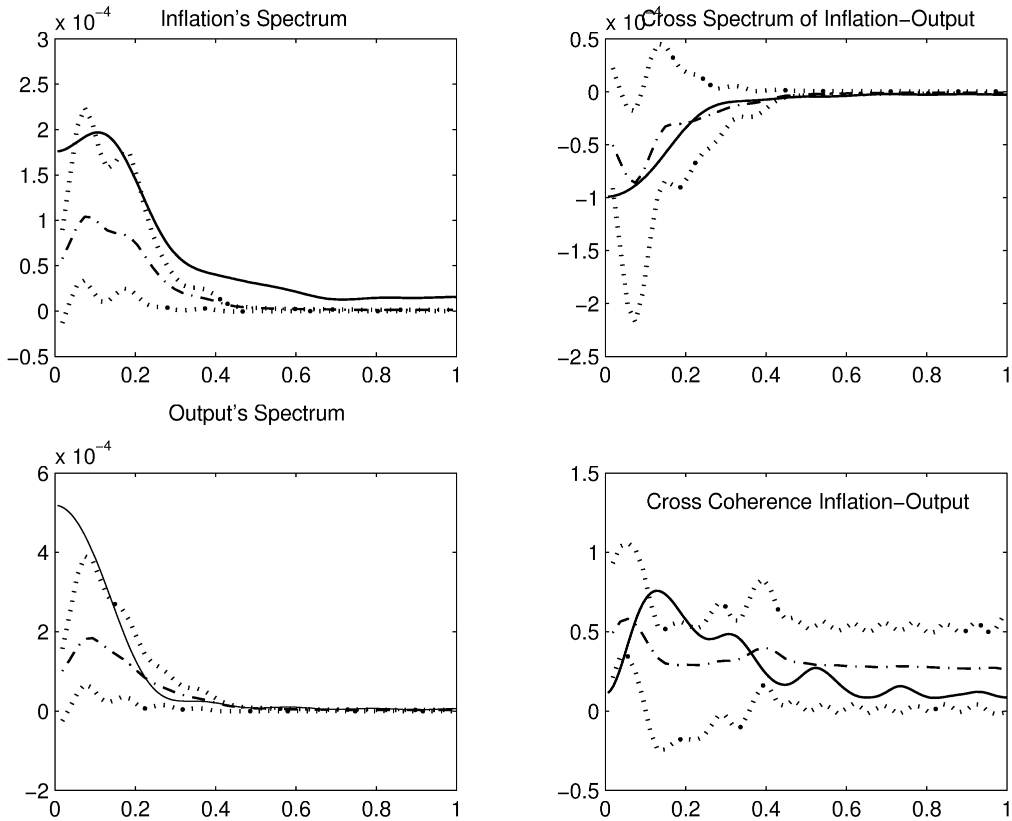
In our case we followed the second methodology. We generated artificial data and filtered it with Hodrick and Prescott, then we took the observed data and filtered it with Hodrick and Prescott as well in order to have two groups of series in the same frequencies to be able to compare them.

5.5 Results

Figure 5 contains, on the upper and lower left panels, the estimated inflation and output gap spectrums along with the corresponding 95% uncertainty bands and the model theoretical spectra. The upper right panel contains the estimated inflation and output gap cross spectral density together with its uncertainty bands and the theoretical cross spectrum. On the lower right panel we find the estimated coherence of inflation and output gap, its uncertainty and the theoretical one.

From the two left panels, that is the univariate spectra, we find that the spectral density is statistically significant for frecuencies between $0,04\pi$ and $0,4\pi$ which correspond to periods

Figure 5: Spectrums and Coherence



The solid line corresponds to the model theoretical spectrum. The dotted line corresponds to the estimated data spectrum. The dashed lines corrspond to the 95% upper and lower bands of the data spectrum respectively.

between 2 and 25 quarters, and variations along these frequencies explain at least 80% of the observed sample variability. The estimated spectra shows a peak for frequencies between $0,08\pi$ and $0,1\pi$ which correspond to periodic movements between 10 and 12 quarters. The estimated cross periodogram is negative for all frequencies and significant for frequencies between $0,04\pi$ and $0,1\pi$, that is for periods between 10 and 25 quarters. The population coherence is statistically significant at frequencies of up to $0,092\pi$, that is for periodic movements beyond 11 quarters, and is not dominated by any particular frequency although it presents a peak at $0,05\pi$, with a correlation of 0.74 for movements arround 20 quarters. It is also scattered significant for some high frequencies .

From these figures we derive the salient features of the data that the model has to mimic. First, inflation and output gap are dominated by periodic movements between 2 and 25 quarters with a peak between 10 and 12 quarters, which could show some degree of stickiness or persistence. The cross spectrum and coherence show results in the same direction. The population coherence does not seem to be dominated by a particular set of frequencies. However, there is a peak correlation of 0.74 for movements around 20 quarters.

The theoretical model frequency analysis shows some persistence both in the univariate spectra as well as in the cross spectrum, with monotone spectrum for output gap and cross spectrum. The inflation spectrum peaks at a frequency of $0,10\pi$, that is, for periodic movements between 9 and 10 quarters. The model's theoretical coherence presents clear dominance in frequencies between $0,05\pi$ and $0,45\pi$, that is periodic movements between 2 and 20 quarters, with a maximum coherence at $0,12\pi$, that is periodic movements between 8 and 9 quarters.

The comparison between sample and theoretical spectra and cross spectra reveals important similarities. The theoretical spectra and cross spectra fall into the sample uncertainty bands for frequencies beyond $0,05\pi$, that is for periodic movements of inflation and comovements of inflation and output gap of up to 20 quarters, that is 5 years, and for periodic movements of output gap of up to 10 quarters (2 and a half years). For shorter frequencies the spectra and cross spectra of the model are significantly different from the sample ones. The model's coherence falls into the uncertainty bands for most of the frequencies but the ones sorrounding the peak of the model's coherence, and very long run periodic movements.

We conclude that the model theoretical spectra and cross spectra does not differ statistically from the respective population quantities for, at least, frequencies beyond $0,05\pi$, which correspond to periodic movements of up to at least 10 quarters. Population's coherence is not statistically different from the model's coherence at most of the frequencies, it is only statistically different at the peak of the model's theoretical coherence and for very short frequencies (very long run period movements).

6 Final Remarks

This paper evaluates *quantitatively* how much of the observed dynamics of the Colombian macroeconomic data can be explained by a model in which the presence of nominal rigidities is important in the context of a small open economy. We explore the macroeconomic effects of different types of shocks (productivity, preference, government expenditure and risk premium shocks) and the implications for monetary policy when operating under an Inflation Targeting framework. The main monetary policy instrument is the nominal interest rate. We also study the macroeconomic effects of higher price flexibility and a lower sensitivity of the risk premium to the debt to output ratio.

We find that as prices become more flexible, inflation becomes more responsive to all types of shocks and so the Central Bank has to respond with higher interest rates. However, the effects of the shocks are less persistent although this difference is not substantial. In addition, the dynamic response of output, net foreign assets and the external interest rate doesn't change significantly. When the risk premium is more sensitive to external debt to output ratio, net foreign assets become less sensitive and nominal depreciation becomes more sensitive to all shocks, but the productivity one.

We go further and take the model to data. In particular, we evaluate the ability of our calibrated model to reproduce the behavior of observed cyclical inflation and output gap, when productivity shocks are the main source of fluctuations. We follow a frequency domain comparison methodology proposed by Diebold et al. (1998,[19]). The Colombian data is characterized by: first, inflation and output gap are dominated by periodic movements between 2 and 25 quarters with a peak between 10 and 12 quarters. The cross spectrum and coherence show results in the same direction. Second, the coherence does not show any significant dominance of frequencies for the cross movements but the correlation jumps to $0,6$ for periodic movements around 5 quarters. These facts are compared to the data simulated from the model. We conclude that the simulated data spectra and cross spectra does not differ statistically from the respective population quantities for at least 10 quarters. The model spectra presents more persistance than the observed data and the population coherence is captured for most frequencies but the ones around the peak of the model's theoretical coherence and very long run periodic movements. It is also possible that the data displays a high degree of persistence due to the fact that the Colombian economy has suffered a long gradual disinflation period. A long disinflation period may induce a high degree of persistence in the data, that may not be present in the future. Subsequent research will address these issues.

At the theoretical level, a number of extensions are left for future work. First, one can evaluate the efficiency and welfare effects of alternative monetary policy rules. We have focused here on Taylor rules, but we can explore alternative specifications such as Inflation Forecast rules. In fact some recent work of Laxton and Pesenti (2003, [13]) and Levin, Wieland and Williams (2001, [14]) has evaluated the efficiency of alternative monetary policy rules. Second, we have used this model for an Emerging Market economy like Colombia. However, there is nothing particular in our model that pertains to an Emerging Economy. In fact our model can also be used for a small open developed economy. Emerging Markets are characterized by a number of imperfections, like borrowing constraints, domestic financial markets imperfections, a high share of non-tradable sector and balance sheet effects of nominal depreciations to name a few. It would be interesting to explore many of these issues.

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Appendix 1: Demand for the diferentiated consumption good

The following is the problem that has to be solved in order to find the demand function:

$$\max_{c(z)_t} P_t^c * c_t$$

s.t.

$$\int_0^1 p^c(z)_t c(z)_t dz$$

or what is the same

$$\max_{c(z)_t} P_t^c \left[\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

s.t.

$$\int_0^1 p^c(z)_t c(z)_t dz$$

deriving with respect to $c(z)$

$$P_t^c \left[\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right]^{\frac{1}{\theta-1}} c(z)^{\frac{-1}{\theta}} = p^c(z)_t$$

$$\frac{P_t^c}{p^c(z)_t} \left[\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right]^{\frac{1}{\theta-1}} = c(z)^{\frac{1}{\theta}}$$

$$\left(\frac{P_t^c}{p^c(z)_t} \right)^{\theta} \left[\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} = c(z)$$

as

$$c_t = \left[\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

then

$$c(z)_t = \left(\frac{P_t^c}{p^c(z)_t} \right)^{\theta} c_t$$

Appendix 2: Optimal price chosen by retailers

$$\max_{p^c(z)_{t+j}} E_t \sum_{j=0}^{\infty} (1-\varepsilon)\varepsilon^j \Delta_{t+j} c_{t+j} \left[\left(\frac{p^c(z)_{t+j}}{P_{t+j}^c} \right)^{1-\theta} - \varphi_{t+j} \left(\frac{p^c(z)_{t+j}}{P_{t+j}^c} \right)^{-\theta} \right]$$

In period t the firm is going to choose a price for the whole horizon of time so $p^c(z)_{t+j} = p^c(z)_t$(they choose prices from now on):

$$\max_{p^c(z)_{t+j}} E_t \sum_{j=0}^{\infty} (1-\varepsilon)\varepsilon^j \Delta_{t+j} c_{t+j} \left[\left(\frac{p^c(z)_t}{P_{t+j}^c} \right)^{1-\theta} - \varphi_{t+j} \left(\frac{p^c(z)_t}{P_{t+j}^c} \right)^{-\theta} \right]$$

deriving with respect to $p^c(z)_t$

$$E_t \sum_{j=0}^{\infty} (1-\varepsilon)\varepsilon^j \Delta_{t+j} c_{t+j} \left[\frac{(1-\theta)}{P_{t+j}^c} \left(\frac{p^c(z)_t}{P_{t+j}^c} \right)^{-\theta} - \theta \frac{\varphi_{t+j}}{P_{t+j}^c} \left(\frac{p^c(z)_t}{P_{t+j}^c} \right)^{-\theta-1} \right] = 0$$

$$E_t\sum_{j=0}^{\infty}(1-\varepsilon)\varepsilon^j\Delta_{t+j}c_{t+j}\theta\frac{\varphi_{t+j}}{P^c_{t+j}}\left(\frac{p^c(z)_t}{P^c_{t+j}}\right)^{-\theta-1}+E_t\sum_{j=0}^{\infty}(1-\varepsilon)\varepsilon^j\Delta_{t+j}c_{t+j}\frac{(1-\theta)}{P^c_{t+j}}\left(\frac{p^c(z)_t}{P^c_{t+j}}\right)^{-\theta}=0$$

$$(1-\varepsilon)\left(p^c(z)_t\right)^{-\theta-1}\theta E_t\sum_{j=0}^{\infty}\varepsilon^j\Delta_{t+j}c_{t+j}\frac{\varphi_{t+j}}{\left(P^c_{t+j}\right)^{-\theta}}+(1-\varepsilon)(1-\theta)\left(p^c(z)_t\right)^{-\theta}E_t\sum_{j=0}^{\infty}\varepsilon^j\Delta_{t+j}c_{t+j}\left(P^c_{t+j}\right)^{\theta-1}$$

$$\left(p^c(z)_t\right)^{-\theta-1}\theta E_t\sum_{j=0}^{\infty}\varepsilon^j\Delta_{t+j}c_{t+j}\frac{\varphi_{t+j}}{\left(P^c_{t+j}\right)^{-\theta}}=(\theta-1)\left(p^c(z)_t\right)^{-\theta}E_t\sum_{j=0}^{\infty}\varepsilon^j\Delta_{t+j}c_{t+j}\left(P^c_{t+j}\right)^{\theta-1}$$

rewriting for $p^c(z)_t$ to obtain the optimal price

$$p_t^{opt}=\frac{\theta}{\theta-1}E_t\left[\frac{\sum_{j=0}^{\infty}\varepsilon^j\Delta_{t+j}c_{t+j}\varphi_{t+j}\left(P^c_{t+j}\right)^{\theta}}{\sum_{j=0}^{\infty}\varepsilon^j\Delta_{t+j}c_{t+j}\left(P^c_{t+j}\right)^{\theta-1}}\right]$$

dividing both sides by P^c_t and multiplying and dividing by $\frac{1}{(P^c_t)^{\theta}}$

$$\frac{p_t^{opt}}{P^c_t}=\frac{\theta}{\theta-1}E_t\left[\frac{\sum_{j=0}^{\infty}\varepsilon^j\Delta_{t+j}c_{t+j}\varphi_{t+j}\left(\frac{P^c_{t+j}}{P^c_t}\right)^{\theta}}{\sum_{j=0}^{\infty}\varepsilon^j\Delta_{t+j}c_{t+j}\left(\frac{P^c_{t+j}}{P^c_t}\right)^{\theta-1}}\right]\tag{29}$$

From the numerator: $E_t\sum_{j=0}^{\infty}\varepsilon^j\Delta_{t+j}c_{t+j}\varphi_{t+j}\left(\frac{P^c_{t+j}}{P^c_t}\right)^{\theta}$, so we define

$$E_t\Theta_{t+1}=E_t\left(\Delta_{t+1}c_{t+1}\varphi_{t+1}\left(\frac{P^c_{t+1}}{P^c_{t+1}}\right)^{\theta}\right)+\varepsilon E_t\left(\Delta_{t+1+1}c_{t+1+1}\varphi_{t+1+1}\left(\frac{P^c_{t+2}}{P^c_{t+1}}\right)^{\theta}\right)$$
$$+\varepsilon^2E_t\left(\Delta_{t+1+2}c_{t+1+2}\varphi_{t+1+2}\left(\frac{P^c_{t+3}}{P^c_{t+1}}\right)^{\theta}\right)+......$$

and

$$\Theta_t=\Delta_tc_t\varphi_t\left(\frac{P^c_t}{P^c_t}\right)^{\theta}+\varepsilon E_t\left(\Delta_{t+1}c_{t+1}\varphi_{t+1}\left(\frac{P^c_{t+1}}{P^c_t}\right)^{\theta}\right)+\varepsilon^2E_t\left(\Delta_{t+2}c_{t+2}\varphi_{t+2}\left(\frac{P^c_{t+2}}{P^c_t}\right)^{\theta}\right)+......$$

so

$$E_t\left(\left(P_{t+1}^c\right)^\theta \Theta_{t+1}\right) = E_t\left(\Delta_{t+1}c_{t+1}\varphi_{t+1}\left(P_{t+1}^c\right)^\theta\right) + \varepsilon E_t\left(\Delta_{t+1+1}c_{t+1+1}\varphi_{t+1+1}\left(P_{t+2}^c\right)^\theta\right) + \varepsilon^2 E_t\left(\Delta_{t+1+2}c_{t+1+2}\varphi_{t+1+2}\left(P_{t+3}^c\right)^\theta\right) + ...$$

$$\left(P_t^c\right)^\theta \Theta_t = \Delta_t c_t \varphi_t \left(P_t^c\right)^\theta + \varepsilon E_t\left(\Delta_{t+1}c_{t+1}\varphi_{t+1}\left(P_{t+1}^c\right)^\theta\right) + \varepsilon^2 E_t\left(\Delta_{t+2}c_{t+2}\varphi_{t+2}\left(P_{t+2}^c\right)^\theta\right) + ...$$

$$\left(P_t^c\right)^\theta \Theta_t = \Delta_t c_t \varphi_t \left(P_t^c\right)^\theta + \varepsilon E_t\left(\left(P_{t+1}^c\right)^\theta \Theta_{t+1}\right)$$

dividing both sides of the equation by $\left(P_t^c\right)^\theta$:

$$\Theta_t = \Delta_t c_t \varphi_t + \varepsilon E_t\left(\left(\frac{P_{t+1}^c}{P_t^c}\right)^\theta \Theta_{t+1}\right)$$

$$\Theta_t = \Delta_t c_t \varphi_t + \varepsilon E_t\left((1+\pi_{t+1}^c)^\theta \Theta_{t+1}\right)$$

In a similar way, from the denominator of 29 $E_t \sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \left(\frac{P_{t+j}^c}{P_t^c}\right)^{\theta-1}$ one can obtain:

$$\Psi_t = \Delta_t c_t + \varepsilon E_t\left((1+\pi_{t+1}^c)^{\theta-1} \Psi_{t+1}\right)$$

Appendix 3: The Complete Model

$$c_t + x_t + g_t + F_{t+1} - \left(1 + i_{t+1}^f\right) F_t = A_t k_t^\alpha h_t^{1-\alpha}$$

$$u_{c_t}\left(c_t, H_t, h_t\right) + \eta_t \rho = \lambda_t 1 + \Phi_{c_t}\left(c_t, m_{t+1}, x_t\right)$$

$$u_{h_t}\left(c_t, H_t, h_t\right) + \lambda_t q_t A_t (1-\alpha) k_t^\alpha h_t^{-\alpha} = 0$$

$$\beta E_t\left(\lambda_{t+1} q_{t+1} A_{t+1} \alpha k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} + \gamma_{t+1}\left(f\left(\frac{x_{t+1}}{k_{t+1}}\right) + \frac{\partial\left(f\left(\frac{x_{t+1}}{k_{t+1}}\right) k_{t+1}\right)}{\partial\left(k_{t+1}\right)}\right) + \gamma_{t+1}(1-\delta)\right) = \gamma_t$$

$$\beta E_t\left(\frac{\lambda_{t+1}}{\left(1+\pi_{t+1}^c\right)\left(1+\Phi_{m_{t+1}}\left(c_t, m_{t+1}, x_t\right)\right)}\right) = \lambda_t$$

$$\beta E_t \left(\frac{\lambda_{t+1} \left(1 + i_{t+1}\right)}{\left(1 + \pi^c_{t+1}\right)} \right) = \lambda_t$$

$$\beta E_t \left(\lambda_{t+1} \left(1 + i^f_{t+1} \right) q_{t+1} \right) = \lambda_t q_t$$

$$\beta E_t \left(\eta_{t+1} + U_{H_{t+1}} \left(c_{t+1}, H_{t+1}, h_{t+1} \right) - \eta_{t+1} \rho \right) = \eta_t$$

$$\lambda_t \left(\Phi_{x_t} \left(c_t, m_{t+1}, x_t \right) + q_t \right) = \gamma_t \left(c_1 + \frac{2 c_2 x_t}{k_t} \right)$$

$$k_{t+1} - (1+\delta) k_t - f \left(\frac{x_t}{k_t} \right) k_t = 0$$

$$H_{t+1} - H_t - \rho (c_t - H_t) = 0$$

$$i_t = i + \zeta \left(\pi^c_t - \overline{\pi}^c \right) + \xi \left(y_t - \overline{y} \right)$$

$$\left(1 + i^f_t \right) = \left(1 + i^*_t \right) \left(1 + \vartheta \left(\frac{F_t}{y_t} \right) \right)$$

$$P^c_t = \left[\varepsilon (p^{rule}_t)^{1-\theta} + (1-\varepsilon)(p^{opt}_t)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

$$(1 + \pi^c_t) = \left(\varepsilon (1 + \pi^c_{t-1})^{(1-\theta)} + (1-\varepsilon) \left(\frac{p^{opt}_t}{P^c_t} \right)^{(1-\theta)} (1 + \pi^c_t)^{(1-\theta)} \right)^{\frac{1}{1-\theta}}$$

$$p^{rule}_t = p^c_{t-1} (1 + \pi^c_{t-1})$$

$$\frac{p^{opt}_t}{P^c_t} = \frac{\theta}{\theta - 1} \frac{\Theta_t}{\Psi_t}$$

$$\Theta_t = \Delta_t c_t q_t + \varepsilon E_t \left((1 + \pi^c_{t+1})^{\theta} \Theta_{t+1} \right)$$

$$\Psi_t = \Delta_t c_t + \varepsilon E_t \left((1 + \pi^c_{t+1})^{\theta - 1} \Psi_{t+1} \right)$$

Appendix 4: Figures for Sensitivity Analysis of Impulse Responses

This appendix presents the figures used for sensitivity analysis of section 4.3.

Figure 6: Productivity shock when prices are more flexible

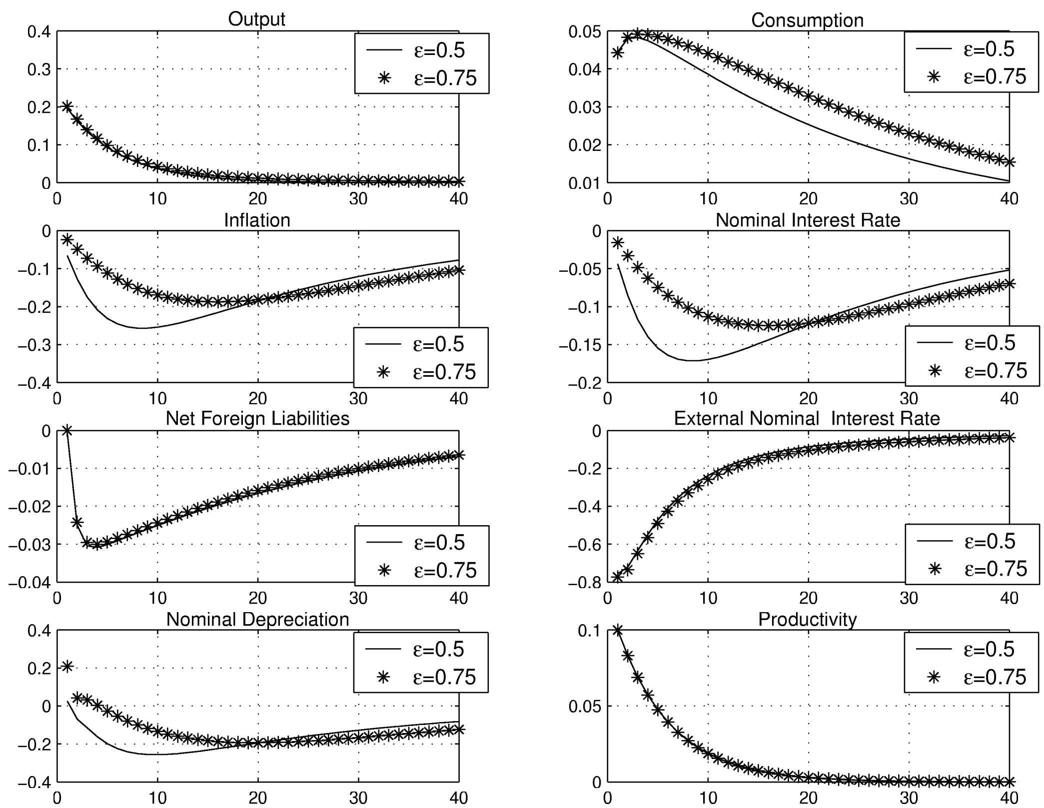

Figure 7: Preferences shock when prices are more flexible

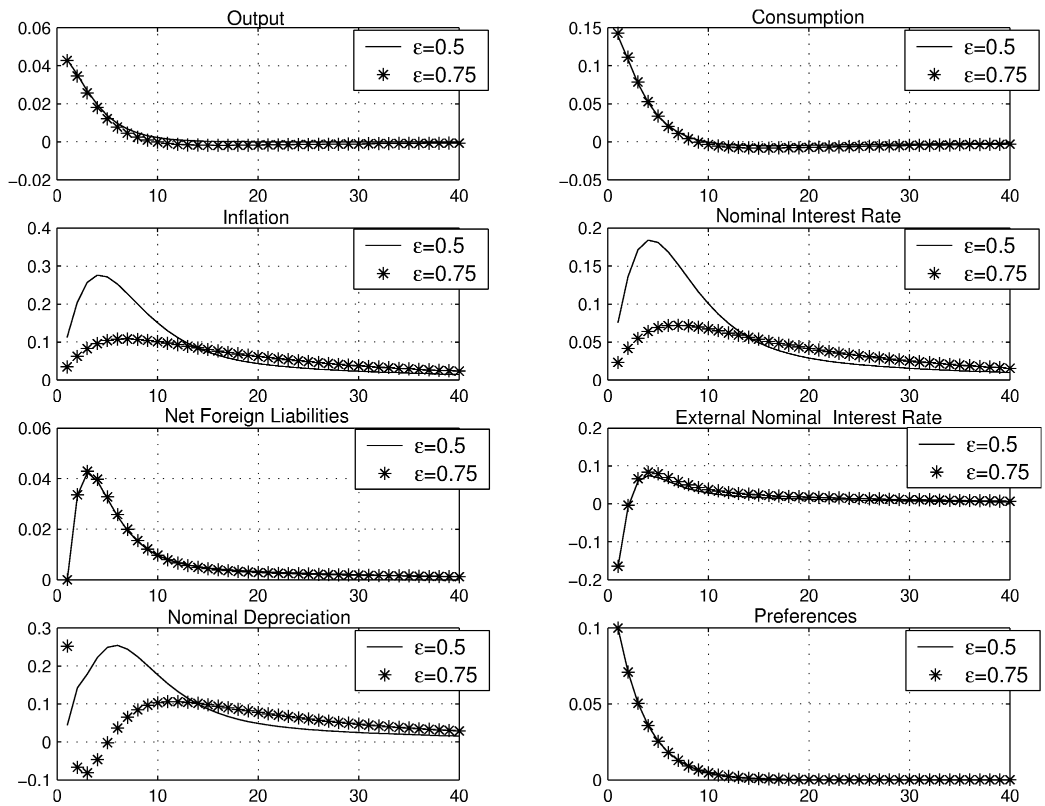

Figure 8: Government Expenditures shock when prices are more flexible

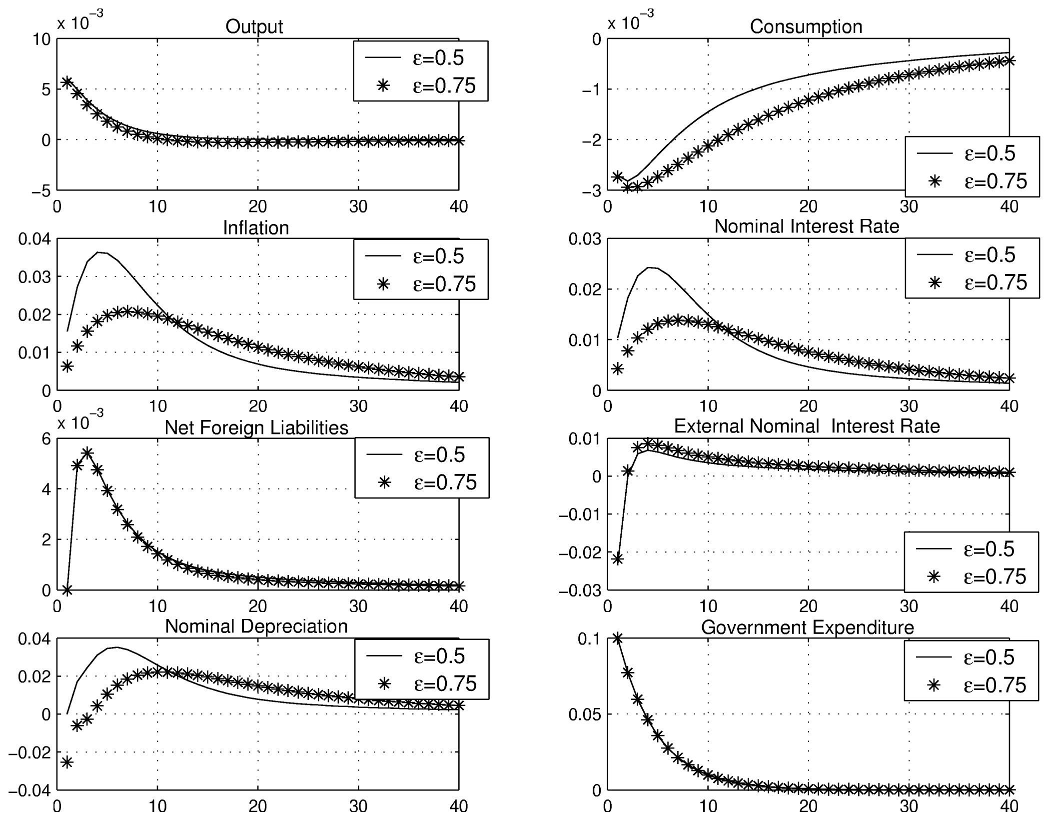

Figure 9: Productivity shock when the risk premium is more elastic

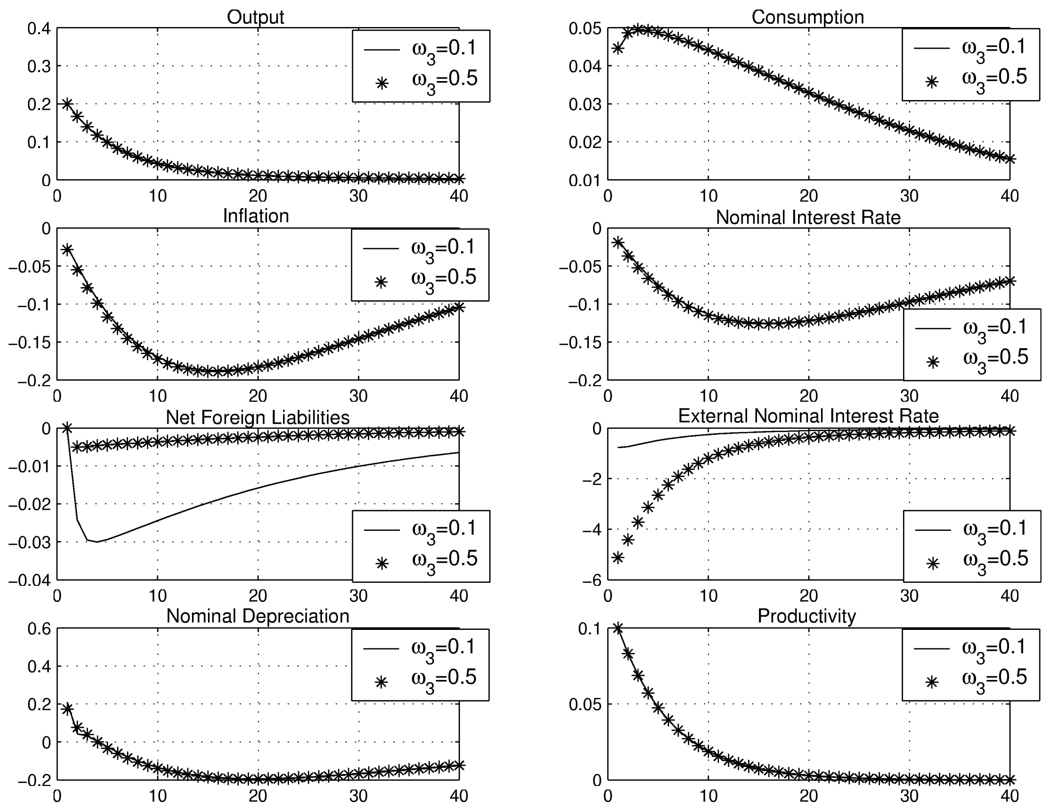

Figure 10: Preferences shock when the risk premium is more elastic

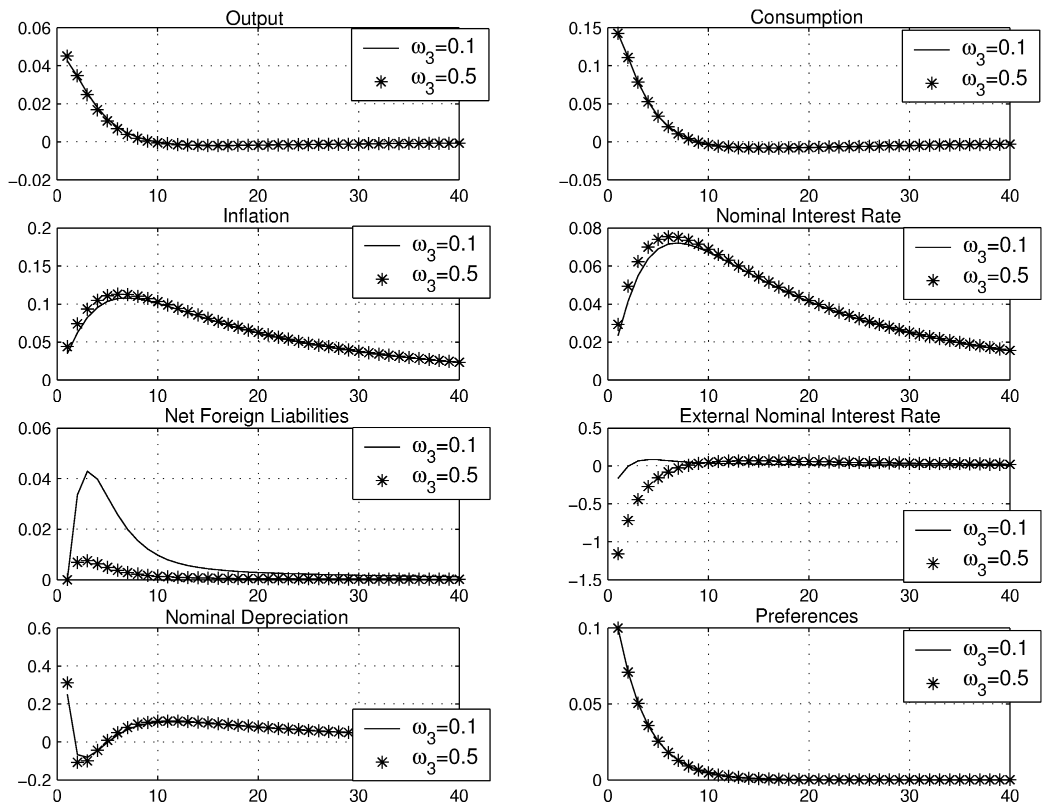

Figure 11: Government Expenditures shock when the risk premium is more elastic