Capital Flows and Monetary Policy

VERSIÓN ACTUALIZADA

Javier Guillermo Gómez*

December 2, 2009

Abstract

Capital inflows and outflows often remind policymakers of the monetary policy "trilemma" and the several associated dilemmas. To tackle these dilemmas, an equilibrium model of capital flows is proposed. The model captures bouts of capital inflows and outflows with shocks to the emerging-market country risk premium. From the equilibrium conditions of the model, an expression for the accounting of net foreign assets is derived. This expression helps study the evolution of foreign debt, during capital inflows and outflows, under fixed and floating exchange rates. A policy experiment is conducted for the case of a capital outflow. It shows that during a capital outflow an interest rate defense of the exchange rate can deliver a recession even in financially resilient economies. This is one possible explanation of the puzzle in Chari, Kehoe and McGrattan [AER Vol. 25 No. 2, 2005]

JEL classification: F41; F32; G15; H62; H63

Keywords: Capital outflows; Sudden stops; Exchange rate regimes; Country credit risk; Debt sustainability

*Banco de la República (the central bank of Colombia) jgomezpi@banrep.gov.co. The author thanks Martin Seneca of the Central Bank of Iceland and two anonymous referees for their comments.

This paper studies capital inflows and outflows under fixed and floating exchange rates in a stochastic dynamic general equilibrium (DSGE) model. In addition, the paper proposes a dynamic equation for the evolution of net foreign assets.

The theoretical literature on capital flows in emerging market economies have focused on unanticipated capital outflows, the so called "sudden stops" (Calvo, 1998). Models of sudden stops include Chari, Kehoe and McGrattan (2005), hereafter CKM, Mendoza (2004), Mendoza and Smith (2002) and Uribe (2006). In these models sudden stops are triggered by a constraint on foreign borrowing that suddenly becomes binding. While this mechanism enables the study of capital outflows, for the same reason that a string cannot be pushed, it cannot be used to study capital inflows.

In other papers, e.g. CCV and GGN, the increase in the cost of borrowing and exchange rate depreciation is a consequence of shocks to the foreign interest rate. But shocks to foreign interest rates and to the country country risk premium are distinct sources of volatility as was suggested by Krugman (2000) and Roubini (2002) (p. 597).

In this paper capital outflows and inflows are the result of changes in risk aversion in international financial markets and hece to the price of rks, in particular, the price of investing in emerging markets. Risk aversion can be captured by measures such as CDX spreads and the EMBI spread. As shocks in the model may be positive or negative (around a theoretical steady-state risk premium) they can capture capital outflows as well as inflows. Even though the model does not have a banking sector, nor realistic features such as banking crisis, when it is subject to large upward shocks to the country risk premium it can capture the machanisms involved during sudden stops−in particular those related to the behavior of output, absorption and the trade balance.

An important point about capital outflows is the behavior of output. CKM show that sudden stops should theoretically increase net exports and output and then pose the question as to why sudden stops cause output drops. In answering this question, they review the literature on models of sudden stops and show that existing models deliver recessions as the outcome of mechanisms that are not completely transparent and that lack empirical evidence.

In this paper, sudden stops can result in output drops even under balance sheet conditions that are not critical. Here recessions do not depend on balance sheet effects alone, but, primarily, on the response of monetary policy. The point is that not only balance sheet effects but also interest rate defenses of the exchange rate can explain why sudden stops cause recessions.[1]

[1] Interest rate defenses are not exactly equal to "fear of floating." On the one hand, interest rate defenses (in the

Besides the behaviro of output, among the many macroeconomic issues at stake during bouts of capital inflows and outflows in emerging economies, other two issues stand out: the monetary policy regime, and the currency denomination of foreign debt. As suggested by Fisher (2003), these issues are related since exchange rate rigidity may cause borrowers to underestimate currency risk. Chang and Velasco (2006) and Chamon and Haussman (2002) have dealt with the link between the monetary policy regime and the currency denomination of foreign debt. The papers point to the following conclusions: if foreign debt is denominated in foreign (domestic) currency, the central bank finds optimal to peg (float) the exchange rate and if the exchange rate is fixed (floating), private agents find optimal to hold foreign debt in foreign (domestic) currency. For analytical convenience, these models are highly stylized; in particular, they are one period models.

These papers do convey the message that the choice of monetary policy needs to be closely related to the choice of the currency denomination of foreign debt. Among the papers that have studied the topic of the choice of exchange rate regime in a small open economy are Céspedes, Chang and Velasco (2004) (henceforth CCV), Gertler, Gilchrist and Natalucci (2001) (GGN), Cook (2004), and Devaraux Lane and Xu (2006). These papers have in common that they are based on the financial accelerator and that they do not deal with the currency denomination of foreign debt. The model in this paper embeds the currency denomination of foreign debt and the exchange rate regime in a DSGE model.

Besides this introduction, the paper has three sections: the model, results and conclusions. The section on the model has three main subsections: the first one is the problem of each agent and the general equilibrium, the second one, the complete model; and the third one, an extended model with the financial accelerator. The section on results presents the monetary-policy experiment under a capital outflow. An appendix presents a model with an equilibrium with flexible prices. Finally, a mathematical appendix presents more detail of the model (the problems of the firm and the foreign household, the steady state and the model in state space form), and some mathematical derivations.[2]

language of Lahiri and Vegh (2003) or Jeanne and Zettlemeyer (2002)) involve changes in interest rates to avoid exchange rate volatility. On the other hand, "fear of floating" in Calvo and Reinhart (2000) also involves large changes in international reserves.

[2] Another appendix with detailed step by step mathematial derivations is available from the author on request.

1 The model

The model consists of two economies, one domestic, one foreign. The domestic economy is a small open economy. The foreign economy is approximately closed. Each economy produces one good, both goods are tradable. In the domestic economy there are four agents: a representative household, a representative firm, a representative distributor firm, and the central bank. In the foreign economy there is one agent, the household.

The household issues three types of debt instruments: domestic bonds, foreign bonds denominated in foreign currency and foreign bonds denominated in domestic currency. Domestic bonds are in zero net supply. Foreign bonds denominated in domestic and foreign currencies are in zero net supply in the world economy but the domestic economy has a negative net foreign asset position and rolls over foreign debt forever. As the foreign economy is large, it is approximately closed and its net foreign asset position tends to zero.

Foreign and domestic currency denominated bonds are traded at spreads $\phi_{F,t}$ and $\phi_{H,t}$ over the foreign interest rate. Country and exchange rate risks are compensated by risk premiums $\phi_{F,t}$ and $\phi_{S,t}$.[3]

1.1 The household

The household solves three optimization problems. In the first problem, it chooses absorption and hours worked. In the second problem, the household splits absorption between domestically produced and imported goods. From the first two problems the household indirectly chooses savings and net foreign assets. In the third one, the household allocates absorption of domestically produced goods along a continuum of an infinite number of goods.

[3]Following a long tradition in small open economy models, the domestic or home economy is denoted without asterisks and the foreign economy, with asterisks. The goods $C_{H,t}$ and $C_{F,t}$ are the home and foreign goods consumed in the domestic economy. The goods $C^*_{H,t}$ and $C^*_{F,t}$ are the home and foreign goods consumed in the foreign economy. Needless to say, $C_{H,t}$ and $C^*_{F,t}$ are the same good. The same is true for the goods $C_{F,t}$ and $C^*_{H,t}$. Thus, the words home and foreign refer primarily to each of the two economies and are denoted with and without asterisk. These words also refer to the two goods but in this case they depend on the context of the economy that is being referred to. When denoting goods, these words are denoted with subindices H and F. The notation should be clear since the discussion deals primarily with the small open economy and not with the large, approximately closed economy. The words home and foreign are also used to qualify the currency denomination of foreign bonds.

1.1.1 The household's first optimization problem

In the first problem, the household maximizes expected utility

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(C_{t+i}^{\Delta})^{1-\sigma}}{1-\sigma} - \frac{\left(L_{t+i}^{\Delta}\right)^{1+\eta}}{1+\eta} \right] \tag{1}$$

where C_t is absorption and L_t is labor, and C_t^{Δ} and L_{t+i}^{Δ}, the quasi difference of absorption and labor are defined as $C_t^{\Delta} = \frac{C_t}{C_{A,t-1}^{\gamma}}$ and $L_t^{\Delta} = \frac{L_t}{L_{A,t-1}^{\gamma}}$ where the habits are aggregate per capita absorption and labor, $C_{A,t}$ and $L_{A,t}$ (since all households are alike, aggregate per capita absorption and leisure are equal to absorption and labor themselves: $C_{A,t} = C_t$, $L_t = L_{A,t}$). Utility (1) is maximized subject to:

$$C_t = \frac{W_t}{P_t} L_t + \Pi_t + A_t^T + (1+i_{t-1}) \frac{B_{t-1}}{P_t} - \frac{B_t}{P_t} \tag{2}$$

$$+ \left(1+i_{t-1}^*\right) \left(1+\phi_{F,t-1}\right) \frac{B_{F,t-1}^* S_t}{P_t} - \frac{B_{F,t}^* S_t}{P_t}$$

$$+ (1+i_{t-1}^*) \left(1+\phi_{H,t-1}\right) \frac{B_{H,t-1}}{P_t} - \frac{B_{H,t}}{P_t},$$

where the household's balance sheet is:[4]

$$\frac{B_t}{P_t} + \frac{B_{F,t}}{P_t} + \frac{B_{H.t}}{P_t} = N_t, \tag{3}$$

Notation is as follows: the W_t is the nominal wage; P_t is the price level; S_t is the nominal exchange rate; i_t is the one-period central-bank nominal policy interest rate, equal to the return on one period bonds; i_t^* is the one-period foreign interest rate; $\phi_{F,t}$ is the country risk premium, defined as the spread of a bond denominated in foreign currency, paid over and above the foreign interest rate i_t^*; the $\phi_{H,t}$ is the spread of a foreign bond denominated in domestic currency; A_t^T is net transfers in the balance of payments; Π_t is the firm's profits; B_t is the domestic one period bond; and N_t is net worth. The variable $B_{F,t}^*$ denotes the household's foreign, one-period bonds denominated in foreign currency and measured in foreign currency, the variable $B_{H,t}$ denotes the

[4] Since there is no physical capital in the model, net worth is equal to net foreign assets. Also, since the domestic firm does not hold any assets, the household's net worth is also equal to the economy's assets.

household's foreign, one-period bonds denominated in domestic currency and valued in domestic currency.[5] [6] [7]

Among the first order conditions for the household's problem are the flow budget constraint (2) and the marginal conditions:[8]

$$\left(C_t^{\Delta}\right)^{-\sigma} = \beta \left(C_{t+1|t}^{\Delta}\right)^{-\sigma} (1+i_t)\frac{P_t}{P_{t+1|t}} \tag{4}$$

$$\left(C_t^{\Delta}\right)^{\sigma} \left(L_t^{\Delta}\right)^{\eta} = \frac{W_t}{P_t} \tag{5}$$

$$(1+i_t^*)(1+\phi_{F,t})\frac{S_{t+1|t}}{S_t} = 1+i_t \tag{6}$$

$$(1+i_t^*)(1+\phi_{H,t}) = 1+i_t \tag{7}$$

$$1+\phi_{H,t} = \left(1+\phi_{F,t}\right)\frac{S_{t+1|t}}{S_t} \tag{8}$$

Equilibrium conditions (4) and (5) are standard. Equations (6) to (7) are the first order conditions with respect to B_t and $B_{F,t}^*$ and $B_{H,t}$. They are the arbitrage conditions among the three types of bonds; one of these conditions is redundant.

Following Bernanke, Gertler and Gilchrist (1999), CCV and GGN, country risk follows the accelerator equation

$$1+\phi_{F,t} = (N_{t-1})^{-\zeta}\left(1+\varepsilon_t^{\phi_F}\right) \tag{9}$$

The source of volatility in the model is changes in investors' sentiment or risk appetite $\varepsilon_t^{\phi_F}$.

Define the exchange rate risk premium as

$$1+\phi_{S,t} \equiv \frac{S_{t+1|t}}{S_t} \tag{10}$$

[5] For S_t the nominal exchange rate, $B_{F,t} = S_t B_{F,t}^*$ is the household's foreign bonds denominated in foreign currency and expressed in units of domestic currency, and $B_{H,t} = S_t B_{H,t}^*$ is the household's bonds denominated in domestic currency and expressed in domestic currency.

[6] To help make contact with the concepts of national accounts, note that $Y_t = \frac{W_t}{P_t}L_t + \Pi_t$ is gross *domestic* product, GDP (gross and net since there is no capital stock), and that $Y_t^S = \frac{W_t}{P_t}L_t + \Pi_t + A_t^T + \left(i_{t-1}^* + \phi_{F,t-1}\right)\frac{B_{F,t-1}^* S_t}{P_t} + \left(i_{t-1}^* + \phi_{H,t-1}\right)\frac{B_{H,t-1}}{P_t}$ is gross *national* product, GNP.

[7] When the variables $B_{H,t}$ and $B_{F,t}^*$ are less than zero the emerging economy is a net debtor. When B_t, $B_{H,t}$ and $B_{F,t}^*$ are bigger than zero the emerging economy is a net creditor.

[8] Use is being made of the notation $E_t X_{t+1} = X_{t+1|t}$. Also, up to a first order linear approximation $E_t(X_{t+1}\,Y_{t+1}) = X_{t+1|t}Y_{t+1|t}$.

Using definition (10), equations (6) to (8) may be written:

$$(1+i_t^*)(1+\phi_{F,t})(1+\phi_{S,t}) = 1+i_t \tag{11}$$

$$(1+\phi_{H,t}) = (1+\phi_{F,t})(1+\phi_{S,t}) \tag{12}$$

Equation (11) reveals that the risk free rate, i_t^*, is free of country risk and exchange rate risk, or in other words, the risky interest rate, i_t, has country and exchange rate risk.

Equation (12) shows that the spread on foreign bonds denominated in home currency is equal to the sum of the country and exchange rate risk premiums.

Given the policy experiment in the paper, it is convenient to write the UIP condition in nominal terms.[9] Defining the rate of depreciation of the nominal exchange rate as $1+s_t^{\Delta} = \frac{S_t}{S_{t-1}}$ the lagged UIP condition (6) may be written:

$$1+s_t^{\Delta} = \frac{1+i_{t-1}}{(1+i_{t-1}^*)(1+\phi_{F,t-1})}(1+\varepsilon_{t|t-1}^s) \tag{13}$$

where $1+\varepsilon_{t|t-1}^s = \frac{S_t}{S_{t|t-1}}$ follows (see mathematical appendix).

$$(1+\varepsilon_{t|t-1}^s) = \frac{(1+\hat{i}{}_t^*)(1+\overset{\wedge}{\phi}_{F,t})}{(1+\hat{i}_t)}(1+\varepsilon_{t+1|t}^s) \tag{14}$$

Define two measures of the real exchange rate, first, the real *bilateral* real exchange rate, $Q_{B,t} = \frac{S_t P_t^*}{P_t}$; second, the price of imported goods relative to the price of domestically produced goods, $Q_t = \frac{P_{F,t}}{P_t}$. The first measure may be written as:

$$Q_{B,t} = Q_{B,t-1}\frac{(1+s_t^{\Delta})(1+\pi_t^*)}{1+\pi_t} \tag{15}$$

where $1+\pi_t = \frac{1+P_t}{1+P_{t-1}}$ and $1+\pi_t^* = \frac{1+P_t^*}{1+P_{t-1}^*}$.

The second measure, defining the gap of the law of one price as $\Psi_t \equiv \frac{S_t P_t^*}{P_{F,t}}$ is:

[9] As the nominal exchange rate is not stationary, running the model with the UIP condition in nominal terms poses a challenge for the stationarity of the model. An alternative is to solve the model for the expected depreciation of the exchange rate using Equation (6) as $1+s_{t+1|t}^{\Delta} = (1+i_t)/(1+i_t^*)(1+\phi_{F,t})$. However, $s_{t+1|t}^{\Delta}$ is not necessarily equal to s_{t+1}^{Δ}; in particular, these variables are not equal to each other when there are unexpected shocks to the UIP equation−which is the policy experiment in the paper. The UIP condition may be solved for the actual depreciation of the exchange rate provided the expectation error of the UIP equation is taken into account explicitly. This is what we do with Equations (13) and (14).

$$Q_t = \frac{Q_{B,t}}{\Psi_t} \tag{16}$$

A time t partial equilibrium for the household is a set of allocations C_t and L_t, given $\phi_{F,t}$, $\phi_{H,t}$, A_t^T, i_t, i_t^*, P_t, r_t^* W_t, Π_t^j, $B_{F,t-1}^*$, $B_{H,t-1}$, B_{t-1}, such that utility is maximized (Equations (4), (5), (6) and (8) hold), and markets clear−which means that Equation (2) holds, and the market clearing condition in the bond market also holds:

$$B_t = 0. \tag{17}$$

By plugging equilibrium condition (17) into the balance sheet equation (3) the following equation obtains:

$$N_t = S_t B_{F,t}^* + B_{H,t} \tag{18}$$

This shows that, in equilibrium, the household's net worth is equal to net foreign assets, which is foreign debt denominated in foreign and domestic currencies.

The household takes the monetary policy reaction function as a given and chooses the currency denomination of foreign debt.

1.1.2 The household's second optimization problem

In the second optimization problem, the household allocates absorption between home and foreign goods by minimizing:

$$P_{H,t} C_{H,t} + P_{F,t} C_{F,t} \tag{19}$$

subject to the definition of the composite good:

$$C_t^{\Delta} = \left[(1 - \bar{c}_F)^{\frac{1}{\upsilon}} (C_{H,t}^{\Delta})^{\frac{\upsilon-1}{\upsilon}} + \bar{c}_F^{\frac{1}{\upsilon}} (C_{F,t}^{\Delta})^{\frac{\upsilon-1}{\upsilon}} \right]^{\frac{\upsilon}{\upsilon-1}} \tag{20}$$

where

$$C_{H,t}^{\Delta} = C_{H,t} - \gamma C_{A,H,t-1}$$

$$C_{F,t}^{\Delta} = C_{F,t} - \gamma C_{A,F,t-1}$$

In Equation (20), $\bar{c}_F$ is the share of imports in aggregate demand in the steady state, and υ is the elasticity of substitution between domestic and foreign goods. Using the definition of the habit in overall absorption, the habit in domestic and imported absorption is also defined as lagged aggregate per capita absorption of the domestic and imported goods respectively.

The first order conditions for the second problem are:

$$C^{\Delta}_{H,t} = (1 - \bar{c}_F) Q_t^{\upsilon\delta} C^{\Delta}_t \tag{21}$$

$$C^{\Delta}_{F,t} = \bar{c}_F Q_t^{-\upsilon} C^{\Delta}_t \tag{22}$$

where $\delta = \bar{c}_F/(1 - \bar{c}_F)$.[10]

The solution of the problem gives a price index $P_t = \left[(1 - \bar{c}_F)(P_{H,t})^{1-\upsilon} + \bar{c}_F (P_{F,t})^{1-\upsilon}\right]^{\frac{1}{1-\upsilon}}$ that converges to $P_t = P_{H,t}^{1-\bar{c}_F} P_{F,t}^{\bar{c}_F}$ when $\upsilon \to 1$. The inflation rate, defined as $1 + \pi_t \equiv \frac{P_t}{P_{t-1}}$, may be written,

$$1 + \pi_t = \left(\frac{P_{H,t}}{P_{H,t-1}}\right)^{1-\bar{c}_F} \left(\frac{P_{F,t}}{P_{F,t-1}}\right)^{\bar{c}_F}$$

1.1.3 The household's third optimization problem

In the third optimization problem, the household allocates absorption of home goods along a continuum of an infinite number of differentiated goods. Absorption of the domestically produced good is a composite of an infinite number of goods indexed in the interval (0,1) and produced by an infinite number of firms that operate under monopolistic competition.

Let $C_{H,j,t}$ and $P_{H,j,t}$ be the quantity consumed and the price paid for domestically produced good j. The household's problem is to minimize, by the choice of $C_{H,j,t}$, $j \in (0,1)$:

$$\int_0^1 P_{H,j,t} C_{H,j,t} dj \tag{23}$$

subject to:

$$C_{H,t} = \left[\int_0^1 C_{H,j,t}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \tag{24}$$

Let λ_t be the Lagrange multiplier associated with constraint (24). The solution to this problem is

$$C_{H,j,t} = \left(\frac{P_{H,j,t}}{P_{H,t}}\right)^{-\theta} C_{H,t} \tag{25}$$

[10] Relative prices of home and imported goods in terms of the real exchange rate Q_t are derived in the appendix.

$$P_{H,t} \equiv \left(\int_0^1 P_{H,j,t}^{1-\theta} dj \right)^{\frac{1}{1-\theta}} \tag{26}$$

where the price index of domestically produced goods has been defined as $P_{H,t} \equiv \lambda_t$.

1.2 The firm

There is an infinite number of firms that produce a continuum of differentiated goods under monopolistic competition.

As the problem solved by the firm is standard, it is left to the mathematical appendix. The main result of the firm's optimization problem can be expressed as a Phillips curve that, in deviation form, and in terms of the quasi difference of inflation, $\pi_{H,t}^{\Delta} = \pi_{H,t} - \gamma\pi_{H,t-1}$, may be written as:

$$\hat{\pi}_{H,t}^{\Delta} = \beta\hat{\pi}_{H,t+1|t}^{\Delta} + \kappa_H \hat{\varphi}_t \tag{27}$$

where marginal cost $\hat{\varphi}_t$ is:

$$\hat{\varphi}_t = \eta(\hat{y}_t - \hat{z}_t) - \gamma\eta(\hat{y}_{t-1} - \hat{z}_{t-1}) + \sigma(\hat{c}_t - \gamma\hat{c}_{t-1}) + \delta\hat{q}_t - \hat{z}_t \tag{28}$$

1.3 The distributor firm

There is an infinite number of monopolistically competitive firms that distribute a continuum of differentiated imported goods. Following Monacceli (2003), the problem of the distributor gives a Phillips curve for inflation of imported goods:

$$\hat{\pi}_{F,t}^{\Delta} = \beta\hat{\pi}_{F,t+1|t}^{\Delta} + \kappa_F \hat{\psi}_t \tag{29}$$

where $\pi_{F,t}^{\Delta} = \pi_{F,t} - \gamma\pi_{F,t-1}$ is the quasi difference of inflation and where the gap of the law of one price follows

$$\hat{\psi}_t = \hat{\psi}_{t-1} + \hat{s}_t^{\Delta} + \hat{\pi}_t^* - \hat{\pi}_{F,t} \tag{30}$$

where $1 + s_t^{\Delta} = \frac{S_t}{S_{t-1}}$.

1.4 The central bank

In the experiments presented in Section 2 the central bank either does not respond to the shock to the risk premium or perfectly stabilizes the nominal exchange rate. The first case can be assimilated to inflation targeting when the central bank does not respond to the shock because it is transitory. The second one is the case of a fixed nominal exchange rate.

The central bank follows the rule:

$$i_t = \lambda(\overset{\wedge}{\phi}_{C,t} + \hat{i}^*_t) + \epsilon \hat{y}_t \tag{31}$$

with $\lambda = 0$ for the floating-exchange-rate regime and $\lambda = 1$ for the fixed-exchange-rate regime. The term $\epsilon \hat{y}_t$ is required for the convergence only and does not have any meaningful effect on the interest rate or the transmission mechanisms of monetary policy.

1.5 The foreign household

For simplicity, the demands for the goods are postulated here as an assumption and their derivation is presented in the mathematical appendix. The demand for the domestic and foreign goods is

$$C^{\Delta *}_{H,t} = \overline{c}^*_F Y^{\Delta *}_t \tag{32}$$

$$C^{\Delta *}_{F,t} = (1 - \overline{c}^*_F) Q_t^{\frac{\upsilon\delta}{\overline{c}_F}} Y^{\Delta *}_t \tag{33}$$

and the demand for foreign good j is:

$$C^*_{F,j,t} = \overline{c}_F \left(\frac{P_{H,j,t}}{P_{H,t}} \right)^{-\theta} C^*_{F,t}. \tag{34}$$

1.6 Market clearing

The market clearing conditions for the home and the j goods are:

$$Y_t = C_{H,t} + C^*_{F,t} = C_t + C^*_{F,t} - C_{F,t} \tag{35}$$

$$Y_{j,t} = C_{H,j,t} + C^*_{F,j,t} \tag{36}$$

In addition, plugging into (36), the equations (25) and (110) (derived in the mathematical appendix), gives an aggregate demand equation for good j:

$$Y_{j,t} = \left(\frac{P_{H,j,t}}{P_{H,t}} \right)^{-\theta} Y_t \tag{37}$$

1.7 General equilibrium

A general equilibrium at time t for the small open economy is a set of absorption and labor allocations (C_t, $C_{H,t}$, $C_{F,t}$, L_t, $C_{H,j,t}$, $L_{t,j}$ for $j \in (0,1)$), assets ($B^*_{F,t}$, $B_{H,t}$, B_t), and prices (i_t, $\phi_{H,t}$, $\phi_{S,t}$, P_t, $P_{H,t}$, $P_{F,t}$, W_t, S_t, $P_{H,j,t}$ for $j \in (0,1)$) such that:

1. households maximize utility, (Equations (4), (5), (6), (8) and (2) are satisfied), minimize the cost of allocating absorption between home and foreign goods (Equations (21), (22), (20) hold), minimize the cost of allocating domestic absorption along a continuum of home goods (Equations (25) and (24) hold), and hold foreign debt in foreign currency if $\lambda = 1$ and in domestic currency if $\lambda = 0$.

2. firms (see mathematical appendix) maximize profits (Equations (91, (90), (93) hold);

3. the central bank follows the ad-hoc rule (31);

4. markets clear (conditions (2), (17), (35) and (36) hold);

given $\phi_{F,t}$, A^T_t, Z_t, $B^*_{F,t-1}$, $B_{H,t-1}$, B_{t-1}, i^*_t, and Y^*_t.

1.8 The three key equations of the model

1.8.1 The law of evolution of net worth

In order to obtain the law of evolution of net worth, combine the household's balance sheet and budget constraint, Equations (3) and (2):

$$N_t = (1+i_{t-1})\frac{B_{t-1}}{P_t} + \left(1+i^*_{t-1}\right)\left(1+\phi_{F,t-1}\right)\frac{B^*_{F,t-1}S_t}{P_t} + \left(1+i^*_{t-1}\right)\left(1+\phi_{H,t-1}\right)\frac{B_{H,t-1}}{P_t} + A_t + A^T_t \tag{38}$$

where $A_t = Y_t - C_t = C_{F,t} - C^*_{F,t}$ is the trade balance and $Y_t = \frac{W_t}{P_t}L_t + \Pi_t$ is gross *domestic* product.

Using equilibrium condition (17) , and defining the share of foreign currency denominated debt in overall foreign debt as $\alpha \equiv S_t D^*_{F,t}/N_t$, Equation (38) becomes,

$$N_t = \left[\alpha\frac{\left(1+i^*_{t-1}\right)\left(1+\phi_{F,t-1}\right)}{1+\pi_t}\frac{S_t}{S_{t-1}} + (1-\alpha)\frac{\left(1+i^*_{t-1}\right)\left(1+\phi_{H,t-1}\right)}{1+\pi_t}\right]N_{t-1} + A_t + A^T_t \tag{39}$$

This equation states that net worth at time t is equal to the economy's savings or the trade balance, A_t, plus transfers, plus the gross return on the previous period net worth. The gross return on assets (the term in braquets) is in turn equal to the (gross, percent) cost of servicing foreign debt weighted by currency denomination.

Using equilibrium conditions (6) and (7) and defining $\frac{S_t}{S_{t|t-1}} \equiv 1+\varepsilon^s_{t|t-1}$ the equilibrium condition for the evolution of net worth (39) becomes:

$$D_t = \frac{(1+i_{t-1})}{(1+\pi_t)}(1+\alpha\varepsilon^s_{t|t-1})D_{t-1} - A_t - A_t^T \tag{40}$$

where $D_t = -N_t$, is foreign debt in real terms.

Further intuition about the evolution of foreign debt can be obtained with a first order approximation:

$$\hat{d}_t = \frac{(1+\bar{r})}{(1+\bar{\gamma})}(\hat{i}_{t-1} - \hat{\pi}_t + \alpha\varepsilon^s_{t|t-1} + \hat{d}_{t-1}) - \frac{\bar{a}}{\bar{d}}\hat{a}_t - \frac{\bar{a}^T}{\bar{d}}\hat{a}_t^T \tag{41}$$

where a bar denotes steady state share, a hat denotes percent deviation from the steady state, and $1+\bar{r} = \frac{(1+\bar{i})}{(1+\bar{\pi})}$, $\hat{d}_t \equiv \frac{\Delta d_t}{\bar{d}_t}$, $\hat{i}_t \equiv \frac{\Delta i_t}{1+\bar{i}}$ and $\bar{\gamma}$ is the rate of growth of steady state output.[11]

According to (41), among the factors that define the evolution of foreign debt are a cost effect, $\frac{(1+\bar{r})}{(1+\bar{\gamma})}(\hat{i}_{t-1} - \hat{\pi}_t)$, and two valuation effects, $\frac{1}{(1+\bar{\gamma})}\alpha\varepsilon^s_{t|t-1}$ and $\frac{\bar{r}}{(1+\bar{\gamma})}\alpha\varepsilon^s_{t|t-1}$. The former valuation effect is the impact of the exchange rate on foreign debt, the latter −which is Lane and Milessi-Ferreti's (2005) case−, is the effect of the exchange rate on the cost of servicing foreign debt.

The UIP condition has an important implication on the law of evolution of net worth (41). To see the effect of a shock to market sentiment on foreign debt, plug (9) and (14) into (41) to obtain:

$$\hat{d}_t = \frac{(1+\bar{r})}{(1+\bar{\gamma})}\left[(\hat{i}_{t-1} - \hat{\pi}_t) + \alpha(\hat{i}^*_t + \varsigma\hat{d}_{t-1} + \varepsilon^{\phi_F}_t - \hat{i}_t + \varepsilon^s_{t+1|t}) + \hat{d}_{t-1}\right] - \frac{\bar{a}}{\bar{d}}\hat{a}_t - \frac{\bar{a}^T}{\bar{d}}\hat{a}_t^T \tag{42}$$

Equation (42) shows that if monetary policy follows the fixed-exchange-rate regime: $\alpha = 1$, $\hat{i}_t \simeq \varepsilon^{\phi_F}_t$, the effect of a shock to market sentiment on foreign debt is at $t+1$ and of size $\hat{d}_{t+1} \simeq \frac{(1+\bar{r})}{(1+\bar{\gamma})}\varepsilon^{\phi_F}_t$. If monetary policy follows the floating-exchange-rate regime $\alpha = 0$ and $\hat{i}_t \simeq 0$, there is no cost or valuation effect on foreign debt.

[11] Note that not only the exchange rate but also the inflation rate can cause surprises to foreign debt at time t. The reason we decided to show the rational expectations error of the exchange rate explicitly and not that of inflation is that the issue at stake is the shock to investors' risk aversion and its impact on foreign debt through the exchange rate.

A different situation is when the authorities are faced with a currency crisis and, given a fixed exchange rate, it is known that $\alpha = 1$ and nonetheless the exchange rate may be mantained fixed of may be devalued. If the peg is mantained the effect of the shock to market sentiment on foreign debt is again at $t+1$ and of the amount $\hat{d}_{t+1} \simeq \frac{(1+\bar{r})}{(1+\bar{\gamma})}\varepsilon_t^{\phi_F}$. If the currency crashes the effect of the shock on foreign debt is at t and of the amount $\hat{d}_t \simeq \frac{(1+\bar{r})}{(1+\bar{\gamma})}\varepsilon_t^{\phi_F}$. The conclusion is that defending the currency or letting it depreciate does not make any difference for the initial impact of the shock on foreign debt.[12]

1.8.2 Output

Let small letters denote logarithms, bars denote share of output in the steady state and hats, log deviation from the steady state. Aggregate demand (derived in the mathematical appendix) is:

$$\hat{y}_t^{\Delta} = \hat{y}_{t+1|t}^{\Delta} - (\vartheta + \sigma_H^{-1})\hat{r}_t + \vartheta\overset{\wedge}{\phi}_{F,t} + (\vartheta - \sigma_X^{-1})\hat{r}_t^{*} \tag{43}$$

where $\sigma_H = \frac{\sigma}{\bar{c}_H}$, $\sigma_X = \frac{\sigma}{\bar{c}_X}$, $\vartheta = (\bar{c}_F + \delta\frac{\bar{c}_X}{\bar{c}_F})\upsilon$, and $\bar{c}_H$, $\bar{c}_X$, and $\bar{c}_F$ are the shares of home goods, exports and imports in aggregate demand.

The aggregate demand equation states that output depends on the domestic interest rate, the foreign interest rate, and the country risk premium. The effect of the domestic interest rate on output, $-(\vartheta + \sigma_H^{-1})$, is negative[13]. An increase in the domestic interest rate decreases output as the result of two forces. On the one hand, it appreciates the exchange rate and hence decreases net exports (the coefficient ϑ). On the other hand, it discourages consumption of the good produced by the domestic economy (the coefficient σ_H^{-1}).

The effect of the foreign interest rate on output, $\vartheta - \sigma_X^{-1}$, is positive. An increase in the foreign interest rate depreciates the exchange rate and this depreciation stimulates net exports, ϑ. There is an offsetting effect since the increase in the foreign interest rate decreases foreign output and then reduces net exports, σ_X^{-1}. The overall effect on domestic aggregate demand is positive.[14]

The effect of the country risk premium on output is positive. The transmission mechanism involves an exchange rate depreciation caused by a credit risk premium and, as a result, an increase in net exports.

[12] The monetary and exchange rate regime does make a difference for the bahavior of foreign debt in the medium term due to the larger response of the trade balance when the exchange rate depreciates.

[13] All coefficients are nonnegative.

[14] Using $\bar{c}_X \simeq \bar{c}_F$, the condition $\widetilde{\upsilon} - \sigma_X^{-1} > 0$ is met if $\sigma > \frac{1}{\upsilon(1+\frac{1}{1-\bar{c}_F})}$, this is satisfied for reasonable parameter values.

1.8.3 Capital flows

Capital flows are modelled as the inverse of the trade balance. A positive trade balance is a capital outflow, a negative trade balance an inflow. The trade balance equation (derived in the mathematical appendix) is:

$$\hat{a}_t^{\Delta} = \hat{a}_{t+1|t}^{\Delta} - \frac{1}{\bar{a}}(\vartheta - \sigma_F^{-1})\hat{r}_t + \frac{1}{\bar{a}}\vartheta\hat{\phi}_{F,t} + \frac{1}{\bar{a}}(\vartheta - \sigma_X^{-1})\hat{r}_t^{*} \tag{44}$$

where $\sigma_F = \frac{\sigma}{\bar{c}_F}$.

The effect of the domestic interest rate on the trade balance, $-(\vartheta - \sigma_F^{-1})$, is negative.[15] An increase in the domestic interest rate appreciates the exchange rate and tends to decrease the trade balance (the coefficient $-\vartheta$). However, the increase in the domestic interest rate tends to increase the trade balance because it discourages overall absorption; in particular, it discourages the imported component of absorption (the coefficient σ_F^{-1}).

1.9 The complete model in deviation form

The complete model consists of a price block, Equations (45) to (49), an exchange-rate, interest-rate and risk-spread block, (50) to (58), a flow block, (59) to (61), a stock block (62), and a block of linking equations, (63) to (67):[16]

$$\hat{\pi}_t^{\Delta} = (1 - \bar{c}_F)\hat{\pi}_{H,t}^{\Delta} + \bar{c}_F\hat{\pi}_{F,t}^{\Delta} \tag{45}$$

$$\hat{\pi}_{H,t}^{\Delta} = \hat{\pi}_{H,t+1|t}^{\Delta} + \kappa_H\hat{\varphi}_t \tag{46}$$

$$\hat{\pi}_{F,t}^{\Delta} = \hat{\pi}_{F,t+1|t}^{\Delta} + \kappa_F\hat{\psi}_t \tag{47}$$

$$\hat{\varphi}_t = \eta(\hat{y}_t - \hat{z}_t) - \gamma\eta(\hat{y}_{t-1} - \hat{z}_{t-1}) + \sigma(\hat{c}_t - \gamma\hat{c}_{t-1}) + \delta\hat{q}_t - \hat{z}_t \tag{48}$$

$$\hat{\psi}_t = \hat{\psi}_{t-1} + \hat{s}_t^{\Delta} + \hat{\pi}_t^{*} - \hat{\pi}_{F,t} \tag{49}$$

[15] The condition is also $\sigma > \frac{1}{\upsilon(1+\frac{1}{1-\bar{c}_F})}$.

[16] The trade balance is obtained as the residual between output and absorption. For further intuition, the trade balance can also be obtained as: $\hat{a}_t^{\Delta} = \hat{a}_{t+1|t}^{\Delta} - \frac{1}{\bar{a}}(\vartheta - \sigma_F^{-1})\hat{r}_t + \frac{1}{\bar{a}}\vartheta\hat{\phi}_{C,t} + \frac{1}{\bar{a}}(\vartheta - \sigma_X^{-1})\hat{r}_t^{*}$

$$\hat{s}_t^{\Delta} = \hat{i}_{t-1} - \hat{\phi}_{F,t} - \hat{i}^*_{t-1} + \varepsilon^s_{t|t-1} \tag{50}$$

$$\varepsilon^s_{t|t-1} = -\hat{i}_t + \hat{\phi}_{F,t} + \hat{i}^*_t + \varepsilon^s_{t+1|t} \tag{51}$$

$$\hat{q}_{B,t} = \hat{q}_{B,t-1} + \hat{s}_t^{\Delta} + \hat{\pi}^*_t - \hat{\pi}_t \tag{52}$$

$$\hat{q}_t = \hat{q}_{B,t} - \hat{\psi}_t \tag{53}$$

$$\hat{i}_t = \lambda(\hat{\phi}_{F,t} + \hat{i}^*_t) + \epsilon\hat{y}_t \tag{54}$$

$$\hat{r}_t = \hat{i}_t - \hat{\pi}_{t+1|t} \tag{55}$$

$$\hat{\phi}_{H,t} = \hat{\phi}_{F,t} + \hat{\phi}_{S,t} \tag{56}$$

$$\hat{\phi}_{S,t} = \hat{s}^{\Delta}_{t+1|t} \tag{57}$$

$$\hat{\phi}_{F,t} = \zeta\hat{d}_{t-1} + \varepsilon_t^{\phi_F} \tag{58}$$

$$\hat{y}_t^{\Delta} = \hat{y}^{\Delta}_{t+1|t} - (\vartheta + \sigma_H^{-1})\hat{r}_t + \vartheta\hat{\phi}_{F,t} + (\vartheta - \sigma_X^{-1})\hat{r}^*_t \tag{59}$$

$$\hat{a}_t^{\Delta} = \frac{1}{\bar{a}}\hat{y}_t^{\Delta} - \frac{\bar{c}}{\bar{a}}\hat{c}_t^{\Delta} \tag{60}$$

$$\hat{c}_t^{\Delta} = \hat{c}^{\Delta}_{t+1|t} - \sigma^{-1}\hat{r}_t \tag{61}$$

$$\hat{d}_t = \frac{(1+\bar{r})}{(1+\bar{\gamma})}[(\hat{i}_{t-1} - \hat{\pi}_t) + \alpha\varepsilon^s_{t|t-1} + \hat{d}_{t-1}] - \frac{\bar{a}}{\bar{d}}\hat{a}_t - \frac{\bar{a}^T}{\bar{d}}\hat{a}_t^T \tag{62}$$

$$\hat{y}_t = \gamma\hat{y}_{t-1} + \hat{y}_t^{\Delta} \tag{63}$$

$$\hat{a}_t = \gamma \hat{a}_{t-1} + \hat{a}{}_t^{\Delta} \tag{64}$$

$$\hat{c}_t = \gamma \hat{c}_{t-1} + \hat{c}{}_t^{\Delta} \tag{65}$$

$$\overset{\wedge}{\pi}_{H,t} = \gamma \overset{\wedge}{\pi}_{H,t-1} + \overset{\wedge}{\pi}{}_{H,t}^{\Delta} \tag{66}$$

$$\overset{\wedge}{\pi}_{F,t} = \gamma \overset{\wedge}{\pi}_{F,t-1} + \overset{\wedge}{\pi}{}_{F,t}^{\Delta} \tag{67}$$

1.10 The extended model

The extended model incorporates accelerator effects on aggregate demand.

The financial accelerator was originally developed by Bernanke, Gertler and Gilchrist (1999) to study the effect of the price of physical capital on the balance sheet of entrepreneurs. The accelerator was later extended to the open economy by CCV and Gertler, Gilchrist and Natalucci (2003). Other papers have used the accelerator with prices other than that of physical capital and for agents other than entrepreneurs. Choi and Cook (2004) use the accelerator to deal with the effect of the exchange rate on the balance sheet of banks, Aoki, Proudman and Vliegue (2004) (henceforth APV), use the accelerator to study the price of housing and the balance sheet of households.

A completely realistic model of the cycle in credit and asset prices and of the financial accelerator would include many asset prices and several balance sheets. The reason is that during booms and busts in credit and asset prices all asset prices and balance sheets typically move together. But for simplicity in the model of this paper the household's net worth depends only on one asset price, the exchange rate.

In the extended version of the model, following APV, the household consists of two members, a permanent income consumer who solves the household's maximization problem of the basic model, and a financially constrained consumer who is risk neutral, who borrows abroad and whose consumption depends on net worth. Absorption by the restricted consumer stands for all balance-sheet related effects on aggregate demand during the cycle in credit and asset prices.

1.10.1 Restricted consumption

Here I outline the agency problem between the domestic borrower and the foreign lender.

The household borrows at home at the rate i_t and abroad at the rates $i_t^* + \phi_{F,t}$ and $i_t^* + \phi_{H,t}$. As monetary policy is autonomous, the household's cost of borrowing at home, is determined by the central bank exogenously. In terms of domestic currency, the household's expected cost of borrowing abroad is $i_t^* + \phi_{F,t} + s_{t+1|t} - s_t$ and $i_t^* + \phi_{H,t}$ depending on the currency denomination of debt. No matter the response of the central bank to a shock to country risk, the behavior of the exchange rate, endogenous to the augmented UIP condition, is such that expected depreciation equalizes the (risk premium adjusted) cost on domestic and foreign debt. In expected terms, the household is indifferent to the choice of portfolio among the three types of bonds.

The lender charges a premium on the foreign interest rate. The premium varies negatively with the financial condition of the domestic household. The household finances total excess expenditure (absorption plus debt service minus output, or the current account) with its wealth (including price and valuation effects), net of financial wealth left to the next period:

$$D_t = (1 + \alpha \varepsilon^s_{t|t-1}) D_{t-1} + A_t^S \tag{68}$$

where $A_t^S = -i_{t-1}(1 + \alpha \varepsilon^s_{t|t-1}) D_{t-1} + A_t + A_t^T$ is the current account.[17]

Also, restricted consumption is a function of net worth (including price and valuation effects):

$$C^{\Delta}_{R,t} = \chi(\varpi) \tag{69}$$

where $\varpi = (1 + \alpha \varepsilon^s_{t|t-1}) D_{t-1}$ and $\chi'(\varpi) < 0$.

We use $\chi(\varpi) = \varpi^{-\mu}$.

1.10.2 The main flows in the extended model

The restricted member of the household minimizes the cost of buying the home and imported goods subject to a given level of absorption. Optimization gives the demands for the goods:

$$C^{\Delta}_{RH,t} = (1 - \bar{c}_F) Q_t^{\upsilon\delta} C_t^{\Delta R} \tag{70}$$

$$C^{\Delta}_{RF,t} = \bar{c}_F Q_t^{-\upsilon} C_t^{\Delta R} \tag{71}$$

Unrestricted consumers follow the Euler equation:

[17] The notation A_t^S stands for secondary savings.

$$\left(C^{\Delta}_{U,t}\right)^{-\sigma} = \beta \left(C^{\Delta}_{U,t+1|t}\right)^{-\sigma} (1+i_t)\frac{P_t}{P_{t+1|t}} \tag{72}$$

Restricted consumers follow:

$$C^{\Delta}_{R,t} = \left[(1+\alpha\varepsilon^{s}_{t|t-1})D_{t-1}\right]^{-\mu} \tag{73}$$

According to (73), if foreign debt is above (below) the steady state, absorption decreases (increases) and foreign debt decreases (increases) towards the steady state.

In log deviation form, absorption is the sum of the restricted and unrestricted components:

$$\hat{c}{}^{\Delta}_{t} = \frac{\overline{c}_U}{\overline{c}}\hat{c}{}^{\Delta}_{U,t} + \frac{\overline{c}_R}{\overline{c}}\hat{c}{}^{\Delta}_{R,t} \tag{74}$$

The aggregate demand equation can be obtained by combining (22), (35), (74), and (107).

$$\hat{y}{}^{\Delta}_{t} = \hat{y}{}^{\Delta}_{t+1|t} - (\vartheta + \sigma^{-1}_{UH})\hat{r}_t + (\vartheta - \sigma^{-1}_{X})\hat{r}{}^{*}_{t} + \vartheta\overset{\wedge}{\phi}_{F,t} - \overline{c}_{RH}\mu\upsilon_t \tag{75}$$

where $\upsilon_t = [(\hat{d}_{t-1} - \hat{d}_t) + \alpha(\varepsilon^{s}_{t|t-1} - \varepsilon^{s}_{t+1|t})]$ and $\sigma_{UH} = \frac{\sigma}{\overline{c}_{UH}}$.

The trade balance and absorption equations are:

$$\hat{a}{}^{\Delta}_{t} = \hat{a}{}^{\Delta}_{t+1|t} + \frac{1}{\overline{a}}\left(\sigma^{-1}_{UF} - \vartheta\right)\hat{r}_t - \frac{1}{\overline{a}}\left(\sigma^{-1}_{X} - \vartheta\right)\hat{r}{}^{*}_{t} + \frac{1}{\overline{a}}\vartheta\overset{\wedge}{\phi}_{F,t} + \frac{\overline{c}_{RF}}{\overline{a}}\mu\upsilon_t \tag{76}$$

$$\hat{c}{}^{\Delta}_{t} = \hat{c}{}^{\Delta}_{t+1|t} - \frac{\overline{c}_U}{\overline{c}}\sigma^{-1}\hat{r}_t - \frac{\overline{c}_R}{\overline{c}}\mu\upsilon_t \tag{77}$$

The complete model with a financial accelerator consists of the basic model, replacing the flow block, (59) and (61), with (75) and (77).

1.11 Parameterization

The results are meant to be quantitatively relevant for emerging economies in general.

From the equilibrium conditions of the model, and based on the results of Chang and Velasco (2006) and Chamon and Haussman (2002) that were mentioned in the introduction, $\alpha = \lambda$.

Net foreign assets, $\bar{n}$, are -2 in the steady state. This value corresponds to a share of 50% of GDP. The share of imported goods in aggregate demand, $\overline{c}_F$, is 0.3. For the central bank reaction function we used $\epsilon = 1.0\text{E}^{-4}$.

In regards to the fundamental parameters, the inter-temporal elasticity of substitution, β, is set at 0.99, which is equivalent to a steady state real interest rate $\overline{r}$ of 0.01 or 4% in real terms at an annual rate. The probability that firms do not optimize their price, ω_H, is 0.75, which is the standard value used in the literature (see for instance Christiano, Eichembaum and Evans, 2001, hereafter CEE). If the pass-through is immediate, $\omega_F \to 0$, if it is sluggish, $\omega_F = 0.25$. The degree of persistence in absorption, γ, is 0.9. This is calibrated so that half an output cycle lasts about four years. The same value is used for the persistence in inflation, for simplicity. The preference parameter, η, is 0.5. The results are robust to a wide range of values in this parameter. The preference parameter, σ, is set at 3 so that a one percent point shock to the interest rate for one year causes a one percentage point drop in output. The elasticity of substitution between domestically produced and imported goods, υ, is 1. The response of the risk premium to net worth, ζ, is 0.005. This parameter was calibrated so that, after a shock, net worth would return to equilibrium in about 8 years.

Finally, other steady state parameters are calibrated as follows. The spread, $\bar{\phi}_{F,t}$ is calibrated as 0.005 which corresponds to an annual rate of 2%. The foreign interest rate, $\bar{r}^*$, is set at 0.005, which is an annual rate of 2%. The growth of trend output, $\bar{\gamma}$, is 0.0075, an annual growth of 3%. $\bar{Y}$ is 1 by definition and $\bar{a}^T$ is 0 for simplicity.

In the extended model, the share of restricted consumption of $\bar{c}_R$ is assumed to be 0.25. The effect of net worth on restricted consumption, μ, is calibrated as 0.03 a small effect as the one found for wealth effects on aggregate demand in the empirical literature.

2 Results

In this section, we study the effect of an unexpected one percentage point positive shock to the country risk premium, $\hat{\varepsilon}_t^{\phi_C}$ (0.25 percentage points in a quarterly basis). We report the results of the extended model.[18] By Equation (9), the country risk premium at time t is equal to the shock to the country credit risk, $\hat{\phi}_{F,t} = \varepsilon_t^{\phi_F}$.

2.1 The shock and the policy response

2.1.1 Fixed-exchange-rate policy: shock absorbed by the interest rate

The policy interest rate moves along with the sum of the credit risk premium and the foreign interest rate. In response to the shock to the country risk premium, the central bank reaction function (54) with $\lambda = 1$ stabilizes the real exchange rate.

[18] The results are robust to the inclusion of the financial accelerator.

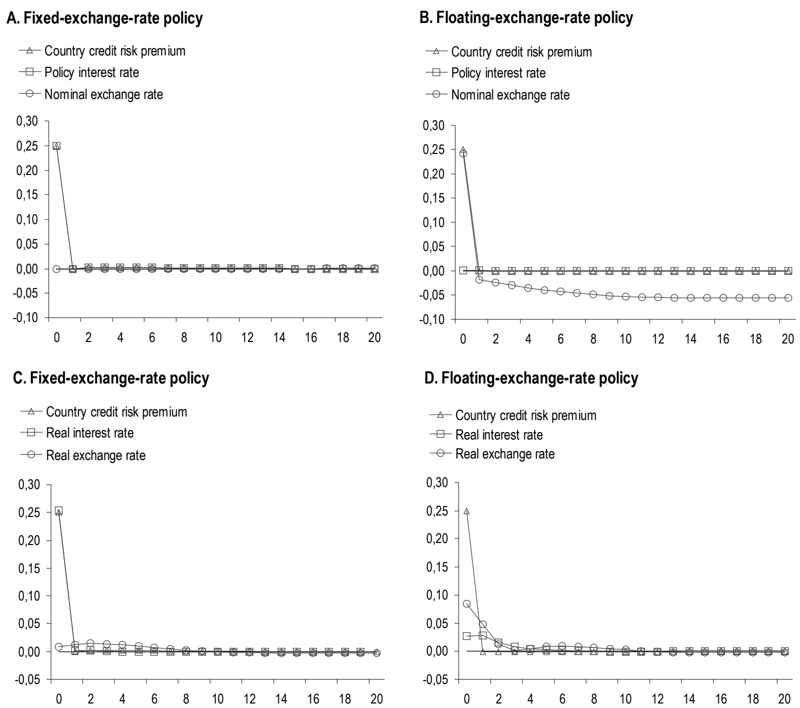

Figure 1: The shock and the policy response

This result is obtained from the uncovered interest parity condition (6). If the nominal exchange rate is fixed, $\hat{i}_t = \hat{\phi}_{F,t}$ and $\hat{s}_{t+1|t} - \hat{s}_t = 0$.

This is shown in Figures 1-A and 1-B.[19]

2.1.2 Floating-exchange-rate policy: shock absorbed by the exchange rate

The policy interest rate does not respond to the shock and the exchange rate depreciates (Figure 1-B). Analytically, from first order condition (6), $\hat{i}_t = 0$ implies $\hat{s}_t^{\Delta} = \hat{\phi}_{F,t}$.

2.2 Output, absorption and capital flows

Define the trade balance and absorption, measured as a share of trend output, as: $\tilde{a}_t^{\Delta} = \bar{a}\hat{s}_t^{\Delta}$ and $\tilde{c}_t^{\Delta} = \bar{c}\hat{c}_t^{\Delta}$.[20] Equations (59) to (61) become:

[19] In the simulations, the size of the shock is one percentage point in annual terms or 25 basis points in quarterly terms. In the figures, price variables are shown in deviation from the steady state, flow and stock variables, in percent of trend GDP.

[20] Since $\bar{y} = 1$, $\tilde{y}_t = \hat{y}_t$.

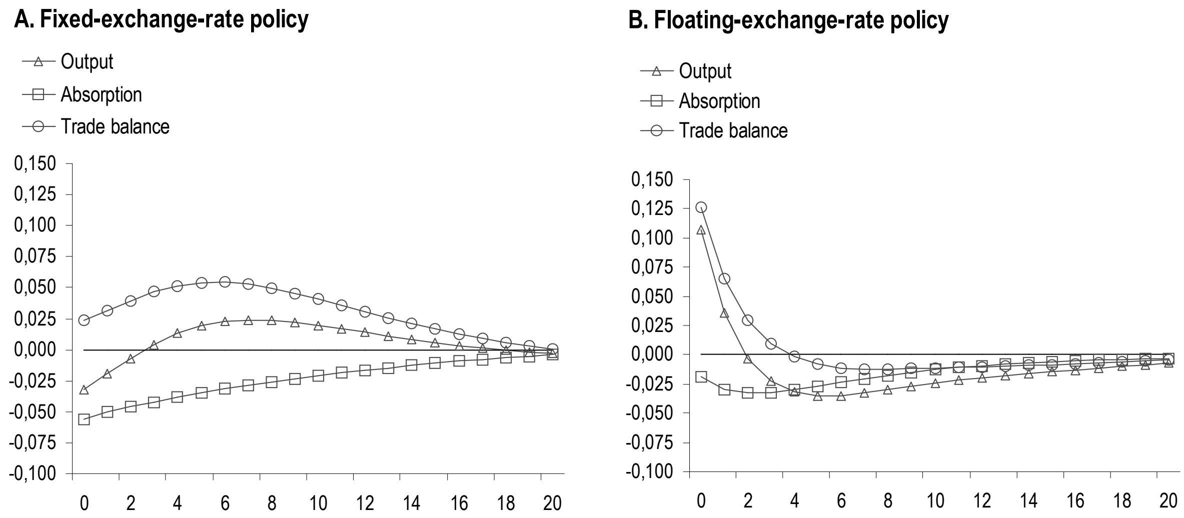

Figure 2: The main flow variables

$$\tilde{y}^{\Delta}_t = \tilde{y}^{\Delta}_{t+1|t} - (\vartheta + \sigma_H^{-1})\hat{r}_t + \vartheta\hat{\phi}_{C,t} + (\vartheta - \sigma_X^{-1})\hat{r}^*_t \tag{78}$$

$$\tilde{a}^{\Delta}_t = \tilde{a}^{\Delta}_{t+1|t} - (\vartheta - \sigma_F^{-1})\hat{r}_t + \vartheta\hat{\phi}_{C,t} + (\vartheta - \sigma_X^{-1})\hat{r}^*_t \tag{79}$$

$$\tilde{c}^{\Delta}_t = \tilde{c}^{\Delta}_{t+1|t} - \sigma^{-1}\hat{r}_t \tag{80}$$

The advantage of analyzing the flows and the stock in units of trend output instead of in log deviation form is that this measure enables us to compare all deviations from the steady state in the intuitive metric of units of GDP.[21]

2.2.1 Fixed-exchange-rate policy: output drops

Figure 2-A shows the behavior of the main flows under a shock to country risk and under a fixed real exchange rate. The graph shows an increase in the trade balance and drops in absorption and output, the drop in absorption being larger than the drop in output.

Consider these results analytically. Under a policy of a fixed real exchange rate, a positive shock to country risk requires an increase in the domestic interest rate: $\hat{i}_t = \varepsilon_t^{\phi_F} > 0$. Plugging conditions

[21] With this measure, in the language of, for example, Galí, López Salido and Vallés (2004), all flow and stock variables are measured "in deviation from the steady state and normalized by steady state GDP".

$\hat{i}_t = \hat{\phi}_{F,t} > 0$ and $\hat{i}^*_t = 0$ into (78) to (79) reveals that the impact of the shock on output, the trade balance and absorption is $-\sigma_H^{-1}$, σ_F^{-1} and $-\sigma^{-1}$ respectively. Then, a shock to the country risk premium involves, first, a drop in output, $\tilde{y}^{\Delta}_t < 0$, –for a large shock, which is the case of a sudden stop, this is Krugman's (2000) "decapitation of the entrepreneurial class"; second, an increase in the trade balance,[22] $\tilde{a}^{\Delta}_t > 0$; third, a drop in absorption, $\tilde{c}^{\Delta}_t < 0$.

The drop in absorption, $-\sigma^{-1}\hat{\phi}_{F,t}$, is larger that the drop in output, $-\sigma_H^{-1}\hat{\phi}_{F,t}$.[23] That the drop in absorption is larger than the drop in output is also implied by the fact that the trade balance increases. In sum, the results are $\tilde{a}_t > 0$ and $\tilde{c}_t < \tilde{y}_t < 0$.

An important point to make here is that the tightening of monetary policy is necessary to keep the exchange rate fixed; this reconciles sudden stops and recessions. The transmission mechanism that causes the recession is the effect of the interest rate on aggregate demand.

2.2.2 Floating-exchange-rate policy: output increases

The model simulation appears in Figure 2-B. As the exchange rate depreciates, output increases with the trade balance. Absorption is relatively constant.[24]

Analytically, if the central bank faces an upward shock to country risk with exchange rate flexibility, the conditions are $\hat{\phi}_{F,t} > 0$, $i_t = i^*_t$, $\hat{r}_t \simeq \hat{r}^*_t = 0$. Plugging these conditions into Equations (78) to (79) it is evident that a shock to country risk leads to increases in output and in the trade balance, while absorption remains unchanged.

The effect of the shock on output (in units of trend output), is similar to the effect of the shock on the trade balance: $\vartheta\hat{\phi}_{F,t}$. The fact that the rise in output and in the trade balance are similar is also implied by the fact that the response of absorption to the shock is small.

2.2.3 Discussion

The result of the basic model that under a fixed exchange rate output drops and under a floating exchange rate it decreases, is robust to different specifications of the model such as degree of openness, indebtedness and currency denomination of foreign debt. This is because the effect of the shock on output in the first case is $-\sigma_H^{-1} = -\frac{\bar{c}_H}{\sigma} < 0$, and in the second case $\vartheta = (\delta\frac{\bar{c}_X}{\bar{c}_F} + \bar{c}_F)\upsilon > 0$.

[22] This increase in the trade balance is known as a transfer. For a treatment of the transfer problem see Eichengreen (1994).

[23] $\bar{c} = \bar{c}_H + \bar{c}_F$ and $\bar{c}_F > 0$ mean $\bar{c} > \bar{c}_H$ and $\sigma_C^{-1} > \sigma_H^{-1}$.

[24] In the graph, absorption increases slightly. This is because by (58), the country risk premium depends in part on net foreign assets.

Under a floating exchange rate, the sudden stop has the same consequence on output as in CKM: sudden stops involve output increases. In CKM there is an output increase because net exports increase. The reason for the increase in output is that in their paper the policy interest rate follows the constant foreign interest rate. Also in this paper output increases because net exports increase. Absorption is constant because the central bank does not change the domestic interest rate. The puzzle in CKM is explained here since recessions can occur during sudden stops provided monetary policy "defends" the exchange rate.

Another point to make is that if there is an upward shock to the country risk premium, there is a transfer or capital outflow no matter whether the exchange rate is fixed or floating.

Finally, note that the drop in output ocurrs with balancesheet conditons that are not critical. This is unlike CCV where critical balancesheets are necesary for the drop in output. Here output drops ocurr even in resilient economies due to the effect of the interest rate through the aggregate demand channel.

2.3 A capital inflow

A downward shock to the country risk premium provides a rationale for a capital inflow. A central conclusion is that no matter whether the exchange rate is rigid or flexible, during a downward shock to the country risk premium the trade balance is in deficit or, in other words, there is a capital inflow.

Another conclusion is that during positive shocks to country risk there is a capital outflow and during negative shocks to country risk there is a capital inflow. This result led us to treat upward and downward shocks to the credit spread and capital outflows and inflows the same.

3 Conclusion

Sudden stops typically result in drops in output. As shown by CCV, this can be the consequence of balance sheet effects in financially fragile economies. In this paper it is shown that this can also be the consequence interest rate defenses of the exchange rate, even in financially resilient economies. Fear of floating creates recessions during capital outflows because the central bank increases interest rates to "defend" the exchange rate. The tight stance of monetary policy decreases aggregate demand. Unlike the mechanisms surveyed by CKM, the mechanism in this paper that makes a capital outflow cause an output drop is the simple and transparent effect of higher interest rates on aggregate demand.

The case of CKM where sudden stops cause output increases takes place when the interest rate faced by consumers follows the constant foreign interest rate. The increase in net exports, hence, increases output. Here output decreases because, notwithstanding the increase in net exports, the increase in the interest rate faced by households decreases absorption and then output.

From the equilibrium conditions of the model, an expression was proposed for the evolution of net foreign assets.

A limitation of the model is the risk structure of interest rates. In the model the risk premium enters the (short-term) domestic interest rate through the response of monetary policy when attempting to maintain the exchange rate fixed. In the real world, however, credit risk is embedded in short-term rates only to a very limited amount, while it is incorporated in long term rates to a larger extent (see for instance the empirical paper by Galindo and Hofstetter, 2008). Output drops may be present even with no response of monetary policy to changes to the country risk premium, because long term rates −the ones that are relevant for aggregate demand− do incorporate the country risk premium.

Another limitation is that in the model all balance sheets have been aggregated into one, but the effect of balance sheets in the real world may be much larger due to domino effects; particularly when the banking sector is involved. This complication may be a rationale behind interest rate defenses of the exchange rate: the policy has a negative effect on aggregate demand, but any effect of increased rates on the exchange rate would help tame an ensuing currency and banking crisis with perhaps more devastating effects on the economy.

Finally, the focus of the paper is in the effects of the response of monetary policy. In the real world other policies are also typically put to use in response to large bouts of capital inflows and outflows: capital controls and changes in international reserves. The study of the effects of these policies in a DSGE model seems currently outside the reach of the profession.

References

[1] Aoki, Kosuke, James Proudman and Gertjan Vliegue. “House Prices and Monetary Policy: a Financial Accelerator Approach.” *Journal of Financial Intermediation*, October, 2004, *13*(4), pp. 414-435.

[2] Benigno, Pierpaolo and Michael Woodford. "Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach." National Bureau of Economic Research, Inc., NBER Working Papers: No. 9905, 2003.

[3] Benigno, Pierpaolo and Michael Woodford. "Optimal Inflation Targeting Under Alternative Fiscal Regimes." Discussion Papers Series, Columbia University. No. 0506-09, January, 2003.

[4] Bernanke, Ben, Mark Gertler, and Simon Gilchrist, "The Financial Accelerator in a Quantitative Business Cycle Framework," in John Taylor and Michael Woodford, eds., *Handbook of Macroeconomics.* Vol. 1. Amsterdam: Elsevier North Holland, 1999. pp. 1341-1393.

[5] Bordo, Michael D. and Christopher M. Meissner. "The Role of Foreign Currency Debt in Financial Crises: 1880-1913 vs. 1972-1997." National Bureau of Economic Research, Inc., NBER Working Papers: No. 11897, 2005.

[6] Botman, Dennis, Douglas Laxton, Dirk Muir and Andrei Romanov. "A New-Open-Economy-Macro Model for Fiscal Policy Evaluation." International Monetary Fund, IMF Working Paper, No. 06/45, 2006.

[7] Calvo, Guillermo. "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics,* September, 1983, *12*(3), 383-398.

[8] Calvo, Guillermo. "Capital Flows and Capital-Market Crises: The Simple Economics of Sudden Stops." *Journal of Applied Economics*, November 1998, *1*(1), pp. 35-54.

[9] Calvo, Guillermo and Carmen Reinhart. "Fear of Floating." *Quarterly Journal of Economics*, May, 2002, *107*(2), pp. 379-408.

[10] Calvo, Guillermo, Alejandro Izquierdo and Ernesto Talvi. "Sudden Stops, the Real Exchange Rate, and Fiscal Sustainability: Argentina's Lessons." National Bureau of Economic Research, Inc., NBER Working Papers: No. 9828, 2003.

[11] Calvo, Guillermo and Carmen Reinhart. "Fear of Floating." National Bureau of Economic Research, Inc., NBER Working Papers: No. 7993, 2000.

[12] Céspedes, Luis F., Roberto Chang, and Andrés Velasco, "Balance Sheets and Exchange Rate Policy," *The American Economic Review,* September, 2004, *94* (4), pp. 1183-1193.

[13] Chamon, Marcos and Rocardo Haussmann. "Why DCountries Borrow the Way They Borrow." In B. Eichengreen and R. Haussmann (Eds.), Other People's Money: *Debt Denomination and Financial Instability in Emerging Market Economies*. Chicago: University of Chicago Press.

[14] Chang, Roberto and Andrés Velasco. "Currency Mismatches and Monetary Policy: A Tale of Two Equilibria." *Journal of International Economics*, June, 2006, *69* (1), pp. 150-175.

[15] Chari, Varadarajan V. and Patrick J. Kehoe "Optimal Fiscal and Monetary Policy." in John B. Taylor and Michael Woodford. eds., *Handbook of Macroeconomics*, Vol. 1, pp. 1671-1745 Amsterdam: Elsevier North Holland, 1999.

[16] Chari, V.V., P. Kehoe and E. R. McGrattan. "Sudden Stops and Output Drops." *American Economic Review*, Vol. 95 No. 2, May 2005, pages 381-387.

[17] Choi, Won G., and David Cook, "Liability Dollarization and the Bank Balance Sheet Channel," *Journal of International Economics,* December, 2004, *64*(2), pp. 247-275.

[18] Christiano, Lawrence J., Martin Eichembaum and Charles Evans. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." National Bureau of Economic Research, Inc., NBER Working Papers: No. 8403, 2001.

[19] Clarida, Richard, Jordi Galí and Mark Gertler. "Optimal Monetary Policy in Open vs. Closed Economies." *American Economic Review,* May, 2001, *91*(2), pp. 248-252.

[20] Cook, David. "Monetary Policy in Emerging Markets: Can Liability Dollarization Explain Contractionary Devaluations?" *Journal of Monetary Economics,* September, 2004, *51*(6), pp. 1155-1181.

[21] Deveraux, Michael, Phillip Lane and Juanyi Xu. "Exchange Rates and Monetary Policy in Emerging Market Economies." *The Economic Journal*, April, 2006, *116*(511), pp. 478-506.

[22] Eichembaum, Martin and Joanas D. M. Fisher. "Evaluating the Calvo Model of Sticky Prices." National Bureau of Economic Research, Inc., NBER Working Papers: No. 10617, 2004.

[23] Eichembaum, Martin and Joanas D. M. Fisher. "Testing the Calvo Sticky Price Model." *Economic Perspectives* 2003:2 pages 40-53.

[24] Eichengreen, Barry, 1994, Transfer Problem, In: Peter Newman, Murray Milgate and John Eatwell, eds., *The New Palgrave Dictionary of Money and Finance*, New York: MacMillan Press.

[25] Feldstein, Martin. "Economic and Financial Crises in Emerging Market Economies, An Overview of Prevention and Management," in: Martin Feldstein ed., *Economic and Financial Crises in Emerging Market Economies.* Chicago and London: The University of Chicago Press, 2003, pp. 1-30.

[26] Fischer, Stanley. "Globalization and Its Challenges." Richard Ely Lecture. *Papers and Proceedings of the American Economic Review.* Vol. 93. No. 2. May 2003.

[27] Fuhrer, Jeffrey C. "Habit Formation in Consumption and its Implications for Monetary-Policy Models." *The American Economic Review*, June, 2000, *90*(3), pp. 367-390.

[28] Galí, Jordi, J. David López-Salido and Javier Vallés. "Understanding the Effects of Government Spending on Consumption." The Federal Reserve Board. International Finance Discussion Papers: No. 805, 2004.

[29] Galindo, Arturo J. and Marc Hofstetter. "Mortgage Interest Rates, Country Risk and Maturity Matching in Colombia," Documentos Cede No. 004544, Universidad de los Andes, 2008.

[30] Gertler, Mark., Simon Gilchrist and Fabio M. Natalucci, "External Constraints on Monetary Policy and the Financial Accelerator," National Bureau of Economic Research, Inc., NBER Working Papers: No. 10128, 2003.

[31] Jeanne, Olivier D. and Jeromin Zettlemeyer, "'Original Sin,' Balance Sheet Crises and the Roles of International Lending." International Monetary Fund, Working Paper No. 02/234, 2002.

[32] Krugman, Paul. "Balance Sheets, the Transfer Problem, and Financial Crises," in: Peter Isard Assaf Razin, A. K. Rose, eds., *International Finance and Financial Crises: Essays in Honor of Robert P. Flood.* Boston: Kluwer Academic, 2000, pp. 31-44.

[33] Lahiri, Amartya and Carlos Vegh. "Delaying the Inevitable: Interest Rate Defense and Balance of Payments Crises." *Journal of Political Economy.* April, 2003, *111*(2), pp. 404-424.

[34] Lane, Phillip R. and Gian M. Milesi-Ferretti. "The Dynamics of External Wealth: A Portfolio Approach." Unpublished paper, 2005.

[35] Lucas, Robert. *Models of Business Cycles.* Oxford: Basil Blackwell, 1986.

[36] Marcet, Albert and Ramon Marimon. "Recursive Contracts." Unpublished paper, 1998.

[37] Mendoza, Enrique G. and Katherine A. Smith. "Margin Calls, Trading Costs, and Asset Prices in Emerging Markets: The Financial Mechanics of the Sudden Stop Phenomenon." National Bureau of Economic Research, Inc., NBER Working Papers: No. 9286, 2002.

[38] Mendoza, Enrique G. and Katherine A. Smith. "Quantitative Implications of a Debt-Deflation Theory of Sudden Stops and Asset Prices." *Journal of International Economics* Vol. 70 No. 1, September 2006 pages 82-114.

[39] Monacelli, T., 2003, Monetary policy in a low pass-through environment, Innocenzo Gasparini Institute for Economic Research, Universita' Bocconi, Mimeo, March.

[40] Nielsen, Mette E. B. "Exchange Rate Targeting in a Small Open Economy." Banco de la República Working Papers No. 367, 2006.

[41] Roubini, Nouriel. "Comment," in: Sebastián Edwards and Jeffrey A. Frenkel, eds., *Preventing Currency Crises in Emerging Markets.* Chicago and London: The University of Chicago Press, 2002, pp. 591-598.

[42] Schmitt-Grohe, Stephanie and Martín Uribe. "Closing Small Open Economy Models." National Bureau of Economic Research, Inc., NBER Working Papers: No. 9270, 2002.

[43] Svensson, Lars E. O. "Open-Economy Inflation Targeting. " *Journal of International Economics* February, 2000, *50*(1), pp. 155-183.

[44] Svensson, Lars. "Optimization Under Commitment and Discretion, the Recursive Saddlepoint Method, and Targeting Rules and Instrument Rules: Lecture Notes." Unpublished paper, 2005.

[45] Uribe, Martín. On Overborrowing." *American Economic Review* Vo. 96, No. 2., May 2006. pages 417-421.

[46] Yun, Tack. "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles." *Journal of Monetary Economics.* 1996, 37(2), pp. 345-370.

4 Appendix: The flexible price equilibrium

The core model has been simplified by assuming a simple definition of the output gap. Here, the model is extended with an output gap defined as the deviation of output from the level of output that would hold if prices were flexible.

Based on well known results, if prices are flexible, marginal cost is constant. In log deviation form Equation (96) becomes

$$0 = \eta(y_t^o - z_t) - \gamma\eta(y_{t-1}^o - z_{t-1}) + \sigma(c_t^o - \gamma c_{t-1}^o) + \delta q_t^o - z_t \tag{81}$$

Solving this equation for flexible price output gives:

$$\hat{y}_t^{\Delta o} = -\frac{\sigma}{\eta}(\hat{c}_t^o - \gamma \hat{c}_{t-1}^o) - \frac{\delta}{\eta}\hat{q}_t^o + \left(1 + \frac{1}{\eta}\right)\hat{z}_t - \gamma \hat{z}_{t-1}^o \tag{82}$$

where technology follows $z_t = \gamma z_{t-1} + \varepsilon_t^z$.

Solving the aggregate demand equation for absorption gives an expression for absorption in the flexible price equilibrium:

$$\hat{c}_t^{\Delta o} = \frac{1}{\overline{c}_H}\hat{y}_t^{\Delta o} - \frac{\overline{c}_X}{\overline{c}_H}\hat{y}_t^{\Delta *} - \delta \upsilon \hat{q}_t^o \tag{83}$$

To complete the flow block of the flexible price equilibrium, the quasi difference of output is given by $y_t^{\Delta o} = y_t^o - \gamma y_{t-1}^o$ and of absorption by $c_t^o = \gamma c_{t-1}^o + c_t^{\Delta o}$.

Consider the flexible price real interest rate. Solving (4) for the real interest rate and expressing the result in units of trend output gives an expression for the Wicksellian interest rate:

$$\hat{r}_t^o = \sigma(\hat{c}_{t+1|t}^{\Delta o} - \hat{c}_t^{\Delta o}) \tag{84}$$

Regarding the real exchange rate, in the flexible price equilibrium it follows a UIP condition where the domestic return is the Wicksellian interest rate:

$$\hat{q}_t^o = \hat{q}_{t+1|t}^o - (\hat{r}_t^o - \hat{r}_t^* - \overset{\wedge}{\phi}_{F,t}) \tag{85}$$

The country risk premium follows the risk premium equation:

$$\overset{\wedge}{\phi}_{F,t} = \mu \hat{d}_{t-1}^o + \varepsilon_t^{\phi_F} \tag{86}$$

Finally, foreign debt follows

$$\hat{d}^o_t = \frac{(1+\bar{r})}{(1+\gamma)}(\hat{r}^o_{t-1} + \alpha\varepsilon^{qo}_{t|t-1} + \hat{d}^o_{t-1}) \tag{87}$$

The complete model with a flexible price equilibrium consists of a model for equilibrium with rigid prices and a model for equilibrium with flexible prices. The equilibrium with flexible prices is given by Equations (82) to (87). The model for equilibrium with rigid prices consists of Equations (46) to (62) replacing Equation (48) with

$$\hat{\varphi}_t = \eta(\hat{y}_t - \hat{y}^o_t) - \eta\gamma(\hat{y}_{t-1} - \hat{y}^o_{t-1}) + \sigma(\hat{c}_t - \hat{c}^o_t) - \sigma\gamma(\hat{c}_{t-1} - \hat{c}^o_{t-1}) + \delta(\hat{q}_t - \hat{q}^o_t) \tag{88}$$

This equation indicates that in the open economy, marginal cost depends on the deviation of output from the level of output if prices were flexible −the definition of the output gap−, and also depends on the gaps of absorption and of the exchange rate.[25]

5 Mathematical appendix

5.1 The firm

There is an infinite number of firms that produce a continuum of differentiated goods under monopolistic competition. Each firm solves two optimization problems. In the first problem, the representative firm chooses labor demand to minimize cost:

$$\frac{W_t}{P_{H,t}} L_{t,j} \tag{89}$$

subject to a given level of output, $Y_{j,t}$, and given the production technology

$$Y_{j,t} = Z_t L_{t,j} \tag{90}$$

where $L_{t,j}$ is the amount of labor hired by firm j and Z_t is an aggregate technology factor identical for all firms.

The optimal condition for labor demand yields an expression for marginal cost:

$$\Phi_t = \frac{W_t/P_{H,t}}{Z_t}. \tag{91}$$

In the second optimization problem, the monopolistic firm chooses the price of the good it produces. Following Calvo (1983), Yun (1996) and CEE the firm re-optimizes its price, each

[25] To obtain (88), subtract (81) from (48).

period, with probability ω. If the firm does not re-optimize, it changes the price by a proportion γ of lagged inflation:

$$P_{H,j,t} = (1 + \pi_{H,t-1})^{\gamma} P_{H,j,t-1}$$

The firm's problem is to pick an optimal price $\widetilde{P}_{H,j,t}$ to maximize expected profits:

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{P_{H,j,t}}{P_{H,t+i}} \right) Y_{j,t+i} - \Phi_{t+i} Y_{j,t+i} \right] \tag{92}$$

subject to the demand for good j (Equation (37)).

CEE show that the optimal price chosen by the firm satisfies:[26]

$$\widetilde{p}_{H,t} = E_{t-1}\{\hat{s}_t + \sum_{l=1}^{\infty} \left[\beta \left(1 - \omega_H\right)\right]^l \left(\hat{\varphi}_{t+l} - \hat{\varphi}_{t+l-1}\right) \tag{93}$$

$$+ \sum_{l=1}^{\infty} \left[\beta \left(1 - \omega_H\right)\right]^l \left(\pi_{H,t+l} - \pi_{H,t+l-1}\right)\}$$

From (26), the aggregate price level may also be written,

$$P_{H,t} = \left[\omega_H \widetilde{P}_{H,t}^{1-\theta} + \left(1 - \omega_H\right) P_{H,t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{94}$$

Linearizing (94) and combining this with (93) gives a Phillips curve that, in terms of the quasi difference of inflation, $\pi^{\Delta}_{H,t} = \pi_{H,t} - \gamma \pi_{H,t-1}$, is:

$$\pi^{\Delta}_{H,t} = \beta \pi^{\Delta}_{H,t+1|t} + \frac{(1 - \beta\omega_H)(1 - \omega_H)}{\omega_H} \hat{\varphi}_t \tag{95}$$

and where

$$\hat{\varphi}_t = \eta(\hat{y}_t - \hat{z}_t) - \gamma\eta(\hat{y}_{t-1} - \hat{z}_{t-1}) + \sigma(\hat{c}_t - \gamma\hat{c}_{t-1}) + \delta\hat{q}_t - \hat{z}_t \tag{96}$$

is marginal cost in log deviation from the steady state.[27]

5.2 The foreign household

The foreign economy consists of a large number of households. The representative household maximizes expected utility, which is a function of consumption. In turn, consumptionis a composite of domestic and foreign goods and the foreign good is a composite of an infinite number of goods.

[26] Since all firms that re-optimize choose the same price, subscript j is omitted.

[27] Define the product and consumption wages as $W_t/P_{H,t}$, and W_t/P_t. Using $P_t = P_{H,t}^{1-\bar{c}_F} P_{F,t}^{\bar{c}_F}$, the product and consumption wages satisfy $W_t/P_{H,t} = (W_t/P_t)\, Q_t^{\delta}$. To obtain (96), combine this expression with (5), (91) and the production function $Y_t = Z_t L_t$.

The foreign representative household maximizes expected utility,

$$E_t \sum_{t=0}^{\infty} \beta^i \left[\frac{(C_{t+i}^{\Delta *})^{1-\sigma}}{1-\sigma} \right]$$

by the choice of $\left\{ C_t^*,\ B_t,\ D_{F,t}^*,\ D_{H,t} \right\}_{t=0}^{\infty}$, subject to:

$$C_t^* = Y_t^* - \left(1 + i_{t-1}^*\right) \frac{B_{t-1}^*}{P_t^*} + \frac{B_t^*}{P_t^*} \tag{97}$$

$$-(1-\xi_t) \left(1 + i_{t-1}^*\right) \left(1 + \phi_{F,t-1}\right) \frac{B_{F,t-1}^*}{P_t^*} + \frac{B_{F,t}^*}{P_t^*}$$

$$-(1-\xi_t) \left(1 + i_{t-1}^*\right) \left(1 + \phi_{H,t-1}\right) \frac{B_{H,t-1}}{P_t^* S_t} + \frac{B_{H,t}}{P_t^* S_t}$$

given the endowment $\{Y_t^*\}_{t=0}^{\infty}$ where ξ_t is the probability of default, and where $C_t^{\Delta *}$ and $C_{A,t}^*$ are defined the same way they were defined in the domestic economy.

The foreign economy is an open economy that is large. In the limit, openness tends to zero and the economy is approximately closed. Also in the limit, net foreign assets tend to zero and absorption tends to output:

$$D_{F,t}^* \simeq 0,\ D_{H,t} \simeq 0,\ N_t^* \simeq 0, C_t^* \simeq Y_t^* \tag{98}$$

The relevant first order and market clearing conditions are:

$$\left(C_t^{\Delta *}\right)^{-\sigma} = \beta \left(C_{t+1|t}^{\Delta *}\right)^{-\sigma} (1 + i_t^*) \frac{P_t^*}{P_{t+1|t}^*} \tag{99}$$

$$C_t^* \simeq Y_t^* \tag{100}$$

First order conditions (6), (7) and (8) of the problem of the household of the domestic economy may also be derived from the problem of the foreign household.

From the equilibrium conditions it is obtained that the country risk premium is the inverse of the probability of no default:

$$1 + \phi_{F,t} = \frac{1}{1-\xi_t} \tag{101}$$

For simplicity, we do not consider the probability of default in the problem of the domestic economy.

In addition, at any time t, the foreign household minimizes the cost of purchasing home and foreign goods:

$$P_{H,t}^{*}C_{H,t}^{*}+P_{F,t}^{*}C_{F,t}^{*} \tag{102}$$

by the choice of $C_{H,t}^{*}$ and $C_{F,t}^{*}$, subject to

$$C_t^{\Delta *}=\left[(1-\overline{c}_F^{*})^{\frac{1}{\upsilon}}(C_{H,t}^{\Delta *})^{\frac{\upsilon-1}{\upsilon}}+\overline{c}_F^{*\frac{1}{\upsilon}}(C_{F,t}^{\Delta *})^{\frac{\upsilon-1}{\upsilon}}\right]^{\frac{\nu}{\nu-1}}, \tag{103}$$

given $C_t^{\Delta *}$ and Q_t^{*}, and where the quasi differences $C_{H,t}^{\Delta *}$ and $C_{F,t}^{\Delta *}$ are defined in a way that should now be obvious.

The first order conditions are:

$$C_{H,t}^{\Delta *}=(1-\overline{c}_F)\left(\frac{P_{H,t}^{*}}{P_t^{*}}\right)^{-\upsilon}C_t^{\Delta *} \tag{104}$$

$$C_{F,t}^{\Delta *}=\overline{c}_F\left(\frac{P_{F,t}^{*}}{P_t^{*}}\right)^{-\upsilon}C_t^{\Delta *} \tag{105}$$

Using $\left(\frac{P_{H,t}^{*}}{P_t^{*}}\right)^{-\upsilon}=1$ and $\left(\frac{P_{F,t}^{*}}{P_t^{*}}\right)^{-\upsilon}=Q_t^{\upsilon\frac{\delta}{\overline{c}_F}}$ (see appendix 5.4.3) and using (100):

$$C_{H,t}^{\Delta *}\simeq Y_t^{\Delta *} \tag{106}$$

$$C_{F,t}^{\Delta *}=Q_t^{\frac{\upsilon\delta}{\overline{c}_F}}C_t^{\Delta *} \tag{107}$$

Finally, at any time t, the foreign household minimizes the cost of purchasing a bundle of the goods produced by the domestic economy:[28]

$$\int_0^1 P_{H,j,t}C_{F,j,t}^{*}dj \tag{108}$$

by the choice of $C_{F,j,t}^{*}$, $j\in(0,1)$, subject to:

$$C_{F,t}^{*}=\left[\int_0^1\left(C_{F,j,t}^{*}\right)^{\frac{\theta-1}{\theta}}dj\right]^{\frac{\theta}{\theta-1}} \tag{109}$$

taking $C_{F,t}^{*}$, $P_{H,t}$, $P_{H,j,t}$, $j\in(0,1)$ as given.

The solution to this problem, using $\lambda_t=P_{H,t}$, is:

$$C_{F,j,t}^{*}=\left(\frac{P_{H,j,t}}{P_{H,t}}\right)^{-\theta}C_{F,t}^{*}. \tag{110}$$

[28] Note that the price of good j imported by the foreign economy is $P_{H,j,t}$, without an asterisk.

The equilibrium for the foreign economy is a set of allocations, C_t^*, $C_{F,t}^*$, $C_{F,j,t}^*$, for $t = 0...\infty$ and $j \in (0,1)$, such that foreign households maximize utility (Equations (99) and (97) hold), minimize the cost of allocating consumption between home and foreign goods (conditions (103), (106) and (107) hold) and minimize the cost of allocating consumption of the foreign good along the continuum of differentiated goods (Equations (109) and (110) hold), given Y_t^*, Q_t, $P_{H,t}$, $P_{H,j,t}$, $j \in (0,1)$. In equilibrium, the markets of the good and of the bond both clear (Equations (98) and (100) hold).

Combining equilibrium conditions (100), (106) and (107), gives the demand for home and foreign goods by the foreign economy, (32) and (33).

5.3 The steady state

The steady state is given by the equations:

$$\overline{r} = \overline{i} - \overline{\pi} \tag{111}$$

$$\overline{i} = \overline{\phi} + \overline{i}^* \tag{112}$$

$$\overline{r}^* = \overline{i}^* - \overline{\pi}^* \tag{113}$$

$$\overline{\phi}_H = \overline{\phi}_F + \overline{\phi}_S \tag{114}$$

$$\overline{\phi}_S = \overline{s}^\Delta \tag{115}$$

$$\overline{s}^\Delta = \overline{i} - \overline{i}^* - \overline{\phi}_F \tag{116}$$

$$\overline{\phi}_F = \zeta \overline{d} \tag{117}$$

$$\overline{c}_X = \overline{c}_F + \overline{a} \tag{118}$$

$$\overline{a} = -\overline{a}^r - \overline{a}^T \tag{119}$$

$$\bar{c} = 1 + \bar{a}^r + \bar{a}^T \tag{120}$$

$$\bar{a}^r = (\bar{r} - \bar{\gamma})\bar{n} \tag{121}$$

$$\bar{c}_H = \bar{c} - \bar{c}_F \tag{122}$$

$$\bar{c}_R = \bar{c} - \bar{c}_U \tag{123}$$

$$\bar{c}_{RH} = \frac{1}{\bar{c}}\bar{c}_R\bar{c}_H \tag{124}$$

$$\bar{c}_{UH} = \frac{1}{\bar{c}}\bar{c}_U\bar{c}_H \tag{125}$$

$$\bar{c}_{RF} = \frac{1}{\bar{c}}\bar{c}_R\bar{c}_F \tag{126}$$

$$\bar{c}_{UF} = \frac{1}{\bar{c}}\bar{c}_U\bar{c}_F \tag{127}$$

Given $\bar{\pi}$, $\bar{\pi}^*$, $\bar{\phi}$, $\bar{i}^*$.

5.4 Mathematical derivations

5.4.1 Output

Aggregate demand, in log deviation from the steady state, is $y_t^\Delta = \bar{c}\hat{c}_t^\Delta + \bar{c}_X\hat{c}_{F,t}^{\Delta*} - \bar{c}_F\hat{c}_{F,t}^\Delta$. Plugging in the demands for goods (22), (107) and (35) and using $\bar{c} - \bar{c}_F = \bar{c}_H$, gives:

$$y_t^\Delta = \bar{c}_H\hat{c}_t^\Delta + (\bar{c}_F + \delta\frac{\bar{c}_X}{\bar{c}_F})\upsilon q_t + \bar{c}_X\hat{c}_t^{\Delta*} \tag{128}$$

Using the lead of (128) and plugging in the UIP and Euler conditions results in aggregate demand (59) where $\sigma_H = \frac{\sigma}{\bar{c}_H}$, $\sigma_X = \frac{\sigma}{\bar{c}_X}$ and $\vartheta = (\bar{c}_F + \delta\frac{\bar{c}_X}{\bar{c}_F})\upsilon$.

5.4.2 The trade balance

In units of trend output, the trade balance is $a_t^{\Delta} = \bar{c}_X \hat{c}_{F,t}^{\Delta *} - \bar{c}_F \hat{c}_{F,t}^{\Delta}$. Plugging in the demands for goods (22) and (107),

$$a_t^{\Delta} = \bar{c}_X \upsilon \frac{\delta}{\bar{c}_F} q_t + \bar{c}_X \hat{c}_t^{\Delta *} + \bar{c}_F \upsilon q_t - \bar{c}_F \hat{c}_t^{\Delta} \tag{129}$$

Combining the lead of (129), the UIP condition (6) and the Euler equation (4) gives the expression for the trade balance (60), where $\sigma_F = \frac{\sigma}{\bar{c}_F}$.

5.4.3 Relative prices in terms of the real exchange rate

The optimal choice of the two goods made by the domestic and foreign households gives expressions for the demand for goods in terms of their relative prices. Here we write those relative prices in terms of the real exchange rate.

The domestic price of the good produced by the foreign economy is

$$P_{F,t} = S_t P_{H,t}^{*} \tag{130}$$

and the foreign price of the good produced by the domestic economy is:

$$P_{F,t}^{*} = P_{H,t} / S_t \tag{131}$$

Using the equation for the pass-through (130) and the definitions for the real exchange rate and of the CPI, $Q_t = \frac{S_t P_t^*}{P_t}$ and $P_t = P_{H,t}^{1-\bar{c}_F} P_{F,t}^{\bar{c}_F}$, gives

$$Q_t = \left(\frac{P_{F,t}}{P_{H,t}} \right)^{\frac{\delta}{\bar{c}_F}} \tag{132}$$

The relative prices of the goods in terms of the real exchange rate are obtained as follows: (132) and the definition of the CPI give: $\left(\frac{P_{H,t}}{P_t} \right)^{-\upsilon} = Q_t^{\upsilon \delta}$. From the definition of the real exchange rate the result is: $\left(\frac{P_{F,t}}{P_t} \right)^{-\upsilon} = Q_t^{-\upsilon}$. And from (130): $\left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\upsilon} = 1$. Finally, (130) and (131), give $\left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\upsilon} = Q_t^{\upsilon \frac{\delta}{\bar{c}_F}}$.

5.4.4 Uncovered interest rate parity

Lagged UIP is $S_{t-1} = S_{t|t-1} \frac{(1+i_{t-1}^*)(1+\phi_{F,t-1})}{(1+i_{t-1})}$. Rearranging and multiplying by S_t :

$$\frac{S_t}{S_{t|t-1}} = \frac{S_t}{S_{t-1}} \frac{(1+i_{t-1}^*)(1+\phi_{F,t-1})}{(1+i_{t-1})} \tag{133}$$

Rearranging and defining $1+\varepsilon^s_{t|t-1} = \frac{S_t}{S_{t|t-1}}$ and $1+s^\Delta_t \equiv \frac{S_t}{S_{t-1}}$ gives equation (13).

Let $R_t = \frac{(1+i^*_t)(1+\phi_{F,t})}{(1+i_t)}$, $\bar{R} = \frac{(1+\bar{i}^*)(1+\bar{\phi}_C)}{(1+\bar{i})}$ and $\hat{R}_t = \frac{(1+\hat{i}^*_t)(1+\hat{\phi}_{F,t})}{(1+\hat{i}_t)}$. Iterating (6) forward:

$$S_t = R_t R_{t+1} ... R_{t+k-1} S_{t+k|t} \tag{134}$$

As shocks are known at time t, at time $t-1$ the best guess of S_t is

$$S_{t|t-1} = \bar{R}_t \bar{R}_{t+1} ... \bar{R}_{t+k-1} S_{t+k|t-1} \tag{135}$$

If k is large enough so that the effect of shocks on the exchange rate have vanished out, then $S_{t+k|t} \simeq S_{t+k|t-1}$, and dividing (134) by (135) gives $\frac{S_t}{S_{t|t-1}} = \hat{R}_t \hat{R}_{t+1} ... \hat{R}_{t+k-1}$, $\frac{S_{t+1}}{S_{t+1|t}} = \hat{R}_{t+1} \hat{R}_{t+2} ... \hat{R}_{t+k-1}$and

$$\frac{S_t}{S_{t|t-1}} = \hat{R}_t \frac{S_{t+1}}{S_{t+1|t}}$$

Using $1+\varepsilon^s_{t|t-1} = \frac{S_t}{S_{t|t-1}}$ and $\hat{R}_t = \frac{(1+\hat{i}^*_t)(1+\hat{\phi}_{F,t})}{(1+\hat{i}_t)}$ gives Equation (14).