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Prices: an endogenous analysis using
an SVAR approach

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Monetary Policy and Commodity Prices: an endogenous analysis using an SVAR approach[†]

Luz Adriana Flórez¹

Abstract

This work analyzes the relationship between real interest rates and commodity prices. According to Frankel's hypothesis (1986-2006): "low real interest rates lead to high real commodity prices". However, some empirical evidence suggests that commodity prices can predict monetary policy. In this way, there is an endogeneity between commodity prices and monetary policy. Using Frankel's model we include a Taylor rule equation in this theoretical model, which let us analyze the endogeneity problem. In order to find empirical support of this model, we estimate SVAR and, using quarterly data from 1962:Q1 to 2009:Q1, we find that the overshooting of commodity prices to 1% increase of real interest rate can be a minimum of 2.86% and a maximum of 5.97% depending on the chosen model. The increase of real interest rate given a 1% increase in commodity prices is positive and significant but of small magnitude (0.20% - 0.05%).

JEL Classification: C32, E31, E52, G14

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1. Introduction:

As Webb (1988) suggests, the interest of Monetary Policy to study commodity prices is given for two prepositions. The first is that “commodity prices are determined in auction markets; they will therefore change quickly in response to monetary policy actions”. And second, “changes in commodity prices are good predictors of future aggregate price change”. If this is true then “commodity prices might well be a useful guide for monetary policy, possibly serving as an intermediate target or at least as an important indicator variable” p. 3.

Commodity prices are one of the most flexible prices in the economy; they can most accurately reflect the effect of monetary policy (Bordo, 1980). In this sense, by understanding the behaviour of commodity prices, we can understand monetary policy behaviour, and vice versa. Notice that the commodity price is a global price and should therefore reflect the global monetary policy, or at least the monetary policy of the most important economies (US, UK, Euro zone and Japan).

The second reason to study commodity prices is that they have a huge influence on the behaviour of the majority of economies. The most recent boom in commodity prices, which occurred during 2003 to mid-2008, had a strong effect on the national income, exchange rate, current account and fiscal balance of developing and developed countries. As some authors describe, this boom was explained by an important increase in global demand, as a result of globally low interest rates during previous years (Arango *et al.*(2008), Askari and Krichene (2007), and Cechetti and Moessener (2008))²

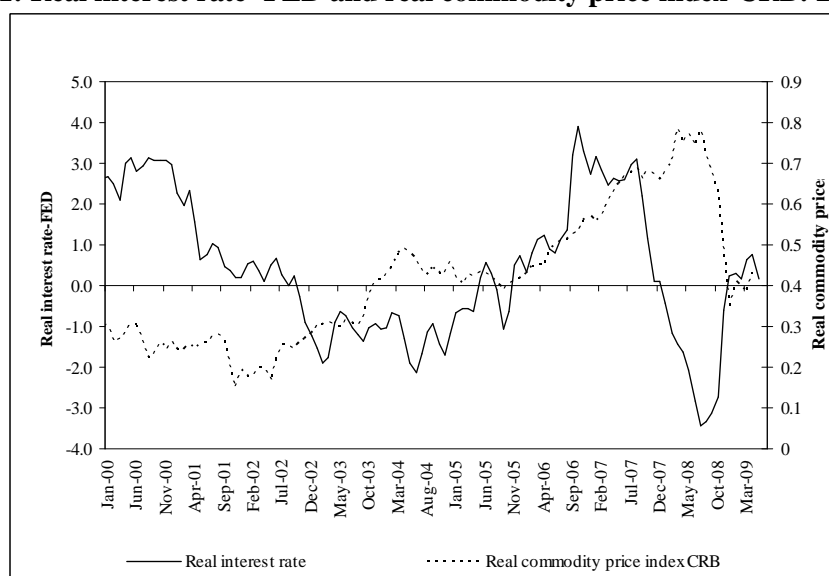
Currently, with a recession in the most important countries of the world (United States, Europe, UK, etc), the increase in commodity prices has halted. However, this appears not to be for long term because the Federal Reserve Bank and the Bank of England, along with other banks in the world, have greatly reduced interest rates. As a consequence, commodity prices have recently started increasing again (**Figure 1**).

It sees that there is a very important relationship between commodity prices and interest rates. How can economic theory explain this behaviour? How do changes in commodity prices influence monetary policy decisions and how do monetary policy decisions affect commodity prices? The aim of this paper is to answer these questions

² Other factors as supply disruptions and speculation had done an important role.

and give more information about the impact of monetary policy on the behaviour of commodity prices and vice versa.

Figure 1: Real interest rate- FED and real commodity price index-CRB: 2000-2009



Source: Datastream, own calculations

This paper is divided into seven sections. The first section is being with this introduction. The second section presents a short brief of the literature on studies of commodity prices and monetary policy along with their most important conclusions. The third section examines the theoretical Frankel’s model which is the basis for the empirical SVAR approach presented in the fourth section. The fifth and sixth sections show the results and robustness of our model, and finally the seventh section summarises the findings and conclusions.

2. Literature review

There was considerable research in to relationship between monetary policy, interest rates and commodity prices during the 80’s. Bordo (1980) showed empirically that prices of raw goods respond more quickly to monetary growth than prices of manufactured goods. This is because the contract structure is different between these two sectors. For example, the auction market, which is an example of commodity prices, is characterized by relative price flexibility.

Frankel and Hardouvelis (1983, 1985) use commodity prices as a potential measure of the market’s perfection of the current monetary policy. According to them, commodity prices are like assets, because the prices are free to adjust from day to day. They looked at the reactions to money supply announcements by observing the prices of nine commodities (gold, silver, sugar, cocoa, cattle, feeders, wheat, soybeans, and

corn). They found that during the period 1980-1982, the market had confidence in the Fed's commitment to stick to its money growth targets.

Following on from this work, Frankel (1986) develops in more detail his overshooting model of commodity prices which follows Dornbusch's (1976) model. According to Frankel, a decline in the nominal money supply implies a decline in the real money supply in the short run, because prices are sticky. As a consequence, this increase in the real interest rate will depress real commodity prices: "They overshoot their new equilibrium in order to generate an expectation of future appreciation sufficient to offset the higher interest rate" p. 344. Finally, Frankel (2006) finds more evidence to support the relationship between real interest rates and real commodity prices. In this case, he develops the concept of inventory cost, and explores empirically the inventory cost effect on commodity prices.

Some current papers have continued to explore empirically Frankel's idea. Arango, Arias and Flórez (2008) find evidence in support of the fact that interest rates seem to maintain a negative relationship with commodity prices. They use a Panel data of 28 commodity prices, and take into account other variables like productivity, traded quantities of commodities and lags in interest rates. Nonetheless, this relationship is not clear for the period 1980-2009. According to Arango *et al.* (2008), the effect of interest rates on commodity prices can take more than one period to become evident.

In addition to the view of commodity prices being a measure of the market's perfection of current monetary policy suggested by Frankel, Webb (1988) suggests that commodity prices are an important predictor of future aggregate price changes. This view has been analyzed extensively in the literature, for example by Garner (1989), Awokuse and Yang (2003), Cody and Mills (1991) and Pecchenino (1992), among others. In general they have found evidence supporting the idea of commodity prices being an important predictor of inflation and future monetary policy. Awokuse and Yang (2003) for example, using the methodology of Toda and Yamamoto (1995) for an alternative procedure of the Granger Causality test, found evidence that commodity prices do not move with changes in lagged macroeconomic variables; however commodity prices give a significant explanation of the future path of the federal fund rate, CPI and industrial production.

Cody and Mills (1991), using a SVAR model, found that the response of monetary policy to commodity prices³ was small, and not statistically significant for the period between 1959:1 and 1987:12. However they showed that if the Federal Reserve gives more weight for stabilizing inflation, then the optimal response requires tighter policy when commodity price inflation accelerates. Nevertheless, some authors have suggested that commodity prices lost the ability to predict inflation after the mid-1980. This is the belief of Blomberg and Harris (1995), Furlong and Ingenito (1996) and Cecchetti and Moesser (2008). The latter found that during the last 15 years, commodity prices have not produced stronger second-round effects in headline inflation for the 19 countries considered⁴.

Moreover, there is a highly esteemed group of researchers who study monetary policy shocks using VAR and SVAR models. In general, these models have included the commodity price variable in order to get a more specific idea of the reaction of monetary policy function. As mentioned by Brissimas and Magginas (2004), the most common empirical problem found in these studies has been the price puzzle⁵. This is evidence of a serious misspecification problem, in particular in the model's equation describing the monetary policy reaction function. As a solution for the price puzzle problem, some authors have proposed adding the commodity price index. The inclusion of this variable has been justified by the fact that commodity prices contain information on the future expectation of inflation.

Kim (1999) reported that “after including some variables representing inflationary pressure such as the commodity price index in the monetary reaction functions, research has resolved the price puzzle” p. 389. However, as argued by Christiano, Eichenbaum and Evans (1998) the assumptions about the relationship between commodity prices and monetary policy are more difficult to assess on theoretical grounds given the absence of an explicit monetary general equilibrium model that incorporates a market for commodity prices.

The use of the commodity price index in VAR models as a predictor of future prices is not unique. There is some work that instead of using commodity prices has

³ The authors do not analyze the contemporaneous effect of commodity prices given a change in monetary policy rule.

⁴ Canada, Denmark, the euro area, Japan, Norway, Sweden, Switzerland, the United Kingdom, The United States, China, Chinese Taipei, Hong Kong SAR, Hungary, Indonesia, Korea, Mexico, Singapore, South Africa and Thailand.

⁵ This is known as a positive response of the price level to a monetary policy tightening (reported by Brissimas, Magginas (2004), Kim (1999), Christiano, Eichenbaum and Evans (1998) and other authors).

used the exchange rate⁶ or leading composite indicators. This is the case of Sousa and Zaghini (2007a)⁷, Peersman and Smets (2001) and Brissimis and Magginas (2004). Sousa and Zaghini (2007a) analyze the international transmission of monetary policy shocks focusing on the effects of foreign liquidity to the euro area. Instead of commodity prices they use the real exchange rate as a variable in the reaction monetary policy function. In this case, the monetary policy responds contemporaneously to monetary aggregate and exchange rate.

Brissimis and Magginas (2004) show that augmenting a standard VAR with a small number of variables which have forward looking information (federal funds futures and leading composite indicator), allows them to produce a theory-consistent response function to monetary policy shocks. In this case the use of commodity prices is substituted by the leading composite indicator. As we can see, the use of a commodity prices index in the specification of the monetary policy reaction function has been more arbitrary and more for convenience than for theoretical arguments. Moreover, in these models the overshooting of commodity prices given a shock in the monetary policy is usually not analyzed.

Some of the papers that have recently emphasized the endogeneity of commodity prices and monetary policy have been Browne and Cronin (2007) and Askari and Krichene (2007). The former showed that there are long run and short run relationships between commodity prices, consumer prices and money. Using a co-integrated VAR model they found that commodity prices initially overshoot their new equilibrium values in response to a money supply shock and that this effect is finally reflected in consumer price inflation. Askari and Krichene (2007) found that during the last boom in commodity prices, 2003-2008, the increase in prices was a result of a monetary shock; this means a low interest rate. The increase in commodity prices was not reflected in the consumer price index (the relationship between consumer and commodity prices seemed to have weakened), causing a policymaker to be a wrongly influenced about the price stability. "Neglecting information for commodity prices may result in unsustainable monetary policy" Askari and Krichene (2007) p. 3.

In summary we can observe from the literature an endogenous relationship between commodity prices and monetary policy. Even though in the literature we find

⁶ The justification for the exchange rate is that being an asset price, it reacts immediately to changes in all the other variables, as commodity prices.

⁷ However, in Sousa and Zaghini (2007b) the authors use commodity prices and a measure of global liquidity for the G5.

empirical studies that analyze this relationship, there are no presentations of a theoretical framework. For that reason in this paper we present a theoretical model that permits us to support the endogenous empirical relationship between commodity prices and monetary policy. Using Frankel's model (1986-2006) and including a monetary policy rule, we test this relationship by an SVAR approach. In contrast with the general studies, we impose the contemporaneous identification restrictions given by our theoretical model and in a non-arbitrary way. The contemporaneous coefficient estimations and the impulse response function of the variables, support our model.

3. Structural Approach

3.1 Intuition of Frankel model

As suggested by Okun (1975) and Bordo (1980), there is a distinction between the prices of manufactured goods and those of commodities:

“The former are the ones with sticky prices: they are differentiated products traded in imperfectly competitive markets where there is no instantaneous arbitrage to insure perfect price flexibility. But the latter do have flexible prices: they are homogeneous products traded in competitive markets where arbitrage does insure instantaneous price adjustment. Commodities are more like assets in this respect. Since their prices are free to adjust from day to day, and even from minute to minute, they offer a potential measure of the market's perception of current monetary policy”⁸, reported by Frankel and Hardouvelis (1983) p. 2.

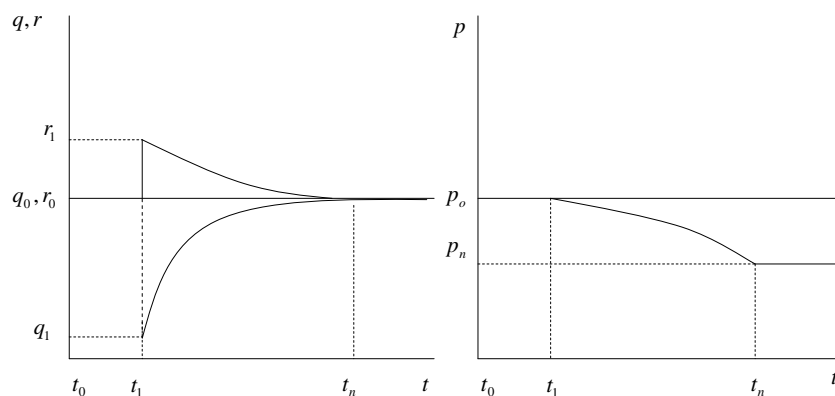
Following this intuition, Frankel *et al.* (1983-2006) explain the overshooting model as follows: Suppose that the economy presents a drop of 1% in money supply and that this is expected to be permanent. Then in the long run all the prices should fall by 1%. But given that in the short run the manufacture prices are fixed, the reduction in the nominal supply is a reduction in real money supply. To equilibrate the money demand the interest rate may increase. But given that commodity goods are storable, they should follow the arbitrage condition (the rate of return on Treasury bills can be no greater than the expected rate of increase of commodity prices, minus the storage cost). Then the commodity prices must fall today by more than one percent.

Frankel (2006) explains that this temporary increase in the real interest rate can be given whether via an increase in the nominal interest rate, a fall in the expected

⁸ Additionally, Frankel *et al.* (1983) reports that: “Okun himself recognized that commodity prices would be sensitive indicators of inflationary expectations. It is not just that commodity prices are free to adjust and others not. Commodities tend to be more easily stored and resold, so that they take on the speculative quality of assets as well. An expectation of future inflation will raise demand for commodity prices, and thus drive up the price today” p.2.

inflation or both. The fall in commodity prices would be given until they generate an expectation of future appreciation that incentivizes the firms to hold inventories despite the high carrying cost. “In the long run, the general price level adjusts to the change in money supply. As a result, the real money supply, real interest rate, and real commodity prices eventually return to where they were” Frankel (2006) p. 5 (see **figure 2**). The reason for the overshooting in commodity prices is because they adjust rapidly, while most other prices adjust slowly, Bordo (1980) and Frankel (1984).

Figure 2: Overshooting of commodity prices
Shock of an increase in the real interest rate (monetary contraction)



Note: q : real price of commodity; p : economy-wide price index; r : real interest rate.

3.2 The model

The formal model is taken from Frankel (1986-2006): The economy is characterized by two types of goods: commodities and manufactured goods. The first are flexible and the second are fixed. Then the overall price level is an average of manufacture prices p_m , with weighting α , and commodity prices p_c , with weighting $(1 - \alpha)$:

$$p = \alpha p_m + (1 - \alpha) p_c \quad (1)$$

As showed by Frankel (2006, 1986) the equation that characterizes the dynamic of the commodity prices follows Dornbusch’s overshooting model (1976): (for more details of the model see **Appendix A**):

$$\dot{p}_c = -\theta(p_c - \bar{p}_c) + \dot{p}^e \quad (2)$$

Where \dot{p}_c is the change of the log of commodity prices, \dot{p}^e the expected change of the log of overall price level, and \bar{p}_c the long-run equilibrium commodity price.

We can re-express the equation (2) in real terms as:

$$\dot{p}_c^e = -\theta(q - \bar{q}) + \dot{p}^e \quad (3)$$

Where q is the real price of the commodity ($q = p_c - p$), and \bar{q} is the long run equilibrium real price of the commodity. This equation means that if today the real price of a commodity is lying above or below its long-run value, in the future it should regress back to its equilibrium over time, at an annual rate (θ).

The second equation is the arbitrage condition that represents the decision between holding the commodity for another period or selling it at today's price and receiving interest. In equilibrium the expected return of these two alternatives should be the same⁹:

$$\dot{p}_c^e + c = i \quad (4)$$

Where c is the net benefit of holding inventories and i the interest rate. According with Frankel (2006) the net benefit of holding inventories is compound by cy : the *convenience yield* from holding the stock, sc : the *storage costs* and rp : the *risk premium* of holding inventories.

$$c \equiv cy - sc - rp$$

Combining (3) and (4)

$$q = \bar{q} - \frac{1}{\theta}(i - \dot{p}^e - c) \quad (5)$$

Assuming the Fisher equation: $i - \dot{p}^e = r$, where (r) express the real interest rate we have:

$$q = \bar{q} - \frac{1}{\theta}(r - c) \quad (5')$$

Equation (5') illustrates that the commodity real price is inversely proportional to the real interest rate and positive to the net benefit of holding inventories. "When the real interest rate is high, as in the 1980, money flows out of commodities, just as it flows out of foreign currencies, emerging markets and other securities... Conversely, when the real interest rate is low, as in 2001-2005, money flows into commodities, just as it flows into foreign currencies, emerging markets and other securities" page 8, Frankel (2006). The reason why this happens is because agents want to protect their investment from inflation (Frankel and Hardouvelis, 1985).

Using the equation (5') in time t :

$$q_t = \bar{q} - \frac{1}{\theta}(r_t - c_t) \quad (5'')$$

⁹ This arbitrage condition has also been studied by Deaton and Laroque (1996)

Notice that in Frankel's model the change in money supply is exogenous. However assuming that there is a central bank that wants to control the money supply according to his target of inflation, we can include in the model a monetary policy rule. Following Taylor (1993) a simple monetary policy rule can be defined as:

$$i_t = \bar{p}_t + \bar{r} + \omega(\hat{p}_t - \bar{p}) + \psi(y_t - \bar{y}) \quad (6)$$

The equation (6) indicates that monetary policy increases the nominal interest rate if there is any observed deviation of the inflation rate and growth above his long-run tendency (target).

We can rewrite the Taylor rule in real terms as:

$$r_t = \bar{r} + \omega(\hat{p}_t - \bar{p}) + \psi(y_t - \bar{y}) \quad (7)$$

However we can assume that there is some smooth in the interest rate (see Batini and Haldane (1999)¹⁰:

$$r_t = \rho r_{t-1} + (1 - \rho)\bar{r} + \omega(\hat{p}_t - \bar{p}) + \psi(y_t - \bar{y}) + \zeta(y_{t-1} - \bar{y}) \quad (8)$$

Rewriting the equation (5') and (8) to one period in advance (time $t+1$), we have:

$$q_{t+1} = \bar{q} - \frac{1}{\theta}(r_{t+1} - c_{t+1}) \quad (9)$$

$$r_{t+1} = \rho r_t + (1 - \rho)\bar{r} + \omega(\hat{p}_{t+1} - \bar{p}) + \psi(y_{t+1} - \bar{y}) + \zeta(y_t - \bar{y}) \quad (10)$$

And calculating the difference in $t+1$ and t of these two equations we get:

$$q_{t+1} - q_t = -\frac{1}{\theta}[(r_{t+1} - r_t) - (c_{t+1} - c_t)] \quad (11)$$

$$r_{t+1} - r_t = \rho(r_t - r_{t-1}) + \omega(\hat{p}_{t+1} - \hat{p}_t) + \psi(y_{t+1} - y_t) + \zeta(y_t - y_{t-1}) \quad (12)$$

The terms: $(c_{t+1} - c_t)$ is the change of the net benefit to hold inventories. As suggested by Frankel (2006) we can use the growth of the real economy activity as a proxy of the *convenience yield term* (*cy*) $c_{t+1} - c_t = y_{t+1} - y_t$. However given that we do not have any variable to measure the *storage cost term* (*sc*) and the *risk premium* (*rp*), we use the structural error term u^q in the equation in an attempt to try to capture this effect. Substituting this assumption in equation (11) we have:

$$q_{t+1} - q_t = -\frac{1}{\theta}[(r_{t+1} - r_t) - (y_{t+1} - y_t)] + u^q \quad (12')$$

¹⁰This type of transformations in monetary policy rule have been used by a lot of different authors: Clarida, Galí and Gertler (2000) and (1998); Judd, and Rudebusch (1998); Rudebusch and Svensson (1999); Levin, Wieland, and Williams (1999), and others. Additionally, a lot of works propose a forward looking Taylor rule, but the estimation of VAR models make it more complicated. A proposal of this can be seen in Brissimis and Magginas (2004), who propose an augmented VAR model.

The structural error term u^q is assumed to be white noise. On the other hand, from equation (1) we have that the annual inflation is given by:

$$\dot{p}_t = \alpha p_{t,m} + (1-\alpha)p_{t,c} \text{ and } \dot{p}_t = p_t - p_{t-4}^{11}$$

Using these conditions and $q = p_c - p$ we find that

$$\dot{p}_{t+1} - \dot{p}_t = (\dot{p}_{t+1,m} - \dot{p}_{t,m}) + \frac{1-\alpha}{\alpha}(q_{t+1} - q_t) - \frac{1-\alpha}{\alpha}(q_{t-3} - q_{t-4}) \quad (13)$$

Substituting (13) in (12) and including the structural error term:

$$\begin{aligned} r_{t+1} - r_t = & \rho(r_t - r_{t-1}) + \omega(\dot{p}_{t+1,m} - \dot{p}_{t,m}) + \omega \frac{(1-\alpha)}{\alpha}(q_{t+1} - q_t) - \omega \frac{(1-\alpha)}{\alpha}(q_{t-3} - q_{t-4}) \\ & + \psi(y_{t+1} - y_t) + \zeta(y_t - y_{t-1}) + u^r \end{aligned} \quad (14)$$

Then the equations (12') and (14) are the basic equations of our model. In the former we can see how one change in the real interest rate has a negative effect in the real change of commodity prices, and in the latter equation we see how one change in real commodity prices has a positive effect on the change of the real interest rate. This positive relationship can be explained by the fact that if the increase in the real commodity prices is reflecting an increase in the future expectations of inflation, then the real interest should increase to control the expectations.

The last equation is a simple Phillips curve; this equation assumes that the inflation in t depends on the output gap and past inflation¹².

$$\dot{p}_t = \nu(y_t - \bar{y}) + \dot{p}_{t-1} \quad (15)$$

Rewriting the equation in difference ($t+1$ and t) and including the structural error term we have:

$$\dot{p}_{t+1} - \dot{p}_t = \nu(y_{t+1} - y_t) + (\dot{p}_t - \dot{p}_{t-1}) + u^p \quad (16)$$

4. Empirical Approach

4.1 Structural VAR (SVAR)

The structural VAR is a vector autoregressive model that permits contemporaneous relationships between the elements of vector x_t . In this way, we can model dynamic

¹¹ We use this definition given that we are working with quarterly data.

¹² The New Keynesian models propose a Phillips curve as a function of the future expectations of the inflation. This type of work should model the rational expectations as is done in Keating (1990), Hansen and Sargent (1979) and others. This works are very complex for that reason we are assuming a simple Phillips curve.

and contemporaneous endogeneity between variables. In matrix form we can write the SVAR (Hamilton, 1994, Section 11.6):

$$B_0 x_t = k + B_1 x_{t-1} + B_2 x_{t-2} + \dots + B_p x_{t-p} + u_t \quad (17)$$

where u_t is white noise. This means that the structural disturbances are serially uncorrelated, then $E[u_t u_t'] = D$, when D is a diagonal matrix. Pre-multiplying by

B_0^{-1} , we have the reduced form (VAR) of the dynamic structural model:

$$\begin{aligned} x_t &= B_0^{-1} (k + B_1 x_{t-1} + B_2 x_{t-2} + \dots + B_p x_{t-p} + u_t) \\ x_t &= c + \Phi_1 x_{t-1} + \Phi_2 x_{t-2} + \dots + \Phi_p x_{t-p} + \varepsilon_t \end{aligned} \quad (18)$$

Where: $\Phi_s = B_0^{-1} B_s$ ($s = 1, 2, \dots, p$), $c = B_0^{-1} k$ and $\varepsilon_t = B_0^{-1} u_t$

The variance-covariance matrix is given by:

$$E[\varepsilon_t \varepsilon_t'] = B_0^{-1} E[u_t u_t'] (B_0^{-1})' = B_0^{-1} D (B_0^{-1})' = \Omega$$

Then we use the equations (12'), (14), (16) to represent our structural VAR model in a matrix form: $B_0 x_t = k + B_1 x_{t-1} + B_2 x_{t-2} + \dots + B_p x_{t-p} + u_t$ ¹³.

$$\begin{aligned} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\beta_{21}^0 & 1 & 0 & 0 \\ -\beta_{31}^0 & 0 & 1 & \beta_{34}^0 \\ -\beta_{41}^0 & -\beta_{42}^0 & -\beta_{43}^0 & 1 \end{bmatrix} \begin{bmatrix} y_{t+1} - y_t \\ p_{t+1} - p_t \\ q_{t+1} - q_t \\ r_{t+1} - r_t \end{bmatrix} &= \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} + \begin{bmatrix} \beta_{11}^1 & \beta_{12}^1 & \beta_{13}^1 & \beta_{14}^1 \\ \beta_{21}^1 & \beta_{22}^1 & \beta_{23}^1 & \beta_{24}^1 \\ \beta_{31}^1 & \beta_{32}^1 & \beta_{33}^1 & \beta_{34}^1 \\ \beta_{41}^1 & \beta_{42}^1 & \beta_{43}^1 & \beta_{44}^1 \end{bmatrix} \begin{bmatrix} y_{t-1} - y_t \\ p_{t-1} - p_t \\ q_{t-1} - q_t \\ r_{t-1} - r_t \end{bmatrix} + \dots \\ &+ \begin{bmatrix} \beta_{11}^p & \beta_{12}^p & \beta_{13}^p & \beta_{14}^p \\ \beta_{21}^p & \beta_{22}^p & \beta_{23}^p & \beta_{24}^p \\ \beta_{31}^p & \beta_{32}^p & \beta_{33}^p & \beta_{34}^p \\ \beta_{41}^p & \beta_{42}^p & \beta_{43}^p & \beta_{44}^p \end{bmatrix} \begin{bmatrix} y_{t-p+1} - y_{t-p} \\ p_{t-p+1} - p_{t-p} \\ q_{t-p+1} - q_{t-p} \\ r_{t-p+1} - r_{t-p} \end{bmatrix} + \begin{bmatrix} u^y \\ u^p \\ u^q \\ u^r \end{bmatrix} \end{aligned}$$

As reported by some authors the non-recursive structure of the VAR, gives an advantage over the recursive method by permitting the modelling of a realistic economic structure. As in Kim (1999) we allow the feedback between commodity prices and monetary policy using the non-recursive structure. "...in this generalized method, it is possible to construct a structure allowing for the current mutual effects between monetary policy shocks and a variables such as commodity prices, which are expected not only to affect monetary policy contemporaneously but also to be affected by monetary policy contemporaneously" p. 392.

¹³ To simplify the empirical estimation we substitute $\hat{p}_{t,m}$ by \hat{p}_t in equation (14). Moreover we are not imposing any restriction in the lags.

Notice that we are assuming the equation for the GDP's growth just depends of the lags of variables in the economy. This mean that the shocks in growth are not related contemporaneously with any of the other shocks. This assumption is used in the majority of literature that analyzes the shock of monetary policy (Kim, 1999), Peersmand and Smets (2001) and Sims (1992), Sousa and Zaghini (2007), Brissimis and Magginas (2004), among others). They justify this assumption by the adjustment cost. As reported by Sousa and Zaghini (2007) "within a quarter, firms do not change their output and prices in response to unexpected changes in financial variables or monetary policy due to adjustment costs" p. 6. The nature of adjustment costs can be menu costs, adjustment costs in investment and employment, etc (Kim, 1999).

This system is stable if all the values of z that satisfy the following condition lie outside the unit circle (Hamilton, chapter 10, p. 259):

$|I_n - \Phi_1 z - \Phi_2 z^2 \dots - \Phi_p z^p| = 0$. This condition is guaranteed if our variables are stationary - $I(0)$.

4.2 Identification problem

Notice that the estimation of the structural VAR model has n^2 more parameters than the VAR, then, in order to find a unique solution we require two conditions to be satisfied: the order and the rank condition. The order condition requires that the number of free parameters in matrices B_o and D should be less than the number of free parameters in matrix Ω . Since Ω is a symmetric matrix then, the number of free parameter of matrix Ω is defined by $(n(n+1)/2)$.

Assuming that D is a diagonal matrix, then B_o can have no more free parameters than: $n(n-1)/2$. We can impose two different restrictions on matrix B_o . The first is the normalisation restriction that aims to assign the value of 1 to variables $x_{i,i}$ in each i equation. And the second is the exclusion restriction that aims to assign zero to some variables in the equation (especially contemporaneous relations). These restrictions are defined by the theoretical model.

The rank condition for identification of a structural VAR is more complex. This requires that the columns of the matrix J be linearly independent; which is defined as (see Hamilton, section 11, 1994):

$$J = \begin{bmatrix} \frac{\partial vech(\Omega)}{\partial \theta'_B} & \frac{\partial vech(\Omega)}{\partial \theta'_D} \end{bmatrix}$$

The operator $vech(\cdot)$ picks out the distinct elements of Ω . This condition is sufficient for local identification¹⁴.

Imposing the restrictions suggested by the theoretical model, we construct the matrix B_o and find the relationship between the error terms of the reduced form and the structural disturbances: $\varepsilon_t = B_o^{-1}u_t$

$$\begin{bmatrix} \varepsilon^y \\ \varepsilon^p \\ \varepsilon^q \\ \varepsilon^r \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\beta_{21}^0 & 1 & 0 & 0 \\ -\beta_{31}^0 & 0 & 1 & \beta_{34}^0 \\ -\beta_{41}^0 & -\beta_{42}^0 & -\beta_{43}^0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} u^y \\ u^p \\ u^q \\ u^r \end{bmatrix}$$

Then, according to our theoretical model, the matrix B_o has 6 free parameters to estimate, which are exactly the same parameters we require for the order condition to be satisfied.

In general the majority of models studying monetary policy shocks use the specification proposed by Kim (1999). He estimates a VAR model with 5 variables (nominal interest rate, monetary aggregate, CPI, industrial production and nominal commodity prices). In contrast to Kim's specification, we do not include the variable of monetary aggregate, because we are not interested in modelling the money demand. The money supply in our model is given by the monetary policy reaction function. In Kim's model, the interest rate responds to the commodity prices and the money aggregate. A difference with our model the monetary policy function does not react to the current value of output and price level. According to the author, in the moment that the monetary authority takes the decision about the interest rate, they do not have all the relevant information available (information delay). However, we are not making this assumption. In the case of the commodity price's equation, Kim's specification is arbitrary. Given that the commodity describes an asset, he assumes that all variables have a contemporaneous effect on the world export commodity prices. Nevertheless, in our model the commodity price's equation is clearly identified.

5. Results

5.1 Data

To estimate our model we will use quarterly data from 1962-Q1 to 2009-Q1. Given that we do not have a global variable of interest rate as for commodity prices, we will

¹⁴ The Eviews 6 program evaluates numerically this condition at the starting values, checking the invertibility of the "augmented" information matrix suggested by Amisano and Giannini, (1997).

use data from the United States as a substitute for world data: Then we will use the US Gross Domestic Product- GDP, US Inflation- INF and different nominal interest rates. The Federal Funds Rate -FF, and the nominal interest rate from the Treasury Bonds at 1, 5, and 10 years, we will call 1Y, 5Y and 10Y respectively. For the world commodity index prices we use different indexes: the first is the Commodity Research Bureau Index-CRB index, aggregated and by subgroups: Metals-CRBM, Oil-CRBO and Raw materials-CRBR. The last two indexes used are Moody's (MOO) and S&P index (S&P). The information available for these two indexes is from 1976:Q1 and 1970:Q1 respectively¹⁵. To transform the interest rates and the commodity prices in real terms we use the CPI and the prefix R to indicate *real terms*.

To check if our variables are stationary- $I(0)$ we use the Augmented Dickey-Fuller Test (ADF) and Phillips-Perron Test . In general the variables in levels have a unit root, however we are interested in the difference at quarterly frequency ($t - (t-1)$) and the difference at annual frequency ($t - (t-4)$) which are in fact stationary (see table B1 in the **Appendix B**). All the variables are in natural logarithms except the interest rates and inflation. We use different criteria to select the number of lags (LR, FPE, AIC, SC, HQ).

5.2 Estimating the contemporaneous coefficients.

Table 1 presents the estimation of contemporaneous relationships of our model in difference at quarter and annual frequency. The first column indicates the estimation using the real Fed Funds Rate- RFF, the second, third and fourth column indicate the estimation using the real interest rate of Treasury Bonds at 1, 5 and 10 years, respectively.

For the model in difference at quarter frequency we find that the contemporaneous reaction of inflation to economic activity is positive. However this coefficient is not significant. The reaction of real commodity prices to economic activity is positive, and to the real interest rate is negative, as our model predicts and both coefficients are significant. Finally, the contemporaneous reaction of real interest rate is positive with the economic activity and commodity prices, but negative with the inflation. Nevertheless our model predicts a positive reaction with inflation. But as we will see later, this reaction turns out to be positive after 4 or 5 quarters and is still significant (**Table 1**)

¹⁵ See the source of the data in Appendix B

For the model in difference at annual frequency we get better results, since in this case the majority of coefficients are significant (except for the model using the Fed Fund rate). In general we get the same dynamic as the model at quarterly frequency, including the negative reaction of the real interest rate to the inflation, but as in the first model the reaction returns positive and still significant after some quarters.

Table 1 Model with different real interest rates: estimation of the contemporaneous relationships (RCRB)

Difference at quarterly frequency $(t - (t - 1))^*$				
Parameters	RFF	R1Y	R5Y	R10Y
$-\beta_{21}^0$	-0.0637 (0.0477) ¹	-0.0571 (0.0476) ¹	-0.0533 (0.0469) ¹	-0.0520 (0.0464) ¹
$-\beta_{31}^0$	-2.4947 (0.7826)	-2.0940 (0.6380)	-1.6910 (0.5007)	-1.4861 (0.4642)
β_{34}^0	5.9727 (2.0615)	4.9446 (1.4889)	4.1230 (1.0638)	3.8764 (0.9652)
$-\beta_{41}^0$	-0.0052 (0.1049) ¹	-0.0632 (0.0829) ¹	-0.0835 (0.0594) ¹	-0.0745 (0.0498) ¹
$-\beta_{42}^0$	1.1283 (0.1983)	1.0915 (0.1531)	1.0063 (0.1058)	0.9715 (0.0878)
$-\beta_{43}^0$	-0.2098 (0.0459)	-0.1656 (0.0322)	-0.0995 (0.0192)	-0.0753 (0.0147)
Difference at annual frequency $(t - (t - 4))^{**}$				
$-\beta_{21}^0$	-0.1043 (0.0518)	-0.0948 (0.0517)	-0.1063 (0.0493)	-0.1082 (0.0481)
$-\beta_{31}^0$	-2.1063 (0.6670)	-1.5623 (0.5587)	-1.2955 (0.4769)	-1.1399 (0.4502)
β_{34}^0	4.0980 (1.5133)	3.2854 (1.1098)	2.8391 (0.9129)	2.8663 (0.8988)
$-\beta_{41}^0$	-0.1140 (0.0994) ¹	-0.1953 (0.0752)	-0.1815 (0.0550)	-0.1437 (0.0461)
$-\beta_{42}^0$	0.9577 (0.1634)	0.9449 (0.1221)	0.9306 (0.0896)	0.8991 (0.0762)
$-\beta_{43}^0$	-0.1750 (0.0422)	-0.1253 (0.0274)	-0.0722 (0.0163)	-0.0546 (0.0128)
^{1/} Level of significance below five percent.				
* Number of lags 8. The value in parenthesis is the standard error				
**Number of lags 12. The value in parenthesis is the standard error				

Moreover, the overshooting of commodity prices to real interest rate is still negative and significant. Comparing the magnitude of the overshooting in both models we can see that the reaction in the difference quarterly frequency model is bigger than in the annual frequency model, especially in the case with the real Fed Fund rate. For example, the overshooting of commodity price to a 1% increase in the real Fed Fund rate is -5.97% in the first, and -4.09% in the second model. Nevertheless, in both models, the overshooting of commodity prices begins to decline with long-term interest rates. The reaction of monetary policy rule to a 1% increase of commodity prices is just 0.20% and 0.17% respectively. Even though this reaction is

significant, the magnitude is reduced (the same results are achieved by Cody and Mills (1991)), and in comparing the models with different interest rates the results are found to be similar. To understand better the dynamic of our model we should analyze the impulse response functions.

5.3 Impulse response function

Notice that from the equation: $\varepsilon_t = B_0^{-1}u_t$, the VAR innovations ε_{jt} is a linear combination of the structural disturbances u_t , then:

$$\frac{\partial \varepsilon_t}{\partial u_t} = B_0^{-1}$$

And, the impulse response function for the SVAR is given by:

$$\frac{\partial x_{t+s}}{\partial u_{jt}} = \frac{\partial x_{t+s}}{\partial \varepsilon_t} \frac{\partial \varepsilon_t}{\partial u_{jt}} = \Psi_s b^j \quad (21)$$

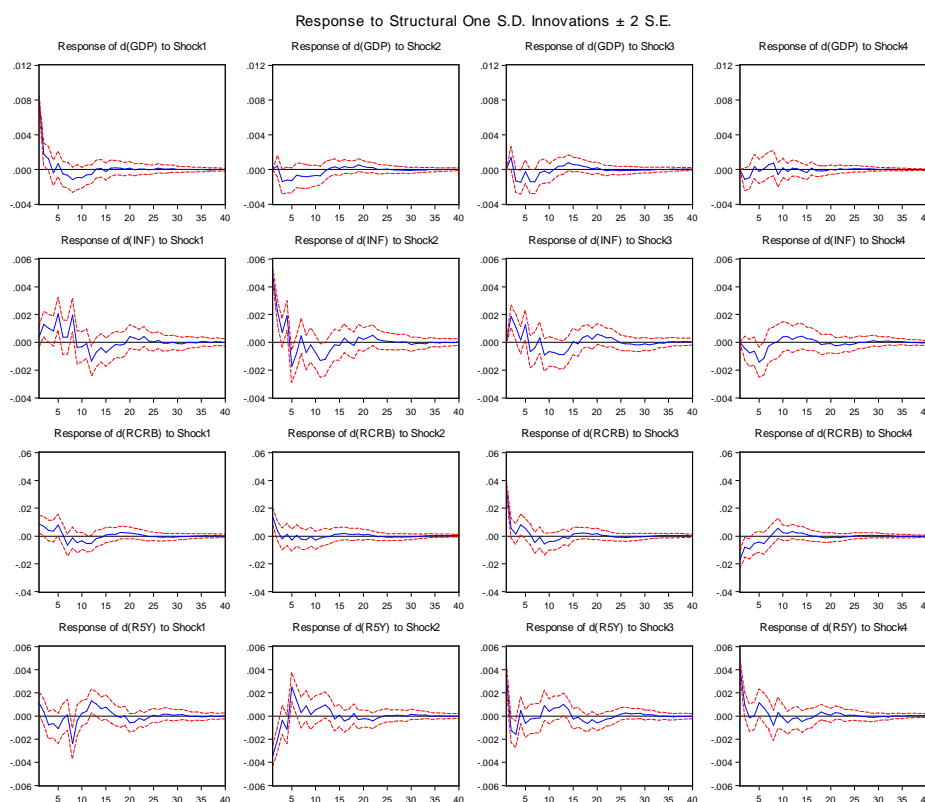
Where b^j is the j th column of B_0^{-1} . The impulse response function describes the response of x_{t+s} to one-time unit change in the structural error u_{jt} . As we can see, we have four structural shocks in our model: u^y : shock in GDP, u^p : shock in inflation, u^q : shock in real commodity prices, and finally u^r : shock in monetary policy or real interest rate.

In **Figure 3** we have the impulse response functions given by the model in difference at quarterly frequency using the 5 year real interest rate. Remember that since our variables are in logs, this difference can be seen as a quarterly growth. The first column shows us the reaction function of all the variables given a shock in GDP. As we can see, a shock in economic activity has a positive effect on the change in inflation (we can see this as an acceleration in inflation) and is significant until the 6th or 7th quarter. Moreover, the response function of real commodity prices and real interest rate to GDP shock, is positive, but disappears rapidly.

The second column shows us the dynamic given a shock in inflation. The impulse response of economic activity is negative and significant between the third and fourth quarter but of small magnitude. The reaction function of real interest rate is negative and significant during the first 5 quarters. However, the response of commodity prices is positive in the first period but disappears rapidly. The third column shows us the dynamic given a shock in commodity prices. The response of economic activity is negative around the fifth quarter but still not significant; Moreover, the response on

inflation is positive and significant until the 5th quarter and the response of real interest rate is positive but again this effect disappears earlier.

Figure 3: Model in difference at quarterly frequency (R5Y)

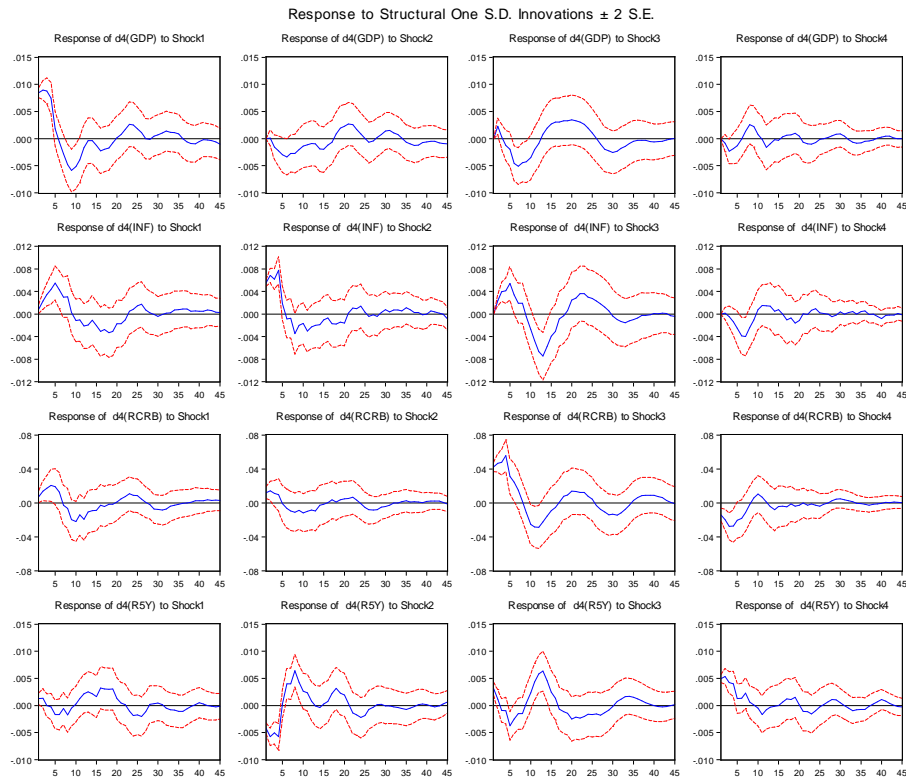


Note: shock 1: represent a shock in GDP, shock 2: represent a shock in inflation, shock 3 represent a shock in real commodity prices, and shock 4: represent a shock in monetary policy or real interest rate.

Finally the fourth column shows us the dynamic given a shock in monetary policy or real interest rate. The effect in economic activity again is not significant; however, the impulse response of inflation is negative and significant around the fifth quarter. The response in real commodity prices is negative, of a significant magnitude, and remains significant after the 7th quarter. In summary, the dynamic for the model in difference at quarterly frequency is as we expected, however the effects disappear earlier and are not always significant.

In the **Figure 4** we have the dynamic for the model in difference at annual frequency and using the real interest rate at 5 years. We can understand this difference as an annual growth in variables. The dynamic of all the variables, given a shock in GDP and a shock in inflation, is very similar to the model in difference at quarterly frequency. However, the dynamic given a shock in commodity prices becomes more significant.

Figure 4: Model in difference at annual frequency (R5Y)



Note: shock 1: represents a shock in GDP, shock 2: represents a shock in inflation, shock 3 represents a shock in real commodity prices, and shock 4: represents a shock in monetary policy or real interest rate.

For example, the impulse response of economic activity is now negative and significant between the 5th and 10th quarter. In the literature there are some authors that have reported this negative effect. Hamilton (1983) argues that the evidence presented from the period 1948-72 supported the argument that oil shocks were a contributing factor in at least some of the U.S. recessions prior to 1972. Moreover Herrera and Pesavento (2009) report evidence in this way as well:

“We find that a one-time 10% increase in the real oil price had a larger and longer-lived effect on output growth... In addition, the historical decomposition suggests an important contribution of oil prices to economic fluctuations, particularly during the years following the Arab-Israeli War and the Persian Gulf War. The contribution declined in the late 1990s, but appears to have increased somewhat during 2006” p.131.

Even though in our case we are analyzing the aggregate commodity prices, the negative effect in GDP’s growth is significant. On the other hand, the impulse response of inflation to a shock in commodity prices is positive and significant during

the first 10 quarters, however before disappearing, its effect returns to negative.¹⁶ The impulse response of real interest rate is positive and significant during the first quarter and between the 8th -17th quarters approximately.

The dynamic of the difference at annual frequency, given an increase in the real interest rate, is much better. The impulse response of the economic activity is now negative and significant during the first five quarters. As before, the reaction of inflation is negative and significant around the seventh quarter and finally the negative overshooting of commodity prices is again significant roughly until the sixth quarter. Notice that as predicted by our model, given an increase in the real interest rate, the commodity prices immediately show a negative overshooting from the equilibrium value and then some quarters later, this negative effect is reflected in the inflation.

Our results are not different from other authors, even if our model is more reduced and we use variables in real terms. For example, Christiano, Eichenbaum and Evans (1998) found that: “ in response to a contractionary policy shock, the federal funds rate rises, monetary aggregates decline (although some with a delay), the aggregate price level initially responds very little, aggregate output falls, displaying a hump shaped pattern, and commodity prices fall” p.24. Then as Kim (1999) argues these results supports the validity of our identifying assumption.

Furthermore, as has been done by other authors, we compare the dynamic of our model with the impulse response function using a Cholesky decomposition¹⁷ (the results are presented in the **Appendix C**). As we can see the results do not change significantly, supporting again, the validity of our model.

In terms of variance decomposition the results are similar to those found in the literature (**Table 2**). The monetary policy shocks do not have an important role in explaining the variability of the GDP. The same is reported by Sousa and Zaghini (2007), Kim (1999) and Peersman and Smets (2003). Moreover, even though the variance in real commodity prices is in general explained by the innovation of itself, at 20 quarters the shock in real interest rate and economic activity can explain a significant magnitude of increase (15% in each).

¹⁶ This result can be explained by the seasonality of the series that we are modelling. To remove some noise as this, some works report the average of impulse response functions.

¹⁷ The Cholesky decomposition is used when we are estimating VAR models , the comparison permit us to see how much our dynamic change when we are imposing the contemporaneous restrictions (the order of the variables still the same: GDP, INF, RCRB, R5Y).

Table 2 Variance decomposition: model in difference at annual frequency

Steps	Variance decomposition of GDP				Variance decomposition of inflation				Variance decomposition of real commodity prices				Variance decomposition of real interest rate			
	<i>S1</i>	<i>S2</i>	<i>S3</i>	<i>S4</i>	<i>S1</i>	<i>S2</i>	<i>S3</i>	<i>S4</i>	<i>S1</i>	<i>S2</i>	<i>S3</i>	<i>S4</i>	<i>S1</i>	<i>S2</i>	<i>S3</i>	<i>S4</i>
5	87.4	8.3	2.4	1.7	15.4	59.8	22.8	1.8	9.3	3.5	72.4	14.7	2.8	50.4	9.1	37.5
10	66.7	16.1	15.7	1.4	22.5	46.9	21.2	9.2	11.8	3.8	68.7	15.6	10.0	52.2	9.8	27.8
15	65.3	16.9	16.0	1.7	20.8	44.0	27.0	8.1	14.9	3.8	66.4	14.7	14.7	45.8	15.3	24.0
20	63.3	17.1	17.6	1.8	21.5	42.9	27.5	7.9	14.7	4.3	66.0	14.7	16.6	44.2	16.0	23.0

Note: the variance decomposition shows the percentage of *k*-step-ahead forecast error variance
S1: shock in GDP, *S2*: shock in inflation, *S3*: shock in commodity prices, *S4*: shock in real interest rate.

The variance in real interest rate is generally explained by innovations in the inflation (44% at 20 quarters). However, the innovation in real commodity prices explains 16% of the variance, similar to the magnitude explained by the GDP.

Finally the variance in inflation is explained by its own innovation. However, the second most important innovation which explains the variance in inflation is given by the real commodity prices, which explains the 27.5% at 20 quarters. Garner (1989) reported that “innovations in the CRB index explain about 25 percent of the prediction error variance for the CPI after forty eight months” p. 513.

6 Robustness

In the next section we present the results of our model using the disaggregated CRB index of commodity prices: Oil prices, Metal prices and Raw material prices; and other indexes as in Moody’s and Standard & Poor’s index. In tables 3 and 4 we present the result of the model in difference at annual frequency¹⁸.

In general we can see that the signs of all the coefficients are still the same, however the contemporaneous reaction of real commodity prices given an increase in the economic activity is still positive but not significant, except when we are using the FED real interest rate (Table 3).

Table 3 Model with desegregated commodity price indexes: estimation of the contemporaneous relationships: difference at annual frequency ($t - (t - 4)$)

(OIL- RCRBO) *				
Parameters	RFF	R1Y	R5Y	R10Y
$-\beta_{21}^0$	-0.1319 (0.0529)	-0.1293 (0.0537)	-0.1396 (0.0503)	-0.1463 (0.0492)
$-\beta_{31}^0$	-2.9247 (1.4280)	-1.7471 (1.1422) ¹	-1.4831 (0.9249) ¹	-1.1773 (0.8869) ¹
β_{34}^0	10.5029 (3.6529)	8.1970 (2.4518)	6.1186 (1.7990)	5.7422 (1.7652)
$-\beta_{41}^0$	-0.2425 (0.1084)	-0.2316 (0.0802)	-0.1960 (0.0558)	-0.1638 (0.0460)

¹⁸ The result for the model in difference at quarterly frequency are in Appendix D

$-\beta_{42}^0$	0.9194 (0.2153)	0.9088 (0.1363)	0.8820 (0.0912)	0.8483 (0.0746)
$-\beta_{43}^0$	-0.0847 (0.0297)	-0.0550 (0.0152)	-0.0255 (0.0083)	-0.0174 (0.0062)
(Metals- RCRBM)*				
$-\beta_{21}^0$	-0.0414 (0.0546) ¹	-0.0366 (0.0536) ¹	-0.0568 (0.0508) ¹	-0.0635 (0.0496) ¹
$-\beta_{31}^0$	-2.3078 (0.9956)	-1.5994 (0.8899) ¹	-0.7290 (0.7637) ¹	-0.4450 (0.7301) ¹
β_{34}^0	5.1237 (1.9539)	4.8922 (1.6572)	4.1568 (1.4049)	4.3423 (1.4141)
$-\beta_{41}^0$	-0.1815 (0.0895)	-0.2296 (0.0721)	-0.1876 (0.0530)	-0.1456 (0.0454)
$-\beta_{42}^0$	0.9345 (0.1407)	0.9556 (0.1155)	0.9451 (0.0852)	0.9061 (0.0741)
$-\beta_{43}^0$	-0.0984 (0.0225)	-0.0799 (0.0162)	-0.0492 (0.0097)	-0.0377 (0.0078)
(Raw Materials- RCRBR)*				
$-\beta_{21}^0$	-0.0646 (0.0521) ¹	-0.0455 (0.0518) ¹	-0.0625 (0.0496) ¹	-0.0692 (0.0483) ¹
$-\beta_{31}^0$	-1.4274 (0.5902)	-1.0202 (0.5774) ¹	-0.5845 (0.5112) ¹	-0.4774 (0.4821) ¹
β_{34}^0	2.6179 (1.1653)	2.6192 (1.0772)	2.2945 (0.9380)	2.3948 (0.9480)
$-\beta_{41}^0$	-0.1571 (0.0841) ¹	-0.2145 (0.0699)	-0.1965 (0.0527)	-0.1536 (0.0452)
$-\beta_{42}^0$	0.9043 (0.1290)	0.9333 (0.1103)	0.9352 (0.0844)	0.8935 (0.0741)
$-\beta_{43}^0$	-0.1349 (0.0306)	-0.1172 (0.0229)	-0.0740 (0.0143)	-0.0591 (0.0119)
^{1/} Level of significance below five percent.				
*Number of lags 12. The value in parenthesis is the standard error				

Additionally, the reaction of inflation to an increase in the GDP is just significant in the case when we use the oil prices (this results are the same in the difference at quarterly frequency model, see **Appendix D**).

Comparing the reaction of the different commodity prices given an increase in the real interest rate, we can see that the overshooting is high in oil prices. Moreover, the reaction of oil, metal and raw material is higher using the FED real interest rate. However, for the case of metal and raw material prices, the magnitude of the reaction is very similar using the interest rates at 5 and 10 years. On the other hand, as we have seen previously, the reaction of real interest rate to commodity prices is significant but small. Nevertheless using raw material prices this magnitude increases, especially in the case of FED interest rate and 1 year interest rate.

The estimations with the Moody's and Standard & Poor's index are presented in **Table 4**. Using the S&P index, we find that the reaction of commodity prices to real interest rate is still negative and significant. Additionally, the magnitude of the

overshooting is high with all the different interest rates¹⁹. The reaction of real interest rate to commodity prices is positive, significant and small.

**Table 4 Estimation of the contemporaneous relationships:
Difference at annual frequency ($t - (t - 4)$)**

Model with Moody's Index: (RMOO) *				
Parameters	RFF	R1Y	R5Y	R10Y
$-\beta_{21}^0$	-0.0350 (0.0821) ¹	-0.0402 (0.0806) ¹	-0.0898 (0.0734) ¹	-0.1119 (0.0709) ¹
$-\beta_{31}^0$	-3.6118 (0.9156)	-2.9056 (0.8074)	-3.0195 (0.7262)	-3.0365 (0.7055)
β_{34}^0	2.1004 (1.1262) ¹	1.5136 (0.9099) ¹	1.5812 (0.9037) ¹	1.7400 (0.9380) ¹
$-\beta_{41}^0$	-0.2832 (0.1239)	-0.2584 (0.1046)	-0.2386 (0.0761)	-0.1711 (0.0648)
$-\beta_{42}^0$	0.8812 (0.1243)	1.0162 (0.1113)	1.0257 (0.0880)	1.0292 (0.0778)
$-\beta_{43}^0$	-0.0710 (0.0253)	-0.0716 (0.0201)	-0.0532 (0.0135)	-0.0480 (0.0108)
Model with S&P index: (RS&P) *				
$-\beta_{21}^0$	-0.1848 (0.0677)	-0.1957 (0.0642)	-0.2038 (0.0603)	-0.2115 (0.0583)
$-\beta_{31}^0$	-5.2142 (1.2866)	-4.0594 (1.0631)	-3.4002 (0.9795)	-2.9487 (0.9543)
β_{34}^0	9.7250 (1.8548)	8.8614 (1.5500)	9.0491 (1.5382)	9.2720 (1.5723)
$-\beta_{41}^0$	-0.1520 (0.1573) ¹	-0.1939 (0.1013) ¹	-0.2186 (0.0698)	-0.1674 (0.0587)
$-\beta_{42}^0$	1.8474 (0.3229)	1.5423 (0.1918)	1.3150 (0.1288)	1.2343 (0.1070)
$-\beta_{43}^0$	-0.1287 (0.0337)	-0.0846 (0.0186)	-0.0547 (0.0112)	-0.0441 (0.0087)
^{1/} Level of significance below five percent.				
*Number of lags 12. The value in parenthesis is the standard error				

The results using the Moody's index show that the contemporaneous reaction of commodity prices given an increase in the real interest rate is small and not significant. This result is explained by the shortage of data in 1980:Q1, and are similar to Arango *et al.*(2008). To check the robustness of this result we estimate the model with the CRB index and the S&P index for the same period. The results are reported in **Appendix E**. As we can see for CRB and S&P index, the contemporaneous reaction of commodity prices given an increase of real interest rate is still significant.

7. Conclusions

The most important results have been reported in this section. In general we find the empirical evidence to support our model. We estimate two models: one in difference

¹⁹ In Appendix D we can see the impulse response function of each model - the dynamic of both is still the same.

at quarterly frequency ($t-(t-1)$) and the other in difference at annual frequency ($t-(t-4)$). For both models we find that the reaction of real commodity prices to economic activity is positive, and to the real interest rate is negative, as our model predicts. Both coefficients are significant. We find that the overshooting of commodity prices to 1% increase of real interest rate can be between 3.87% and 5.97% in the first model and between 2.86% and 4.09% in the second model, depending on the real interest rate used. In general, the overshooting in the difference at quarterly frequency model is bigger, especially in the case which uses the real Fed Funds rate.

The second important result is that as our model predicts, we find a positive contemporaneous reaction of real interest rate with the economic activity and commodity prices. We find that the increase of the real interest rate given a 1% increase in commodity prices can be between 0.20% and 0.07% in the difference at quarter frequency model and between 0.17% and 0.05% in the difference at annual frequency model. This result is significant even though the magnitude is reduced.

On the other hand, the dynamic of our model is better in the difference at annual frequency model, especially because we find a negative and significant impulse response of economic activity to a shock in commodity prices between the 5th and 10th quarters. This result is widely reported in the literature. Additionally, the dynamic of our model given an increase in the real interest rate or monetary policy shock is as we expected, particularly the immediately negative overshooting of commodity prices from the position of equilibrium, which some quarters later is reflected in a low inflation.

The third interesting result is that using a disaggregate index of commodity prices: oil, metals and raw materials, we find that the overshooting of oil prices is the highest. Additionally, the overshooting for all the commodity prices is especially high when we use the real Fed Funds rate.

And finally we found that using the aggregate index of Standard and Poor's index S&P the reaction of commodity prices to real interest rate is still negative and significant. However these results are still negative but not significant using Moody's index. One of the reasons for this is the shortage of data. However, using the CRB and S&P index for this shorter period (1980-2009) we find that our result is still significant.

In summary we can see that as has been reported by Frankel, commodity prices are flexible and react immediately to monetary policy actions. One of the recommendations of monetary policy is that according with this evidence, the commodity prices can be seen as an indicator of the current monetary policy position. Then high real commodity prices can be seen as an expansionary monetary policy and low real commodity prices can be seen as contractionary monetary policy. Additionally, they help to predict inflation, and even monetary policy rules react to an increase in commodity prices, this reaction is relatively small. In general, and as suggested by Cody and Mills (1991) this reaction should be higher if the monetary policy are more compromised with his target.

Some future research can be carried out by replacing data from the United States with global data such as: world inflation, world GDP and world interest rates. One possible way to do that is making a weight average of different countries. Furthermore, other ways to check the robustness of our model is by using the GDP deflator as an alternative to CPI in order to convert the interest rate and commodity prices in real terms. Moreover, the development of more theoretical models can be very useful in understanding the dynamics of commodity prices and monetary policy. Finally, the literature on VAR models offers different ways to overcoming this problem. One of these is by using, for example, long-run identification restrictions instead of contemporaneous restrictions. This model was proposed by Blanchard and Quah (1989) and has been widely used in the literature. The use of FAVAR or Factor Augmented VAR models has appeared recently. This VAR models allows us to increase the number of variables without losing grades of freedom. The idea is to take the principal component of a large number of variables and use it as one variable in the VAR model. In this way, we are taking into account the majority of information available, and modelling the economy more accurately.

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Appendixes

Appendix A: Frankel model (1986) and (2006)

This model is taken from Frankel (1986) and (2006). Frankel follow the idea of Okun (1975) that presumed that in the economy there are two different prices: the prices of manufactured goods (p_m in log form) and the prices of basic commodities (p_c in log form). The former are sticky and the latter are flexible. He assumes that if commodities are homogeneous and storable then, they are subject to the arbitrage condition:

$$\dot{p}_c^e + c = i \quad (1)$$

where $c \equiv cy - sc - rp$, cy is the convenience yield from holding the stock, sc is the storage costs, rp is the risk premium and i is the interest rate. (Frankel, 2006)

Equation (1) represents the expected return from holding the commodity for another period as inventories should have the same value as if the commodity were sold at today's price and the proceeds deposited in the bank to earn interest.

The level of manufacture price is fixed by each commodity's past history. It can adjust itself in response to excess demand only gradually over time, in accordance with an expectations-augmented Phillips curve:

$$\dot{p}_m = \nu(y_m - \bar{y}_m) + \dot{p}^e \quad (2)$$

where y_m is the log of demand for manufactures, \bar{y}_m is the log of potential output in manufactures and \dot{p}^e is the expected rate of inflation.

Excess demand is defined as an increasing function of the price of commodities relative to manufacture and a decreasing function of the real interest rate:

$$y_m - \bar{y}_m = \delta(p_c - p_m) - \sigma(i - \dot{p}^e - \bar{r}) \quad (3)$$

where \bar{r} is constant term. The long-run equilibrium is defined when there is zero excess demand ($y_m = \bar{y}_m$) and the relative price of the two commodities ($p_c - p_m$) settles down to a given value ($\bar{p}_c - \bar{p}_m$) and the real interest rate ($i - \dot{p}^e$) becomes \bar{r} .

Substituting (3) in (2)

$$\dot{p}_m = \nu[\delta(p_c - p_m) - \sigma(i - \dot{p}^e - \bar{r})] + \dot{p}^e \quad (4)$$

Assuming that the money demand equation is given by:

$$m - p = \Phi y - \lambda i \quad (5)$$

where m is the log of the nominal money supply, p is the log of the overall price level, y is the log of total output, Φ is the elasticity of money demand with respect to output and λ is the semi-elasticity of money demand with respect to the interest rate. The overall price level is an average of manufacture prices, with weight α , and commodity price $(1 - \alpha)$:

$$p = \alpha p_m + (1 - \alpha) p_c \quad (6)$$

Substituting (6) in (5)

$$m - \alpha p_m - (1 - \alpha) p_c = \Phi y - \lambda i \quad (7)$$

The long-run equilibrium version of the money demand equation is as follows:

$$\bar{m} - \alpha \bar{p}_m - (1 - \alpha) \bar{p}_c = \Phi \bar{y} - \lambda \bar{i}$$

Using the result $i - \dot{p}^e = \bar{r}$, we have:

$$\bar{m} - \alpha \bar{p}_m - (1 - \alpha) \bar{p}_c = \Phi \bar{y} - \lambda (\bar{r} + \dot{p}^e) \quad (8)$$

Taking the difference of the two equations (7) and (8)

$$\alpha (p_m - \bar{p}_m) + (1 - \alpha) (p_c - \bar{p}_c) = \lambda (i - \dot{p}^e - \bar{r}) \quad (9)$$

Where it is assumed that no changes in the money supply ($m = \bar{m}$) and ($y = \bar{y}$) are expected.

Now combining equation (1) and (9)

$$\dot{p}_c^e = \frac{\alpha}{\lambda} (p_m - \bar{p}_m) + \frac{1 - \alpha}{\lambda} (p_c - \bar{p}_c) + \dot{p}^e + \bar{r} - c \quad (10)$$

Combining equation (4) and (9) and using the normalization ($\bar{p}_c - \bar{p}_m = 0$)

$$\begin{aligned} \dot{p}_m &= \nu \left[\delta [(p_c - \bar{p}_c) - (p_m - \bar{p}_m)] - \frac{\sigma}{\lambda} [\alpha (p_m - \bar{p}_m) + (1 - \alpha) (p_c - \bar{p}_c)] \right] + \dot{p}^e \\ \dot{p}_m &= \nu (p_c - \bar{p}_c) \left[\delta - \frac{\sigma(1 - \alpha)}{\lambda} \right] - \nu (p_m - \bar{p}_m) \left[\delta + \frac{\sigma\alpha}{\lambda} \right] + \dot{p}^e \end{aligned} \quad (11)$$

The model is closed assuming that expectations are formed rationally:

$$\dot{p}^e = \dot{p} \quad \text{and} \quad \dot{p}_c^e = \dot{p}_c$$

The differential equations (10) and (11) can be written in a matrix form, and solving the system $|A - \theta I| = 0$, Frankel (1986) found the characteristic roots. Using just the negative characteristic roots ($-\theta$) that guarantee that the system is stable, the solution can be written as:

$$\begin{aligned} \dot{p}_m &= -\theta (p_m - \bar{p}_m) + \dot{p}^e \\ \dot{p}_c &= -\theta (p_c - \bar{p}_c) + \dot{p}^e + \bar{r} - c \end{aligned} \quad (12)$$

Notice that using the arbitrage condition and the equilibrium equation $i - \dot{p}^e = \bar{r}$ the equation (15) can be written as:

$$\dot{p}_c = -\theta(p_c - \bar{p}_c) + \dot{p}_c^e$$

And assuming that in equilibrium $\dot{p}_c = p_m = \dot{p}$ we have

$$\dot{p}_c = -\theta(p_c - \bar{p}_c) + \dot{p}^e \quad (13)$$

This equation is the same as the classic Dornbusch overshooting model, but with the price of commodities substituted for the price of foreign exchange and with the convenience yield substituted for the foreign interest rate (Frankel, 2006).

Appendix B: Source of data and Unit Root Test Results

GDP: Gross Domestic Product of US: Bureau of Economic Analysis:

<http://www.bea.gov/>

INF: Annual inflation of US: Bureau of Labour Statistics, Datastream.

CRB: Commodity Research Bureau: Datastream.

FF: Federal Funds rate: Datastream

1Y, 5Y, 10Y: Treasury yields rates: Department of Treasury, Datastream.

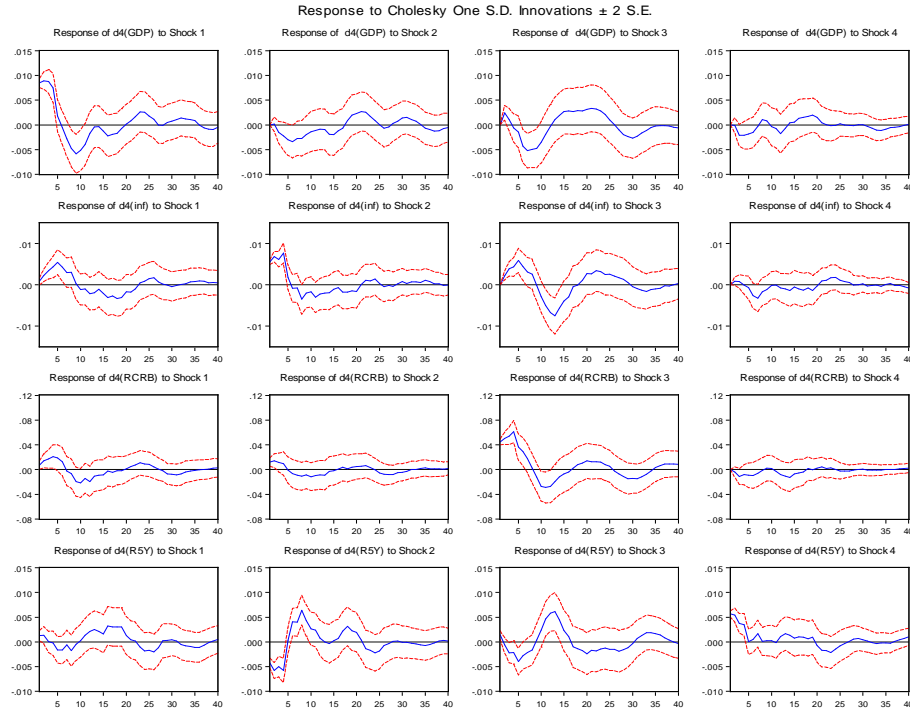
MOO: Moody's Index: Datastream

S&P: Standard and Poor's index, Datastream

Table B1: Unit Root Test Results

	GDP	INF	RCRB	RFF	R1Y	R5Y	R10Y
Levels							
ADF (AIC)*	0.3215	0.4999	0.8061	0.2333	0.0379	0.0380	0.0624
ADF (SIC)	0.2452	0.4319	0.7445	0.0679	0.0737	0.0380	0.0546
Phillips-Perron	0.2633	0.1540	0.7000	0.0219	0.0308	0.6550	0.0716
Levels							
	RCRBM	RCRBO	RCRBR	RMOO	RS&P		
ADF (AIC)	0.1165	0.8243	0.7334	0.1373	0.5715		
ADF (SIC)	0.1466	0.6026	0.5411	0.6572	0.5636		
Phillips-Perron	0.2602	0.5749	0.5671	0.5087	0.4734		
*All the tests give us the <i>p-value</i> to accept the null hypothesis (H_0 : there is a unit root)							
Note: All the variables, in quarterly and annual difference, are $I(0)$.							

Appendix C: Impulse response function: Cholesky decomposition: model in difference at annual frequency with R5Y



Note: shock 1: represents a shock in GDP, shock 2: represents a shock in inflation, shock 3 represents a shock in real commodity prices, and shock 4: represents a shock in monetary policy or real interest rate.

Appendix D: Estimation with disaggregated indexes: Oil, Metal and Raw material and other indexes (Moody's and S&P index)

Table D1: Model with disaggregated commodity price indexes: difference at quarterly frequency $(t - (t - 1))$

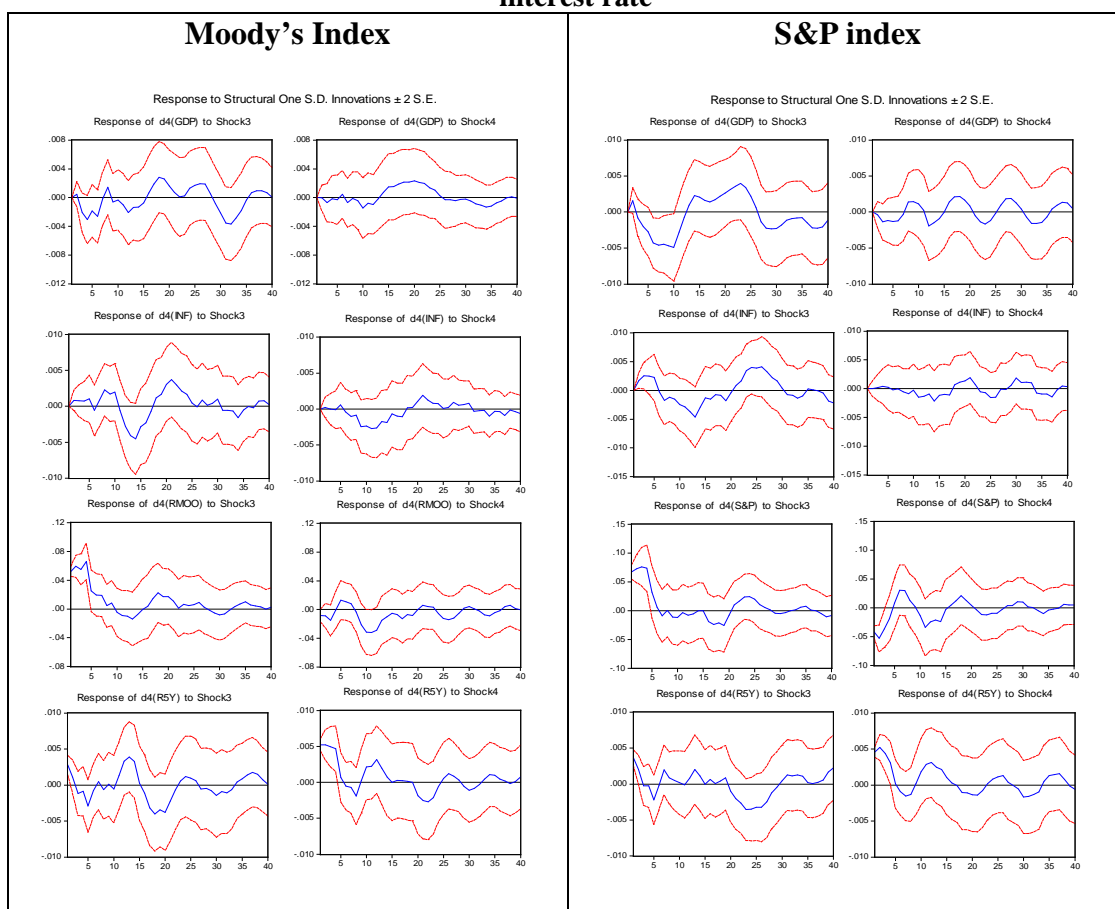
(OIL- RCRBO)*				
Parameters	RFF	R1Y	R5Y	R10Y
$-\beta_{21}^0$	-0.0995 (0.0495)	-0.0966 (0.0492)	-0.0961 (0.0484)	-0.0782 (0.0472) ¹
$-\beta_{31}^0$	-4.2455 (1.5546)	-2.6390 (1.1892)	-1.9747 (0.9740)	-1.9503 (0.9427)
β_{34}^0	13.3991 (4.1658)	10.3866 (2.7482)	8.3057 (1.9852)	8.8350 (1.8678)
$-\beta_{41}^0$	-0.1248 (0.1194) ¹	-0.1499 (0.0832) ¹	-0.1317 (0.0570)	-0.1057 (0.0495)
$-\beta_{42}^0$	1.1492 (0.2726)	1.0542 (0.1654)	0.9513 (0.1018)	0.9804 (0.0923)
$-\beta_{43}^0$	-0.1019 (0.0327)	-0.0663 (0.0169)	-0.0336 (0.0087)	-0.0257 (0.0070)
(Metals- RCRBM)*				
$-\beta_{21}^0$	-0.0698 (0.0490) ¹	-0.0704 (0.0487) ¹	-0.0733 (0.0476) ¹	-0.0279 (0.0456) ¹
$-\beta_{31}^0$	-2.3830 (1.0826)	-1.7900 (0.8923)	-1.2634 (0.7768) ¹	-1.6802 (0.8453)
β_{34}^0	6.5569 (2.6328)	5.4372 (1.9212)	4.9743 (1.5834)	6.5195 (1.8093)
$-\beta_{41}^0$	-0.1744 (0.0913) ¹	-0.1754 (0.0745)	-0.1433 (0.0551)	-0.1005 (0.0497)
$-\beta_{42}^0$	0.8994	0.9440	0.9403	0.9754

	(0.1644)	(0.1291)	(0.0939)	(0.0919)
$-\beta_{43}^0$	-0.1049 (0.0270)	-0.0801 (0.0178)	-0.0520 (0.0106)	-0.0468 (0.0088)
(Raw Materials- RCRBR)*				
$-\beta_{21}^0$	-0.0507 (0.0476) ¹	-0.0447 (0.0472) ¹	-0.0462 (0.0464) ¹	-0.0300 (0.0455) ¹
$-\beta_{31}^0$	-1.6513 (0.6668)	-1.3440 (0.5707)	-1.0703 (0.4929)	-1.3072 (0.5046)
β_{34}^0	3.7242 (1.6312)	3.2222 (1.2764)	2.8285 (1.0326)	3.5594 (1.0462)
$-\beta_{41}^0$	-0.1146 (0.0876) ¹	-0.1313 (0.0746) ¹	-0.1274 (0.0561)	-0.0754 (0.0497) ¹
$-\beta_{42}^0$	0.8727 (0.1503)	0.9022 (0.1280)	0.8977 (0.0954)	0.9830 (0.0887)
$-\beta_{43}^0$	-0.1572 (0.0376)	-0.1258 (0.0285)	-0.0803 (0.0176)	-0.0750 (0.0138)
^{1/} Level of significance below five percent.				
* Number of lags 8. The value in parenthesis is the standard error				

Table D2: Model with Moody's and Standard and Poor indexes: difference at quarterly frequency ($t - (t - 1)$)

Moody's Index: (RMOO) *				
Parameters	RFF	R1Y	R5Y	R10Y
$-\beta_{21}^0$	-0.1645 (0.0674)	-0.1704 (0.0679)	-0.1567 (0.0679)	-0.1561 (0.0672)
$-\beta_{31}^0$	-3.8621 (0.9092)	-3.7863 (0.8897)	-3.2995 (0.7767)	-3.0264 (0.7456)
β_{34}^0	4.0671 (1.6227)	3.6666 (1.4815)	3.5766 (1.3326)	3.5347 (1.2761)
$-\beta_{41}^0$	-0.1738 (0.1055) ¹	-0.1613 (0.0995) ¹	-0.0851 (0.0824) ¹	-0.0572 (0.0719) ¹
$-\beta_{42}^0$	0.8857 (0.1329)	0.9589 (0.1239)	0.9328 (0.1037)	0.9302 (0.0922)
$-\beta_{43}^0$	-0.0859 (0.0236)	-0.0927 (0.0206)	-0.0679 (0.0159)	-0.0571 (0.0132)
S&P index: (RS&P) *				
$-\beta_{21}^0$	-0.1653 (0.0599)	-0.1614 (0.0592)	-0.1387 (0.0590)	-0.1366 (0.0583)
$-\beta_{31}^0$	-6.6195 (1.5775)	-5.3340 (1.2696)	-4.4123 (1.1436)	-3.7394 (1.0602)
β_{34}^0	12.8253 (2.9342)	11.3891 (2.1839)	11.4872 (1.9962)	11.1871 (1.8311)
$-\beta_{41}^0$	0.0771 (0.2121) ¹	-0.0619 (0.1317) ¹	-0.1078 (0.0903) ¹	-0.0845 (0.0726) ¹
$-\beta_{42}^0$	2.2171 (0.5080)	1.8447 (0.2944)	1.4839 (0.1889)	1.3653 (0.1451)
$-\beta_{43}^0$	-0.1806 (0.0526)	-0.1226 (0.0279)	-0.0792 (0.0165)	-0.0615 (0.0119)
^{1/} Level of significance below five percent.				
* Number of lags 8. The value in parenthesis is the standard error				

Figure D3: Impulse response function: difference at annual frequency model using R5Y interest rate



Note: shock 3 represent s a shock in real commodity prices, and shock 4: represent s a shock in monetary policy or real interest rate.

Appendix E: Estimation with S&P index and CRB from 1980-2009

Table E1. Contemporaneous relationships model in fourth difference

RCRB – difference at annual frequency $(t - (t - 4))^{**}$				
Parameters	RFF	R1Y	R5Y	R10Y
$-\beta_{21}^0$	-0.1098 (0.0735) ¹	-0.0962 (0.0733) ¹	-0.1331 (0.0673)	-0.1389 (0.0657)
$-\beta_{31}^0$	-3.7051 (0.9533)	-2.6080 (0.6904)	-2.3847 (0.5932)	-2.2557 (0.5583)
β_{34}^0	3.0008 (1.3541)	2.2060 (0.9399)	1.9646 (0.8310)	1.9451 (0.8210)
$-\beta_{41}^0$	-0.2801 (0.1509)	-0.2873 (0.1119)	-0.3044 (0.0797)	-0.2536 (0.0686)
$-\beta_{42}^0$	0.8732 (0.1691) ¹	0.9843 (0.1364)	1.0191 (0.1049)	1.0060 (0.0923)
$-\beta_{43}^0$	-0.1599 (0.0502)	-0.1292 (0.0343)	-0.0765 (0.0206)	-0.0564 (0.0165)
RS&P- difference at annual frequency $(t - (t - 4))^{**}$				
$-\beta_{21}^0$	-0.0932 (0.0827)	-0.1358 (0.0665) ¹	-0.1110 (0.0585)	-0.0998 (0.0572) ¹
$-\beta_{31}^0$	-4.4974 (1.3559)	-3.1981 (1.1112)	-3.7490 (1.2164)	-3.5998 (1.2114)
β_{34}^0	7.0477 (1.4114)	6.0809 (1.3556)	8.2556 (1.8496)	9.2061 (1.9917)
$-\beta_{41}^0$	-0.2320	-0.3202	-0.3729	-0.3186

	(0.1171)	(0.0906)	(0.0688)	(0.0628)
$-\beta_{42}^0$	1.4838 (0.1688)	1.5152 (0.1542)	1.3641 (0.1364)	1.3034 (0.1285)
$-\beta_{43}^0$	-0.0617 (0.0154)	-0.0632 (0.0140)	-0.0571 (0.0111)	-0.0496 (0.0098)
^{1/} Level of significance below five percent.				
**Number of lags 12. The value in parenthesis is the standard error				