Estimation of Conditional Time-Homogeneous Credit Quality Transition Matrices for Commercial Banks inColombia

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Estimation of Conditional Time-Homogeneous Credit Quality Transition Matrices for Commercial Banks in Colombia[1]

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Abstract

This paper presents an estimation of credit quality transition matrices for commercial banks in Colombia, using a duration hazard function model, and following the methodology proposed by Gómez-González et al (2009). Using a test developed by Weißbach et al (2005), we test for the time-homogeneity of transition matrices estimated this way, after conditioning on firm-specific and macroeconomic variables. We found that 70% of the time we could not reject the null hypothesis of time homogeneity. We also found that obtaining matrices for different subsamples was not necessary, given the similarities of the survival function.

JEL Classification: C12; C41; E44; G21

Keywords: Credit risk, transition probabilities, hazard functions.

1. Introduction

The declining credit quality of debtors is a cause for concern for banks as it becomes a source of credit risk. The estimation of credit quality transition matrices is at the core of credit risk measures, therefore credit ratings, internal or external, become an important input in credit risk valuation.

Agency ratings play an important role in credit risk valuation as they provide an overview of the default risk of a firm. This information is widely used by different

[1] The contents of this document are exclusive responsability of the authors. They do not correspond to oficial positions of of the Banco de la República, Fogafin, or their respective boards of directors.

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methodologies (i.e. JP Morgan's Credit Metrics) for the estimation of credit quality transition matrices. Nonetheless, internal ratings contain important information that can be used for credit risk valuation (Treacy and Carey 2000), hence for the estimation of transition matrices.

In this paper we use the internal ratings that banks provide to the Financial Superintendence[4], to estimate credit quality transition matrices following a methodology developed in Gómez-González et al (2009), which overcomes several inconveniences of traditional transition matrix estimation methodologies. In particular, it allows to test whether the Markovian assumption holds, and allows the inclusion of macroeconomic and microeconomic variables in order to obtain conditional time homogeneity.

Results show that the covariates included in the estimations are jointly significant; therefore they help explain transition intensities / probabilities. This result confirms the non – markovian behavior of rating transitions, implying that we obtain more accurate credit quality transition matrices, for this sample, when we condition them on economic variables. In addition, following Weißbach et al (2005), we performed a test to check for the time homogeneity assumption, after controlling for the included covariates. Results shows that, most of the time, the null hypothesis of time homogeneity cannot be rejected at the 1 percent level when we use the matrices estimated from the duration models.

The importance of finding time homogeneous matrices is that they allow us to make statistical inference. Matrices with this characteristic are useful for forecasting future transition probabilities under macro and microeconomic shocks. Therefore, they prove to be an important tool for the measurement of credit risk in the institutions that are part of the financial safety net, and for financial intermediaries themselves.

For instance, banks can estimate how the probability of default of their credits will behave due to a change in economic activity or in the financial condition of their debtors and therefore, they can infer the level of provisions for non performing loans they should hold for the future.

[4] The Financial Superintendence is the Bank's Supervisory Agency in Colombia.

The outline of the paper is as follows. Section 2 briefly describes the data used for the estimations; section 3 describes and estimates the duration model. Section 4 presents a sensitivity analysis and, finally, section 5 outlines some conclusions.

2. Description of the Data

In this paper we use data from two different sources. We use the data banks report to the Financial Superintendence about their commercial loans operations. This data contains specific information about each credit, like debtor identification, size of the credit, type of guarantee and most importantly the credit rating commercial banks assign to each debtor. It is important to note that in our estimations individuals are credits rather than firms or banks.

We also use data from the Corporations Superintendence[5]. In particular, we use data from the financial statements firms report annually to this Superintendence. This information contains basic balance sheet data that enables us to calculate financial ratios in order to use them as debtor-specific covariates for the econometric models. We use data from December 1999 to December 2007.

Given that Format 341 contains credit id’s, it is possible to match a good proportion of credits (nearly 40%) to its balance sheet data. The sub-sample used throughout this paper contains commercial loans of all commercial banks whose debtors are supervised by the Corporations Superintendence and that are reported quarterly in the 341 Format to the Financial Superintendence. This is, we do not use the full sample of commercial loans but those which are granted to debtors that are supervised by the Corporations Superintendence and therefore fulfill some specific characteristics, in particular, their assets are above from a certain threshold.

For this reason, we recommend that our results are not used for inference out of this specific sample.

[5] The Corporations Superintendence is the Firm’s / Corporation’s Supervisory Agency in Colombia.

3. Construction of a duration hazard model

Using the same database used in this study, Gómez-González et al (2009) showed that rating dynamics vary over time. This result has been obtained also in studies using similar databases from other countries. It has also been shown that different covariates influence significantly transition probabilities. Jonker (2002), using a data set of ratings of banks in Europe, USA and Japan, finds that the country of origin of the bank matters in the downgrading process. Bangia et al (2002), using data from the Standard & Poor's CreditPro 3.0 database, show that the business cycle influences significantly credit migration matrices, by separating the economy into two states (contraction and expansion) and computing transition matrices for these states separately. Lando and Skodeberg (2002), and Kavvathas (2000) use survival analysis techniques to show the influence of migration matrices on previous rating and waiting time effects, and on macroeconomic variables, respectively. Gómez-González and Kiefer (2009) introduce macroeconomic variables and bank specific variables (summarized by the capitalization ratio) to explain bank rating dynamics in Colombia.

In this paper, we include firm-specific and macroeconomic variables to explain credit rating dynamics. The way of introducing these covariates in the estimation of transition intensities is the following. Let $\lambda_{ij}^{n}(t)$ denote the transition intensity from category i to category j of bank n. Then,

$$\lambda_{ij}^{n}(t) = Y_i^{n}(t)\alpha_{ij}^{n}\left(\beta_{ij}, t, X^{n}(t)\right) \qquad (1)$$

where $Y_i^{n}(t)$ is an indicator function which takes the value 1 if the firm is rated in category i at time t and 0 otherwise; $\alpha_{ij}^{n}\left(\beta_{ij}, t, X^{n}(t)\right)$ is a function both of time and of a vector of covariates of bank n at time t, denoted $X^{n}(t)$. In this study, we use time varying covariates; however, if time varying covariates are not available or if the covariates to be included do not vary during the observation period, a vector of fixed covariates can be used. It is assumed that the function $\alpha_{ij}^{n}\left(\beta_{ij}, t, X^{n}(t)\right)$ has the multiplicative (proportional hazards) form, as in Cox (1972):

$$\alpha_{ij}^{n}\left(\beta_{ij}, t, X^{n}(t)\right) = \alpha_{ij}^{0}(t)\phi\left(\beta_{ij}, X^{n}(t)\right) \qquad (2)$$

where $\alpha_{ij}^{0}(t)$ represents the baseline intensity, common to all banks, which captures the direct effect of time on the transition intensity. For estimation purposes, a functional form is specified for $\phi(\beta_{ij}, X^n(t))$, while the baseline intensity is let unspecified (the only restriction is that it is non-negative). A functional form which is frequently chosen for $\phi(\cdot)$ is an exponential form, $\phi(\beta_{ij}, X^n(t)) = exp(\beta_{ij} X^n(t))$, which has the advantage of guaranteeing non-negativity without imposing any restrictions on the values of the parameters of interest (β_{ij}'s). The model is estimated by the method of partial likelihood estimation, developed by Cox (1972).

3.1. Description of Covariates

The debtor specific characteristics included in this study are variables widely used in the literature, especially in estimations of a firm probability of bankruptcy. For example, Bhattacharjee, Higson, Holly and Kattuman (2002) find that increases in firm size and in profitability reduce the probability of a firm entering in bankruptcy. Geroski and Gregg (1997) find that an increase in the debt to assets ratio increases the likelihood of failure, in our estimations, as a leverage ratio we use the debt (liabilities) to equity ratio. Lennox (1999), using probit and logit models finds that an increase in liquidity and in earnings reduces the likelihood of failure, whereas an increase in capital gearing increases the bankruptcy probability. Bunn and Redwood (2003) find a nonlinear effect of profitability and also that lower liquidity levels measured by the current ratio imply a higher bankruptcy probability.

For the case of Colombia, Gómez-González, Orozco and Zamudio (2006) use a duration model to estimate a firm's probability of default, and find that an increase in size, measured as the logarithm of sales, and having positive return on assets reduces default probability. Gómez-González et. al (2009) find that liquidity, size and debt composition determine transition intensities for the credit quality of commercial loans in Colombia. Zamudio (2007) uses an ordered logit model to estimate default probabilities for firm's commercial loans and finds that higher loan maturity[6] results in a higher default

[6] We use the ratio of short term liabilities to total liabilities as a proxy for a firms debt structure and maturity. Our results are consistent with those of Zamudio (2007).

probability and that higher liquidity levels imply lower default probabilities. Finally, Arango, Zamudio and Orozco (2005) estimate a probit model with random effects to estimate bankruptcy probabilities and found a nonlinear effect of profitability in bankruptcy probabilities.

The credit specific covariates and the macroeconomic variables used in our estimations are described in Table 1.

Table 1: Covariates Description

Variable	Description
Liquidity	$\frac{Current\ Assets + Long\ Term\ Investments\ and\ De}{Current\ Liabilities + Long\ Term\text{:}\ financial\ and\ laboral\ oblig}$
Leverage	$\frac{Liabilities}{Equity}$
Size	$Assests\ of\ the\ institution\ divided\ by\ the\ assets\ of\ the\ largest\ institution\ in\ each\ year$
Operational Costs	$Operating\ \frac{Expenses}{Assets}$
Debt Composition	$Short\ Term\ \frac{Liabilities}{Short}\ Term + Long\ Term\ Liabilities$
Number of operations	$Number\ of\ operations\ each\ firm\ has\ with\ a\ commercial\ bank$
Economic Activity	$Annualized\ quarterly\ growth\ rate\ of\ real\ GDP$
Active Rate	$Average\ lending\ interest\ rate\ in\ real\ terms$

3.2. Estimation Results

Tables 2 through 6 present the estimation results. In all the estimations, the coefficients are jointly significant at the 99% level, providing evidence of non-markovian behavior in the underlying credit rating process of commercial loans.

The variables that seem to have the most important effects are firm size and debt composition. These variables are significant most of the times and present the expected sign. This is, greater firm size implies a higher upgrade probability. On the other hand, firms with high levels of short term debt have lower downgrade probabilities.

Table 2: Transitions out of Risk Category A

	AB		AC		AD		AE	
	Coef	Std Error	Coef	Std Error	Coef	Std Error	Coef	Std Error
Liquidity	0.00008	0.00002	0.00011	0.00003	-0.36508	0.06305	-0.33915	0.08035
Leverage	0.00008	0.00001	0.00011	0.00001	0.00007	0.00008	0.00004	0.00021
Size	-0.15810	0.34815	-1.93622	1.59293	-6.78982	4.37088	-12.40884	7.76551
Op. Costs	-0.31179	0.02763	-0.43421	0.08687	-0.49253	0.10675	-0.63922	0.08313
Debt Compos.	-0.79331	0.03667	-1.17591	0.10302	-2.17489	0.16114	-2.49187	0.20755
Number of Op.	-0.00189	0.00142	-0.01850	0.00792	-0.08341	0.02089	-0.09494	0.03008
Econ. Activity	-0.50965	0.00572	-0.54174	0.01720	-0.65423	0.03605	-0.58773	0.03911
Active Rate	-0.11716	0.00877	-0.03488	0.02525	0.09638	0.04088	0.03962	0.04980
Log-Likelihood	-135466.66		-15323.486		-4994.9294		-2939.6112	
LR. $\chi 2$ (8)	9429.54		1314.98		735.49		510.88	
Prob > $\chi 2$	0.00%		0.00%		0.00%		0.00%	
No. Obs	518170		518170		518170		518170	

Table 3: Transitions out of Risk Category B

	BA		BC		BD		BE	
	Coef	Std Error	Coef	Std Error	Coef	Std Error	Coef	Std Error
Liquidity	0.00008	0.00001	-0.00319	0.00727	-0.11459	0.04303	-0.37043	0.11414
Leverage	0.00000	0.00002	0.00006	0.00001	0.00001	0.00015	-0.00100	0.00314
Size	1.40009	0.35830	-2.40461	1.46436	-10.70588	5.17448	-40.45815	19.93072
Op. Costs	0.11479	0.01992	-0.16779	0.07811	-0.59722	0.19302	-0.05488	0.25804
Debt Compos.	0.56876	0.04216	-0.43213	0.08355	-0.89584	0.16240	-1.97405	0.27931
Number of Op.	0.00118	0.00098	-0.00611	0.00408	-0.00281	0.00671	-0.08757	0.03809
Econ. Activity	-0.50167	0.00667	-0.57691	0.01478	-0.60158	0.04187	-0.47266	0.05439
Active Rate	-0.12879	0.00948	-0.04196	0.01933	-0.23992	0.04002	-0.06989	0.07076
Log-Likelihood	-79045.183		-16457.062		-3956.8148		-1224.5123	
LR. $\chi 2$ (8)	6282.67		1848.07		352.88		162.99	
Prob > $\chi 2$	0.00%		0.00%		0.00%		0.00%	
No. Obs	26242		26242		26242		26242	

Table 4: Transitions out of Risk Category C

	CA		CB		CD		CE	
	Coef	Std Error	Coef	Std Error	Coef	Std Error	Coef	Std Error
Liquidity	-0.00194	0.01206	0.00011	0.00002	-0.00005	0.00036	-0.22977	0.08289
Leverage	0.00003	0.00001	-0.00229	0.00340	0.00002	0.00001	-0.00004	0.00020
Size	3.93489	0.69689	3.99287	0.97731	-18.58535	4.26960	0.54414	3.00561
Op. Costs	0.25130	0.08006	-0.02203	0.12804	-0.26676	0.09758	-0.55512	0.31595
Debt Compos.	0.97228	0.13185	0.06822	0.13857	0.26241	0.09351	-0.74164	0.24049
Number of Op.	0.00661	0.00155	-0.01564	0.01037	-0.01195	0.00638	-0.00274	0.01121
Econ. Activity	-0.50142	0.02262	-0.57577	0.02591	-0.53665	0.01772	-0.45552	0.04538
Active Rate	-0.08445	0.03029	0.00812	0.03140	-0.11113	0.02297	-0.11981	0.06367
Log-Likelihood	-5899.8264		-4849.5742		-10409.861		-1491.2276	
LR. $\chi2$ (8)	606.08		622.05		1093.35		143.64	
Prob > $\chi2$	0.00%		0.00%		0.00%		0.00%	
No. Obs	6967		6967		6967		6967	

Table 5: Transitions out of Risk Category D

	DA		DB		DC		DE	
	Coef	Std Error	Coef	Std Error	Coef	Std Error	Coef	Std Error
Liquidity	-0.05001	0.03924	-0.00006	0.00127	-0.00006	0.00126	0.00008	0.00003
Leverage	0.00005	0.00002	-0.00194	0.00406	-0.00003	0.00016	0.00004	0.00002
Size	0.91905	4.25573	12.44029	3.14397	9.76747	3.28380	-12.08474	4.14776
Op. Costs	0.18435	0.16200	-0.06747	0.29945	0.19992	0.19155	-0.41992	0.14559
Debt Compos.	0.98214	0.18021	0.42146	0.25044	0.46147	0.20481	0.20343	0.10584
Number of Op.	0.00201	0.00593	0.00735	0.00433	-0.05425	0.02535	-0.00704	0.00727
Econ. Activity	-0.52565	0.03541	-0.61903	0.05141	-0.54715	0.03810	-0.56488	0.02093
Active Rate	-0.21240	0.04023	-0.04516	0.05446	-0.04922	0.04567	-0.22060	0.02711
Log-Likelihood	-2830.7322		-1416.9411		-2069.6012		-7203.7142	
LR. $\chi2$ (8)	308.75		203.4		252.02		941.85	
Prob > $\chi2$	0.00%		0.00%		0.00%		0.00%	
No. Obs	6488		6488		6488		6488	

Table 6: Transitions out of Risk Category E

	EA		EB		EC		ED	
	Coef	Std Error	Coef	Std Error	Coef	Std Error	Coef	Std Error
Liquidity	-0.02889	0.02753	-0.01358	0.03519	-0.07492	0.09354	0.01020	0.00864
Leverage	-0.00060	0.00142	-0.00651	0.00824	-0.00234	0.00485	-0.00308	0.00263
Size	5.72429	7.93459	6.83608	12.66133	20.86756	7.94909	15.62112	3.82279
Op. Costs	0.09482	0.23558	0.18057	0.43732	0.67556	0.36155	-0.04088	0.24298
Debt Compos.	1.19137	0.20724	0.38210	0.36184	-0.46106	0.41367	-0.11537	0.17283
Number of Op.	-0.00353	0.01170	0.00630	0.00506	-0.05139	0.05387	0.00896	0.00214
Econ. Activity	-0.56632	0.04071	-0.58920	0.07164	-0.43718	0.06424	-0.62793	0.04534
Active Rate	-0.03872	0.04878	0.16473	0.08347	0.12239	0.09620	-0.31916	0.04849
Log-Likelihood	-2107.1333		-664.31135		-568.82818		-2583.0692	
LR. $\chi2$ (8)	270.24		97.73		60.16		310.18	
Prob > $\chi2$	0.00%		0.00%		0.00%		0.00%	
No. Obs	8008		8008		8008		8008	

* Cox Proportional Hazards estimation for the credit quality of commercial loans. Sample: December 1999 to December 2007. Quarterly Data. Source: Financial Superintendence, Corporations Superintendence, Banco de la República.

One striking result is the sign of the coefficient for the economic activity. This variable presents a negative sign in all estimations. In duration models the effect of the covariates might be affected by the baseline hazard, which can be capturing other effects, even more when the variable is macro and is the same for all individuals. This problem might be solved by using a larger dataset with the whole universe of commercial loans, but in this case we would not be able to use credit specific covariates.

Given that covariates are jointly significant in all estimations, we are able to obtain transition matrices from the duration models, following a four-step procedure, similar to the one proposed by Gómez-González et al (2009):

1. With the estimated coefficients and the covariates we can calculate $\exp(X^n(t)'\hat{\beta}_{ij})$.
2. Following Kalblfleisch and Prentice (2002), we can recover the baseline hazard $\lambda_0(t)$ once we have estimated the $\hat{\beta}_{ij}$ coefficients.
3. Therefore, we can calculate the hazard function (transition intensities) with $\lambda_{ij}(t) = \lambda_0(t)\exp(X^n(t)'\hat{\beta}_{ij})$.
4. With the transition intensities we can form the generator $\Pi(t,s)$ and obtain the transition probability matrix: $\Pi(t,s) = \exp\Lambda(t,s)$.

Tables 7 and 8 present the average transition probability matrices estimated from the duration models for years 2000 and 2007, respectively. Results show more stable matrices, with more probability mass concentrated in the diagonal elements.

Table 7: 2000 Credit Quality Transition Matrix estimated from the Duration Model

	A	B	C	D	E
A	99.58%	0.36%	0.06%	0.001%	0.001%
B	4.51%	94.23%	1.24%	0.02%	0.00%
C	1.86%	2.30%	93.00%	2.79%	0.04%
D	0.15%	1.07%	1.07%	95.15%	2.56%
E	0.93%	0.34%	0.18%	0.63%	97.93%

Table 8: 2007 Credit Quality Transition Matrix estimated from the Duration Model

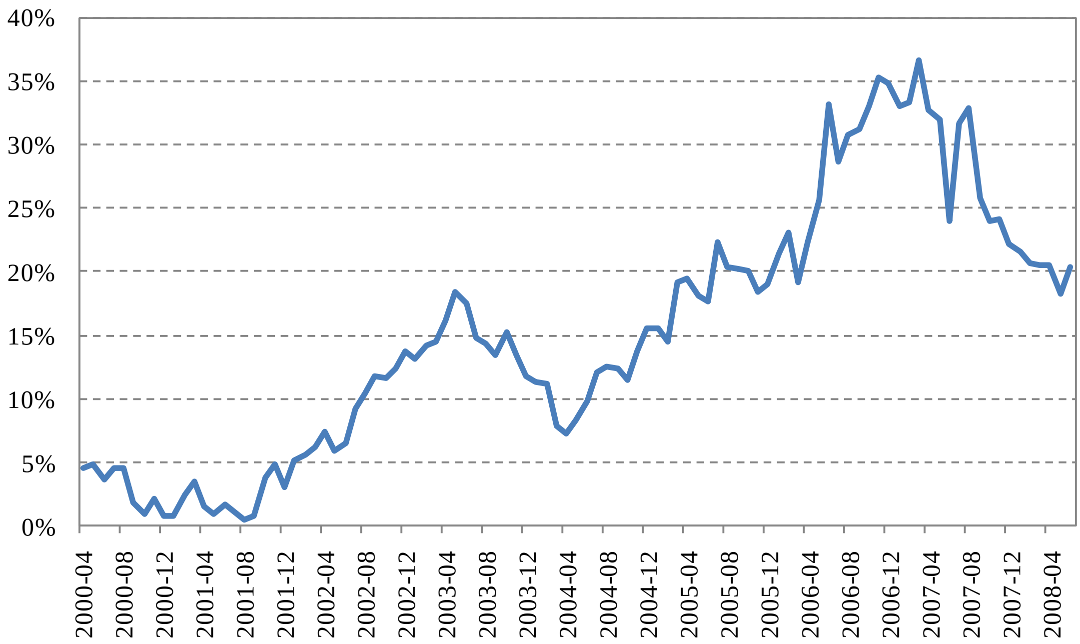
	A	B	C	D	E
A	99.37%	0.49%	0.12%	0.01%	0.01%
B	2.31%	96.48%	0.99%	0.08%	0.14%
C	3.44%	1.90%	92.87%	1.20%	0.60%
D	0.85%	0.64%	1.17%	96.96%	0.38%
E	1.54%	0.55%	0.22%	0.10%	97.59%

The 2007 average matrix presents higher credit quality conditions indicating lower credit risk. For instance, the probability mass concentrated above the diagonal (the downgrade probabilities) is lower in 2007 (3.64% versus 4.51% in 2000). However, the upgrade probability mass is higher for the 2000 matrix. This can be explained by the fact that transition events are more frequent, in general, after 2002.

One important finding in the transition matrices estimated by the duration models is the evolution of the matrices since the end of 2004. Colombia experienced a credit boom since the last quarter of 2004 till the last quarter of 2006 (see Figure 1). This boom is captured by the average transition matrices of those years, which present higher levels of upgrade probabilities (26.44% in 2005 and 51.14% in 2006). Table 9 presents the 2006 average transition probability matrix, which shows that the credit boom is consistent with better quality of the debtors.

However, in 2007 commercial loans growth is slower and the upward trend stops. This process is also captured by the 2007 matrix, which shows smaller transition probabilities in general.

Figure 1. Annual Growth of Commercial Loans - Commercial Banks



Source: Financial Superintendence, author calculations.

Table 9: 2006 Credit Quality Transition Matrix estimated from the Duration Model

	A	B	C	D	E
A	97.58%	1.84%	0.42%	0.11%	0.04%
B	10.75%	84.21%	4.03%	0.51%	0.49%
C	12.18%	7.16%	73.24%	5.28%	2.14%
D	4.32%	2.87%	4.31%	86.25%	2.25%
E	6.42%	1.78%	0.69%	0.64%	90.46%

This methodology allows us to estimate average transition matrices for different subsamples. For instance, one can obtain transition matrices for each commercial bank, economic sector, or for a group with specific characteristics. To prove if estimating transition matrices for different subsamples was worthy, we estimated the survival function for the different groups using the Kaplan-Meier non-parametric estimator:

$$S(t) = \prod_{t_i \le t} \left[1 - \frac{d_i}{N_i} \right] \qquad (3)$$

Where d_i represents the number of credits that changed from risk category *i* to *j* at time t_i, and N_i is the total number of credits that were in category *i* at time t_i.

The estimated survival functions for the subsample of foreign banks versus domestic banks and for creditors in the industry sector versus creditors in the other economic sectors show that the survival functions for the different groups are very similar; therefore, there is no evidence that suggest that we should obtain transition matrices for different subgroups.

3.3. Time-homogeneity test

Following Weißbach et al (2005), the time-homogeneity assumption was tested over the average quarterly transition intensity matrix (the generator matrix) using a roll-over technique. The test was performed to prove time homogeneity over a four quarter period (one year). Results show that we cannot reject the null hypothesis of time homogeneity 70% of the time. This finding supports, first, that once transition intensities are conditioned on relevant covariates; and second, when duration models are used, one can obtain matrices with more desirable conditions, like time-homogeneity.

The importance of finding that in most quarters the matrices are time-homogeneous is that it allows us to make statistical inference and stress testing exercises over the matrices. For instance, one could estimate how a shock to any of the covariates can affect transition probabilities over the next year. This type of analysis can be very useful for the institutions that are part of the financial safety net, given that it can provide them an overview of future credit quality due to the deterioration of the financial condition of debtors; hence it could be a valuable credit risk tool.

3.3.1 Specificities of the time-homogeneity test

The objective is to check whether rating transitions can be adequately modeled by a first-order homogeneous Markov model, after conditioning them on firm-specific and macroeconomic variables. The null hypothesis one wants to test is

$$\lambda_{ij}(t) = \lambda_{ij}, \qquad for\ all\ i, j\ and\ t \in [t', t'], \tag{4}$$

where $t'\ and\ t'$ are arbitrary. The alternative hypothesis is that transition intensities are time-dependent. The time-dependence of transition probabilities can be approximated by structural breaks of the transition intensities, as shown by Weißbach et al (2005).

The time-homogeneity test corresponds to a likelihood ratio test that compares the value of the maximized likelihood function under the null hypothesis of time-homogeneity to the value of the maximized likelihood function under the alternative hypothesis of structural breaks of the transition intensities. The test statistic of this likelihood ratio test is then

$$\theta = -2Ln(LR) = -2\left(\frac{logL_0}{logL_1}\right), \tag{5}$$

where L_0 corresponds to the value of the maximized likelihood function under the null hypothesis, and L_1 corresponds to the value of the maximized likelihood function under the alternative hypothesis. Using standard arguments of likelihood theory, under fairly general regularity conditions θ is asymptotically distributed χ^2 with $r_0 - r_1$ degrees of freedom, where r_0 is the dimension of the parameter space under the null hypothesis and r_1 is the dimension of the parameter space under the alternative hypothesis.

4. Sensitivity Analysis

In order to highlight the importance of estimating transition matrices of the type presented in section 3, here a sensitivity analysis is presented along with the estimation of credit quality transition matrices under a hazard function model with debtor-specific variables only.

Currently in Colombia, the Financial Superintendence requires that banks provision for non performing loans according to an expected loss model. In particular, provisions are the result of the expected loss calculated as:

$$\text{Provisions}_{\text{i}} = \text{Expected} \quad \text{Loss}_{\text{i}} = PD * LGD_i * Exposure_i \qquad (6)$$

Where PD is the probability of default, specifically the probability of migrating to the default category from any of the rating classes; LGD is the loss given default of each credit; and Exposure equals the total amount of the credit (capital).

The Financial Superintendence publishes the PD every year and banks can calculate their provisions taking into account the other elements of the equation. The exercises and the results presented here could be very useful for banks. Given that we have found time-homogeneous matrices conditioned on different covariates, banks can create different scenarios for the evolution of covariates and estimate PDs one-year ahead. In this way, they can have an approximation of the level of provisions that the Financial Superintendence will require. Furthermore, they can do stress testing exercises by worsening the conditions of debtors to determine the levels in which provisions might fluctuate given such shocks.

Moreover, banks can estimate a model for the whole sample of credits given that they have specific information about any debtor related to its financial condition, type of business, etc. With this they may have more accurate estimations and will be taking into account the whole universe of credits that need to be provisioned.

As was stated before, results here may be useful only for the sub-sample used for these exercises.

4.1. Duration Model with debtor-specific variables

With the purpose of performing a sensitivity analysis, transition matrices were estimated via a hazard function model using only microeconomic variables. We used the same debtor-specific variables of section 3 and include a new variable, the return on assets (ROA), which was not significant when the macro variables were included, but proves to be very important under this specification.

We did not estimate transition probabilities out of risk category E (the default category), given that it has been established by regulators as an absorbing state and therefore, the default probability has been defined as 100%. However, as results throughout this paper show, for this subsample risk category E is not an absorbing state and transition probabilities out of it are greater than zero. Tables 10 to 13 present the estimation results.

Table 10: Transitions out of Risk Category A

	AB		AC		AD		AE	
	Coef	Std Error	Coef	Std Error	Coef	Std Error	Coef	Std Error
Liquidity	-0.00130	0.00008	0.00005	0.00010	-0.33755	0.06056	-0.31882	0.07681
Leverage	0.00007	0.00001	0.00010	0.00001	0.00006	0.00008	0.00003	0.00021
Size	-0.29730	0.34952	-1.70337	1.51640	-4.76043	3.92636	-9.55435	7.27293
Op. Costs	-0.50985	0.02882	-0.50683	0.08979	-0.78206	0.07082	-0.80380	0.07023
ROA	-0.54477	0.02705	-0.08442	0.03727	-0.84354	0.07091	-0.85350	0.07782
Debt Compos.	-0.71339	0.03664	-1.13446	0.10279	-2.12598	0.16170	-2.47092	0.20798
Number of Op.	-0.00439	0.00173	-0.02520	0.00839	-0.09663	0.02145	-0.11129	0.03092
Log-Likelihood	-139693.5		-15869.374		-5200.9659		-3066.5743	
LR. $\chi 2$ (8)	975.84		223.21		323.41		256.96	
Prob > $\chi 2$	0.00%		0.00%		0.00%		0.00%	
No. Obs	518170		518170		518170		518170	

Table 11: Transitions out of Risk Category B

	BA		BC		BD		BE	
	Coef	Std Error	Coef	Std Error	Coef	Std Error	Coef	Std Error
Liquidity	0.00009	0.00001	-0.02141	0.00857	-0.11471	0.04084	-0.36922	0.11087
Leverage	-0.00001	0.00002	0.00005	0.00001	-0.00001	0.00018	-0.00145	0.00328
Size	2.22795	0.32531	0.05914	1.06752	-6.79098	4.52183	-28.05051	17.78786
Op. Costs	0.05930	0.02110	-0.62529	0.09198	-1.25473	0.22242	-0.38382	0.31722
ROA	0.54110	0.09582	-0.99351	0.05912	-1.34880	0.13638	-0.82845	0.25752
Debt Compos.	0.43125	0.04160	-0.53200	0.08133	-0.93319	0.15860	-1.98744	0.27360
Number of Op.	0.00102	0.00106	-0.01181	0.00528	-0.00463	0.00793	-0.09930	0.03864
Log-Likelihood	-82082.424		-17254.25		-4068.641		-1261.1295	
LR. $\chi 2$ (8)	208.18		253.69		129.23		89.76	
Prob > $\chi 2$	0.00%		0.00%		0.00%		0.00%	
No. Obs	26242		26242		26242		26242	

Table 12: Transitions out of Risk Category C

	CA		CB		CD		CE	
	Coef	Std Error	Coef	Std Error	Coef	Std Error	Coef	Std Error
Liquidity	-0.00413	0.01380	0.00011	0.00002	-0.00005	0.00041	-0.25035	0.08207
Leverage	0.00002	0.00001	-0.00300	0.00324	0.00000	0.00001	-0.00006	0.00023
Size	2.96369	0.67084	3.09285	0.88262	-15.75328	3.89401	0.36380	2.65906
Op. Costs	0.16118	0.08315	-0.17023	0.13447	-0.56446	0.10129	-0.73694	0.31397
ROA	0.62942	0.25407	-0.00190	0.25215	-0.90227	0.08836	-0.85386	0.28441
Debt Compos.	0.85137	0.12930	-0.06768	0.13457	0.14630	0.09107	-0.86412	0.23519
Number of Op.	0.00604	0.00171	-0.02260	0.01089	-0.02055	0.00707	-0.00655	0.01236
Log-Likelihood	-6162.7345		-5148.1341		-10893.003		-1542.1594	
LR. $\chi2$ (8)	80.27		24.93		127.07		41.77	
Prob > $\chi2$	0.00%		0.08%		0.00%		0.00%	
No. Obs	6967		6967		6967		6967	

Table 13: Transitions out of Risk Category D

	DA		DB		DC		DE	
	Coef	Std Error	Coef	Std Error	Coef	Std Error	Coef	Std Error
Liquidity	-0.08282	0.04462	-0.00904	0.03058	-0.00914	0.02354	0.00011	0.00003
Leverage	0.00004	0.00002	-0.00107	0.00368	-0.00004	0.00016	0.00002	0.00002
Size	0.52795	4.18029	11.66538	3.17126	10.04690	3.25896	-10.70633	3.86816
Op. Costs	0.15871	0.17130	-0.21903	0.31883	0.05157	0.20471	-0.61168	0.14688
ROA	0.51650	0.34153	0.39533	0.52711	-0.33563	0.26646	-0.65799	0.14151
Debt Compos.	0.82304	0.17665	0.26592	0.24161	0.35583	0.19963	0.11400	0.10256
Number of Op.	0.00083	0.00722	0.00825	0.00499	-0.07581	0.02674	-0.01787	0.00983
Log-Likelihood	-2965.8808		-1511.0776		-2184.5522		-7648.0684	
LR. $\chi2$ (8)	38.45		15.13		22.11		53.14	
Prob > $\chi2$	0.00%		3.44%		0.24%		0.00%	
No. Obs	6488		6488		6488		6488	

* Cox Proportional Hazards estimation for the credit quality of commercial loans, micro variables. Sample: December 1999 to December 2007. Quarterly Data. Source: Financial Superintendence, Corporations Superintendence.

Once again evidence of non-markovian behavior is found given that in all estimations covariates are jointly significant. Additionally, the variables present, most of the times the expected sign, and when not they are not statistically significant.

In particular, a higher return on assets is associated with a higher upgrade probability and a lower downgrade probability. The same holds for size, liquidity, debt composition, number of operations and efficiency. This last variable may be interpreted as employment; this is the higher the labor expenses as a proportion of assets imply a higher level of labor force in the firm. On the contrary, higher leverage levels imply lower upgrade probabilities and higher downgrade probabilities.

We also tested these matrices for the time-homogeneity assumption, using the same methodology as before. Results show that, most of the times (70%), we cannot reject the null hypothesis of time-homogeneity[7].

Tables 14 and 15 present the 2000 and 2007 transition probability matrix, respectively. On the one hand, the 2000 matrix presents a high level of probability mass concentration in the diagonal elements and low probability levels of far transitions. On the other hand, the 2007 matrix is less stable than the 2000 matrix. This may be due to the lack of transition frequencies in the early years of the sample.

Table 14: 2000 Credit Quality Transition Matrix estimated from the Duration Model with micro variables

	A	B	C	D	E
A	87.64%	12.33%	0.03%	2.22E-06	1.01E-08
B	1.92%	97.85%	0.23%	2.17E-05	1.08E-07
C	0.68%	1.13%	96.76%	1.42%	0.01%
D	0.01%	0.27%	0.29%	98.12%	1.31%
E	0.00%	0.00%	0.00%	0.00%	100.00%

Table 15: 2007 Credit Quality Transition Matrix estimated from the Duration Model with micro variables

	A	B	C	D	E
A	98.52%	1.18%	0.19%	0.08%	0.02%
B	33.26%	60.12%	3.04%	2.96%	0.62%
C	14.60%	5.70%	59.33%	16.27%	4.10%
D	4.48%	1.35%	1.88%	83.77%	8.52%
E	0.00%	0.00%	0.00%	0.00%	100.00%

Figures 2 and 3 present the evolution of the sum of the downgrade probabilities from category A, and the sum of the upgrade probabilities from category D, respectively. These probabilities evolve according to the financial cycle: the downgrade probabilities are higher in years 2000 and 2001 and the upgrade probabilities have grown

[7] For the test results see Appendix 1.

substantially since 2005. This also seems to be consistent with the credit cycle, as evidenced by Figure 1.

Figure 2: Evolution of downgrade probability from risk category A

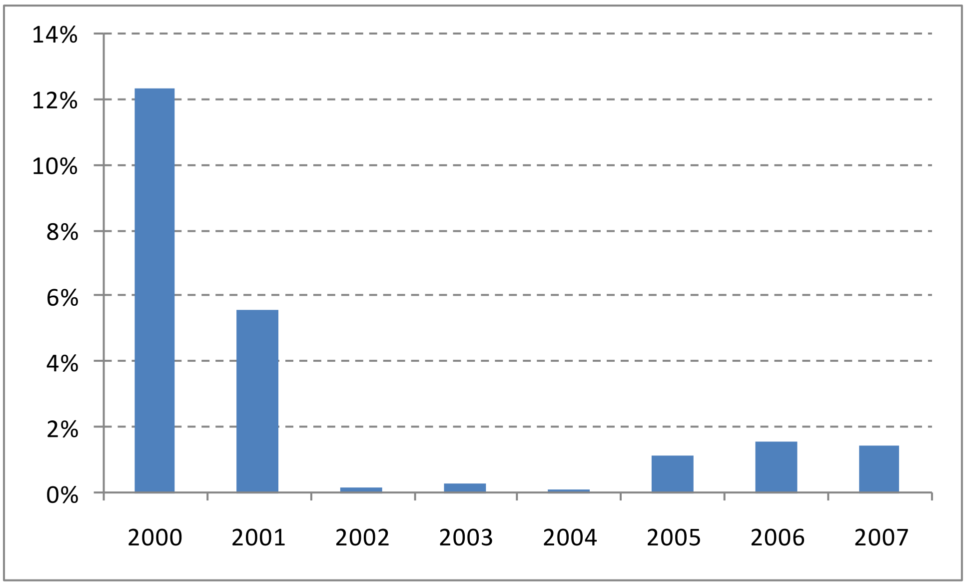
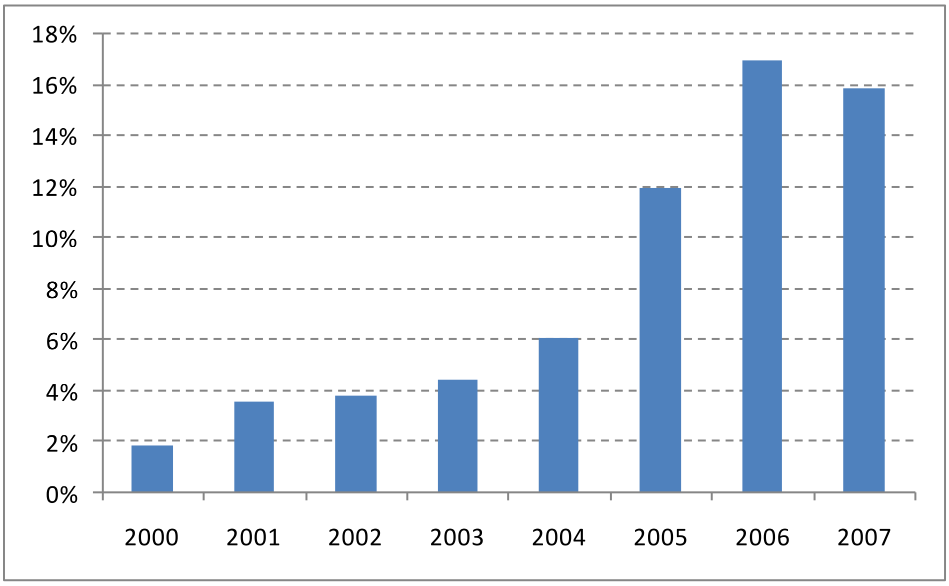


Figure 3: Evolution of upgrade probability from risk category D



4.2. Sensitivity Analysis Exercise

Since we have found time-homogenous matrices with this model, and we have been able to condition transitions probabilities on covariates, it is possible to build average transition probability matrices according to different scenarios of the independent variables.

For instance, we have decided to forecast the transition matrix one-year ahead, assuming that the debtor-specific variables behave like they did in 1999. We calculated the one year growth of all covariates, except number of operations and size, from 1998 to 1999 using a homogeneous sample, this is we only took into account firms that appear in both periods. Table 16 presents the growth rates found. For size we assumed that it fell 3% on average and for number of operations we use the average of the 2000 and 2001 values.

Table 16: Growth rates assumed for covariates

Covariate	% Change
Liquidity	-6%
Leverage	8%
Size	-3%
Operational Costs	-6%
Return on Assets	-130%
Debt Composition	-2%
Number of Operations	Average years 2000 and 2001

Source: Corporations Superintendence, Financial Superintendence, authors calculations

Table 17 presents the forecasted transition probability matrix. If compared to the 2007 matrix we observe:

1. The sum of the downgrade probabilities from every risk category is higher.

2. The sum of the upgrade probabilities from every risk category is lower except for risk category D, due to a higher upgrade probability from D to C after the shock, although values are very similar.

These results are consistent with a worsening of the financial conditions of debtors, as was the scenario adopted for this exercise. It is important to note that the forecast is done using the last baseline hazard value estimated; hence, results may change if any other assumption about the baseline is made.

Table 17: Credit Quality Transition Matrix under the assumed scenario

	A	B	C	D	E
A	98.43%	1.25%	0.20%	0.10%	0.03%
B	32.96%	60.07%	3.13%	3.12%	0.73%
C	14.14%	5.82%	58.67%	16.94%	4.43%
D	4.36%	1.32%	2.09%	83.20%	9.03%
E	0.00%	0.00%	0.00%	0.00%	100.00%

With this type of exercise banks can generate a set of different scenarios and therefore a probability distribution function for each transition in order to establish with some level of confidence the default probability of its debtors in the near future.

5. Conclusions

This paper presents an estimation of credit quality transition matrices for commercial banks in Colombia, using a duration hazard function model, and following the methodology proposed by Gómez-González et al (2009). Using a test developed by Weißbach et al (2005), we test for the time-homogeneity of transition matrices estimated this way, after conditioning on firm-specific and macroeconomic variables. We found that 70% of the time we could not reject the null hypothesis of time homogeneity. We also found that obtaining matrices for different subsamples was not necessary, given the similarities of the survival function.

However, the sign of the macroeconomic variables was not the expected in some of the estimations. Hence, we estimated the transition probability matrices conditioning only on debtor-specific variables. Results show that better financial conditions are associated with higher upgrade probabilities and lower downgrade probabilities.

A sensitivity analysis exercise was performed and we found that a worsening of the financial condition of debtors will result in higher credit risk as evidenced by higher downgrade probabilities and lower upgrade probabilities for this specific sample.

Being able to conduct this type of exercise becomes an important credit risk tool for banks, as they will be able to estimate and forecast the default probability of their debtors and therefore the level of provisions they must hold. Moreover, banks will be able to have more accurate estimations as they have more specific information about their debtors.

The importance of finding time-homogeneous matrices and being able to condition transitions on covariates is that they allow us to make statistical inference. Matrices with these characteristics are useful for forecasting future transition probabilities under macro and microeconomic shocks. Therefore, they prove to be an important tool for the measurement of credit risk in the institutions that are part of the financial safety net, and for financial intermediaries themselves.

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