Unequal Opportunities and Human Capital Formation*

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Abstract

This paper develops a tractable, heterogeneous agents general equilibrium model where individuals have different endowments of the factors that complement the schooling process. The paper explores the relationship between inequality of opportunities, inequality of outcomes, and aggregate efficiency in human capital formation. Using numerical solutions we study how the endogenous variables of the model respond to two different interventions in the distribution of opportunities: a mean-preserving spread and a change in the support. The results suggest that a higher degree of inequality of opportunities is associated with lower average level of human capital, a lower fraction of individuals investing in human capital, higher inequality in the distribution of human capital, and higher wage inequality. In particular, the model does not predict a trade-off between aggregate efficiency in human capital formation (as measured by the average level of human capital in the economy) and equality of opportunity.

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1. Introduction

The importance of human capital accumulation as an engine of economic growth and development has been widely recognized in theoretical and empirical studies.[1]

Most of the literature that studies the effects of income inequality on economic growth through its effects on human capital accumulation has focused on the role of credit constraints. The main idea of this line of research is the following: relatively poor individuals don't have the means to finance the accumulation of human capital, and, because they are credit constrained (that is, there is no way to finance the costs of human capital accumulation using future earnings as the collateral for a loan to pay the tuition fees and living expenses), they end up either not investing in human capital or investing very little. Furthermore, if in addition to credit constraints there are decreasing returns to the accumulation of human capital, the final outcome does not maximize the size of the economic pie. Consequently there may be space for redistribution of resources from rich to poor individuals which, in turn, increases the size of the pie. This redistribution would reallocate resources towards more profitable investments given that the marginal returns to human capital accumulation are higher for those individuals (the relatively poor ones) who have less human capital. The theoretical idea has been extensively developed in the literature since the work by Galor and Zeira (1993) and Banerjee and Newman (1993).[2], [3]

But the accumulation of human capital involves other complementary factors as well. This has been extensively documented in a number of recent empirical studies, some of which will be reviewed in the next section. While some of these complementary factors are either non-purchasable or beyond the individual's control once the time to make human capital investment decisions comes (family background, parental education, socioeconomic characteristics, race, genes, culture, provision of social connections, installation of preferences and aspirations in children, etc.), others are (neighborhood effects, distance to schools, and different qualities of books, teachers and schools).[4] The explanation provided in this paper for how inequality affects aggregate efficiency in human capital formation does not rely on credit market imperfections because, as we will argue in the next section, there

[1] The reader is referred to the seminal contributions of Lucas (1988) on the theoretical side, and those of Mankiw, Romer and Weil (1992), Benhabib and Spiegel (1994, 2003) and Barro (2001) for the empirical evidence supporting the importance of human capital in explaining growth rates across countries.

[2] See Aghion et al. (1999) for a thorough review of this literature. Further developments have been proposed by De Gregorio (1996) and Bénabou (1996, 2000).

[3] Empirical evidence has been found in favor of the hypothesis that inequality and credit constraints affect investment in human capital by Flug et al. (1998), De Gregorio (1996) and Mejía (2003).

[4] See Schultz (1988), Roemer (2000), Bénabou (2000), Carneiro and Heckman (2002) and Dardanoni et al. (2003), among others.

are crucial complementary factors to the schooling process that are non-purchasable in the market *once the time to make investment decisions in human capital formation comes.* More precisely, this paper explores another, perhaps complementary, explanation for the negative relation between economic inequality and the average level of human capital which does not rely on credit market imperfections and is based on differences in the rates of return to time investment in human capital accumulation, the latter being determined by each individual's endowment of the complementary factors to the schooling system. Despite the fact that it is sometimes difficult to disentangle exactly which factors cannot be attributed to income or wealth differences, the next section documents a series of empirical results where non-purchasable factors such as race, the composition of the household (e.g. no mother or father in the household), ethnic group, and parental schooling significantly affect different measures of educational attainment (even after one controls for family income or wealth). These non-purchasable factors that complement time and effort in the formation of human capital can be thought of as John Roemer's set of "pre-determined circumstances" or aspects of an individual that are beyond her control once the time to make human capital investment decision comes, and for which society should not hold the individual responsible (Roemer, 2000, 2002, and 2005).

If the previously mentioned factors are important in determining differences in educational attainment across individuals, the distribution of these (non-puchasable) "socio-economic characteristics" across individuals matters. In other words, if the distribution of factors that complement the schooling process matters, one should encounter differences in educational attainment across individuals even in economies with universally free public schools.[5] The model is capable of generating two stylized fact relationships observed in the data which will be described in detail in the next section. These are, first, the negative relationship between average human capital and inequality in the distribution of human capital, and second, the positive correlation between educational opportunities and educational and income outcomes. The truncated shape of the human capital Lorenz curves is also a motivating fact that the model is capable of generating and which, in some sense, is behind the negative relationship between inequality in the distribution of human capital and the average level of human capital. In sum, Roemer meets Hanushek in a simple general equilibrium model.[6]

This paper is related to the literature that links economic inequality and human capital accumulation (see, among others, Galor and Zeira, 1993, and Bénabou, 1996, 2000a, 2000b,

[5] This does not rule out the importance of the lack of financial resources to pay for the (monetary) costs of education. In fact, family income has been found to have large explanatory power on longitudinal studies of educational outcomes across individuals.

[6] We thank one of the anonymous referees for pointing this phrase out.

Ferreira, 2001, Glomm and Ravikumar, 1992, and Fernández and Galí, 1999). While some of these papers emphasize the role of credit constraints and/or indivisibilities in human capital investment in driving the negative relation between inequality and aggregate human capital formation (Galor and Zeira, 1993, Ferreira, 2001), others have emphasized the choice of public versus private schooling made through a political channel as a key determinant of how inequality affects human capital formation (Glomm and Ravikumar, 1992, and Ferreira, 2001).

The paper is organized as follows: the second section presents the stylized facts that motivate the construction of the model. Namely, the negative relationship between the degree of inequality in the distribution of human capital and the average level of human capital across countries, and the positive relationship between inequality of opportunities and inequality of outcomes. Also, this section reviews the empirical evidence regarding the importance of the (non-purchasable) complementary factors to the schooling system on educational outcomes, on which the model is based. The third section presents the model, and the fourth section the results of the model's numerical solution using a distribution function that allows us to simulate different degrees of inequality of opportunity, while keeping the mean of the distribution constant. The last section presents some concluding remarks.

2. Stylized Facts

2.1. The Macro Picture

The main focus of this paper has to do with the relationship between the degree of inequality in the distribution of the complementary factors to the schooling system across individuals and the average level of human capital. Although we do not have a direct measure of the degree of inequality in the distribution of the complementary factors to the schooling process across countries, we do know from a recent paper by Thomas et. al (2002) that the human capital Gini coefficient and the average years of schooling among the adult population are negatively associated in the cross country data (see Figure 1, taken from Thomas et. al, 2002).[7] Although it is true that the number of years of schooling is a censored variable, as hardly anyone goes to school for more than 16 years (complete tertiary), the negative relationship between average years of schooling and education Gini can also be linked to the observed (truncated) form of Lorenz curves for education in most countries in the world

[7] The authors show that the relation between human capital inequality and average human capital follows the same pattern if the Theil Index is used as the measure of human capital inequality (see Thomas et. al, 2002).

(see Figure 2 where the education Lorenz curves are shown for India and Korea in 1960 and 1990). The model, as will be shown below, is also capable of generating the patterns observed in the education Lorenz curves.

[INSERT Figure 1 here]

Those countries with the highest degree of inequality in the distribution of human capital (as measured by the human capital Gini coefficient) have the lowest average years of schooling across the adult population.

[INSERT Figure 2 here]

Some papers have argued that the relationship between inequality in the distribution of human capital and average human capital follows a "Kuznetian" curve. In other words, inequality in the distribution of human capital first increases with average human capital and then declines. However, this relation is observed only when the standard deviation of schooling is used as a measure of inequality (see Thomas et. al, 2002 for a review of the evidence, and the main problems associated with the use of the standard deviation as a measure of human capital inequality).

2.2. The Micro Evidence on the Importance of the Complementary Factors to the Educational Process

Since the publication of the Coleman Report (Coleman et al., 1966), hundreds of papers have studied the relationship between school expenditure and the complementary factors to the schooling process on different measures of educational outcomes in the United States. More precisely, the Coleman Report found that the socioeconomic composition of the student body had a significant effect on test scores after controlling for student background, school, and teacher characteristics (Ginther et al., 2000). These findings attracted the attention of scholars and policy makers, as one of its main conclusions was that school characteristics were relatively unimportant in determining achievement, while family characteristics were the main determinant of student success or failure (Hanushek, 1996). Since then, many studies have used different data sets and econometric techniques to improve the estimates of the effects of family background, parental education, neighborhood effects and many other socioeconomic characteristics on educational outcomes.[8] Importantly, many of these studies have found that non-purchasable complementary factors to the schooling process (such as ethnic background, parental education, region of origin, household composition, etc.) have a first order impact on different measures of educational achievement

[8]For a review of the literature, as well as the main findings (and econometric specification problems) the reader is refered to Ginther et al. (2000) and Hanushek (1986 and 1996). The paper by Durlauf (2002) presents a complete review of how social interactions play an important role on the perpetuation of poverty, although not only through the human capital channel.

after controlling for family income and other material resources involved in the educational process.

In a study with more than 5,000 undergraduates at UC San Diego, Betts and Morell (1997) found that personal background (such as race) and the demographic characteristics of former high school classmates, significantly affect students' GPAs. This result was obtained after controlling for the degree program in which the students were enrolled, the resources of the high school attended, and family income. Moreover, they found that school characteristics partially reflected the incidence of poverty and the educational level among adults in students' high-school neighborhood. Goldhaber and Brewer (1997) found that family background characteristics had a significant effect on test scores achieved by 18,000 students in the 10th grade, even after controlling for school characteristics and the results of a previously taken math test by the same students. They found that, for instance, years of parental education and family income were positively related to test scores. Also, black or Hispanic children, and children with no mother in the household had, on average, a lower predicted score in the math test. A study by Groger (1997) found empirical evidence of the negative (and significant) effects of local violence on the likelihood of graduating from high school. While the average dropout rate in his sample was 21 percent, minor violence increased the dropout rate by 5 percentage points, moderate levels of violence raised it by 24 percent, and substantial violence by 27 percent.

Data requirements for longitudinal studies constitute the main constraint in estimating the effects of the complementary factors to the schooling process on educational outcomes in developing countries. However, the use of randomized experiments to estimate the effects of changes in the complementary factors (such as improving health conditions, providing educational inputs, and lowering the costs associated with school attendance) on different measures of educational outcomes has become one of the most popular topics in the recent development literature.[9] The list of recent papers that evaluate the effects of improving the accessibility of these complementary factors is growing rapidly, but a thorough survey of their findings is not the purpose of this article. Some examples, however, are worth mentioning.

One of these randomized experiments evaluates the effects of mass deworming in seventy five school populations in Kenya. The results are clear: "Health and school participation improved not only at program schools, but also at nearby schools, due to reduced disease transmission. Absenteeism in treatment schools was 25% (or 7 percentage points) lower than in comparison schools. Including this spill-over effect, the program increased schooling by 0.15 years per person treated" (Kremer and Miguel, 2001). The same pattern of results

[9] The reader is referred to Duflo and Kremer (2003) and Kremer (2003) for a review of the methodology of randomized experiments as well as their main findings.

was found in a similar randomized experiment in India (see Bobonis et al., 2002).[10]

Handa (2002) shows that building more schools and raising adult literacy have a larger impact on enrollment rates in primary school in Mozambique than interventions that raised household income. Also, different dimensions of school quality (such as the number of trained teachers) and access to school have a positive and significant impact on school enrollment rates.

A recent paper by Bourguignon et al. (2003) studies the relationship between inequality of opportunities in human capital formation and earnings inequality in Brazil. According to the authors, parental schooling level explains between 35 and 47 percent of children's schooling. This paper also finds that inequality of opportunities (that is, individual circumstances such as parental levels of education, parental occupation, race, and region of origin) accounts for 8-10 percentage points of earnings inequality. According to the authors, between half and three fourths of this share can be attributed to parental schooling level alone.

In the following section we will construct a tractable general equilibrium model with heterogeneous agents that accounts for some of the stylized facts described in this introduction. Namely, for the negative relation between the average level of human capital in the economy and the degree of inequality in the distribution of human capital, the observed patterns of the educational Lorenz curves, and for the positive relation between the degree of inequality of opportunities and the degree of inequality of outcomes (human capital and wage distribution).

3. The Model

Consider an economy operating under perfectly competitive markets. The production of the (single) final good is determined by a neoclassical production function that combines physical capital, human capital and unskilled labor.

Individuals are identical regarding their preferences and cognitive abilities, but may differ on their endowments of the complementary factors to the schooling process. Each individual's endowment of the complementary factors can be thought of as a composite index of parental level of education, child nourishment, neighborhood and peer effects, and the degree of accessibility to the formal schooling system, among other things. The distribution of the complementary factors to the schooling process across individuals is

[10] Other randomized experiments include: PROGRESA in Mexico (Schultz, forthcoming), school vouchers in Colombia (Angrist et al., 2003), school meals in Kenya (Kremer and Vermeersch, 2002), provision of uniforms, textbooks and classroom construction in Kenya (Kremer et al., 2002), provision of a second teacher (if possible, female) in one-teacher schools in India (Banerjee and Kremer, 2002).

assumed to be exogenously given, and we will refer to the degree of inequality in this distribution as the degree of "inequality of opportunities". Given her endowment of the complementary factors, each individual in the economy decides how much time she would invest in human capital formation (if any), and then compares the income she would receive if she decides to work as a skilled worker, with the wage she would receive if she decides not to invest time in human capital formation and to work as an unskilled worker.

Although unequal access to the complementary factors of the educational system can be partially linked to wealth or income inequality, there are some pre-determined characteristics of individuals that are beyond their control and/or cannot be purchased in the market once the time to make investment decisions in education comes, e.g. pre- and post-natal care, parental level of education, family background, race, genes, culture, etc.. In order to concentrate on the effects of inequality of opportunities on human capital investment decisions, it will be assumed that all individuals are endowed with an equal share of the total capital stock of the economy and the production of human capital uses only the individual's time. In other words, we will assume that investment in human capital does not involve any monetary payment, and as a result our explanation for the negative relation between human capital and inequality will not rely on the existence of credit market imperfections to finance educational investments as in Galor and Zeira (1993). However, it will be assumed that the amount of time of labor force participation that an individual sacrifices per unit of time invested in education varies with the individual's endowment of the complementary factors to the educational process. Summarizing the previous ideas, our model explores another explanation for the negative relation between average human capital and human capital inequality based on differences in the rates of return to time investment in human capital. The latter, in turn, are determined by each individual's endowment of the complementary factors to the schooling process.

3.1. Production Technology and Firms' Optimization Conditions

The technology of production of the final good combines unskilled labor, skilled labor (human capital) and physical capital according to a neoclassical production function characterized by aggregate constant returns to scale and diminishing marginal returns to each one of these factors (equation 1).

$$Y = F(L^u, H, K) = (L^u)^{\alpha} H^{\beta} K^{1-\alpha-\beta}, \tag{1}$$

where $0 < \alpha, \beta < 1$, L^u is the number of individuals who work as unskilled labor; H is total human capital in the economy, given by $H = L^s \bar{h}^s$, with L^s being the number of individuals that acquire human capital and $\bar{h}^s$ being the average level of human capital across those individuals who invest a positive amount of their time in human capital formation and work as skilled workers; K is the aggregate capital stock, which is assumed to be exogenously given. For the sake of simplicity it is assumed that the total population consists of a continuum of individuals of size 1. That is, it will be assumed that: $L^u + L^s = 1$.

Markets are assumed to be perfectly competitive and firms choose the number of unskilled and skilled workers they hire as well as physical capital in order to maximize profits. The inverse demand for each one of the factors of production is given by equations 2 to 4.

$$w^u = \alpha(L^u)^{\alpha-1} H^{\beta} K^{1-\alpha-\beta} \tag{2}$$

$$w^s = \beta\,(L^u)^{\alpha}\, H^{\beta-1} K^{1-\alpha-\beta} \tag{3}$$

$$r = (1-\alpha-\beta)\,(L^u)^{\alpha}\, H^{\beta} K^{-\alpha-\beta} \tag{4}$$

Where w^u is the unskilled wage rate, w^s is the wage rate per unit of human capital, and r is the rental rate of capital.

3.2. Individual's Human Capital Decision

Individuals are identical in their preferences and cognitive abilities and each of them is endowed with one unit of time which they allocate between labor force participation and investment in human capital (if any). The fraction of time allocated by individual i to the accumulation of human capital will be denoted by u_i, where $0 \leq u_i < 1$. Also, we will assume that for each unit of time, u_i, that individual i devotes to human capital accumulation, she will acquire a level of human capital equal to $b(u_i)$. In words, human capital formation uses only individuals' time, and the function $b(u)$ captures the technology of human capital formation. It will be assumed that $b(u)$ is an increasing and concave function of the fraction of time invested in human capital formation, u. That is, $b'(u) > 0$ and $b''(u) < 0$. In other words, we will assume that there are decreasing returns to time investment in human capital formation. For the sake of simplicity we will use the following functional form for the technology of human capital formation:

$$b(u_i) = u_i^{\gamma} \qquad \text{with} \qquad 0 < \gamma < 1, \tag{5}$$

where γ measures the elasticity of human capital with respect to time devoted to its accumulation.

In addition to an endowment of one unit of time, each individual in the economy has a given endowment of the complementary factors to the educational process. We will denote individual i's endowment of the complementary factors by θ_i, where $\theta_i \geq 0$.[11] Furthermore, it will be assumed that θ_i is distributed across individuals according to the distribution function $F(\theta, \phi)$, that is $\theta \sim F(\theta, \phi)$, with the support of θ being: $[\underline{\theta}, \overline{\theta}]$, and $\underline{\theta} \geq 0$. The parameter ϕ will be used later to capture an inverse measure of inequality in the distribution of endowments of the complementary factors to the schooling process across individuals.

Each individual i's endowment of the complementary factors to the schooling process, θ_i, determines the effective time cost (in terms of labor force participation) per unit of time devoted to human capital formation.[12] This idea will be introduced in the model in, perhaps, the simplest way: for each unit of time that individual i allocates to the accumulation of human capital, she sacrifices a fraction of time of labor force participation equal to $\frac{1}{1+\theta_i}$. As discussed in the introduction, this assumption captures the idea that different individuals face different costs of acquiring human capital. Note that the larger the endowment of the complementary factor that individual i has, the lower the fraction of time of labor force participation that she sacrifices per unit of time invested in the formation of human capital.

The total effective time of labor force participation sacrificed by an individual i, who invests a fraction u_i of her time in human capital accumulation, is given by: $\frac{u_i}{1+\theta_i}$. The remaining fraction of time, $1 - \frac{u_i}{1+\theta_i}$, is devoted to (skilled) labor force participation.

Summarizing, individual i's supply of human capital in the labor market is given by:

$$h_i = (1 - \frac{u_i}{1+\theta_i})u_i^{\gamma}, \tag{6}$$

where the term in parenthesis in the right hand side of equation 6 is the amount of time devoted to (skilled) labor force participation, and the second term is the amount of human capital acquired by individual i.

Each individual in the economy takes the skilled wage rate, w^s, as given, and chooses the fraction of time investment in human capital accumulation, u_i, in order to maximize her total wage income. That is, each individual i solves the following problem:

$$\max_{u_i} \ w^s h_i \ = \ \max_{u_i} \ w^s(1 - \frac{u_i}{1+\theta_i})u_i^{\gamma} \tag{7}$$

$$\text{subject to: } 0 \leq u_i \leq 1$$

[11]For instance, θ_i can be thought of as a composite index of different complementary factors to the schooling process, such as parental level of education and health (see footnote 14 and Appendix A1).

[12]Note that the endowment θ_i determines individual $i's$ rate of return of time investment in human capital accumulation.

The solution to the above constrained maximization problem is given by:[13]

$$u_i^* = \frac{\gamma}{(1+\gamma)}(1+\theta_i) \tag{8}$$

Not surprisingly, the higher the endowment of complementary factors to the schooling system for individual i is, the larger is her time investment in human capital formation.[14]

Replacing the result obtained in equation 8 into equation 6, individual i's supply of human capital in the labor market is given by:

$$h(\theta_i) = \frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}(1+\theta_i)^\gamma \tag{9}$$

The higher individual i's endowments of the complementary factors to the schooling system is, the greater is her supply of human capital in the labor market.

3.3. Educational (occupational) Choice

Given individual i's endowment of the complementary factors, and the corresponding optimal time investment in human capital accumulation derived in the previous subsection (equation 8), she compares the income she would receive under the two alternative occupations: skilled or unskilled.

On the one hand, she can become an unskilled worker, which implies no time investment in human capital formation. Under this alternative she would supply one unit of unskilled labor in the market and her income would be given by the unskilled wage rate. That is:

$$\text{Income of an unskilled worker} = w^u \tag{10}$$

On the other hand, if she decides to become a skilled worker, she would optimally invest a fraction u_i^* of her time in the accumulation of human capital, and her total wage income would be given by the skilled wage rate multiplied by the amount of human capital she supplies in the labor market (as given by equation 9). That is:

$$\text{Income of skilled worker } i = w^s\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}(1+\theta_i)^\gamma \tag{11}$$

Individual i chooses the occupation that yields the highest income. Comparing the incomes under the two alternative occupations (expressions 10 and 11), there exists a threshold value of the endowment, which we will denote by θ^*, that determines which individuals

[13] We will assume throughout that $\bar{\theta} < \frac{1}{\gamma}$ to ensure that the solution found is interior for all individuals.

[14] Note that from the optimization conditions an econometric specification can be derived if the researcher has some hypothesis about the factors that determine θ_i (see the Appendix (A1) for an example).

decide to invest in human capital and work as skilled workers, and which individuals decide to devote no time to human capital formation and work as unskilled workers. In other words, those individuals with an endowment $\theta_i = \theta^*$ are indifferent between investing a fraction of time u_i^* in human capital formation and working as skilled workers, and devoting all their time to working as unskilled workers. Equating the two alternative incomes given in expressions 10 and 11, the threshold endowment of the complementary factors to the schooling process is given by:

$$\theta^* = \left(\frac{(1+\gamma)^{1+\gamma}}{\gamma^\gamma} \frac{w^u}{w^s} \right)^{1/\gamma} - 1 \tag{12}$$

Those individuals who have an endowment lower than the threshold endowment θ^* will not invest any time in human capital formation and will work as unskilled labor, whereas those individuals with an endowment of the complementary factors larger than the threshold endowment will invest a fraction u_i^* (equation 8) of their time in human capital formation and will work as skilled labor.[15] Although one might be worried about a lower bound for the wage ratio to guarantee an interior solution, the existence and uniqueness of the general equilibrium established in Proposition 1 below ensures this. Since this is a general equilibrium model, the lower bound for the ratio of wages $\frac{w^u}{w^s}$ would never be reached because, as unskilled workers become scarce, the relative wage of unskilled workers would rise.[16] The optimal occupational choice by individual i is summarized by the following two expressions:

$$If \quad \theta_i < \theta^* \Rightarrow \text{ Work as unskilled worker and receive income} = w^u \tag{13}$$

$$If \quad \theta_i \geq \theta^* \Rightarrow \text{Invest } u_i^* \text{ in the acquisition of human capital,}$$

$$\text{work as skilled worker and receive income} = \; w^s \frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}} (1+\theta_i)^\gamma \tag{14}$$

Given the occupational choice of each individual, and the assumption that the endowments of the complementary factors to the schooling process are distributed across the

[15] This result follows from the fact that the income of skilled workers increases in θ (equation 11).

[16] In particular, if all individuals decided to be skilled the relative wage of unskilled workers would be infinite (see equations 2 and 3).

population according to the distribution function $F(\theta, \phi)$, the number of unskilled individuals, L^u, is given by the fraction of the population with an endowment of the complementary factors lower than the threshold endowment. Similarly, the number of skilled individuals, L^s, is given by the fraction of individuals with an endowment of the complementary factors larger than the threshold endowment. Summarizing, the numbers of unskilled and skilled individuals are given by the following expression:

$$L^u = F(\theta^*, \phi) \qquad \text{and} \qquad L^s = 1 - F(\theta^*, \phi), \tag{15}$$

where $F(\theta^*, \phi)$ is the fraction of individuals with an endowment of the complementary factors lower than the threshold endowment.

3.4. Human Capital

The total amount of human capital supplied in the labor market, H, is given by the sum of individuals' human capital supplied in the labor market. This sum can be obtained by integrating equation 9 over those endowments larger than the threshold endowment because, as we have already shown, these are the endowments for which individuals acquire positive levels of human capital. Furthermore, note that given that the size of the population has been normalized to one, total human capital in the economy, H, is also equal to the average human capital across all individuals, which will be denoted by: $\bar{h}$. Consequently, the expression for both, total (H) and average human capital $(\bar{h})$, is given by:

$$H = \bar{h} = \frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}} \int_{\theta^*}^{\bar{\theta}} (1+\theta)^\gamma \, dF(\theta, \phi) \tag{16}$$

From the above expression, the total level of human capital in the economy depends, among other things, on the distribution of endowments of the complementary factors to the educational process across individuals. More precisely, note that average level of human capital depends on the parameter ϕ, which later on will be used as an inverse measure of inequality in the distribution of endowments.

The average level of human capital among the skilled individuals is given by the total human capital in the economy (equation 16) divided by the number of skilled individuals. That is, average human capital among the skilled individuals is given by:

$$\bar{h}^s = \frac{\dfrac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}} \displaystyle\int_{\theta^*}^{\bar{\theta}} (1+\theta)^\gamma \, dF(\theta, \phi)}{1 - F(\theta^*, \phi)} \tag{17}$$

3.5. Labor Market Equilibrium

The labor market equilibrium is a pair of wages (w^u, w^s) for which both labor markets clear (skilled and unskilled). In order to determine the equilibrium wages we need to solve for the equilibrium threshold endowment of the complementary factors, θ^*, as a function of the parameters of the model.

Using equations 2 and 3, note that the ratio of unskilled to skilled wages is given by:

$$\frac{w^u}{w^s} = \frac{\alpha}{\beta}\frac{H}{L^u} \tag{18}$$

Recall from expression 15, that the proportion of individuals who decide to work as unskilled workers, L^u, is equal to $F(\theta^*, \phi)$. Using this fact, and equation 16 to substitute for total human capital in equation 18, the the ratio of unskilled to skilled wages (the ratio of marginal productivities) is given by:

$$\frac{w^u}{w^s} = \frac{\alpha}{\beta}\frac{\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\int_{\theta^*}^{\bar{\theta}} (1+\theta)^\gamma \, dF(\theta, \phi)}{F(\theta^*, \phi)} \tag{19}$$

Equations 12 and 19 together determine the labor market equilibrium. More precisely, replacing the ratio of unskilled to skilled wages from equation 19 into equation 12, and, after doing some algebra, the equilibrium threshold endowment of the complementary factors to the educational process, θ^*, is determined implicitly by the following equation:

$$\theta^* = \left[\frac{\alpha}{\beta}\frac{\int_{\theta^*}^{\bar{\theta}} (1+\theta)^\gamma \, dF(\theta, \phi)}{F(\theta^*, \phi)}\right]^{1/\gamma} - 1 \tag{20}$$

The following proposition states that an equilibrium threshold endowment exists (and is unique) under very general conditions on the distribution of endowments of the complementary factors to the schooling process.

Proposition 1: Suppose that there exists a function $f : D \times E \to R$, where $D = [\underline{\theta}, \overline{\theta}]$, and E is an interval on the real line, such that the restriction f_ϕ defined as $f_\phi(\theta) = f(\theta, \phi)$ $\forall \theta \in D$ is a probability density function for the endowments of the complementary factors $\forall \phi \in E$. If f_ϕ is (Riemann) integrable, then there exists a unique equilibrium threshold $\theta^*(\phi)$.

Proof: See Appendix A2.

Intuitively, θ^* can never be equal to $\underline{\theta}$ (the lower bound of the support of $F(\theta)$) because in this case the number of unskilled workers would be zero and the relative wage of unskilled workers, $\frac{w^u}{w^s}$, would rise to infinity. The same argument applies for the upper bound for θ^*. As θ^* approaches its upper bound $(\overline{\theta})$, that is, as all individuals become unskilled workers, the relative wage of skilled workers, $\frac{w^s}{w^u}$, would rise to infinity. Riemann integrability of the density function f_ϕ is required to guarantee that its integral (and hence the number of individuals supplying either skilled or unskilled labor) is a continuous function of the threshold endowment.[17] In other words, Riemann integrability of the density function is a continuity requirement for the relative supply of the labor market.[18]

Note that the decentralized solution of the model derived above is Pareto Optimal. That is, the occupational choice made independently by income maximizing individuals yields the maximum level of aggregate output possible.[19]

Having found the equilibrium threshold endowment, θ^*, we can, in principle, determine the equilibrium values of all the endogenous variables by replacing θ^* from equation 20 in the relevant equations derived above. However, given that the solution found for the equilibrium threshold endowment in equation 20 does not have a closed form, we will need to use numerical solutions of the model to solve for the endogenous variables. As explained in the introduction of the paper, we are particularly interested in the (equilibrium) relation between average human capital and the degree of inequality of opportunities, and the relationship between inequality of opportunities and inequality of outcomes (human capital and wage inequality). Before turning to the numerical solution and the simulations we need to first define the measures of inequality in the distribution of human capital and wages.

3.6. Human Capital Distribution

As discussed before, those individuals whose endowment of the complementary factors is larger than the equilibrium threshold endowment (given implicitly in equation 20) invest a positive fraction of time in human capital formation. Using equation 15, the proportion of these individuals is equal to $1 - F(\theta^*, \phi)$. The remaining individuals, $F(\theta^*, \phi)$, do not invest any time in human capital formation and therefore their supply of human capital in the labor market is equal to zero. With this information in mind we can construct a measure of human capital inequality, that is, the human capital Gini coefficient.

[17] In particular, all continuous PDFs are permissible.

[18] Note that continuity of relative demand is guaranteed by the particular choice of the production technology of the final good (equation 1).

[19] In other words, for a given inverse measure of inequality in the distribution of endowments, ϕ, the corresponding equilibrium level of output is maximized. The proof of this statement is available from the authors upon request.

Figure 3 depicts the human capital Lorenz curve implied by the model. To compute the human capital Lorenz curve, we first order individuals' human capital by magnitude, starting with the lowest. Then, we plot the cumulative proportion of the population so ordered (from zero to one along the horizontal axis) against the cumulative proportion of total human capital (from zero to one along the vertical axis).[20] A fraction $F(\theta^*, \phi)$ of individuals do not accumulate any human capital and as a result the human capital Lorenz curve is truncated at zero for a cumulative proportion of the population equal to $F(\theta^*, \phi)$. As explained before, starting with the individuals whose endowments of the complementary factors are equal to the equilibrium threshold endowment, that is the individuals with $\theta_i = \theta^*$, individuals supply positive amounts of human capital in the labor market. As a result, in Figure 3, after the individual with $\theta_i = \theta^*$, the cumulative proportion of total human capital is greater than zero and increasing in θ_i, and therefore increasing as we move to the right of the graph.[21], [22]

[INSERT Figure 3 here]

Using the equation that relates each individual's supply of human capital in the labor market to her endowments of the complementary factors (equation 9), and the distribution of these endowments across the population, we can derive the distribution of human capital across individuals using the change of variable technique. However, note that those individuals in the population with a lower endowment than the equilibrium threshold endowment do not accumulate any human capital. Formally, human capital is distributed across individuals according to the following probability distribution:

$$\begin{aligned} \Pr(h = 0) &= F(\theta^*, \phi) \\ \Pr(h < \hat{h}) &= F(\theta^*, \phi) + \int_{h(\theta^*)}^{\hat{h}} g(h, \phi) dh \qquad \text{for} \qquad \hat{h} \in [h(\theta^*), h(\bar{\theta})] \end{aligned} , \tag{21}$$

where: $g(h, \phi) = f[\theta(h), \phi] \left| \dfrac{d\theta}{dh} \right|$; $h(\theta^*)$ is the human capital supplied in the labor market by the individual with an endowment of the complementary factors equal to the equilibrium threshold endowment, that is, the individual with $\theta_i = \theta^*$; $h(\bar{\theta})$ is the human capital supplied in the labor market by the individual with the highest endowment of the complementary factors in the population, that is, the individual with $\theta_i = \bar{\theta}$;[23] $f(.)$ is the density

[20] For a more detailed explanation on the computation of the Lorenz curve the reader is referred to Lambert, 2001, p. 24.

[21] Figure 2 presents the education Lorenz curves for two countries (Korea and India) in two points in time (1960 and 1990) for each country. Note that the pattern of the Lorenz curves predicted in the model (the fact that they are truncated) is observed in the actual data. The reader is referred to Thomas et al. (2002) for details.

[22] Using Figure 3, the human capital Gini coefficient is defined as: $Gini_h = \frac{A_h}{A_h + B_h}$

[23] $h(\theta^*)$ and $h(\bar{\theta})$ are obtained by evaluating equation 9 at $\theta_i = \theta^*$ and $\theta_i = \bar{\theta}$ respectively.

function associated with the CDF $F(.)$; $\theta(h)$ and $\dfrac{d\theta}{dh}$ can be obtained from equation 9.

Having found the distribution of human capital across individuals, the human capital Gini coefficient is defined by:[24]

$$Gini_h = 2\int_{h(\theta^*)}^{h(\bar{\theta})} hG(h,\phi)g(h,\phi)dh - 1, \tag{22}$$

where: $G(\hat{h},\phi) = \displaystyle\int_{h(\theta^*)}^{\hat{h}} g(h,\phi)dh.$

Equation 22 will be used in the next section when we solve the model numerically and examine how inequality in the distribution of opportunities affects inequality in the distribution of human capital. In other words, how inequality of opportunities affects inequality of outcomes.

3.7. Wage Income Distribution

We have assumed that agents differ only regarding their endowment of the complementary factors to the educational process. Although this is a strong assumption it is valid if we want to concentrate on the effects of inequality of opportunities on human capital accumulation and inequality of outcomes. A more complete model would have to assume also heterogeneity of wealth, and one can plausibly expect the two sources of heterogeneity across individuals to be highly correlated. For the moment, however, we will concentrate only on wage income distribution and leave aside the distribution of the physical capital stock.

Replacing by the equilibrium threshold endowment, θ^*, in equations 13 and 14, and using the distribution of human capital across individuals (equation 21), we can construct the wage income Lorenz curve (see Figure 4).[25]

[Figure 4 here]

From Figure 4, a fraction $F(\theta^*,\phi)$ of individuals work as unskilled labor and all receive the same wage income, given by the unskilled wage rate, w^u. Individuals with an endowment of the complementary factors larger than the equilibrium threshold endowment receive wage income equal to $w^s \dfrac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}(1+\theta_i)^\gamma$.[26]

[24] See Lambert (2001), chapter 2.

[25] The computation of the wage income Lorenz curve follows the same steps as the computation of the human capital Lorenz curve described above.

[26] Using the information in Figure 4, the wage income Gini coefficient is given by: $Gini_w = \frac{A_w}{A_w+B_w}$

3.8. Inequality of Opportunities and Average Human Capital

This section examines the relation between inequality of opportunities and average human capital in the economy.

Recall that the distribution of the complementary factors to the educational process is determined by the cumulative distribution function $F(\theta, \phi)$, where the parameter ϕ captures an inverse measure of inequality in the distribution of endowments (the degree of inequality of opportunity).

After replacing for the equilibrium threshold endowment (θ^*) in equation 16, the marginal change in average human capital that results from a marginal change in the degree of inequality of opportunity is given by:

$$\frac{dh}{d\phi} = \Lambda \left[\int_{\theta^*(\phi)}^{\bar{\theta}(\phi)} (1+\theta)^{\gamma} \frac{\partial f(\theta, \phi)}{\partial \phi} d\theta - (1+\theta^*)^{\gamma} f(\theta^*, \phi) \frac{d\theta^*}{d\phi} + (1+\bar{\theta})^{\gamma} f(\bar{\theta}, \phi) \frac{d\bar{\theta}}{d\phi} \right], \quad (23)$$

where $\Lambda = \dfrac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}$, and $\dfrac{d\theta^*}{d\phi}$ can be obtained from equation 20 using the implicit function theorem.

The first term of the bracketed expression on the right hand side of equation 23 captures the marginal change in human capital (across the skilled individuals) that results from the marginal change in the density of the distribution of endowments caused, in turn, by a marginal change in the degree of inequality. The second term captures the marginal change in human capital that results from a marginal change in the equilibrium threshold endowment (i.e., how much human capital is accumulated by the individuals with $\theta_i = \theta^*$, times the corresponding marginal change in θ^*). The third term captures the marginal change in human capital that results from a marginal extension/contraction of the upper bound of the support of the distribution of endowments that results from a marginal change in the degree of inequality. The last term measures how much human capital is accumulated by those individuals with the highest endowment, times the corresponding change in this endowment level that results from a marginal change in degree of inequality in the distribution of endowments.

In the following section we will use a specific distribution function for the endowments of the complementary factors that allows for changes in the degree of inequality in the distribution while keeping the mean endowment in the population constant. This exercise will allow us to understand how average human capital changes as the degree of inequality in the distribution of endowments increases, while keeping the mean endowment in the population constant. Also, the numerical simulations will tell us how each of the components in equation 23 affects the average level of human capital as the degree of inequality of opportunity increases.

4. Numerical simulations

This section presents the numerical solution of the model as well as the main results of the simulation of two different kind of interventions on the distribution of endowments. We begin by specifying a (well behaved) distribution function for the endowments of the complementary factors to the schooling process and then implement two kind of interventions. First, we simulate a change in the degree of inequality in the distribution of endowments keeping the mean endowment in the population constant (a mean preserving spread in the distribution of endowments), and second, we simulate a change in the support of the distribution keeping the other parameters of the model constant.[27] Once the two different interventions are simulated, we will be able to explain how human capital and its distribution across individuals changes as the degree of inequality in the distribution of endowments of the complementary factors changes. In other words, using the equations derived in the previous section, the numerical simulations will allow us to disentangle the equilibrium relationships between inequality of opportunities (that is, inequality of endowments of the complementary factors to the schooling process), inequality of outcomes (inequality in the distribution of human capital and of wage income), and the degree of aggregate efficiency in the accumulation of human capital (as measured by average human capital in the economy).[28]

Recall that the cumulative distribution function of endowments of the complementary factors to the schooling process across the population is denoted by $F(\theta, \phi)$. Let $F(\theta, \phi)$ take the following functional form:

$$F(\theta, \phi) = \begin{cases} 0 & for \quad \theta < 0 \\ \left[\dfrac{\phi}{1+\phi}\right]^{\phi} \theta^{\phi} & for \quad \theta \in \left[0, \frac{1+\phi}{\phi}\right] \\ 1 & for \quad \theta > \dfrac{1+\phi}{\phi} \end{cases}, \tag{24}$$

with $\phi \in [\frac{\gamma}{1-\gamma}, 1]$.[29]

[27] Altough the two interventions impose strong restrictions in the kind of changes in the distribution that we permit, they allow us to isolate changes in the dispersion of the distribution from changes in the mean, and viceversa.

[28] It should be stressed here that aggregate efficiency in the accumulation of human capital refers to the aggregate (and average) level of human capital in the economy, and not to the Pareto efficiency of the equilibrium, which, by Proposition 1, is granted regardless of the initial level of inequality of opportunity. We thank one of the referees for pointing this out.

[29] Recall that we had the following restriction: $\overline{\theta} < \frac{1}{\gamma}$ (see footnote # 13). We will assume that the upper

Some of the characteristics of the cumulative distribution function, $F(\theta,\phi)$, described in equation 24 are:

(i) *Mean*: $E(\theta) = 1 \quad \forall\phi$

(ii) *Median*: $F(\theta_m) = \dfrac{1}{2} \;\Rightarrow\; \theta_m = \dfrac{1+\phi}{\phi 2^{1/\phi}}$

(iii) As $\phi \to 1$, the distribution function in equation 24 approaches the Uniform distribution.

(iv) Define the first measure of inequality in the distribution of θ as: $\Omega = \dfrac{median}{mean}$. That is:

$$\Omega = \frac{1+\phi}{\phi 2^{\frac{1}{\phi}}} \quad \text{where:} \quad \Omega \in [\frac{1}{\gamma 2^{\frac{1-\gamma}{\gamma}}}, 1]. \tag{25}$$

A higher value of Ω corresponds to a lower degree of inequality (because the median of the distribution is closer to the mean). Note also that:

$$\frac{\partial\Omega}{\partial\phi} > 0 \;\; \text{for} \quad \phi \in [\frac{\gamma}{1-\gamma}, 1]$$

Therefore, both ϕ and Ω are measures of inequality in the distribution of endowments of the complementary factors. As ϕ and Ω increase, inequality decreases.

(v) Define the Gini coefficient of the distribution θ as:[30]

$$Gini_\theta = 2\int_0^{\frac{1+\phi}{\phi}} \theta F(\theta,\phi) f(\theta,\phi) d\theta - 1 \tag{26}$$

Solving the previous equation using the distribution given in equation 24 we have:

$$Gini_\theta = \frac{2(1+\phi)}{2\phi+1} - 1$$

Using the last equation, note that: $\dfrac{\partial Gini_\theta}{\partial\phi} < 0$. As the parameter that captures the degree of inequality in the distribution of endowments increases, the Gini coefficient, which is a measure of inequality in the distribution of endowments, decreases.

4.1. A Change in Inequality of Opportunities

Given that the mean of the distribution specified in equation 24 is constant for all values of the parameter ϕ, any change in this last parameter modifies the shape (dispersion) of the distribution while leaving the mean unchanged. We will make use of this characteristic of the distribution to carry out the first simulation.

bound of the domain satisfies: $\frac{1+\phi}{\phi} < \frac{1}{\gamma}$, which is equivalent to : $\frac{\gamma}{1-\gamma} \leq \phi$.

[30] See Lambert (2001), chapter 2.

This exercise will allow us to concentrate on the changes in the endogenous variables of the model that arise from changes in the degree of inequality of the distribution of endowments while maintaining fixed the mean endowment .

To carry out the simulation we begin by fixing some parameters of the model,[31] and solve for the equilibrium values of the endogenous variables for different values of ϕ, for $\phi \in [\frac{\gamma}{1-\gamma}, 1]$.

The first (and main) step to solve the model numerically is to find the value of θ^* that solves equation 20 using the distribution function in equation 24 for different values of the parameter ϕ. Once we have the numerical solution for the equilibrium threshold endowment, θ^*, for each value of ϕ in the interval $[\frac{\gamma}{1-\gamma}, 1]$, we replace it in the relevant equations derived in the previous section, along with the other parameter values used, to obtain the corresponding values of the endogenous variables of the model.

The results of the simulation of the first intervention are presented in the panels of Figure 5.[32] From Panel (A), as the measure of inequality of opportunity in the distribution of endowments, as captured by the parameter Ω (see equation 25), decreases, average human capital in the population also increases. Conversely, a more unequal distribution of opportunities is associated with a lower level of total (and average) human capital across individuals. Panel (B) depicts the relationship between average human capital and a measure of inequality in the distribution of human capital across individuals, the human capital Gini coefficient. A lower level of average human capital is associated with a more unequal distribution of human capital (a higher human capital Gini). Panel (C) graphs the relationship between inequality of opportunities and inequality of outcomes in human capital formation. The higher the degree of inequality of opportunities is, the higher the degree of inequality in the distribution of human capital across individuals. In other words, a higher inequality of opportunities leads to a higher inequality of outcomes (in terms of the accumulation of human capital). Panel (D) shows the relationship between inequality of opportunities, as measured by the Gini coefficient of the distribution of endowments, and

[31] We use the following parameter values for the simulation: $\alpha = 0.3$, $\beta = 0.3$, $K = 10$ and $\gamma = 0.15$. The first two parameters measure the elasticities of output with respect to unskilled labor and human capital respectively. The values chosen for these two parameters are close enough to those found in the empirical growth literature (see Mankiw, Romer and Weil, 1992). The qualitative results of the simulation do not change with the size of the capital stock chosen, $K = 10$. Regarding the parameter γ, which measures the elasticity of human capital with respect to time investment, we chose a value such that the technology of human capital formation was sufficiently concave. However, the results of the simulation are maintained for different values of this parameter that satisfy the restriction imposed on this parameter in footnote # 29.

[32] The different curves presented in Figure 5 are not 'smooth' because the simulation of the model involves numerical approximations of integrals and of the solutions to non-linear equations.

wage inequality. A higher degree of inequality of opportunities is associated with a more unequal distribution of wage income. Panel (E) shows that the higher the inequality of opportunities is, the larger is the fraction of individuals who decide not to acquire human capital and work as unskilled workers. Also, a more unequal distribution of human capital across individuals is associated with a higher fraction of the population not investing time in human capital formation and working as unskilled workers (panel F). Changes in ϕ induce changes in the relative supply of skills and, as a result, also in relative wages. In particular, an increase in ϕ (a decrease in inequality of opportunities) increases the number of individuals accumulating human capital (L^s) and reduces the number of unskilled individuals (L^u) (panel G), and, as a result, increases the ratio of unskilled to skill wages (panel H).

[Figure 5 here]

Summarizing the results obtained so far, a higher degree of inequality of opportunities is associated with lower average human capital, higher inequality in the distribution of human capital, higher wage inequality, a lower fraction of individuals in the population investing in human capital formation, and a higher ratio of unskilled to skilled wages.

The main finding of this section is the lack of a trade-off between inequality of opportunities and aggregate efficiency in the accumulation of human capital (as measured by the total and average level of human capital in the economy). In other words, the relation between average human capital in the economy and the degree of inequality in the distribution of human capital obtained from the simulation of the model is negative.[33] Also, according to the numerical simulations, there is a direct relationship between inequality of opportunities and inequality of outcomes, not only in terms of human capital but also in terms of wage income.

From the simulation of the model we can also disentangle the different forces behind the main result of this section, namely, the different forces behind the negative relationship between inequality of opportunities and the average level of human capital in the economy. Recall that the change in human capital that results from a change in the degree of inequality of opportunity can be decomposed into three different factors (see equation 23 and the explanation thereafter). From equation 24, we know that the last term in the bracketed expression in equation 23 is negative, that is: $\frac{d\bar{\theta}}{d\phi} = -\frac{1}{\phi^2} < 0$. Also, we know from the results of the simulation that the second term is positive, since we find that: $\frac{d\theta^*}{d\phi} > 0$. Therefore, given that we have found that human capital (h) increases with equality of opportunity

[33] This finding matches the main stylized fact regarding the relation between these two variables described in the introductory section of the paper (see Figure 1).

(ϕ), the first term in the right hand side of equation 23 is positive. In words, this means that the increase in average human capital in the economy results from two counteracting forces. On the one hand, average human capital increases as the measure of inequality of opportunities decreases (that is, as ϕ increases) because the human capital among the skilled individuals increases. On the other hand, average human capital decreases because, first, the equilibrium threshold endowment increases with equality and therefore the economy "loses" the human capital accumulated by those individuals with $\theta_i = \theta^*$ times the size of the increase in θ^*. And second, the upper bound of the support decreases, so the economy also "loses" the human capital of those individuals with the highest endowments, that is the human capital of those individuals with $\theta_i = \bar{\theta}$, times the size of the decrease in $\bar{\theta}$ that results form the change in the degree of inequality of opportunity. We also know from the results of the simulation that the change in the density of individuals that results from a change in inequality of opportunity is sufficiently large to offset the two negative effects just described.

The intuition behind the negative relationship found between inequality of opportunities and the average level of human capital is the following: first, a lower degree of inequality in the distribution of opportunities implicitly implies a reallocation of the complementary factors to the schooling process from individuals with a high endowment towards individuals with a low endowment,[34] keeping the mean endowment fixed. Second, under the assumption that the returns to time investment in human capital formation are decreasing, individual's human capital supplied in the labor market is a concave function of her endowment of the complementary factors (equation 9). And third, note that the general equilibrium model takes into account the endogenous labor supply response that results from a higher degree of inequality of opportunity in human capital formation.[35] The results from the simulations of the model, which combine the three elements described above, show that a lower degree of inequality of opportunity is associated with a higher average level of human capital in the economy. Conversely, the human capital "forgone" by decreasing the endowments of those individuals who are relatively better off is more than offset by the additional human capital that is acquired by those individuals who, after the implicit redistribution of resources, choose to invest a positive fraction of their time in human capital formation.

[34] Our model, however, doesn't specify the mechanism by which this redistribution takes place. The models by Benabou (2000) and Galor and Moav (2003) explicitly specify this mechanism.

[35] That is, the change in the equilibrium threshold endowment that results from a change in the degree of equality.

4.2. A Change in the Support of the Distribution of Opportunities

In the second exercise we fix the parameter ϕ and shift the whole distribution of endowments. That is, we change the support of the distribution $F(\theta, \phi)$ while keeping the parameter ϕ fixed. The cumulative distribution function is now given by:

$$F(\theta, \phi) = \begin{cases} 0 & for \quad \theta < z \\ \left[\dfrac{\phi}{1+\phi}\right]^{\phi} (\theta - z)^{\phi} & for \quad \theta \in \left[z, z + \frac{1+\phi}{\phi}\right] \\ 1 & for \quad \theta > z + \dfrac{1+\phi}{\phi} \end{cases}, \tag{27}$$

where we allow the parameter z to vary between $[0, 1]$.

We assume throughout this section the same parameter values that we used in the first exercise and, as a benchmark, we fix $\phi = 0.5$.

When we change the parameter z, it is as if each individual in the economy were given an additional fixed quantity of the complementary factors to the schooling process. The mean endowment in this case changes linearly with z.[36] In this exercise, changes in average human capital across all individuals are expected to be positive as z increases, because each agent has a higher endowment of the complementary factors to the schooling process. However, we are interested in the relationship between average human capital, inequality of opportunities, inequality in the distribution of human capital, and wage inequality. Given that each individual, under this exercise, has a higher endowment of the complementary factors as we increase z, although ϕ is held constant, inequality in the distribution of human capital and wage inequality are expected to change because, given that human capital is a concave function of the endowment of the complementary factor, an increase of one unit in the endowment has a higher effect for a poor individual than it has for a relatively rich individual. The change of support being simulated here is a translation of the entire distribution by z. Although one of the measures of inequality in the space of opportunities, ϕ, is held constant, other measures such as the relative Gini coefficient and the ratio of median to mean endowment - that are scale invariant but not translation invariant measures of inequality - will change as the parameter z changes. Therefore, this second simulation involves changes in both the first and the second moments of the distribution $F(\theta, \phi)$.

The reader can easily derive the characteristics of the new distribution function that correspond to points (i) through (v) in the last subsection.

The simulation in this case is very similar to the one carried out in the first exercise, except that we now fix the parameter ϕ and allow the parameter z to change in the interval $[0, 1]$.

[36] The mean endowment in this case is given by: $E(\theta) = 1 + z$

As in the previous simulation, the first step to solve the model numerically is to find the value θ^* that solves equation 20 using the distribution function described in equation 27, for each value of the parameter z in the interval $[0,1]$. Once we have the solution for θ^* for the different values of z we can use the equations derived in the previous section to obtain the corresponding values of the endogenous variables of the model.

The results of the second simulation are presented in the panels of Figure 6. From Panel (A), a lower degree of inequality of opportunity, as captured by the ratio median over mean, is associated with a higher level of average human capital. This results is confirmed by Panel (B), where the relationship between human capital inequality (as measured by the human capital Gini) and average human capital is negative. Panels (C) and (D) say that more inequality of opportunities is associated with more inequality in the distribution of human capital and more wage inequality. Panel (E) says that the higher the inequality of opportunities is, the higher is the number of individuals that do not invest time in human capital formation and work as unskilled labor. Panel (F) says that higher inequality in the distribution of human capital is associated with a larger number of unskilled individuals in the population. Finally, in panels (G) and (H) we plot the mean endowment in the population $(1+z)$ against the relative supply of skilled workers and the relative wages of unskilled workers. As the mean endowment in the population increases, more individuals acquire human capital (panel G), and, as a result, the ratio of unskilled to skilled wages increases (panel H).

[Figure 6 here]

Summarizing, the results in this section match the results obtained in the first simulation. Namely, they confirm the negative relationship between inequality of opportunities and the average level of human capital in the economy, and the positive relation between inequality of opportunities and inequality of outcomes, also found in the simulation of the first intervention. However, the two interventions are different in nature. While the first one changes the degree of inequality keeping the mean endowment constant, the second one shifts the whole distribution of endowments to the right while leaving the shape of the cumulative distribution function unchanged.

5. Concluding Remarks

This paper develops a heterogeneous agent general equilibrium model with unequal opportunities in human capital formation. The model presented explains, among other things, the negative relation between average human capital and human capital inequality. While most of the existing literature suggests an explanation for the negative relation between these two variables based on the existence of credit market imperfections (that prevent poor individuals from investing in human capital), our model explores a different, and, perhaps complementary explanation. Namely, our explanation is based on the existence of different rates of return to time invested in the accumulation of human capital across individuals, which are, in turn, determined by each individual's endowment of the complementary factors to the schooling process. In other words, the model specifies inequality of opportunities in human capital formation across individuals as a differential endowment of the factors that complement the schooling process.

In equilibrium, the endogenous variables of the model are determined, among other parameters, by the degree of inequality in the distribution of the endowments that complement the schooling process. In order to study the relationship between the endogenous variables of the model and the parameters, we solve the model numerically using a distribution function for the endowments of the complementary factors that allows us to isolate changes in the degree of inequality from changes in the mean endowment. Using numerical simulations we examine how the endogenous variables of the model respond to two different interventions in the distribution of opportunities: a mean-preserving spread, and a change in the support of the distribution. Among the main results, we find that a higher degree of inequality of opportunities is associated with a lower average human capital in the population, a lower fraction of individuals investing in human capital, a higher degree of inequality in the distribution of human capital, and a higher degree of wage inequality.

Appendix

(A1)

Note that from equation 8, $b(u_i^*) = \left(\frac{\gamma}{1+\gamma}(1+\theta_i)\right)^\gamma$ denotes the optimal level of human capital for individual i. Recall that θ_i is different for all individuals and is determined by each agent's endowment of the complementary factors to the educational process . Let, only as an example, the endowment be a weighted average[37] of two characteristics: parental level of education (ε_i) and an a measure of health $(\varkappa_i)$. That is, let: $1+\theta_i = \delta\varepsilon_i^\lambda \varkappa_i^{1-\lambda}$, with δ and λ being unknown parameters to the researcher. If parental level of education and health characteristics are observed for each individual and we are able to proxy $b(u_i^*)$ with test scores, or by an indicator of years of schooling for each individual (s_i) then, the effects of parental education and health can be estimated from the log-linearization of the optimal amount of human capital derived above. That is:

$$\ln s_i = const + \gamma\lambda \ln \varepsilon_i + \gamma(1-\lambda)\ln \varkappa_i. \tag{A1}$$

From the estimation of the above equation, a researcher can estimate the effects of different characteristics of the individual on observed educational outcomes. In many of the empirical studies reviewed in the introduction, this is the form that is estimated.

(A2)

Proof of Proposition 1

There exists a unique equilibrium solution under a fairly general context. The market solution exists if there is an equilibrium threshold endowment, θ^*, which is a root of the following non linear function in θ for any given parameters $\gamma, \alpha, \beta \in (0,1)$, and $\phi \in E$.

$p(\theta, \alpha, \beta, \gamma, \phi) = k(\theta, \alpha, \beta, \gamma, \phi) - g(\theta, \gamma, \phi)$,

where $k(\theta, \alpha, \beta, \gamma, \phi) = \frac{\beta}{\alpha}(1+\theta)^\gamma F_\phi(\theta)$, $g(\theta, \gamma, \phi) = \int_\theta^{\overline{\theta}} (1+y)^\gamma f_\phi(y)dy$, $f_\phi(.)$ is a probability density function of the endowments and $D = [\underline{\theta}, \overline{\theta}]$ is the support of the distribution $f_\phi(.)$.

(**Existence**) First, note that $k(.)$ is strictly increasing in θ and that $g(.)$ is non-increasing in θ. Thus, $p(.)$ is strictly increasing in θ. If $f_\phi(.)$ is Riemann integrable, $p(.)$ is continuous, and the image of D is a compact and a connected subset of the real numbers. That is, the

[37] It is not necessary that the weights add up to 1, but is a hypothesis that can be tested.

image is a bounded and closed interval, which we will denote by $I = [a, b] \subset R$. Since $p(.)$ is strictly increasing, $a = p(\underline{\theta}, \alpha, \beta, \gamma, \phi) = k(\underline{\theta}, \alpha, \beta, \gamma, \phi) - g(\underline{\theta}, \gamma, \phi) = -\int_{\underline{\theta}}^{\overline{\theta}} (1+y)^{\gamma} f_{\phi}(y) dy < 0$ and $b = p(\overline{\theta}, \alpha, \beta, \gamma, \phi) = \frac{\beta}{\alpha}(1 + \overline{\theta})^{\gamma} > 0$. Therefore, $0 \in I$ so that there exists $\theta^* \in (\underline{\theta}, \overline{\theta})$ for which $p(\theta^*, \alpha, \beta, \gamma, \phi) = 0$.

(**Uniqueness**) Since $p(.)$ is strictly increasing on D, it is injective and, therefore, a unique equilibrium solution exists. Q.E.D.

Remark: The only assumption we impose on $f_{\phi}(.)$ is Riemann integrability. Continuous or piecewise continuous functions are particular cases of Riemann integrable functions.

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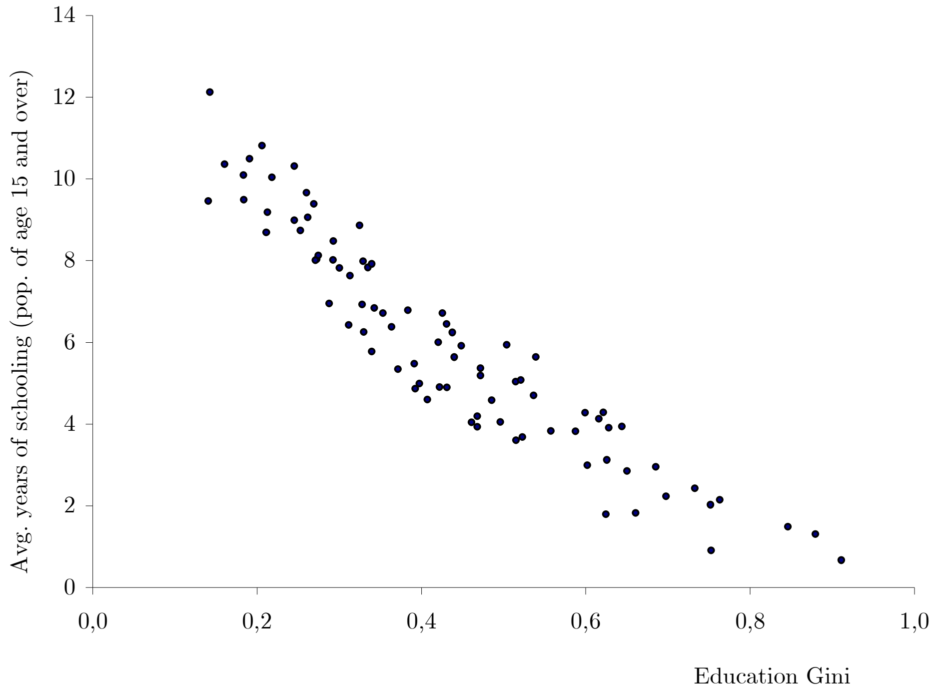
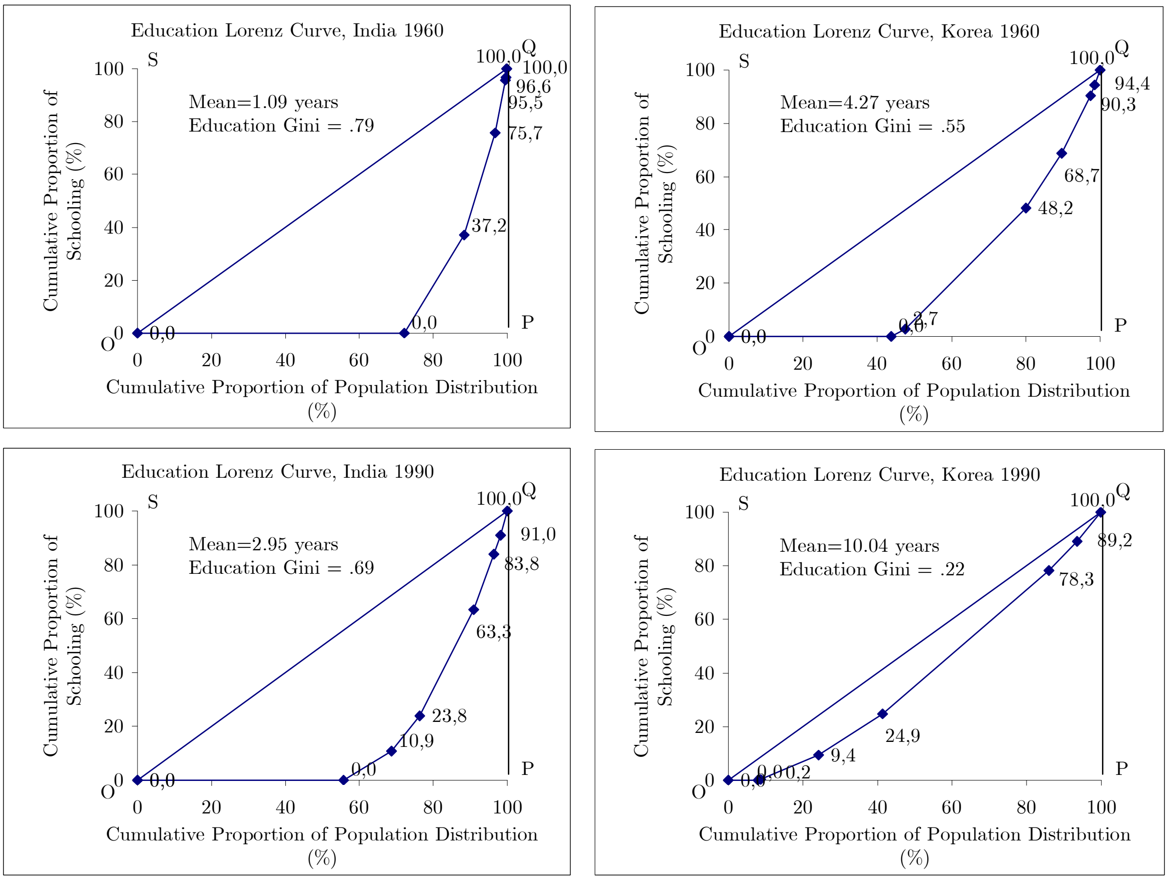


Source: Thomas et. al (2002)

Figure 1: Average years of schooling and education Gini coefficient.



Source: Thomas et al. (2002)

Figure 2: Human capital Lorenz curves for India and Korea (1960 and 1990).

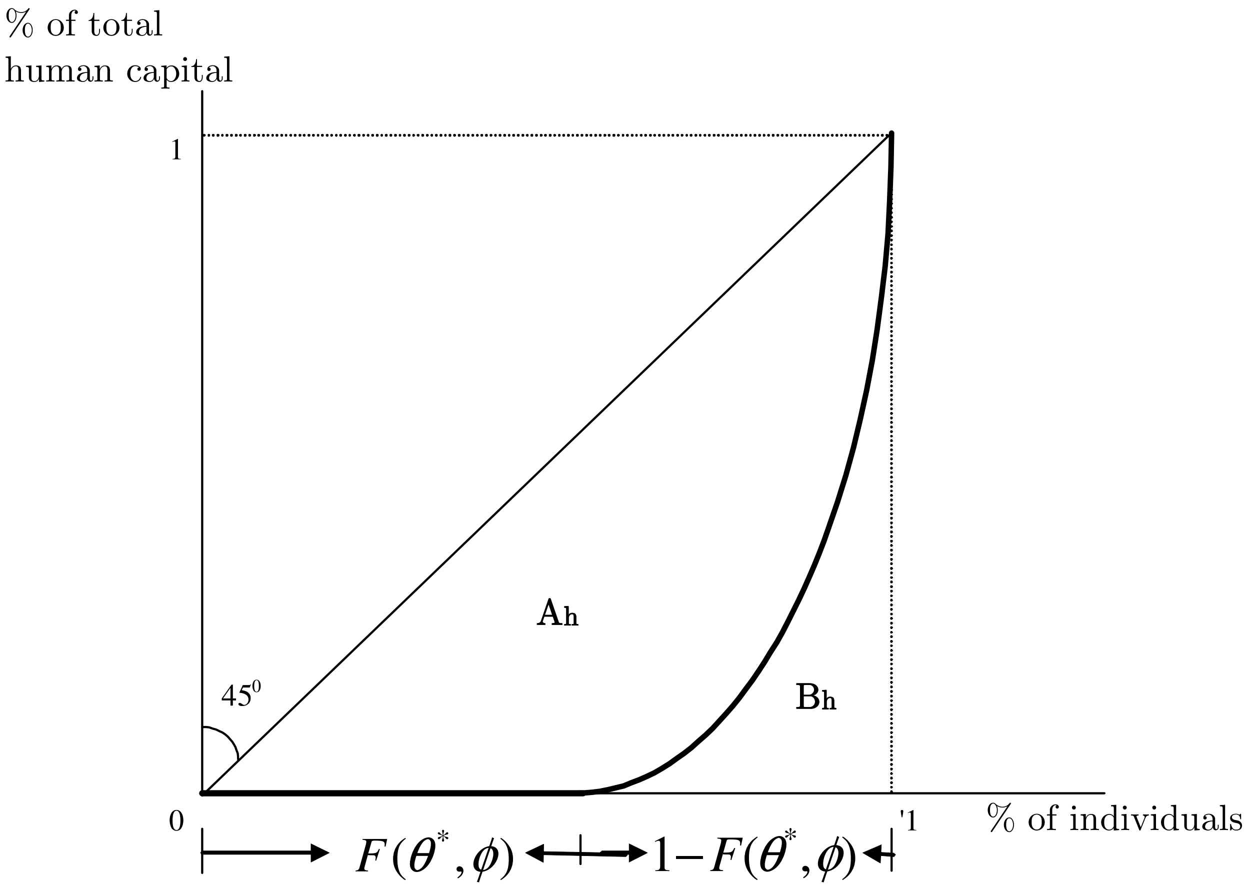


Figure 3: Human capital Lorenz curve.

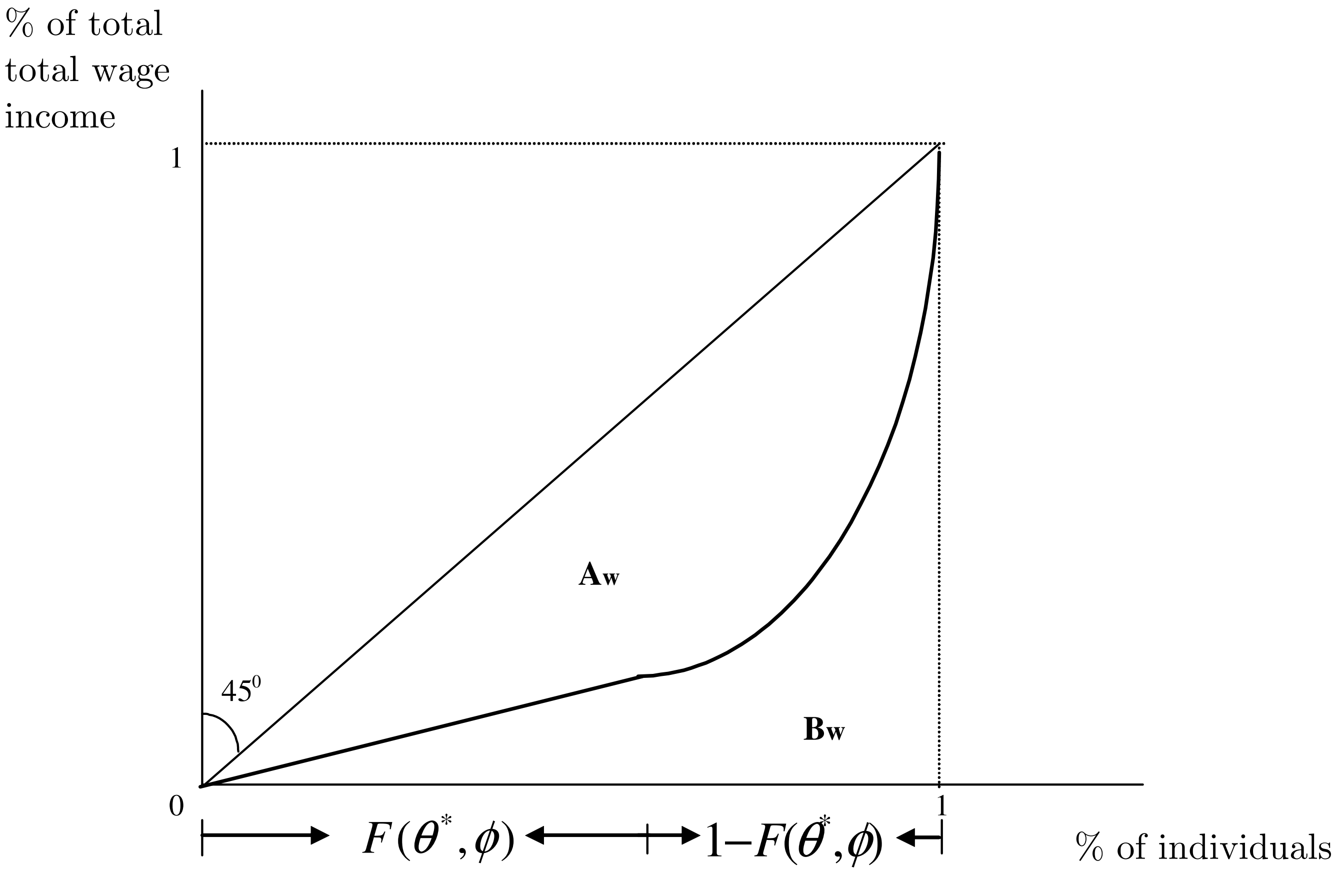


Figure 4: Wage income Lorenz curve.

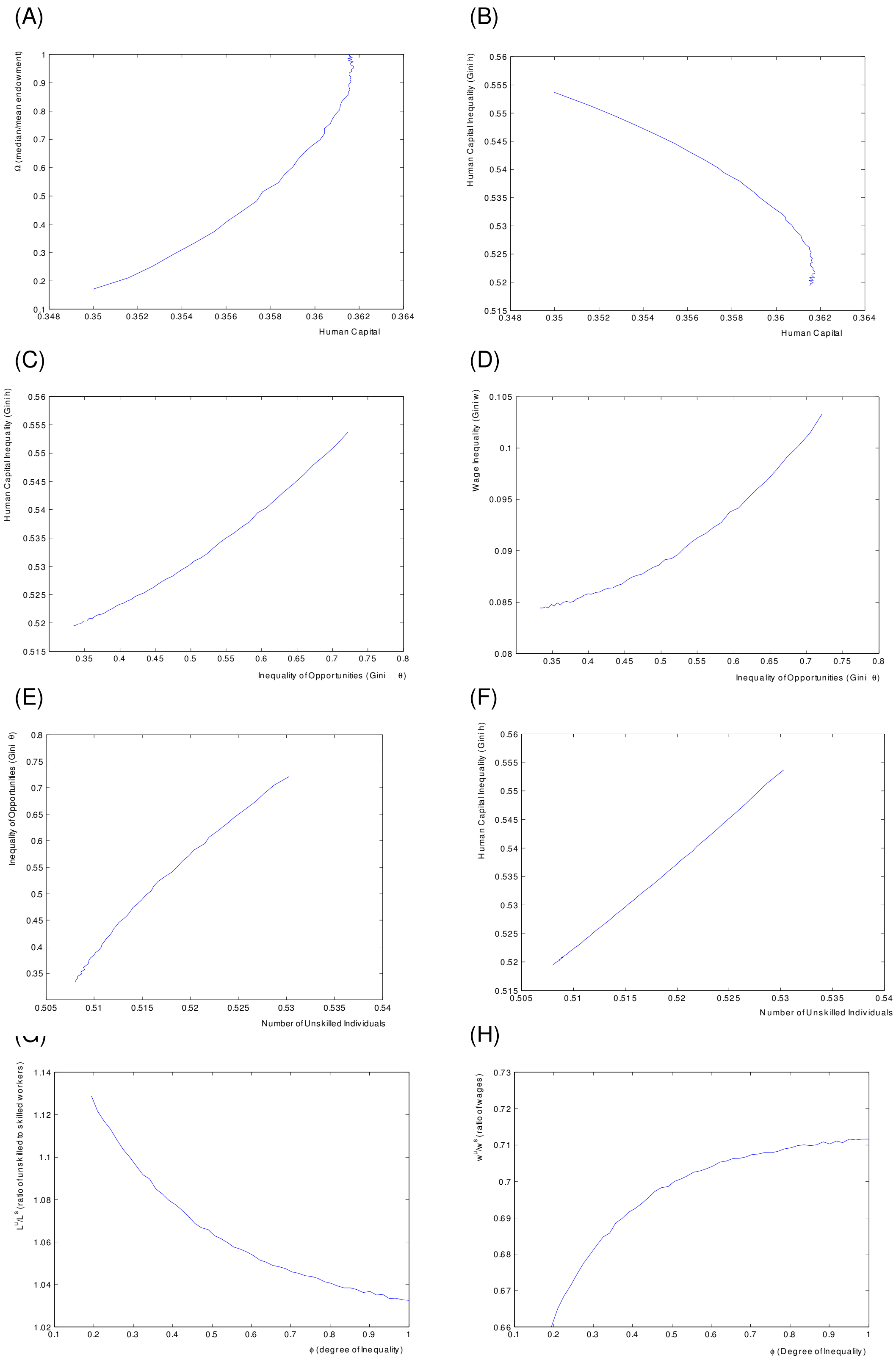


Figure 5: Effects of a mean-preserving spread in the distribution of endowments.

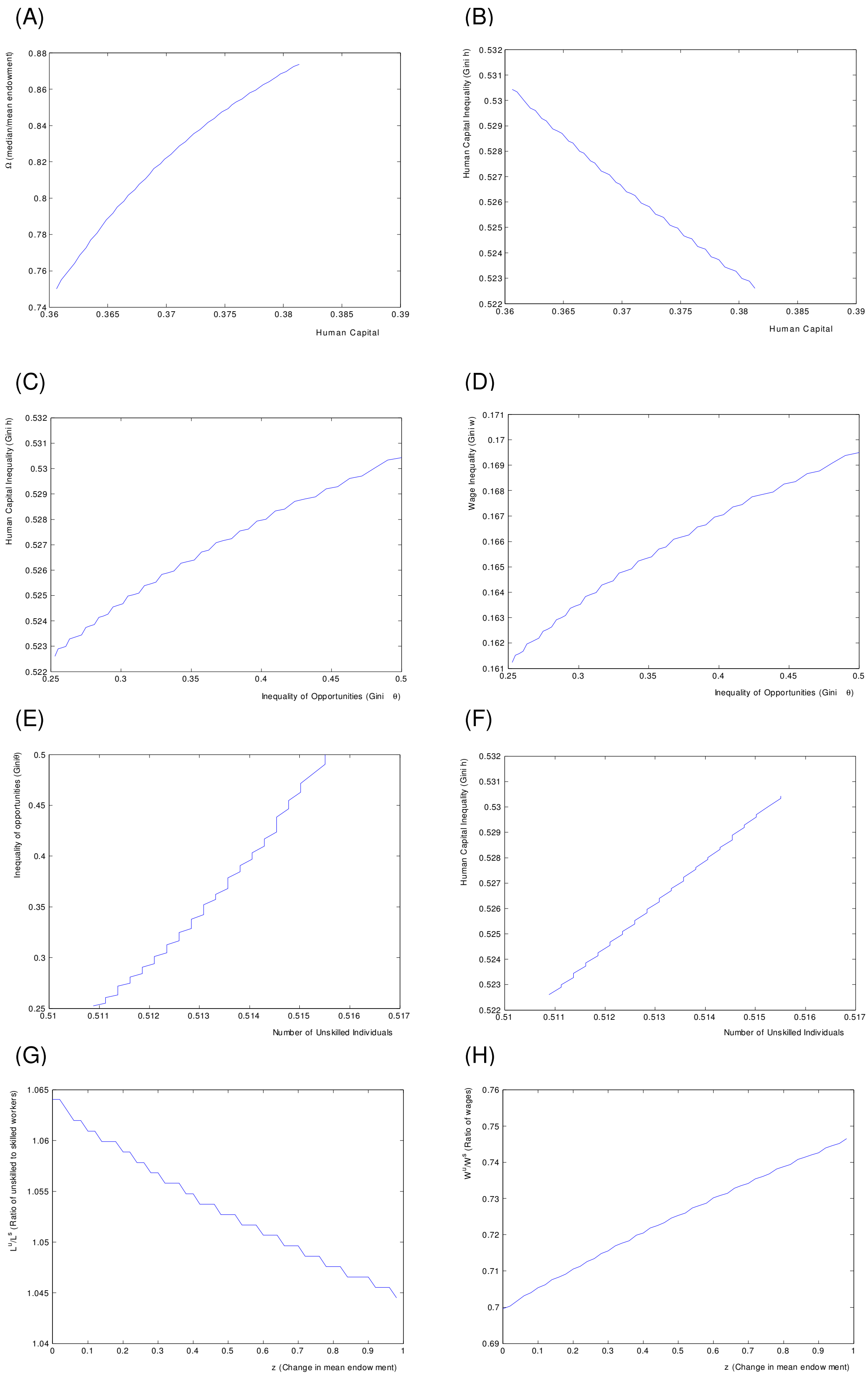


Figure 6: Effects of a change in the support of the distribution of endowments.