Exchange Rate Targeting in a Small Open Economy

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Abstract

The paper develops a New Keynesian Small Open Economy Model characterized by external habit formation and Calvo price setting with dynamic inflation updating. The model is used to analyze the effect of nominal exchange rate targeting on optimal policy and impulse responses. It is found that even moderate exchange rate concerns are capable of changing both sign and magnitude of the optimal instrument response to variables, and that whether the concern is with respect to the level or first difference has much impact on monetary policy. Also, the cost of exchange rate stabilization in terms of output and inflation is evident in the model, and impulse responses under moderate exchange rate targeting are not simple combinations of those under a float and a regime that cares almost only for meeting the exchange rate target.

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1 Introduction

In recent years, it has come to attention that many countries, though announcing flexible exchange rate regimes, actually do intervene considerably to stabilize their exchange rate. These interventions imply a concern for exchange rate stabilization which generally arises from a desire to lower exchange rate risk and transaction costs, and for very open economies also from the pass through to CPI-inflation. Concerns are especially strong for emerging markets, where depreciations tend to be contractionary, increase service on foreign debt, and adversely affect credit market access. However, Calvo and Reinhart (2000) documents how "fear of floating" prevails even among some developed countries.[1]

With a clear motivation for exchange rate concerns, several papers[2] have joined the quest of understanding whether open economy Central Banks *should* stabilize the exchange rate under an optimal policy. This paper turns the table around and asks: Given that a country "fears floating" as documented above, what are the effects on monetary policy and the economy?

The question is answered for a small open economy inflation targeting Central Bank. The regime is highly relevant as it has been adopted by several countries, including Sweden, New Zealand, Canada, the UK, Finland, Brazil, Chile, and Colombia. Under flexible inflation targeting, the Central Bank is given independence to pursue the goals set out in the loss function. A high degree of transparency and accountability furthermore characterizes the regime. This allows the Central Bank to use all available information to achieve its goals, and makes clear to the public the objectives of monetary policy.

The main contribution of the paper is hence to demonstrate how the inclusion of nominal exchange rates in the Central Bank loss function affects the economy. This way of modelling "fear of floating" seems natural: The loss function reflects the

[1] Also Reinhart and Rogoff (2002) and Levy-Yeyati and Sturzenegger (2002) classify exchange rate regimes on a defacto (as opposed to a dejure) basis.

[2] See for instance Benigno and Benigno (2004), Clarida, Gali, and Gertler (2001), (2002), Monacelli (2003) and Taylor (2000).

specific objectives of monetary policy, and if they include a concern for exchange rate swings, then so should the loss function. The weight attached to exchange rate objectives is easily adjusted to reflect any level of importance relative to other objectives such as output and inflation stabilization. Finally, the inclusion of exchange rate objectives in the loss function increases transparency (by making clear those objectives to the public) and credibility (by explaining policy changes aimed at lowering exchange rate volatility).

In addition, the paper offers a tractable framework with an improved modelling of the foreign economy for determining the impact on a small country of targeting a range of variables in a flexible inflation targeting regime. This improved setting turns out to be necessary under risk sharing.

The exposition is made within a dynamic general equilibrium model with micro-foundations, and the optimal policy is determined using the Recursive Saddlepoint Method of Marcet and Marimon. Two types of "fear of floating"- arising due to concerns about respectively the level and the first difference of the nominal exchange rate- are described and compared to both a completely flexible exchange rate, a peg, and a policy which is effectively aimed only at smoothing the nominal exchange rate. The paper then considers implications for optimal policy and impulse responses from a shock to the world interest rate.

Turning to the relevant literature, the investigation of monetary policy aspects in economies under influence of the rest of the world has led to a large expansion in the so-called New Open Economy Macroeconomics (NOEM) literature. Contributions within the field are generally characterized by a dynamic general equilibrium framework with micro foundations and real and/or nominal rigidities causing monetary policy to have short run real effects.

Most closely related to the model of the current paper is the work by Clarida, Gali, and Gertler (2001) , Gali and Monacelli (2005), and Svensson (2000). The two former share the basic structure of the model including risk sharing. However, they exhibit simpler dynamics than the current model due to the absence of habit formation and inflation updating. This enables the reduction of the models to the standard canonical form of the closed economy,[3] allowing the qualitative results from that case to carry over to the open economy. In particular, Clarida, Gali, and Gertler (2001) demonstrates the optimality of targeting domestic inflation and the output gap only, while in Gali and Monacelli (2005), a domestic inflation based Taylor rule dominates both a peg and a CPI-based Taylor rule in all simulations.

Svensson (2000) uses a similar model with richer dynamics (but with foreign variables following an AR(1) process) to compare strict and flexible domestic and CPI-inflation targeting to Taylor rules. As in the current paper, the comparison is carried out by solving for the optimal policy subject to the full rational expectations model and looking at impulse responses. Only discretionary policy is considered. It is found that when the policymaker cares for real activity in addition to the level of inflation, CPI-inflation targeting is a good alternative.

Finally, a part of the NOEM literature deals more explicitly with emerging market economies. Models are augmented with specific features of emerging markets (such as financial vulnerability and balance sheet effects) to better understand the special importance of exchange rates in these economies. Contributions in the area include Céspedes, Chang, and Velasco (2004) and Morón and Winkelried (2005).

The paper is organized as follows: section II presents the model, and section III discusses monetary policy objectives. The solution method is briefly explained in section IV. Section V describes optimal policy under commitment, and section VI presents impulse responses following a shock to the foreign interest rate. Finally, section VII summarizes the results and proposes some avenues for further research.

2 The Model

A New Keynesian model of a Small Open Economy is augmented with external habit formation and Calvo staggered price setting with dynamic inflation updating in order to ensure inertia in both consumption and inflation.

[3] That is, a New Keynesian Phillipscurve and an IS curve.

The model builds on Clarida, Gali, and Gertler (2001), (2002). There are two countries, Home and Foreign, with Home being a small open economy and Foreign representing the rest of the world. As in Gali and Monacelli (1999), the foreign economy is modelled as a limiting case of the small open economy with negligible openness. The economies are otherwise symmetric, each consisting of a continuum of households normalized to $[0, 1]$ and intermediate and final good firms. The former supply labour in an imperfectly competitive labour market, receive wages and a lump sum transfer, and buy and consume the final good. Intermediate good firms use labour as the only input to produce for the final good firms, which combine a continuum of intermediate goods into a single consumption composite that is sold under perfect competition. For simplicity, only final goods are traded between countries, and there is immediate pass-through. An assumption of complete asset markets closes the model.

2.1 Households

The representative household (indexed by j) maximizes utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\left(C_{jt}/C_{t-1}^h\right)^{1-\sigma} - 1}{1-\sigma} - \frac{N_{jt}^{1+\varphi} - 1}{1+\varphi} \right) \tag{1}$$

where $\beta \in [0, 1)$, C_{jt} is the consumption of household j in period t, C_t is average consumption $\left(C_t = \int_0^1 C_{jt} dj\right)$, and N_{jt} is the amount of labour supplied by household j in period t. The utility function exhibits external habit formation as measured by the parameter h; large h indicates strong habits. Average consumption is taken for granted by all households when optimizing.

Household consumption, C_{jt}, is composed of Home (C_{jHt}) and Foreign (C_{jFt}) final goods according to the index

$$C_{jt} = C_{jHt}^{1-\gamma} C_{jFt}^{\gamma} \tag{2}$$

where $\gamma \in [0, 1]$ measures the degree of openness of the economy with the approximately closed foreign economy having $\gamma = 0$.

For simplicity, assume that asset markets are complete.[4] Let D_{t+1} denote the random payoff at time $t+1$ of a portfolio bought at time t, and let $Q_{t,t+1}$ be the corresponding stochastic discount factor. From cost minimization, the consumer price index is given by

$$\begin{aligned} P_t &= \gamma^{-\gamma}(1-\gamma)^{-(1-\gamma)} P_{Ht}^{1-\gamma} P_{Ft}^{\gamma} \\ &= \gamma^{-\gamma}(1-\gamma)^{-(1-\gamma)} P_{Ht} S_t^{\gamma} \end{aligned} \tag{3}$$

where P_{Ht} is the domestic price of the domestically produced final good, P_{Ft} the domestic price of the foreign final good, and S_t the terms of trade ($\equiv P_{Ft}/P_{Ht}$, the relative price of imports in the home country). The wage of household j at time t is denoted W_{jt}. Finally, the overall lump sum transfer to household j is given by T_{jt}, representing government net transfers as well as accrued profits from the monopolistically competitive intermediate good firms.

With this notation in place, the budget constraint of the representative household is

$$P_t C_{jt} + E_t\left(Q_{t,t+1} D_{j,t+1}\right) = W_{jt} N_{jt} + D_{jt} + T_{jt} \tag{4}$$

The optimization is furthermore subject to the intermediate good firms' demand for individual labour. This is derived from cost minimization below, and is given by

$$N_{jt} = \left(\frac{W_{jt}}{W_t}\right)^{-\eta_t} N_t \tag{5}$$

where $\eta_t > 1$ is the elasticity of demand for the labour services of worker j (assumed to follow an exogenous stationary stochastic process),[5] W_t is the relevant wage index

$$W_t = \left(\int_0^1 W_{jt}^{1-\eta_t} dj\right)^{\frac{1}{1-\eta_t}} \tag{6}$$

and N_t is per capita employment.

[4] See Schmit-Grohe and Uribe (2003) for alternative ways of closing small open economy models.

[5] The exogenous process is assumed for simplicity as an alternative to modelling fully the frictions (for instance efficiency wages) causing the time variation in η_t.

Expenditure minimization given C_{jt} determines the domestic demand for domestic and foreign goods as

$$C_{Ht} = (1-\gamma)\left(\frac{P_t}{P_{Ht}}\right) C_t \tag{7}$$

$$C_{Ft} = \gamma\left(\frac{P_t}{P_{Ft}}\right) C_t \tag{8}$$

Finally, the optimal choice of consumption, labour, and portfolio of each household yields the first order conditions

$$\frac{W_{jt}}{P_t} = (1+\mu_t^w)\, N_{jt}^{\phi} C_{jt}^{\sigma} C_{t-1}^{h(1-\sigma)} \tag{9}$$

$$Q_{t,t+1} = \beta\left(\frac{C_{j,t+1}^{-\sigma} C_t^{h(\sigma-1)} P_t}{C_{jt}^{-\sigma} C_{t-1}^{h(\sigma-1)} P_{t+1}}\right) \tag{10}$$

with μ_t^w being the optimal (stochastic) wage markup, $1+\mu_t^w = \frac{\eta_t}{\eta_t - 1}$. Because asset markets are complete, consumption is identical across consumers so that $C_{jt} = C_t\ \forall\ j$. Flexible wages ensure that also $W_{jt} = W_t$ and $N_{jt} = N_t\ \forall\ j$, so that equations (9) and (10) hold in the aggregate as well as for each household. Symmetric conditions hold in the rest of the world.

Complete asset markets furthermore guarantee that (10) holds for each possible state in period $t+1$ instead of in expectations only. Let e^{i_t} be the nominal interest rate in the economy. Then $E_t Q_{t,t+1} = \frac{1}{e^{i_t}}$, and taking expectations of (10) for foreign and domestic assets, and log linearizing, one obtains the familiar uncovered interest rate parity:

$$i_t = i_t^* + E_t \Delta e_{t+1} \tag{11}$$

with e_t being the (log) nominal exchange rate, and i_t (i_t^*) the nominal yield on a riskless one period discount bond which pays one unit of domestic (foreign) currency in period $t+1$.

2.2 Firms

2.2.1 Final goods sector

The final goods sector is perfectly competitive and assumed able to adjust prices immediately without costs. Firms in the sector use a continuum of intermediate goods to produce the consumption composite according to the CES technology

$$Y_t = \left(\int_0^1 Y_t(f)^{\frac{\xi-1}{\xi}} df \right)^{\frac{\xi}{\xi-1}} \tag{12}$$

where $\xi > 1$, $Y_t(f)$ is the input of intermediate goods from firm $f \in [0,1]$, and Y_t is (aggregate) output. Profit maximization along with perfect competition on the output market yields the demand curve for each intermediate good

$$Y_t(f) = \left(\frac{P_{Ht}(f)}{P_{Ht}} \right)^{-\xi} Y_t \tag{13}$$

and the domestic price index

$$P_{Ht} = \left(\int_0^1 P_{Ht}(f)^{1-\xi} df \right)^{\frac{1}{1-\xi}} \tag{14}$$

2.2.2 Intermediate goods sector

The monopolistically competitive intermediate good firms use labour as the sole input, and produce according to the linear production function

$$Y_t(f) = A_t N_t(f) \tag{15}$$

where $Y_t(f)$ is output of firm f, A_t is productivity assumed to follow an exogenous stationary stochastic process, and $N_t(f)$ is firm f's composite labour input given by

$$N_t(f) = \left(\int_0^1 N_{jt}^{\frac{\eta_t-1}{\eta_t}} dj \right)^{\frac{\eta_t}{\eta_t-1}} \tag{16}$$

Cost minimization by the intermediate good firms yields the labour demand function (5) used in the household optimization problem as well as the wage index (6). The real marginal cost of production is

$$MC_t = \frac{W_t}{A_t P_{Ht}} = \gamma^{-\gamma} (1-\gamma)^{-(1-\gamma)} \frac{W_t S_t^{\gamma}}{A_t P_t} \tag{17}$$

While prices of the consumption composite and wages are perfectly flexible, the intermediate good firms set prices on a staggered basis following Guillermo Calvo

(1983). Let $(1-\theta)$ be the probability of a given firm adjusting its price $(P_{Ht}(f))$ in each period, and assume that if a firm f is not allowed to change its price in period t, $P_{H,t-1}(f)$ is updated according to $P_{Ht}(f) = P_{H,t-1}(f) \times \Pi_{t-1}$ with Π_{t-1} being lagged gross domestic inflation. The optimal price setting then requires solving

$$\max_{P_{H,t}^{0}} \quad E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k}(f) \left(P_{Ht}^{0} \Psi_{tk} - P_{H,t+k} MC_{t+k} \right)$$

subject to the downward sloping demand curve facing each intermediate good producer (13), and with

$$\Psi_{tk} \equiv \begin{cases} \Pi_t \times \Pi_{t+1} \times ... \times \Pi_{t+k-1} & \text{for } k \geq 1 \\ 1 & \text{for } k = 0 \end{cases}$$

The first order condition is

$$E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k}(f) \left(P_{Ht}^{0} \Psi_{tk} - \frac{\xi}{\xi - 1} P_{H,t+k} MC_{t+k} \right) = 0 \tag{18}$$

Loglinearizing this first order condition and carrying out some algebra[6] results in the Phillipscurve

$$\pi_t = \frac{\beta}{1+\beta} E_t \pi_{t+1} + \frac{1}{1+\beta} \pi_{t-1} + \delta mc_t \tag{19}$$

where inflation is log domestic inflation measured in deviation from steady state $(= 0)$, $\delta \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta(1+\beta)}$, and mc_t is log deviation from steady state real marginal cost (which is also zero).

For future reference, note that when prices are flexible, $\theta = 0$, the first order condition for optimal price setting reduces to

$$\frac{P_{Ht}^{0}}{P_{Ht}} = \frac{\xi}{\xi - 1} MC_t \tag{20}$$

which is the standard result that the producer sets his real price as a markup on real marginal cost.

[6] A more detailed derivation is available in Christiano, Eichenbaum, and Evans (2001).

2.3 Equilibrium relations

In equilibrium, consumption is divided between domestic and foreign goods according to

$$c_t = (1-\gamma)\, c_{Ht} + \gamma c_{Ft} \tag{21}$$

where lowercase variables are in log deviations from steady state. This is the sense in which γ measures the degree of openness of the economy. Letting c_t^{H*} denote consumption of domestic goods in the foreign economy, goods market clearing implies

$$y_t = (1-\gamma)\, c_{Ht} + \gamma c_t^{H*} \tag{22}$$

By (8), import demand is

$$\begin{aligned} c_{Ft} &= c_t - (1-\gamma)\, s_t \\ &= c_{Ht} - s_t \end{aligned} \tag{23}$$

while by (7), domestic demand for domestic goods is

$$c_{Ht} = c_t + \gamma s_t \tag{24}$$

Because the two economies are symmetric in all aspects except for their degree of openness, $c_t^* = y_t^*$ at all times, and export demand for the domestic good is given by

$$c_t^{H*} = y_t^* + s_t \tag{25}$$

The relation makes it clear how disturbances to foreign output affect export demand directly.

Combining the above equilibrium relations gives us consumption as a function of production at home and abroad, and the terms of trade:

$$c_t = \frac{1}{1-\gamma} y_t - \frac{(2-\gamma)\,\gamma}{1-\gamma} s_t - \frac{\gamma}{1-\gamma} y_t^* \tag{26}$$

2.3.1 Risk sharing

Because of the assumption of complete markets, the first order condition (10) holds exactly for each state of the world at all times instead of in expectations only. This is true for both the domestic and foreign economy. Combining the two Euler equations, assuming that the law of one price holds,[7] and defining the real exchange rate

$$\begin{aligned} q_t &\equiv e_t + p_t^* - p_t \\ &= (1-\gamma)\, s_t \end{aligned} \tag{27}$$

we have the risk sharing result

$$c_t - \frac{h(\sigma-1)}{\sigma} c_{t-1} = c_t^* - \frac{h(\sigma-1)}{\sigma} c_{t-1}^* + \frac{1-\gamma}{\sigma} s_t \tag{28}$$

While the standard risk sharing result depends on the terms of trade alone, this model exhibits additional dependence on past consumption in both countries due to habit formation; the standard result is recovered when $h = 0$.

2.3.2 Flex Price Equilibrium

In the flex price equilibrium, firms are free to set a new price every period, and it follows from (20) that (log) marginal cost is zero. As in Clarida, Gali, and Gertler (2002), let the wage markup be fixed at its steady state value in the flex price equilibrium - as noted in their paper, this makes sense if the wage markup represents unmodelled wage rigidities. Assume in addition that the flex price equilibrium is conditional on foreign output and all past variables. Then combining the first order condition for labour (9) with the log-linearized production function ($y_t = a_t + n_t$), the equilibrium relation (26), and the expression for marginal cost (17), one obtains

[7] Assuming the law of one price is a simplification which hardly holds true empirically. See Monacelli (2003) for a New Keynesian Small Open Economy model with Calvo price setting and imperfect pass through, and Lindé, Nessén, and Söderström (2004) for a model with imperfect financial integration and gradual pass through. Flamini (2003) analyzes the effect of imperfect pass-through on the transmission of CPI-inflation targeting optimal monetary policy.

the following expression for flex price output $\overline{y}_t$

$$\begin{aligned}\overline{mc}_t &= 0 \Rightarrow \\ \overline{y}_t &= \frac{(1-\gamma)(\varphi+1)}{\varphi(1-\gamma)+\sigma}a_t + \frac{\sigma\gamma}{\varphi(1-\gamma)+\sigma}y_t^* - \gamma\left(\frac{1-\gamma-\sigma(2-\gamma)}{\varphi(1-\gamma)+\sigma}\right)\overline{s}_t \\ &\quad - \frac{h(1-\sigma)}{\varphi(1-\gamma)+\sigma}\left(y_{t-1}-\gamma y_{t-1}^* - \gamma(2-\gamma)s_{t-1}\right)\end{aligned} \tag{29}$$

where $\overline{mc}_t$ and $\overline{s}_t$ denote flex price values of respectively marginal cost and terms of trade. The expression that determines $\overline{s}_t$ is derived from the risk sharing condition (28) using (26), and is given by

$$\overline{s}_t = \frac{\sigma}{\sigma_1}\left(\overline{y}_t - y_t^*\right) - \frac{h(\sigma-1)}{\sigma_1}\left(y_{t-1} - y_{t-1}^*\right) + \frac{\gamma h(\sigma-1)(2-\gamma)}{\sigma_1}s_{t-1} \tag{30}$$

with $\sigma_1 \equiv \gamma(2-\gamma)(\sigma-1)+1$.

Finally, the flex price natural rate, $\overline{r}_t$, is determined from the Euler condition (10):

$$\begin{aligned}\overline{r}_t &= \frac{\sigma}{1-\gamma}\left(E_t\Delta\overline{y}_{t+1} - (2-\gamma)\gamma E_t\Delta\overline{s}_{t+1} - \gamma E_t\Delta y_{t+1}^*\right) \\ &\quad - \frac{h(\sigma-1)}{1-\gamma}\left(\overline{y}_t - y_{t-1} - (2-\gamma)\gamma\left(\overline{s}_t - s_{t-1}\right) - \gamma\Delta y_t^*\right)\end{aligned} \tag{31}$$

2.4 Reduced Form

Due to external habit formation in consumption, it is not possible to rewrite the model in the standard canonical form of the closed economy. However, one can come close by rewriting the model as an IS and AS curve for each economy combined with a terms of trade (or risk sharing) equation.

To obtain the IS-curve, combine the Euler equation (10) with (26) and let $\sigma_0 \equiv$

$\sigma + h(\sigma - 1)$. Then

$$\begin{aligned} y_t &= \frac{\sigma}{\sigma_0} E_t y_{t+1} + \left(1 - \frac{\sigma}{\sigma_0}\right) y_{t-1} \\ &\quad - \frac{(2-\gamma)\gamma}{\sigma_0} \left(\sigma E_t \Delta s_{t+1} - h(\sigma - 1) \Delta s_t\right) \\ &\quad - \frac{\gamma}{\sigma_0} \left(\sigma E_t \Delta y^*_{t+1} - h(\sigma - 1) \Delta y^*_t\right) \\ &\quad - \frac{1-\gamma}{\sigma_0} \left(i_t - E_t \pi_{t+1} - \gamma E_t \Delta s_{t+1}\right) \end{aligned} \tag{32}$$

For $\gamma = 0$, this reduces to the standard IS-curve $y_t = \frac{\sigma}{\sigma_0} E_t y_{t+1} + \left(1 - \frac{\sigma}{\sigma_0}\right) y_{t-1} - \frac{1}{\sigma_0}\left(i_t - E_t \pi_{t+1}\right)$, making it clear how the Central Bank is able to influence output immediately through the effect of the nominal interest rate on the real rate.

When the economy opens up, the direct response of output to interest rate changes is reduced. Expected and current foreign output and terms of trade growth comes to influence output too, the former through exports/imports, the latter through the exchange rate channel: Lines 2 and 3 capture the indirect effect through consumption while expected terms of trade growth in the 4th line is due to its effect on expected CPI-inflation alone.

The Phillips curve follows from (19) using the log linearized marginal cost equation (17) along with the first order condition for labour, the log linearized production function ($y_t = a_t + n_t$), the equilibrium condition (26), and the flex price values of output and the terms of trade. The result is

$$\begin{aligned} \pi_t &= \frac{\beta}{1+\beta} E_t \pi_{t+1} + \frac{1}{1+\beta} \pi_{t-1} + \delta \left(\varphi + \frac{\sigma}{1-\gamma}\right) (y_t - \overline{y}_t) \\ &\quad + \delta\gamma \left(1 - \frac{\sigma(2-\gamma)}{1-\gamma}\right) (s_t - \overline{s}_t) + \delta \mu^w_t \end{aligned} \tag{33}$$

As in the closed economy, inflation increases when output exceeds its flex price level, or when there is a shock to the stochastic wage markup (cost push shock). What is different is that domestic inflation is seen to respond stronger to variations in the (domestic) output gap when the economy is open ($\gamma > 0$): The effect working through employment is unaffected by the degree of openness, but the response working through consumption is not. This is due to some of the goods produced being exported.

Terms of trade now enters the Phillips curve with a positive or negative coefficient depending on the parameter σ: The positive effect is through the direct effect on CPI-inflation changing the labour/leisure trade off in (9) while the negative effect works indirectly through consumption, cf (26). Finally, due to inflation updating of prices, the long run Phillipscurve is vertical.

The Central Bank is able to affect inflation immediately through the real interest rate, with the effect of policy working through the aggregate demand channel. An interest rate change is furthermore bound to change inflation expectations, which again affects inflation directly through optimal price setting of intermediate firms' products. This is the expectations channel of monetary policy.[8]

Though not immediately evident from (33), foreign output also has an immediate effect on inflation through export demand and consumption. Hence disturbances in the foreign economy affect inflation at once with the effect entering through the flex price values of domestic output and terms of trade.

The equation determining the terms of trade as a function of output and past variables follows directly from risk sharing (28) when using (26). The relation is

$$s_t = \frac{\sigma}{\sigma_1}\left(y_t - y_t^*\right) - \frac{h\left(\sigma-1\right)}{\sigma_1}\left(y_{t-1} - y_{t-1}^*\right) + \frac{h\left(\sigma-1\right)\left(2-\gamma\right)\gamma}{\sigma_1} s_{t-1} \tag{34}$$

Terms of trade is seen to exhibit serial dependence due to habit formation in consumption, and is in addition determined by current and past relative output with an increase in the former causing a real depreciation of the domestic currency. The inclusion of this equation and its flexprice counterpart in the model is what makes a difference from the standard model which emphasizes an isomorphism between the open and closed economy. In those models, the openness of the economy changes

[8] The immediate effect of monetary policy on output and inflation facilitates the solution of the model and is used for simplicity. More realistically, one could impose that output and inflation is predetermined (in the sense of having exogenous forecast errors) with monetary policy having a faster effect on output than inflation. See for instance Svensson (2000).

only the *coefficients* in the canonical form.[9]

Finally, the foreign economy is symmetric except that it is approximately closed so that $\gamma = 0$. For simplicity, I let the foreign Central Bank follow a Taylor rule (Taylor (1993)) with standard coefficients, and allow for a policy shock by adding a zero-mean iid error term $\varepsilon^*_{i,t}$. The rest of the world is hence described by the equations

$$\begin{aligned}
y^*_t &= \frac{\sigma}{\sigma_0} E_t y^*_{t+1} + \left(1 - \frac{\sigma}{\sigma_0}\right) y^*_{t-1} - \frac{1}{\sigma_0}\left(i^*_t - E_t \pi^*_{t+1}\right) && (35) \\
\pi^*_t &= \frac{\beta}{1+\beta} E_t \pi^*_{t+1} + \frac{1}{1+\beta}\pi^*_{t-1} + \delta\left(\varphi + \sigma\right)\left(y^*_t - \overline{y}^*_t\right) + \delta \mu^{*w}_t \\
\overline{y}^*_t &= \frac{\varphi + 1}{\varphi + \sigma} a^*_t + \frac{h\left(\sigma - 1\right)}{\varphi + \sigma} y^*_{t-1} \\
\overline{r}^*_t &= \sigma E_t \Delta \overline{y}^*_{t+1} - h\left(\sigma - 1\right)\left(\overline{y}^*_t - y^*_{t-1}\right) \\
i^*_t &= \overline{r}^*_t + \frac{3}{2}\pi^*_t + \frac{1}{2}\left(y^*_t - \overline{y}^*_t\right) + \varepsilon^*_{i,t}
\end{aligned}$$

The Taylor rule applied is sophisticated in two ways; it depends on the current natural interest rate (rather than its long run average), and it reacts to current as opposed to past inflation and output gap. An important implication is that $\frac{di^*_t}{d\varepsilon_{i,t}} \neq 1$. Rather, the change in the foreign interest rate brought about by a policy shock causes a change in output and inflation which again affects the interest rate. The system of equations in (35) above gives the equilibrium relation.

The model is closed by assuming stationary AR(1) processes for the stochastic wage markup and productivity:

$$\begin{aligned}
a^*_t &= \alpha_a a^*_{t-1} + \varepsilon^*_{a,t} && (36) \\
a_t &= \alpha_a a_{t-1} + \varepsilon_{a,t} \\
\mu^{*w}_t &= \alpha_\mu \mu^{*w}_{t-1} + \varepsilon^*_{\mu,t} \\
\mu^{w}_t &= \alpha_\mu \mu^{w}_{t-1} + \varepsilon_{\mu,t}
\end{aligned}$$

[9]For instance Clarida, Gali, Gertler (2001) find that the terms of trade is proportional to current relative output. It is therefore easy to substitute out the terms of trade from the model. See also Clarida, Gali, Gertler (2002) and Gali and Monacelli (2005).

where all coefficients are nonnegative and less than unity, and the shocks zero-mean iid.

To summarize, the model consists of $(32)-(34)$, the domestic flex price equilibrium $(29)-(31)$, the foreign economy (35), and the AR(1) processes for productivity and the stochastic wage markup (36).

3 Policy Objective

The paper considers variants of flexible inflation targeting with the Central Bank minimizing a loss function which is quadratic in output deviations from the flex price level, CPI-inflation $\left(\pi_t^{cpi} = \pi_t + \gamma \Delta s_t \text{ by } (3)\right)$, and nominal exchange rate changes and levels:

$$
\begin{aligned}
L &= (1-\beta) E_t \sum_{\tau=0}^{\infty} \beta^{\tau} L_{t+\tau} \qquad (37) \\
L_t &= \frac{1}{2}\left(\lambda_{cpi}\left(\pi_t^{cpi}\right)^2 + \lambda_y \left(y_t - \overline{y}_t\right)^2 + \lambda_{\Delta e}\left(e_t - e_{t-1}\right)^2 + \lambda_e \left(e_t - \overline{e}\right)^2\right)
\end{aligned}
$$

where β is the Central Bank discount factor, assumed equal to the subjective discount factor of consumers. A different weight (λ_*) is attached to each term in the period loss function to reflect the relative importance for the monetary policy authority of the relevant targets being met, and the overall function is scaled by $(1-\beta)$ to make it a weighted average of expected losses in all future periods. $\overline{e}$, the target for the level of the nominal exchange rate, is set to zero. (37) demonstrates the explicit objectives for monetary policy.

The main contribution of the paper is to show how the inclusion of the nominal exchange rate in the monetary policy objectives affects the economy. It turns out that whether the desired exchange rate stabilization relates to the level or first difference of the nominal rate matters a good deal for policy and hence for the implied dynamics of the economy. To illustrate these differences, both the change in and level of the nominal rate have been included in (37).

As argued in the introduction, there are several reasons why targeting the nominal exchange rate may be desirable. This is especially so for emerging market economies where exchange rate movements heavily affecting CPI-inflation, foreign debt service, and market access gives reason to "fear of floating" in addition to more general considerations such as lowering exchange rate risks and transaction costs. Also, empirics show that many countries actively try to manage their exchange rate in some way. It is therefore important to understand the economic trade-offs involved in doing so, as well as to compare moderate exchange rate interventions to regimes where the exchange rate is close to being the sole policy objective.

The remaining terms in the loss function are standard: CPI targeting (as opposed to domestic inflation targeting) is the norm among inflation targeting countries, and output is targeted at its flexprice level.[10] Having both CPI-inflation and nominal exchange rate depreciation in the loss function is redundant to the extent that CPI-inflation is a linear combination of inflation of domestic and imported goods. However, both objectives are here included because of their relevance from the policymaker's point of view. It is assumed throughout that $\lambda_{cpi} = 1$ and $\lambda_y = 0.5$.

To illustrate the effects of nominal exchange rate targeting, the paper considers five different loss functions: 1) A pure float, 2) A peg, 3) Heavy smoothing, 4) Moderate stabilization, and 5) Moderate smoothing.

1) is modelled as the small open economy targeting only inflation and output, ie all other weights in the loss function are zero. Note that the float is not pure in the sense that the exchange rate drifts on its own, unaffected by monetary policy. Rather, interest rate changes affect the exchange rate, but due to the form of the loss-function, policy-makers are only concerned with these changes insofar they affect output and inflation. Because of free international capital mobility, monetary policy and exchange rate policy can no longer be distinguished.

2) is modelled by attaching a very large weight ($\lambda_e = 100$) to the term $(e_t - \overline{e})^2$, so that the Central Bank is effectively concerned only with keeping $e = \overline{e} = 0$. 3) is modelled with a similarly large weight on the term $(e_t - e_{t-1})^2$. The remaining two cases represent "fear of floating" with a moderate concern for exchange rate

[10] The general form of the loss function and solution method allows for other targets to be easily added, corresponding to for instance interest rate smoothing or real exchange rate concerns.

targeting. 4) and 5) are therefore modelled as having respectively $\lambda_e = 0.5$ and $\lambda_{\Delta e} = 0.5$. This corresponds to caring as much about the nominal exchange rate target as the output target.

4 Solution Method

The model is solved numerically using the Recursive Saddlepoint Method of Marcet and Marimon as described in Svensson (2005). This solution method reformulates the non-recursive problem of minimizing the loss function (37) subject to the model equations for the home and foreign economy into a recursive problem which can be solved using standard methods. The appendix shows the model in its state space form along with the matrices defining the loss function and a brief explanation of the solution method under commitment.

Parameters are set to reasonable values as follows: In the utility function, a fairly strong external habit formation with $h = 0.9$, and a labour supply elasticity of 1/3 ($\varphi = 3$) is assumed. Sigma is set to 7. The discount factor of the utility and loss functions are set equal at a value of 0.99 corresponding to an annual riskless return of 4% in steady state when time is measured in quarters. The stochastic wage markup has parameter $\alpha_\mu = 0.5$ in the AR(1) process. Letting firms set new prices once a year on average, and using the calibration of productivity from Gali and Monacelli (2005), I set $\theta = 0.75$ and $\alpha_a = 0.66$. The economy is assumed fairly open with $\gamma = 0.4$, and as already mentioned the flexible inflation targeting loss function gives weights of 1 on CPI-inflation, and 0.5 on output deviations from flexprice level always. In addition, some positive weight is associated with nominal exchange rate targeting depending on the regime considered.

5 Optimal Policy

Under commitment, the Central Bank is able to credibly bind itself to a state contingent policy and hence solve the optimization problem once and for all in the initial period.[11] The case is interesting as its solution gives us the *optimal policy function*, the optimal way to conduct policy given the goals set out in the loss function. It is hence possible to understand better the effects of exchange rate targeting under the best possible conditions for monetary policy.

This section addresses the optimal policy function under the five alternative specifications of the loss function presented above: 1) A pure float, 2) A peg, 3) Heavy smoothing, 4) Moderate stabilization, and 5) Moderate smoothing. For comparison, the reaction coefficients in the optimal instrument rule under these alternative specifications are presented in Table 1.[12]

Specification	1	2	3	4	5
Loss function	–	$\lambda_e = 100$	$\lambda_{\Delta e} = 100$	$\lambda_e = 0.5$	$\lambda_{\Delta e} = 0.5$
e_{t-1}	-0.0000	0.2362	-0.0000	-0.1630	-0.0000
i^*_{t-1}	0	0	0	0	0
π_{t-1}	0.1339	-0.0230	-0.0219	0.0238	0.0863
y_{t-1}	-0.0261	-0.0696	-0.0716	-0.0019	-0.0555
s_{t-1}	-0.1467	-0.1018	-0.1031	-0.0892	-0.1396
$\overline{y}_{t-1}$	0	0	0	0	0
π^*_{t-1}	0.2533	0.8362	0.8296	0.7598	0.4202
y^*_{t-1}	2.8997	3.1054	3.1121	2.8461	3.0198
a_t	-0.0234	0.0968	0.1002	-0.0452	0.0438
μ^w_t	0.0028	-0.0180	-0.0178	-0.0128	-0.0035
μ^{*w}_t	-0.0013	0.1382	0.1371	0.1119	0.0425
$\varepsilon^*_{i,t}$	0.5758	0.6126	0.6141	0.5552	0.5983
$\overline{y}^*_t$	-3.9414	-4.2419	-4.2506	-3.8869	-4.1095
l_1	-0.0000	-0.0000	-0.0000	0.0000	-0.0000
l_2	-0.0000	-0.0000	-0.0000	0.0000	-0.0000
l_3	-0.0172	0.0010	-0.0006	0.0334	-0.0210
l_4	0.0524	0.0013	0.0002	0.0526	0.0273
l_5	0	0	0	0	0

Table 1. Optimal instrument rule reaction coefficients.

[11] Woodford considers "commitment in a timeless perspective" which requires any commitment made by the Central Bank to be as if made in the remote past. See Svensson and Woodford (2005).

[12] The variables $\overline{y}^*_t, a^*_t$, and y^*_{t-1} included in the state space form are linearly dependent. To ensure that coefficients are comparable across regimes, the relationship $\overline{y}^*_t = \frac{\varphi+1}{\varphi+\sigma}a^*_t + \frac{h(\sigma-1)}{\varphi+\sigma}y^*_{t-1}$ has been used to express reaction coefficients in terms of $\overline{y}^*_t$ and y^*_{t-1} only.

$l_1 - l_5$ denote the Lagrange multipliers associated with the equations for the forward-looking variables; (41), (42), (43), (44), and (45) in the appendix. It is due to the specific linear combination of equations that all reaction coefficients to l_5 are zero.

The table makes it clear that exchange rate targeting in any of the cases under consideration is capable of generating significant changes in both sign and magnitude of the optimal reaction coefficients. Though the coefficients are generally difficult to interpret due to interaction between variables and effects running through expectations, Table 1 does reveal several interesting features.

First, it is immediately evident that the optimal policy rate reacts only to variables that influence the loss function *and* are predetermined. This reflects the fact that the Central Bank responds to all relevant information available at time t.[13] The dependence on a set of predetermined Lagrange multipliers associated with the equations for the forward-looking variables reflects history dependence of the commitment solution.

Second, and in connection to the previous point since e_{t-1} affects e_t only, the reaction to lagged nominal exchange rates is negligible unless the nominal exchange rate level is specifically targeted. The positive coefficient under a peg is also intuitive: When e_{t-1} is large, the currency is depreciated, and it is desirable to cause an appreciation by increasing the policy rate. The negative coefficient on e_{t-1} when there is only moderate stabilization is more of a puzzle.

Third, as predicted by UIP (11), the response to foreign interest rate shocks when there is a large weight on exchange rate targeting is very close to the net impact on the foreign interest rate: The latter is 0.6141 (in response to a 1% shock),[14] and coefficients on $\varepsilon_{i,t}^{*}$ are respectively 0.6126 and 0.6141. Even when there is less or no weight on nominal rates is the reaction coefficient between 0.55 and 0.6. This is an attractive feature of the model, as also empirically interest rate changes in large countries is important information which often leads to domestic interest rate

[13] That it is advantageous for the Central Bank to do so is a more general insight: The instrument should react to variables determining target variables rather than the target variables themselves. See for instance Svensson (2003) which compares Taylor rules to optimal monetary policy.

[14] In the foreign interest rate rule (38) below, the foreign reaction coefficient to $\varepsilon_{i,t}^{*}$ is 1 rather than 0.6141. As already mentioned, the latter is what results after taking into account the reduction in the interest rate from deflation, a negative output gap, and changes in the foreign natural rate. Since the domestic interest rate is confined to react to $\varepsilon_{i,t}^{*}, a_t^{*}, i_{t-1}^{*}, \pi_{t-1}^{*}, y_{t-1}^{*}$, and $\overline{y}_t^{*}$, and the five latter variables are unaffected by $\varepsilon_{i,t}^{*}$, it is reasonable for the domestic reaction coefficient to be closer to 0.6141 than 1.

changes in the same direction.

Fourth, the reaction coefficients on y^*_{t-1} and $\overline{y}^*_t$ are much larger than on any domestic variables. This stems to a large degree from the optimality of partly following movements in the foreign interest rate: Increases in the foreign interest rate induce a depreciation of the domestic currency unless the small open economy also tightens its monetary policy. Using (35) and the parameterization of the model, the foreign Taylor rule can be rewritten as

$$\begin{aligned} i^*_t &= -0.952\, a^*_t + 4.28\left(y^*_t - \overline{y}^*_t\right) + 1.5\pi^*_t + 1.62\left(y^*_{t-1} - \overline{y}^*_t\right) + \varepsilon^*_{i,t} \qquad (38) \\ &= 4.28 y^*_t - 8.28 \overline{y}^*_t + 1.5\pi^*_t + 2.9052 y^*_{t-1} + \varepsilon^*_{i,t} \end{aligned}$$

The reduced form makes it clear that the foreign policy rate reacts strongly to y^*_t, y^*_{t-1} and $\overline{y}^*_t$ with the rate increasing in the two former and decreasing in the latter just as in the domestic optimal policy function.

Finally, as is intuitive, the reaction to foreign variables is generally stronger the larger is the weight on exchange rate targeting. The coefficients on $\varepsilon^*_{i,t}$, $\overline{y}^*_t$ and y^*_{t-1} under moderate stabilization are the only exceptions.

6 Impulse Responses

To illustrate the impact on the economy of differences in optimal policies, this section considers impulse responses following a one-time shock to the foreign interest rate of a quarter of a percentage point ($\varepsilon^*_{i,0} = 0.25$, $\varepsilon^*_{i,t} = 0 \ \forall \ t \neq 0$). The foreign economy response is considered first, followed by three subsections comparing respectively moderate stabilization to regimes with different weights on the $e = 0$ target, moderate smoothing to regimes with different weights on the $e_t = e_{t-1}$ target, and finally moderate smoothing to moderate stabilization. This allows us to get a picture of how "fear of floating" affects the economy compared to more extreme cases of exchange rate targeting, and how the type of "fear of floating" matters.

6.1 The Foreign Economy

The foreign economy disturbance affects the home country in several ways as previously discussed: Foreign deflation is transmitted directly to domestic CPI inflation due to perfect pass through, and the change in foreign output affects the domestic economy through risk sharing, export demand and import supply. To understand the reaction of the domestic economy to the foreign policy shock, it is therefore important to first look at the effect of the disturbance abroad. To this end, this section considers impulse responses for the foreign economy as illustrated in Figure 1 below. Because the foreign country is approximately closed, these impulse responses are independent of the small open economy's policy regime.

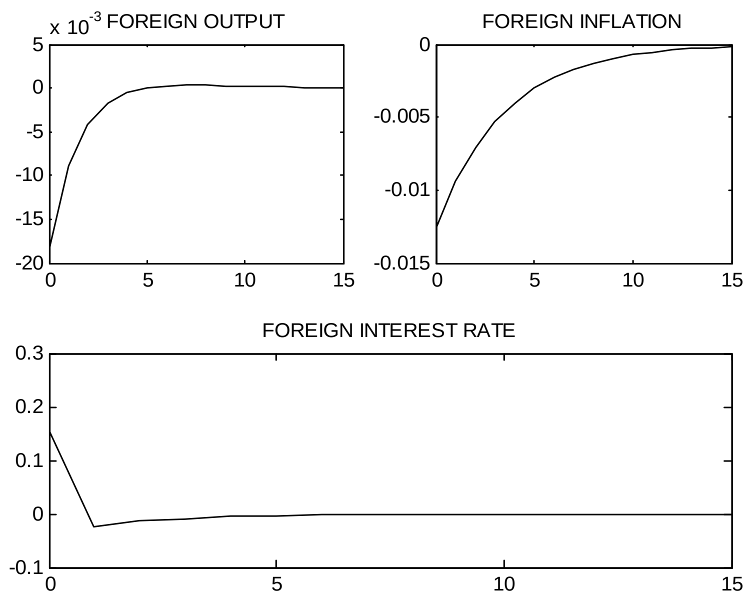

Figure 1. The foreign economy.

From the Figure, it is seen that the immediate effect of the shock is a rise in the real interest rate causing recession and deflation. The initial jump in the nominal rate is less than the shock because of the immediate response of the foreign Central Bank to deflation, a negative output gap, and a fall in the natural interest rate.

In the period following the shock, the foreign instrument rate is decreased considerably to stimulate the economy. Slight stimulation continues for several periods, gradually bringing inflation and output back to their natural levels. Overall, the adjustment process lasts around 5 periods for output and slightly longer for inflation. For the foreign economy, CPI inflation is simply domestic inflation, and consumption equals production. Hence, these variables also follow the paths outlined in Figure 1.

Though the interest rate setting of the Central Bank looks reasonable, it should of course be kept in mind that it is the result of a Taylor rule rather than optimal policy behaviour as modelled for the small open economy. This is also what keeps the Central Bank from completely offsetting the shock in the initial period.

6.2 Moderate Stabilization versus Float and Peg

The consequences of moderate "fear of floating" with concern for nominal exchange rate levels are best displayed in comparison with two extreme cases, namely floating exchange rates and a peg. The comparison makes evident the cost of stabilization of the exchange rate level and how that cost is optimally spread over time depending on the weight given to the exchange rate target in the loss function. Impulse responses to the foreign interest rate shock under the three regimes are illustrated in Figure 2.

The responses under a float and peg are not surprising: The initial monetary tightening abroad entails a real depreciation of the domestic currency, which under floating rates results in a nominal depreciation and under a peg is avoided by a prolonged domestic deflation and recession. Concern for the nominal exchange rate level induces the return of e to target over time in contrast to the nonstationarity of the nominal exchange rate under a float. As in the foreign country, there is a tightening of the instrument rate followed by monetary stimulation, making evident the optimality of keeping the domestic interest rate closely in line with the foreign.

The cost of exchange rate stabilization can also be seen in Figure 2: Under a peg, there is prolonged domestic and CPI deflation, and an immediate recession and larger negative output gap. Though both regimes experience an initial increase in net exports (not shown) and a fall in consumption, the latter is greater under the peg due to a larger increase in the interest rate on impact. Overall, though, the economy is close to stabilized within 10 quarters in both cases, and there is a permanent nominal depreciation of approximately 0.05 under floating rates.

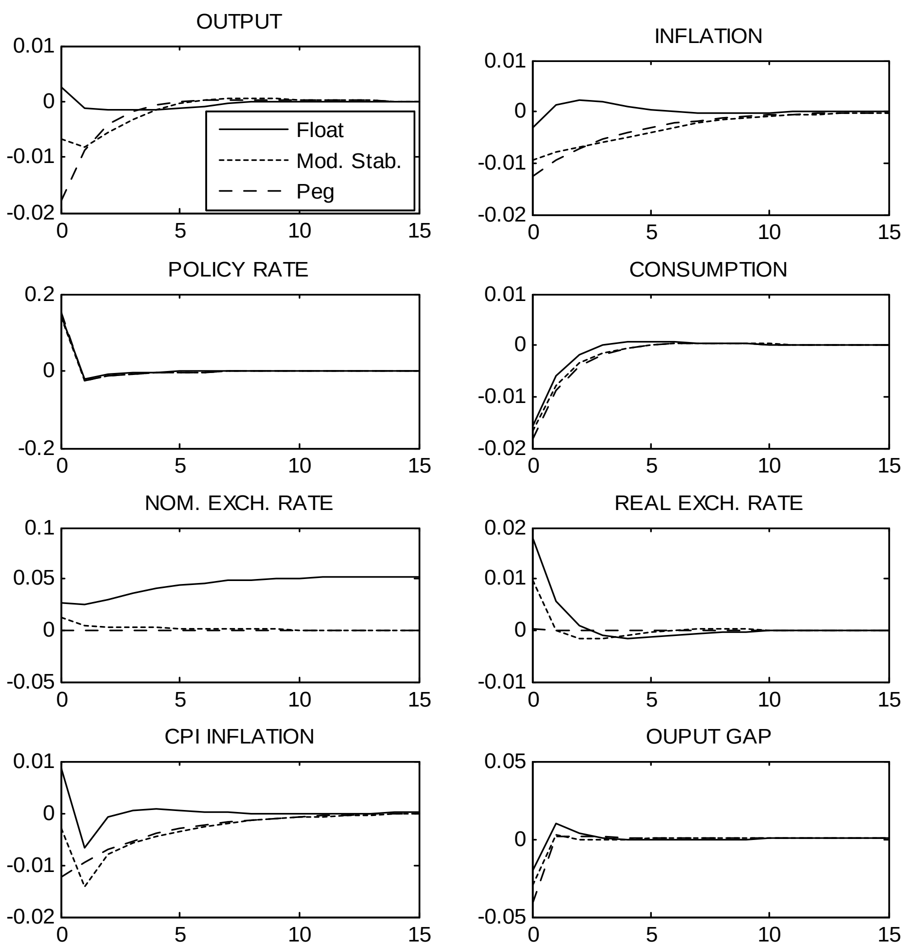

Figure 2. Moderate stabilization, float, and peg.

Turning to moderate stabilization, it is interesting that this case does not necessarily produce paths of variables inbetween those of the two extremes as one might have expected. What happens is rather that it still pays to bring back the nominal exchange rate to target in the long run, but the recession and deflation necessary to do so is optimally spread over time to avoid the large initial negative output gap and CPI deflation of the fixed exchange rate regime. Hence, we see an initial nominal depreciation which is quickly reversed.

As a result, the output impulse response function inherits the hump shape of the floating regime, but is shifted down so that output is below that of both other regimes for some periods. Also, domestic deflation decreases smoothly towards zero as under a peg, but the curve is now less steep. The path of the real exchange rate is much as under flexible rates, though the real appreciation occurs more quickly to ensure the return of the nominal rate to target. It is the similarity between real exchange rate paths under a float and moderate stabilization that causes CPI inflation under the latter regime to also have a downward spike in the quarter immediately following the shock. As is the case for domestic inflation, the curve is shifted down so that there is a prolonged deflation in the consumer price index.

In summary, moderately stabilizing the nominal exchange rate causes a smaller initial recession and higher consumption than under a peg, and smaller real and nominal exchange rate swings than under a float. Also, the nominal exchange rate is returned to close to target fairly quickly given the relatively small weight devoted to exchange rate stabilization.

6.3 Moderate Smoothing versus Float and Heavy Smoothing

The comparison of regimes assigning different weights to exchange rate smoothing gives a picture similar to that of the comparison carried out in the previous section. In particular, the practical difference between a peg and heavy smoothing is small because the policy rate under the latter also follows the foreign interest rate closely as expected from the UIP condition (11). Also, the pursuit of exchange rate smoothing to a moderate degree decreases the initial output gap and deflation compared to heavy smoothing as illustrated in Figure 3 below.

One difference to the analysis in the previous section is that moderate smoothing produces paths of variables almost inbetween those of the more extreme cases because the nominal exchange rate is still nonstationary. In fact, the lasting depreciation of the domestic currency is only reduced to approximately 70% of what it is under a float.

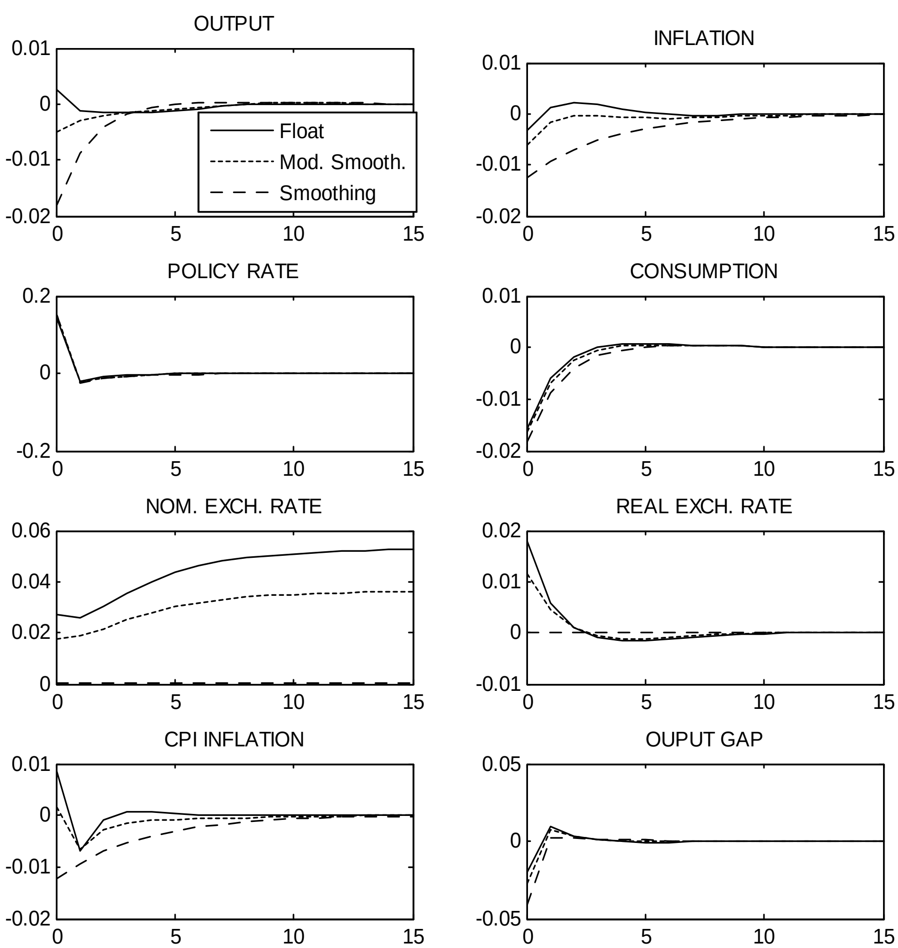

Figure 3. Moderate smoothing, float, and heavy smoothing.

The path of output is much as under heavy smoothing but with a decreased slope. There is initial deflation which is quickly brought close to zero. CPI inflation displays an initial positive value due to the real depreciation, then a negative spike as the real exchange rate appreciates. There is subsequently a gradual return towards steady state as swings in domestic inflation and the real exchange rate decrease. Finally, the real exchange rate follows a path similar to that of a floating regime, but with less variability.

The overall effect of moderately smoothing the exchange rate is thus less initial deflation and recession than under heavy smoothing, a smaller initial jump in real and nominal exchange rates, and a smaller permanent depreciation of the domestic currency than under a float.

6.4 Fear of Floating- What is the Difference?

The results above indicate that exchange rate targeting induces a more stable exchange rate at the cost of increased variability in output and inflation. Where the previous sections have compared differences in impulse responses resulting from different weigths on the same policy objective, this section takes the analysis a step further by looking at differences in impulse responses arising from assigning the same moderate weight to different exchange rate targets. Figure 4 illustrates the paths of important variables under the two types of "fear of floating" outlined above.

Most obviously, there is great variation in the impulse response of the nominal exchange rate, which is brought back on target in the long run under stabilization, but is nonstationary and allowed to permanently depreciate under smoothing. The optimality of bringing the nominal exchange rate back on target under moderate stabilization causes the recession and domestic deflation to be more pronounced, and the real exchange rate to appreciate and return to steady state faster than under smoothing. For the Central Bank, the cost of pursuing stabilization is mainly in terms of increased CPI-deviations from target for the entire simulation period. There is also an increase in the initial output gap.

In summary, the nature of "fear of floating" determines whether monetary policy should primarily be used to bring back the nominal exchange rate to zero (while also caring for inflation and output), or to stabilize inflation and output (while also keeping nominal exchange rate swings low).

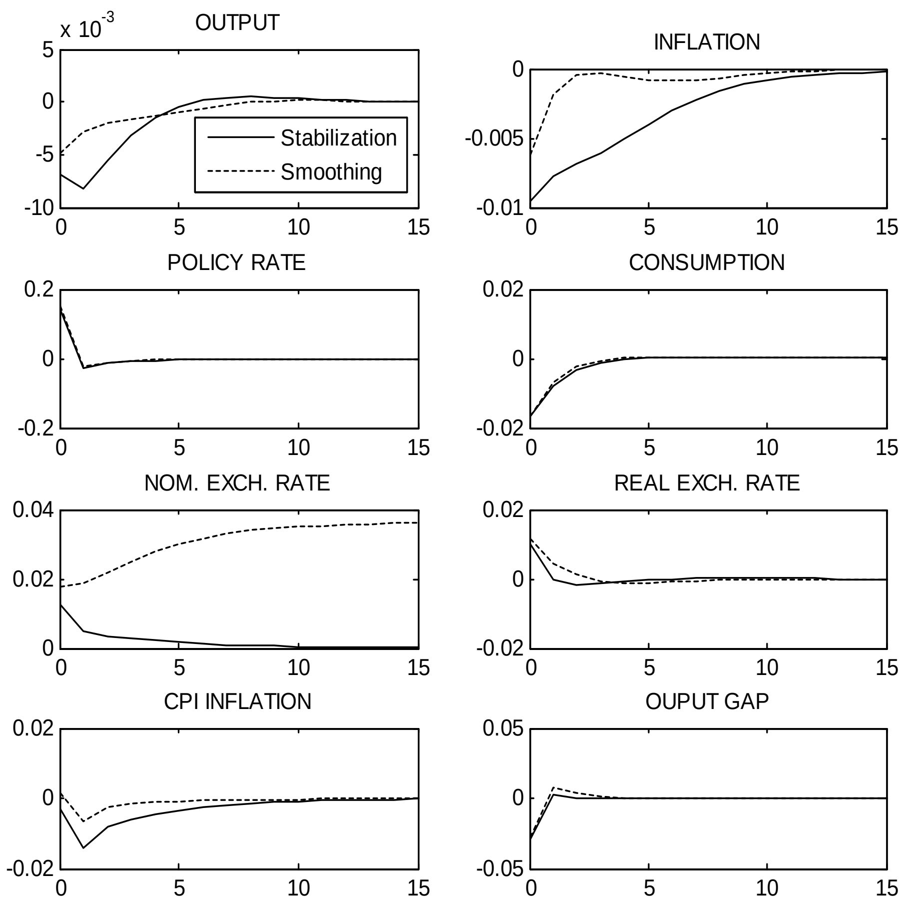

Figure 4. Fear of Floating.

7 Conclusion

In a Small Open Economy Model, it is demonstrated how targeting the nominal exchange rate in levels or first differences affects the economy. In particular, the paper considers optimal policy and impulse responses following a shock to the foreign interest rate under five alternative regimes: 1) A pure float, 2) A peg, 3) Heavy smoothing, 4) Moderate stabilization, and 5) Moderate smoothing.

It is found that even moderate exchange rate concerns are capable of turning around the sign of reaction coefficients to both domestic and foreign variables in the optimal policy rule, and that the absolute value of the response to foreign variables is usually increasing in the weight attached to nominal exchange rates. An attractive feature of the model is that the instrument is kept close to the foreign rate under every regime, and especially so under a peg and heavy smoothing as predicted by UIP. The fact that optimal policy reacts to all relevant information is evident in the instrument rule having nonzero coefficients on all variables affecting the loss function under each regime, and zero coefficients on all other variables.

The cost of exchange rate targeting is also evident in the model. However, though "fear of floating" generally stabilizes exchange rates at the cost of output and inflation compared to a float, impulse responses are not simply inbetween those of a pure float and regimes assigning more weight to the exchange rate target. For instance, the recession and deflation necessary to bring the nominal exchange rate back on target under moderate stabilization is optimally spread over time compared to a peg to avoid part of the initial deflation and decrease in output.

Because even a small weight on exchange rate stabilization makes it optimal to bring back the nominal exchange rate to zero, the nature of "fear of floating" has much impact on the focus of monetary policy. In particular, monetary policy can stabilize CPI-inflation and the output gap better under moderate smoothing because it need not generate a nominal appreciation to make up for the initial jump in the nominal exchange rate. The result is much smaller CPI-deviations from target for the entire simulation period and a decrease in the initial output gap.

The general framework presented in the paper lends itself to several extensions. Most importantly, realism could be increased by imposing predeterminedness of output and inflation, imperfect pass through, and by considering discretionary policy rather than commitment. The latter is especially important if a large part of the concern for currency instability is attributable to lack of credibility (as for instance Calvo and Reinhart (2000) argue could be the case for emerging markets). It would then also be sensible to close the model by assuming a risk premium on small open economy borrowing rather than risk sharing.

An interesting theoretical extension is to construct a fully optimizing framework by allowing the Foreign Central Bank to also optimize subject to the full rational expectations model and discretion. To understand whether the decrease in exchange rate variability is worth the costs, one must make welfare calculations based on approximations to the representative consumer's utility in a model that explicitly takes into account the reasons to care about exchange rates.

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9 Appendix

9.1 State space form and loss function

This subsection shows how the model equations (29) – (36) can be rewritten in the convenient state space form

$$\begin{bmatrix} X_{t+1} \\ HE_t x_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_t \\ x_t \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} i_t + \begin{bmatrix} C \\ 0 \end{bmatrix} \varepsilon_{t+1} \tag{39}$$

where $t \geq 0$, X_{t+1} is an n_X vector of predetermined variables (variables with exogenous one-period-ahead forecast error) with X_0 given, x_t is an n_x vector of forward-looking variables, i_t is the instrument, and ε_{t+1} is an n_ε vector of exogenous zero-mean iid shocks as described in Svensson (2005). The matrices A and B are partitioned conformably with X_t and x_t.

To rewrite the model equations in the form of (39), let $n_X = 15$, $n_x = n_\varepsilon = 5$, and

$$\begin{aligned} X_t &= \left(e_{t-1}, i_{t-1}, i^*_{t-1}, \pi_{t-1}, y_{t-1}, s_{t-1}, \overline{y}_{t-1}, \pi^*_{t-1}, y^*_{t-1}, a_t, \mu^w_t, a^*_t, \mu^{*w}_t, \varepsilon^*_{i,t}, \overline{y}^*_t\right) \\ x_t &= \left(\pi_t, y_t, s_t, \pi^*_t, y^*_t\right) \\ \varepsilon_t &= \left(\varepsilon_{a,t}, \varepsilon_{\mu,t}, \varepsilon^*_{a,t}, \varepsilon^*_{\mu,t}, \varepsilon^*_{i,t}\right) \end{aligned} \tag{40}$$

The covariance matrix of ε_t is assumed to be the identity matrix for the solution method to be valid. In particular, shocks are assumed to be uncorrelated.[15]

To obtain the state space form, simply rewrite the equations of the domestic and foreign economies separately. Defining the composite parameters

$$\begin{aligned} \sigma_0 &\equiv \sigma + h(\sigma - 1) \\ \sigma_1 &\equiv \gamma(2-\gamma)(\sigma - 1) + 1 \\ \sigma_2 &\equiv -\varphi\sigma_1 - \sigma \end{aligned}$$

[15] Because the model is linear-quadratic there is certainty equivalence and the optimal policy is independent of the variance of shocks.

the equations specifying the foreign economy

$$\begin{aligned}
y_t^* &= \frac{\sigma}{\sigma_0}E_t y_{t+1}^* + \left(1-\frac{\sigma}{\sigma_0}\right)y_{t-1}^* - \frac{1}{\sigma_0}\left(i_t^* - E_t\pi_{t+1}^*\right) \\
\pi_t^* &= \frac{\beta}{1+\beta}E_t\pi_{t+1}^* + \frac{1}{1+\beta}\pi_{t-1}^* + \delta\left(\varphi+\sigma\right)\left(y_t^* - \overline{y}_t^*\right) + \delta\mu_t^{*w} \\
\overline{y}_t^* &= \frac{\varphi+1}{\varphi+\sigma}a_t^* - \frac{h\left(1-\sigma\right)}{\varphi+\sigma}y_{t-1}^* \\
\overline{r}_t^* &= \sigma E_t\Delta\overline{y}_{t+1}^* - h\left(\sigma-1\right)\left(\overline{y}_t^* - y_{t-1}^*\right) \\
i_t^* &= \overline{r}_t^* + \frac{3}{2}\pi_t^* + \frac{1}{2}\left(y_t^* - \overline{y}_t^*\right) + \varepsilon_{i,t}^*
\end{aligned}$$

yield

$$\begin{aligned}
\frac{\beta}{1+\beta}E_t\pi_{t+1}^* &= \pi_t^* - \frac{1}{1+\beta}\pi_{t-1}^* - \delta\left(\varphi+\sigma\right)y_t^* + \delta\left(\varphi+\sigma\right)\overline{y}_t^* - \delta\mu_t^{*w} && (41) \\
\sigma E_t y_{t+1}^* + E_t\pi_{t+1}^* &= \frac{3}{2}\pi_t^* + \left(\frac{1}{2} - \frac{\sigma h\left(1-\sigma\right)}{\varphi+\sigma} + \sigma_0\right)y_t^* + \frac{\sigma h\left(1-\sigma\right)}{\varphi+\sigma}y_{t-1}^* && (42) \\
&\quad + \frac{\sigma\left(\varphi+1\right)\left(\alpha_a-1\right)}{\varphi+\sigma}a_t^* - \left(h\left(\sigma-1\right)+\frac{1}{2}\right)\overline{y}_t^* + \varepsilon_{i,t}^* \\
\overline{y}_{t+1}^* &= \frac{\left(\varphi+1\right)\alpha_a}{\varphi+\sigma}a_t^* - \frac{h\left(1-\sigma\right)}{\varphi+\sigma}y_t^* + \frac{\varphi+1}{\varphi+\sigma}\varepsilon_{a,t+1}^* \\
i_t^* &= -\frac{\sigma\left(\varphi+1\right)\left(1-\alpha_a\right)}{\varphi+\sigma}a_t^* + \left(\frac{1}{2} + \frac{\sigma h\left(\sigma-1\right)}{\varphi+\sigma}\right)y_t^* + \frac{3}{2}\pi_t^* \\
&\quad + \frac{\left(\sigma-1\right)h\varphi}{\left(\sigma+\varphi\right)}y_{t-1}^* - \left(h\left(\sigma-1\right)+\frac{1}{2}\right)\overline{y}_t^* + \varepsilon_{i,t}^*
\end{aligned}$$

where for simplicity the natural foreign interest rate has been substituted out.

The domestic economy requires slightly more algebra due to the unusual form where it has not been possible to simply substitute out the terms of trade. The resulting state space form is therefore slightly messy. Use the relations

$$\begin{aligned}
y_t &= \left(2-\gamma\right)\gamma s_t + \gamma y_t^* + \frac{\sigma}{\sigma_0}E_t\left(y_{t+1} - \left(2-\gamma\right)\gamma s_{t+1} - \gamma y_{t+1}^*\right) \\
&\quad + \left(1-\frac{\sigma}{\sigma_0}\right)\left(y_{t-1} - \left(2-\gamma\right)\gamma s_{t-1} - \gamma y_{t-1}^*\right) - \frac{1-\gamma}{\sigma_0}\left(i_t - E_t\pi_{t+1} - \gamma E_t\Delta s_{t+1}\right) \\
\pi_t &= \frac{\beta}{1+\beta}E_t\pi_{t+1} + \frac{1}{1+\beta}\pi_{t-1} + \delta\left(\varphi + \frac{\sigma}{1-\gamma}\right)\left(y_t - \overline{y}_t\right) \\
&\quad + \delta\left(\gamma - \frac{\sigma\gamma\left(2-\gamma\right)}{1-\gamma}\right)\left(s_t - \overline{s}_t\right) + \delta\mu_t^w
\end{aligned}$$

$$
\begin{aligned}
s_t &= \frac{\sigma}{\sigma_1}\left(y_t - y_t^*\right) - \frac{h\left(\sigma-1\right)}{\sigma_1}\left(y_{t-1} - y_{t-1}^*\right) + \frac{h\left(\sigma-1\right)\left(2-\gamma\right)\gamma}{\sigma_1} s_{t-1} \\
\overline{y}_t &= \frac{\left(1-\gamma\right)\left(\varphi+1\right)}{\varphi\left(1-\gamma\right)+\sigma} a_t + \frac{\sigma\gamma}{\varphi\left(1-\gamma\right)+\sigma} y_t^* - \gamma\left(\frac{1-\gamma-\sigma\left(2-\gamma\right)}{\varphi\left(1-\gamma\right)+\sigma}\right)\overline{s}_t \\
&\quad - \frac{h\left(1-\sigma\right)}{\varphi\left(1-\gamma\right)+\sigma}\left(y_{t-1} - \gamma y_{t-1}^* - \gamma\left(2-\gamma\right)s_{t-1}\right) \\
\overline{s}_t &= \frac{\sigma}{\sigma_1}\left(\overline{y}_t - y_t^*\right) - \frac{h\left(\sigma-1\right)}{\sigma_1}\left(y_{t-1} - y_{t-1}^*\right) + \frac{\gamma h\left(\sigma-1\right)\left(2-\gamma\right)}{\sigma_1} s_{t-1}
\end{aligned}
$$

to obtain the state space form equations

$$
\begin{aligned}
\frac{\beta}{1+\beta} E_t \pi_{t+1} &= \pi_t - \frac{1}{1+\beta}\pi_{t-1} - \delta\left(\varphi + \frac{\sigma}{1-\gamma}\right) y_t - \delta\left(\gamma - \frac{\sigma\gamma\left(2-\gamma\right)}{1-\gamma}\right) s_t \\
&\quad + \left(\varphi+1\right)\delta a_t + \frac{\delta\gamma\sigma}{\left(1-\gamma\right)} y_t^* + \frac{\left(\sigma-1\right)h\delta}{\left(1-\gamma\right)} y_{t-1} - \frac{\left(\sigma-1\right)h\gamma\delta}{\left(1-\gamma\right)} y_{t-1}^* \\
&\quad + \frac{\left(\gamma-2\right)\left(\sigma-1\right)h\gamma\delta}{\left(1-\gamma\right)} s_{t-1} - \delta\mu_t^w
\end{aligned}
\tag{43}
$$

$$
\begin{aligned}
&\frac{\sigma}{\sigma_0} E_t y_{t+1} + \gamma\left(\frac{1-\gamma-\left(2-\gamma\right)\sigma}{\sigma_0}\right) E_t s_{t+1} - \frac{\gamma\sigma}{\sigma_0} E_t y_{t+1}^* + \frac{1-\gamma}{\sigma_0} E_t \pi_{t+1} \\
= \; & y_t + \gamma\left(\frac{\left(1-\gamma\right)}{\sigma_0} - \left(2-\gamma\right)\right) s_t - \gamma y_t^* - \left(1 - \frac{\sigma}{\sigma_0}\right) y_{t-1} \\
&+ \frac{1-\gamma}{\sigma_0} i_t + \left(1 - \frac{\sigma}{\sigma_0}\right)\gamma y_{t-1}^* + \left(1 - \frac{\sigma}{\sigma_0}\right)\left(2-\gamma\right)\gamma s_{t-1}
\end{aligned}
\tag{44}
$$

$$
0 = \sigma y_t - \sigma y_t^* - h\left(\sigma-1\right) y_{t-1} + h\left(\sigma-1\right) y_{t-1}^* + h\left(\sigma-1\right)\left(2-\gamma\right)\gamma s_{t-1} - \sigma_1 s_t \tag{45}
$$

$$
\begin{aligned}
\overline{y}_t &= \frac{-\sigma_1\left(\varphi+1\right)}{\sigma_2} a_t + \frac{\left(2-\gamma\right)\left(\sigma-1\right)\gamma\sigma}{\sigma_2} y_t^* - \frac{\left(\sigma-1\right)h}{\sigma_2} y_{t-1} \\
&\quad + \frac{\left(\gamma-2\right)\left(\sigma-1\right)^2\gamma h}{\sigma_2} y_{t-1}^* + \frac{\left(2-\gamma\right)\left(\sigma-1\right)\gamma h}{\sigma_2} s_{t-1}
\end{aligned}
$$

Also, use the fact that from (27), $e_t = e_{t-1} + \left(1-\gamma\right)s_t - \left(1-\gamma\right)s_{t-1} + \pi_t - \pi_t^*$. It then easily follows that the matrices H, A, B, C defining the system are given by

$$
A_{11} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & \gamma-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{(\sigma-1)h\varphi}{(\sigma+\varphi)} & 0 & 0 & \frac{\sigma(\varphi+1)(\alpha_a-1)}{\varphi+\sigma} & 0 & 1 & -\left(h\left(\sigma-1\right)+\frac{1}{2}\right) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{(\sigma-1)h}{\sigma_2} & \frac{(2-\gamma)(\sigma-1)\gamma h}{\sigma_2} & 0 & 0 & \frac{(\gamma-2)(\sigma-1)^2\gamma h}{\sigma_2} & \frac{-\sigma_1(\varphi+1)}{\sigma_2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_a & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_\mu & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_a & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_\mu & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{(\varphi+1)\alpha_a}{\varphi+\sigma} & 0 & 0 & 0
\end{bmatrix}
$$

$$A_{12}=\begin{bmatrix} 1 & 0 & 1-\gamma & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{2} & \left(\frac{1}{2}-\frac{\sigma h(1-\sigma)}{\varphi+\sigma}\right) \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(2-\gamma)(\sigma-1)\gamma\sigma}{\sigma_2} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{h(1-\sigma)}{\varphi+\sigma} \end{bmatrix}, \quad B_1=\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad C=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{\varphi+1}{\varphi+\sigma} & 0 & 0 \end{bmatrix}$$

$$A_{21} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{1+\beta} & 0 & 0 & 0 & 0 & -\delta & 0 & \delta(\varphi+\sigma) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sigma h(1-\sigma)}{\varphi+\sigma} & 0 & 0 & \frac{\sigma(\varphi+1)(\alpha_a-1)}{\varphi+\sigma} & 0 & 1 & -\left(h(\sigma-1)+\frac{1}{2}\right) \\ 0 & 0 & 0 & -\frac{1}{1+\beta} & \frac{(\sigma-1)h\delta}{(1-\gamma)} & \frac{(\gamma-2)(\sigma-1)h\gamma\delta}{(1-\gamma)} & 0 & 0 & -\frac{(\sigma-1)h\gamma\delta}{(1-\gamma)} & (\varphi+1)\delta & -\delta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\left(1-\frac{\sigma}{\sigma_0}\right) & \left(1-\frac{\sigma}{\sigma_0}\right)(2-\gamma)\gamma & 0 & 0 & \left(1-\frac{\sigma}{\sigma_0}\right)\gamma & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -h(\sigma-1) & h(\sigma-1)(2-\gamma)\gamma & 0 & 0 & h(\sigma-1) & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{22}=\begin{bmatrix} 0 & 0 & 0 & 1 & -\delta(\varphi+\sigma) \\ 0 & 0 & 0 & \frac{3}{2} & \left(\frac{1}{2}-\frac{\sigma h(1-\sigma)}{\varphi+\sigma}+\sigma_0\right) \\ 1 & -\delta\left(\varphi+\frac{\sigma}{1-\gamma}\right) & -\delta\left(\gamma-\frac{\sigma\gamma(2-\gamma)}{1-\gamma}\right) & 0 & \frac{\delta\gamma\sigma}{(1-\gamma)} \\ 0 & 1 & \gamma\left(\frac{(1-\gamma)}{\sigma_0}-(2-\gamma)\right) & 0 & -\gamma \\ 0 & \sigma & -\sigma_1 & 0 & -\sigma \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1-\gamma}{\sigma_0} \\ 0 \end{bmatrix}, \quad H = \begin{bmatrix} 0 & 0 & 0 & \frac{\beta}{1+\beta} & 0 \\ 0 & 0 & 0 & 1 & \sigma \\ \frac{\beta}{1+\beta} & 0 & 0 & 0 & 0 \\ \frac{1-\gamma}{\sigma_0} & \frac{\sigma}{\sigma_0} & \left(\frac{(1-\gamma)\gamma-(2-\gamma)\gamma\sigma}{\sigma_0}\right) & 0 & -\frac{\gamma\sigma}{\sigma_0} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

To complete the state space form, the period loss function, L_t, must be expressed in terms of a vector of target variables

$$Y_t = D \begin{bmatrix} X_t \\ x_t \\ r_t \end{bmatrix}$$

and a weighting matrix Λ so that $L_t = \frac{1}{2} Y_t' \Lambda Y_t$.

In the current model, the loss function is quadratic in the terms π_t^{cpi}, $y_t - \overline{y}_t$, $e_t - e_{t-1}$ and e_t.[16] To get the model into the desired form, rewrite these variables as

$$\begin{aligned}
\pi_t^{cpi} &= \pi_t + \gamma s_t - \gamma s_{t-1} \\
y_t - \overline{y}_t &= y_t + \frac{\sigma_1(\varphi+1)}{\sigma_2} a_t - \frac{(2-\gamma)(\sigma-1)\gamma\sigma}{\sigma_2} y_t^* + \frac{(\sigma-1)h}{\sigma_2} y_{t-1} \\
&\quad - \frac{(\gamma-2)(\sigma-1)^2\gamma h}{\sigma_2} y_{t-1}^* - \frac{(2-\gamma)(\sigma-1)\gamma h}{\sigma_2} s_{t-1} \\
e_t - e_{t-1} &= (1-\gamma) s_t - (1-\gamma) s_{t-1} - \pi_t^* + \pi_t \\
e_t &= e_{t-1} + (1-\gamma) s_t - (1-\gamma) s_{t-1} + \pi_t - \pi_t^*
\end{aligned}$$

Then, dropping the constant $(1-\beta)$, the matrices D and Λ describing the loss function are:

$$D = \left[\begin{matrix} 0 & 0 & 0 & 0 & 0 & -\gamma & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(\sigma-1)h}{\sigma_2} & -\frac{(2-\gamma)(\sigma-1)\gamma h}{\sigma_2} & 0 & 0 & -\frac{(\gamma-2)(\sigma-1)^2\gamma h}{\sigma_2} & \frac{\sigma_1(\varphi+1)}{\sigma_2} \\ 0 & 0 & 0 & 0 & 0 & -(1-\gamma) & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -(1-\gamma) & 0 & 0 & 0 & 0 \end{matrix} \right.$$
$$\left. \begin{matrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & \gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -\frac{(2-\gamma)(\sigma-1)\gamma\sigma}{\sigma_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1-\gamma & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1-\gamma & -1 & 0 & 0 \end{matrix} \right]$$

[16] The Matlab programme accompanying this paper also allows a nonzero weight on interest rate smoothing, $(i_t - i_{t-1})^2$.

$$
\Lambda = \begin{bmatrix} \lambda_{cpi} & 0 & 0 & 0 \\ 0 & \lambda_y & 0 & 0 \\ 0 & 0 & \lambda_{\Delta\varepsilon} & 0 \\ 0 & 0 & 0 & \lambda_\varepsilon \end{bmatrix}
$$

9.2 Model Solution Under Commitment

This section considers the solution to

$$
\min_{\{i_t\}_{t=0}^{\infty}} \ (1-\beta) E_0 \sum_{t=0}^{\infty} \beta^t L_t \tag{46}
$$

subject to the model (39) when the Central Bank can commit and hence solve the problem once and for all in the initial period. I use the Recursive Saddlepoint Method of Marcet and Marimon as described in Svensson (2005).[17] The main idea of this method is to reformulate the problem (46) as a recursive problem to which solutions to the standard linear quadratic regulator problem can be applied.

Under initial conditions $X_0 = \overline{X}_0$, this is done by first setting up the Lagrangian

$$
\begin{aligned}
L_0 \quad = \quad & (1-\beta) E_0 \sum_{t=0}^{\infty} \beta^t \left[L_t + \begin{pmatrix} \xi'_{t+1} \left(X_{t+1} - A_{11} X_t - A_{12} x_t - B_1 i_t - C\varepsilon_{t+1} \right) \\ +\Xi'_t \left(H x_{t+1} - A_{21} X_t - A_{22} x_t - B_2 i_t \right) \end{pmatrix} \right] \\
& + \frac{1-\beta}{\beta} \xi'_0 \left(X_0 - \overline{X}_0 \right)
\end{aligned}
$$

Here, ξ_{t+1} is the n_X vector of forwardlooking Lagrange multipliers corresponding to the upper block of the model equations, ie the predetermined variables, while Ξ_t is the n_x vector of predetermined Lagrange multipliers corresponding to the remaining n_x model equations.

One can then show that the problem can be reformulated as a recursive saddlepoint problem

$$
\begin{aligned}
& \max_{\{\varsigma_t\}_{t=0}^{\infty}} \ \min_{\{x_t, i_t\}_{t=0}^{\infty}} (1-\beta) E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{L}_t \\
\text{s.t. } X_{t+1} \quad &= \quad A_{11} X_t + A_{12} x_t + B_1 i_t + C\varepsilon_{t+1} \\
\Xi_t \quad &= \quad \varsigma_t \\
X_0 \quad &= \quad \overline{X}_0, \Xi_{-1} = 0
\end{aligned}
$$

[17] See also Marcet and Marimon (1998).

where $\widetilde{L}_t$ is the modified loss function $\widetilde{L}_t \equiv L_t + \varsigma_t' (-A_{21}X_t - A_{22}x_t - B_2 i_t) + \frac{1}{\beta}\Xi_{t-1}' H x_t$, and that the standard solution for the linear quadratic regulator problem can be used. Mathematically, this is done by solving the Riccati equation

$$\widetilde{V} = Q + \beta \widetilde{A}'\widetilde{V}\widetilde{A} - \left(\beta \widetilde{B}'\widetilde{V}\widetilde{A} + N'\right)' \left(\beta \widetilde{B}'\widetilde{V}\widetilde{B} + R\right)^{-1} \left(\beta \widetilde{B}'\widetilde{V}\widetilde{A} + N'\right)$$

where

$$\begin{aligned} Q &= \begin{bmatrix} W_{XX} & 0 \\ 0 & 0 \end{bmatrix}, \quad \widetilde{A} = \begin{bmatrix} A_{11} & 0 \\ 0 & 0 \end{bmatrix}, \quad \widetilde{B} = \begin{bmatrix} A_{12} & B_1 & 0 \\ 0 & 0 & I \end{bmatrix} \\ N &= \begin{bmatrix} W_{Xx} & W_{Xi} & -A_{21}' \\ \frac{1}{\beta}H & 0 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} W_{xx} & W_{xi} & -A_{22}' \\ W_{xi}' & W_{ii} & -B_2' \\ -A_{22} & -B_2 & 0 \end{bmatrix} \end{aligned}$$

and W_{**} are defined by partitioning the matrix $W \equiv D'\Lambda D$ conformably with X_t, x_t, and i_t such that

$$W = \begin{bmatrix} W_{XX} & W_{Xx} & W_{Xi} \\ W_{Xx}' & W_{xx} & W_{xi} \\ W_{Xi}' & W_{xi}' & W_{ii} \end{bmatrix}$$

The solution to the problem is then given by

$$Z_t \equiv \begin{bmatrix} \widetilde{X}_{t+1} \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} M & 0 & 0 \\ F_x & 0 & 0 \\ F_i & 0 & 0 \end{bmatrix} \begin{bmatrix} \widetilde{X}_t \\ x_{t-1} \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} \widetilde{C} \\ 0 \\ 0 \end{bmatrix} \varepsilon_{t+1} \tag{47}$$

where $\widetilde{X}_{t+1} \equiv \begin{pmatrix} X_{t+1} \\ \Xi_t \end{pmatrix}$, F_x is the n_x top rows of the matrix F, F_i denotes the subsequent n_i rows of F, n_i is the number of instruments (=1), $M \equiv \widetilde{A} + \widetilde{B}F$, and F is given by

$$F = -\left(\beta \widetilde{B}'\widetilde{V}\widetilde{B} + R\right)^{-1} \left(\beta \widetilde{B}'\widetilde{V}\widetilde{A} + N'\right)$$

The system can be written more compactly as $Z_t = \overline{A}Z_{t-1} + \overline{C}\varepsilon_{t+1}$ where the definition of the matrices $\overline{A}$ and $\overline{C}$ should be obvious from (47). In the current model, the vector Z_t contains mainly variables dated at time t (since many variables in the vector X_{t+1} are lagged one period to make them predetermined). However, the variables $a, \mu^w, a^*, \mu^{*w}, \varepsilon_i^*$, and $\overline{y}^*$are timed at $t+1$. The bottom row of (47) is the *optimal policy function*.