Modelling autoregressive processes with a shifting mean

Timo Teräsvirta*
Department of Economic Statistics
Stockholm School of Economics

Andrés González
Banco de la República, Colombia
Unidad de investigaciones económicas

November 9, 2006

Abstract

This paper contains a nonlinear, nonstationary autoregressive model whose intercept changes deterministically over time. The intercept is a flexible function of time, and its construction bears some resemblance to neural network models. A modelling technique, modified from one for single hidden-layer neural network models, is developed for specification and estimation of the model. Its performance is investigated by simulation and further illustrated by two applications to macroeconomic time series.

Keywords: deterministic shift, nonlinear autoregression, nonstationarity, nonlinear trend, structural change
JEL Classification Codes: C22, C52

Acknowledgement:: This work has been supported by Jan Wallander's and Tom Hedelius's Foundation, Grants No. J02-35 and P2005-0033:1. The paper has been presented at the workshop "Nonlinear Dynamical Methods and Time Series Analysis", Udine, August/September 2006 and at the 14th SNDE conference, Washington University, March 24-25, 2006. Material from the paper has also been discussed in seminars at the European Central Bank, Frankfurt am Main, Bank of Finland, Helsinki, and Swedish School of Economics and Business Administration, Helsinki. Comments from participants are gratefully acknowledged. Our warmest thanks also to Birgit Strikholm for useful comments. The responsibility for any errors and shortcomings in this work remains ours.

*Department of Economic Statistics, Stockholm School of Economics, Box 6501, SE-113 83 Stockholm, Sweden, email: Timo.Terasvirta@hhs.se

1 Introduction

There exist several methods of decomposing time series into components. Typically, one of the components is called "trend", and there may also be a cyclical component. When a series is divided into these components, the "trend" is often extracted from the series using a filtering procedure such as a two-sided moving average. The dynamic behaviour of the trend-adjusted series is then modelled separately, and the results are conditional on the filtered trend. For a recent overviews of filtering time series, see Canova (1998), Baxter and King (1999), Morley (2000) and Morley, Nelson and Zivot (2003). Massmann, Mitchel and Weale (2003) provide an overview of both smoothing, using a Kalman filter, and filtering. It is also possible to assume that the series has a linear or perhaps quadratic trend and estimate it jointly with the stochastic dynamic fluctuations in the series. The fluctuations are then movements around this deterministic trend. In cases where this is done, the series under study is most often a realization of a trending nonstationary process.

Sometimes a similar idea may be applied to series that do not "look" nonstationary in the sense that they would seem to have a linear or quadratic trend. They may rather be viewed as having a shift in the unconditional mean. In those cases a popular assumption in econometrics has been that the underlying process has a break or breaks in the mean, so the process is piecewise stationary. Some of the series may, however, be viewed as having a smooth transition in the mean. These series may sometimes be relatively well described by stationary autoregressive processes that contain a stationary root close to the unit circle, although the data-generating process involves a deterministic shift in the mean of the process.

Monthly European inflation series beginning around 1980 are a case in point. Their early values are high but settle down in the 1990s, when the inflation rate keeps fluctuating at a rather low level. This process may be characterized by an autoregressive model with a near unit root and a starting-point far above the mean of the series. The rôle of this root is to pull down the realization to the level where it fluctuates in the 1990s. Another view would be to interpret the decrease in inflation as a downward shift in the unconditional mean. This would imply that there have been changes in economic policy that are hard to quantify and that have brought the inflation down. These changes are then proxied by time and represented by a shifting mean in the autoregressive process. For more discussion, see (González, Hubrich and Teräsvirta 2006). In the present paper we consider an autoregressive model with a shifting mean that may be seen as a generalization of a

corresponding model with a single break or several breaks in the mean.

The autoregressive model with a shifting mean of the type we shall consider is similar to a linear model with a polynomial trend in the sense that the parameters in the trend are estimated jointly with other parameters. It differs from the smoothing and filtering approaches, in which the series is first smoothed (components removed) or filtered, and the remaining analysis is conditional on this step. The approach closest to ours is that of Bierens (1997, 2000). Bierens (1997) derived a Dickey-Fuller test (Dickey and Fuller (1979)) against the hypothesis of nonlinear trend stationarity in which the nonlinear trend is approximated prior to testing by Chebyshev polynomials. Bierens (2000) developed nonparametric tests of co-trending, a situation that occurs when a pair of series that are stationary around a nonlinear trend follow each other closely, so that they share a common trend. Our approach will also allow us to compare different series with shifting means and see if the shifts are in some sense similar across series. This extension is, however, left for further work.

The plan of the paper is as follows. The autoregressive model with a shifting mean is presented in Section 2, and specification issues are dealt with in Section 3. Section 4 contains results of a Monte Carlo experiment, and empirical examples can be found in Section 5. Section 6 concludes.

2 Nonlinear deterministic trend stationary models

The stationary autoregressive (AR) model with a shifting mean (SM) of order p [SM-AR(p)] model, can be written as

$$y_t = \delta(t) + \sum_{j}^{p} \theta_j y_{t-j} + \varepsilon_t \tag{1}$$

where the roots of the lag polynomial $1 - \sum \theta_j L^j$ lie outside the unit circle, $\{\varepsilon_t\}$ is the sequence of normal independent $(0, \sigma^2)$ errors and $\delta(t)$ is a deterministic nonlinear shift function. It is often assumed that $\delta(t)$ is linear function of t in which case (1) is typically called "trend-stationary". Sometimes, however, the functional form of $\delta(t)$ is unknown. In this paper we define $\delta(t)$ as follows:

$$\delta(t) = \delta_0 + \sum_{i=1}^{q} \delta_i g(\gamma_i, c_i, t/T) \tag{2}$$

where δ_i, $i = 1, \ldots, q$, are parameters and $g(\gamma_i, c_i, t/T)$, $i = 1, \ldots, q$, are logistic transition functions:

$$g(\gamma_i, c_i, t/T) = \big(1 + \exp\big(-\gamma_i(t/T - c_i)\big)\big)^{-1} \tag{3}$$

with $\gamma_i > 0$, $i = 1, ..., q$. The components in the shift function (2) are exchangeable, and identification is achieved for example by assuming $c_1 < \cdots < c_q$. In applications, it may be sometimes assumed that $0 \leq c_i \leq 1$, $i = 1, ..., q$, but that restriction is not required by the statistical theory.

The shifting mean of y_t at time t equals

$$E_t y_t = (1 - \sum_{j=1}^{p} \theta_j L^j)^{-1} \delta(t).$$

One may also parameterize the SM-AR model as follows:

$$y_t = \delta(t) + \sum_{j=1}^{p} \theta_j \{y_{t-j} - \delta(t-j)\} + \varepsilon_t$$

in which case $E_t y_t = \delta(t)$.

The specification of $\delta(t)$ in (2) with (3) bears resemblance to the so called "single hidden-layer" artificial neural network model. It follows that theoretically, any function $h(t)$ satisfying rather mild regularity conditions can be approximated arbitrarily accurately by $\delta(t)$ such that $q \leq q_0 < \infty$ in (2). This has been discussed in several papers including Cybenko (1989) and Hornik, Stinchcombe and White (1989).

The parametric form of (2) is very flexible and contains as special cases well known examples of nonlinear trends. For instance, when $\delta_1 = \cdots = \delta_q = 0$, (2) becomes constant, and when $q = 1$, $\delta(t)$ changes smoothly from δ_0 to $\delta_0 + \delta_1$ as a function of t, with the centre of the change at $t = c_1 T$. The smoothness of the change is controlled by γ_1: the larger γ_1 the faster the transition. When $\gamma_1 \to \infty$, $\delta(t)$ collapses into a step function, so there is a single break in the intercept. On the contrary, when γ is small, $\delta(t)$ represents a slow shift that around c is approximately linear. Values $q > 1$ add flexibility to $\delta(t)$. For example, when $q = 2$, $\gamma_1 = \gamma_2$, $c_1 < c_2$ and $\delta_1 = -\delta_2$ in (2), $\delta(t)$ changes from δ_0 at the beginning of the sample to $\delta_0 + \delta_1$ for $t \in (c_1 T, c_2 T)$ and back towards δ_0 for $t > c_2 T$. This kind of symmetric change in the intercept can also be approximated with a logistic function with a second-order exponent as explained in Teräsvirta (1998) and Jansen and Teräsvirta (1996).

3 Model specification

In practical modelling situations, the form of the SM-AR model has to be determined from the data. This implies selecting p and q. There is no natural order in which the choice is made. It may be expected, however, that if p is selected first and there is a shifting mean,

appropriate selection criteria may sometimes choose a large p in order to accommodate the shift. This may leave less room for a time-varying intercept. But then, if q is selected first, it may in turn be large, as some of the stochastic variation may be ascribed to the shift function. The order of selection may thus depend on the problem at hand. For example, if there is economic theory suggesting that having a shifting mean may be an appropriate solution, one may want to select q first. If this is not a case, choosing p first may be the more appealing alternative of the two. The decision is left to the model builder. Nevertheless, when q is selected first, one may use a heteroskedasticity-autocorrelation consistent (HAC) estimator for the covariance matrix of the estimators throughout the model selection process and thus account for the fact that the conditional mean may also be time-varying.

3.1 QuickShift: a procedure for determining the number of transition functions

Determining q in (2) may be done either by a sequence of specification tests or by model selection criteria. We shall suggest a procedure that we call QuickShift. It is a simplified form of a procedure that White (2006) recently proposed for specifying and estimating artificial neural network models and that he called QuickNet. QuickShift as well as QuickNet have two useful properties. First, they transform the model selection problem into a problem of selecting variables, which simplifies the computations. Second, overfitting is avoided.

The origins of QuickNet lie in a result by Bierens (1990), saying that under very general conditions $\mathrm{E}(\varepsilon_t|x_t) \neq 0$ implies that $\mathrm{E}(\varepsilon_t \exp(x_t'\theta)) \neq 0$ for $\theta \in \Theta$. This means that when a model is misspecified, the disturbance ε_t is correlated with functions of the form $\exp(x_t'\theta)$ for any $\theta \in \Theta$. Stinchcombe and White (1998) generalized this result and showed that it holds for a very general family of functions, including the logistic function. It follows that $\exp(x_t'\theta)$ can be replaced by any function of the form $G(x_t'\theta) = (1 + \exp\{-x_t'\theta\})^{-1}$. In fact, polynomial functions form perhaps the most important exception to this rule.

White (2006) uses the results of Stinchcombe and White (1998) to construct a sequential procedure for specifying a neural network model. The idea is simple: given a finite set of hidden units that are logistic functions, the algorithm selects the subset of them that gives the largest in-sample predictive power. The selection procedure is sequential: the hidden unit with the largest correlation with the predictive error is selected at each step. The algorithm stops when the predictive power of the marginal unit is sufficiently small.

QuickNet requires specifying *a priori* a maximum number of hidden units and a large set of candidate functions. The maximum number of hidden units $\bar{q}$ can be set to any value such that the model can be estimated, given the sample size. Applying White's idea to the present situation results in QuickShift. The hidden units are simply replaced by logistic transition functions of type (3). The set of transition functions is obtained by evaluating (3) over a fixed grid for γ and c. A feasible grid may be defined as follows: $\Theta_N = \{(\Gamma_{N_\gamma} \times C_{N_c})\}$ with $\Gamma_{N_\gamma} = \{\gamma_s : \gamma_s = \kappa\gamma_{s-1}, s = 1, \ldots, N_\gamma, \kappa \in (0,1)\}$ and $C_{N_c} = \{c_s : c_s = c_{s-1} + (1/N_c), s = 1, \ldots, N_c\}$.

Given $\bar{q}$ and Θ_N, QuickShift consists of the following steps:

1. Estimate model (1) assuming $\delta(t) = \delta_0$, save the residuals $\hat{\varepsilon}_{t,0}$.

2. After selecting $q-1$ transitions, $q > 1$, choose the transition function that has the largest correlation with $\hat{\varepsilon}_{t,q-1}$ that is, let

$$(\hat{\gamma}, \hat{c})_q = \text{argmax}_{(\gamma_s, c_s) \in \Theta_N} [r(g(\gamma_s, c_s, t/T), \hat{\varepsilon}_{q-1,t})]^2$$

   where $r(.,.)$ is the sample correlation between $g(\gamma_s, c_s, t/T)$ and

$$\hat{\varepsilon}_{q-1,t} = y_t - \hat{\delta}_0 - \sum_{i=1}^{q-1} \hat{\delta}_i g(\hat{\gamma}_i, \hat{c}_i, t/T) - \sum_{j=1}^{p} \hat{\theta}_j y_{t-j}$$

3. Given $(\hat{\gamma}, \hat{c})_q$, obtain the estimates $(\hat{\delta}_0, \ldots, \hat{\delta}_q, \hat{\theta}_1, \ldots, \hat{\theta}_q)'$ by OLS. Save $\hat{\varepsilon}_{q,t}$ and compute the value of the Bayesian Information Criterion (BIC), call it BIC(q).

4. Continue until $q > \bar{q}$ and choose $\hat{q}$ such that $\hat{q} = \text{argmin}_{q \in (1,\ldots,\bar{q})} \text{BIC}(q)$.

The above description of QuickShift uses BIC as the criterion for determining $\hat{q}$, but other decision rules are possible. White (2006) uses a consistent cross-validation criterion proposed by Racine (1997, 2000). In the present context this cross-validation criterion has two problems. First, it increases the computational burden of the algorithm. Second, in some situations computing the value of the cross-validation criterion is not possible. The reason is that when the validation sample is large, the moment matrix for the estimation sample can be near-singular. Near-singularity in the moment matrix is likely to occur when two or more transition functions have similar location parameters and the slope parameters are not sufficiently different from each other.

In another variant of QuickShift, tests for parameter nonconstancy are used as the criterion for determining q. In this case, after adding a transition to the model a test

for remaining intercept nonconstancy is carried out and testing is continued sequentially until the first non-rejection of the constancy hypothesis. We consider two tests available for testing intercept nonconstancy. They are the Taylor expansion based test by Lin and Teräsvirta (1994) and the Neural Network test by Lee, White and Granger (1993).

The increasing sequence of dependent tests in applying QuickShift means that the nominal size of the overall sequence is not known, although an upper bound for it can be computed. In order to favour parsimony we suggest a decreasing sequence of significance levels. That is, the algorithm is started with a relatively large significance level $\alpha_0 = 0.5$, say, and is subsequently decreased following the rule $\alpha_{s+1} = \tau\alpha_s$ where s is the current step of the algorithm, and $\tau \in (0, 1)$. When α_0 is large, τ should be relatively small, $\tau = 0.5$, for example.

3.1.1 Taylor expansion based parameter constancy test

Assume for a moment that p has been specified and consider testing for a time-varying intercept in (1). This testing situation is a nonstandard one, because under the null hypothesis there are unidentified nuisance parameters in the model. Under the null hypothesis $\mathrm{H}_0 : \gamma_i = 0, i = 1, \ldots, q$, and so c_i, $i = 1, \ldots, q$, are unidentified. Furthermore, while $\delta_i^* = \delta_0 + (1/2)\sum_{i=1}^{q} \delta_i$ can be estimated consistently, this is not true for the elements of the sum. Lin and Teräsvirta (1994) circumvent this identification problem by replacing $\delta(t)$ in (2) with Taylor expansions of $g(\gamma_i, c_i, t/T)$ around $\gamma_i = 0$, reparameterizing (1), and testing $\mathrm{H}_0 : \delta(t) = \delta_0$ as $\mathrm{H}_0^1 : \phi_i = 0$, $i = 1, \ldots, m$, in the auxiliary regression

$$y_t = \delta_0^* + \sum_{i=1}^{m} \phi_i (t/T)^i + \theta' w_t + \varepsilon_t^* \tag{4}$$

where $w_t = (y_{t-1}, \ldots, y_{t-p})'$, $\varepsilon_t^* = \varepsilon_t + R(t/T)$, and $R(t/T)$ is the remainder of the Taylor expansion. Note that the asymptotic theory is not affected by the Taylor expansion, because under H_0^1; $\varepsilon_t^* = \varepsilon_t$. The null hypothesis H_0^1 can be tested using the standard LM statistic or its F-version. Under H_0^1 the LM statistic has an asymptotic χ^2 distribution with m degrees of freedom, and the F-statistic is approximately distributed as $F(m, T - kp - m - 1)$. See Lin and Teräsvirta (1994) and Granger and Teräsvirta (1993) for details on how to compute these statistics.

In order to carry out the test, one has to choose m. Lin and Teräsvirta (1994) suggest $m = 3$, but in the present context larger values of m may be considered depending on how smooth the deterministic component has to be in order to be an adequate description of the shifting mean. For example, it may be that $m = 3$ does not lead to a rejection

of the null hypothesis, whereas $m = m_0 > 3$ will. A similar argument is used in Bierens (1997) to determine the order of the Chebyshev polynomial of time used to approximate the nonlinear trend. This procedure assumes that there is no serial correlation in the errors. If the specification search is initiated by erroneously assuming $p = 0$, then the test statistic should be computed using an HAC estimator for the error covariance matrix.

The parameter constancy test based on (4) can be used sequentially for determining the number of transition functions in (2). This is possible because the test can be applied to a model in which $\delta(t)$ has been already estimated, and the question then is whether this specification adequately captures the intercept nonconstancy. Eitrheim and Teräsvirta (1996) generalized the test by Lin and Teräsvirta (1994) to this situation. For ease of presentation, assume that model (1) with $q = 1$ has been estimated and adding a new transition function to it is considered. The extended model is

$$y_t = \delta_0 + \delta_1 g(\gamma_1, c_1, t/T) + \delta_2 g(\gamma_2, c_2, t/T) + \theta' w_t + \varepsilon_t^* \tag{5}$$

where $g(\gamma_2, c_2, t/T)$ is defined in (3). The null hypothesis of no remaining intercept nonconstancy is $\mathrm{H}_0 : \gamma_2 = 0$. The testing problem is again nonstandard because δ_2 and c_2 are not identified when H_0 is true. The Taylor expansion approximation is applied again and the null hypothesis $\mathrm{H}_0 : \gamma_2 = 0$ becomes $\mathrm{H}_0^1 : \delta_{2i}^* = 0$, $i = 1, \ldots, m$, in the auxiliary regression,

$$y_t = \delta_0^* + \delta_1 g(\gamma_1, c_1, t/T) + \sum_{i=1}^{m} \delta_{2i}^* (t/T)^i + \theta' w_t + \varepsilon_t^*. \tag{6}$$

As before, the test can be computed using the standard LM test or its F-approximation. Under the null hypothesis, the LM statistic is asymptotically χ^2-distributed with m degrees of freedom while the corresponding F statistic has an approximate $\mathrm{F}(m, T - kp - m)$ distribution.

3.1.2 The neural network test

The neural network test is an LM test of $\mathrm{H}_0 : \delta_i = 0$, $i = 1, \ldots, q$ in (2). This choice of null hypothesis leaves γ_i, and c_i, $i = 1 \ldots, q$, unidentified when H_0 holds. Lee et al. (1993) solve the identification problem by drawing the unidentified parameters γ_i, c_i, $i = 1, \ldots, q$, from a uniform distribution and testing H_0 with an LM test for each draw where the LM statistic has an asymptotic χ^2 distribution with q degrees of freedom. As the neural network tests involves several draws of γ_i and c_i, $i = 1, \ldots, q$, one ends up with a sequence of dependent p-values. In this case, the Bonferroni inequality provides an upper bound for the p-value of the composite statistic. Let $p_1, \ldots, p_s$ be the p-values corresponding to

s test statistics, and $P_{(1)}, \ldots, P_{(s)}$ the ordered p-values. The Bonferroni inequality leads to rejection of H_0 at α level if $P_{(1)} \leq \alpha/s$. Since it is based on the smallest p-value it may cause a power loss in the test. In applications the number of draws is not usually large. Lee et al. (1993) use $s = 5$ in their empirical application.

In this paper the Neural Network test is used for detecting remaining intercept non-constancy after a number of transition functions have been added to (2). This implies that in order to carry out the test one has to define a ceiling $\bar{q}$ for the number of transition functions that can be included in the model. In this paper, $\bar{q} = 5$.

4 Estimation

Once the number of transitions in (2) has been selected, the parameters of the model can be estimated. However, full estimation of the model parameters can be a challenging task. The reason is that in small samples the log-likelihood function of an SM-AR(p) model typically has a complicated surface with nearly flat regions and a large number of local maxima.

Fortunately, full estimation of parameters may not be necessary in the present context, because QuickShift in general provides good approximations to maximum likelihood estimates. However, when exact maximum likelihood estimates are considered necessary, potential numerical problems may be solved by applying a global optimization algorithm such as simulated annealing and using the vector of parameters $(\boldsymbol{\gamma}', \mathbf{c}')'$ where $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_q)'$ and $\mathbf{c} = (c_1, \ldots, c_q)'$, selected by QuickShift as starting-values. Brooks and Morgan (1995) and Goffe, Ferrier and Rogers (1994) contain useful expositions of simulated annealing.

5 Monte Carlo Experiment

In this section we investigate the properties of QuickShift by simulation. In particular, we concentrate ourselves on the effect that different criteria for selecting the number of transitions will have on the performance of the algorithm. As mentioned in Section 3.1, BIC, Cross-Validation criteria or a sequence of parameter constancy tests may be used in connection with QuickShift. As a benchmark to our algorithms we present result based on the sequential procedure by Bai and Perron (1998), designed for selecting the number of breaks.

When parameter constancy tests are applied, setting up QuickShift requires deciding upon the order on which p and q are selected. If one selects q first, an HAC estimator

for the co-variance matrix should be used when computing the value of the test statistic. The second decision to be taken is whether or not use a rapidly decreasing sequence of significance levels. If this is done, the selection may started with a rather large initial nominal level. Alternatively, a low nominal level or a sequence of slowly decreasing levels may be used throughout the selection procedure. Examples of both cases will be given. Furthermore, we consider the situation in which $p = 1$ prior to selecting q and another case in which q is selected assuming $p = 1$. In simulations where the significance level is sequentially decreased, the algorithm is started with $\alpha_0 = 0.5$ and is decreased by a factor $\tau = 0.5$ after each rejection. In the constant size simulations $\alpha_0 = 0.05$. Finally, since the test by Lin and Teräsvirta can be computed using different values of m in (4), we simulate the procedure with both $m = 3$ and $m = 6$ in order to investigate the effect that the polynomial approximation has on QuickShift.

The data-generating processes are defined as follows:

$$y_t = 0.1+0.7g(t/T, \gamma_1 = 3, c_1 = 0.33)-0.7g(t/T, \gamma_2 = 2, c_2 = 0.67)+\rho y_{t-1}+\epsilon_t \quad \text{(Model 1)}$$

$$y_t = 0.1+0.3g(t/T, \gamma_1 = 3, c_1 = 0.25)+0.5g(t/T, \gamma_2 = 3, c_2 = 0.75)+\rho y_{t-1}+\epsilon_t \quad \text{(Model 2)}$$

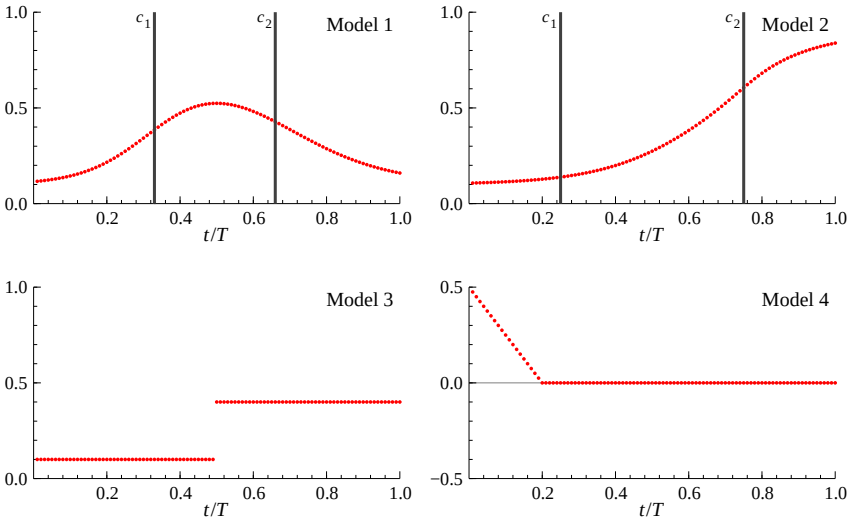
$$\begin{aligned} y_t =& 0.1 + \epsilon_t \quad \text{for} \quad 0 < t/T \leq 0.5, \\ =& 0.3 + \epsilon_t \quad \text{for} \quad 0.5 < t/T \leq 1, \end{aligned} \quad \text{(Model 3)}$$

$$\begin{aligned} y_t =& \delta_0 + \delta_1(t/T) + \epsilon_t \quad \text{for} \quad 0 < t/T \leq 0.2, \\ =& \delta_0 + 0.2\delta_1 + \epsilon_t \quad \text{for} \quad 0.2 < t/T \leq 1, \end{aligned} \quad \text{(Model 4)}$$

where $\delta_0 = 0.5$, $\delta_1 = -0.2\delta_0$, ρ is either 0 or 0.5, $g(.)$ is defined in (3) and $\epsilon_t \sim N(0, 0.2)$. In Models 1 and 2, the shift in the mean is smooth, whereas it is abrupt in Model 3. Model 4 is a mixture of these two types in the sense that the shift is smooth, except that there is a point of discontinuity at $t/T = 0.2$. The sample sizes are $T = 150$ and $T = 300$. The results are based on 1000 replications from each model.

Figure 1 presents the shifting means for the four models when $\rho = 0$. The mean shift in Model 1 starts and ends at the same value and the mean is largest in the middle of the sample. This type of shift should be easily estimated by QuickShift as it satisfies (2) when $q = 2$. The shift in Model 2 is somewhat more difficult to approximate because it may already be well approximated by a single transition function instead of two, the number of transitions in the data-generating mechanism. As already mentioned, Model 3 has a

Figure 1: Generated shifting means when $\rho = 0$



Note: The graph displays the generated shift functions for Models 1 to 4. The vertical lines in graphs Model 1 and Model 2 represent the values of c_i, $i = 1, 2$. For Model 1 $c_1 = 0.33$ and $c_2 = 0.66$ whereas for Model 2 $c_1 = 0.25$ and $c_2 = 0.75$.

single break in mean, and it is used for investigating the performance of QuickShift when the true data-generating process contains a structural break instead of smooth change. This is a case in which the algorithm by Bai and Perron (1998) has an advantage, but the behaviour of QuickShift in such a situation must be of considerable interest. Finally, the shift in Model 4 may be difficult to capture by any of the procedures.

Throughout the Monte Carlo experiment we consider models with up to five transition functions, $\bar{q} = 5$. The pool of potential transition functions is created as explained in Section 3.1 using a fixed grid for γ and c. We use 500 values of γ between 0.1 and 10 and 100 values of c between 0.1 and 0.9. Since γ is not scale-free, we divide it by the "standard deviation" of (t/T) when creating the transition functions. Finally, to compute the cross-validation criteria we use $v = 5$ as the size of the validation sample and $h = 2$ to account for possible autocorrelation. The value of h reflects the amount of correlation in the data and concecuently it is added to the validation sample. The results based on cross-validation can be affected by the small values of h and v. But then, large values of these parameters may cause numerical problems, because the moment matrix of observations that remain after excluding $2h+v$ validation observations could already be ill-conditioned.

Table 1 consists of three panels. Panel (a) contains results for all models when $\rho = 0$. Panels (b) and (c) have the results for Models 1 and 2 when $\rho = 0.5$. The difference between Panels (b) and (c) is that in the former, p is selected before applying QuickShift,

whereas in the latter, q is selected first. It is seen from Panel (a) that when $\rho = 0$ and $T = 150$ in Model 1, both BIC and CV find $q = 2$ more often than the other alternatives. However, when $T = 300$, both criteria find more transitions, and for CV this tendency is quite pronounced. For the other models, the outcome is somewhat different in that the most frequent number of transitions is one and for BIC it does not increase with the sample size. Even here, CV leads to less parsimonious models than BIC. For Model 2 the correct number of transitions is two, but although CV often chooses $q > 1$, the dispersion around $q = 2$ is large.

Selecting q under the assumption $p = 0$ makes a large difference compared to assuming $p = 1$ when BIC is concerned, but CV yields rather similar results in both cases, at least for Model 1. BIC is again more parsimonious than CV.

Results for QuickShift based on parameter constancy tests appear in Tables 2-4. The tables contain two panels. In Panel (a), the significance level $\alpha_0 = 0.5$ for all tests. In Panel (b), $\alpha_0 = 0.05$ and $\tau = 0.5$, so the significance level is halved at each step. Three test statistics are considered. LT_j is the Lin and Teräsvirta (LT) statistic with $m = j$, $j = 3, 6$, and NN is the Lee, White and Granger neural network test with the Bonferroni bound. In Table 2, it is assumed that $\rho = 0$. Table 3 contains results for the case $\rho = 0.5$ when the lag length in the autoregressive null model equals one. In Table 4, $\rho = 0.5$ and the test statistics are calculated using Andrews's HAC estimator for the covariance matrix.

The results in these tables indicate that parameter constancy tests perform better than either BIC or CV. In this experiment, the constant significance level leads to more parsimonious models than a sequence of decreasing levels. This result does not generalize, however, as it is dependent on the initial significance level α_0 which is quite high, $\alpha_0 = 0.5$. Lowering it increases parsimony. From Table 2 it is seen that generally there is no big difference between the LT and NN tests. There is one exception, however. When there is a single break in the mean (Model 3), NN finds $q = 1$ more often than the two LT tests. This is due to the difficulty of approximating a break with a polynomial of time. LT_3 is generally more parsimonious than LT_6, but in some situations (Model 4 with a discontinuity in the mean, Panel (b)) a sixth-order polynomial seems superior to a third-order one. Comparing Tables 3 and 4 it seems that applying the HAC covariance matrix sharpens the results. The choice is more focused on a single value of q than it is if the presence of y_{t-1} in the model is ignored.

Table 5 contains results from the sequential selection procedure of Bai and Perron

(1998) (BP). This table is also divided into two panels. In Panel (a), the results concern all models, assuming $\rho = 0$ in Models 1 and 2. Data for results in Panel (b) have been generated only from Models 1 and 2 with $\rho = 0.5$. The alternative before selecting q are $p = 0$ with the HAC estimator and $p = 1$ without it. Two significance levels, $\alpha = 0.05$ and $\alpha = 0.1$ are used in this experiment. The results are sensitive to the choice between $p = 0$ (with HAC) or $p = 1$ (without HAC), the latter leading to a greater number of breaks than the former. An interesting fact is that a single break (Model 4) is equally easily found by, say, the NN test and QuickShift, as it is with the BP procedure. For Model 1, BP selects either $q = 0$ or $q = 2$ when $T = 150$ but favours $q \geq 2$ when $T = 300$. This type of shift is obviously not easy to approximate by breaks.

So far we have reported results concerning the number of transitions found using QuickShift as well breaks detected by BP. There are, however, qualitative differences between the former and the latter. Figure 2 displays estimated shifting intercepts based on a single time series generated from each of the four models. We also present the estimated shift obtained using tenth-order Chebyshev polynomials because these polynomials are commonly applied to approximating nonlinear functions. As is seen from the figure, the approximating properties of our flexible intercept (2) are generally quite good compared to the Chebyshev polynomial and BP. In particular, our model is vastly superior to the polynomial approximation at both ends of the sample. Naturally, BP is superior to the other methods when it comes to Model 3 that contains a break in the mean. It offers a rather crude approximation when the shifting mean is smooth as in Models 1 and 2 and is not very good on the downward sloping stretch of Model 4. A general conclusion is that our SM-AR model is a useful tool when it comes to parameterizing autoregressive processes with a shifting mean.

6 Applications

6.1 The series

In this section we present two applications of QuickShift in order to illustrate its properties. The first application is related to Garcia and Perron (1996), Bai and Perron (2003) and Zeileis and Kleiber (2005). It consists of estimating the shifting mean of a US ex-post real interest rate series. More precisely, the time series is the US ex-post real interest rate, defined as the three-month Treasury bill rate deflated by the consumer price index (CPI) inflation rate. The series contains 103 quarterly observations for the period

Table 1: Performance of QuickShift based on BIC and Cross-Validation

	Model 1				Model 2				Model 3				Model 4			
	BIC		CV		BIC		CV		BIC		CV		BIC		CV	
q/T	150	300	150	300	150	300	150	300	150	300	150	300	150	300	150	300
Panel (a): $\rho = 0.0$, $p = 0$																
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	8	0
1	1	0	0	0	861	883	631	307	895	910	667	389	888	956	641	486
2	621	392	316	61	66	44	178	100	86	75	252	275	94	39	232	221
3	202	246	245	111	56	63	156	326	14	13	60	172	16	4	79	148
4	160	356	408	663	12	9	31	191	3	2	19	97	1	1	31	74
5	16	6	30	165	5	1	4	76	2	0	2	67	1	0	9	71
Panel (b): $\rho = 0.5$, q is selected assuming $p = 0$																
0	0	0	0	0	0	0	0	0								
1	0	0	0	0	344	293	651	331								
2	86	20	261	43	40	28	71	49								
3	158	74	227	93	240	317	188	280								
4	496	701	459	644	191	223	69	221								
5	260	205	53	220	185	139	21	119								
Panel (c): $\rho = 0.5$, q is selected assuming $p = 1$																
0	0	0	6	0	0	0	0	0								
1	0	0	4	0	409	619	314	168								
2	402	265	274	55	319	245	340	237								
3	318	309	321	152	200	115	256	313								
4	254	417	379	651	43	19	68	189								
5	26	9	16	142	29	2	22	93								

Note: The table contains the numbers of times out of 1000 when QuickShift selects a given value for q. CV stands for Cross-Validation. We use $h = 3$ and $v = 5$ to compute the consistent cross-validation criterion by Racine (2000). The value of v is smaller than the optimum but, larger values may cause numerical problems. The maximum number of transitions in QuickShift is 5. The simulations are done without first testing for intercept non constancy which means that when $q = 0$ the BIC or CV criterion select a linear model with constant intercept.

Table 2: Performance of QuickShift based on different parameter constancy tests assuming $\rho = 0$

	Model 1			Model 2			Model 3			Model 4		
q	LT_3	LT_6	NN	LT_3	LT_6	NN	LT_3	LT_6	NN	LT_3	LT_6	NN
						Panel (a): $\alpha_0 = 0.05, \tau = 1$						
	$T = 150$											
0	0	1	0	0	0	0	0	1	1	0	3	4
1	2	7	15	962	920	958	939	943	971	965	967	980
2	913	864	902	27	50	29	50	34	23	34	25	15
3	64	97	71	9	24	12	9	12	3	1	3	1
4	19	26	12	2	5	1	2	9	0	0	1	0
5	2	5	0	0	1	0	0	1	2	0	1	0
	$T = 300$											
0	1	5	0	0	1	0	0	2	2	0	0	0
1	0	0	0	942	883	927	898	923	938	981	983	992
2	882	503	656	40	58	50	79	51	43	18	9	6
3	67	365	272	17	48	20	22	15	13	1	8	2
4	49	115	70	1	9	3	1	6	3	0	0	0
5	1	12	2	0	1	0	0	3	1	0	0	0
						Panel (b): $\alpha_0 = 0.5, \tau = 0.5$						
	$T = 150$											
0	0	0	0	0	0	0	0	1	1	0	0	0
1	0	1	1	823	691	835	738	722	872	798	841	926
2	835	688	830	131	196	119	215	211	100	183	129	65
3	132	257	157	43	99	43	40	49	17	18	25	9
4	31	49	12	2	13	3	6	15	6	1	5	0
5	2	5	0	1	1	0	1	2	4	0	0	0
	$T = 300$											
0	0	0	0	0	0	0	1	1	1	0	0	0
1	0	0	0	782	592	765	621	668	790	845	875	930
2	795	287	468	151	242	164	293	255	158	139	104	55
3	123	528	438	64	147	66	78	57	41	14	19	14
4	81	175	93	3	16	5	7	15	9	4	2	1
5	1	10	1	0	3	0	0	4	1	0	0	0

Note: The columns LT_3 and LT_6 contain results for Taylor expansion based test when $m = 3$ or $m = 6$ in (4), whereas results for the Neural Network test are reported in column NN. α_0 is the initial nominal level in the sequential procedure, τ is the adjustment factor. The nominal level is adjusted following $\alpha_s = \tau\alpha_{s-1}$, $s = 1, \ldots, 5$. Data for models 1 and 2 was generated assuming $\rho = 0.0$ and all tests are computed without including lags of y_t into the null model.

Table 3: Performance of QuickShift based on parameter constancy tests, $\rho = 0.5$ and $p = 1$

	Model 1			Model 2		
q	LT_3	LT_6	NN	LT_3	LT_6	NN
	Panel (a): $\alpha_0 = 0.05, \tau = 0$					
	$T = 150$					
0	2	8	1	1	5	1
1	19	7	26	639	614	701
2	789	677	733	258	226	186
3	141	229	183	90	120	86
4	46	71	51	9	18	18
5	3	8	6	3	17	8
	$T = 300$					
0	0	9	2	0	2	0
1	0	0	0	793	691	767
2	810	377	484	170	204	181
3	118	445	409	31	85	45
4	70	157	101	6	13	7
5	2	12	4	0	5	0
	Panel (b): $\alpha_0 = 0.5, \tau = 0.5$					
	$T = 150$					
0	1	2	0	0	3	0
1	1	1	1	325	307	459
2	687	470	607	473	420	351
3	239	407	321	182	214	151
4	69	107	67	18	44	33
5	3	13	4	2	12	6
	$T = 300$					
0	0	1	1	0	0	0
1	0	0	0	476	340	541
2	694	188	328	405	427	326
3	195	582	530	105	205	121
4	110	216	140	14	24	12
5	1	13	1	0	4	0

Note: The test statistics are computed using the standard OLS estimator for the variance-covariance matrix. The model under the null includes y_{t-1} so the errors are assume uncorrelated. The column names and other symbols are defined as in Table 1.

Table 4: Performance of QuickShift based on parameter constancy tests $\rho = 0.5$ with the HAC covariance matrix

	Model 1			Model 2		
q	LT_3	LT_6	NN	LT_3	LT_6	NN
	Panel (a): $\alpha_0 = 0.05, \tau = 1$					
	$T = 150$					
0	0	10	67	0	0	0
1	150	538	834	965	974	1000
2	811	427	99	28	16	0
3	29	21	0	6	8	0
4	8	4	0	1	2	0
5	2	0	0	0	0	0
	$T = 300$					
0	0	3	0	0	0	0
1	0	0	1	897	844	935
2	863	590	834	59	80	44
3	73	273	137	39	73	20
4	58	123	28	5	3	1
5	6	11	0	0	0	0
	Panel (b): $\alpha_0 = 0.5, \tau = 0.5$					
	$T = 150$					
0	0	0	0	0	0	0
1	0	11	154	730	669	925
2	847	803	835	186	247	71
3	127	175	11	79	78	4
4	25	11	0	5	6	0
5	1	0	0	0	0	0
	$T = 300$					
0	0	1	0	0	0	0
1	0	0	0	659	465	731
2	727	278	630	184	281	178
3	146	529	317	137	237	90
4	125	185	53	20	17	1
5	2	7	0	0	0	0

Note: The table contains the numbers of times that QuickShift finds a given q. The tests statistics are computed using the consistent estimator of the variance-covariance by Andrews. The data is generated with $\rho = 0.5$ and no lags y_t are included in the tests. α_0 is the initial nominal level, τ is the adjustment coefficient, $\alpha_i = \tau\alpha_{i-1}$, $i = 0, \ldots, 5$. The column names are defined as follows: LT_3 and LT_6 stand for the test based on (4) when $m = 3$ or 6, whereas NN corresponds to Neural Network test when the Bonferroni bound is used.

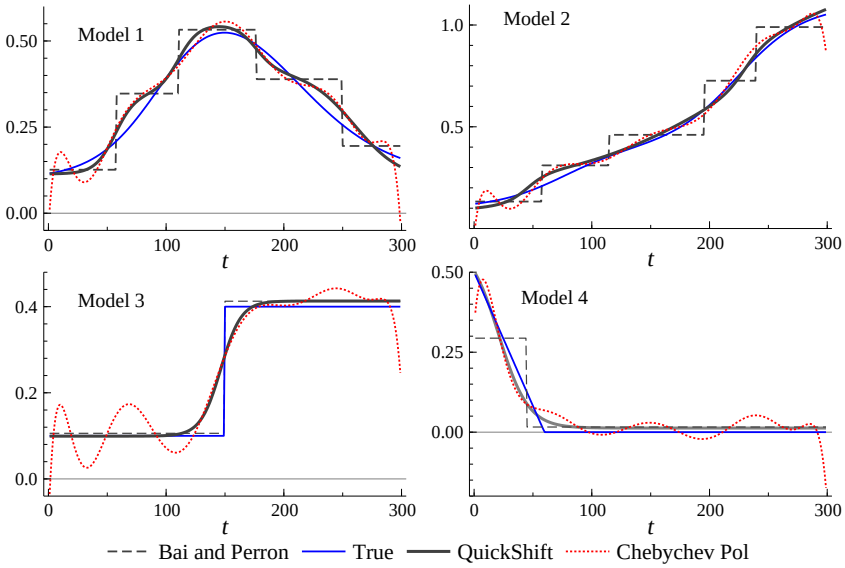
Table 5: Performance of the Bai and Perron for selecting breaks procedure

Panel (a): $\rho = 0.0$ and $p = 0$								
	Model 1		Model 2		Model 3		Model 4	
q	0.1	0.05	0.1	0.05	0.1	0.05	0.1	0.05
$T = 150$								
0	0	12	0	0	0	0	0	3
1	4	9	0	0	919	956	941	963
2	771	836	190	259	80	44	55	34
3	204	133	728	699	1	0	4	0
4	21	10	82	42	0	0	0	0
5	0	0	0	0	0	0	0	0
$T = 300$								
0	0	0	0	0	0	0	0	0
1	0	0	0	0	942	972	956	990
2	507	609	9	25	55	28	44	10
3	365	317	549	641	3	0	0	0
4	128	74	431	329	0	0	0	0
5	0	0	11	5	0	0	0	0

Panel (b): Data is generated with $\rho = 0.5$								
	Without HAC				With HAC			
	Model 1		Model 2		Model 1		Model 2	
q	0.1	0.05	0.1	0.05	0.1	0.05	0.1	0.05
$T = 150$								
0	375	628	0	0	127	273	0	0
1	19	30	4	12	16	37	0	0
2	455	285	281	439	593	528	92	147
3	134	54	636	508	218	143	648	670
4	17	3	79	41	45	19	255	181
5	0	0	0	0	1	0	5	2
$T = 300$								
0	0	27	0	0	0	2	0	0
1	0	0	0	0	0	0	0	0
2	561	636	44	82	532	620	13	29
3	330	278	565	642	328	293	474	554
4	109	59	386	275	139	84	492	405
5	0	0	5	1	1	1	21	12

Note: Panel (a) contains results for all models when the data is generated without autocorrelation. For models 1 and 2 this means that $\rho = 0.0$. Consequently, we apply the sequential procedure by Bai and Perron (1998) that uses the standard estimator for the covariance-matrix. In panel (b) data is generated from Models 1 and 2 with $\rho = 0.5$. The left hand side of panel (b) presents results for the case in which $p = 1$ whereas results for $p = 0$ when Andrews's HAC estimator is used are reported on the right-hand side. The results in this table are computed using Perron's GAUSS code procedures available on his web page.

Figure 2: Estimated shifting intercepts from a single realization



Note: The setup for QuickShift is the following: The number of transitions q is selected using sequential testing with LT_3 as the test, where $\alpha_0 = 0.05$ and $\tau = 1$. For Bai and Perron we used a 5% significance level. We generate one draw from each model with $T = 300$ and $\rho = 0$ in Models 1 and 2. All test statistics are computed with the OLS estimator for the covariance matrix. In order to compare results with an alternative method for approximating nonlinear trends we also include the estimated trend with a Chebychev polynomial of order 10.

1961(1)-1986(3), and the same series has been used in the three papers just mentioned. The second application concerns the quarterly annualized Colombian inflation rate for the period 1960(2)-2005(4). Inflation is computed as quarterly changes of the seasonally adjusted CPI. The index is obtained from the Banco de la República database and seasonal adjustment has been carried out using X12-ARIMA. Figures 3 and 4 present the graphs of the series. Of interest in both applications is to describe the shifting mean, so we set $p = 0$ and account for serial correlation using Andrews's estimator for the covariance matrix when computing the test statistics.

The setup of QuickShift is the following. The maximum number of transition functions in (2) is 15. The grid for constructing the pool of potential transition functions consists of 500 different values of γ and c defined as explained in Section 3.1. With γ's between $(0.1/\sigma_T)$ and $(10/\sigma_T)$ and c are between $0.05T$ and $0.95T$. γ is standardized with $\sigma_T = T/\sqrt{(T^2-1)/12}$ to make it scale-independent. See van Dijk, Teräsvirta and Franses (2002) for details. In both applications we use QuickShift with a decreasing sequence of significance levels starting with $\alpha_0 = 0.5$ and adjusting α_s after every rejection by a fraction $\tau = 0.5$.

6.2 Shifting mean of the US ex-post real interest rate

The number of transitions selected for the US interest rate series is reported in Table 6.3. It has been selected assuming $p = 0$ and by carrying out the LT and NN test sequences using Andrews's estimator for the covariance matrix. As can be seen, QuickShift with BIC selects a large number of transitions, six in all, whereas CV and the LT tests select two and NN tests only find one. The fact that BIC selects a large number of transition may be expected given the simulation results in Section 5. When we consider breaks, the Bai and Perron procedure chooses three of them. This number of breaks in the US interest rate series was also found by Bai and Perron (2003) and Zeileis and Kleiber (2005). It differs from the outcome in Garcia and Perron (1996) who located two breaks.

Table 7 presents the selected transitions together with the estimated break dates. The columns labelled "Start", "Centre" and "End" indicate the dates at which values of the qth transition function begins to differ appreciably from zero, where the function takes value 0.5 and where it reaches one, respectively. For instance, the transition function of the first transition begins to obtain values different from zero at 1968(1), the transition is centred at 1972(2) and the function is practically equal to one after 1975(4). In other words, a complete shift in the mean or regime change takes approximately seven years. The second shift takes place in the second half of the sample. It has its centre at 1980(4) and spans over a period of nine years. It is interesting to note that the centres of the transitions lie close to the estimated break dates obtained by the Bai and Perron technique. Figure 3 shows the estimated nonlinear means together with the selected transition functions (dashed lines). The main difference between Bai and Perron's mean and the one estimated with QuickShift lies in the rate of adjustment towards a new level rather than in the levels themselves.

6.3 Shifting mean of the Colombian quarterly inflation rate

Results for the Colombian quarterly inflation are summarized in Tables 6.3 and 7 and Figure 4. In this case, two transition functions are needed to capture the shifting mean. The first one is centred at 1973(1) and the other at 1999(2). In the 1970s inflation increased from around 10% to about 20% in more or less seven years, whereas the deceleration process in the 1990s covers between 12 to 13 years and is about to be completed at the end of the sample. A problem of interpretation arises when a transition is not completed by the end of the observation period: will it continue or has a new level already been reached? This is a forecasting problem not present when the shifting mean is characterized by breaks in the intercept. But then, both approaches have a different problem: when will a new

Table 6: Number of transitions and breaks using QuickShift and Bai and Perron

	QuickShift				Bai and Perron
	BIC	CV	LT(3)	NN	$\alpha = 0.05$
US ex-post real interest rate	6	2	2	1	3
Colombian quarterly inflation rate	4	2	2	2	2

Note: q selected assuming $p = 0$.

Table 7: Selected transitions and break dates

	QuickShift					Bai Perron
q	γ	c	Start	Center	End	Breaks
			US ex-post real interest rate			
1	10	0.43	1968(1)	1972(1)	1975(4)	1966(4)
2	10	0.78	1977(1)	1980(4)	1986(3)	1972(3)
3	-	-				1980(3)
			Colombian quarterly inflation rate			
1	10	0.28	1965(4)	1973(1)	1979(4)	1972(2)
2	10	0.86	1992(3)	1999(2)	2005(4)	1998(2)

transition begin or, when does the next break occur? Nevertheless, even in this application the centres of transitions and the break-points match each other quite well. The centre of the first shift in the mean is eleven months apart from the first estimated break, and the difference between the latter ones is one year.

7 Conclusions

In this paper we have modified a linear autoregressive model to describe situations in which the data-generating process is affected by outside influences that cannot be easily observed and are being proxied by time. This is done by making the intercept of the process a flexible function of time. The QuickNet process of White (2006) is employed (under the name QuickShift) to select the form of the flexible intercept. This technique of modelling smooth change is different in nature from filtering in that the autoregressive structure of the process is determined simultaneously with the smooth changes and not conditionally on the results of filtering.

The empirical examples show how the technique works in practice. Inflation is typically a phenomenon that is affected by factors that are difficult to quantify, such that the policies

Figure 3: US ex-post real interest rate

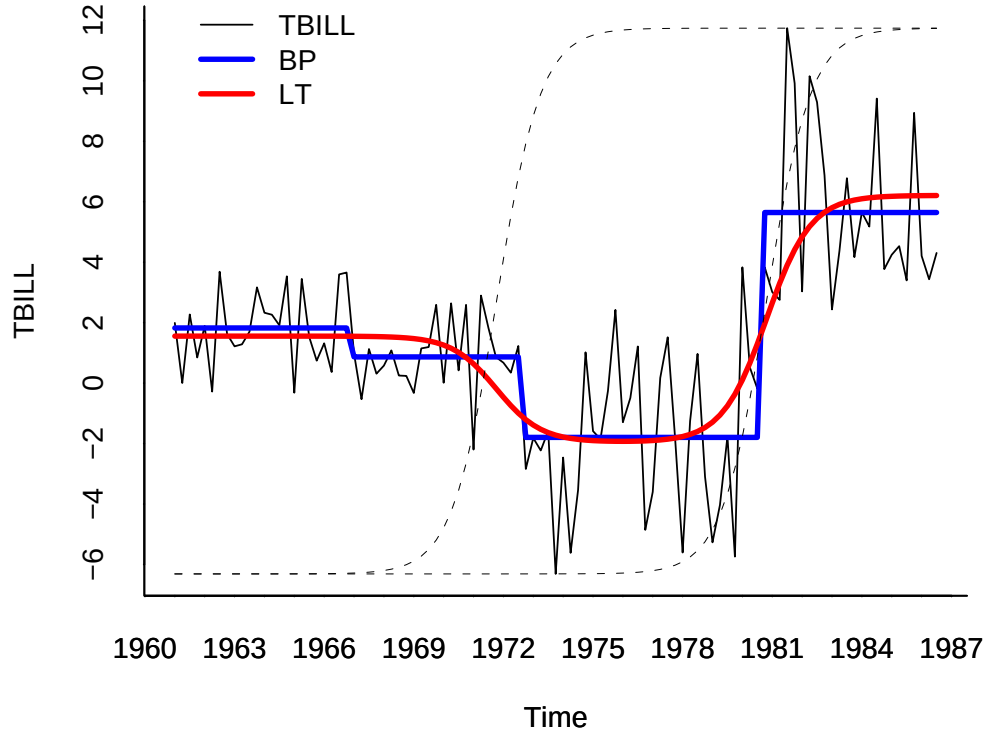
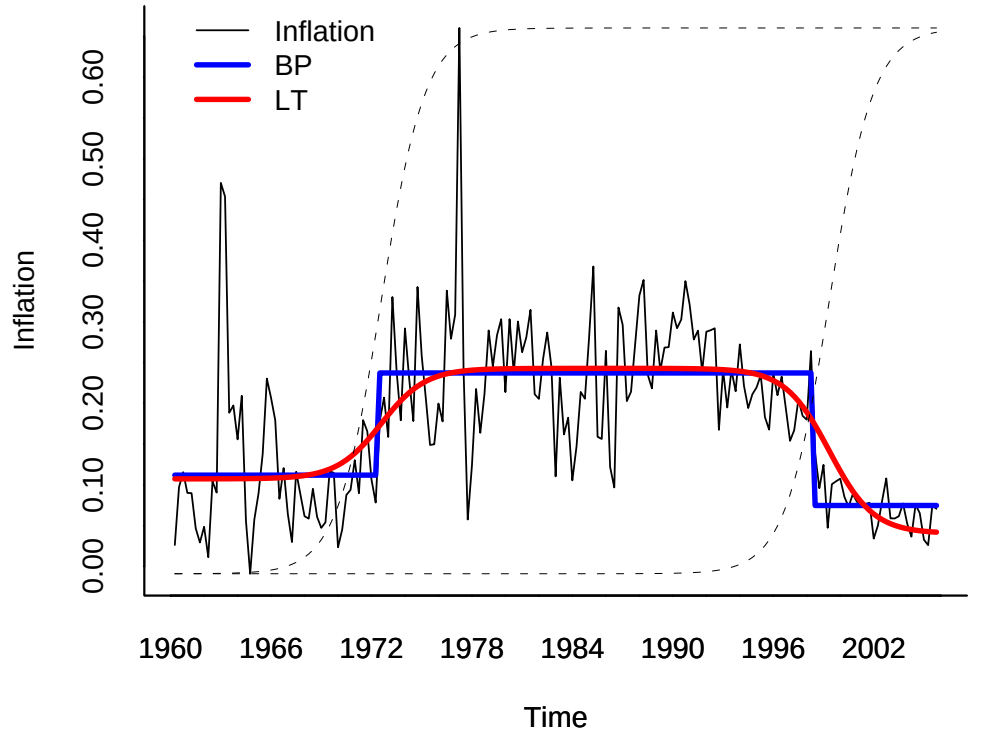


Figure 4: Colombian quarterly inflation rate



of the central bank or institutional changes. The same is true for interest rates. The idea that macroeconomic time series such as inflation and interest rates contain breaks has been quite popular in econometric modelling, where the assumption of breaks has been used to account for external influences such as changes in institutions. However, although institutional changes in some situations can be abrupt, their effects on the series under consideration may be distributed over a number of periods. The present applications show how it is possible to obtain a more nuanced picture of deterministic changes in the series by assuming that these changes can be smooth instead of just occurring abruptly at a given moment of time.

References

Bai, J. and Perron, P.: 1998, Estimating and testing linear model with multiple structural changes, *Econometrica* **66**, 47–78.

Bai, J. and Perron, P.: 2003, Computation and analysis of multiple structural change models, *Journal of Applied Econometrics* **18**, 1–22.

Baxter, M. and King, R. G.: 1999, Measuring business cycles: approximate band-pass filters for economic time series, *Review of Economics and Statistics* **81**, 575–593.

Bierens, H. J.: 1990, A consistent conditional moment test of functional form, *Econometrica* **58**, 1443–1458.

Bierens, H. J.: 1997, Testing the unit root with drift hypothesis against nonlinear trend stationarity, with an application to us price level and interest rate, *Journal of Econometrics* **81**, 29–64.

Bierens, H. J.: 2000, Nonparametric nonlinear cotrending analysis, with an application to interest and inflation in the united states, *Journal of Business and Economic Statistics* **18**, 323–337.

Brooks, S. P. and Morgan, B.: 1995, Optimization using simulated annealing, *The Statistician* **44**, 241–257.

Canova, F.: 1998, Detrending and business cycle facts, *Journal of Monetary Economics* **41**, 475–512.

Cybenko, G.: 1989, Approximation by superposition of sigmoidal functions, *Mathematics of Control, Signals, and Systems* **2**, 303–314.

Dickey, D. A. and Fuller, W. A.: 1979, Distribution of the estimators for autoregressive time series with a unit root, *Journal of the American Statistical Association* **74**, 427 – 431.

Eitrheim, Ø. and Teräsvirta, T.: 1996, Testing the adequacy of smooth transition autoregressive models, *Journal of Econometrics* **74**, 59–75.

Garcia, R. and Perron, P.: 1996, An analysis of the real interest rate under regime shifts, *Review of Economics and Statistics* **78**, 111–125.

Goffe, W., Ferrier, G. D. and Rogers, J.: 1994, Global optimization of statistical functions with simulated annealing, *Journal of Econometrics* **60**, 65–99.

González, A., Hubrich, K. and Teräsvirta, T.: 2006, Inflation dynamics in the presence of structural change, *Work in progress* .

Granger, C. W. J. and Teräsvirta, T.: 1993, *Modelling nonlinear economic relationships*, Oxford University Press, Oxford.

Hornik, K., Stinchcombe, M. and White, H.: 1989, Multi-layer feedforward networks are universal approximators, *Neural Networks* **2**, 359–366.

Jansen, E. S. and Teräsvirta, T.: 1996, Testing parameter constancy and super exogeneity in econometric equations, *Oxford Bulletin in Economics and Statistics* **58**, 735–763.

Lee, T.-H., White, H. and Granger, C. W. J.: 1993, Testing for neglected nonlinearity in time series models: A comparison of neural network methods and alternative tests, *Journal of Econometrics* **56**, 269–290.

Lin, C.-F. J. and Teräsvirta, T.: 1994, Testing the constancy of regression parameters against continuous structural change, *Journal of Econometrics* **62**, 211–228.

Massmann, M., Mitchel, J. and Weale, M.: 2003, Business cycles and turning points: a survey of statistical techniques, *National Institute Economic Review* **183**, 90–106.

Morley, J. C.: 2000, A state-space approach to calculating the Beveridge-Nelson decomposition, *Economic Letters* **75**, 173–127.

Morley, J. C., Nelson, C. R. and Zivot, E.: 2003, Why are Beveridge-Nelson and unobserved-components decompositions of the GDP so different?, *Review of Economics and Statistics* **85**, 235–243.

Racine, J.: 1997, Feasible cross-validatory model selection for general stationary processes, *Journal of Applied Econometrics* **12**, 169–179.

Racine, J.: 2000, Consistent cross-validatory model selection for dependent data: *hv*-block cross-validation, *Journal of Econometrics* **99**, 39–61.

Stinchcombe, M. and White, H.: 1998, Consistent specification testing with nuisance parameters present only under the alternative, *Econometric Theory* **14**, 295–325.

Teräsvirta, T.: 1998, Modeling economic relationships with smooth transition regressions, *in* A. Ullah and D. E. A. Giles (eds), *Handbook of Applied Economic Statistics*, Dekker, New York, pp. 507–552.

van Dijk, D., Teräsvirta, T. and Franses, P. H.: 2002, Smooth transition autoregressive models - a survey of recent developments, *Econometric Reviews* **21**, 1–47.

White, H.: 2006, Approximate nonlinear forecasting methods, *in* G. Elliott, C. W. J. Granger and A. Timmermann (eds), *Handbook of Economic Forecasting*, Vol. 1, Elsevier, Amsterdam, pp. 459–512.

Zeileis, A. and Kleiber, C.: 2005, Validating multiple structural change models-a case study, *Journal of Applied Econometrics* **20**, 685–690.