Colombian economic growth under Markov switching regimes with endogenous transition probabilities♣

Martha Misas
mmisasar@banrep.gov.co
Senior Researcher
Unidad de Investigaciones
Banco de la República, Colombia

Maria Teresa Ramirez
mramirgi@banrep.gov.co
Senior Researcher
Unidad de Investigaciones
Banco de la República, Colombia

ABSTRACT

We modelled the Colombian long-run per capita growth under Markov switching regimes with time-varying transition probabilities (TVTP) to explain regime changes in the economic growth. We found evidence of nonlinearity in the per capita economic growth, and identified two different levels in the data associated with depression and sustainable growth regimes. The hypothesis of fixed transition probabilities (FTP) is rejected in favour of the time-varying transition probabilities. Then, TVTP model gives more information than the FTP model because the probabilities have changed significantly during the period under analysis and the explanatory variables are very informative in dating the evolution of the state of the economy, especially those associated with external shocks. In particular, the probability of remaining in the sustainable growth regime increases with a rise in terms of trade and decreases with a rise in government expenditures. Increases in government expenditures and terms of trade reduce the probability of being in the depression state while an increase in capital outflows raises the probability.

JEL classification: O40, C22, E32, N16

Keywords: Markov endogenous switching regime model, time-varying transition probabilities, economic growth, Colombia

♣We are grateful to Juan Carlos Parra for excellent research assistance. We also thank Andrés Gonzales, Munir Jalil and Yanneth Rocio Betancourt for helpful discussions and Gretchen C. Weinbach, whose original code in Matlab was used to cross check our code The views expressed are those of the authors and not of the Central Bank of Colombia or its Board of Directors. The usual disclaimer applies to any remaining errors or omissions.

1. Introduction

The Markov switching regime model (MSRM) and its extensions has become extensively used to study nonlinearities, especially in macroeconomics and economic growth.[1] As Hamilton and Raj (2002) point out, the purpose of the regime switching model is to capture the asymmetry presented in the business cycle. In this context, the transition from one state of the cycle to another is modelled as a regime switch, and the probability of changing regime is inferred from the data. In the Hamilton (1989)'s original model, the transition probabilities were constants.[2] However, constant or fixed transition probabilities are too restrictive to explain the behaviour of economic growth since economic variables are not allowed to affect transitional probabilities.

An extension of Hamilton (1989) allows time-varying transition probabilities.[3] As explained by Filardo (1994) and Diebold et al. (1999), the Markov switching model with time-varying transition probability (TVTP) has the advantage over the fixed transition probabilities (FTP) in terms of flexibility. It can recognize systematic changes in the transition probabilities before and after turnings points, capture more complex temporal persistence and allow expected duration to vary across time. In this context, economic fundamentals and policy shocks can influence the regime transition probabilities.[4]

The purpose of this paper is to model Colombian long-run per capita economic growth using a Markov switching regime model with time-varying transition probabilities, in order to determine the effect of some economic variables over the transition probabilities to explain regime changes in Colombian growth. To this end, we allow probabilities to be affected by policy variables, and analyse the asymmetric influence of these variables on the different growth regimes.[5] This paper is an extension of Misas and Ramírez (2007), who used a first-order MSRM with fixed transition probabilities to study Colombian's long run economic growth[6]. They found evidence of nonlinearity in the annual rate of growth and identified two different states depression and sustainable growth.[7]

Our main results can be summarized as follows: the hypothesis of fixed probabilities is rejected in favour of the time-varying transition probabilities. The TVTP model is superior to the FTP model since the probabilities have changed significantly during the period under analysis and the explanatory variables are very informative in dating the evolution of the state of the economy. The probability of remaining in the sustainable growth regime increases with a rise in terms of trade and decreases with a rise in government expenditures. Increases in government expenditures and terms of trade decrease the probability of being in the depression state, while an increase in capital outflows raises such probability.

The paper is organized as follows. Section 2 describes the econometric model, section 3 presents the data and some stylized facts of Colombian long-run per capita economic growth. Section 4 reports and discusses the estimation results, first those from the model with fixed transition probabilities and then those from the model with time-varying transition probabilities. Section 5 concludes.

2. The Markov switching regime model with time-varying transition probabilities

We begin by applying the basic Hamilton (1989) model to the Colombian per capita economic growth. Let y_t be the real per capita GDP annual rate of growth and s_t an unobserved discrete variable that represents the state or regime the economy is in; we assume two states: 0 = depression and 1 = sustainable growth, such that:[8]

$$y_t = \phi_{s_t} X_t^{'} + \varepsilon_t \qquad s_t \in \{0,1\} \qquad (1)$$

$$\varepsilon_t \sim N\left(0, \sigma^2_{s_t}\right) \qquad (2)$$

$$\sigma^2_{s_t} = \sigma^2_0 + \sigma^2_1 s_t, \quad \sigma^2_0 > 0 \qquad (3)$$

$$\phi_{s_t} = \phi_0 + \phi_1 s_t, \qquad (4)$$

$$\begin{array}{ll} P\left[s_t = 0 \mid s_{t-1} = 0\right] = p & P\left[s_t = 1 \mid s_{t-1} = 1\right] = q \\ P\left[s_t = 1 \mid s_{t-1} = 0\right] = 1 - p & P\left[s_t = 0 \mid s_{t-1} = 1\right] = 1 - q \end{array} \qquad (5)$$

From equation (1), the growth rate (y_t) depends on X_t, which includes lags of y_t and an *iid* random variable (ε_t), which follows a normal distribution with zero mean and $\sigma^2_{s_t}$ state-dependent variance (equations 2 and 3). Equation 4 describes the behaviour of the parameter ϕ, which is also state-dependent. In equation 5 we specify that the switching of regimes follows a first-order Markov chain. Probabilities are noted by *p* and *q*, where *p* is the probability of remaining in state 0 at *t,* given that the economy is in regime 0 at *t-1*, and *q* is the probability of staying in regime 1 at *t,* given that the economy is in state 1 at *t-1*; *1-p* and *1-q* are the transition probabilities for switching from one regime to the other.

In the basic Hamilton (1989) model, transition probabilities are assumed to be fixed; in that case, they are very restrictive in explaining changes in regimes. For this reason, we allow transition probabilities to depend on some macroeconomic variables in order to explain the probability of switching from one regime to another. We then analyse the results to determine whether changes in the economic variables can cause changes of regimes.

Following Diebold et al. (1999), we endogenized probabilities of changes of regime by incorporating economic variables as their determinants. Then, equation (5) becomes:

$$\begin{array}{ll} P\left[s_t = 0 \mid s_{t-1} = 0\right] = p(z_{t-1}) & P\left[s_t = 1 \mid s_{t-1} = 1\right] = q(z_{t-1}) \\ P\left[s_t = 1 \mid s_{t-1} = 0\right] = 1 - p(z_{t-1}) & P\left[s_t = 0 \mid s_{t-1} = 1\right] = 1 - q(z_{t-1}) \end{array} \qquad (6)$$

where z_{t-1} is a set of information variables. The transition probabilities are modelled as a logistic functional form such as (7):

$$
\begin{array}{cc}
 & \text{Time (t)} \\
 & \begin{array}{cc} \qquad\qquad\text{State 0} \qquad\qquad & \qquad\qquad \text{State 1} \qquad\qquad \end{array} \\
\begin{array}{c} \text{State 0} \\ \\ \\ \text{Time (t-1)} \\ \\ \\ \text{State 1} \end{array} &
\left[\begin{array}{c:c}
\begin{array}{c} P_t^{00} \\ P\left(s_t = 0 \middle| s_{t-1} = 0,\ Z_{t-1};\ \beta_0\right) \\ \dfrac{\exp\left(Z_{t-1}^{'}\beta_0\right)}{1+\exp\left(Z_{t-1}^{'}\beta_0\right)} \end{array} &
\begin{array}{c} P_t^{01} = \left(1 - P_t^{00}\right) \\ P\left(s_t = 1 \middle| s_{t-1} = 0,\ Z_{t-1};\ \beta_0\right) \\ 1 - \dfrac{\exp\left(Z_{t-1}^{'}\beta_0\right)}{1+\exp\left(Z_{t-1}^{'}\beta_0\right)} \end{array} \\
\hdashline
\begin{array}{c} P_t^{10} = \left(1 - P_t^{11}\right) \\ P\left(s_t = 0 \middle| s_{t-1} = 1,\ Z_{t-1};\ \beta_1\right) \\ 1 - \dfrac{\exp\left(Z_{t-1}^{'}\beta_1\right)}{1+\exp\left(Z_{t-1}^{'}\beta_1\right)} \end{array} &
\begin{array}{c} P_t^{11} \\ P\left(s_t = 1 \middle| s_{t-1} = 1,\ Z_{t-1};\ \beta_1\right) \\ \dfrac{\exp\left(Z_{t-1}^{'}\beta_1\right)}{1+\exp\left(Z_{t-1}^{'}\beta_1\right)} \end{array}
\end{array}\right]
\end{array}
\qquad (7)
$$

Note: $Z_{t-1} = \left(1, Z_{1t-1}, Z_{2t-1}, \cdots, Z_{(k-1)t-1}\right)^{'}$ y $\beta_{\mathrm{i}} = \left(\beta_{i0}, \beta_{i1}, \cdots, \beta_{i(k-1)}\right)^{'}$

To estimate this regime switching model, we must specify the complete data likelihood function. Following Diebold et al. (1999), let y_t be the sample path of a time series conditional upon s_t as follows:

$$\left(y_t \middle| s_t = i\,; \alpha_{\mathrm{i}}\right) \overset{iid}{\sim} N\left(\mu_i, \sigma_i^2\right) \qquad \text{where} \quad \alpha_{\mathrm{i}} = \left(\mu_i, \sigma_i^2\right)^{'} \ i = 0{,}1 \qquad (8)$$

Thus, the conditional density of y_t is:

$$f\left(y_t \middle| \mathrm{s}_t = i\,; \alpha_{\mathrm{i}}\right) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(\frac{-\left(y_t - \mu_i\right)^2}{2\sigma_i^2}\right) \qquad i = 0{,}1 \qquad (9)$$

In the likelihood function, a quantity of particular interest is $P(s_1)$, which denotes $P(S_1 = s_1)$ or the unknown initial state of the system. Regarding z_t, there are two cases to consider, stationary and nonstationary. When conditioning variables are stationary, $P(s_1)$ is simply the long-run probability of $S_1 = s_1$, which in turn is determined by β. However, when the conditioning variables are nonstationary, the long-run probability does not exist and $P(S_1 = s_1)$ must be treated as an additional parameter that needs to be estimated. Diebold et al. (1999) show that $P(S_1 = 1)$, which denotes (ρ) is all that is needed to construct the first likelihood term.

Let $\theta = (\alpha', \beta', \rho)'$ be the vector of all model parameters. The complete data likelihood function for a sample of size T can be expressed as:

(10)

$$
\begin{aligned}
f\left(\underset{\leftarrow}{y_T}, \underset{\leftarrow}{\mathrm{s}_\mathrm{T}} \middle| \underset{\leftarrow}{z_T}; \theta\right) &= f\left(y_1, \mathrm{s}_1 \middle| \underset{\leftarrow}{z_T}; \theta\right) \prod_{t=2}^{T} f\left(y_t, s_t \middle| \underset{\leftarrow}{y_{t-1}}, \underset{\leftarrow}{s_{t-1}}, \underset{\leftarrow}{z_T}; \theta\right) \\
&= f\left(y_1 \middle| s_1, \underset{\leftarrow}{z_T}; \theta\right) P(s_1) \prod_{t=2}^{T} f\left(y_t \middle| s_t, \underset{\leftarrow}{y_{t-1}}, \underset{\leftarrow}{s_{t-1}}, \underset{\leftarrow}{z_T}; \theta\right) \mathrm{X}\, P\left(s_t \middle| \underset{\leftarrow}{y_{t-1}}, \underset{\leftarrow}{s_{t-1}}, \underset{\leftarrow}{z_t}; \theta\right)
\end{aligned}
$$

where f denotes any density and $(\leftarrow)$ denotes past history of the variable from t=1 to the variable subscript. Taking logs, the expression given above can be written in terms of indicator functions for the states, $I(s_i)$. Thus, the complete data log likelihood function becomes:

(11)

$$
\begin{aligned}
\log\ f\left(\underset{\leftarrow}{y_T}, \underset{\leftarrow}{\mathrm{s}_\mathrm{T}} \middle| \underset{\leftarrow}{z_T}; \theta\right) &= I(s_1 = 1)\left[\log f\left(y_1 \middle| \mathrm{s}_1 = 1; \alpha_1\right) + \log \rho\right] \\
&\quad + I(s_1 = 0)\left[\log f\left(y_1 \middle| \mathrm{s}_1 = 0; \alpha_0\right) + \log(1-\rho)\right] \\
&\quad + \sum_{\mathrm{t}=2}^{\mathrm{T}} \left\{
\begin{array}{l}
I(s_t = 1) \log f\left(y_t \middle| s_t = 1; \alpha_1\right) \\
+ I(s_t = 0) \log f\left(y_t \middle| s_t = 0; \alpha_0\right) \\
+ I(s_t = 1, s_{t-1} = 1) \log\left(p_t^{11}\right) \\
+ I(s_t = 0, s_{t-1} = 1) \log\left(1 - p_t^{11}\right) \\
+ I(s_t = 1, s_{t-1} = 0) \log\left(1 - p_t^{00}\right) \\
+ I(s_t = 0, s_{t-1} = 0) \log\left(p_t^{00}\right)
\end{array}
\right\}
\end{aligned}
$$

The complete data log likelihood cannot be constructed in practice because it is not observed. However, the incomplete data log likelihood function can be obtained by summing over all possible state sequences, as in equation (12), and then maximizing with respect to the vector of parameters θ.

$$\log f\left(\underset{\leftarrow}{y_T}\,\middle|\,\underset{\leftarrow}{z_T};\theta\right)=\log\left(\sum_{s_1=0}^{1}\sum_{s_2=0}^{1}\cdots\sum_{s_T=0}^{1} f\left(\underset{\leftarrow}{y_T},\underset{\leftarrow}{s_T}\,\middle|\,\underset{\leftarrow}{z_T};\theta\right)\right) \qquad (12)$$

As mentioned by Filardo (1994) and Diebold et al. (1999), in the TVTP model the vector of parameters θ, which includes the mean and variances of each state $(\mu_i,\sigma_i^2)\forall i=0,1$, the transition probabilities (P_t^{00},P_t^{11}), their determinants (β') and the initial conditions (ρ) are jointly estimated with Maximum Likelihood (ML) methods.[9]

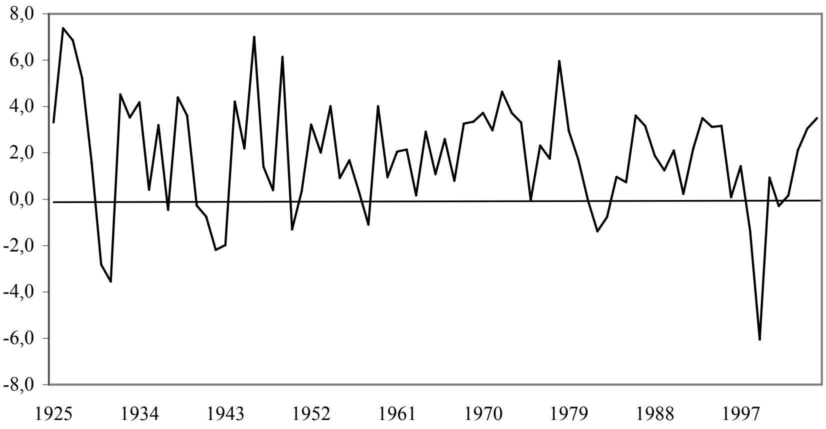
In this particular case, we used the Expectation Maximization (EM) algorithm, which is a very general iterative procedure for maximising the incomplete data log likelihood when some of the random variables involved are not observed. The EM algorithm proceeds, from an analytical point of view, in two stages. In the first, expectations of the smoothed transition probabilities are computed, conditional upon the parameters. In the second, the parameters are updated, conditional upon those smoothed transition probabilities. The algorithm iterates until the maximisation is reached.

3. Data

The above models are estimated with annual data of the real per capita GDP growth for the period 1925-2005 as a proxy of y_t (graph 1). For information variables in z_t, we chose those that we consider to have been the main causes of variation in Colombian per capita economic growth during the twentieth century.[10] Some variables reflect external shocks, such as terms of trade, international coffee prices, capital inflows and the USA real per capita GDP growth, and some comprise fiscal and monetary shocks. Graph 2 presents the evolution of these variables.

Graph 1

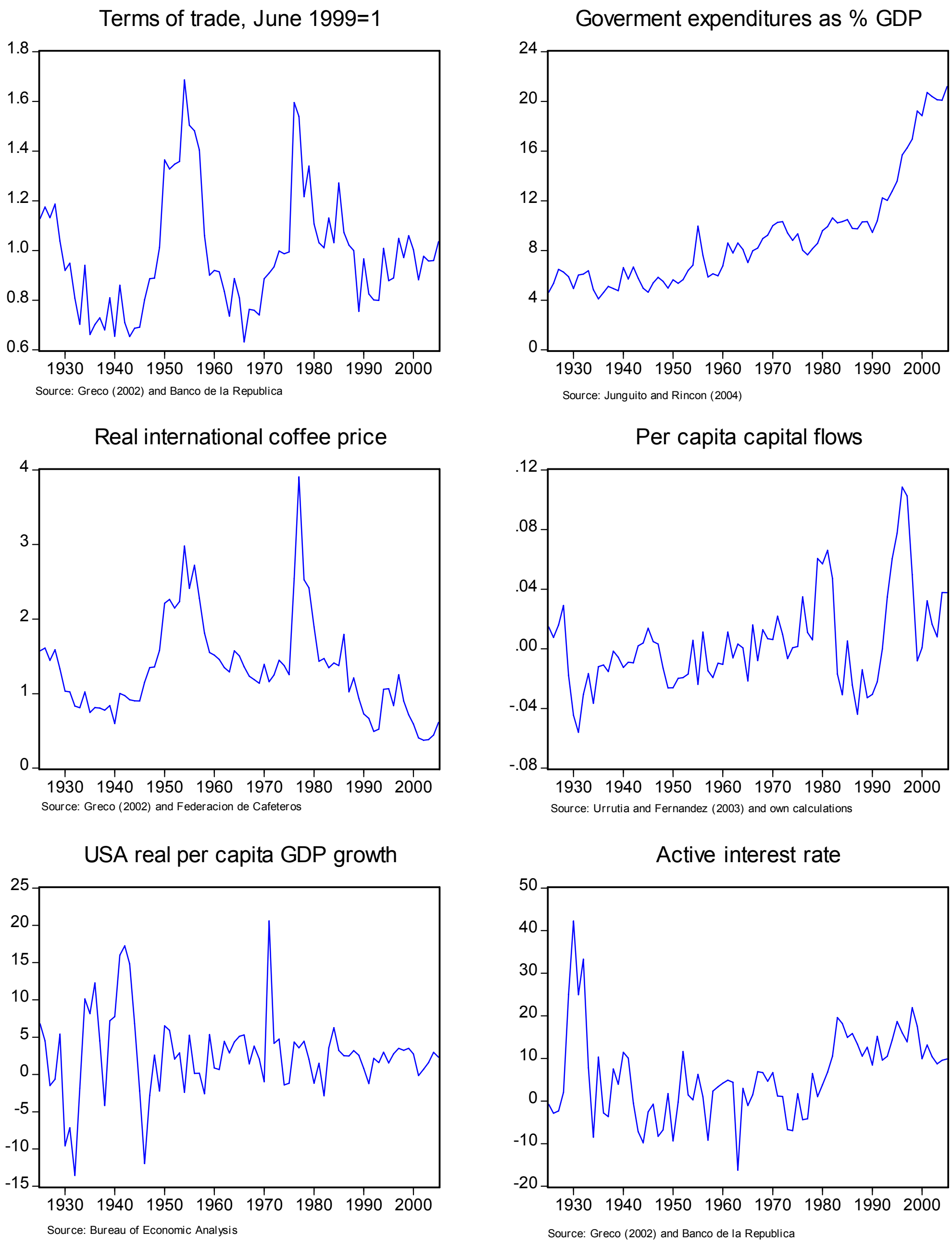
Colombian real per capita GDP growth (%): 1925-2005



Source: GRECO and DANE

Colombian real per capita GDP grew, on average 1.9 percent per year between 1925 and 2005. As mentioned in Misas and Ramírez (2007), in general, the Colombian economy has stayed on a path of sustainable growth. In particular, positive per capita growth has characterized the Colombian economy for long periods, the longest being between 1959 and 1974. However, there have been fluctuations throughout the period. In fact, between 1925 and 2005 the economy has suffered six main slowdowns. The first one, between 1930 and 1931, reflects the worldwide Great Depression. The second occurred between 1940 and 1943 as a consequence of World War II. The economy again registered negative per capita rates of growth in 1950 and 1951 and between 1957 and 1958. With the decline of international coffee prices at the end of the seventies and the Latin American Debt Crisis, beginning in 1982, the Colombian economy fell by 1.4 per cent in 1982 and 0.8 percent in 1983. Nevertheless, the worst contraction of the twentieth century occurred between 1998 and 1999, as a result of the international financial crisis and the macroeconomic imbalances caused by excessive aggregate demand in previous years.[11] In fact, in 1999, the economy declined in per capita terms by more than 6 percent.

Graph 2



As Misas and Ramírez (2007) shows the causes of fluctuations are principally based on external shocks that influenced the evolution of the terms of trade, capital inflows and international coffee prices, among other factors. As mentioned before, the purpose of the present paper is to determine precisely the effect of these and other economic variables over the transition probabilities in order to explain regime changes in Colombian economic growth.

4. Results

4.1 Fixed transition probabilities (FTP)

We first present the results from the fixed transition probabilities model[12] in order to compare these results with those from the time-varying transition probabilities estimation.

In the estimation, we included in X_t an intercept and the first four lags of the dependent variable y_t, and a random variable ε_t with a state-dependent variance. However, the lags of the dependent variable were not statistically significant. We then estimated the equation including only the intercept in X_t. The results of this specification show that the random variable ε_t is a state-dependent variance.

A first-order two-state Markov switching model was estimated for the Colombian per capita economic growth. In table 1 the maximum likelihood estimates of parameters are shown, which are significant at 5 per cent. The results indicate that the two different levels presented in the data, μ_0 and μ_1, are statistically different and the ε_t process is a state-dependent variance. The average annual per capita growth is -1.09 percent in regime 0 (depression) and 2.52 percent in regime 1 (sustainable growth). Furthermore, the probability ($p=P_{11}$) of staying in a depression at time (t), given that the economy is in the same state at time $(t-1)$, is 0.60. The probability ($q=P_{22}$) of being in sustainable growth in time (t), given that the economy was in the sustainable growth path at time $(t-1)$ is large, 0.92 greater than ($p=P_{11}$). These high probabilities indicate that if the economy is in either sustainable growth or depression, it is likely to remain in such regime. In addition, the probability of switching from a depression state to sustainable growth (*1-p=P_{12}*) is almost 0.4, while the probability of changing from sustainable growth to depression (*1-q=P_{21}*) is close to 0.09, which indicates change from depression to sustainable growth more likely than change from sustainable growth to depression.

Table 1
Maximum likelihood estimation of the parameters
and asymptotic standard errors

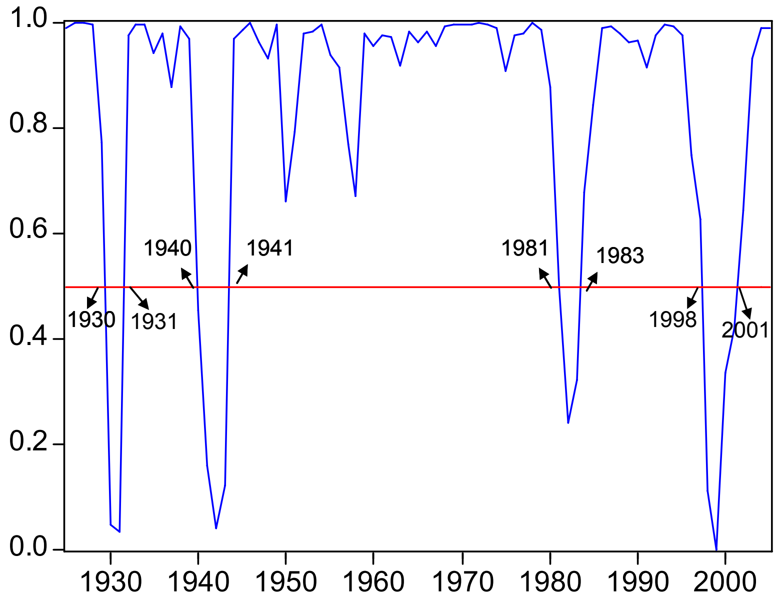
Parameters $\{\mu_0, \mu_1, \sigma_0^2, \sigma_1^2, p, q\}$	Estimation	Standard errors
μ_0	-1.0855	0.7065
μ_1	2.5178	0.3356
σ_0^2	4.1955	2.1292
σ_1^2	3.6945	0.7652
$p = P_{11}$	0.604	0.1723
$q = P_{22}$	0.9155	0.0575
$P(S_1 = 1 \mid y_1, \cdots, y_T; \hat{\theta})$=0.9906 Objective Function: -200.41		

As in Misas and Ramírez (2007), we found that the average length of sustainable growth is twelve years, whereas the expected duration of a depression regime is approximately three years[13].

Graph 3 plots the probability of being in sustainable growth at each date in the sample; i.e, it depicts the evolution of the smoothed probabilities of state 1. The inference is based on the full sample and the estimated maximum likelihood parameters. The years in which the economy switched from each regime, based on $P(s_t = 1 \mid y_1, \cdots, y_T; \hat{\Theta}) \leq 0.5$, are shown in the graph.

Graph 3

Probabilities of being in sustainable growth (state 1) FTP



According to the graph, the economy stays in sustainable growth for long periods, and the switching between regimes is sudden, deep and sporadic. As we can see, the graph indicates four major changes from sustainable growth to depression in the sample, when major shocks, especially external, occurred.

Table 2 presents some specification tests proposed by Hamilton (1996) in order to verify the performance of the model. First, the White autocorrelation test[14] suggests no evidence of autocorrelation. Second, the White specification test[15] indicates that the Markov model not can be rejected against the alternative that there are no changes in regime. Therefore, evidence of nonlinearity in the Colombian economic growth is found. LM tests confirm the results of no autocorrelation. Similar results are also obtained when we examine each regime separately, and the LM test on ARCH effects shows that there is no indication of the presence of such effects. Summing up, the tests suggest that there is no evidence of model misspecification.

Table 2

Specification tests

White autocorrelation test $\chi^2(4)$	5.452 *P-Value* 0.243
White Markov specification test $\chi^2(4)$	4.748 *P-Value* 0.314
LM test on autocorrelation in state 0, $\chi^2(1)$	3.959 *P-Value* 0.046
LM test on autocorrelation in state 1, $\chi^2(1)$	0.481 *P-Value* 0.488
LM test on autocorrelation across states, $\chi^2(1)$	0.663 *P-Value* 0.415
LM test on ARCH effects, $\chi^2(1)$	1.503 *P-Value* 0.220

4.2 Time-varying transition probabilities (TVTP)

We estimate the model under transitional endogenous probabilities, allowing a set of economic variables to explain the evolution of such probabilities. As mentioned above, from an initial set of possible explanatory variables we include the real international price of coffee, terms of trade, capital inflows, USA per capita real GDP growth, government expenditures as a percentage of GDP and the real interest rate. With this information we established different models to select the one that presents the smooth transition probabilities consistent with the economic history of the country.[16]

Table 3 presents the results of the selected model, which includes terms of trade, per capita capital inflows and government expenditure as percentage of GDP. As expected, the external shocks have significantly affected the evolution of GDP growth. The model was selected based on the gradients and on a likelihood test that compares the Hamilton model, with fixed transition probabilities, with the model with time-varying transition probabilities.[17]

The initial values associated with $\mu_0, \mu_1, \sigma_0^2, \sigma_1^2$ and ρ are obtained through the Hamilton model. It is worth mentioning, that the algorithm is robust to different initial values of these parameters. In addition, the initial values of the coefficients in the vector β, equation 7, are taken from the OLS regressions. In those regressions the dependent variable is the smooth probability vector obtained from the Hamilton model, as a proxy for the endogenous transition probabilities and explanatory of the selected variables.

The TVTP estimations from table 3 also indicate that two different states of the economy with magnitudes that differ considerably can be identified: a depression regime with a negative μ_0, and a sustainable growth regime with a positive μ_1. Allowing TVTP the average annual per capita economic growth rates in regime 0 and 1 are similar to those from the FTP estimations.

We observe that the explanatory variables of the transition probabilities present, in almost all cases, the sign suggested by economic intuition. In fact, the probability of remaining in the sustainable growth period increases with a rise in terms of trade and decreases with increments in government expenditures. If the economy is in depression, an increase in government expenditures and terms of trade decreases the probability of remaining in this state, while a rise in capital outflows increases this probability.[18]

Following Diebold et al. (1999), we conducted a likelihood test that compares the model of time-varying transition probabilities with the model of fixed probabilities. The first model is the unrestricted one and the second is the restricted one. We obtain a *p-value* of 1.7E-06, which allows us to reject the null hypothesis of constant probabilities in favor of the TVTP model. Therefore, the right model is the one with endogenous transition probabilities.

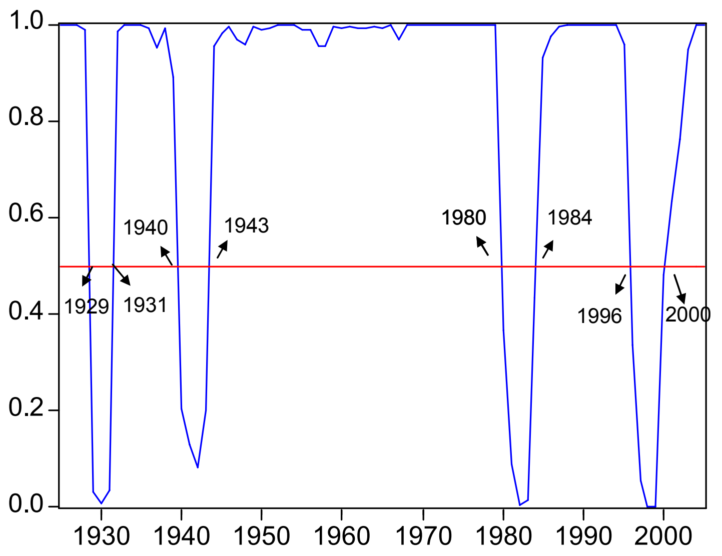
Table 3

Parameter	**Estimation**
μ_0	-0.879
μ_1	2.536
σ_0^2	4.338
σ_1^2	3.749
Regime 0 (depression)	
β_{00} (intercept)	0.175
β_{01} (terms of trade)	3.032
β_{02} (capital flows)	-0.174
β_{03} (government expenditures)	4.676E1
Regime 1 (sustainable growth)	
β_{10} (intercept)	-1.947
β_{11} (terms of trade)	1.337
β_{12} (capital flows)	0.663
β_{13} (government expenditures)	-1.056E2
ρ	0.98
Convergence: 8.16E-9 Objective Function: -185.56	

Graph 4 presents the smoothed TVTP obtained from the model of endogenous probabilities. As we see, the length of both depression and sustainable growth states are longer than in the FTP (graph 3). In particular, the probabilities of being in sustainable growth decrease earlier in time in the TVTP model than in the FTP model. It also captures the date of the turning points better. Therefore, the variables included in the set of explanatory variables, z_t, are informative in dating the state of the economy.

Graph 4

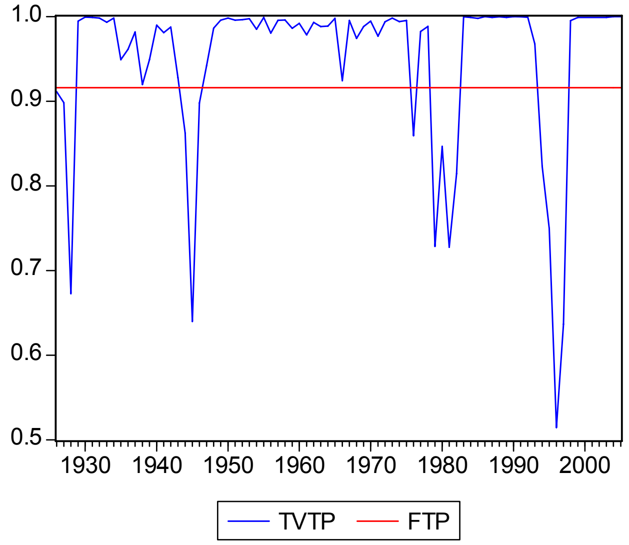
Probabilities of being in sustainable growth (state 1) TVTP



Finally, graph 5 shows the time-varying transition probabilities against the fixed probabilities from Hamilton's model. As we can see, terms of trade, capital inflows and government expenditures are important determinants of transition probabilities, and of the switching between regimes. Clearly, the TVTP model gives more information than the FTP model since the probabilities have changed significantly during the period under analysis.

Graph 5

Long-Run Probability of being in a sustainable Growth



5. Conclusions

In this paper, we modelled the Colombian long-run per capita economic growth using a Markov switching regime model with time-varying transition probabilities to explain regime changes in Colombian growth. In particular, we allowed probabilities to be affected by policy variables to analyse the asymmetric influence of these variables on the different growth states.

We found that the hypothesis of fixed probabilities can be rejected in favor of the time-varying transition probabilities, which means that the adequate model is the one with endogenous transition probabilities. Then, the TVTP model is superior to the FTP model since the probabilities have changed considerably during the period under analysis and the explanatory variables are very informative in dating the evolution of the state of the economy. In particular, the probability of remaining in the sustainable growth regime increases with a rise in terms of trade and decreases with a rise in government expenditures. Also, if the economy is in the depression state, an increase in government expenditures and terms of trade decreases the probability of remaining in this state, while an increase in capital outflows increases this probability.

References

Arango L, Melo L. Expansion and contractions in Brazil, Colombia and Mexico: a view through non-linear models. Journal of Development Economics 2006; 80; 2; 501-517.

Beyaert A, García-Solanes J, Pérez-Castejón, J.J. Uncovered interest parity with switching regimes. Economic Modelling 2007; 24; 2; 189-202.

Buckle R, Haugh D, Thomson P. Markov Switching Models for GDP growth in a Small Open Economy: The New Zealand Experience. Journal of Business Cycle Measurement and Analysis 2004; 1; 227-257.

Chen S-W, Shen C-H. A sneeze in the U.S., a cough in Japan, but pneumonia in Taiwan? An application of the Markov-Switching vector autoregressive model. Economic Modelling 2007; 24; 1; 1-14

Cruz M. A three-regime business cycle model for an emerging economy. Applied Economics Letters 2005; 12; 399-402.

Diebold F, Lee J-H, Weinbach G.C. Regime switching with time-varying transition probabilities. In Business cycles: durations, dynamics and forecasting, Princeton University Press; 1999.

Diebold F, Rudebusch G. Measuring Business Cycles: A modern perspective. In Business cycles: durations, dynamics and forecasting, Princeton University Press; 1999.

Durland J.M, Mc Curdy T. Duration-Dependent Transitions in a Markov Model of U.S. GNP Growth. Journal of Business & Economic Statistics 1994; 12; 3.

Fernandez C, González A. Integración y vulnerabilidad externa en Colombia. Borradores de Economía, Banco de la República 2000; 156, 1-27.

Filardo A. Business-cycle phases and their transitional dynamics. Journal of Business and Economic Statistics 1994; 12; 3; 299-308.

Filardo A, Gordon, S. Business cycle durations. Journal of Econometrics 1998; 85; 99-123.

Frömmer M, MacDonald R, Menkhoff, L. Markov switching regimes in a monetary exchange rate model. Economic Modelling 2005; 22; 3; 485-502.

GRECO. El crecimiento económico Colombiano en el siglo XX, Edited by Banco de la República and Fondo de Cultura Económica, Chapter. 2. Bogotá; 2002; p. 39-58.

Hamilton J. A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. Econometrica 1989; 57; 357-384.

Hamilton J. Specification testing in Markov-switching time-series models. Journal of Econometrics 1996; 70; 127-157.

Hamilton J, Raj B. New directions in business cycle research and financial analysis: Introduction and overview. Empirical Economics 2002; 27; 2; 149-162.

Höppner F, Assenmacher-Wesche K. Non-linear effects of fiscal policy in Germany: A Markov-Switching Approach, unpublished manuscript; 2001.

Junguito R, Rincón H. La política fiscal en el siglo XX en Colombia. Borradores de Economía, Banco de la República 2004; 318.

Kim C-J, Morley J, Piger, J. Nonlinearity and the permanent effects of recessions. Journal of Applied Econometrics 2005; 20; 291-309.

Layton A, Katsuura M. A new turning point signalling system using the Markov switching model with application to Japan, the USA and Australia. Applied Economics 2001; 33; 59-70.

Mills T, Wang P. Estimating the permanent and transitory components of the U.K. Business cycle. Economic Issues 2003; 8; Part.1; 1-13.

Mills T, Wang P. Modelling regime shift behaviour in Asian real interest rates. Economic Modelling 2006; 23; 6; 952-966

Misas M, Ramírez M.T. Depressions in the Colombian economic growth during the XX century: A Markov Switching Regime Model. Applied Economics Letters 2007; 14; 11; 803-808

Ming Chien L, Piger J. Is the Response of Output to Monetary Policy Asymmetric? Evidence from a Regime-Switching Coefficients Model. Journal of Money, Credit and Banking 2005; 37; 5; 865-86.

Ming-Yuan L-L, Hsiou-Wei W, Hisu-Hua R.J. The performance of the Markov-switching model on business cycle identification revisited. Applied Economics Letters 2005; 12; 513-520.

Moolman E.A. Markov switching regime model of the South African business cycle. Economic Modelling 2004; 21; 631-646.

Ocampo J. Historia económica de Colombia, Edited by Fedesarrollo-Siglo Veintiuno. Bogotá; 1987.

Pok-Sank L. A Markov-switching model of GNP growth with duration dependence. International Economic Review; 2004; 45; 175-204.

Posada C. Los ciclos económicos en el siglo XX. Borradores de Economía, Banco de la República 1999; 126; 1-73.

Simpson P, Osborn D, Sensier M. Modelling Business Cycle Movements in the UK economy. Economica 2001; 69; 243-267.

Soto R. Switching regimes, macroeconomic policies and sustained growth. Instituto de Economía, Pontificia Universidad Católica de Chile 2002, unpublished manuscript.

Stanca L. Asymmetries and nonlinearities in Italian macroeconomic fluctuations. Applied Economics 1999; 31; 483-491.

Urrutia M, Fernández C. Política Monetaria Expansiva en épocas de crisis: El caso Colombiano en el siglo XX. ¿No se quiso o no se pudo ser contracíclico?. Revista del Banco de la República 2003; 908; 16-44.

U.S. Department of Commerce, Bureau of Economic Analysis, http://www.bea.gov/national/index.htm#gdp, October 2006.

Notes

[1] See for example: Misas and Ramírez (2007), Beyaert et al. (2007), Chen and Shen (2007), Mills and Wang (2006), Kim et al. (2005), Frömmel et al. (2005), Ming-Yuan et al. (2005), Cruz (2005), Pok-Sank (2004), Buckle et al. (2004), Moolman (2004), Mills and Wang (2003), Soto (2002), Layton and Katsuura (2001), Stanca (1999) and Filardo and Gordon (1998), among others.
[2] See Hamilton (1989).
[3] For other extensions, see Diebold and Rudebusch (1999), Durland and Mc Curdy (1994), Filardo and Gordon (1998), Hamilton and Raj (2002).
[4] For details see Filardo (1994) and Diebold et al. (1999).
[5] For an application of Markov switching regime models with time-varying transition probabilities to study business cycles see, for example, Filardo (1994), Moolman (2004), Simpson et al. (2001), Höppner and Assenmacher-Wesche (2001), Ming Chien and Piger (2005), Soto (2002), among others.
[6] According to Misas and Ramírez (2007), most studies of Colombian business cycles, with the exception of Arango and Melo (2006), assume that the growth rate follows a linear process; for instance, see Posada (1999), Fernández and Gonzáles (2000) and Urrutia and Fernández, (2003).
[7] Misas and Ramírez (2007) found that transitions between states were sudden and sporadic. In particular, the economy remained in the sustainable regime for most of the period and the turning points from the Markov switching model adequately capture the behaviour of Colombian real output through time.
[8] This notation closely follows Misas and Ramírez (2007)
[9] Diebold et al. (1999) used the EM algorithm.
[10] The review of the Colombian business cycle literature was useful in identifying the variables used in the estimation. See Ocampo (1987), Posada (1999), Fernández and Gonzáles (2000), and Misas and Ramírez (2007), among others.
[11] See Misas and Ramírez (2007).
[12] Misas and Ramírez (2007) estimated the fixed transition probabilities model using the real GDP growth instead of the real per capita GDP growth. As it can be expected the results are very similar.
[13] The larger duration of the sustainable growth regime can be explained by the fact that we defined sustainable growth as times when the economy experienced persistent growth, including periods of booms and very small slowdowns.
[14] The white autocorrelation test verifies the score correlation at time (t) with respect to μ_i and the score of time $(t-1)$ with respect to μ_j with $i, j = 1,2$.
[15] The Markov assumption that $P(s_t = i)$ depends only on whether the state in $(t-1)$ can be tested against two alternative hypotheses: $P(s_t = i)$ depends on several previous states or $P(s_t = i)$ depends on the realization of y_{t-1}. The test verifies if the score with respect to the transition probabilities can be forecasted by its lags or by the score with respect to the average.
[16] The estimation was made with a code developed by the authors in SAS version 9, IML procedure. In addition, Gretchen C. Weinbach provided us with her original code in Matlab, which was used to cross-check our code.
[17] It is important to mention that the likelihood function (equation 11) in our paper presents some issues regarding the indeterminacy of the logarithmic function. This problem was overcome by replacing the indeterminacy by values close to zero or 1.
[18] During almost all the period under analysis, this variable corresponds mainly to capital outflows.