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Testing a DSGE model and its partner database*

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Abstract

There is now an impetus to apply dynamic stochastic general equilibrium models to forecasting. But these models typically rely on purpose-built data, for example on tradable and nontradable sector outputs. How then do we know that the model will forecast well, in advance? We develop an early warning test of the database-model match and apply that to a Colombian model. Our test reveals where the combination should work (consumption) and where not (in investment). The test can be adapted to look at many likely sources of DSGE model failure.

Keywords: Monetary Policy, Sectoral Model, DSGE, Forecast Performance, Kalman Filter

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1 Introduction

When modelling monetary policy in developing countries such as Colombia, a popular strategy is to distinguish between tradable and nontradable sectors. Intuitively tradable sectors are those that export, import, compete with and raise finance directly from foreigners. The nontradable sectors in contrast operate in closed factor, product and capital markets. There are many variants on this theme. For example one popular version divides production into two tradable sectors, one that transforms imported inputs so that they can be traded domestically and one that exports, and one nontradable sector that uses only domestic inputs to produce domestic consumption.

There are two reasons why tradable/nontradable sector models have a long and continuing history developing country monetary policy. First, on an aggregate level, tradable/nontradable sector models produce more plausible implications than models which simplify all production into one tradable sector, such as the monetary approach to the balance of payments. Allowing for a nontradable sector whose prices do not follow world prices breaks the rigid assumption that purchasing power parity ties down all domestic prices always, so that the current account need not adjust as much in order to restore the economy to equilibrium. Instead the exchange rate adjusts more. Second, tradable/nontradable sector models are also motivated by a policy interest in where the burden of adjustment to shocks and to policy actions falls: as the real exchange rate responds more and possibly with overshooting, the tradable sector is typically more vulnerable to shocks than the nontradable sector. For frameworks and references see Sebastian Edward's Real Exchange Rates in Developing Countries (Edwards, 1991) or Corden's survey, The Economics of the Booming Sector and Dutch Disease (Corden, 1984). Levy Yeyati and Sturzenegger (2007) discuss whether current monetary policies in developing countries are shaped by mercantilism, a focus on promoting growth in the tradable sector.

Recently there is a impetus among central banks in developing countries to build dynamic stochastic general equilibrium (DSGE) versions of tradable and nontradable sector models. One example is Benes, Catello Branco, and Vavra (2007). A DSGE model would link the tradable and nontradable sector through an optimizing problems for consumers, producers and financial intermediaries. Well-defined demand and supply functions relate the price and quantity of consumption items whose production are more sensitive to international conditions to those items whose production is predominantly a domestic affair. The path of aggregate consumption over time is consistent with an intertemporal consumption problem. Thus the current account, the trade balance and the capital account are related to savings, returns and a sustainable accumulation of net assets. Demand, supply and wealth effects combine. The clearing of the capital and good markets across both sectors jointly determines the real exchange rate which can be considered as the price of items produced from the nontradable sector relative to the price of tradable sector output. See Obstfeld and Rogoff (1996), and Galí and Monacelli (2005) for well known examples. The literature is surveyed in Lane (2001).

This paper is about what we then see as the most important obstacle to overcome before tradable/nontradable sector monetary policy models can be graduated on to forecasting. The problem is that National Accounts offices do not publish separate real volume data and deflators for theoretically differentiated aggregates, such as for a tradable and a nontradable sector. How then one can build database that reliably supports a sectoral DSGE monetary policy model in practice? And how can one know if that marriage will work, in advance?

Constructing a sectoral database is always feasible and usually undemanding. National Accounts measure the value of nominal output and real output of individual sectors at high levels of disaggregation (often up to six digits). Some combination of this data, index number formulas, other data sources and heroic assumptions will always deliver a database that distinguishes the real and nominal output of theoretically consistent definitions of tradable and nontradable sectors even at a quarterly frequency. The standard recipe is to classify each highly disaggregated sector into an aggregate sector (most typically the goods producing sectors are classified as tradables and services as nontradables) and then calculate real volume and deflators series of the aggregates from their components.

But the ease with which a database can be built does not guarantee that the conjunction of database and model will forecast well. In particular we should worry about the match when there is a large intermediate trade between tradable and nontradable sectors. Intermediate trade across tradable and nontradable sectors represents a likely source of forecast failure both because of data mismeasurement and model misspecification. Data error creeps in because National Accounts authorities find it difficult to quantify the values and especially the volumes of intermediate commerce. For example the uncaptured outsourcing of an inhouse service (an intermediate input) in manufacturing can lead to downward bias in the value-added of a tradable sector and an upward bias in that of a nontradable sector. Model misspecification can also be expected beause intermediate trade is complex. It is in reality a myriad of flows of different types (investment, services and materials) and in different directions. Trying to fully incorporate this intermediate trade into a forecasting model would mean relying heavily on poor data and substantially increasing model size. Therefore even if the theoretical literature recognises the importance of intermediate trade (Basu, 1995), it is usually excluded or drastically simplified in forecasting models.

The numbers are not trivial. We estimate that the intermediate trade from good to services in Colombia was about 15% of GDP and about 17% going the other way[1]. Burstein, Neves, and Rebelo (2003) have highlighted that in particular the domestic distribution sector (a nontradable sector) receives just less than

[1] Calculated as the average for 1990 to 2005, with goods being all sectors with an NIC code less than 38 on a two digit classification, except for utilities.

half of the value of goods destined for consumption for Argentina and the US. In Colombia, we calculate that same margin to be 36% (on average for 1990-2005). And Colombia and Argentina are relatively closed to imports in consumption. So the developing country central banks that are contemplating using sectoral models to forecast need to take this risk of forecast failure from not modelling intermediate trade seriously.

Of course the ultimate test of any forecasting model and data set is how well the marriage serves during at least a few years of service in actual policy analysis. But by then it could be too late. This is because even if an inappropriate combination of a model and a database does create forecast error and could lose precious credibility, we only find that out long after the large sunk costs of making the model have been paid. Once made, forecasting models can only be patched and are never replaced until many years later. A crucial question in model design then is assessing how well the model and its purpose-built database match early on.

In this paper we develop a test of the model-database combination that provides for this need. The test can be applied even before the steady state of the model is built. All one needs to know for the basic test are the price and volume aggregators that are consistent with the theory underlying the model.

The test is based on the post-estimation sample forecast performance of the supply and demand equations in the model, taken and tested in isolation[2]. These equations link the relative prices and quantities across tradable and nontradable sectors. Rather than using the theory of the model to derive data on price and volume series which National Accounts series do not publish, we use more general index numbers. This allows for a residual in these key equations which would then have to be forecasted. If a single equation forecasts poorly, it is likely that the forecasts from the full model for important prices or for volumes (or for both) will have unpredictable residuals. Forecast failure in these equations would therefore reveal problems with the database model combination in a sectoral model. In a nutshell this is the rationale for our test.

There are of course other more generic problems to be expected with any forecasting models and not just DSGE models. For any model, some data will often built partly by assumption, for example the capital stock and margins series. Then these models are often calibrated around a balanced growth steady state which rule out the strong relative price trends we observe in the data (Whelan, 2005). Finally the functional forms used in macroeconomic models are much simpler than what microeconometricians would use to test the same theories. For example macroeconomic models impose homotheticity. If directed, our test could pick these possible errors up too.

The test takes account of many other prosaic adaptations that are made in forecasting, especially with DSGE models.

[2] Johri and Letendre (2007) advocate an insample test of the residuals of first order conditions from DSGE models.

- First the test takes account of shock extrapolation. In an older generation of models, forecasts were improved by adjustments that extrapolated recent equation errors, conferring an automatic immunity against possible shift structural breaks. In DSGE model forecasts, we have shocks which are assigned an exogenous process and then also forecasted. The forecast at policy relevant horizons is sensitive to the parameters defining these shock processes, and they seem to contribute heavily to the goodness of fit of DSGE forecasting models. Of course, providing the DSGE model is well identified, the shocks have a structural interpretation, and then perhaps can be considered part of the economic structure. Leaving aside this controversial issue, what matters for us is that this common practice has the potential to improve the forecast. We should therefore take account of it in a fair test and we do, by allowing for a time-varying parameter in our demand and supply equations, estimated as unobserved components within a state-space model.

- Second it is important that our test of a forecasting model is based on forecast performance and not only goodness of fit. The general argument for this is well established (Dawid, 1984 and West and McCracken, 1998). This matters especially in this class of models because even if there are significant in sample residuals, a forecaster might extrapolate them. Thus a poor goodness of fit may not always imply forecast errors if in sample errors take a predictable shape. Some typical errors in database construction can be expected to lead to constant, and thus treatable, residuals. Our test reveals only when insample residuals cannot be easily forecasted. In addition, goodness of fit weighs less with DSGE models simply because as we have explained the data for these models often has to be purpose-built. When the theory of the model is fully imposed on the creation of the data, the fit is often perfect, but then there is a large risk of that particular theory being wrong. Finally if the objective of the whole exercise is to forecast, it is natural that we should test post sample forecast performance even at an early stage. These are all very convincing reasons for an out of sample test.

- The third advantage of the test is that it also deals with time-varying parameters. Forecasting DSGE models feature time-varying parameters because they cannot capture the full extent of structural change that we see in the data and being more theory based unlike the earlier generation of forecasting models, they have less recourse to arbitrary constants and time trends. See Harrison, Nikolov, Quinn, Ramsay, Scott, and Thomas (2005), page 96 for a justification of time-varying parameters in the context of the Bank of England's DSGE forecasting model and Fernández-Villaverde and Rubio-Ramírez (2007) for a formal analysis. Although this tactic is designed to keep residuals small, it then transforms the problem from one of forecasting residuals to one of forecasting time-varying parameters but with presumably gaining some advantage on the way. The unobserved component also accounts for this.

- Fourth, our test uses a bootstrapped distribution of the RMSE that takes account of parameter estimation uncertainty. Parameter estimation uncertainty is an important source of forecast error. One reason is that estimated constants tend to be poorly determined when shift structural breaks in the data are not built into the model. Parameter estimation uncertainty matters more in DSGE models because when parameter values are more determined by theory than by data, they are more sensitive to the risk that theory is inadequate, for example, poorly identified (Canova and Sala, 2006).

- Fifth the method can easily be extended to test for sensitivity of the database-model match to revisions in the data. We can compare the RMSE and the distributions of parameter estimates also across different vintages of data. Similarly we can test for robustness to the database construction assumptions. These issues really matter when the dataset has to be bent more towards theoretical concepts and away from published numbers.

We apply our test to a particular open economy model for Colombia. Given the significant intermediate trade between tradable and nontradable sectors in Colombia, we steered away from building a model that fully split the tradable and nontradable sectors. Instead we focused on another open economy sectoral model which is especially relevant in a world where production is vertically disintegrated across national boundaries. The model and database we build and test separates the direct import transmission channel in Colombia.

Domestic production divided into two broad sectors. One sector carries out all domestic production, and the other imports and commercialises international products for domestic consumption or for intermediate use as investment goods. Domestic consumption and domestic investiment are our equivalents of the tradable sector. All other sectors have some mixture of tradable and non tradable elements. Domestic production is itself split in two stages. In a first stage, labour, capital and raw materials produce a generic domestic output. Then that output is transformed into different forms of domestic production, including exports. One progenitor of this model could be that of Galí and Monacelli (2005), which is popular in the literature on small open economies.

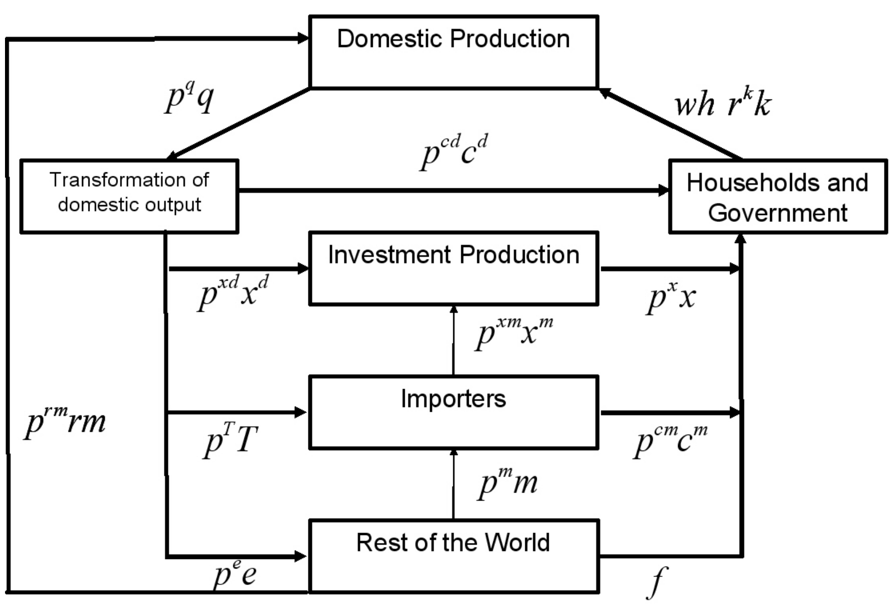
Although this model suits available data better than a full tradable non-tradable split, we show that building a data set that matches all of its theoretical concepts still involves substantial compromise. But after testing this combination of model and database, we find that some important parts of the model database promise to forecast well. This particularly applies to the demand for domestically produced and foreign produced consumption items. As the equations for consumption demand matter directly in forecasting the key variables of consumption and consumer prices, this is very reassuring. However, we find that the modelling of investment could prove to be very problematic. The parts of the model where generic domestic output is split into different forms is also identified as potentially difficult. We show that the database-model combination is robust to a revision in consumption data of the scale that we typically observe in Colombian National Accounts. The dataset is, though, surprisingly sensitive to the particular series for raw material prices that we use. These results are very useful in guiding the construction of a model which seeks to compromise available data with the need to capture key aspects of a developing country.

The paper is as follows. The model is explained in the next section, Section 2. In Section 3 we describe in gory detail how we adapt Colombian National Accounts data to prepare a database for this model. Section 4 motivates our test of the model database match in broad terms and then uses a Monte Carlo simulations to justify the particular design of our test. Test results are reported for five individual demand relationships in Section 5. Section 6 reports results for three supply relations which form part of a system and hence require an adapted version of the test. Section 7 applies the test to explore the robustness of the model-database match to GDP revisions and Section 8 tests for robustness to a key assumption in database construction. Section 9 concludes.

2 The model

Figure 1 on page 7 shows our model diagrammatically.

Figure 1: A model of import transformation



Imports are split into three types: raw materials, investment goods and consumption goods. Imported investment and consumption goods, once across the national frontier, are transformed by a distribution sector into products that can be sold to the final and intermediate consumer. Raw materials may also be transformed but without input from the distribution sector. The idea is that they are more of a standard product and require less marketing and distribution, but their domestic price may differ significantly from the world price when for example the importer absorbs international price fluctuations in his margins. In this way the model distinguishes the pass-through of different types of foreign prices into the domestic economy.

Our presentation of the model is directed by three objectives. First we want to derive all the supply and demand relationships that we can potentially apply our test to and especially those that straddle the classification into different sectors. Second we want to show that these equations fit into a complete DSGE model. Third we want to show that as this model is for forecasting, the theory of the model is at least in part dictated by the need to match and suit available National Accounts data. Details of the derivations that are left our here will be presented in González, Mahadeva, Prada, and Rodríguez (2008).

2.1 The consumer

In what follows, the convention is to denote per capita volumes by lower case and aggregate volumes by upper case. The utility function for a representative member of the economy is:

$$E_t \sum_{t=1}^{\infty} (\beta (1+n))^t \frac{z_t^u}{1-\sigma} \left[c_t - \frac{z_t^h}{1+\eta} A_t \left(h_t TBP_t (1-u_t) \right)^{1+\eta} \right]^{1-\sigma} \tag{1}$$

where c_t is consumption per head, h_t is average hours worked by an employee. Labour supply adjusts for unemployment, u_t, and participation, TBP_t, although we only model the choice of hours worked as endogenous. E_t denotes expectations formed with information available at time t. z_t^u and z_t^h reflect exogenous shifts in utility, with the latter possibly capturing relative technology in leisure. Consistently, A_t is labour-embodied technological progress in output production. σ is the intertemporal elasticity of substitution, and β $(0 < \beta < 1)$ is the discount rate. $\frac{1}{\eta}$ is the Frisch income elasticity of labour supply.

Population grows at the rate, n :

$$N_t = N_0 (1+n)^t .$$

The consumption of that member is a CES aggregate of the consumption of domestically produced $\left(c_t^d\right)$ and foreign produced items (c_t^m),

$$c_t = c\left(c_t^d, c_t^m\right) = \left[\gamma_t^{\frac{1}{\omega}} \left(c_t^d\right)^{\frac{\omega-1}{\omega}} + (1-\gamma_t)^{\frac{1}{\omega}} \left(c_t^m\right)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} . \tag{2}$$

ω is the elasticity of substitution between the two items. If $\omega = 0$, the goods are Leontieff complements, if $\omega = 1$, they are Cobb-Douglas complements, and if $\omega = \infty$, they are perfect substitutes.

The representative agent faces a budget constraint that balances expenditures on consumption $(P_t^c c_t)$, investment ($P_t^x x_t$) and gross purchases of domestic bonds (b_t) against returns on capital $\left(R_t^k \frac{k_{t-1}}{1+n}\right)$, from labour income ($W_t h_t TBP_t (1-u_t)$), securing new loans from abroad ($S_t f_t$), transfers from abroad ($S_t tr_t$), and profits and dividends (π_t). Domestic bond holdings earn net interest of $\frac{b_{t-1}}{1+n}(1+i_{t-1})$, and foreign loans need net repayment of $S_t \frac{f_{t-1}}{1+n}\left(1+i_{t-1}^e\right)$:

$$\begin{aligned} P_t^c c_t + P_t^x x_t + b_t = {} & R_t^k \frac{k_{t-1}}{1+n} + W_t h_t TBP_t (1-u_t) + S_t f_t + S_t tr_t + \pi_t \\ & + \frac{b_{t-1}}{1+n}(1+i_{t-1}) - S_t \frac{f_{t-1}}{1+n}\left(1+i_{t-1}^e\right). \end{aligned} \tag{3}$$

Note that assets are depreciated by population growth as assets are transferred to new members of the population in equal shares. P_t^c is the price of consumption, P_t^x is the price of investment, R_t^k is the return on capital, S_t is the nominal exchange rate, and i_t and i_t^e are the interest rates faced by Colombians on net domestic and net foreign assets respectively.

Capital per individual, k_t, is accumulated from investment per individual, x_t, according to the function:

$$k_t = x_t - \frac{\psi}{2}\left(\frac{x_t(1+n)}{k_{t-1}} - (1+n)(1+g) + (1-\delta)\right)^2 \frac{k_{t-1}}{1+n} + (1-\delta)\frac{k_{t-1}}{1+n} \tag{4}$$

which allows for depreciation at a rate, δ, and a quadratic adjustment cost function. g is the constant rate of technical progress:

$$A_t = A_0 (1+g)^t .$$

Once built, it takes capital one period to earn returns. We do not model firms' inventories separately; we are assuming that this is subsumed in investment.

The representative agent's problem is to maximise its utility with respect to consumption, financial and physical asset investments and hours supplied, given transversality conditions on the three forms of net wealth:

$$\begin{aligned} \lim_{t\to\infty} (\beta(1+n))^t \lambda_t f_t = 0, \qquad & \lim_{t\to\infty} (\beta(1+n))^t \lambda_t b_t = 0 \\ \text{and} \qquad & \lim_{t\to\infty} (\beta(1+n))^t \gamma_t k_t = 0. \end{aligned} \tag{5}$$

The first-order conditions can be rearranged to link the price of investment and the cost of capital:

$$\frac{P_t^x}{P_t^c} = E_t \frac{(1+n)}{\left(\frac{1+i_t}{P_{t+1}^c} P_t^c\right)} \left(R_{t+1}^k + \frac{P_{t+1}^x}{P_{t+1}^c} \frac{\left(\frac{\psi}{2}\left(\left(\frac{x_{t+1}(1+n)}{k_t}\right)^2 - D^2\right)\right)}{\left(1-\psi\left(\frac{x_{t+1}(1+n)}{k_t} - D\right)\right)} \right) \tag{6}$$

with

$$D \equiv (1+n)(1+g) - (1-\delta). \tag{7}$$

The choice of working compared to leisure is related to real wage rate by:

$$h_t = \frac{1}{TBP_t(1-u_t)} \left(\frac{W_t}{P_t^c z_t^h A_t} \right)^{\frac{1}{\eta}}.$$

The first-order conditions also imply the following relationships between the share parameters in the CES consumption function and the relative price and nominal share of spending on domestically produced items, which we exploit for our tests:

$$\frac{P_t^{cd} C_{Dt}}{P_t^c C_t} = \left(\frac{P_t^{cd}}{P_t^c} \right)^{1-\omega} \gamma_t \tag{8}$$

and

$$\frac{P_t^{cm} C_{Mt}}{P_t^c C_t} = \left(\frac{P_t^{cm}}{P_t^c} \right)^{1-\omega} (1-\gamma_t). \tag{9}$$

Combining equations 2 (in aggregate), 8 and 9 implies that the prices of imported and domestically produced consumption $\left(P_t^{cd}\text{ and }P_t^{cm}\text{ respectively}\right)$ combine to give the price of total consumption of government and households:

$$P_t^c = \left[\gamma_t \left(P_t^{cd}\right)^{1-\omega} + (1-\gamma_t)\left(P_t^{cm}\right)^{1-\omega} \right]^{\frac{1}{1-\omega}}. \tag{10}$$

Our model combines private sector consumers with the government. Given the complex tax system, separating out government would be an extremely difficult project. But we still want the model to solve for a household-only consumption price because monetary policy targets this series. Our compromise is to assume that the price paid by private consumers for domestically produced items is the same as that paid by the government, and likewise for imported items. Then the household-only consumption price is:

$$P_t^{cp} = \left[\gamma_t^p \left(P_t^{cd}\right)^{1-\omega^p} + (1-\gamma_t^p)\left(P_t^{cm}\right)^{1-\omega^p} \right]^{\frac{1}{1-\omega^p}} \tag{11}$$

with the private sector domestic consumption demand given as:

$$\frac{P_t^{cd} C_{Dt}^p}{P_t^{cp} C_t^p} = \left(\frac{P_t^{cd}}{P_t^{cp}} \right)^{1-\omega^p} \gamma_t^p. \tag{12}$$

The path of aggregate consumption is given by combining and then aggregating the first-order conditions for bonds and for consumption:

$$E_t \frac{C_{t+1}}{C_t} = \beta E_t \left(\frac{1+i_t}{P_{t+1}^c} P_t^c \right)^{\frac{1}{\sigma}}. \tag{13}$$

Finally foreign and domestic interest rates are connected through an uncovered interest rate parity relationship:

$$E_t \frac{S_{t+1}}{S_t} = \frac{(1 + i_t^e)}{(1 + i_t)}. \tag{14}$$

2.2 Production of domestic output

There are a continuum of firms indexed by $z \in (0, 1)$. Each use the following production function:

$$Q_t(z) = z_t^q \left[\alpha_t^{\frac{1}{\rho}} (V_t(z))^{\frac{\rho-1}{\rho}} + (1 - \alpha_t)^{\frac{1}{\rho}} z_t^{rm} (RM_t(z))^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \text{ and}$$

$$V_t(z) = z_t^v \left[(\alpha_{vt})^{\frac{1}{\rho_v}} (A_t N_t (1 - u_t) TBP_t h_t(z))^{\frac{\rho_v - 1}{\rho_v}} + (1 - \alpha_{vt})^{\frac{1}{\rho_v}} (N_t k_{t-1}(z))^{\frac{\rho_v - 1}{\rho_v}} \right]^{\frac{\rho_v}{\rho_v - 1}} \tag{15}$$

where RM_t is the volume of imported raw materials whose price is P_t^{rm}. The price of gross output of the z^{th} firm is $P_t^q(z)$. z_t^q, z_t^{rm} are z_t^v relative technical progress terms, and A_t is labour embodied technical progress.

In order to allow for a markup, the model features a monopolistically competitive market where each firm ex ante thinks it has some market power:

$$P_t^q(z) = \left(\frac{Q_t(z)}{Q_t} \right)^{-\theta_t} P_t^q. \tag{16}$$

The first-order conditions for solving this problem are:

$$\frac{P_t^{rm} RM_t}{P_t^c Q_t} = (\zeta_t)^\rho (z_t^q)^{\rho-1} \left(\frac{P_t^{rm}}{P_t^c} \right)^{1-\rho} \alpha_t, \tag{17}$$

$$\frac{P_t^v V_t}{P_t^c Q_t} = (\zeta_t)^\rho (z_t^q)^{\rho-1} (z_t^{rm})^\rho \left(\frac{P_t^v}{P_t^c} \right)^{1-\rho} (1 - \alpha_t), \tag{18}$$

$$\frac{W_t N_t h_t (1 - u_t) TBP_t}{A_t P_t^c V_t} = (\zeta_t^v)^{\rho_v} (z_t^v)^{\rho_v - 1} \left(\frac{W_t}{A_t P_t^c} \right)^{1-\rho_v} \alpha_{vt}, \tag{19}$$

and

$$\frac{R_t^k N_t k_{t-1}}{P_t^c V_t} = (\zeta_t^v)^{\rho_v} (z_t^v)^{\rho_v - 1} \left(\frac{R_t^k}{P_t^c} \right)^{1-\rho_v} (1 - \alpha_{vt}), \tag{20}$$

as firms are all identical. Then real marginal costs of all inputs are:

$$\zeta_t = \frac{1}{z_t^q} \left[\left(\alpha_t (\zeta_t^v)^{1-\rho} + (1 - \alpha_t)(z_t^{rm})^\rho \left(\frac{P_t^{rm}}{P_t^c} \right)^{1-\rho} \right) \right]^{\frac{1}{1-\rho}} \tag{21}$$

within which the real marginal costs of value-added component are:

$$\zeta_t^v = \frac{1}{z_t^v}\left[\alpha_{vt}\left(\frac{W_t}{A_t P_{ct}}\right)^{1-\rho_v} + (1-\alpha_{vt})\left(\frac{R_t^k}{P_{ct}}\right)^{1-\rho_v}\right]^{\frac{1}{1-\rho_v}}. \tag{22}$$

The value of gross output is a markup on the unit costs of production,

$$P_t^q Q_t = \frac{P_t^q}{P_t^c}\frac{1}{\zeta_t}\left(P_t^c V_t + P_t^{rm} RM_t\right), \tag{23}$$

with the nominal value of the value-added component as:

$$P_t^c V_t = \frac{1}{\zeta_t^v}\left(R_t^k N_{t-1} k_{t-1} + W_t N_t h_t (1-u_t) TBP_t\right),$$

and nominal profits as:

$$\begin{aligned}\pi_t &= P_t^q Q_t - R_t^k N_t k_{t-1} - W_t N_t h_t (1-u_t) TBP_t - P_t^{rm} RM_t \\ &= \left(\frac{P_t^q}{P_t^c}\frac{1}{\zeta_t} - 1\right)\left(\frac{1-\zeta_t^v}{\zeta_t^v}\left(R_t^k N_t k_{t-1} + W_t N_t h_t (1-u_t) TBP_t\right) + P_t^{rm} RM_t\right).\end{aligned} \tag{24}$$

2.3 Monopolistic competition and price setting

We assume that there are nominal rigidities in setting output prices of a Calvo form. Following Céspedes, Ochoa, and Soto (2005), each period a fixed proportion $(1-\varepsilon)$ of firms are allowed to optimally adjust their prices. The rest have to set prices according to a rule: the firms that cannot optimally adjust prices between time t and $t+i$ have to set their prices in $t+i$ as $P_t^q \Gamma_t^i$ with

$$\Gamma_t^i \begin{cases} \equiv \prod_{j=1}^{i}\left(1+\pi_{t+j-1}^q\right)^{\kappa}\left(1+\pi_{t+j}^*\right)^{1-\kappa} & \text{if } i \geq 1; \\ \equiv 1 \text{ if } i = 0. \end{cases} \tag{25}$$

$\pi_t^q \left(\equiv \frac{P_t^q}{P_{t-1}^q} - 1\right)$ is the inflation rate of output prices at time t, $\pi_t^* \left(\equiv \frac{P_t^*}{P_{t-1}^*} - 1\right)$ is the target rate at time t, and κ is a parameter that determines the degree of inflation stickiness relative to credibility. After log linearisation (see the Appendix), this produces a Phillips curve of the form:

$$\pi_t^q = \frac{\upsilon}{1+\kappa\upsilon}E_t\pi_{t+1}^q + \frac{\kappa}{1+\kappa\upsilon}\pi_{t-1}^q + \frac{(1-\varepsilon)(1-\upsilon\varepsilon)}{\varepsilon(1+\upsilon\kappa)}\zeta_t + \varsigma_t \tag{26}$$

where ς_t is a term related to changes in the inflation target,

$$\varsigma_t = \frac{\kappa \upsilon}{1+\kappa\upsilon} E_t \Delta\pi^*_{t+1} - \frac{\kappa}{1+\kappa\upsilon} E_t \Delta\pi^*_t \tag{27}$$

with the parameter υ representing a discount at the steady-state real rate of interest (r^{ss}):

$$\upsilon \equiv \frac{1}{1+r^{ss}}. \tag{28}$$

2.4 Transforming production

In order to model different outputs from domestic production without splitting factor markets, we make use of the transformation of domestically produced output into domestic consumption, C_t^d, domestically produced investment, X_t^d, exports, E_t, and the commerce and transport margin to importers of consumption and investment goods, T_t. We assume the functional form:

$$Q_t = z_t^d D\left(C_t^d, X_t^d, T_t, E_t\right) = z_t^d \left[\nu_{ct}^{\frac{1}{\nu}} \left(C_t^d\right)^{\frac{\nu-1}{\nu}} + \nu_{xt}^{\frac{1}{\nu}} \left(X_t^d\right)^{\frac{\nu-1}{\nu}} + \nu_{Tt}^{\frac{1}{\nu}} \left(T_t\right)^{\frac{\nu-1}{\nu}} + \nu_{\exp t}^{\frac{1}{\nu}} \left(E_t\right)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}} \tag{29}$$

with $\nu < 0$, $\nu_{ct}, \nu_{xt}, \nu_{Tt}, \nu_{\exp t} > 0$.

The problem of these firms is then:

$$\begin{aligned} \max_{\left\{C_t^d, X_t^d, T_t, E_t\right\}} \quad & P_t^{cd} C_t^d + P_t^{xd} X_t^d + P_t^T T_t + P_t^{exp} E_t \\ & - P_t^q z_t^d \left[\nu_{ct}^{\frac{1}{\nu}} \left(C_t^d\right)^{\frac{\nu-1}{\nu}} + \nu_{xt}^{\frac{1}{\nu}} \left(X_t^d\right)^{\frac{\nu-1}{\nu}} + \nu_{Tt}^{\frac{1}{\nu}} \left(T_t\right)^{\frac{\nu-1}{\nu}} + \nu_{\exp t}^{\frac{1}{\nu}} \left(E_t\right)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}. \end{aligned} \tag{30}$$

The deflators of these different outputs are related to the gross output price by

$$P_t^q = \frac{1}{z_t^d} \left[\nu_{ct} \left(P_t^{cd}\right)^{1-\nu} + \nu_{xt} \left(P_t^{xd}\right)^{1-\nu} + \nu_{Tt} \left(P_t^T\right)^{1-\nu} + \nu_{\exp t} \left(P_t^{\exp}\right)^{1-\nu}\right]^{\frac{1}{1-\nu}}. \tag{31}$$

The first-order conditions can be rearranged as

$$\frac{P_t^{cd} C_{Dt}}{P_t^q Q_t} = \left(z_t^d\right)^{\nu-1} \left(\frac{P_t^{cd}}{P_t^q}\right)^{1-\nu} \nu_{ct}, \tag{32}$$

$$\frac{P_t^{xd} X_t^d}{P_t^q Q_t} = \left(z_t^d\right)^{\nu-1} \left(\frac{P_t^{xd}}{P_t^q}\right)^{1-\nu} \nu_{xt}, \tag{33}$$

$$\frac{P_t^T T_t}{P_t^q Q_t} = \left(z_t^d\right)^{\nu-1} \left(\frac{P_t^T}{P_t^q}\right)^{1-\nu} \nu_{Tt}, \tag{34}$$

and

$$\frac{P_t^{\exp} E_t}{P_t^q Q_t} = \left(z_t^d\right)^{\nu-1} \left(\frac{P_t^{\exp}}{P_t^q}\right)^{1-\nu} \nu_{\exp t}. \tag{35}$$

2.5 Importers of consumption/investment

Importers buy consumption and investment goods from abroad, and combining that with the output of the distribution sector transform those raw imports into consumer and investment goods that are available for final and intermediate consumption. M_t^d is the volume of imported consumption and investment goods at the point of use, and M_t^p is the volume of these before transformation. M_t^p is what we assume the National Accounts measure as imports of consumption and investment goods in CIF prices. The transformation function is:

$$M_t^d = z_t^m M\left(T_t, M_t^p\right) = z_t^m \left[\iota_{Tt}^{\frac{1}{a_m}} \left(T_t\right)^{\frac{a_m-1}{a_m}} + \left(1-\iota_{Tt}\right)^{\frac{1}{a_m}} \left(M_t^p\right)^{\frac{a_m-1}{a_m}}\right]^{\frac{a_m}{a_m-1}} \tag{36}$$

with $a_m > 0$. The maximisation problem of these firms is therefore

$$\begin{aligned} \max_{\{T_t,\, M_t^p\}} \quad & P_t^{md} M_t^d - P_t^T T_t - P_t^{mp} M_t^p \\ \text{s.t} \quad & M_t^d \leq z_t^m \left[\iota_{Tt}^{\frac{1}{a_m}} \left(T_t\right)^{\frac{a_m-1}{a_m}} + \left(1-\iota_{Tt}\right)^{\frac{1}{a_m}} \left(M_t^p\right)^{\frac{a_m-1}{a_m}}\right]^{\frac{a_m}{a_m-1}} \end{aligned} \tag{37}$$

which implies that

$$\frac{P_t^T T_t}{P_t^{md} M_t^d} = \left(z_t^m\right)^{a_m-1} \left(\frac{P_t^T}{P_t^{md}}\right)^{1-a_m} \iota_{Tt} \tag{38}$$

and

$$\frac{P_t^{mp} M_t^p}{P_t^{md} M_t^d} = \left(z_t^m\right)^{a_m-1} \left(\frac{P_t^{mp}}{P_t^{md}}\right)^{1-a_m} \left(1-\iota_{Tt}\right). \tag{39}$$

We also assume that imported consumption and imported investment goods are perfect substitutes even once transformed

$$P_t^{xm} = P_t^{cm} = P_t^m,$$

such that the volumes of the components add up in volume terms, just as they do in value terms:

$$M_t^d = X_t^m + C_t^m.$$

2.6 Importers of raw materials

Unlike imported capital and consumption, raw materials are assumed not to be transformed by any domestic sector but to be taken from port directly into production. Hence

$$\frac{P_t^{rm}}{P_t^c} = \frac{S_t}{P_t^c} P_t^{rm*} \tag{40}$$

and therefore

$$P_t^{um} M_t^u = P_t^{rm} RM_t + P_t^{pm} M_t^p. \tag{41}$$

P_t^{um} is the total imports deflator from national accounts, that is, the price of imports not transformed yet. Similarly, M_t^u are the volume of imports before transformation. To obtain the price of consumption and capital imports before transformation (P_t^{mp}) we assume that consumption and capital imports on one hand, and raw materials on the other are split out from total imports before transformation by the following maximisation problem:

$$\begin{aligned} \max_{\{M_t^p,\, RM_t\}} \quad & P_t^{um} M_t^u = P_t^{rm} RM_t + P_t^{pm} M_t^p. \\ \text{s.t} \quad & M_t^u \leq z_t^{rm} \left[(\iota_{rmt})^{\frac{1}{a_r}} (RM_t)^{\frac{a_r - 1}{a_r}} + (1 - \iota_{rmt})^{\frac{1}{a_r}} (M_t^p)^{\frac{a_r - 1}{a_r}} \right]^{\frac{a_m}{a_m - 1}} \end{aligned} \tag{42}$$

The solution implies,

$$\iota_{rmt} = (z_t^{rm})^{1-a_r} \left(\frac{P_t^{rm}}{P_t^{um}} \right)^{a_r - 1} \left(\frac{P_t^{rm} RM_t}{P_t^{um} M_t^u} \right), \tag{43}$$

$$1 - \iota_{rmt} = (z_t^{rm})^{1-a_r} \left(\frac{P_t^{pm}}{P_t^{um}} \right)^{a_r - 1} \left(\frac{P_t^{pm} M_t^p}{P_t^{um} M_t^u} \right), \tag{44}$$

and therefore

$$P_t^{um} = \frac{1}{z_t^{rm}} \left[\iota_{rmt} \left(P_t^{rm} \right)^{1-a_r} + (1 - \iota_{rmt}) \left(P_t^{pm} \right)^{1-a_r} \right]^{\frac{1}{1-a_r}}. \tag{45}$$

Equation 45 permits a solution for capital and consumption import prices within the model conditional on data and separate (off-model) forecasts of raw material prices and total import prices (both presumably in foreign currency terms so that they can be modelled as exogenous world market prices). But the system permits alternatives depending on what data is available. One could instead consider capital and consumption import prices and total import prices as exogenous series and solve for raw materials prices from Equation 45. Or, take both the capital and consumption import prices and the raw materials prices as exogenous, and forecast total import prices, and compare the insample fit to the National Accounts imports deflator.

2.7 World markets for exports

In the world market for Colombian exports, Colombian goods compete with foreign produced items for a share of the world market GDP $\left(P_t^{WGDP} WGDP_t\right)$. The price and volume of Colombia's competition is $P_t^{WD}$and X_t^{WD}. The problem of a consumer in this world market is therefore:

$$\begin{aligned} \max \quad & P_t^{WGDP} WGDP_t - \frac{P_t^{\exp}}{S_t} E_t - P_t^{WD} X_t^{WD} \\ \text{s.t} \quad & WGDP_t \leq z_{wt} \left[\left(\gamma_t^W\right)^{\frac{1}{\omega^W}} \left(E_t\right)^{\frac{\omega^W - 1}{\omega^W}} + \left(1 - \gamma_t^W\right)^{\frac{1}{\omega^W \iota}} \left(X_t^{WD}\right)^{\frac{\omega^W - 1}{\omega^W}} \right]^{\frac{\omega^W}{\omega^W - 1}} . \end{aligned} \tag{46}$$

The demand for domestic exports in world markets is represented by a first-order condition:

$$\frac{P_t^{\exp} E_t}{P_t^{WGDP} WGDP_t} = z_{wt}^{\omega^W - 1} \left(\frac{P_t^{\exp}}{S_t P_t^{WGDP}} \right)^{1 - \omega^W} \gamma_t^W . \tag{47}$$

2.8 Investment

Households create an aggregate investment good by combining domestic and foreign investment:

$$X_t = z_t^x X\left(X_t^d, X_t^m\right) = z_t^x \left[\left(\varkappa_t\right)^{\frac{1}{\iota}} \left(X_t^d\right)^{\frac{\iota - 1}{\iota}} + \left(1 - \varkappa_t\right)^{\frac{1}{\iota}} \left(X_t^m\right)^{\frac{\iota - 1}{\iota}} \right]^{\frac{\iota}{\iota - 1}} \tag{48}$$

where X_t is aggregate gross investment, X_t^d and X_t^m are the volumes of domestic and imported investment goods respectively. Their maximisation problem is:

$$\begin{aligned} \max \quad & P_t^x X_t - P_t^{xd} X_t^d - P_t^m X_t^m \\ \text{s.t} \quad & x_t \leq z_t^x \left[\left(\varkappa_t\right)^{\frac{1}{\iota}} \left(X_t^d\right)^{\frac{\iota - 1}{\iota}} + \left(1 - \varkappa_t\right)^{\frac{1}{\iota}} \left(X_t^m\right)^{\frac{\iota - 1}{\iota}} \right]^{\frac{\iota}{\iota - 1}} . \end{aligned} \tag{49}$$

This gives us two demand equations:

$$\frac{P_t^{xd} X_t^d}{P_t^x X_t} = \left(z_t^x\right)^{\iota - 1} \left(\frac{P_t^{xd}}{P_t^x} \right)^{1 - \iota} \varkappa_t , \tag{50}$$

and

$$\frac{P_t^m X_t^m}{P_t^x X_t} = \left(z_t^x\right)^{\iota - 1} \left(\frac{P_t^m}{P_t^x} \right)^{1 - \iota} \left(1 - \varkappa_t\right) . \tag{51}$$

2.9 National Accounts GDP

The nominal value-added of the whole economy at market prices is the nominal value-added of the domestic production sector:

$$
\begin{aligned}
& P^{GDP}_{MPt} GDP^{q}_{MPt} \\
& = P^q_t Q_t - P^{rm}_t RM_t \\
& = P^{cd}_t C^d_t + P^{xd}_t X^d_t + P^T_t T_t + P^{\exp}_t E_t - P^{rm}_t RM_t \\
& = P^c_t C_t + P^x_t X_t + P^T_t \ T_t + P^{\exp}_t E_t - P^{cm}_t C^m_t + P^{xm}_t X^m_t - P^T_t \ T_t - P^{rm}_t RM_t \\
& = P^c_t C_t + P^x_t X_t + P^{\exp}_t E_t - P^{pm}_t M^p_t - P^{rm}_t RM_t \\
& = P^c_t C_t + P^x_t X_t + P^{\exp}_t E_t - P^{um}_t M^u_t
\end{aligned}
\tag{52}
$$

remembering that consumption includes that of households and government together and that investment includes changes in inventories. A decomposition of National Accounts GDP at market prices according to factor incomes is then:

$$P^{GDP}_{MPt} GDP^{q}_{MPt} = W_t N_t h_t + \left[R^k_t N_{t-1} k_{t-1} + \pi_t \right].$$

Then the first term on the righthandside could be the sum of National Accounts salaries and mixed income. The second term in brackets could represent National Accounts return to capital, which comprises true returns to capital in the model and profits. We assume that the factor cost adjustment (τ_t) is included in these profits, the idea being that the government has some monopoly rights over the firm and the factor cost adjustment is its charge for that:

$$\tau_t = P^{GDP}_{MPt} GDP^{q}_{MPt} - P^{GDP}_{FCt} GDP^{q}_{FCt}.$$

There is no equivalent to the real National Accounts GDP concept within the model up until now. We can use a Tornqvuist approximation to a Divisia index to forecast a National Accounts GDP deflator within the model

$$P^{GDP}_{MPt} = \left(\frac{P^q_t}{P^q_{t-1}} \right)^{\frac{1}{1-sr_t}} \left(\frac{P^{rm}_t}{P^{rm}_{t-1}} \right)^{\frac{-sr_t}{1-sr_t}} P^{GDP}_{MPt-1}$$

with the weights given as

$$sr_t = \left(\frac{P^{rm}_{t-1} RM_{t-1}}{P^q_{t-1} Q_{t-1}} \right)^{0.5} \left(\frac{P^{rm}_t RM_t}{P^q_t Q_t} \right)^{0.5}. \tag{53}$$

The model's measure of the real National Accounts GDP series will then be the measure of National Accounts nominal GDP divided by this deflator. In this way the model can be used to forecast and simulate National

Accounts concepts.

2.10 Monetary policy and the flexible-price state

To close the model we assume a monetary policy rule of the form:

$$i_t = \rho_s i_{t-1} + (1-\rho_s)\left(i_t^{FP} + \varphi_\pi \left(\frac{P_t^{cp}}{P_{t-1}^{cp}} - 1 - \Delta\pi_t^*\right)\right) + \varphi_y \left(\frac{GDP_{MPt}}{GDP_{MPt}^{FP}} - 1\right). \quad (54)$$

Here i_t^{FP} is the flexible-price nominal rate of interest and GDP_{MPt}^{FP} is the flexible price level of GDP. To simulate these variables the entire model has to be solved again but with the nominal rigidities removed. If we refer to all flexible-price variables with the superscript FP, in the flexible-price state we replace equations 25, 26 and 27 with:

$$\frac{P_t^{FP,q}}{P_t^{FP,c}} = \frac{\theta_t}{1-\theta_t}\zeta_t^{FP}$$

and

$$\frac{P_t^{FP,cp}}{P_{t-1}^{FP,cp}} = 1 + \Delta\pi_t^*$$

and replace equation 54 with a rearrangement of the aggregate consumer budget constraint

$$i_t^{FP} = \frac{E_t^{FP} - M_t^{FP} + S_t^{FP}\left(F_t^{FP} - F_{t-1}^{FP}\right) + S_t^{FP} TR_t}{F_{t-1}^{FP}} - 1. \quad (55)$$

Net foreign assets are exogenous but must satisfy at least its terminal condition. All the other equations are as they are in the actual state but with a flexible-price superscript added to variable names. The solution to this flexible-price version of the model produces the paths for the two variables we need to complete and close the entire model.

3 The database

The first step is to decide which of the variables in our model have data already directly available in the National Accounts and which require construction. Of course even if there is a National Accounts data series corresponding to a theoretical variable rarely will the National Accounts concept match exactly what the model requires. Thus there may always be some residual and sometimes, for example in the capital account, those discrepancies may be large. Nevertheless we may expect more errors in the data constructed by the model builder than those by the National Accounts office. And as the former are the exclusive responsibility of the model builder, they are the sole focus of this paper.

We divide our database into three categories. Table 1 on page 19 includes the National Accounts expenditure aggregates only. All of these are available as published series as both nominal and real values.

Table 1: National accounts data for the model

Variable	**Explanation**	**National Accounts available**
$P_t^c, C_t, P_t^c C_t$	Consumption of households and government	Nominal, real volumes
$P_t^X, X_t, P_t^X X_t$	Gross fixed capital formation and changes in inventories	Nominal, real volumes
$P_t^{um}, M_t^u, P_t^{um} M_t^u$	Total imports before transformation	Nominal, real volumes
$P_t^{\exp}, E_t, P_t^{\exp} E_t$	Exports	Nominal, real volumes
$P_t^Y, Y_t, P_t^Y Y_t$	GDP value-added	Nominal, real volumes
P_t^{cp}	Price of private sector consumption	CPI data

A second table includes only the sectoral variables.

Table 2: Sectoral data for the model

Variable	**Explanation**
$P_t^{cm}, C_t^m, P_t^{cm} C_t^m$	Consumption by households and government of direct imports
$P_t^{cd}, C_t^d, P_t^{cd} C_t$	Consumption by households and government of domestic production
$P_t^{md}, M_t^d, P_t^{md} M_t^d$	Aggregate capital and consumption imports after transformation
$P_t^{mp}, M_t^p, P_t^{mp} M_t^p$	Aggregate capital and consumption imports before transformation
$P_t^T, T_t, P_t^T T_t$	Distribution sector input into transforming consumption and capital imports
$P_t^{rm}, RM_t, P_t^{rm} RM_t$	Raw material imports
$P_t^{xm}, X_t^m, P_t^{xm} X_t^m$	Imported physical investment
$P_t^{xd}, X_t^d, P_t^{xd} X_t^d$	Domestically produced physical investment

In contrast to the variables in Table 1 on page 19, none of the variables in Table 2 on page 19 are directly published by the National Accounts. But we need these series because we have chosen to work with a tradable/nontradable model. The focus of our paper is on the construction of the series and the testing of that part of the database and model.

For completeness a third table includes all the other series, whose data are partially available. For example, the real volume and user cost of capital require a lot of construction and assumptions.

Table 3: Other data for the model

Variable	**Explanation**
N_t	Population
h_t	Average hours worked by employee
u_t	Unemployment rate
TBP_t	Participation rate
$R_{kt}, K_t, R_{kt}K_t$	Physical capital stock
W_t	Hourly wage rate
B_t	Net domestic currency asset holdings
F_t	Net foreign currency liabilities
i_t^d	Interest rate on net domestic assets
i_t^e	Interest rate on net foreign liabilities of government and households
TR_t	Remittances and other capital account items assumed exogenous
$P_t^{WGDP}, WGDP_t$	World GDP, World GDP deflator

3.1 Summary of the database construction

What was then our strategy to construct the missing data in Table 2 on page 19?

We will lay the answer in full detail shortly, but our method can be summarised in five steps:

- First we used the input-ouput data where available to construct some annual nominal shares for the sectoral series;
- Second we interpolated and extrapolated those shares to cover our whole sample at a quarterly frequency;
- Next we obtained some data on the price and volume split of at least one component from other parts of National Accounts data or from other sources.
- Finally we combined that with data on the aggregate concept on prices and volumes that was also available (see Table 1 on page 19) to derive a series on the missing components price. To do this we need to bring in some economic theory to extract the separate price and volume of the other component. Thus even if we had data for the series P_{1t}, P_t, Z_t and $\frac{P_{1t}Z_{1t}}{P_tZ_t}$ in

$$P_{1t}Z_{1t} + P_{2t}Z_{2t} = P_tZ_t,$$

that would not be enough to derive P_{2t} and Z_{2t} separately.

There are three solutions available to economists facing this fundamental problem. First one could make some strong assumptions about relative prices, for example that they are fixed ($P_{1t} = P_{2t}$). But if the assumption is not supported by the data, a large approximation error would appear in the initial values of the forecast. A second solution is to use the theoretical price aggregators consistent with the theoretical model to derive the volume or price of the component. This would correspond to assuming that there is no error in the model's in-sample solution. But the quality of the model's forecasts would then depend very much on that theory being realistic.

A third solution is to employ index numbers to construct those series. Index numbers are designed to be compatible with a wider range of utility and production functions than the specific CES form taken in our model (Diewert and Nakamura, 1993). Our strategy was therefore to rely on index numbers as much as possible, and in a few cases assume that relative prices are fixed. This strategy gave us a database. As index numbers are more general than the theoretical series, the difference between our data and theoretical equations were then used to test differences between the model and the database.

3.2 Details of the database construction

For the sake of completeness, in this section, we describe step by step how we built our database. The reader who is more interested in our test can skip this subsection.

3.2.1 Step 1: Approximating the extent of import transformation in Colombian production

The first step was to use the input output tables to approximate the nominal value of the input of the import transforming sector. Our aim is to approximate the extent to which the domestic sectors as a whole transforms imported inputs rather than domestic inputs. Clearly the demarcation between the two activities is blurred: all imports can even be thought of as intermediate inputs which domestic production transforms to varying degrees, as in Allsopp, Kara, and Nelson (2006). Perhaps for this reason, National Accounts data rarely provides explicit calculations on how much each import is transformed before it is sold to its final consumer, abroad, or to another producer.

The National Accounts convention is instead to organise production into sectors each of which exclusively imports and produces one particular category of product[3]. Thus imports that are transformed by the distribution and sold to the final consumer are not allocated to the two distribution sectors (domestic transport and commerce) but rather to the sector which produces that same type of good. So an imported

[3]See Lequiller and Blades (2007) page 289.

T-shirt would not show up in the imports or sales of either the haulage firm that took it from Buenaventura (Colombia's main port) to Bogotá nor in the supermarket that sold it once in Bogotá. By National Accounts convention, the T-shirt would be included in the imports and sales of the textile sector, and so would be mixed up with imports of raw fabric and sales of domestically produced T-shirts. The value-added gained by the haulage firm and the supermarket would be allocated under distributors´ margin contribution to the production cost of the textile sector. Similarly if a car is imported by the distributor to help a salesman sell those T-shirts within Colombia that would appear in the imports of the transport equipment sector, and neither in those of the textile sector nor in those of the distribution sector.

Clearly, we need some approximation. Our method was to use the ratio between imports and total intermediate inputs (imports plus other intermediate inputs) into each sector as a guide to how much that sector was transforming imported inputs as opposed to producing domestically. If a sector does not use any significant intermediate inputs, but only labour and capital, we assumed that sector only produces domestically. This qualifier applied to some of the service sectors, including nonmarket services. Multiplying the import transforming ratios by the distributors' margin gives us the value of distribution input that is used to transform imports in that sector, as opposed to transforming domestic production. Adding up those values across the sectors gives us the total value of import transforming distribution ($P_t^T T_t$) for 1994 to 2005. We worked at a two digit level for this gave us the longest time series.

We used the same import transforming ratio of each sector to proportion the sales to consumer of each sector to consumption of domestic production and consumption of direct imports. Once we add up across sectors, and take account both of direct purchases of consumption imports by residents and direct consumption of imports by nonresidents in national territory, we have an approximation to the value of consumption of government and households that is directly imported ($P_t^{cm} C_t^m$). This is taken forward as a share of total consumption for the years 1994 to 2005[4].

Possible sources of error in this first step. We have explained that could be two potential sources of error in this calculation. First that there are intermediate imports of a different type to the product of the importing sector but the National Accounts classification would allocate this to the sectors which produce

[4]As an example to help future researchers retrace our steps, we can derive the accounts for the confectionary sector (cocoa, chocolate and other sweet products using sugar) in 2004. This sector bought \$88 024 million of imported inputs and \$1044498 million of domestic intermediate inputs (including from within the sector itself). Thus the import transformation ratio is 8.43%=88024/1044498. The total distribution sector margin on the sector's supply was \$414452. We therefore estimate that the value of the distribution input in transforming imports in this sector was \$35623 (or 8.43% of \$414452). Final consumption sales of the sector was \$1643487. Then the value of imported consumption coming through the sector is calculated to be \$138.503 (or 8.43% of \$1643487).

As a check on how this balances up, adding the total distribution sector margin to the total intermediate input as well as remuneration for labour including mixed income (\$161895), returns to capital and profits (\$608838), and net subsidies (\$30266) gives total factor costs as \$2351952. This closely matches total revenue earned from sales of \$2347973 which is split into \$168519 sold to other domestic industries as intermediate production, \$505900 of export sales, \$29414 as investment and inventory additions, and the \$1643487 of final consumption.

that output. To formalise this, let us denote an import of product of type i going to sector j by I^{ij} and D^{ij} denote domestically produced inputs of type i purchased by sector j. There are N sectors and N products, with each sector indexed by the type of product it makes. The value of distributors margins going into that sector are DM_j. Then the true measure of the extent of import transformation for sector j would be:

$$P^T T = \sum_j \frac{\sum_i I^{ij}}{\sum_i I^{ij} + \sum_i D^{ij}} DM_j.$$

But the National Accounts allocates all imports of type i to sector i. This is the only source of error as we assume that sector j is the only domestic sector to produce intermediate good of type j. Thus our imperfect measure of import transformation for sector j would be:

$$\widehat{(P^T T)} = \sum_j \frac{\sum_i I^{ji}}{\sum_i I^{ji} + \sum_i D^{ij}} DM_j.$$

The key question is then, how distorted is our measure of aggregate import transformation? The difference between our measure and the true measure would then be:

$$\begin{aligned}
&(P_t^T T_t) - \widehat{(P_t^T T_t)} \\
&= \sum_j \left(\frac{\sum_i I^{ji}}{\sum_i I^{ji} + \sum_i D^{ij}} - \frac{\sum_i I^{ij}}{\sum_i I^{ij} + \sum_i D^{ij}} \right) DM_j \\
&= \sum_j \left(\frac{\left(\sum_i D^{ij}\right)}{\left(\sum_i I^{ij} + \sum_i D^{ij}\right)\left(\sum_i I^{ji} + \sum_i D^{ij}\right)} \right) \left(\sum_i I^{ji} - \sum_i I^{ij}\right) DM_j.
\end{aligned}$$

The first term inside the summation is bounded between zero and one for each sector, and apart from that is hard to measure. We thus focus more on trying to speculate on the size of the last two terms for each sector. It seems the distortions would be large where there is a great discrepancy between the sum of intermediate imports of that sector, and the sum of intermediate imports of that type of product $\left(\sum_i I^{ji} - \sum_i I^{ij}\right)$ for sectors which use a lot of commerce. For example if most intermediate car purchases are imported outside of the transport equipment sector then the bias could affect our calculation if the domestic to imported ratio of that sectors differ from the ratio of imported to total cars. A similar error in our measure of imported consumption would be:

$$
\begin{aligned}
&(P^{cm}C^{m}) - (\widehat{P^{cm}C^{m}}) \\
&= \sum_{j} \left(\frac{\left(\sum_{i} D^{ij}\right)}{\left(\sum_{i} I^{ij} + \sum_{i} D^{ij}\right)\left(\sum_{i} I^{ji} + \sum_{i} D^{ij}\right)} \right) \left(\sum_{i} I^{ji} - \sum_{i} I^{ij}\right) (P^{c}C)_{j}
\end{aligned}
$$

where $(P^{c}C)_{j}$ is the value of production of the sector j destined for final consumption.

To measure this error exactly, we would need an intermediate use table which reveals which products are actually used by which industry. But in Colombia all that is publicly available is a matrix of imports for 1992 and 1993. This matrix tells us, in total, which imports were finally destined for each industry. This information is presented in Table 4 on page 24 as the share of imports going to each aggregate sector in columns 2 and 4 for the two years. In columns 3 and 5 we present the shares for the same sectors as assumed in our calculations. Columns 6 and 7 present the 1992-93 average share of each sector in total margins and in total domestic consumption, respectively. If there is a large discrepancy between columns 2 and 4 on one hand and columns 3 and 5 on the other, and either shares in columns 6 and 7 are large, then we can assume that a error will be in the calculation for that sector.

Table 4: Comparing direct and indirect import data for 1992-1993

	Share of total direct imports	Share of total indirect imports	Share of total direct imports	Share of total indirect imports	Share of total margin	Share of total consumption
	1992		1993		1992/93	1992/93
Economywide	6.81		6.85			
Agriculture, Forestry & Fishing	3.07	5.30	2.62	4.40	15.12	7.08
Mining	2.6	0.50	3.10	0.50	-2.46	0.01
Industry	62.26	79.3	58.27	83.2	87.27	42.59
Utilities	2.15	0.60	2.16	0.60	0.06	1.93
Construction	1.68	0.00	1.37	0.00	0.00	0.12
Consumption	9.84		8.10			
Distribution, hotels & catering	0.76	0.00	1.58	0.00	0.00	4.46
Transport and communications	5.11	7.70	6.28	7.00	0.00	1.84
Financial & real estate services	0.94	3.80	2.25	3.20	0.00	12.07
Public services	1.19	0.10	1.13	0.20	0.00	23.29
Unclassified	3.6		6.3			

Source: DNP and own calculations

Table 4 on page 24 makes it clear that any error would arise only in the case of industry; service sectors do not import much even directly, and have very low margins. As for industry, it seems we could be overestimating the extent to which the sector import inputs by a maximum of 17-25% of total imports[5]. But even an error of this size will only mean that our estimate of the aggregate share of commerce going to import transformation over total non-raw material imports is overstated by a maximum of 4pp (as 30 instead of 26% say). Our share of imported consumption in total consumption would only be overstated by a maximum of 1.5pp (as 16 instead of 14.5%). The scale of this error does not seem large enough to imply a significant difficulty in forecasting variables. In any case, we would hope that our tests in the main body of this paper would pick that up.

There is another source of error, which is perhaps more a question of interpretation. Recall that we have weighted each sector purely on the basis of how much domestic versus imported intermediates they use. But imagine if there were a sector that purchased a large value of domestic labor and domestic capital, a small value of intermediate imports but even less domestic intermediates. According to our method, this sector would have a high import transformation weight whereas one could argue that most of its output is really due to its domestic value-added inputs; thus that it should be a domestically orientated sector. But looking at the data, there are only two sectors which have a large (greater than 40% on average for the 15 year sample) share of value-added inputs in gross sales and a large (greater than 40%) ratio of the value of imported intermediates to domestic intermediates: the water transport sector and the waste sector. Neither of these two have large margins (greater than 5% of total margins per year on average) or a large contribution to final consumption (greater than 5% per year on average). So insofar as we are interested in aggregate measures of the share of distribution in import transformation and the share of imported consumption, we are not distorting much by organising production along the axis of domestic intermediate versus foreign produced intermediate produced transformation.

3.2.2 Steps 2 to 11

2. Input output tables in Colombia are only available annually from 1990 to 2005. We had to extend these shares to 2006Q4 where our database ends and kept the quarterly shares as fixed during each year of the sample. Given that the shares do not move too much, one might hope that our interpolations are quite accurate. But later on we show how our test can be applied to test for robustness to these assumptions.

3. We need to incorporate some component price series from outside the National Accounts expenditure

[5]This is a maximum because the overestimate of the amount of imports going to industry could be offset (at least partially) by an underestimate of imports elsewhere.

data set in Table 1 on page 19. We used an imported raw material price that is compiled by the Banco de la República using a mixture of National Accounts data, CPI data and producer price series. We have National Accounts data on the value of imported raw materials[6] that takes account of volume changes.

4. Given the imported raw material price data and the National Accounts data on the value of raw materials, on the value and volume of total imports, we used a Fisher weighted chained value-added index to calculate the untransformed price and volume of consumption and investment imported items together (P_t^{mp} and M_t^p).

5. Next we assumed that the relative price of transport to commerce is at a constant value, and is the same for transforming domestic production as it is for imports. This gives us price for the distribution sector input as the deflator of commercial sector $\left(P_t^T\right)$. That price corresponds the nominal value of that input we have already derived in step 1.

6. Using data on the price and volume of the two inputs used in transforming consumption and investment imports, and another chained Fisher weighted index we get separate series on the price and the volume of transformed consumption and good imports together (P_t^{md} and M_t^d).

7. Our input output calculations in the first step gave us a value of final consumption imports ($P_t^{cm}C_t^m$). Using this and the calculation in the previous step, we calculate the separate value of transformed investment ($P_t^{xm}X_t^m$) .

8. We keep with the assumption in the theory of the model that the price of transformed consumption imports is the same as the price of transformed investment imports ($P_t^{xm} = P_t^{cm}$). This then gives us the volumes of transformed consumption imports and investment imports, and a common deflator for both ($C_t^m,\ X_t^m,\ P_t^{cm} = P_t^{xm}$).

9. Combining the price and value share of consumption imports with price of aggregate consumption in a Fisher weighted chained index gives us the price of domestically produced consumption (P_t^{cd}). Dividing by the nominal value of domestically produced consumption, from our first step, we then have the corresponding real volume (C_t^d).

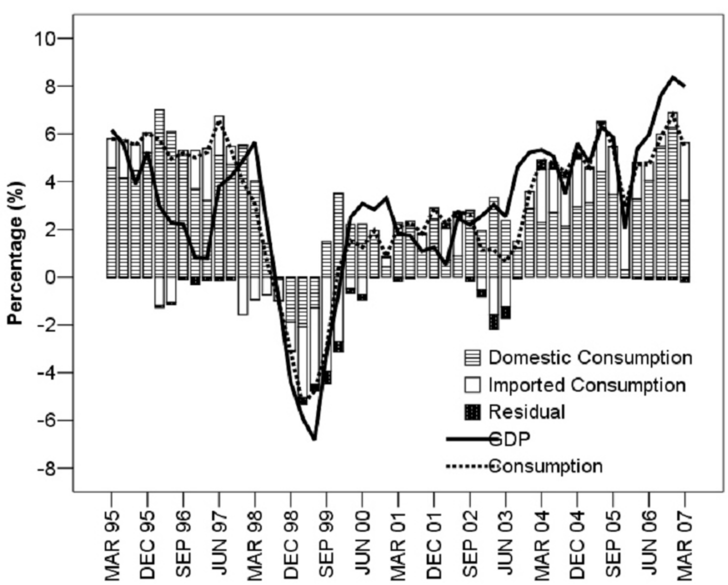
[6] We have not used the equivalent data on the value of imported consumption and capital goods which we think are much less reliable. To explain why, note first that Colombian imports data suffers from five significant source of errors. First illicit imports, which are a large share of Colombian imports, are difficult to capture. Second the imports of services can also be problematic. Third, problems can arise in converting from metric weight to the economic concept of volume. Finally it is not easy to classify consumption imports from intermediate imports. Our presumption is that these measurement problems are much less likely to affect raw materials compared to consumption and capital imports. Most raw material imports are goods (assuming that the service sectors only import services, and as service imports are about 13% of total imports from 1990-2005); most raw material imports are legal (an exception could be the small value of chemicals used in processing cocaine); raw material imports do not experience much qualititave change and the raw materials are straightforward to classify (unlike blurred consumption and capital items such as mobile phones, computers or vehicles).

10. Similarly bringing together the price and value share of investment imports with those of aggregate investment (including inventory accumulation) in a Fisher weighted chained index gives us the price of domestic investment, and thus that volume (P_t^{xd} and X_t^d).

11. Finally we need to calculate some idea of the value of private imported consumption in order to help forecast a CPI price index. To do this we repeat the exercise of calculating the share of imported consumption from the input output tables but now focus only on households and non -profit organizations' consumption. Note though that we do not know how much of this government consumption is actually imported (government imports are allocated to the production sectors that produce that type of good and then sold as an intermediate to the government production sectors).

Looking back at Table 2 on page 19, our database is now completed. Perhaps it is best to judge the new data visually. For example, Figure 2 on page 27 plots the created data for domestic and imported consumption[7].

Figure 2: Domestic and Imported Consumption (contributions to annual growth in real total consumption)



We can see that domestic consumption drives most of consumption due to its larger share. But as imported consumption is more cyclical, the imported component matters more in peaks and troughs. This is very plausible, as services comprise a larger proportion of domestic consumption.

Figure 3 on page 28 plots the domestic and imported contributions to investment. The shares of each are now closer to 50%, and both components are now seen to be very cyclical. Taken with Figure 2 on page 27, Figure 3 on page 28 partly explains why the current account deficit in Colombia is procyclical.

[7] As the contributions formula is a linearisation, there is a small residual.

Figure 3: Domestic and Imported Investment (contributions to annual growth in real total investment)

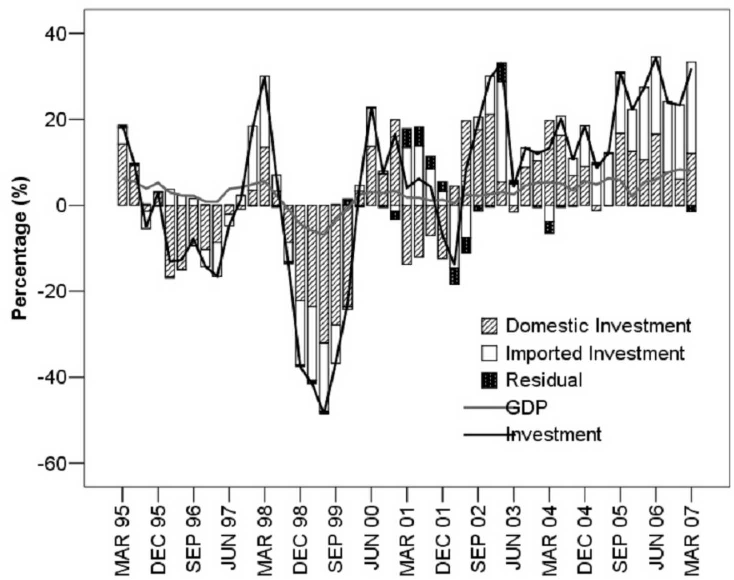


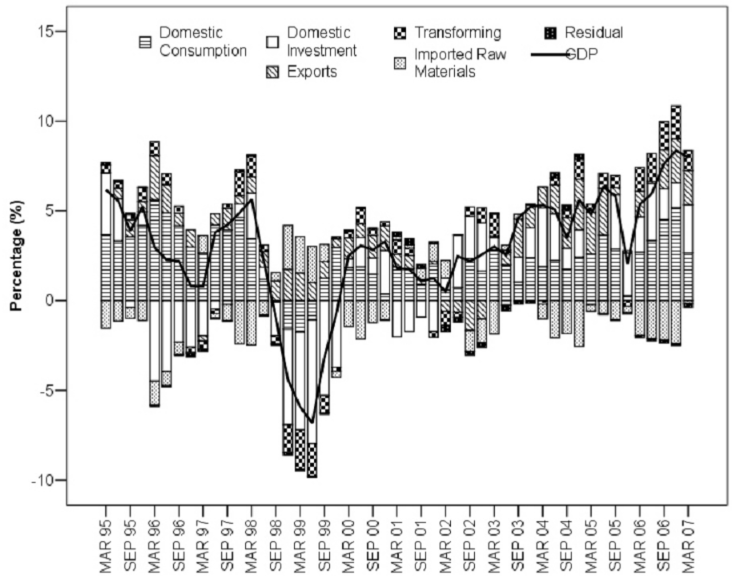
Figure 4 on page 29 plots the contributions to the growth in gross real value-added output of the different types of domestic production. Immediately we can see that the real contribution of exports is acyclical. This corresponds to the intuition that the real supply of Colombia's exports does not respond much to price incentives. Production for domestic consumption, which should include a lot of services, is the next most steady component, and that which takes the largest nominal share. Domestic investment is probably the most procyclical. As this is in large part construction, this makes sense too. The contribution to output of the commerce and distributing sector in transforming inputs also seems procyclical. This could either be because imports are procyclical or, less likely, because margins are procyclical. Finally imported raw materials exert a countercyclical influence on value-added output, again as we would expect.

4 Testing the database

4.1 Motivating our test

We have derived a database that could in principle be consistent with our theoretical model. But it is crucial to pre-test to see if this combination actually works. Our first set of tests are based on the forecasting performance of one equation from the two-good demand and supply relations. In the case of the demand for consumption of domestic production versus imported consumption items that equation is:

Figure 4: Contribution to annual real growth in real value-added output



$$s_{cdt} = \left(\frac{P_t^{cd}}{P_t}\right)^{1-\omega} \gamma_t \tag{56}$$

where s_{cdt} is the nominal share.

These demand and supply relations are a suitable basis from which assess our model-database combination. To see why, remember that in the case of domestic consumption, we did not use the theoretical price aggregator relationship to calculate the price index for domestically produced consumption. Instead we used a more general index number formula. Thus if P_t^{cd} refers to the theoretically consistent domestic good consumption deflator and $P_t^{dat,cd}$ refers to our index, then the share of domestic consumption is given by:

$$s_{cdt} = \left(\frac{P_t^{cd}}{P_t^{c}}\right)^{1-\omega} \gamma_t$$

$$s_{cdt} = \left(\frac{P_t^{dat,cd}}{P_t^{c}}\right)^{1-\omega} \gamma_t \left(\frac{P_t^{cd}}{P_t^{dat,cd}}\right)^{1-\omega}$$

$$s_{cdt} = \left(\frac{P_t^{dat,cd}}{P_t^{c}}\right)^{1-\omega} \vartheta_t \tag{57}$$

and so a composite residual term comprises the measurement error and the true parameter, γ_t,

$$\vartheta_t \equiv \gamma_t \left(\frac{P_t^{cd}}{P_t^{dat,cd}} \right)^{1-\omega}.$$

If there were any mismatch between the database index and theoretically consistent series, that would show up in the residual ϑ_t. One reason why this might happen would be if there are errors in our construction of sectoral data. But ϑ_t may also include misspecifications in the model rather than the data. For example shifts in the technical progress in the production of these goods relative to labour embodied technical progress would enter in ϑ_t, as we can see in equations 32 to 35 for example. Then the imposed model may excessively restrict preferences. For example the homotheticity implied by the CES function may be too restrictive[8].

Yet, in practice, forecasters learn to live with residuals. As there is always a cost to extending a forecasting model with more economic relationships, often forecasters will use exogenous time series models to extrapolate residuals into the forecasts. This is even true in DSGE models, although here residuals have a theoretical meaning as shocks or as theoretical parameters. Therefore what threatens the forecast in practice is not the presence of a residual per se, but whether or not that residual is difficult to forecast. For this reason, our tests are therefore based on the post-sample forecast performance of these demand and supply relationships. They are not based just on whether or not there is a large residual.

Finally we should explain why we test only the demand function for only one item in a two good system. The reason is that the second equation would be redundant under the null that the price aggregator is correct. If we tested both jointly we would running a risk of poor identification.

How does our test work? Our main test is based on a state-space model of equation 57. We favour the state-space format because it can incorporate two important features of our problem. First, as we explained in the introduction, policy forecasters who will be working with the model- database combination are likely to allow ϑ_t to vary over time and are likely to allow for serial correlated residuals. Both practices are captured by modelling ϑ_t as time-varying unobserved component. Here we assume that ϑ_t follows the AR(1) state process:

$$\ln(\vartheta_t) = \phi_{11} \ln(\vartheta_{t-1}) + u_{1t}. \tag{58}$$

Second as we are testing this equation in isolation from the rest of the model, we also need to estimate a state process to forecast the relative price series. The following state-space model accounts for both these features.

[8] Fernández-Villaverde and Rubio-Ramírez (2007) present some other examples.

The observation equation is:

$$\mathbf{y}_t = \mathbf{H}\alpha_t \tag{59}$$

and the state equation is:

$$\alpha_t = \mathbf{\Phi}\alpha_{t-1} + \mathbf{\Xi} + \mathbf{u}_t \tag{60}$$

with

$$\alpha_t \equiv \left[\ln\left(\vartheta_t\right), \ln\left(x_t\right)\right]^T; \tag{61}$$

$$\mathbf{y}_t \equiv \begin{bmatrix} \ln\left(\frac{P_t^{cd}}{P_t^c}\right) & \ln\left(s_{cdt}\right) \end{bmatrix}^T; \tag{62}$$

$$\mathbf{u}_t \equiv \begin{bmatrix} u_{1t} & u_{2t} \end{bmatrix}^T; \tag{63}$$

$$\mathbf{\Phi} \equiv \begin{bmatrix} \phi_{11} & 0 \\ 0 & \phi_{22} \end{bmatrix}; \tag{64}$$

$$\mathbf{\Xi} \equiv \begin{bmatrix} \left(1-\phi_{11}\right)\xi_1 \\ \left(1-\phi_{22}\right)\xi_2 \end{bmatrix}; \tag{65}$$

$$\mathbf{Q} \equiv \begin{bmatrix} \sigma_{u1}^2 & 0 \\ 0 & \sigma_{u2}^2 \end{bmatrix}; \tag{66}$$

and

$$\mathbf{H} \equiv \begin{bmatrix} 0 & 1 \\ 1 & 1-\omega \end{bmatrix}. \tag{67}$$

The first two equations are state equations combining the process for ϑ_t and the relative price, x_t, both in logs. The two observation equations are first a simple definition which links the state process for the relative price to the data series and second, the share demand equation itself. Thus the model allows for a time-varying ϑ_t and a time-varying share, and for the two be cointegrated jointly with the relative price. Note that all unobserved stochastic variation in the relationship is subsumed in ϑ_t, including any measurement error.

But even a simple state-space model such as this can involve some severe identification problems. Using data on the relative price and the share only it is difficult to jointly identify all the three constants and three variances. To overcome this we adopted a two-step approach. We first estimate an AR(1) process for relative prices by OLS:

$$\ln(\frac{P_t^{cd}}{P_t^c}) = \widehat{\xi}_2 + \widehat{\phi}_{22}\ln(\frac{P_{t-1}^{cd}}{P_{t-1}^c}) + \widehat{u}_{2t}^{OLS},$$
$$\text{and } \widehat{u}_{2t} \sim N(0, \widehat{\sigma_{u2}^2}). \quad (68)$$

The values of the parameter estimates for this process $\widehat{\xi_2}$, $\widehat{\phi_{22}}$ and $\widehat{\sigma_{u2}^2}$ were imposed in a second stage where we estimated the values for the remaining parameters ($\phi_{11}, \xi_1, \sigma_{u1}^2$ and ω) by maximum likelihood within the state-space model. The admissible values of parameters were restricted as follows

$$\phi_{11} \in [0, 1], \quad (69)$$

$$\omega \in [0, \infty], \quad (70)$$

$$\sigma_{u1}^2 \in [0, \infty], \quad (71)$$

and

$$\xi_1 \in \left[0.01 + \{\ln(s_{cdt})\}_{12} - \omega * \left\{\ln(\frac{P_t^{cd}}{P_t^c})\right\}_{12}, -0.01 + \{\ln(s_{cdt})\}_{12} - \omega * \left\{\ln(\frac{P_t^{cd}}{P_t^c})\right\}_{12}\right]. \quad (72)$$

Restriction 69 ensures that is a positive autocorrelated process. Restriction 70 keeps the elasticity of demand to its permissible range. $\{\ln(s_{cdt})\}_{12}$ and $\left\{\ln(\frac{P_t^{cd}}{P_t^c})\right\}_{12}$, are the mean values of the last three years of the estimation sample only. Hence restriction 72 implies that the initial value of the mean of ϑ would compensate for any systematic forecast error in the recent residuals of the share demand equation. This mechanical rule incorporate the typical policy forecaster's practice of extrapolating residuals to allow for possible structural breaks.

We also compare these state-space estimates against two simpler models for the share demand equation. The first is a simple OLS estimation where the process for γ_t is assumed to be constant with white noise:

$$\ln(s_{cdt}) = \left(1 - \widehat{\omega}^{OLS}\right) * \ln\left(\frac{P_t^{cd}}{P_t^c}\right) + \ln\widehat{\vartheta}^{OLS} + \widehat{e}_t^{OLS},$$
$$\text{and } \widehat{e}_t^{OLS} \sim N(0, \widehat{\sigma_e^2}). \quad (73)$$

The second simple model allows for an AR(1) error, such that the process for γ_t can be taken to be autoregressive, as in the state-space model:

$$\ln(s_{cdt}) = \left(1 - \widehat{\omega}^{ARML}\right) * \ln\left(\frac{P_t^{cd}}{P_t^c}\right) + \widehat{\vartheta}^{ARML} + \widehat{e}_t^{ARML},$$
$$\widehat{e}_t^{ARML} = \widehat{\rho}\widehat{e}_{t-1}^{ARML} + \widehat{v}_t^{ARML};$$
$$\text{and } \widehat{v}_t^{ARML} \sim N(0, \widehat{\sigma_v^2}). \tag{74}$$

This model is estimated by maximum likelihood taking Cochrane Orcutt estimates as initial values.

We estimate this model for five pairs of relative price and shares: domestic consumption as a share of total consumption (equation 8), the input of the distribution sector in transforming capital and consumer imports (equation 38), domestic investment relative to total investment (equation 50); raw materials relative to total imports (equation 43), and private sector domestic good consumption relative to total private consumption (equation 12). We could have applied our tests to other parts of the model, such as exports (equation 47) and domestic production (equations 17 to 20). But those equations do not straddle the tradable/non-tradable split which is the focus of this paper.

All three models are estimated on a sample which excludes the last two years' observations. In what follows, N is the estimation sample size comprising 50 quarterly observations. Our evaluation is based on the forecast performance of our models in predicting the nominal shares over the last two years of quarterly data without conditioning on any data whatsoever outside the estimation sample. So neither do we use the last two years' data on the relative prices; that series has to be forecasted also. In the state-space model, the relative price is forecasted within the model. For the two single equation models, we use equation 68 to forecast the relative price. We assess the models on the basis of RMSEs in predicting the share series.

4.2 A Monte Carlo experiment to justify our test

We need to demonstrate that our test can identify some typical problems. To do this we carry out a Monte Carlo experiment. In each replication of our Monte Carlo experiment we create fifty quarterly observations each of two nonstationary price series: a price of imported consumption components and a price for domestically produced consumption components. The number of observations in each replication roughly matches our sample of our database. We generate a series of exogenous values for ϑ_t and assume a fixed value of ω. Using this data we create a true aggregate price index and a true share which are consistent with a well defined consumption problem. The data generation processes for the true values in the experiment is as follows:

$$
\begin{aligned}
\ln P_{it}^{cd} &= \ln P_{it-1}^{cd} + 0.01 + e_{1it} \ , \\
\ln P_{it}^{cm} &= \ln P_{it-1}^{cm} + 0.005 + e_{2it} \ , \\
\text{and } \ln \vartheta_t &= 0.6 * 0.85 + 0.4 * \ln \vartheta_{t-1} + e_{3it}. \\
e_{1it} &\sim N(0, 0.01) \, ; \\
e_{2it} &\sim N(0, 0.005) \, ; \\
e_{3it} &\sim N(0, 0.01) \, ; \\
P_{it}^{c} & \text{ as in equation 10 ;} \\
s_{cdt} &\equiv \frac{P_{it}^{cd} C_{Dit}}{P_{it}^{c} C_{it}} \text{ as in equation 8;} \\
\omega &= 1.5; \\
i &= 1, .., 500; \ t = 1, .., 50 \\
\text{and } P_{i1}^{cd} &= P_{i1}^{cm} = 1.
\end{aligned}
$$

In each replication, we derive an approximation to the price for domestically produced items, $P_{it}^{rep,cd}$, exactly as we did to create our data base. We apply this method across three different cases, in two of which we have introduced measurement problems of the type we could have made.

In the first case, we derive our approximation combining the correct series for the aggregate consumer price, the imported consumer price and the correct share data (but only the annual averages) in a Tornqvuist discrete approximation to a Fisher ideal value-added index. Given the favourable properties of a Fisher index on good data, we would expect this first experiment to deliver a reasonably predictable residual series, even if the shares are only updated annually.

In a second experiment we build in a measurement error into the imported consumption price data only; now the measured imported price grows at a 1pp faster trend rate and has extra 10 pp standard deviation of noise around the true series. The measurement error also features a Markov regime switch which can jump back and forth from a 2pp higher level with 50% transition probability in the last ten quarters. This is an interesting challenge because the last eight of those ten quarters will be in the post-estimation sample. This incorrect series is combined with the correct nominal share data and the true aggregate price data to derive an infected domestically produced consumption series (again in a Fisher index).

A third case explores what happens when only the share data is wrong. As in the first case, the share data is altered only once a year. But it is now out of date: the share data is the harmonic mean of the true shares for the previous year, and not the current year. We also allowed for the same Markov regime switching process to affect the measurement in the last ten periods as we did with the price measurement, except now the switch was to a 5pp higher level.

For each set of generated data (500 replications of each of the three cases) we estimate on a sample that excluded the last eight observations. The state-space model and also the two single equation models were tested on the post estimation sample, as explained in Section 4.1. It is worth emphasising that for the experiment to be realistic, the post estimation sample must also be derived from flawed data in the second two cases. Notice also that the experiment allows for parameter estimation uncertainty.

Tables 5 and 6 reports the Monte Carlo mean, standard deviation, mean and mode of the RMSE for the first and the second year of the forecast. The mode RMSEs from the two single equation models are also there. And Figure 5 on page 36 plots the distribution of RMSE of the first case against each of other two over each year. Figure 6 on page 37 plots the distribution of the second two cases for the state-space estimation against the fixed parameter OLS results, again over each year.

Table 5: Monte Carlo results for average of year 1

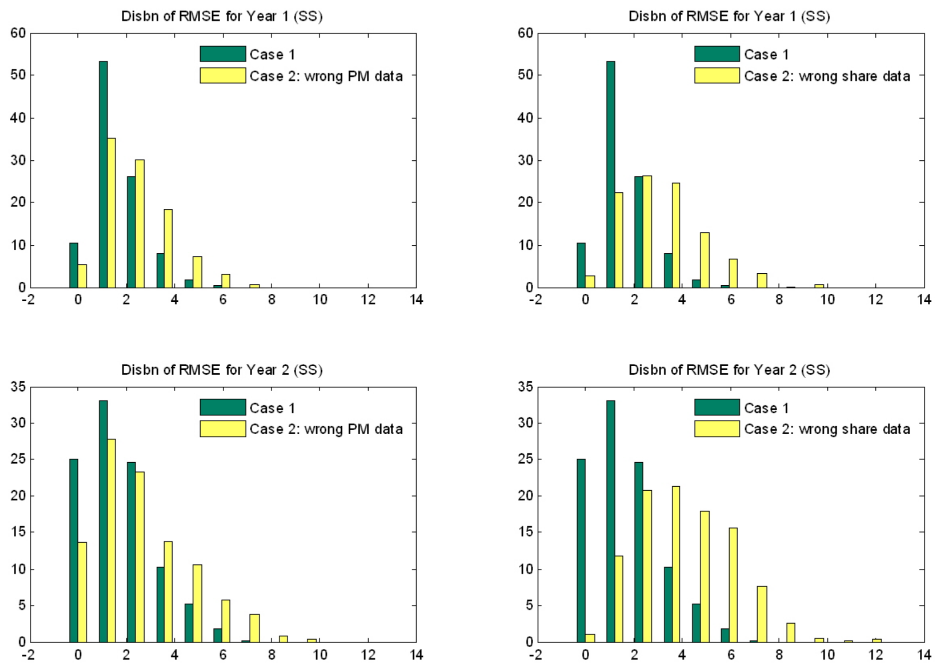
	State-Space			**OLS**	**AR1**
	RMSE (mean)	RMSE (mode)	RMSE (sd)	RMSE (mode)	RMSE (mode)
Case 1	1.64%	0.17%	0.95%	0.04%	0.26%
Case 2	2.33%	0.52%	1.26%	0.09%	0.15%
Case 3	3.09%	0.37%	1.82%	0.19%	0.44%

Table 6: Monte Carlo results for average of year 2

	State-Space			**OLS**	**AR1**
	RMSE (mean)	RMSE (mode)	RMSE (sd)	RMSE (mode)	RMSE (mode)
Case 1	1.88%	0.02%	1.27%	0.01%	0.05%
Case 2	2.67%	0.02%	1.90%	0.03%	0.05%
Case 3	4.10%	0.51%	2.05%	0.94%	0.16%

A first important confirmation is that the model can produce a low RMSE given good data and the correct parameters even though only annually updated shares are available. The mode RMSE of 0.17pp in year one can be interpreted in terms of implying an error of 0.17% in either the relative price or the quantity of domestic consumption at the end of the first year of the forecast. That is not much considering

Figure 5: Monte Carlo distribution of RMSE (state-space Models)



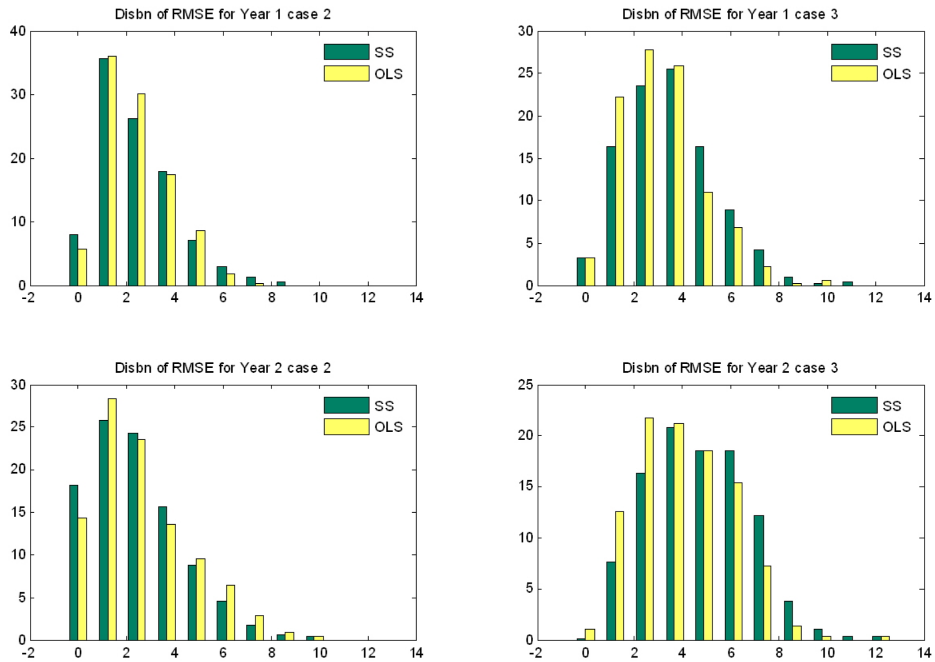
the RMSE has a irreducible component in any estimated model let alone one which has to make do with annually updated share data.

We can compare this best case to the cases where the relative price and the share data are respectively wrong. The comparison reveals that the test works. With the wrong relative price data, the mode RMSE is 0.35 pps larger than in case 1 in year 1, although the same in year 2. It also comes with a larger standard deviation. Comparing the wrong share data case, Case 3, with Case 1, the RMSE is 0.20 pp higher in year 1 but dramatically higher — 0.49 pp — in year 2. What is perhaps remarkable is that the RMSE test works even if the post sample data realistically contains errors in construction. As we shall shortly see this is not the case for insample measures of fit.

Note also that the mode RMSE is not always much worse in the OLS estimate than in the state-space model estimates. But the OLS estimate fares much, much worse in the case of the wrong share data. So the common strategy of allowing for a time-varying parameter does not always buy success but perhaps minimises (but cannot eliminate) the risk of large forecast break downs.

The AR model does not do very well compared to either alternative, possibly because the maximum likelihood estimation is not very robust to shifts in the constant (Pesaran and Timmerman, 1994). So in so

Figure 6: Monte Carlo distribution of RMSE (state-space versus OLS)



far as we want to allow for a time-varying parameter ϑ, we should use a state-space model rather than an AR compromise.

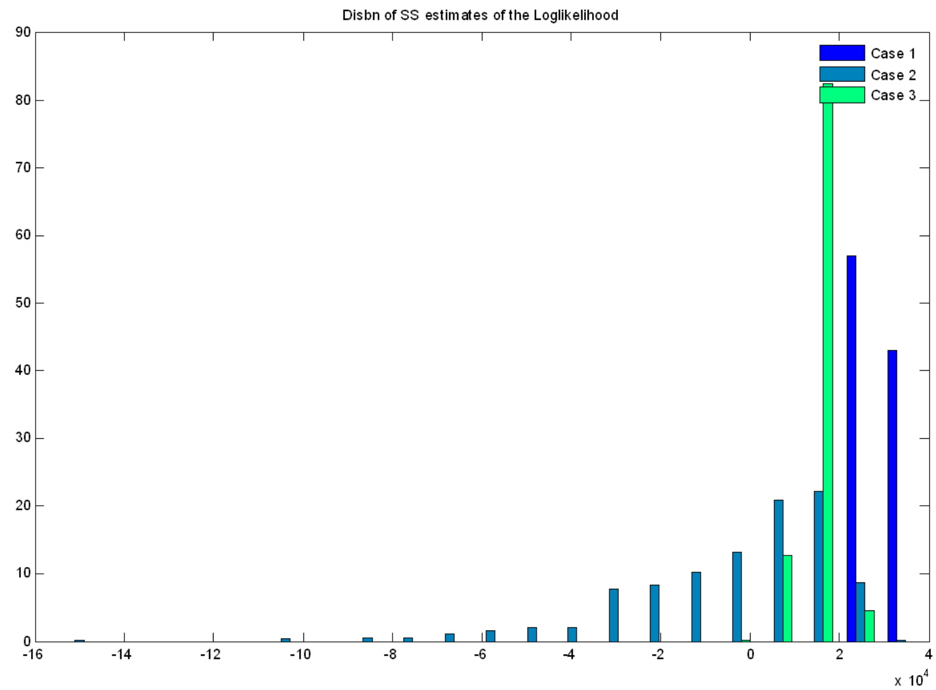
The exercise also demonstrates that none of the three methods can a priori contain the risk of serious forecast error. Comparing the mean and mode RMSEs in the state-space estimations reveals that there is always a large skew. The charts reveal a fat upper tail, especially in the two cases that the data is badly measured (see Figure 6 on page 37). The same is true of the OLS and ARML estimates (Figure 5 on page 36). Thus, it is difficult to identify structural breaks in advance, with or without time-varying parameters.

Figures 7 and 8 plot the Monte Carlo distributions of the loglikelihood and the Schwartz Bayesian Criterion (SBC) of the state-space estimates of the three models. In Cases 2 and 3, these measures of goodness of fit are calculated by fitting a model estimated on the incorrect data to the best (Case 1) data.

Both the log likelihood and the SBC, which penalises the log likelihood for too many parameters, pick out the best (Case 1) model. But remember that here we are comparing log likelihoods on the best (most accurate) Case 1 dataset. Therefore the charts are only confirming that in an ideal world we could discriminate between models using in-sample goodness of fit.

Figures 9 and 10 describe the more realistic scenario where we are using purpose-built and possibly flawed

Figure 7: Monte Carlo distribution of the loglikelihood



datasets to assess models.

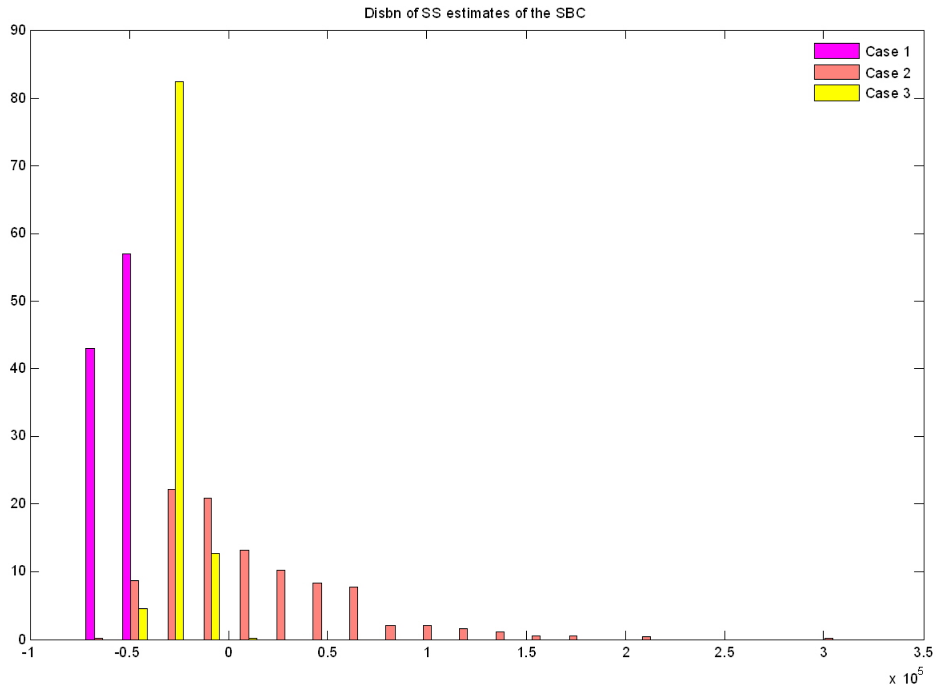
Specifically we now use the Case 2 dataset (which uses flawed imported prices) to compare the best parameter estimates made on the Case 1 data against the Case 2 estimates. We can see that when there are errors in the purpose-built data used for estimation, neither the log likelihood nor the SBC would pick out the model that would forecast the best, on the best dataset[9].

That a measure of insample goodness of fit is not a guarantee of good forecast performance, and often quite the converse, is well established in the literature. See for example Mayer (1975) and more recently Aznar, Ayuda, and García-Olaverri (2001). Here we have only revisited that finding in the different context of models where data is purpose-built.

The distribution of the RMSE is not normal in this small sample. This also has important implications for how we calculate the RMSE. In the introduction we argued that we have to take account of parameter estimation uncertainty in our estimated distributions; it really seems to really matter in forecasting error.

[9]If we were estimating our models by Bayesian methods, the natural analogue to our RMSE test would be the ratios of predictive marginal likelihoods of the RMSEs calculated for a post estimation sample. Our tests suggest that the more common comparison instead based on the ratio of marginal likelihoods calculated for the whole sample could be very misleading. See Geweke (2005) and especially Adolfson, Andersson, Lindé, Villani, and Vredin (2005) or Adolfson, Lindé, and Villani (2005). Given that we have decided not to estimate by Bayesian methods, the RMSE seems the most robust and direct joint test of the model and its partner database.

Figure 8: Monte Carlo distribution of SBC



There are two routes available to do this. The first is to calculate the analytical standard errors of the RMSE which require normality, among other things (Ansley and Kohn, 1986). The alternative approach is to bootstrap the distribution, as explained in Shumway and Stoffer (2000).

Table 7 on page 40 reports calculations of the analytical RMSEs with and without taking account of parameter uncertainty[10]. Although the ranking of the three cases is broadly similar, the scale of RMSEs seem very different to that from our Monte Carlo exercise reported in Tables 5 and 6. So our Monte Carlo experiments point us in the direction of bootstrapping and away from any further inference from analytical RMSEs.

[10]The calculation with parameter uncertainty was made as follows. Let $\mathbf{P}_{N+s|N}$ be the MSE matrix of the forecast of the state variables α_{N+s} at time using information up to time N without taking account of parameter uncertainty and $\mathbf{I}(\psi)$ be the information matrix at parameter values ψ. Ansley and Kohn (1986) offer the following approximation for the MSE that takes account of parameter estimation uncertainty:

$$\mathbf{P}_{N+s|N} + \frac{d\alpha_{N+s|N}}{d\psi} [\mathbf{I}(\psi)]^{-1} \left[\frac{d\alpha_{N+s|N}}{d\psi}\right]^T,$$

which can be evaluated at the maximum likelihood estimates. The expression we used to calculate the MSE matrix for the forecast of the observed variables $\mathbf{y}_{N+s}$ taking account of parameter uncertainty was

$$\mathbf{HP}_{N+s|N}\mathbf{H}^T + \left[\mathbf{H}\frac{d\alpha_{N+s|N}}{d\psi} + \frac{d\mathbf{H}}{d\psi}\alpha_{N+s|N} + \frac{d\mathbf{D}}{d\psi}\right] [\mathbf{I}(\psi)]^{-1} \left[\mathbf{H}\frac{d\alpha_{N+s|N}}{d\psi} + \frac{d\mathbf{H}}{d\psi}\alpha_{N+s|N} + \frac{d\mathbf{D}}{d\psi}\right]^T$$

Harvey (1991) provides one set of recursive algorithms to calculate $\frac{d\alpha_{N+s|N}}{d\psi}$ and $\mathbf{I}(\psi)$ but other (sometimes more efficient and reliable) methods are also available.

Figure 9: Monte Carlo distribution of the loglikelihood on wrong data

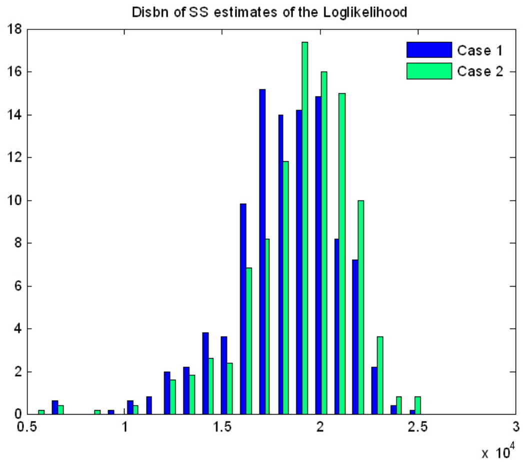


Table 7: Analytical RMSE Monte Carlo results

	without parameter uncertainty		**with parameter uncertainty**	
	RMSE (mean)	RMSE (mode)	RMSE (mean)	RMSE (mode)
Case 1	0.91% (1.14)	0.33% (0.41)	0.93 (1.16)	0.33 (0.42)
Case 2	1.01% (1.27)	0.29% (0.37)	1.05 (1.32)	0.30 (0.39)
Case 3	1.83% (2.30)	1.28% (1.61)	1.93 (2.42)	1.29 (1.62)

Note. Average of quarterly year 1 forecasts (average of year 2 forecasts in brackets)

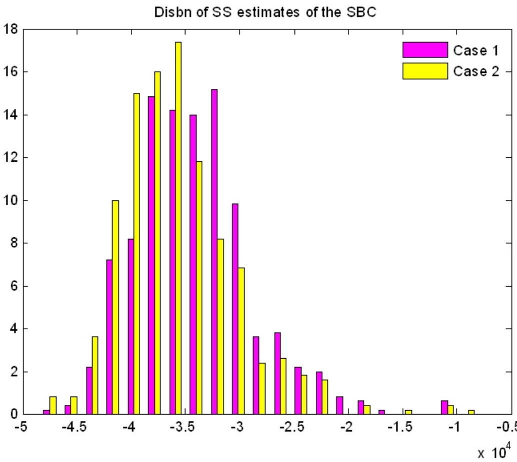
In conclusion our Monte Carlo experiment proves first, that our post sample RMSE diagnostic can pick out some likely problems to do with either the model or the database; and second, we should take account of a time-varying parameters in order to make our test fair; but third that we need to compare at near and far horizons; fourth, that the distribution and not just the mode of the RMSE matters and fifth that we should calculate the bootstrapped distribution of the RMSE which accounts for parameter uncertainty, rather than use analytical standard errors. These valuable lessons are incorporated in our testing of the actual data.

5 Tests of five single demand and supply systems in the model

5.1 The challenge

Our challenge is plotted in Figure 11 on page 42. The estimation dataset on relative prices and nominal shares is restricted to the left of the black line. The economic structure as defined by the CES functions of

Figure 10: Monte Carlo distribution of SBC on wrong data



the model and as generalised by the state-space model is estimated only on that part of the data. The aim is to forecast the data on nominal shares to the right of the line.

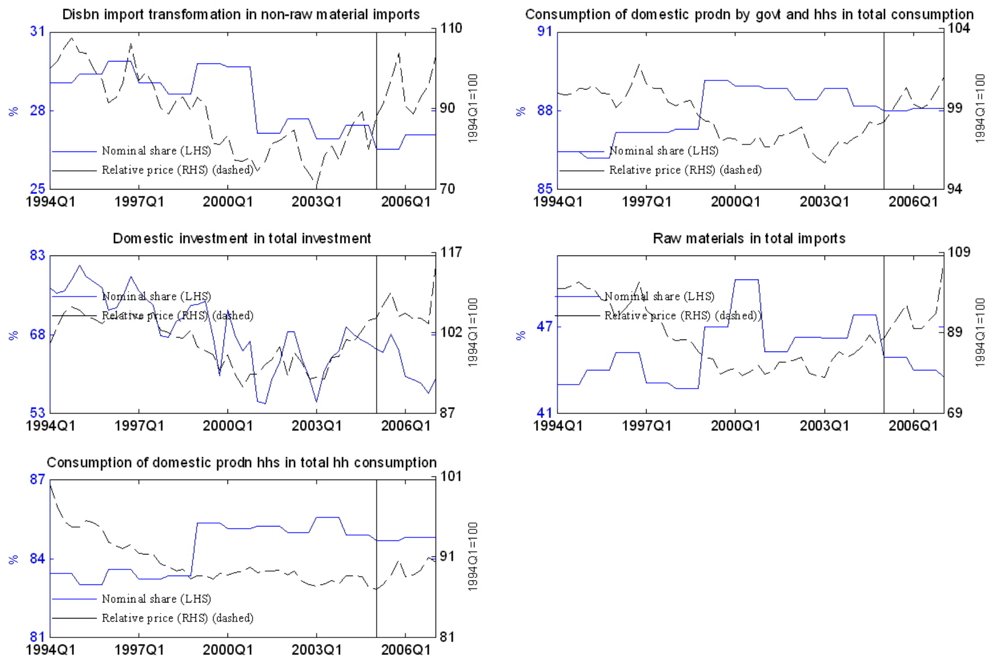
Immediately one can see that this will not always be easy. Typically the share data is characterised by irregular cycles. This makes the trade off between anticipating a turning point or chasing the recent trend in the forecast period difficult. If the model predicts the relative price data well that might help, for example in nearly all series the relative prices are rising towards the end of the sample, implying a fall in share if the two components are complements. But as we shall see, often there still remains a difficult challenge.

Our contention is that this challenge would typically carry over to the full model so that evaluating the forecasts of these systems is very informative for the complete forecast. Looking over the CES demand and supply equations, such as equation 8, we can see that the relationship between shares and relative prices is independent of other variables in the model. Maybe we can forecast relative prices or real quantities more accurately in a full model (possibly because we have more information on nominal and real dynamics). However the literature that evaluates full model forecasting warns us that parsimonious reduced form models are not easily beaten. On these grounds we maintain that this small problem is a useful microcosm of the whole forecasting problem, especially when the data is partly designed for the model.

5.2 Bootstrapping

The Monte Carlo experiment favoured a bootstrapped distribution of our parameter estimates. It is important to clarify how we carried out that bootstrapping. The residuals on which we perform the bootstrap should not be serial correlated nor feature heteroskedasticity (Davidson and MacKinnon, 2006). But esti-

Figure 11: Data on five demand and supply relationships



mates of the one step ahead prediction error in the state, $\widehat{\mathbf{u}}_t$, featured both.

Our solution was to model a general autoregressive, heteroskedastic process for these one step ahead residuals, in standardised form. The model we chose was the Bayesian AR(4) heteroskedastic model as described in Geweke (2005) and LeSage (2003). The residuals that drive that process were estimated, and tested and found to be white noise. Thus the bootstrapping was performed on these underlying residuals and not on the untreated one step ahead state prediction errors. But of course this estimated process for $\mathbf{u}_t$ was taken into account when generating the bootstrapped estimates of the RMSEs and of the parameter values of the state-space model.

For completeness we formally describe the process as follows. Let $\widehat{\mathbf{P}}$ be the estimated MSE matrix of the one step ahead state variables and let $\widehat{\mathbf{K}}$ be that of the Kalman filter gain (both taken at the maximum likelihood values). Then bearing in mind equations 59 to 67, define:

$$\widehat{\mathbf{F}} \equiv \widehat{\mathbf{H}}\widehat{\mathbf{P}}\widehat{\mathbf{H}}^T + \widehat{\mathbf{Q}}.$$

Our process for the residuals was then

$$\widehat{\mathbf{F}}^{-\frac{1}{2}}\widehat{\mathbf{u}}_t = \left[\widehat{\mathbf{F}}^{-\frac{1}{2}}\widehat{\mathbf{u}}_{t-1}, \cdots, \widehat{\mathbf{F}}^{-\frac{1}{2}}\widehat{\mathbf{u}}_{t-4}\right] * \beta + \mathbf{e}_t \text{ with } \beta^T \equiv \begin{bmatrix} \beta_1 & \cdots & \beta_4 \end{bmatrix};$$

$$\mathbf{e}_t \sim N\left(0, \sigma^2\mathbf{V}\right) \text{ with } \mathbf{V} \equiv \begin{bmatrix} v_1 & \mathbf{0} & 0 \\ \mathbf{0} & \ddots & \mathbf{0} \\ 0 & \mathbf{0} & v_N \end{bmatrix};$$

$$\beta \sim N\left(\mathbf{c}, \mathbf{T}\right);$$

$$\frac{1}{\sigma^2} \sim \Gamma\left(\nu, d_0\right);$$

and

$$\frac{r}{v_i} \sim ID\ \frac{\chi^2\left(r\right)}{r}$$

The prior value for r was 1, suggesting much heteroskedasticity. The other priors were diffuse: $\mathbf{c} = \mathbf{0}, \mathbf{T} = \mathbf{0}, \nu = 0$ and $d_0 = 0$. The model was estimated using Monte Carlo Markov Chain sampling, for 10100 draws omitting 100[11]. Stability conditions were imposed on the AR coefficients using Gibb sampling and the mean acceptance rates were all over 80%.

The bootstrapping was performed on the residuals for this process once they have been adjusted for heteroskedasticity; i.e. we bootstrapped the series $\left(E(\mathbf{v}_t)\right)^{-0.5}\mathbf{e}_t$ with $E(\mathbf{v}_t)$ being the estimated posterior mean of parameter the heteroskedasticity standard deviation, $\mathbf{v}_t$. Hence

$$\left[\mathbf{w}_i, \cdots, \mathbf{w}_N\right] = \text{sample with replacement from } \left[\left(E(\mathbf{v}_4)\right)^{-0.5}\mathbf{e}_4, \cdots, \left(E(\mathbf{v}_N)\right)^{-0.5}\mathbf{e}_N\right].$$

These bootstrapped residuals were used to generate different series for the standardised one-step-ahead residuals:

$$\widehat{\mathbf{F}}^{-\frac{1}{2}}\widehat{\mathbf{u}}_{it} = \left[\mathbf{1}, \widehat{\mathbf{F}}^{-\frac{1}{2}}\widehat{\mathbf{u}}_{it-1}, \cdots, \widehat{\mathbf{F}}^{-\frac{1}{2}}\widehat{\mathbf{u}}_{it-4}\right] E\left[\beta\right] + \left(E(\mathbf{v}_t)\right)^{0.5}\mathbf{w}_{it} \text{ for } t = 1, \cdots, N$$

where $E\left[\beta\right]$ is the estimated posterior mean of β and then new data series using

$$\rho_{it} = \begin{bmatrix} \widehat{\mathbf{\Phi}} & \mathbf{0} \\ \widehat{\mathbf{H}} & \mathbf{0} \end{bmatrix} \rho_{it-1} + \begin{bmatrix} \widehat{\mathbf{D}} \\ \widehat{\mathbf{\Xi}} \end{bmatrix} + \begin{bmatrix} \widehat{\mathbf{K}}\widehat{\mathbf{F}}^{\frac{1}{2}} \\ \widehat{\mathbf{F}}^{\frac{1}{2}} \end{bmatrix} \widehat{\mathbf{F}}^{-\frac{1}{2}}\widehat{\mathbf{u}}_{it}$$

[11]We used the econometric toolkit of LeSage (2003).

for $\rho_{it} = \begin{bmatrix} \alpha_{it} \\ \mathbf{y}_{it} \end{bmatrix}$ as in Shumway and Stoffer (2000). All parameters are at their maximum likelihood values and the same initial values for the states as used in calculating the maximum likelihood estimates. This was repeated for $i = 1, ..., 500$ (the number of bootstrapped samples).

In Section 4.1 we described on how some of the parameters had to be pre-estimated and imposed before we calculate the maximum likelihood estimate. That first pre-maximum likelihood step was also bootstrapped; all those parameters were re-estimated on each bootstrapped sample. But for brevity we only report the bootstrapped distribution of the four parameters that were estimated in the second, maximum likelihood, stage.

5.3 Results

We can now turn to the results of our bootstrapped state-space estimations. Table 8 on page 44 reports the RMSE from all five individual demand and supply relationships but the whole distribution is in Figures 15 to 19 in the appendix.

Table 8: Estimates and tests for five supply and demand relationships

RMSE estimates, calculated from the state-space.				
	Max. likelihood	Bootstrap distribution		
	est.	mean	mode	sd
Domestic consumption of gvt and hhds	1.07 (1.33)	1.47 (1.46)	0.12 (0.01)	0.93 (0.94)
Domestic investment	9.76 (21.41)	22.67 (39.16)	8.39 (9.19)	7.67 (9.52)
Distribution in consumption and investment imports	8.20 (4.19)	17.91 (9.58)	1.69 (0.12)	7.48 (7.48)
Raw materials in imports	5.28 (9.05)	12.75 (13.84)	1.54 (0.93)	9.88 (10.52)
Domestic consumption of households	1.87 (1.67)	2.11 (2.05)	0.08 (0.03)	1.34 (1.33)

* average over year one of the forecast (average over year two in brackets).

At first glance, the RMSEs in Table 8 on page 44 all seem large. But these are forecasts made with simple single equation models. We would argue that the lowest RMSE here are consistent with what could be satisfactory performance when the whole model is put to forecast and combined with off model judgement. That said, we would also contend that the results warn us in advance where we would expect problems further down the line.

We can begin with the RMSE for consumption. Here there seems little risk of forecast error originating in the relations in the demand for domestically produced consumption for government and households together. The mode RMSE is very low, especially for the second year, at about 0.12. While there is slightly more forecast error in the consumption problem just for households, the size of the error remains low enough not to cause alarm there either. The greater error might indicate either that the difference between the consumption deflator and the CPI brings with it some cost, or that our assumption that the price of domestic consumption is the same for government and for households. These two results justify this part of our created dataset and are more important because forecasting inflation well matters more.

The mode RMSE in predicting distribution output share in the transformation of non raw material imports is larger at 1.7%. Given all the assumptions we had to employ to get this data, this is also perhaps reassuring. Note however that the bootstrapped mean is much higher than the bootstrapped mode, indicating that there is a risk of some large errors. We can expect a risk of forecast breakdown here. The test also indicates that the separation of raw materials from total imports does involve some unpredictability: the mode RMSE here is 1.54% for one year ahead.

But the greatest error by far is in the disaggregation of investment into its domestically produced and foreign produced components. The mode RMSE is 8.39% for the first year and then 9.18% by the second. The mean RMSE values are much higher and the bootstrapped distribution has a fat upper tail. Clearly the model is missing some of the cyclical behaviour in investment.

Tables 9 to 13 summarise the distribution of parameter estimates. Figures 23 to 27 in the appendix plot the whole bootstrap distributions for the four parameters.

Table 9: Domestic consumption of government and households

	Max. likelihood	Bootstrap distribution			
	est.	mean	mode	sd	90% limits
ϕ_{11}	0.23	0.34	0.24	0.04	
100*σ_{11}	0.93	0.72	0.47	0.13	$[0.67, 1.06]$
100*exp(ξ_1)	87.6	88.04	87.16	0.28	
ω	1.69	1.45	0.99	0.10	

Table 10: Domestic investment

	Max. likelihood	Bootstrap distribution			
	est.	mean	mode	sd	90% limits
ϕ_{11}	0.02	0.24	0.06	0.11	
100*σ_{11}	9.24	12.32	4.55	4.67	$[6.23, 20.46]$
100*exp(ξ_1)	64.24	70.17	60.66	3.02	
ω	0.20	0.54	0.24	0.43	

Table 11: Distribution in consumption and investment imports

	Max. likelihood	Bootstrap distribution			
	est.	mean	mode	sd	90% limits
ϕ_{11}	0.13	0.17	0.13	0.03	
100*σ_{11}	5.21	4.16	1.02	1.57	$[2.03, 7.04]$
100*exp(ξ_1)	28.72	29.37	26.98	0.96	
ω	0.77	0.96	0.35	0.14	

Table 12: Raw materials in imports

	Max. likelihood	Bootstrap distribution			
	est.	mean	mode	sd	90% limits
ϕ_{11}	0.03	0.19	0.17	0.05	
100*σ_{11}	4.55	4.26	1.51	1.86	$[2.12, 8.04]$
100*exp(ξ_1)	44.38	44.30	36.99	1.68	
ω	1.23	1.15	0.24	0.36	

The estimated elasticity of substitutions (ω) all seem quite sensible. For example notice that the value for consumption indicates limited substitutability between domestically produced and foreign made items: the mode values are quite close to the Cobb-Douglas restriction of one. Comparing Table 9 on page 45 and Table 13 on page 47, it appears that households are less likely to substitute in between domestic production and imports than is government. This might seem odd, bearing in mind that the government as a service sector employs a larger proportion of domestic value-added factors of production than a typical tradable sector would do. But the government also imports some of its consumption in Colombia (for example, military consumption). The mode elasticity also indicates complementarity between domestic and imported items in the distribution transformation problem, which seems realistic. Raw material imports are found to be complements to consumption and investment items. That too seems plausible.

Foreign and domestic investment are judged to be very strong complements; the data would have the mean elasticity of substitution close to the permissible lower bound of Leontieoff, and the mean value is at about 0.5. This is some tentative evidence for a strong income effect associated with investment such that when either domestic or foreign investment becomes cheap, spending on both rises. A more general model than the CES form such as an translog system might then feature less forecast error.

In all cases the 90% limits of the standard deviation of the unobserved component lie above zero. Thus the data favour time variation in ϑ over a fixed coefficient. The estimated distributions of the parameter

Table 13: Domestic consumption of households

	Max. likelihood	Bootstrap distribution			
	est.	mean	mode	sd	90% limits
ϕ_{11}	0.23	0.29	0.23	0.05	
100*σ_{11}	1.09	0.82	0.28	0.34	$[0.43, 1.47]$
100*exp(ξ_1)	82.24	83.39	80.98	1.14	
ω	1.29	1.08	0.39	0.16	

ϕ_{11} in each model are also quite revealing. In the case of investment especially this value is quite low. This means that the unobserved state is not autoregressive and then there is very little information from past values that the Kalman Filter can use to build a forecast. It seems that the investment series is a forecaster's nightmare because it is both volatile and not persistent.

Of course, investment is one of the most difficult aspects of macroeconomic forecasting generally. This is reflected in the post sample RMSEs reported by Smets and Wouters (Smets and Wouters, 2007) model for the U.S., for example. But could we adapt our tests to investigate why this is so in Colombia? One possibility is that the large forecast error is due to the inventories component, which is very irregular and also contains the National Accounts residual[12]. We could test for this by repeating our tests on fixed investment only. Although we do not have quarterly data on this separation, some split may be derived from the annual series. But then even if the tests report that the forecast error diminishes substantially, that would only improve the forecast for the model as a whole if the tactic of separating out inventories somehow brings with it more information. So we should also apply our test to an equation in inventories only, and then test for the possibility that the two equations working separately forecast total investment worse than the aggregate equation. Indeed the input-ouput tables report that one group of sectors carry out 95% of investment while all other sectors hold about 95% of inventories. This would suggest that the classification between inventories and investment is made on the basis of sectors, and otherwise they might be economically similar concepts.

In summary our tests reveal where we can more be reassured about the model, and also where we can expect problems. Our distributions of parameter estimates also help us to scout out some possible avenues for solutions. The largest error is expected in the modelling of investment. This may be due to poor fixed investment data; it may be because there are irregular cycles in inventories; it may be because inventories are where the National Accounts office allocates its residual; it may also be due to uncaptured aspects of tastes and technology. We also found some evidence that the homothetic CES functional form may be restricting

[12] Although, a comparison of national accounts inventory data with surveys of inventory accumulation in Colombia indicate that the movements in the national accounts inventories are by no means all due to national accounts residual allocation.

the investment model excessively. The tests could be adapted to test at least a few of these possibilities in advance of building a full model.

6 A test of the transformation of domestic output

The transformation problem in the model (in Section 2.4) implies a system of four supply equations. As the elasticity of substitution is common across the relations, it is not efficient to apply our test four separate state-space models. So in this section we develop a system version of our previous test.

The observation equation is:

$$\mathbf{y}_t = \mathbf{H}\alpha_t \tag{75}$$

and the state equation is:

$$\alpha_t = \mathbf{\Phi}\alpha_{t-1} + \mathbf{\Xi} + \mathbf{u}_t \tag{76}$$

with the state vector given by:

$$\alpha_t \equiv \left[\ln\left(\vartheta_{ct}\right), \ln\left(\vartheta_{xt}\right), \ln\left(\vartheta_{Tt}\right), \ln\left(x_{1t}\right), \ln\left(x_{2t}\right), \ln\left(x_{3t}\right)\right]^T$$

and the observed data vector as:

$$\mathbf{y}_t \equiv \begin{bmatrix} \ln(\frac{P_t^{cd}}{P_t^Q}) & \ln(\frac{P_t^{xd}}{P_t^Q}) & \ln(\frac{P_t^T}{P_t^Q}) & \ln\left(\frac{P_t^{cd}C_{Dt}}{P_t^q Q_t}\right) & \ln\left(\frac{P_t^{xd}X_{Dt}}{P_t^q Q_t}\right) & \ln\left(\frac{P_t^T T_t}{P_t^q Q_t}\right) \end{bmatrix}^T.$$

The error terms are:

$$\mathbf{u}_t \equiv \begin{bmatrix} u_{11t} & u_{12t} & u_{13t} & u_{21t} & u_{22t} & u_{23t} \end{bmatrix}^T \tag{77}$$

with a variance covariance matrix of:

$$\mathbf{Q} \equiv \begin{bmatrix} \sigma_{u11}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{u12}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{u13}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{u21}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{u22}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{u23}^2 \end{bmatrix}; \tag{78}$$

$$\mathbf{\Xi} \equiv \begin{bmatrix} (1-\phi_{111}) * \xi_{11} \\ (1-\phi_{122}) * \xi_{12} \\ (1-\phi_{133}) * \xi_{13} \\ (1-\phi_{211}) * \xi_{21} \\ (1-\phi_{222}) * \xi_{22} \\ (1-\phi_{233}) * \xi_{23} \end{bmatrix}; \tag{79}$$

$$\mathbf{\Phi} \equiv \begin{bmatrix} \phi_{111} & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_{122} & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_{133} & 0 & 0 & 0 \\ 0 & 0 & 0 & \phi_{211} & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_{222} & 0 \\ 0 & 0 & 0 & 0 & 0 & \phi_{233} \end{bmatrix}; \tag{80}$$

and

$$\mathbf{H} \equiv \begin{bmatrix} 0 & I_{3\times3} \\ I_{3\times3} & (1-\omega) * I_{3\times3} \end{bmatrix}. \tag{81}$$

The restrictions on our parameters are different because now we are estimating an elasticity of supply, not demand.

$$\widehat{\phi}_{11} \in [0,1], \tag{82}$$

$$\widehat{\omega} \in [-\infty, 0], \tag{83}$$

$$\widehat{\sigma}^2_{u1} \in [0, \infty], \tag{84}$$

and

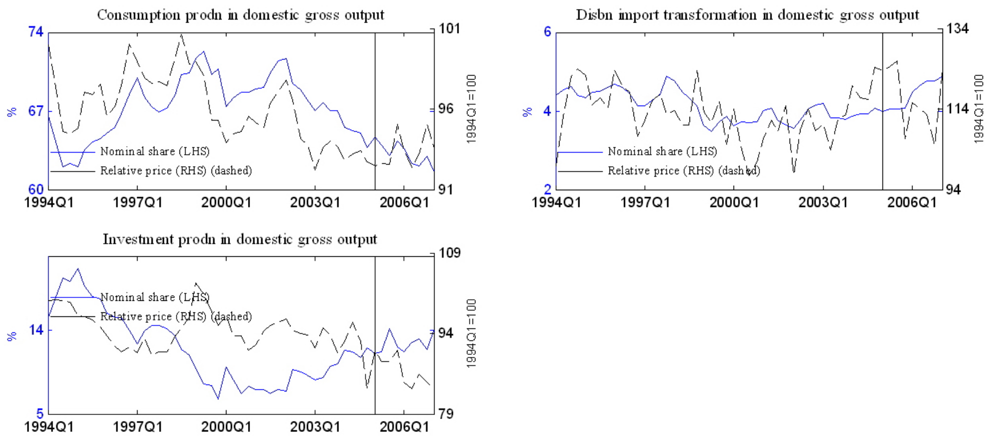
$$\exp(\widehat{\xi}_{11}) \in [0,1], \exp(\widehat{\xi}_{12}) \in \left[0, 1-\exp(\widehat{\xi}_{11})\right], \widehat{\xi}_{13} \in \left[0, 1-\exp(\widehat{\xi}_{11}) - \exp(\widehat{\xi}_{12})\right]. \tag{85}$$

The data is plotted in Figure 12 on page 50 and the RMSE results are in Table 14 on page 50.

The RMSEs of the system (in Table 14 on page 50 and in Figures 20 to 22 in the appendix) are much less reassuring than that for the individual demand relationships of the previous section. The RMSE of consumption has a mode of 2.49 and 6.01 for one and two years out respectively. The RMSE for the domestic production of the distribution sector output is also high, but the RMSE for domestic investment are even worse. Mode values of 22% and 13% there indicate the model will very likely fail spectacularly in picking either the level or price of investment or both. The mean values are enormous.

Tables 15 to 17 summarise the parameter estimates with the whole distributions plotted in the appendix

Figure 12: Data on transformation of domestic production



(Figures 28 to 30).

The elasticity of substitution is close to zero, indicating that domestic production cannot easily switch from one form of output to another. That seems both very plausible and also interesting. Most models do not impose rigidities in switching factors of production (especially labour) across sectors. These results suggest that such restrictions matter.

But the results might also be pointing towards misspecification, possibly in the direction of a more general non-homothetic production function. The other estimates of the other parameters reveal more problems. The bootstrapped standard deviations of all the other parameter estimates are very low. This indicates that the state-space system estimates have not updated much from initial values. In the light of the high RMSEs, this is not reassuring.

In conclusion we adapted and applied our test to the system of transformation of domestic production. The results indicated a great risk of forecast error. This might be because the transformation function is too restrictive. One solution could be to split these sectors entirely. But then we would need to construct

Table 14: Estimates and tests for five supply and demand relationships

	State-space (bootstrap RMSE 1 Year)			
	ML est.	Mean	Mode	sd
Domestic production of consumption	3.93 (11.77)	8.26 (13.21)	2.49 (6.01)	2.60 (1.71)
Domestic production of distribution	13.78 (31.18)	10.65 (28.33)	6.22 (3.48)	3.02 (4.34)
Domestic production of investment	26.87 (43.26)	64.85 (63.45)	21.79 (12.33)	18.93 (9.17)

Table 15: Domestic production of consumption

	ML est.	Mean	Mode	sd	90% limits
ϕ_{11}	0.36	0.31	0.36	0.06	
100*σ_{11}	2.18	3.60	2.18	1.99	$[2.18, 8.21]$
100*exp(ξ_1)	66.99	67.02	66.99	0.035	
ω	-0.068	-0.112	-0.051	0.057	

Table 16: Domestic production of distribution

	ML est.	Mean	Mode	sd	90% limits
ϕ_{11}	0.78	0.78	0.78	0.00	
100*σ_{11}	2.56	2.56	2.56	0.01	$[2.56, 2.58]$
100*exp(ξ_1)	3.91	3.91	3.91	0.00	
ω	-0.068	-0.112	-0.051	0.057	

data on the separate factor markets, using assumptions which could bring more forecast error. A more general version of the output transformation that allows for different elasticities of substitution between different types of production or perhaps just relative taste and technology shifts are other avenues. All these interesting possibilities can be tested prior to use.

7 A test of robustness to data revisions

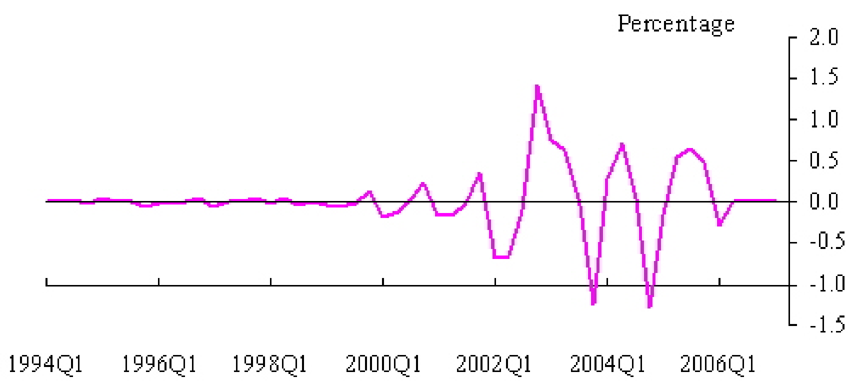
Forecasting models have to work with data that may later be revised, reflecting the fact that data is uncertain. That matters especially for the construction of a sectoral model database because revisions between components that are used to classify sectors can imply large changes in the forecast.

To test this we compared two vintages of Colombian National Accounts data: an old 2006Q1 vintage against the latest vintage we had been working with up to now, of 2007Q1. Unfortunately we only had information on what real GDP volumes were before and after the revision. The supply and demand equations we have tested are actually fairly robust to changes in aggregate real GDP volume and even the GDP deflator data. These parts of the model depend more on data on expenditure components rather than the total GDP number. So to make the exercise interesting, we assumed that the revision in real GDP was entirely due to offsetting revisions in real consumption and the consumption deflator which left nominal consumption unchanged. Figure 13 on page 52 plots the implied percentage revision in the level of consumption. The autocorrelated pattern is due to the National Accounts authority shifting growth between adjacent quarters.

Table 17: Domestic production of investment

	ML est.	Mean	Mode	sd	90% limits
ϕ_{11}	0.83	0.83	0.83	0.00	
100*σ_{11}	99.22	99.19	99.22	0.17	$[99.12, 99.22]$
100*exp(ξ_1)	10.11	10.10	10.11	0.011	
ω	-0.068	-0.112	-0.051	0.057	

Figure 13: Revision in the level of real consumption between 2006Q1 and 2007Q2



To estimate what this would do to our model forecasts, we simply repeated our estimations but on the old data. Table 18 on page 53 compares the RMSEs of domestically produced consumption from the new and old databases.

We can see that the RMSE of household and government consumption has hardly changed between the two databases. Neither is there a change in the RMSE of the domestically produced consumption share of households only. In this sense we can say that our database-model combination is likely to be fairly robust to the type of revision which we typically find in Colombian National Accounts.

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Table 18: Effect of data revisions

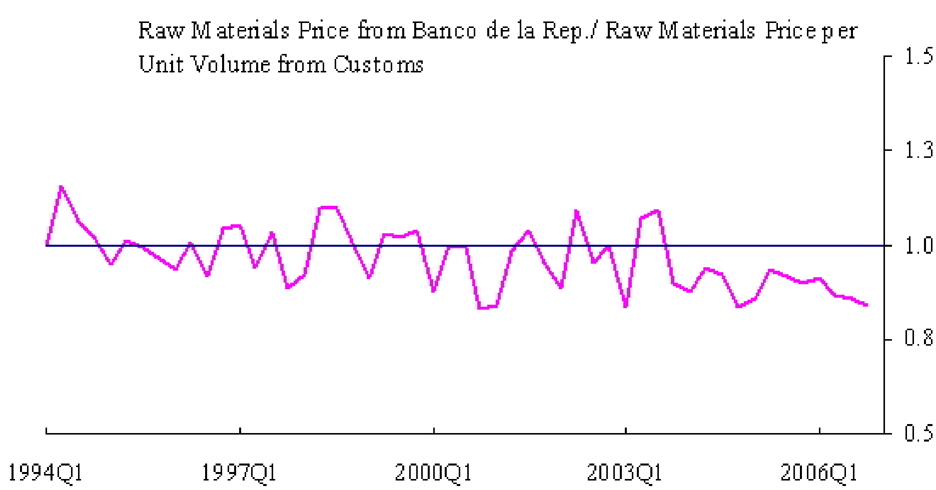
	ML est.	Mean	Mode	sd
Domestically produced consumption of hhs and govt				
RMSE (old data)	1.04	1.37	0.11	0.88
RMSE	1.07	1.47	0.12	0.93
Domestically produced consumption of hhs				
RMSE (old data)	1.83	1.89	0.11	1.20
RMSE	1.87	2.11	0.08	1.34

Note: RMSE for average of year one

8 A test of robustness to construction assumptions

We used many assumptions in building our database. Our test can be used to assess the robustness to these assumptions. One particular assumption was our choice of raw material price data. We used an in house series rather than the unit value series from customs data. Figure 14 on page 53 describes the difference.

Figure 14: Difference in two raw material price series



Source: Banco de la República and DIAN

The choice between the two series is moot. The Banco de la República series is based on producer price data which does not enjoy the same quality of sample reliability as for example National Accounts data, and perhaps less than the Customs data. But the unit value series do not adjust for qualitative change, although that might be less of a problem for raw materials prices than for consumption or capital imports prices. On other hand neither do the unit value series adjust for shifting expenditure shares and this can create serious distortions. Looking at Figure 11 on page 42, we can see that the unit volume series seems to be overestimating raw material prices relative to the inhouse series since the end of 2003, the start of latest period of world energy price rises.

Leaving aside which is the more appropriate measure, our aim is to inform the decision by comparing forecasts made with the inhouse series and those made with the unit value data. Table 19 on page 54 compares the effect of the two again in terms of the consumption and raw material price and volume forecasts.

Table 19: Test of robustness to new raw material price series

	ML est.	Mean	Mode	sd
Domestically produced consumption of hhs and govt				
RMSE (Customs RM series)	0.28	1.05	0.08	0.70
RMSE	1.07	1.47	0.12	0.93
Domestically produced consumption of hhs				
RMSE (Customs RM series)	1.87	2.11	0.08	1.34
RMSE	0.58	2.27	0.10	1.48
Raw materials in imports				
RMSE (Customs RM series)	8.98	13.06	1.73	7.29
RMSE	5.28	12.75	1.54	9.88

Note: RMSE for average of year one

The differences are quite marked, much more than one would perhaps expect, judging only from the perspective the share of raw materials in gross output is about 8%. The higher raw material price series helps improve the mean short-term prediction of consumption considerably, lowering RMSE by two thirds. The effect of a higher raw material price, given a fixed series for the import deflator, is to lower the price of consumption imports and thus raise the forecast of the nominal share of domestically produced consumption. This is what seems to be driving the improvement in the consumption forecasts. The question then is, is that improvement in the forecast a coincidence, or is it because the new series contains better information? One clue is that the change worsens the predictions of raw materials share itself (see the last two rows of Table 19 on page 54). This would lead us to favour the explanation that the improvements brought about by the new series are more of a coincidence than an actual new information.

This exercise builds on the previous sections to demonstrate how our tests can be adapted to examine database and model design choices early on, when they are most needed.

9 Conclusions

We began this paper by explaining why models that distinguish tradable and nontradable sectors are so popular in emerging market countries. We then described a DSGE tradable/nontradable sector model and its accompanying database for Colombia. A key feature of the model is that it separated out different importing sectors, and this meant that the database had to be purpose-built. Our main interest was in developing an early warning test of whether the combination of model and database is likely to forecast well in the future. This is likely to be extremely useful for emerging market central banks who are currently contemplating building and using such models.

Our test revealed some areas where the combination should work (consumption) and some areas where serious problems should be expected (investment and the transformation of domestic production into different types). We also demonstrated how the test can be used to look into robustness to data revisions and particular assumptions.

Our test is very general in the sense that when it reveals failure it is not very specific about the source of that failure. That said this is both its weakness and its strength: more specific testing would require for the full model to be in operation.

For example, while it matters more that we fail to forecast some variables (consumer price inflation, GDP growth and interest rates) than others, our test does not prioritise failure. A complementary extension could be to provide a metric for ranking the importance of forecast errors across equations by quantifying how much measurement error in each data series affects the objective. But then one would have to first calibrate and simulate the model, and therefore this extension would take us away from our idea of giving early warning of model-data failure. This is left for future work.

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10 Appendix of Charts

Figure 15

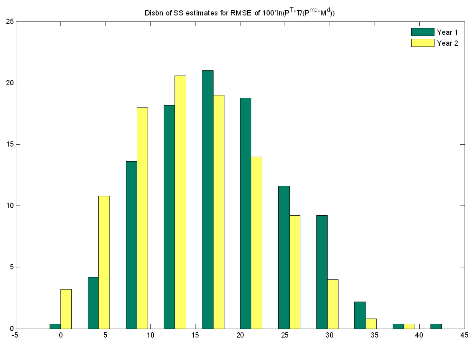


Figure 16

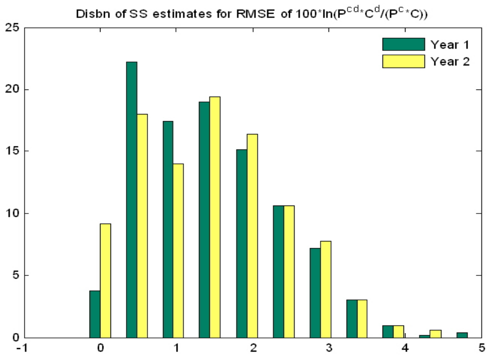


Figure 17

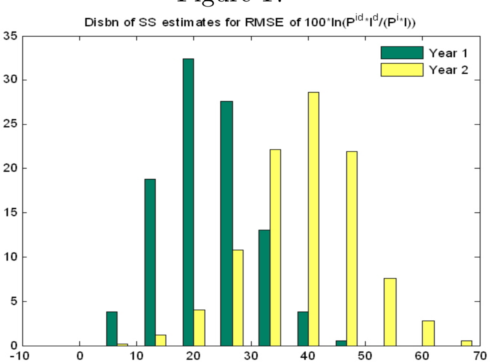


Figure 18

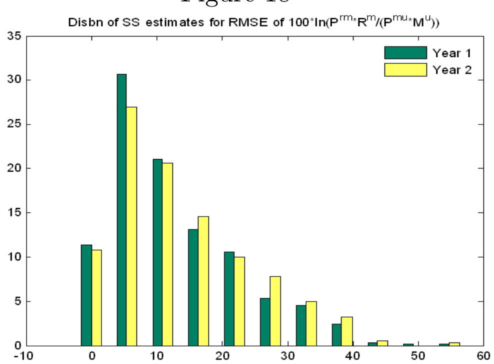


Figure 19

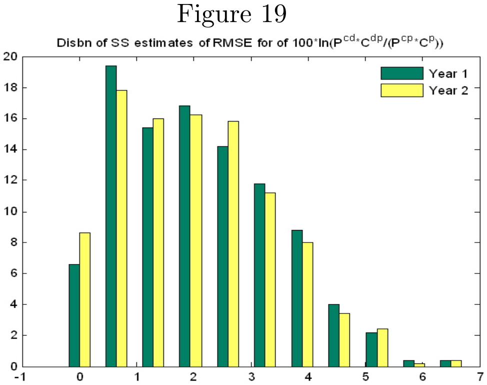


Figure 20

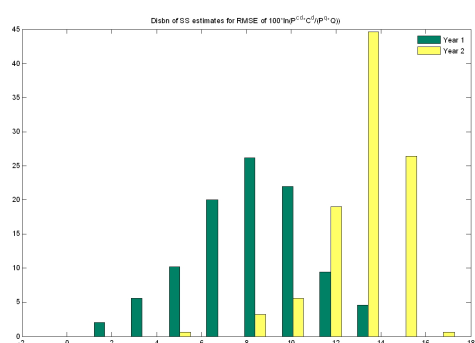


Figure 21

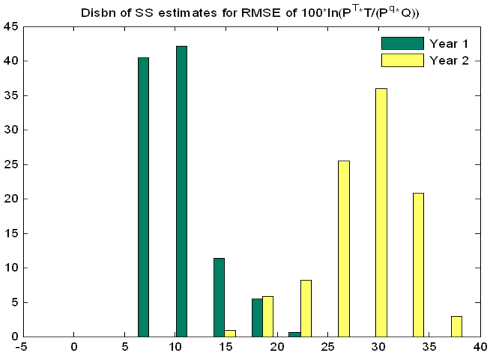


Figure 22

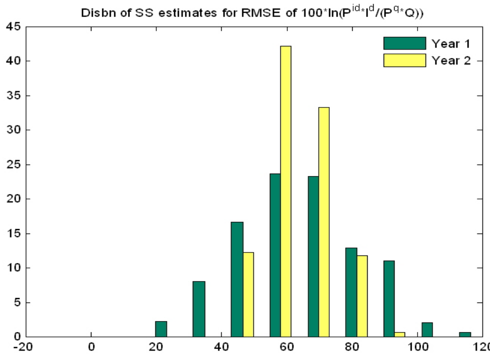


Figure 23. Distribution in consumption and investment imports

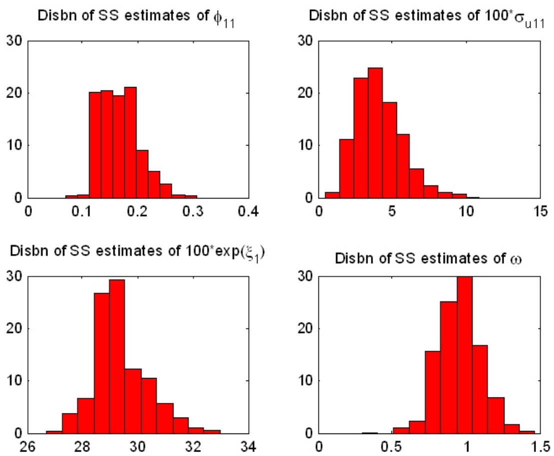


Figure 24. Domestic consumption of government and households

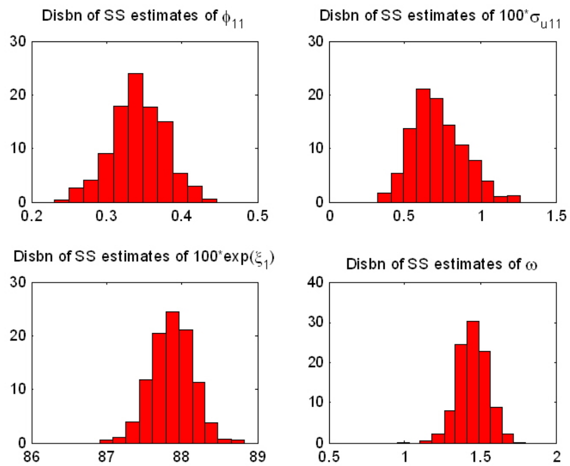


Figure 25. Domestic investment in investment

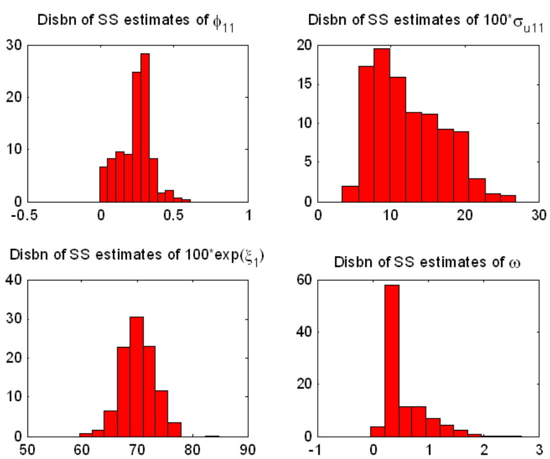


Figure 26. Raw materials in imports

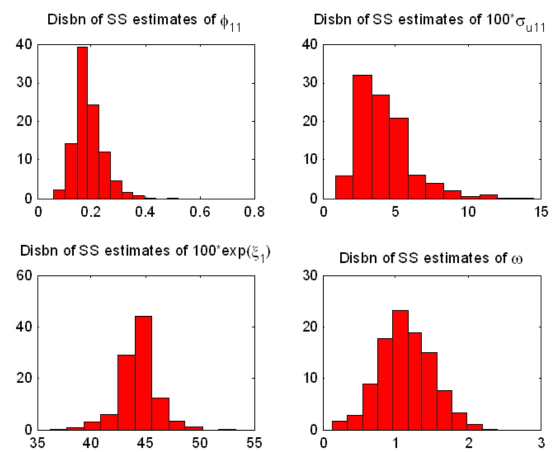


Figure 27. Domestic consumption of households

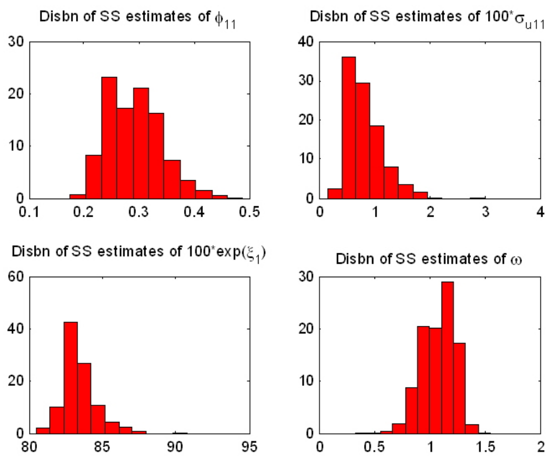


Figure 28. Domestic production of consumption

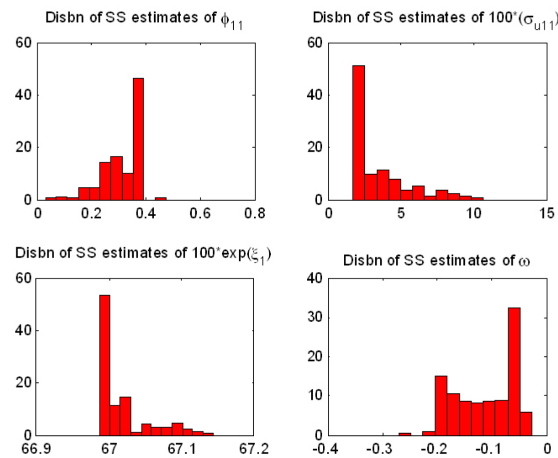


Figure 29. Domestic production of distribution

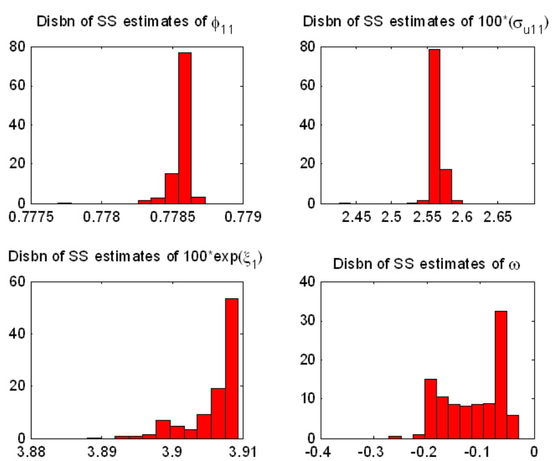
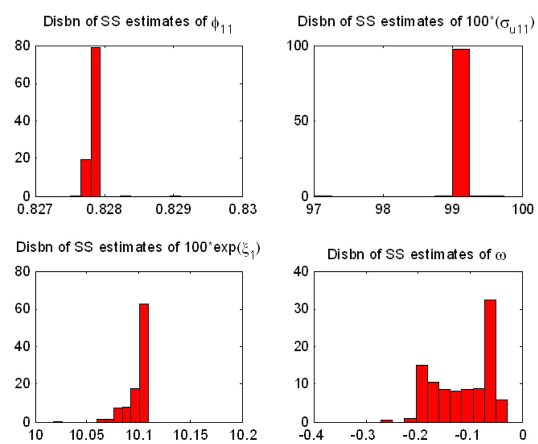


Figure 30. Domestic production of investment



11 Appendix

Each period a fixed proportion $(1-\varepsilon)$ of firms are allowed to optimally adjust their prices. The rest have to set prices according to a rule: the firms that cannot optimally adjust prices between time t and $t+i$ have to set their prices in $t+i$ as $P_t^q \Gamma_t^i$ with

$$\Gamma_t^i \begin{cases} \equiv \prod_{j=1}^{i} \left(1+\pi_{t+j-1}^q\right)^{\kappa} \left(1+\pi_{t+j}^*\right)^{1-\kappa} \text{ if } i \geq 1, \\ \equiv 1 \text{ if } i = 0, \end{cases} \tag{86}$$

$\pi_t^q \left(\equiv \frac{P_t^q}{P_{t-1}^q} - 1\right)$ is the inflation rate of output prices at time t, $\pi_t^* \left(\equiv \frac{P_t^*}{P_{t-1}^*} - 1\right)$ is the target rate at time t, and κ is a parameter that determines the degree of inflation stickiness relative to credibility.

The firms that can change prices choose a price P_t^{opt} to solve the following problem

$$\begin{aligned} &\max_{P_t^q(z)} \left\{ E_t \sum_{i=0}^{\infty} \varepsilon^i \Delta_{t,t+i} \left(\frac{\Gamma_t^i P_t^q(z)}{P_{t+i}^c} - \zeta_{t+i}(z) \right) Q_{t+i}(z) \right\} \\ &\text{s.t } Q_{t+i}(z) = \left(\frac{\Gamma_t^i P_t^q(z)}{P_{t+i}^q} \right)^{-\frac{1}{\theta_{t+i}}} Q_{t+i}, \\ &\text{and } E_t \theta_{t+i} = \theta_t \end{aligned}$$

where the discount rate is determined by the marginal utility of per capita consumption, λ_{t+i}, and also discounted for inheritance:

$$\begin{aligned} \Delta_{t,t+i} &\equiv \frac{\lambda_{t+i}}{(1+n)^{t+i}} \\ &\begin{cases} = \lambda_t \left(\prod_{s=1}^{i} (1+r_{t+s-1}) \right)^{-1} & \text{if } i \geq 1, \\ = \lambda_t \text{ if } i = 0. \end{cases} \end{aligned}$$

The first-order condition to the problem is

$$\frac{P_t^{opt}(z)}{P_t^q} = \frac{1}{(1-\theta_t)} \frac{E_t \sum_{i=0}^{\infty} \varepsilon^i \Delta_{t,t+i} \zeta_{t+i}(z) \left(\frac{P_t^q \Gamma_t^i}{P_{t+i}^q}\right)^{-\frac{1}{\theta_t}} Q_{t+i}}{E_t \sum_{i=0}^{\infty} \varepsilon^i \Delta_{t,t+i} \frac{P_{t+i}^q}{P_{t+i}^c} \frac{P_t^q \Gamma_t^i}{P_{t+i}^q} \left(\frac{P_t^q \Gamma_t^i}{P_{t+i}^q}\right)^{-\frac{1}{\theta_t}} Q_{t+i}}.$$

Imposing symmetry,

$$\frac{P_t^{opt}}{P_t^q} = \frac{1}{(1-\theta_t)} \frac{E_t \sum_{i=0}^{\infty} \varepsilon^i \Delta_{t,t+i} \zeta_{t+i} \left(\frac{P_t^q \Gamma_t^i}{P_{t+i}^q} \right)^{-\frac{1}{\theta_t}} Q_{t+i}}{E_t \sum_{i=0}^{\infty} \varepsilon^i \Delta_{t,t+i} \frac{P_{t+i}^q}{P_{t+i}^c} \frac{P_t^q \Gamma_t^i}{P_{t+i}^q} \left(\frac{P_t^q \Gamma_t^i}{P_{t+i}^q} \right)^{-\frac{1}{\theta_t}} Q_{t+i}} \tag{87}$$

Log-linearising equation 87 about the steady state gives

$$\widehat{p}_t^{opt} = \left(1 - \frac{\varepsilon}{1+r^{ss}}\right) E_t \sum_{i=0}^{\infty} \left(\frac{\varepsilon}{1+r^{ss}}\right)^i \left(\widehat{\zeta}_{t+i} - \widehat{p}_{t+i}^q - \widehat{\pi}_{t,t+i}^q + \widehat{\Gamma}_t^i\right) \tag{88}$$

with variables defined in terms of log deviations from their steady state:

$$\widehat{p}_t^{opt} \equiv \ln\left(\frac{P_t^{opt}}{P_t^q} - \frac{P_t^{ss,opt}}{P_t^{ss,q}}\right),$$

$$\widehat{p}_{t+i}^q \equiv \ln\left(\frac{P_t^q}{P_t^c} - \frac{P_t^{ss,q}}{P_t^{ss,c}}\right),$$

$$\widehat{\zeta}_{t+i} \equiv \ln\left(\zeta_{t+i} - \zeta^{ss}\right),$$

$$\widehat{\pi}_{t,t+i}^q \equiv \ln\left(\frac{P_{t+i}^q}{P_t^q} \frac{P_t^*}{P_{t+i}^*}\right),$$

$$\widehat{\Gamma}_t^i \equiv \ln\left(\Gamma_t^i - \prod_{j=1}^{i} \left(1+\pi_{t+j-1}^*\right)^{\kappa} \left(1+\pi_{t+j}^*\right)^{1-\kappa}\right),$$

and with the steady-state values of relative prices and inflation are given by

$$\frac{P_t^{ss,opt}}{P_t^{ss,q}} = \frac{1}{(1-\theta)} \zeta^{ss} \frac{P_t^{ss,q}}{P_t^{ss,c}} \text{ and } \frac{P_t^{ss,q}}{P_{t-1}^{ss,q}} = 1 + \pi_t^*.$$

Equation 88 can be written as

$$\widehat{p}_t^{opt} = \left(1 - \frac{\varepsilon}{1+r^{ss}}\right) \left(\widehat{\zeta}_t - \widehat{p}_t^q\right) + \frac{\varepsilon}{1+r^{ss}} \left(E_t \widehat{p}_{t+1}^{opt} + E_t \widehat{\pi}_{t+1}^q - \kappa \widehat{\pi}_t^q\right). \tag{89}$$

Since the firms are distributed along a continuum, the aggregate price is given by a CES aggregator of those firms who are allowed to change prices and those who cannot:

$$\begin{aligned} \left(P_t^q\right)^{1-\theta_t} &= (1-\varepsilon) \left(P_t^{opt}\right)^{1-\theta_t} + \varepsilon \left(\Gamma_{t-1}^1 P_{t-1}^q\right)^{1-\theta_t} \\ \Rightarrow \left(\frac{P_t^{opt}}{P_t^q}\right)^{1-\theta_t} &= \frac{1}{(1-\varepsilon)} - \frac{\varepsilon}{(1-\varepsilon)} \left(\frac{\Gamma_{t-1}^1 P_{t-1}^q}{P_t^q}\right)^{1-\theta_t}. \end{aligned} \tag{90}$$

Log-linearising 90 about the steady state gives

$$\widehat{p}_t^{opt} = \frac{\varepsilon}{(1-\varepsilon)} \left(\widehat{\pi}_t^q - \kappa \widehat{\pi}_{t-1}^q + \kappa \left(\pi_t^* - \pi_{t-1}^* \right) \right) \tag{91}$$

Putting 91 into 89 gives

$$\begin{aligned} & \widehat{\pi}_t^q - \kappa \widehat{\pi}_{t-1}^q + \kappa \left(\pi_t^* - \pi_{t-1}^* \right) \\ & = \frac{(1-\varepsilon)}{\varepsilon} \left(1 - \frac{\varepsilon}{1+r^{ss}} \right) \left(\widehat{\zeta}_t - \widehat{p}_t^q \right) \\ & + \frac{\varepsilon}{1+r^{ss}} \left(\frac{1}{\varepsilon} \left(E_t \widehat{\pi}_{t+1}^q - \kappa \widehat{\pi}_t^q \right) + \kappa \left(E_t \pi_{t+1}^* - \pi_t^* \right) \right) \end{aligned} \tag{92}$$

After some rearranging we get a Phillips curve of the form

$$\pi_t^q = \frac{\upsilon}{1+\kappa\upsilon} E_t \pi_{t+1}^q + \frac{\kappa}{1+\kappa\upsilon} \pi_{t-1}^q + \frac{(1-\varepsilon)(1-\upsilon\varepsilon)}{\varepsilon(1+\upsilon\kappa)} \zeta_t + \varsigma_t \tag{93}$$

where ς_t is a term related to changes in the inflation target,

$$\varsigma_t = \frac{\kappa\upsilon}{1+\kappa\upsilon} E_t \Delta \pi_{t+1}^* - \frac{\kappa}{1+\kappa\upsilon} E_t \Delta \pi_t^* \tag{94}$$

and where

$$\upsilon \equiv \frac{1}{1+r^{ss}}. \tag{95}$$

Notice that full credibility ($\kappa = 0$) implies a fully forwarding-looking linearised Phillips curve, and that the weight on past inflation is never more than $\frac{1+r^{ss}}{2+r^{ss}}$.