Purchasing Power Parity and Breaking Trend Functions in the Real Exchange Rate

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Abstract

This paper provides evidence of long run purchasing power parity by performing a recently developed method to test for unit roots in the presence of structural breaks. Data consist of real exchange rate series for 20 countries including developed and developing economies. Structural breaks are detected in 18 countries and real exchange rates are found to be stationary in all countries except Japan. Estimated linear trends are the result of cross-country total factor productivity differentials between tradable and nontradable sectors. Estimated breaks correspond to large and permanent total factor productivity shocks associated with historical events like wars, structural reforms or deep economic recessions. An exercise with total factor productivity data shows that the Balassa-Samuelson effect explains the estimated long run trends in most countries.

Keywords: Purchasing power parity, unit root test, structural change, Balassa-Samuelson effect, real exchange rate.

JEL Classification: C22, F31, F40, N70

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1. Introduction

This paper provides a new method to detect and estimate unit roots, linear trends, and breaks in the real exchange rate. The econometric methods are based on Carrion-i-Silvestre, Kim and Perron (2009). A general equilibrium macroeconomic model is used to show that linear trends and breaks in the real exchange rate can be interpreted in the context of the Balassa-Samuelson effect and that they are compatible with Purchasing Power Parity (PPP) if the implied residuals are stationary.

The econometric methods are applied to real exchange rate series for 20 countries including both developed and developing economies. These annual data span more than 100 years through 2006 in each country. Breaks in either the level or the slope of the long run trend are detected and computed in 18 countries. All real exchange rate series except in the case of Japan are identified to be stationary around their deterministic trend. This finding recovers the positive PPP evidence described by Alan Taylor (2002) as "a century of purchasing power parity" but shows that it holds only if structural breaks are allowed in the linear trend[1].

The empirical evidence is interpreted in a macroeconomic model with tradable and nontradable sectors, perfect labor mobility across sectors, and PPP in the tradable sector which allows for productivity differences between sectors and across countries. These differences create trending behavior for the real exchange rate, a phenomenon which is known in the literature as the Balassa-Samuelson effect. In this model, PPP is defined as the long run equalization of tradable good prices across countries.

The estimated trends and breaks are analyzed country by country. Breaks are associated with permanent total factor productivity shocks at the time of historical events like wars, structural reforms or economic recessions. Implied trends allow computing real exchange rate deviations from its PPP level. Persistence levels for these deviations are estimated and compared with previous results in the empirical literature on PPP.

The econometric methods described in this paper are therefore an alternative approach to estimate the degree of misalignment of a given real exchange series. This is performed by comparing observed versus PPP levels of the series at any moment of time.

[1] Lopez, Murray and Papell (2005) had shown that Taylor's PPP evidence weakens when improved lag selection procedures are included in his unit root tests.

The PPP level of the real exchange rate is defined by its linear trend which possibly includes breaks. Linear trend and breaks can be interpreted in the context of a macroeconomic model for the Balassa-Samuelson effect. Note that the unit root hypothesis should be rejected in order to validate this misalignment analysis which assumes that PPP holds in the tradable sector of the economy. This kind of misalignment analysis for the real exchange rate is particularly relevant for economic policies in developing countries as explained by Edwards (1989).

This paper is organized in the following way. A summary of related research is presented in section 2. The framework for unit root test and structural break analysis is described in section 3. Description of the data and country by country econometric results are presented in section 4. A model of the real exchange rate in the context of the Balassa-Samuelson effect is described in section 5 in order to interpret the estimated trends and breaks. Finally, section 6 concludes.

2. Related Literature

Although the literature on empirical PPP tests is quite substantial, this section will focus on works which are closely related to this paper. The most recent literature review on PPP is presented in Taylor and Taylor (2004). Extensive reviews are also found in Rogoff (1996) and Froot and Rogoff (1995).

Taylor (2002) adds favorable evidence to the PPP debate by studying real exchange rate stationarity and its persistence degree in a panel data with 19 countries spanning 105 years from 1892 to 1996. This evidence is obtained by applying unit root tests to individual series and taking the U.S. dollar as base currency. When the series are allowed to have a linear trend then the null hypothesis of a unit root is rejected for 18 countries. Japan is the only country for which Taylor (2002) does not find any PPP evidence. The empirical exercise I present in Section 4 utilizes an updated version of Taylor's data.

Lopez, Murray and Papell (2005) apply improved unit root test methods to the same long span data set described in Taylor (2002). This improvement in unit root tests corresponds to a new lag selection procedure which corrects for size distortions as described in Ng and Perron (2001). Results show that the evidence presented by Taylor (2002) weakens when improved tests are applied because it is no longer possible to reject the null unit root hypothesis in the case of 7 out of 19 countries.

The paper by Papell and Prodall (2006) is a continuation of Lopez, Murray and Papell (2005) which focuses on those 7 countries where no PPP evidence is found. They apply an extended version of the unit root tests developed by Perron (1997) and Vogelsang and Perron (1998) which are characterized by allowing for one break in either the intercept or the slope of the deterministic trend[2]. With the exception of two countries (Canada and The Netherlands), Papell and Prodall (2006) provide positive PPP evidence after applying unit root tests extended to allow for two structural breaks in each series. For most countries, these structural breaks are characterized by a downward shift of the RER around World War II which is later compensated by and upward shift. This is because they restrict the parameters of the second break to fully compensate the shift estimated in the first break.

In contrast to Papell and Prodall (2006), I apply the structural break analysis to all countries and not only to a subset of them. Furthermore, I use the method developed recently in Perron and Yabu (2009), Kim and Perron (2009) and Carrion-i-Silvestre, Kim and Perron (2009). A new feature of this method is the application of a pretest which allows detecting the presence of breaks before testing for unit root. This idea allows separating the issue of testing for a structural break from the issue of testing for a unit root, and prevents distortions that result from the interaction of both kinds of tests.

The economic model used to interpret why trends and breaks occur in the RER is a slightly modified version of the the Balassa-Samuelson effect model described in Obstfeld and Rogoff (1996, p. 203-214). Alternative versions of this model along with a description of the empirical evidence can also be found in Froot and Rogoff (1995) and Lothian and Taylor (2008)[3]. The latter paper presents positive empirical evidence for the Balassa-Samuelson effect for the US-UK real exchange rate using annual data since 1820. This paper also performs unit root tests in the context of a non-linear adjustment model and interpreting the Balassa-Samuelson effect as the long run trend of the real exchange rate.

3. Econometric Methods

In this section, first I explain a unit root test which assumes no breaks in the deterministic trend. Then, I describe a structural break test which is suitable for both stationary and

[2] Perron and Vogelsang (1992) was the first paper to apply these kinds of tests to PPP analysis.
[3] The seminal papers on this phenomenon are Balassa (1964) and Samuelson (1964).

nonstationary series. Lastly, an improved unit root test which allows for the presence of structural breaks in the deterministic trend is presented.

3.1 Unit Root Test with No Breaks

Following Ng and Perron (2001), I use the MZ_t^{GLS} test which is part of the so-called M-class of unit root tests. This class of tests has an improved method to select the number of lags in auto regressions thus preventing size distortions with too few lags or power distortions with too many lags. This is accomplished by using a Modified Akaike Information Criterion (MAIC) to select the optimal number of lags.

The M-class also includes a new method for long run variance estimation by using an auto regressive spectral density estimator. This method decouples the estimation of the variance from the estimation of the autoregressive parameters, thus preventing some size distortions. When deterministic components are included in the specification, it is necessary to detrend the series before applying the procedure. It is done by applying the so-called GLS detrending which has been shown to maximize the unit root test's local asymptotic power. All these improvements also make the M-class of tests robust to measurement errors and outliers[4].

The MZ_t^{GLS} test is applied to two alternative specifications. The demeaned specification only includes an intercept as deterministic component.

$$q_t = \alpha_0 + u_t \tag{1}$$

The detrended specification includes additionally a linear trend with slope α_1.

$$q_t = \alpha_0 + \alpha_1 t + u_t \tag{2}$$

Throughout this paper q_t is the natural logarithm of the real exchange rate of a foreign currency with respect to the US Dollar. It is measured as the number of US goods necessary to buy one unit of foreign good. Therefore, an increase in q_t means a real appreciation of the foreign currency respect to the US Dollar[5].

The error process $\{u_t\}$ is assumed to obey the following model:

[4] See Haldrup and Jansson (2006) for a detailed presentation of the tests as well as of the GLS detrending method.

[5] See appendix for a detailed description of the computation of log real exchange rates.

$$u_t = \alpha u_{t-1} + \psi(L)\varepsilon_t \tag{3}$$

Equation (3) represents a linear process where $\psi(L) = \sum_{j=0}^{\infty} \psi_j L^j$ is a lag polynomial whose coefficients $\{\psi_j\}$ satisfy $\sum_{j=1}^{\infty} j\left|\psi_j\right| < \infty$. Furthermore, ε_t is assumed to be iid with mean 0 and variance σ_ε^2. Note that this definition is general enough to include any stationary ARMA process. In this context, the null and alternative hypotheses for the unit root tests are $H_0 : \alpha = 1$ and $H_1 : |\alpha| < 1$, respectively.

Assuming that we have $T+1$ observations and $t = 0,1,2,\ldots T$, the MZ_t^{GLS} test can be written as the product of the other two tests in the M-class: $MZ_t^{GLS} = MZ_\alpha^{GLS} \times MSB^{GLS}$ with the following definitions.

$$MZ_\alpha^{GLS} = (T^{-1}q_T^2 - s_{AR}^2)\left(2T^{-2}\sum_{t=1}^{T} q_{t-1}^2\right)^{-1} \tag{4}$$

$$MSB^{GLS} = \left(T^{-2}\sum_{t=1}^{T} q_{t-1}^2 \Big/ s_{AR}^2\right) \tag{5}$$

In Equations (4) and (5), s_{AR}^2 is the auto regressive spectral density estimator. Also note that in these equations we assume q_t to be previously GLS detrended. Finally, the asymptotic distribution of the MZ_t^{GLS} test was computed by Ng and Perron (2001) such that a complete set of critical values is available to evaluate the test.

3.2. A Structural Break Pretest

The test developed in Perron and Yabu (2009), is applied to the series in order to confirm the presence of a structural break in the deterministic component before testing for a unit root. If this pretest rejects the null hypothesis of no breaks, then we apply a procedure to estimate the break date and test for a unit root as described in section 3.3.

Three alternative break models are considered in the application of both the pretest and the unit root test. The first model in Equation (6), assumes an intercept break at break date T_1.

$$q_t = \alpha_0 + \alpha_1 t + \alpha_2 1\{t > T_1\} + u_t \tag{6}$$

In the second model there is a break in the slope of the linear trend at date T_1.

$$q_t = \alpha_0 + \alpha_1 t + \alpha_3 (t - T_1) 1\{t > T_1\} + u_t \tag{7}$$

The third model includes simultaneous breaks both in the intercept and in the slope at time T_1.

$$q_t = \alpha_0 + \alpha_1 t + \alpha_2 1\{t > T_1\} + \alpha_3 (t - T_1) 1\{t > T_1\} + u_t \tag{8}$$

For all three models, the error term u_t is assumed to follow the linear process described in Equation (3). The parameters α_2 and α_3 measure the size of the breaks, T_1 is the break date, and the expression $1\{t > T_1\}$ is an indicator function which takes the value 1 once the break has already happened.

The test in Perron and Yabu (2009) is an exponential aggregation of Wald tests which evaluate all possible break dates using the functional form in Equation (9). The subscript RQF stands for the method of estimation of the parameters related to breaks: Robust Quasi Feasible GLS.

$$ExpW_{RQF} = \log\left[T^{-1} \sum_{\Lambda} \exp\left(\frac{1}{2} W_{RQF}(\lambda_1^{'})\right)\right] \tag{9}$$

In Equation (9), Λ is the set of all possible break dates, $\lambda \equiv T_1/T$ is a break fraction and W_{RQF} is a Wald test for the null hypothesis of no breaks in Equations (6), (7) or (8). The Wald test, for a given break fraction λ_1, is defined in the following equation:

$$W_{RQF}(\lambda_1) = \left[R\left(\tilde{\Psi} - \Psi\right)\right]' \left[\hat{h}_v R\left(X'X\right)^{-1} R'\right]^{-1} \left[R\left(\tilde{\Psi} - \Psi\right)\right] \tag{10}$$

In Equation (10), the null hypothesis to be tested is whether the following restriction on parameters hold: $R\left(\tilde{\Psi} - \Psi\right) = 0$. The parameters related to the structural breaks are $\tilde{\Psi}$ which are estimated from the GLS detrended break models in (6), (7) and (8). The independent detrended variables in each one of the break models are denoted X in Equation (10). Finally, $\hat{h}_v$ is an estimate of (2π times) the spectral density function at frequency zero of the detrended residuals[6].

The models in Equations (6)-(8) are GLS detrended with an autoregressive parameter $\hat{\alpha}$ which is estimated from an $AR(p)$ model where p is selected with a Bayesian

[6] The exact expression for the estimator of h_v varies with the model and the estimated persistence of the residuals. See Perron and Yabu (2007) for details.

information criterion. Furthermore, the estimator $\hat{\alpha}$ is adjusted not only with the bias correction proposed by Roy and Fuller (2001) but also with a "super-efficient" procedure which truncate the estimator when it is close enough to 1[7]. These adjustments improve the asymptotic properties of the Wald test in Equation (10).

Therefore, the test $ExpW_{RQF}$ is an exponential aggregation of Wald tests and can be interpreted as a test for the null hypothesis of no structural breaks in the deterministic trend of q_t when the break date is unknown. The main advantage of the functional form in Equation (9) is that the relevant quantiles of its limit distribution when u_t is I(0) are very similar to the case when u_t is I(1). Therefore, by taking the larger critical value at every relevant significance level, Perron and Yabu (2009) are able to construct a robust test statistic for structural breaks with either stationary or integrated errors.

A rejection of the null hypothesis gives a strong indication of the presence of at least one structural break in the series under analysis. The next step in this case is to apply unit root tests that allow for structural breaks as explained below. If the pretest does not reject, it means that a unit root procedure with no breaks, like the one described in section 3.1, should be performed.

3.3. Testing for a Unit Root with Breaks in the Trend

When the unit root hypothesis needs to be tested in a framework where structural breaks are allowed in the deterministic trend, I apply the recent techniques described in Carrion-i-Silvestre, Kim and Perron (2009). These procedures bring about several improvements with respect to the previous literature[8]. They are as well, an extension of the test for a single break described in Kim and Perron (2009).

The following are the main improvements that Carrion-i-Silvestre, Kim and Perron (2009) included in their procedure compared with previous literature on this kind of test. First, the break is included in the null as well as in the alternative hypotheses. Second, the test is extended to allow for multiple breaks. Third, the test adopts a quasi-GLS detrending

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[7] Detailed descriptions of the bias correction and the "super efficient" estimation are presented in Perron and Yabu (2007).

[8] The seminal paper by Perron (1989) put forward the research agenda on unit root test procedures allowing for breaks in the deterministic trend. Since then, the literature on this kind of unit root test has focused in methods that incorporate unknown break dates. A comprehensive survey paper is Perron (2006).

method, similarly to the tests described in section 3.1, in order to maximize local asymptotic power. Fourth, the procedure extends the M-class of unit root tests, described in section 3.1, to the case of breaks in the deterministic trend.

Specifically, in this paper I apply the M-class test defined in the following equation.

$$MZ_t^{GLS}(\lambda_0) = \left(T^{-1}\tilde{q}_T^2 - s(\lambda_0)^2\right)\left(4s(\lambda_0)^2 T^{-2}\sum_{t=1}^{T}\tilde{q}_{t-1}^2\right)^{-1/2} \tag{11}$$

In Equation (11), λ_0 is the break fraction as explained above for equation (9). $\tilde{q}_t$ denotes the log real exchange rate series once it has been detrended from its deterministic and break components. The autoregressive spectral density estimator is denoted $s(\lambda_0)^2$ and it depends on the break fraction because it is estimated from the detrended series $\tilde{q}_t$. I apply the test in Equation (11) to each of the three different models described in Equations 6-8.

The GLS detrending is performed with the following autoregressive parameter: $\tilde{\alpha} = 1 + \bar{c}/T$, where $\bar{c}$ is a non-centrality parameter which is chosen optimally for each model following the procedure devised by Elliot, Rothenberg and Stock (1996). This procedure requires computing the asymptotic power of the test so that $\bar{c}$ ends up being a function of the number of structural breaks and their break fractions. Only in the case of the intercept break model in Equation (6), the non-centrality parameter is independent of the breaks. For the remaining models, Carrion-i-Silvestre, Kim and Perron (2009) provide a response surface algorithm which allows computing $\bar{c}$ easily given the number of breaks and their location.

I perform the estimation of λ_0 via a global minimization of the sum of squared residuals from the GLS detrended model. For this procedure, a trimming of 15% at the beginning and the end of the sample is applied in order to guarantee the consistency of this estimator. Carrion-i-Silvestre, Kim and Perron (2009) show that this estimation's rate of convergence is fast enough for the limit distribution of the unit root test to coincide with the known break date case.

In the intercept break case, Equation (6), the limit distribution of the test in Equation (11) coincides with the no break case as in Ng and Perron (2001). For the remaining break models, the limit distribution of the test does depend on the location and number of breaks. Carrion-i-Silvestre, Kim and Perron (2009) also provide a response surface algorithm in this case which allows computing the critical values of the test given the number of estimated breaks and their location.

4. Country by Country Econometric Results

In this section, I present the results from the application of the unit root tests and break pretests described in the previous section to data for 20 countries. Then a model selection procedure is developed in order to determine, country by country, the most appropriate model for the deterministic trend of the real exchange rate. Next, a quantitative analysis of the estimated breaks and trends is shown along with a brief description of the historical causes of these breaks.

The data consist of long span real exchange rate series for 20 countries with the US as the base country. These data are an update of the dataset in Taylor (2002) and contain annual observations through 2006 and initial dates as early as 1870 for some countries. A list of countries and their respective sample period is shown below in Table 2. Further details on sources and computations are described in the appendix.

4.1. Unit Root Tests

Table 1
Summary of Econometric Results
(Number of countries out of 20)

Standard Unit Root Tests	PPP evidence	
	No Trend	Linear Trend
	11	12
Test Allowing for a Break	**PPP evidence**	**Break Evidence**
1. Intercept Break	8	17
2. Slope Break	2	2
3. Mixed Breaks	9	18
Overall	19	18
BIC Model Selection	**# Selected**	
A. Constant	2	
B. Linear Trend	5	
C. Intercept Break	5	
D. Slope Break	2	
E. Mixed Breaks	5	

This table shows a summary of the econometric results related to the PPP tests in this paper. It shows the number of countries, out of 20, where either PPP or break evidence is found for each of the three models and overall. It also shows results for model selection using Bayesian information criteria (BIC).

Table 1 shows a summary of all econometric results. By applying the unit root test described in section 3.1, it is found PPP or real exchange rate stationarity evidence in 11 out of 20 countries. If a linear trend is allowed as in Equation (2), such evidence is present in 12 countries. When a break is allowed, the results depend on the specific break model. For the intercept break model, the Perron-Yabu test identifies breaks in 17 countries, of which, 8 show PPP evidence. In the case of the slope break model, only 2 countries were detected to have breaks and both show PPP evidence. The results for the mixed break model are similar to the intercept break model. Overall, taking in account all unit-root tests, it is found PPP evidence for 19 countries and break evidence for 18 countries. There is no evidence of stationarity in the Japanese real exchange rate.

Table 1 also shows the final outcome of the model selection process. Despite having break evidence for 18 countries, a model with breaks is the best selection in only 12 cases; a linear trend without breaks is chosen for 5 countries; and a single constant is the best model for the deterministic trend in the remaining 2 countries.

In Table 2, we can see the results from the unit root test without breaks described in section 3.1 for both available models: intercept and linear trend. Overall, there is no PPP evidence using these tests in the following five countries: Denmark, France, Japan, Portugal and Spain.

Table 2
MZt Unit Root Tests on the Log of Real Exchange Rates

Country	Sample	Demeaned	k_{MAIC}	Detrended	k_{MAIC}
Argentina	1884-2006	-3.029***	4	-3.186**	4
Australia	1870-2006	-2.520**	0	-2.762*	0
Belgium	1880-2006	-2.381**	3	-3.608**	0
Brazil	1889-2006	-2.109**	7	-2.605	0
Canada	1870-2006	-1.803*	0	-2.070	0
Denmark	1880-2006	-1.2913	6	-2.169	6
Finland	1881-2006	-3.927***	0	-4.115***	0
France	1880-2006	-1.128	6	-1.594	7
Germany	1880-2006	-1.953*	2	-2.951**	2
Italy	1880-2006	-3.060***	0	-3.106**	0
Japan	1885-2006	-0.163	2	-2.389	2
Mexico	1886-2006	-1.564	6	-3.346**	2
Netherlands	1870-2006	-1.900*	2	-2.890*	2
Norway	1870-2006	-1.180	5	-2.828*	5
Portugal	1890-2006	-1.338	6	-1.567	6
Spain	1880-2006	-1.609	6	-1.711	6
Sweden	1880-2006	-2.590***	2	-3.204**	0
Switzerland	1892-2006	-0.881	2	-2.618*	6
United Kingdom	1870-2006	-1.800*	4	-2.184	4
Colombia	1923-2006	-1.007	0	-2.662*	0

This table shows the MZt unit root test applied to real exchange rate series for 20 countries. The demeaned model includes a constant in the specification. The detrended model includes both constant and trend. kMAIC denotes the optimal lag in the autoregression which is performed in order to estimate the zero frequency spectral density. This lag is chosen according to the modified information criterion (MAIC). Both the information criterion and the test are described in Ng and Perron (2001).
* denotes significance at 10% level; ** denotes significance at 5% level; *** denotes significance at 1% level.

Table 3 presents results for the pretest and the unit root test that allows a single break as in Equation (6), (7) or (8); these tests were described in both sections 3.2 and 3.3. Note that Table 3 only shows those cases in which a break is detected and the unit root hypothesis is rejected. Detailed results for all countries and each one of the three break models are shown in Tables A1, A2 and A3 at the Appendix.

Table 3
Structural Break and MZt Unit Root Tests
Summary of Cases Where Unit Root Hypothesis is Rejected

Country	Sample	Break Model	Pre-test	Break Date	Unit Root Test	k_{MAIC}
Argentina	1884-2006	Mixed	4.7513**	1974	-3.3533**	0
Australia	1870-2006	Intercept	3.0293**	1919	-3.177**	0
Australia	1870-2007	Mixed	3.4843*	1919	-3.3256**	0
Brazil	1889-2006	Intercept	4.2061***	1947	-2.7117*	0
Denmark	1880-2006	Mixed	3.1364*	1945	-3.6053**	0
France	1880-2006	Intercept	3.0169**	1984	-3.5263***	0
France	1880-2006	Slope	8.6658***	1958	-4.0735***	0
France	1880-2006	Mixed	9.1519***	1984	-3.5724***	0
Germany	1880-2006	Intercept	2.4027**	1933	-2.7614*	2
Italy	1880-2006	Intercept	11.046***	1919	-2.894*	0
Netherlands	1870-2006	Mixed	3.0774*	1948	-3.1019*	0
Norway	1870-2006	Intercept	6.2154***	1919	-2.8883*	0
Norway	1870-2006	Mixed	6.5023***	1919	-2.9904*	0
Portugal	1890-2006	Intercept	6.971***	1919	-2.9071**	0
Portugal	1890-2006	Slope	1.9886*	1959	-3.4672**	0
Spain	1880-2006	Mixed	5.8444***	1948	-3.2671**	0
Sweden	1880-2006	Intercept	1.9978*	1948	-2.9858**	0
Sweden	1880-2006	Mixed	3.2983*	1948	-2.9856*	0
United Kingdom	1870-2006	Mixed	19.095***	1948	-3.8229***	0

This table shows the results for the MZt unit root test in those cases where a break is detected and the unit root hypothesis is rejected. Complete results for all models and countries are presented in Tables A1, A2 and A3 at the appendix. The pre-test allows assesing whether a break is present or not as defined by Perron and Yabu (2009). The break date is endogenous to the estimation procedure as described in Carrion-i-Silvestre, Kim and Perron (2009). The break models are described in Section 3.1. kMAIC denotes the optimal lag in the autoregression which is performed in order to estimate the zero frequency spectral density. This lag is chosen according to the modified information criterion (MAIC) which described in Ng and Perron (2001).
* denotes significance at 10% level; ** denotes significance at 5% level; *** denotes significance at 1% level.

There are seven countries with no PPP evidence when a break is allowed in the deterministic trend: Belgium, Canada, Finland, Japan, Mexico, Switzerland and Colombia. Table 3 also shows 5 countries (Australia, France, Norway, Portugal and Sweden) for which there is break and PPP evidence for more than one break model. A model selection procedure is applied to resolve this ambiguity.

Taking together the results from standard unit root tests and break models (Tables 2 and 3), there is only one country for which no PPP evidence is found at all: Japan. These results show the importance of testing and estimating breaks in the deterministic trend of the real exchange rate in order to obtain evidence of long run PPP. Note that by allowing for a single structural break it is possible to recover the positive evidence pointed out by Taylor (2002) and labeled as "A Century Purchasing Power Parity"[9].

4.2. Analysis of the Implied Trends and Breaks

I implement a model selection procedure for each country by minimizing a Bayesian Information Criterion (BIC). The goal is to find the best model for each country's trend among equations (1), (2), (6), (7) and (8) by minimizing the variance of residuals without adding too many explanatory variables. The final selected models are summarized in Table 4[10]. The results correspond to the classification presented above in Table 1 in which a model including a break is the optimal selection for 12 countries. Figures for the estimated trends, breaks and observed real exchange rates are presented in the Appendix.

Table 4
Estimated Real Exchange Rate Slopes and Break Sizes

Country	Selected Model	Break Date	Initial Slope	Post Break Slope	Break Size
Argentina	Constant	NA	0.00%	0.00%	0.0%
Australia	Intercept Break	1919	0.03%	0.03%	-27.4%
Belgium	Linear Trend	NA	0.53%	0.53%	0.0%
Brazil	Intercept Break	1947	-1.19%	-1.19%	195.2%
Canada	Constant	NA	0.00%	0.00%	0.0%
Denmark	Mixed Breaks	1945	0.19%	1.38%	-27.8%
Finland	Linear Trend	NA	0.15%	0.15%	0.0%
France	Slope Break	1958	-0.63%	0.57%	0.0%
Germany	Intercept Break	1933	0.08%	0.08%	21.2%
Italy	Intercept Break	1919	0.46%	0.46%	-30.0%
Japan	NA	NA	NA	NA	NA
Mexico	Linear Trend	NA	-0.53%	-0.53%	0.0%
Netherlands	Mixed Breaks	1948	0.00%	1.49%	-24.1%
Norway	Mixed Breaks	1919	1.10%	0.62%	-38.8%
Portugal	Slope Break	1959	-1.09%	1.57%	0.00%
Spain	Mixed Breaks	1948	-0.27%	1.53%	-38.0%
Sweden	Intercept Break	1948	0.42%	0.42%	-15.1%
Switzerland	Linear Trend	NA	0.84%	0.84%	0.00%
United Kingdom	Mixed Breaks	1948	0.05%	0.97%	-34.5%
Colombia	Linear Trend	NA	-1.90%	-1.90%	0.0%

This table shows final estimations on break date, initial slope, post break slope and break size for the selected model in each country. Slopes can be interpreted as annual equilibrium rates of real exchange rate appreciation. Break sizes are measured as the percentage deviations from the pre-break real exchange rate levels.
NA: Not available.

Table 4 also shows final estimations of break dates, break sizes and estimated slopes for all countries. Break dates fluctuate between 1919, (Australia, Italy and Norway), and 1959 (Portugal). Slopes can be interpreted as annual equilibrium rates of real exchange rate appreciation[11]. As consequence of breaks, slopes switch from negative to positive in

[9] It is possible to test for two or more structural breaks in each country. In particular, it is interesting to find the optimal number of breaks. However that line of research is beyond the scope of this paper.
[10] Table A4 in the appendix shows further details on these computations.
[11] The next section presents a model which allows understanding the determinants of these long run trends in the real exchange rate.

Portugal, France and Spain. They increase with no change in signs in the Netherlands, Denmark and UK. The most prominent slopes, in absolute magnitude, are observed in Colombia, Portugal, Spain and the Netherlands.

Table 5 provides a brief historical explanation for the estimated breaks. They are the outcome of structural reforms or productivity shocks associated with depressions or wars. In section 5, I describe a model of the Balassa-Samuelson effect in which these breaks are interpreted as results of permanent productivity (TPF) shocks. It is interesting that most of these breaks are associated with one of the world wars.

Table 5
Historical Causes of Estimated Breaks

Country	Selected Model	Break Date	Historical Cause
Argentina	Constant	NA	NA
Australia	Intercept Break	1919	Post WWI economic crisis as consequence of a fall in exports prices
Belgium	Linear Trend	NA	NA
Brazil	Intercept Break	1947	Brazilian industrial revolution and a huge increase in export prices
Canada	Constant	NA	NA
Denmark	Mixed Breaks	1945	Crisis in agricultural sector after WW2 as consequence of lack of trade
Finland	Linear Trend	NA	NA
France	Slope Break	1958	Structural reforms to increase foreign trade and European integration
Germany	Intercept Break	1933	Economic Reforms which focused on the tradable sector
Italy	Intercept Break	1919	Heavy taxes and distortionary intervention After World War I
Japan	NA	NA	NA
Mexico	Linear Trend	NA	NA
Netherlands	Mixed Breaks	1948	Low productivity after WWII as consequence of shortages of inputs
Norway	Mixed Breaks	1919	Fall in productivity as consequence of lack of capital goods after WWI
Portugal	Slope Break	1959	Manufacture development plan implemented by Salazar's Government
Spain	Mixed Breaks	1948	Economic crisis as consequence of autarkic policies and price controls
Sweden	Intercept Break	1948	Large decrease in export industries during WWII due to closed markets
Switzerland	Linear Trend	NA	NA
United Kingdom	Mixed Breaks	1948	Destruction of infrastructure during WWII and tight regulation thereafter
Colombia	Linear Trend	NA	NA

Sources:
Australia: Wilson, Charles. (1988). "Australia 1788- 1988: the Creation of a Nation. Barnes & Noble Books. Pages 177-178.
Brazil: Pereira, Luiz B. (1984). "Development and Crisis in Brazil, 1930-1983". (Translated from Portuguese). Westview Press Inc. Pages 21-25.
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Sweden: Fritz, Martin. (1982). "The Swedish Economy 1939-1945: A Survey." In: "The Adaptable Nation: Essays in Swedish Economy during the Second World War." Almquist & Wiksell International, Stockholm. Pages 5-16.
United Kingdom: May , Trevor. (1987). "An Economic and Social History of Britain: 1760-1970." Longman Group, UK Limited. Pages 374-383.
NA: Not available.

From Tables 4 and 5, we can examine the estimated intercept breaks in terms of their size and their respective cause. The biggest break is estimated in Brazil and amounts to a 195% upward shift in the trend. This break occurred around 1947 as a result of postwar changes in world trade which led to a huge increase in export prices and to an industrial revolution in Brazil. The second biggest intercept break (39%) is a downward shift detected in Norway in 1919. This productivity drop is the outcome of a shortage of capital goods after the First World War. Table 5 summarizes the cause for the remaining breaks and lists the corresponding sources in the economic history literature.

4.3. Deviations from Purchasing Power Parity

Once the best model for the deterministic trend is chosen and estimated, it is possible to compute its residuals and interpret them as deviations of the real exchange rate from its PPP level. We know these deviations are stationary; therefore, any shock eventually disappears as the real exchange rate returns to its long run level. In order to measure how persistent these residuals are, I perform an ARMA selection model procedure based on the Bayesian Information Criterion (BIC). For each country, I select the best model among the nine available ARMA(*p*,*q*) models with $p \le 2$ and $q \le 2$. Table 6 shows the selected models and their estimated parameters[12]. This table also shows results for the usual measure of PPP deviations persistence in the literature: the half life of shocks. This measure provides the estimated length of time that a temporary shock to the real exchange rate takes to get reduced by a half[13].

Table 6
ARMA Model and Half life of Shocks to the Real Exchange Rate Residuals

Country	Selected Model	AR(1)	AR(2)	MA(1)	MA(2)	Half Life (years)
Argentina	ARMA(1,1)	0.615	0.000	0.059	0.000	1.6
Australia	AR(2)	0.997	-0.239	0.000	0.000	3.1
Belgium	ARMA(1,1)	0.640	0.000	0.349	0.000	2.0
Brazil	AR(1)	0.836	0.000	0.000	0.000	3.8
Canada	AR(1)	0.913	0.000	0.000	0.000	7.3
Denmark	AR(2)	0.921	-0.214	0.000	0.000	2.5
Finland	ARMA(1,1)	0.456	0.000	0.487	0.000	1.1
France	AR(2)	0.942	-0.378	0.000	0.000	2.0
Germany	ARMA(1,1)	0.861	0.000	0.394	0.000	6.5
Italy	ARMA(1,2)	0.576	0.000	0.447	0.373	2.0
Japan	NA	NA	NA	NA	NA	NA
Mexico	AR(1)	0.770	0.000	0.000	0.000	2.7
Netherlands	AR(2)	0.998	-0.220	0.000	0.000	3.3
Norway	AR(2)	1.044	-0.220	0.000	0.000	4.1
Portugal	AR(2)	0.968	-0.267	0.000	0.000	2.6
Spain	AR(2)	1.172	-0.460	0.000	0.000	3.0
Sweden	ARMA(1,1)	0.701	0.000	0.410	0.000	2.5
Switzerland	AR(2)	1.181	-0.350	0.000	0.000	4.5
United Kingdom	AR(2)	0.846	-0.256	0.000	0.000	1.9
Colombia	AR(2)	0.808	-0.022	0.000	0.000	3.0

This table shows final estimations for the selected ARMA model for real exchange rate residuals. The model was selected on the basis of choosing the minimum Bayesian information criterion. Half lives of shocks are interpreted as the number of years that it takes for a temporary shock to the real exchange rate to get reduced by a half.
NA: Not available.

[12] Table A4 in the appendix shows all the values for the BIC criterion for each model and country.
[13] A good description of half-life computation in the context of PPP analysis can be found in Taylor and Taylor (2004).

Interestingly, Table 6 shows that for most countries (10 out of 19), an AR(2) model is selected such that its AR(1) estimated parameter is positive and the AR(2) parameter is negative. This finding shows that deviations from PPP are very persistent during the first year but start converging rapidly during the second year.

The estimated half lives of shocks to the RER in Table 6 range from 1.1 year in Finland to 7.3 years in Canada. The average for 19 countries is 3.1 years which accords with the consensus level (3-5 years) established by Rogoff (1996). For many countries, including Japan, the next step is testing for two structural breaks or more and then to select the optimal number of breaks with an information criterion as suggested by Carrion-i-Silvestre, Kim and Perron (2009). Those tests are however out of the scope of this paper which only seeks to provide general evidence of purchasing power parity assuming up to one break.

5. A Model of Long Run Real Exchange Rate Determination

In the first part of this section, I present a general equilibrium long run model of the Real Exchange Rate (RER) which incorporates the Balassa-Samuelson effect. This model follows closely the model presented by Obstfeld and Rogoff (1996, p. 203-214). The second part of this section explains how the model is useful to interpret and understand the determinants of both linear trends and breaks in the real exchange rate. Finally, I describe an empirical exercise with Total Factor Productivity (TFP) data for 11 OECD countries in which some predictions of the theoretical model are contrasted.

5.1. Specification of the Model

Consider a small open economy which produces two composite goods, tradables and non tradables (NT). The production functions, $Y_T = A_T F(K_{T,} L_T)$ and $Y_N = A_N G(K_{N,} L_N)$ are assumed to have constant returns to scale on both capital and labor. The subscripts (*T*) and (*N*) denote the traded and nontraded sectors respectively, and A_T and A_N are total productivity shifters. Labor is internationally immobile but can migrate instantaneously between sectors within each economy. Labor mobility insures that workers earn the same wage (W) in either sector. The traded good is assumed to be the numeraire good.

While domestic labor supply is fixed, $(L = L_T + L_N)$, there is no resource constraint for capital which is internationally mobile, that is, resources can always be borrowed abroad and turned into domestic capital. As usual, it does not matter whether we model capital as being accumulated by individuals and allocated through a rental market, or being accumulated by firms for their own use.

Assume that one unit of tradables can be transformed into a unit of capital at zero cost. The reverse transformation is, similarly, assumed to be costless. Nontradable goods cannot be transformed into capital, however. This assumption is only for simplification and does not affect the relevant outcomes. As a timing assumption, capital must be put in place a period before it is actually used. Also, capital can be used for production and then consumed as a tradable at the end of the same period.

The assumption on perfect capital mobility ties capital's domestic rate of return to the world interest rate. If *r* is the world interest rate in terms of tradables then, under perfect foresight, *r* must also be the marginal product of capital in the traded goods sector. At the same time, *r* must be the value, measured in tradables, of capital's marginal product in the NT goods sector.

I assume for simplicity a constant world interest rate and let *p* be the relative price of nontradable goods in terms of tradables. The representative firm maximizes the present value of profits measured in units of tradables. Equations (8) and (9) describe this present value for a firm in the tradable and nontradable sector respectively.

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[A_{T,s}F(K_{T,s},L_{T,s}) - W_s L_{T,s} - \Delta K_{T,s+1}\right] \tag{8}$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[p_s A_{N,s}G(K_{N,s},L_{N,s}) - W_s L_{N,s} - \Delta K_{N,s+1}\right] \tag{9}$$

I assume no depreciation so that $\Delta K_{i,s+1} = K_{i,s+1} - K_{i,s}$ for $i = T, N$ [14]. The capital - labor ratios in traded and NT goods production are $k_T \equiv K_T/L_T$ and $k_N \equiv K_N/L_N$ respectively. Outputs per worker in each sector are $y_T = A_T f(k_T) \equiv A_T F(k_T,1)$ and $y_N = A_N g(k_N) \equiv A_N G(k_N,1)$.

[14] A positive depreciation rate, (common for both countries), can be interpreted as a lower real interest rate with no further effects on the solutions of this model.

The following are the first order conditions for firm's profit maximization by choosing capital and labor. In the tradable sector we have the following conditions where the marginal products of both capital and labor are expressed in per-capita terms.

$$A_T f'(k_T) = r \tag{10}$$

$$A_T[f(k_T) - f'(k_T)k_T] = W \tag{11}$$

In the NT sector we have:

$$pA_N g'(k_N) = r \tag{12}$$

$$pA_N[g(k_N) - g'(k_N)k_N] = W \tag{13}$$

I assume that unanaticipated shocks cannot occur so that the first order conditions hold ex ante and ex post; this is a long run, perfect foresight setting. It is then possible to solve for the equilibrium value of the four unknowns, (k_T, W, k_N, p), by operating equations (10), (11), (12) and (13). The main result is that consumer's demand has no role in determining *p*, the relative price of NT goods. Variables like government spending or net capital inflows can have an effect on the real exchange rate when the assumption on perfect capital mobility is dropped.

Equation (10) solves for the equilibrium capital-labor ratio in the tradable sector as a positive function of tradable sector productivity and a negative function of the world interest rate: $k_T(A_T, r)$. Then from equation (10) and (11), it is possible to solve for the equilibrium wage:

$$W = A_T f(k_T) - rk_T \tag{14}$$

Equation (14) is also known as the factor-price frontier, $W = W(A_T, r)$, which is a positive function of A_T and a negative function of *r*.

Equation (12) shows a positive relation between the relative price of NT goods *p* and the capital-labor ratio in the NT sector (k_N). From equation (13) and (14) it is possible to compute the following equation for *p* as function of (k_N).

$$p = \frac{W}{A_N\left(g(k_N) - g'(k_N)\right)} \tag{15}$$

Equations (12) and (15) can be solved simultaneously to find the long run equilibrium values of both *p* and k_N. I assume the following Cobb-Douglas production functions for each sector: $Y_T = A_T K_T^{\alpha} L_T^{1-\alpha}$ and $Y_N = A_N K_N^{\alpha} L_N^{1-\alpha}$. Notice that these functions have the same share (α) for capital in the production process in both sectors.

With Cobb-Douglas production functions it is possible to obtain the following equilibrium solutions for the endogenous variables: wage rate, capital-labor ratios and relative price of NT goods in (16), (17) and (18) respectively.

$$W = (1-\alpha) A_T \left(\frac{\alpha A_T}{r} \right)^{\frac{\alpha}{1-\alpha}} \tag{16}$$

$$k_T = k_N = \left(\frac{\alpha A_T}{r} \right)^{\frac{1}{1-\alpha}} \tag{17}$$

$$p = \frac{A_T}{A_N} \tag{18}$$

We can see in these equations that wages and stocks of capital, in both sectors, are positive functions of A_T and negative functions of the real interest rate; they do not depend on A_N. Therefore, the relative price of NT goods (*p*) is the only adjustment variable when productivity shocks occur in the NT sector.

I assume that $0 < \gamma < 1$ is the share of NT goods in total consumption. Furthermore, the consumer price is obtained by a Cobb-Douglas aggregation of good prices. Therefore, since the tradable goods price is assumed to be equal to one, the total price index is:

$$P = p^{\gamma} = \left(\frac{A_T}{A_N} \right)^{\gamma} \tag{19}$$

Now it is possible to compute the real exchange rate as the ratio of consumer price indices between any pair of countries[15]. I assume the base country to be the US and denote its variables with the corresponding superscript. Furthermore, I assume that γ is equal across countries. The following equation represents the long run equilibrium level of the real exchange rate between a foreign country and the US.

$$\frac{P}{P^{US}} = \left(\frac{A_T / A_N}{A_T^{US} / A_N^{US}} \right)^{\gamma} \tag{20}$$

[15] Note that in this real economy without money, the nominal exchange rate is always equal to one. Therefore, the real exchange rate is simply the ratio of consumer price indices.

In equation (20) it is possible to observe the Balassa–Samuelson effect. With PPP holding in the tradable sector, the long run level of the real exchange rate is a function of productivity evolution in both sectors and across both countries. For instance, real exchange rates have a positive (resp. negative) trend if tradable sector productivity in the foreign country grows faster (resp. slower) than in the US and NT sector productivity is constant in both countries.

5.2. Interpreting Linear Trends and Breaks

Assume that log productivities evolve according to the following linear processes for the tradable and NT sectors respectively: $\log A_T = a_T + b_T t$ and $\log A_N = a_N + b_N t$. Thus the slopes of these processes (b_T, b_N) can be interpreted as the productivity's long run growth rates in each sector. Inserting these processes in the logarithm of equation (20), I obtain a linear expression for the real exchange rate which is suitable for the interpretation of structural breaks.

$$\log\left(\frac{P}{P^{US}}\right) = \gamma(a_T - a_N + a_N^{US} - a_T^{US}) + \gamma(b_T - b_N + b_N^{US} - b_T^{US})t \qquad (21)$$

Equation (21) represents the evolution of the equilibrium RER as function of productivity growth in both sectors and in both countries. Breaks in the intercept can be interpreted as shifts in productivity level. Similarly, breaks in the slope can be interpreted as shifts in the productivity growth rate. A positive (resp. negative) slope is the result of a higher (resp. lower) productivity growth differential between tradable and NT sectors in the home country than in the US.

We can use Equation (21) to understand the estimated slopes presented in Table 4 as outcomes of the Balassa-Samuelson effect. For example, the estimated slope breaks in France and Portugal are the effects of structural reforms implemented in these countries during the 1950's which increased the productivity of the manufacturing sector, (see Table 5). These reforms made the slope switch from negative to positive in both countries. Note that a positive slope does not necessarily mean that the tradable sector is more productive in the foreign country with respect to the US; instead, it means that the productivity gap between tradable and nontradable sectors is higher in the foreign country than in the US.

Equation (21) also allows interpreting the estimated intercept breaks. A positive intercept break in the RER corresponds to either an upward productivity shift in the tradable sector or a downward productivity shift in the NT sector; the interpretation of negative breaks is analogous. Table 5 shows that most of the estimated intercept breaks are associated with negative shocks to the tradable sector which are closely related to one of the world wars.

It is interesting to note that in Tables 4 and 5 we can identify four countries (Denmark, the Netherlands, Spain and UK), where a mixed break occurred at the end of World War II and have one common feature: a negative intercept break simultaneously with a positive slope break. This common feature can be explained as a negative effect of the war on the tradable sector along with positive effects from structural reforms implemented right after the war.

Finally, it is important to explain why some countries do not appear to have any structural break in the real exchange rate during World War II. In particular, it appears puzzling that the tests in this paper do not identify any structural break during this period in Germany, France and Italy. The answer to this question can also be explained with equation (21); when similar magnitude shocks affect the productivity in both sectors, the net effect on the trend of the real exchange rate is very small.

5.3. Empirical Exercise with Productivity Data

Using sectoral Total Factor Productivity (TFP) data I compute the theoretical slope of the real exchange rate trend in Equation (21): $\gamma(b_T - b_N + b_N^{US} - b_T^{US})$. Then I compare these computed theoretical slopes with those estimated using actual real exchange rate data, structural break tests and unit root tests. The general result is that the model presented in Section 5.1 allows a good approximation to the actual long run rate of real appreciation in 7 out of 11 countries. In the following, I explain the details of this exercise.

TFP data are obtained from OECD's International Sectoral Database for 12 countries (including the United States) and 14 different sectors. For each sector, OECD computes TFP as Solow residuals assuming Cobb-Douglas production functions and using data for capital stock, employment and value added. Following De Gregorio and Wolf (1994), I define TFP in the tradable sector as the average of agriculture, mining, manufacturing and transportation weighted by their relative value added. Nontradable sector TFP is the weighted average of the remaining sectors. The database's span of data starts in 1970 and ends in the 1990s. Table 7 shows the specific span of data for each country.

I compute average annual TFP growth for both sectors in each country. Table 7 shows that, in each country, TFP growth in the tradable sector is higher than in the nontradable sector. Furthermore, it is observed average negative growth in the nontradable sector in four countries: USA, Canada, Italy and Norway. Finland shows the fastest combined productivity growth among these 12 countries.

Table 7
Equilibrium Real Exchange Rate Growth
Comparison of Estimated Rates Versus Rates Computed with TFP Data

Country	Sample	Annual TFP growth		Equilibrium RER Appreciation		Absolute Error
		Tradable Sector	Nontradable Sector	Implied: TFP data	Estimated	
USA	1970-1993	0.9%	-0.3%	NA	NA	NA
Belgium	1970-1995	3.1%	0.3%	0.8%	0.5%	0.2%
Canada	1970-1997	0.3%	-0.3%	-0.3%	0.0%	0.3%
Denmark	1970-1992	1.5%	0.4%	-0.1%	1.4%	1.5%
Finland	1970-1996	2.9%	1.0%	0.4%	0.2%	0.2%
France	1970-1997	1.8%	0.8%	-0.1%	0.6%	0.7%
Netherlands	1970-1994	1.3%	0.7%	-0.3%	1.5%	1.8%
Italy	1970-1994	2.5%	-0.4%	0.9%	0.5%	0.4%
Norway	1970-1991	0.9%	-0.8%	0.2%	0.6%	0.4%
Sweden	1970-1994	1.6%	0.4%	0.0%	0.4%	0.4%
UK	1970-1990	1.7%	0.2%	0.2%	1.0%	0.8%
Germany	1970-1993	1.3%	0.6%	-0.3%	0.1%	0.4%

This table compares the annual equilibrium real exchange rate growth estimated in Table 4 with that implied by Total Factor Productivity (TFP) data as in Equation (21). It is assumed that the share of non tradable goods in total consumption is 0.5. Sectoral TFP data were retrieved from the OECD International Sectoral Database.
NA: Not available.

Using annual TFP growth for each country and for the US as well as Equation (21), I compute the theoretical annual rates of real exchange rate appreciation (with respect to the US Dollar) which are shown in Table 7. The highest appreciation rates correspond to those countries where the productivity growth gap between tradable and nontradable sectors is greater: Italy and Belgium. These rates are then compared with the equilibrium appreciation rates in Table 4 which are computed with actual real exchange rate data once structural breaks and model selection procedures have been taken in account.

In the last column, Table 7 shows absolute differences between theoretical and empirical appreciation rates. We can notice that in 7 out of 11 countries these errors are equal or below 0.4%. Therefore, for these 7 countries the Balassa-Samuelson effect is a good approximation to the observed long run appreciation rates. We should take in account that theoretical rates have been calculated with sectoral TFP data which is only available after 1970. For this reason, an annual approximation error of 0.4% is considered a close enough approximation in this specific exercise. The Balassa-Samuelson model fails to match the observed appreciation rate in: Denmark, France, Netherlands and UK. In these four cases, the theoretical rate is much lower than the observed appreciation rate[16].

6. Conclusions

This paper provides evidence of long run purchasing power parity by performing a recently developed method to test for unit roots in the presence of structural breaks. The econometric method is based on Carrion-i-Silvestre, Kim and Perron (2009) and allows separating the issue of testing for structural breaks from the issue of testing for a unit root. Improvements in size and power of the tests are brought about by using a structural break pretest based on Perron and Yabu (2009).

Data consist of real exchange rate series for 20 countries including developed and developing economies. Structural breaks are detected in 18 countries and real exchange rates are found to be stationary in all countries except Japan. These results contrast with those obtained by Papell and Prodall (2006) because they obtain positive PPP evidence for 17 countries but find structural break evidence only in 5 countries. Additionally, these results recover the positive PPP evidence pointed out by Taylor (2002) and known as "a century of purchasing power parity".

Using a two-country and two-sector macroeconomic model it is possible to show that the estimated linear trends are the result of cross-country total factor productivity differentials between tradable and nontradable sectors. Furthermore, estimated breaks correspond to large and permanent total factor productivity shocks associated with historical events like wars, structural reforms or deep economic recessions.

An empirical exercise with sectoral total factor productivity data shows that the Balassa-Samuelson model explains the estimated long run trends in most of OECD countries where these data are available. Future work on additional exercises with sectoral

[16] Besides the limited span of TFP data, the fact that the unit root tests only allow for one break can explain the poor approximation of the Balassa-Samuelson model in these countries.

productivity data is necessary to fully understand the extent of the Balassa-Samuelson effect and its impact on the real exchange rate in the remaining countries.

The methods applied in this paper are an alternative approach to study the degree of misalignment of any real exchange rate. This can be done by comparing observed levels of the real exchange rate with its PPP level which corresponds to the estimated linear trend. This kind of analysis is important for policy makers in developing countries who try to detect and prevent balance of payment crises as explained, among others, in Edwards (1989).

Appendix A: Additional Tables

Table A1
Structural Break and MZt Unit Root Tests
Model 1: Break in the Intercept

Country	Sample	Pre-test	Break Date	Unit Root Test	k_{MAIC}
Argentina	1884-2006	0.8611	NA	NA	NA
Australia	1870-2006	3.0293**	1919	-3.177**	0
Belgium	1880-2006	15.925***	1918	-1.7871	0
Brazil	1889-2006	4.2061***	1947	-2.7117*	0
Canada	1870-2006	0.78752	NA	NA	NA
Denmark	1880-2006	4.7271***	1945	-1.6811	7
Finland	1881-2006	2.7368**	1917	-2.5986	0
France	1880-2006	3.0169**	1984	-3.5263***	0
Germany	1880-2006	2.4027**	1933	-2.7614*	2
Italy	1880-2006	11.046***	1919	-2.894*	0
Japan	1885-2006	2.9875**	1931	-2.2591	0
Mexico	1886-2006	12.373***	1981	-2.3768	0
Netherlands	1870-2006	2.7049**	1948	-2.6113	2
Norway	1870-2006	6.2154***	1919	-2.8883*	0
Portugal	1890-2006	6.971***	1919	-2.9071**	0
Spain	1880-2006	2.3457**	1919	-2.5077	0
Sweden	1880-2006	1.9978*	1948	-2.9858**	0
Switzerland	1892-2006	1.4052	NA	NA	NA
United Kingdom	1870-2006	6.2112***	1948	-1.4586	7
Colombia	1923-2006	4.0901***	1956	-2.5876	1

This table shows the MZt unit root test allowing for a structural break as defined in Carrion-i-Silvestre, Kim and Perron (2009). It is applied to long span real exchange rate series for 20 countries. The pre-test allows assesing whether a break is present or not as defined by Perron and Yabu (2009). The break date is endogenous to the estimation procedure as described as well in Carrion-i-Silvestre, Kim and Perron (2009). The break intercept model allows for one break in the intercept of the deterministic trend. kMAIC denotes the optimal lag in the autoregression which is performed in order to estimate the zero frequency spectral density. This lag is chosen according to the modified information criterion (MAIC) which is described in Ng and Perron (2001).
* denotes significance at 10% level; ** denotes significance at 5% level; *** denotes significance at 1% level. **NA:** Not available.

Table A2
Structural Break and MZt Unit Root Tests
Model 2: Slope Break

Country	Sample	Pre-test	Break Date	Unit Root Test	k_{MAIC}
Argentina	1884-2006	-0.0678	NA	NA	NA
Australia	1870-2006	-0.2903	NA	NA	NA
Belgium	1880-2006	-0.1671	NA	NA	NA
Brazil	1889-2006	-0.2463	NA	NA	NA
Canada	1870-2006	-0.1362	NA	NA	NA
Denmark	1880-2006	-0.23857	NA	NA	NA
Finland	1881-2006	-0.22122	NA	NA	NA
France	1880-2006	8.6658***	1958	-4.0735***	0
Germany	1880-2006	-0.2666	NA	NA	NA
Italy	1880-2006	-0.26731	NA	NA	NA
Japan	1885-2006	-0.25247	NA	NA	NA
Mexico	1886-2006	-0.25221	NA	NA	NA
Netherlands	1870-2006	-0.22805	NA	NA	NA
Norway	1870-2006	-0.29821	NA	NA	NA
Portugal	1890-2006	1.9886*	1959	-3.4672**	0
Spain	1880-2006	-0.14984	NA	NA	NA
Sweden	1880-2006	-0.2265	NA	NA	NA
Switzerland	1892-2006	-0.2832	NA	NA	NA
United Kingdom	1870-2006	-0.1114	NA	NA	NA
Colombia	1923-2006	-0.2736	NA	NA	NA

This table shows the MZt unit root test allowing for a structural break as defined in Carrion-i-Silvestre, Kim and Perron (2009). It is applied to long span real exchange rate series for 20 countries. The pre-test allows assesing whether a break is present or not as defined by Perron and Yabu (2009). The break date is endogenous to the estimation procedure as described in Carrion-i-Silvestre, Kim and Perron (2009). The slope break model allows for one break in the slope of the linear deterministic trend. kMAIC denotes the optimal lag in the autoregression which is performed in order to estimate the zero frequency spectral density. This lag is chosen according to the modified information criterion (MAIC) which described in Ng and Perron (2001).
* denotes significance at 10% level; ** denotes significance at 5% level; *** denotes significance at 1% level. NA: Not available.

Table A3
Structural Break and MZt Unit Root Tests
Model 3: Mixed Breaks

Country	Sample	Pre-test	Break Date	Unit Root Test	k_{MAIC}
Argentina	1884-2006	4.7513**	1974	-3.3533**	0
Australia	1870-2006	3.4843*	1919	-3.3256**	0
Belgium	1880-2006	23.859***	1918	-1.9819	0
Brazil	1889-2006	4.2381**	1947	-2.7212	0
Canada	1870-2006	0.9001	NA	NA	NA
Denmark	1880-2006	3.1364*	1945	-3.6053**	0
Finland	1881-2006	6.4932***	1917	-2.6893	0
France	1880-2006	9.1519***	1984	-3.5724***	0
Germany	1880-2006	2.7453*	1933	-2.7891	2
Italy	1880-2006	11.241***	1919	-2.8943	0
Japan	1885-2006	3.2503*	1931	-2.316	0
Mexico	1886-2006	12.989***	1981	-2.2784	6
Netherlands	1870-2006	3.0774*	1948	-3.1019*	0
Norway	1870-2006	6.5023***	1919	-2.9904*	0
Portugal	1890-2006	7.3256***	1919	-2.9696	0
Spain	1880-2006	5.8444***	1948	-3.2671**	0
Sweden	1880-2006	3.2983*	1948	-2.9856*	0
Switzerland	1892-2006	1.336	NA	NA	NA
United Kingdom	1870-2006	19.095***	1948	-3.8229***	0
Colombia	1923-2006	4.1648**	1956	-2.6332	1

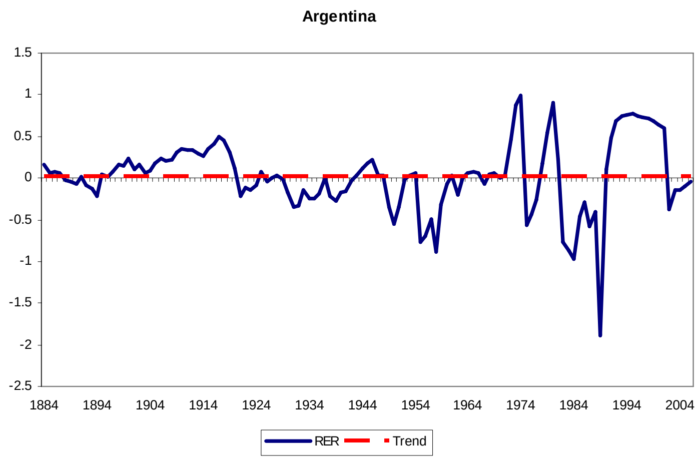
This table shows the MZt unit root test allowing for a structural break as defined in Carrion-i-Silvestre, Kim and Perron (2009). It is applied to long span real exchange rate series for 20 countries. The pre-test allows assesing whether a break is present or not as defined by Perron and Yabu (2009). The break date is endogenous to the estimation procedure as described as well in Carrion-i-Silvestre, Kim and Perron (2009). The mixed break model allows for one simultaneous break in both the intercept and the slope of the linear deterministic trend. kMAIC denotes the optimal lag in the autoregression which is performed in order to estimate the zero frequency spectral density. This lag is chosen according to the modified information criterion (MAIC) which described in Ng and Perron (2001).
* denotes significance at 10% level; ** denotes significance at 5% level; *** denotes significance at 1% level. NA: Not available.

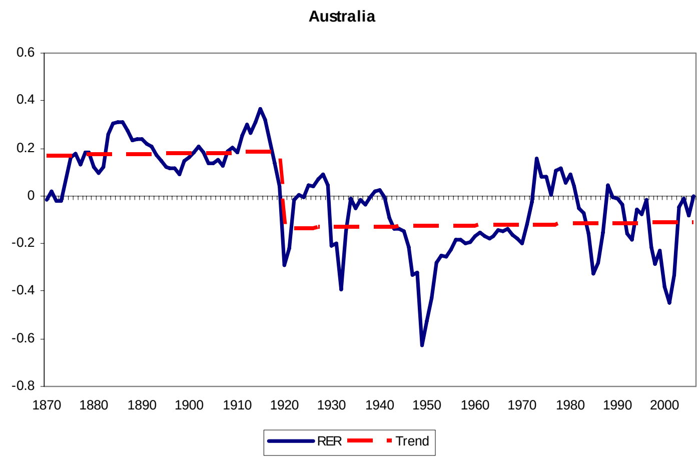
Table A4
Model Selection Based on Bayesian Information Criteria

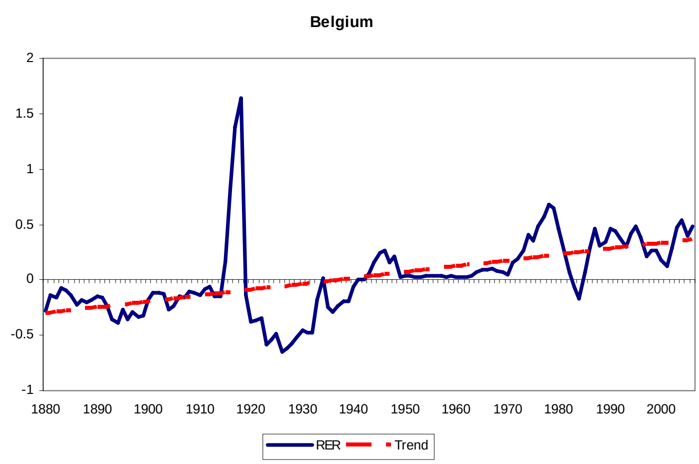
Country/ Model	A. Constant	B. Linear Trend	C. Intercept Break	D. Slope Break	E. Mixed Breaks	Selected
Argentina	-1.8227	-1.673	NA	NA	-1.6667	A
Australia	-3.2211	-3.6215	-3.9304	NA	-3.9196	C
Belgium	-2.0476	-2.3638	NA	NA	NA	B
Brazil	-1.3515	NA	-1.6554	NA	NA	C
Canada	-4.3831	NA	NA	NA	NA	A
Denmark	NA	NA	NA	NA	-3.6811	E
Finland	-2.9080	-2.9281	NA	NA	NA	B
France	NA	NA	-4.0016	-4.2225	-3.9810	D
Germany	-2.9101	-3.1318	-3.1545	NA	NA	C
Italy	-2.7219	-2.7018	-2.8377	NA	NA	C
Japan	NA	NA	NA	NA	NA	NA
Mexico	NA	-2.6506	NA	NA	NA	B
Netherlands	-2.8629	-3.0644	NA	NA	-3.6169	E
Norway	NA	-3.0210	-3.3211	NA	-3.3311	E
Portugal	NA	NA	-3.0920	-3.1546	NA	D
Spain	NA	NA	NA	NA	-3.3451	E
Sweden	-3.3460	-3.5282	-3.5580	NA	-3.5233	C
Switzerland	NA	-3.1829	NA	NA	NA	B
United Kingdom	-3.6858	NA	NA	NA	-4.5625	E
Colombia	NA	-2.5408	NA	NA	NA	B

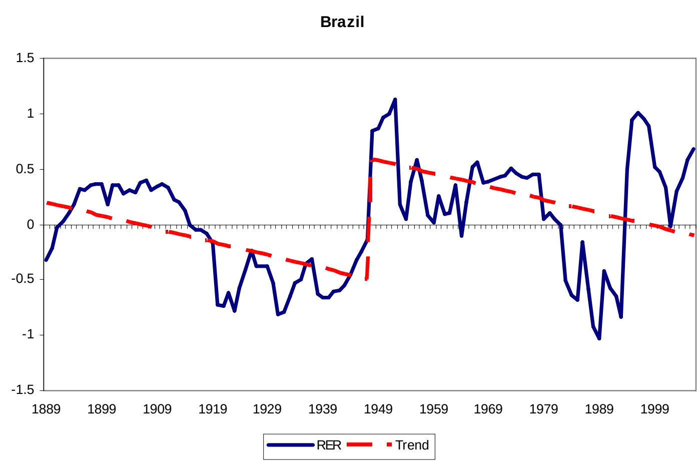
This table shows Bayesian information criteria calculations applied to those models in which PPP evidence is found. The selected model corresponds to the lowest value of the information criterion. The specification of the models is described in Equations (1), (2), (6), (7), and (8).
NA: Not available.

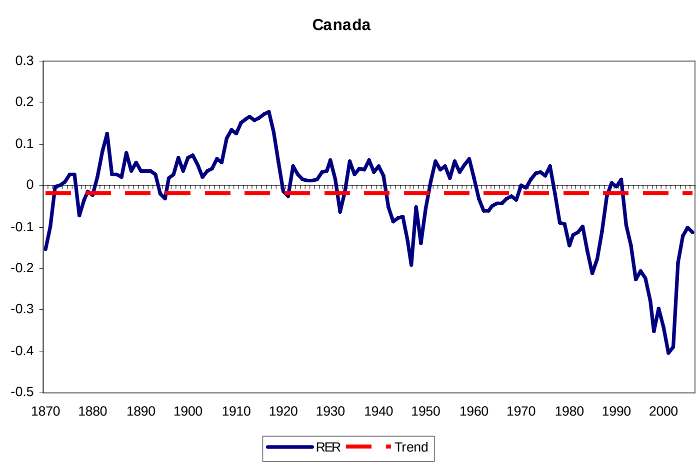
Appendix B: Figures for Real Exchange Rate and Estimated Trends

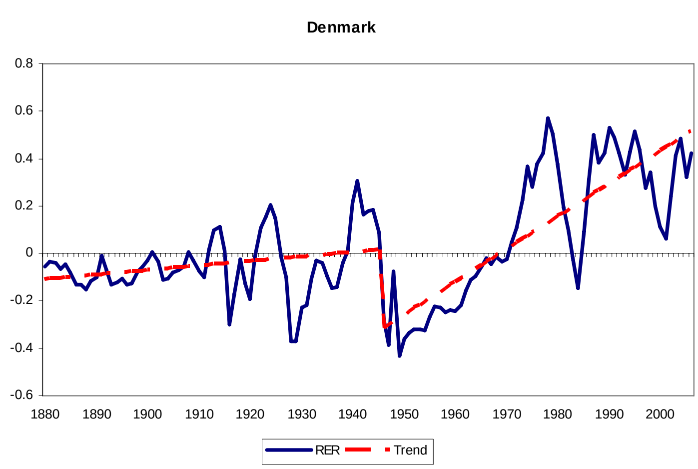


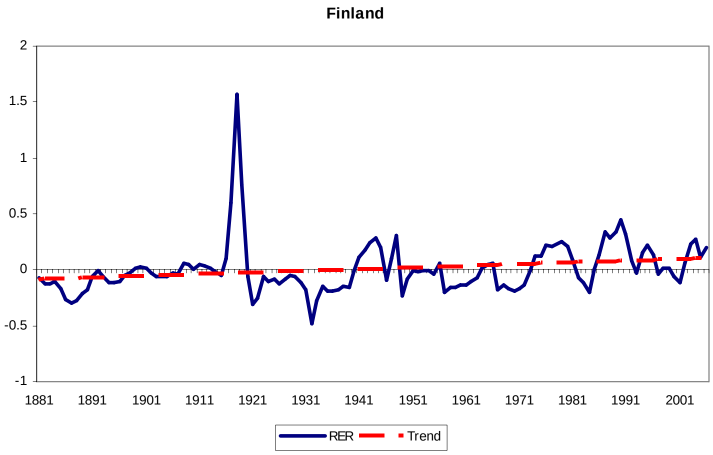








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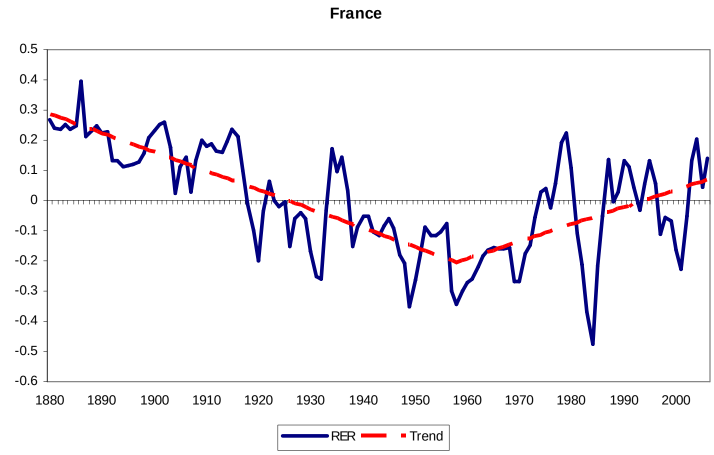
1
0.5
0
-0.5
-1
1881 1891 1901 1911 1921 1931 1941 1951 1961 1971 1981 1991 2001
RER
Trend


France
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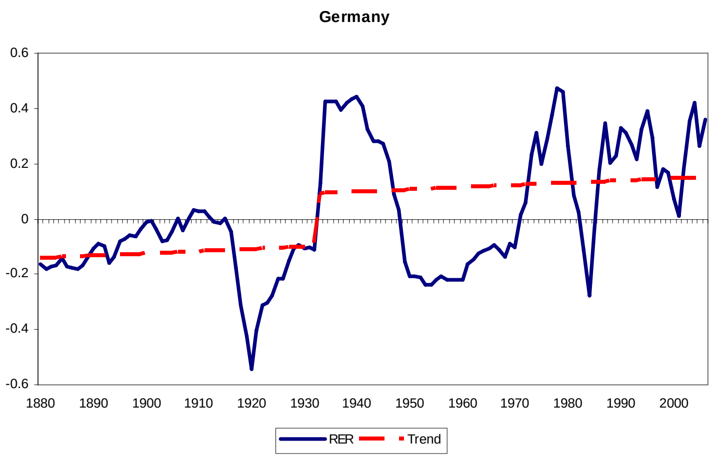
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
1880 1890 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000
RER
Trend


Germany
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0.2
0
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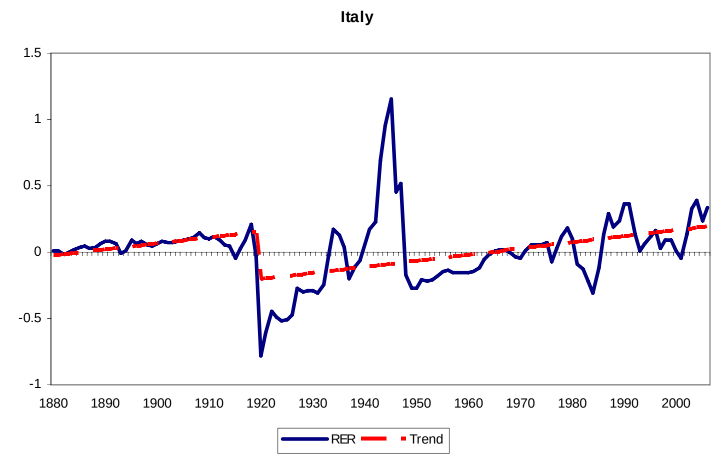
-0.4
-0.6
1880 1890 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000
RER
Trend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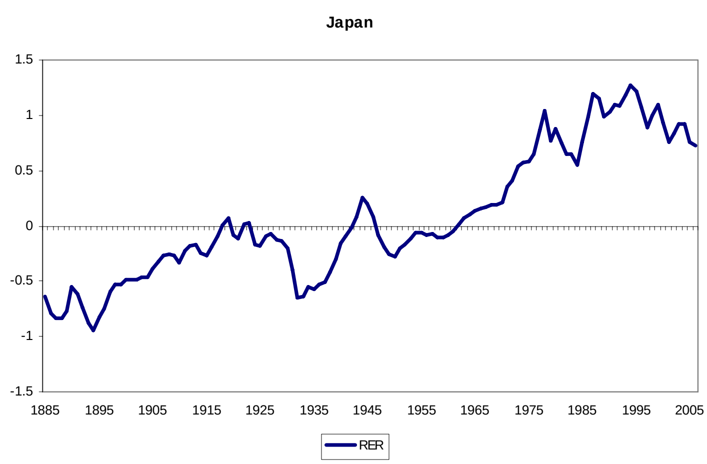
1
0.5
0
-0.5
-1
1880 1890 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000
RER
Trend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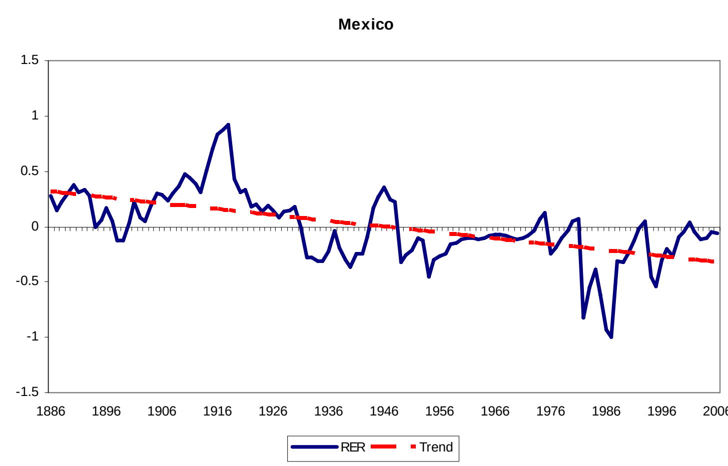
1
0.5
0
-0.5
-1
-1.5
1885 1895 1905 1915 1925 1935 1945 1955 1965 1975 1985 1995 2005
RER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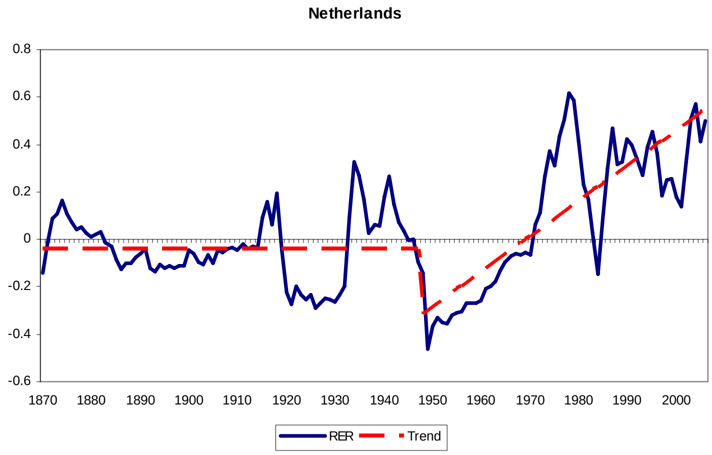
-1
-1.5
1886 1896 1906 1916 1926 1936 1946 1956 1966 1976 1986 1996 2006
RER
Trend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0
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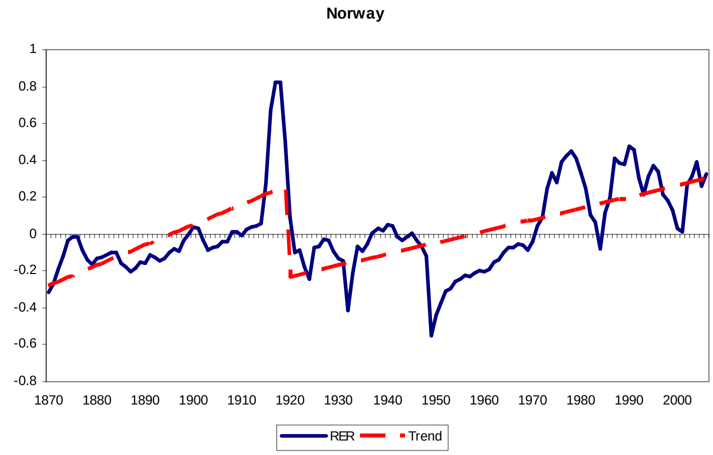
-0.6
1870 1880 1890 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000
RER
Trend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0.6
0.4
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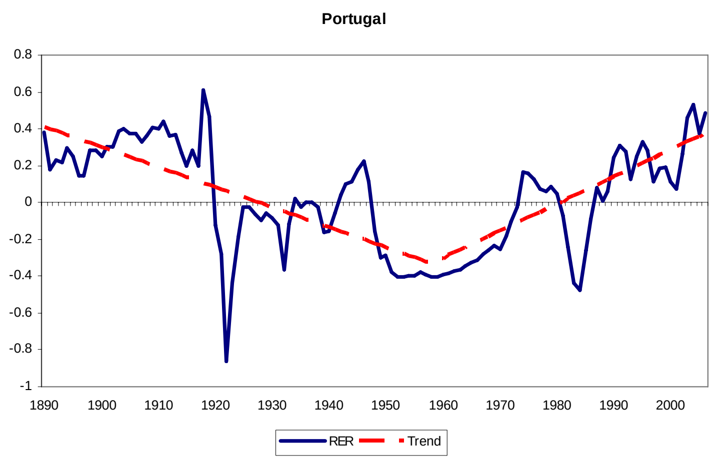
-0.4
-0.6
-0.8
1870 1880 1890 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000
RER
Trend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0.4
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0
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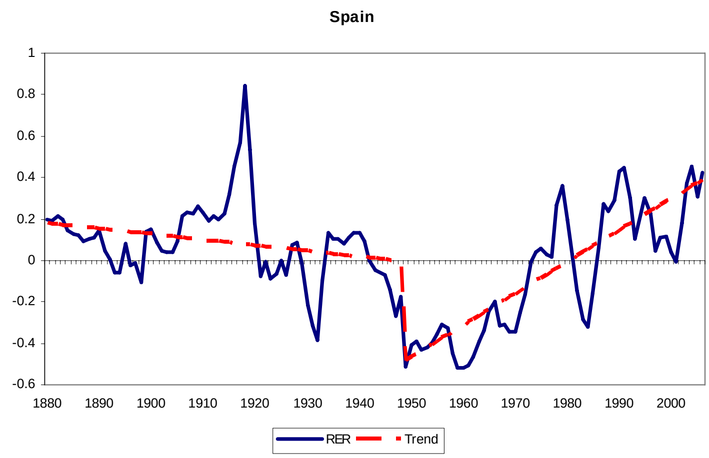
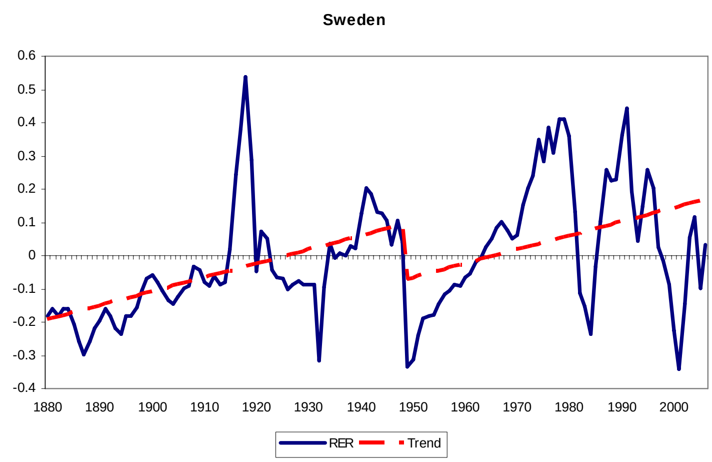
-0.6
-0.8
-1
1890 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000
RER
Trend


Spain
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
1880 1890 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000
RER
Trend


Sweden
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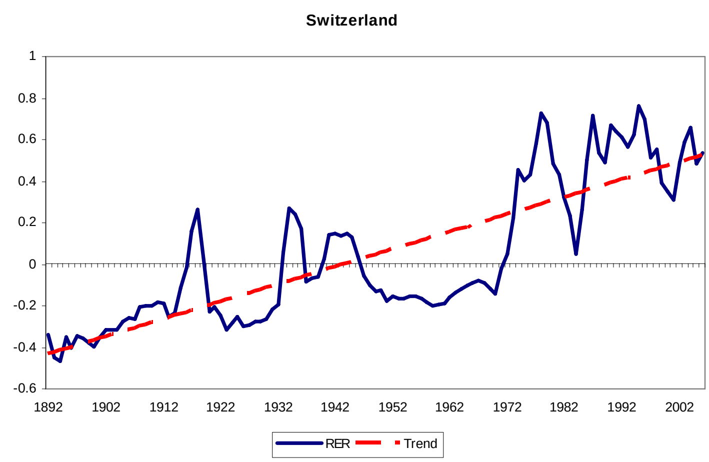
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
1880 1890 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000
RER
Trend


Switzerland
1
0.8
0.6
0.4
0.2
0
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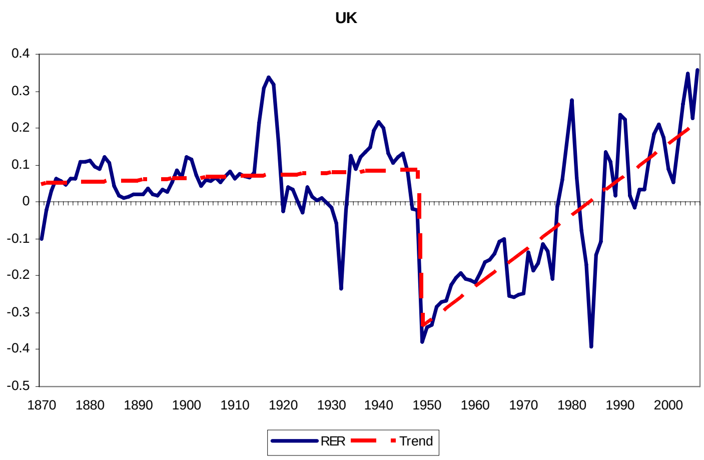
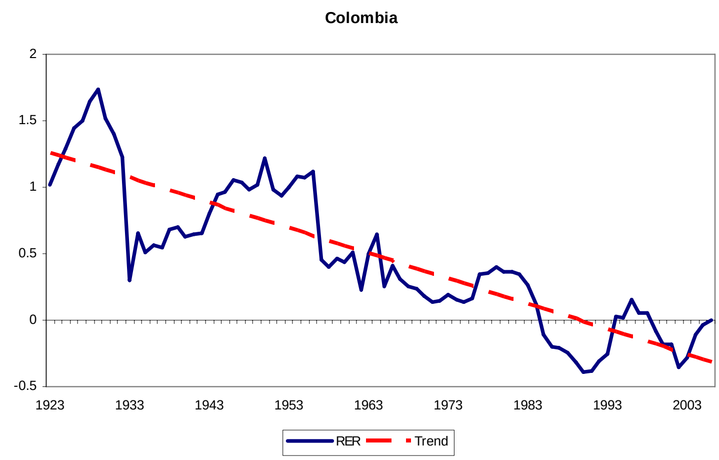
-0.4
-0.6
1892 1902 1912 1922 1932 1942 1952 1962 1972 1982 1992 2002
RER
Trend





Appendix C: Real Exchange Rate Data Description

The data consists of annual real exchange rates for 20 countries. The longest series starts on 1870. The original data set, which contains data through 1996, was provided by Alan Taylor. According to Taylor (2002), these series were constructed with the IMF's International Financial Statistics (IFS) and the historical statistical volumes by Brian Mitchell. I updated the information through 2006 with data from IFS and central banks on observed nominal exchange rates and consumer price indices. See Taylor (2002, p. 140) for a description of some interpolations included in the dataset during world war periods in a few countries. The Colombian RER series was constructed with series for the nominal exchange rate and the consumer price index provided by Banco de la Republica (Colombian Central Bank).

All tests in this paper are computed on the natural logarithm of the real exchange rate index in each country. Let q_t^i be the log real exchange rate for country i; it is computed using the following formula:

$$q_t^i = e_t^i - p_t^{US} + p_t^i$$

Where e_t^i is an index for the log nominal exchange rate in terms of US dollars per unit of i's currency. The logarithm of the Consumer Price Index (CPI) in country i and the US are p_t^i and p_t^{US} respectively.

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