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Policy Analysis Tool Applied to Colombian Needs: PATACON Model Description*

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Abstract

In this document we lay out the microeconomic foundations of a dynamic stochastic general equilibrium model designed to forecast and to advice monetary policy authorities in Colombia. The model is called Policy Analysis Tool Applied to Colombian Needs (PATACON). In companion documents we present other aspects of the model and its platform, including the estimation of the parameters that affect the dynamics and the impulse responses functions.

Keywords: Monetary Policy, DSGE, Small open economy.

JEL codes: E32, E52, F41

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Contents

1 Introduction

In this document we present a dynamic stochastic general equilibrium model designed to forecast and to provide advice for monetary policy in Colombia. We call the model *Policy Analysis Tool Applied to Colombian Needs* (PATACON). The model is similar to models used in other small open economies. For example, the Riksbank, the central Bank of Sweden, uses the model by Christiano, Trabandt, and Walentin (2007). The Bank of Spain uses MEDEA, a DSGE model by Burriel and Rubio-Ramirez (2009). The Bank of Norway and the Bank of Canada constitute other examples.

PATACON is a New Keynesian model constructed on top of a neo-classical growth model in which economic agents optimize the use of their resources over time. The source of growth is exogenous and depends on technological change and the rate of population growth. Following the work of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), this model is augmented to match the date with features such as sticky nominal wages and prices as well as real rigidities such as habit in consumption, adjustment costs in investment, variable capital utilization and endogenous capital depreciation.

As it name implies, PATACON is tailored to match particular Colombian economic circumstances. For instance, Colombia is a net international borrower and consequently, it is influenced by changes in world capital markets. The model has therefore to allow for external world interest rates and perceptions of Colombian risk to affect domestic developments. The first effect is captured by a world interest rate, the second by a sustainable ratio of net foreign assets to GDP that can be altered along with perceptions of risk. World demand also matters, through export demand. Then, the external factors which have been important for the Colombian macroeconomic outlook are present in the model.

Although Colombia is not very open to trade[1], world prices do matter for GDP and inflation. This is probably because imports are complementary in domestic production and consumption. That said, the import price is affected by commercialization within national frontiers. In brief, this calls for a model with different types of imported inputs - capital, raw materials and consumption products - each of which can be complementary in intermediate or final consumption with domestically produced inputs. Naturally there should be a role for domestic margins in affecting the pass-through, as suggested by studies such as Parra (2010) or González, Rincón, and Rodríguez (2010). Another channel by which world prices matter is through export prices and so revenues, a recurrent theme of Colombia's economic history (Mahadeva and Gómez (2010)).

The evidence reported in Julio (2010), Julio, Zárate, and Hernández (2009), Iregui, Melo, and Ramírez (2009), Misas, López, and Parra (2009) and Hofstetter (2010) suggests that wages and prices in Colombia

[1] Imports plus exports are about 45% of the GDP

are very heterogeneous in their degree of stickiness. Some wages in the formal sector and some important regulated prices continue to be indexed but there are other prices which respond and adjust quickly. To account for this fact we have built in many different relative prices with different degrees of stickiness and permitted monopolistic competition in important parts of the production structure.

Equally important in model design are the restrictions imposed by the available data. Reliable data on the amounts and prices of factor inputs employed by different production sectors are not available, although there are data on sectoral output and prices. To get around this, our proposal is to model the production of a composite output in one sector using all labour and capital, and then describe a transformation of that composite output into its different forms in other sectors where these two factor inputs do not feature. Similarly different imported inputs are combined with domestic factors and aggregated without the use of capital and labour. This production structure allows us to avoid depending on problematic data constructions of sectoral factor demands.

As important as designing the economic structure of a model is also to develop a platform in which the model rests. The platform is a set of tools that makes possible to use the model. The main reason for this is the large uncertainty featured in the economic environment, not least for a country such as Colombia, which cannot be anticipated by experience and feasibly put into the model environment. These uncertainties are to do with structural shifts in economic structure, or to do with the measurement error contained in the data that is presented to us. The platform makes it easier to adjust the model to cope with these uncertainties as they happen.

That platform is designed to work in a central bank. Crucially the theoretical structure of the model acts as a constraint on this platform, the discipline of economic theory acts against ad hoc solutions to unpredicted changes. The elements of this platform are contained in other articles published by the Department of Macroeconomic Modelling in the Banco de la República and can be listed as follows:

- Mahadeva and Parra (2008) describe the construction and testing data set that is used to calibrate this model.
- Bonaldi, González, Prada, Rodríguez, and Rojas (2009) describe an efficient algorithm for calibrating the steady state ratios and relative prices.
- González, Mahadeva, Rodríguez, and Rojas (2009) describe how the model can be used taking account of real world features of the data.
- Bonaldi, González, and Rodríguez (2011) and Bonaldi, González, and Rodríguez (2010) describe the estimation of the model and present impulse response analysis.

This paper is organized as follows. Section 2 presents an overview of the model. Section 3 describes the technological progress, the dynamics for population, employment and model units. The household's behavior is described in section 4. In section 5 we present the production structure including intermediate and final good producers. Section 6 discusses demand for Colombian exports in the world market and the debt elastic external interest rate. Monetary policy arrangements are shown in section 7. Section 8 describes the model relation with national accounts and section 9 concludes.

2 The model structure

The model structure is summarized in Figure 2.1 and can be described roughly as follows. Households rent capital and labor to firms, obtain the benefits they generate, receive remittances from abroad and borrow from the international markets at an interest rate that depends on the aggregate level of indebtedness. Regarding expenditure, they purchase imported and domestic goods for both consumption and investment and cover the debt service. The production sector consists of monopolistically competitive firms that hire capital, labour and imported raw materials to produce an homogeneous domestic good. This domestic good is transformed, through a technology, into goods suitable for consumption, investment, exports and distribution. Finally, these domestic goods need to be distributed and commercialized. This is done by firms that combine distribution services with consumption, investment and exports goods. These firms operate in monopolistic competition. Similarly, imported goods are combined with distribution services by firms with some market power. In general, the distribution allows consumer and investment goods, domestic and imported, to be purchased by households and exports to be sold abroad.

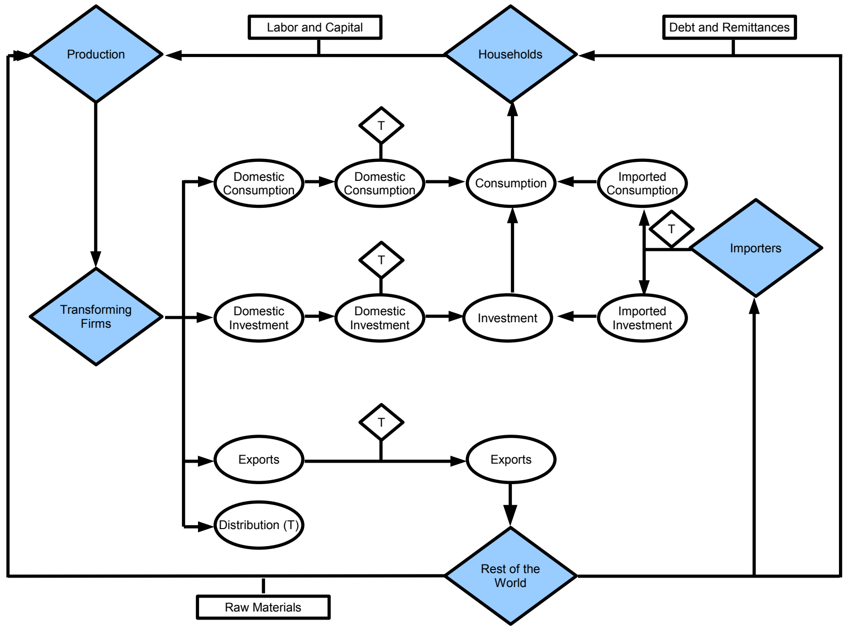
One difference between PATACON and DSGE models estimated by Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007) and Adolfson, Laséen, Lindé, and Villani (2008), is that the former explicitly includes the distribution services. Thus, the final price of imported goods is determined by both the foreign price, and the cost of distribution in the domestic market. Similarly, the final price of exported goods incorporates the distribution costs. Thus, there is incomplete pass-through of the exchange rate to consumer prices. Parra (2010) and González, Rincón, and Rodríguez (2010) show evidence for this hypothesis[2].

3 Technological progress, population, unemployment and model units

Growth in the model is driven by population and trend productivity per worker, per hour worked. The economy is populated by a continuum of households. The total population of size N_t grows at an exogenous

[2] The complete set of variables and the set of equations are presented respectively in appendices A and B.

Figure 2.1: PATACON: Model structure



Note: T stands for distribution.

rate $\overline{n}$. That is, $N_t = (1+\overline{n})\, N_{t-1}$. Technological progress A_t, is exogenous $A_t = (1+g_t)\, A_{t-1}$ where g_t is a stationary process and it is described by

$$\ln g_t = \rho_g \ln g_{t-1} + (1-\rho_g) \ln \overline{g} + \epsilon_t^g$$

$\overline{g}$ is the steady state growth rate of the technological progress, $0 < \rho_g < 1$, and $\epsilon_t^g \sim n\,(0, \sigma^g)$.

To keep things simple, both the rate of labour force participation and the rate of unemployment are also exogenous in this economy. The number of people working during each period is

$$L_t = (1 - TD_t) \times TBP_t \times N_t$$

where TD_t is the unemployment rate from the economically active population and TBP_t is the gross rate of participation from the total population. Both concepts correspond to series reported by the Department of National Statistics (DANE)[3].

The model is solved for the stationary variables and consequently we express all variables in model units. Let J_t, in uppercase, be the total quantity of a real economic variable, such as the volume of consumption.

[3]Departamento Administrativo Nacional de Estadística, Colombia.

The lowercase with a tilde above represents the same variable in per capita terms:

$$\tilde{j}_t \equiv \frac{J_t}{N_t}$$

The variable in model units is expressed without the tilde. It is adjusted not just for population but also for productivity growth, so that it is stationary and therefore constant in the steady state. For convenience, the variable is also adjusted by the total hours available per person[4].

$$j_t \equiv \frac{J_t}{A_t N_t \bar{l}}$$

There are some variables in the model such as the holdings of domestic assets by domestic residents, B_t, where the upper case denotes nominal values. For those variables, the lower case expression is also in real terms and so adjusted for the consumer price index, p_t^{cF}, as well as population, total hours worked and trend productivity growth. For example, $\tilde{b}_t = \frac{B_t}{N_t p_t^{cF}}$ is the real stock of assets per person and $b_t = \frac{\tilde{b}_t}{A_t \bar{l}}$ is the real stock of assets per person and per total hour worked, per unit of technological progress. Wages are also an exception because the stationary real variable is $w_t = \frac{\tilde{w}_t}{A_t}$ where $\tilde{w}_t = \frac{W_t}{p_t^{cF}}$ is the real hourly wage rate.

Finally, external nominal variables such as the external debt, $B_t^\star$ and remittances, $TR_t^\star$ are defined in real terms using the foreign consumer price index. For example, $\tilde{b}_t^\star = \frac{B_t^\star}{N_t p_t^{c\star}}$ is the real stock of external debt per person in foreign currency, and as before, $b_t^\star = \frac{\tilde{b}_t^\star}{A_t \bar{l}}$ is the real stock of external debt per person, per total hour worked and per unit of technological progress in foreign currency.

We are restricted to work with certain functional forms for technology and preferences, as those mentioned by King, Plosser, and Rebelo (1988), which ensures the existence of a balanced growth path. Thus the intertemporal rate of substitution of consumption should be independent of the scale of consumption. Neither should the income and substitution effects associated with greater technological progress affect the supply of labour. The production functions should all feature constant returns to scale, and technological progress should be Harrod-neutral.

4 Households

There is a continuum of households indexed by j. These households solve three problems simultaneously, the first is to maximize utility subject to a budget constraint, the second is to choose the consumption bundle composition, between domestic and imported goods. Finally, since the households offer differentiated labour

[4] 24 hours daily, or approximately 2016 hours each quarter

in a monopolistically competitive labour market, they choose the nominal wage.

4.1 Utility maximization

The household j seeks to maximize the discounted sum of its utility subject to a budget constraint. Its instantaneous utility function describes its preferences over consumption and leisure at any moment in time $u\left(c_t^F(j), l_t(j)\right)$, where $c_t^F(j)$ is the average consumption bundle for each member of the household and $l_t(j)$ is the average level of leisure spent for each member of the household. We assume that each household has a fixed allocation of time, so that the average leisure for each member of the household is given by $l_t(j) = \bar{l} - (1 - TD_t) TBP_t \bar{l} h_t(j)$, where $\bar{l} h_t(j)$ is the total hours that each individual spends working during the quarter and $\bar{l}$ is the total number of hours in an average quarter.

The instantaneous utility function is the following,

$$u(\cdot) = \frac{z_t^u}{1-\sigma}\left(\tilde{c}_t^F(j) - hab\bar{\tilde{c}}_{t-1}^F(1+g_t)\right)^{1-\sigma} - \frac{z_t^h}{1+\eta}\bar{l}^{-\sigma-\eta}A_t^{1-\sigma}\left((1-TD_t)TBP_t\tilde{h}_t(j)\right)^{1+\eta},$$

σ represents the intertemporal elasticity of substitution, hab is the habit consumption parameter, $\bar{\tilde{c}}_{t-1}^F$ is last period per capita final consumption and η is the inverse of the Frisch´s elasticity. In addition, there are exogenous shocks to the marginal utility of consumption, z_t^u, and to the marginal disutility of labour, z_t^h. These exogenous shocks are assumed to follow autoregressive processes,

$$\begin{aligned}
\ln z_t^u &= \rho_{z^u}\ln z_{t-1}^u + (1-\rho_{z^u})\ln \bar{z}^u + \epsilon_t^{z^u} \\
\ln z_t^h &= \rho_{z^h}\ln z_{t-1}^h + (1-\rho_{z^h})\ln \bar{z}^h + \epsilon_t^{z^h}
\end{aligned}$$

where $\bar{z}^u$, $\bar{z}^h$ are long run means, $0 < \rho_{z^u} < 1$, $0 < \rho_{z^h} < 1$, $\epsilon_t^{z^u} \sim n\left(0, \sigma^{z^u}\right)$ and $\epsilon_t^{z^h} \sim n\left(0, \sigma^{z^h}\right)$.

The household owns the production factors, total hours to work h_t and physical capital k_t. From the use of these factors of production each household earns a nominal hourly wage rate, $W_t(j)$, and a nominal rental rate of capital, R_t^k. Additionally, households receive nominal profits, Ξ_t, from the firms and remittances, $tr_t^\star$ from abroad. These remittances are exogenous in foreign currency and follow the autoregressive process,

$$\ln tr_t^\star = \rho_{tr^\star}\ln tr_{t-1}^\star + (1-\rho_{tr^\star})\ln \overline{tr}^\star + \epsilon_t^{tr^\star}$$

where $\overline{tr}^\star$ is the mean parameter , $0 < \rho_{tr^\star} < 1$, and $\epsilon_t^{tr^\star} \sim n\left(0, \sigma^{tr^\star}\right)$.

The household buys the consumption bundle at a price p_t^{cF}, invests in capital stock by buying new investment goods, $x_t^F(j)$, at a price p_t^{xF}. Since investing is costly, we assume that the household covers the investment cost. This cost is proportional to changes in the investment rate as in Christiano, Eichenbaum, and Evans (2005) and it is describe through the following equation:

$$\Psi^X\left(x_t^F(j), x_{t-1}^F(j)\right)=\frac{\psi^X}{2}\frac{\left(x_t^F(j)-x_{t-1}^F(j)\right)^2}{x_{t-1}^F(j)}$$

where ψ^X is the adjustment cost parameter.

The household also chooses how intensively to work the capital. The rate of capital utilization is represented by u_t. Through this decision the household will affect both the rent of capital and its depreciation rate. See Christiano, Eichenbaum, and Evans (2005). Additionally, the household has to cover the debt services in domestic and foreign currency. Net domestic assets, B_t, earn a nominal interest i_t, and net foreign assets, $B_t^\star$, in nominal foreign currency terms, pay an interest rate $i_t^\star$. We denote the nominal exchange rate as s_t and the external consumption bundle price as $p_t^{c\star}$.

Finally, the household buys Arrow-Debreu securities $a_{t+1,t}(j)$ at a real price $p_{t+1,t}^a(j)$. These are state contingent securities that insure the household against idiosyncratic shocks. As we will see, wages are allowed to differ across households, however, with these securities, consumption plans of different households are identical.

Equation 4.1, summarizes the budget constraint faced by household j.

$$\begin{aligned}
c_t^F(j)+\frac{p_t^{xF}}{p_t^{cF}}x_t^F(j)+b_t &+ \frac{s_t p_t^{c\star}}{p_t^{cF}}\frac{b_{t-1}^\star(j)}{(1+\overline{n})(1+g_t)}\left(\frac{1+i_{t-1}^\star}{1+\pi_t^{c\star}}\right)+\int p_{t+1,t}^a(j)\,a_{t+1}(j)\,d\omega_{t+1,t}(j)+\\
\Psi^X\left(x_t^F(j), x_{t-1}^F(j)\right) &= r_t^k u_t(j)\frac{k_{t-1}(j)}{(1+\overline{n})(1+g_t)}+w_t(j)(1-TD_t)TBP_t h_t(j)+\xi_t+a_t(j) \qquad (4.1)\\
\frac{s_t p_t^{c\star}}{p_t^{cF}}tr_t^\star+\frac{s_t p_t^{c\star}}{p_t^{cF}}b_t^\star(j) &+ \frac{b_{t-1}(j)}{(1+\overline{n})(1+g_t)}\left(\frac{1+i_{t-1}}{1+\pi_t^{cF}}\right)
\end{aligned}$$

where ξ_t represent real profits from all firms, p_t^{cF} is the price of the consumption bundle and $\frac{p_t^{c\star}}{p_{t-1}^{c\star}}=(1+\pi_t^{c\star})$ is the external consumption bundle inflation in foreign currency that follows the exogenous process

$$\ln\pi_t^{c\star}=\rho_{\pi^{c\star}}\ln\pi_{t-1}^{c\star}+(1-\rho_{\pi^{c\star}})\ln\overline{\pi}^{c\star}+\epsilon_t^{\pi^{c\star}}$$

where $\overline{\pi}^{c\star}$ is the mean of the foreign inflation, $0<\rho_{\pi^{c\star}}<1$, and $\epsilon_t^{\pi^{c\star}}\sim n\left(0,\sigma^{\pi^{c\star}}\right)$.

A second constraint face by the household is the capital accumulation restriction, defined as:

$$k_t(j)=x_t^F(j)+\frac{(1-\delta(u_t(j)))\,k_{t-1}(j)}{(1+\overline{n})(1+g_t)}. \qquad (4.2)$$

where $\delta\left(u_t\left(j\right)\right)$ is the endogenous depreciation rate, that is a function of the capital utilization,

$$\delta\left(u_t\left(j\right)\right) = \overline{\delta} + \frac{b}{1+\Upsilon}\left(u_t\left(j\right)\right)^{1+\Upsilon}$$

with $\overline{\delta}$ being the steady state rate of depreciation, $b > 0$ a scale parameter and $\Upsilon > 0$ a parameter that affects the dynamics of the capital utilization.

The dynamic problem of the household j, in per capita terms, is[5]:[6]

$$\max_{\left\{\tilde{c}_t^F, \tilde{x}_t, \tilde{k}_t, \tilde{b}_t, \tilde{fd}_t, u_t\right\}} \quad E_t \sum_{s=0}^{\infty} \left(\beta\left(1+\overline{n}\right)\right)^s u\left(\tilde{c}_{t+s}^F\left(j\right), \overline{l} - \left(1 - TD_{t+s}\right) TBP_{t+s}\tilde{h}_{t+s}\left(j\right)\right)$$

$$\begin{aligned}
\text{s.t.} \quad & \tilde{w}_t\left(j\right)\left(1-TD_t\right)TBP_t\tilde{h}_t\left(j\right) + r_t^k u_t\left(j\right)\frac{\tilde{k}_{t-1}\left(j\right)}{1+\overline{n}} + \tilde{\xi}_t + \frac{\tilde{b}_{t-1}\left(j\right)}{1+\overline{n}}\left(\frac{1+i_{t-1}}{1+\pi_t^{cF}}\right) \\
& + \frac{s_t p_t^{c\star}}{p_t^{cF}}\left(\tilde{b}_t^{\star}\left(j\right) + \tilde{tr}_t^{\star}\right) - \int p_{t+1,t}^a\left(j\right)\tilde{a}_{t+1}\left(j\right)d\omega_{t+1,t}\left(j\right) \\
& \geq \tilde{c}_t^F\left(j\right) + \frac{p_t^{xF}}{p_t^{cF}}\tilde{x}_t^F\left(j\right) + \frac{s_t p_t^{c\star}}{p_t^{cF}}\frac{\tilde{b}_{t-1}^{\star}\left(j\right)}{1+\overline{n}}\left(\frac{1+i_{t-1}^{\star}}{1+\pi_t^{c\star}}\right) \\
& + \tilde{b}_t\left(j\right) + \frac{\psi^X}{2}\frac{\left(\tilde{x}_t^F\left(j\right) - \left(1+g_t\right)\tilde{x}_{t-1}^F\left(j\right)\right)^2}{\left(1+g_t\right)\tilde{x}_{t-1}^F\left(j\right)} + \tilde{a}_t\left(j\right)
\end{aligned}$$

$$\tilde{k}_t\left(j\right) = \tilde{x}_t^F\left(j\right) + \left(1 - \delta\left(u_t\left(j\right)\right)\right)\frac{\tilde{k}_{t-1}\left(j\right)}{1+\overline{n}}.$$

where β is the gross subjective discount rate, and must be small enough such that there is always net discounting of the future. In particular, for given values of $\overline{n}$, $\overline{g}$ and σ, β is constrained by $\beta\left(1+\overline{n}\right)\left(1+\overline{g}\right)^{1-\sigma} < 1$.

Since households are insured against idiosyncratic shocks, and have the same preferences then it must be the case that all individuals decisions are identical. Consequently, we can aggregate across the individuals

[5] The household is interested in maximizing utility for each member over an infinite lifetime horizon,

$$N_t u\left(\tilde{c}_t^F\left(j\right), \overline{l} - \left(1-TD_t\right)TBP_t\tilde{h}_t\left(j\right)\right) = \left(1+\overline{n}\right)^i u\left(\tilde{c}_t^F\left(j\right), \overline{l} - \left(1-TD_t\right)TBP_t\tilde{h}_t\left(j\right)\right).$$

See Romer (2005) page 48. Remember that $N_0 = 1$ and that $\frac{N_t}{N_{t-i}} = \left(1+\overline{n}\right)^i$.

[6] The household problem is solved in per-capita terms and the first order conditions are transformed to model units.

and express the first-order conditions for this problem in model units as[7]:

$$
\begin{aligned}
\lambda_t^c &= z_t^u \left(c_t^F - hab\bar{\tilde{c}}_{t-1}^F\right)^{-\sigma} && (4.3)\\
r_t^k &= \frac{\lambda_t^x}{\lambda_t^c}\, b\, u_t^{\Upsilon} && (4.4)\\
\lambda_t^c \frac{p_t^{xF}}{p_t^{cF}} &= \lambda_t^x - \lambda_t^c\, \psi^X\, \frac{x_t^F - x_{t-1}^F}{x_{t-1}^F} && (4.5)\\
&\quad + \beta E_t \left(1+\overline{n}\right)\left(1+g_{t+1}\right)^{1-\sigma} \lambda_{t+1}^c \left(\frac{\psi^X \left(x_{t+1}^F - x_t^F\right) + \Psi^X\left(x_{t+1}^F, x_t^F\right)}{x_t^F}\right)\\
\lambda_t^x &= \beta E_t \left(1+g_{t+1}\right)^{-\sigma} \lambda_{t+1}^c r_{t+1}^k u_{t+1} + \beta E_t \left(1+g_{t+1}\right)^{-\sigma} \lambda_{t+1}^x \left(1-\delta\left(u_{t+1}\right)\right) && (4.6)\\
\lambda_t^c &= \beta E_t \left(1+g_{t+1}\right)^{-\sigma} \lambda_{t+1}^c \left(\frac{1+i_t}{1+\pi_{t+1}^{cF}}\right) && (4.7)\\
\lambda_t^c &= \beta E_t \left(1+g_{t+1}\right)^{-\sigma} \lambda_{t+1}^c \left(1+i_t^{\star}\right)\left(\frac{1+d_{t+1}}{1+\pi_{t+1}^{cF}}\right) && (4.8)
\end{aligned}
$$

where $1+d_t = \frac{s_t}{s_{t-1}}$ denotes the nominal devaluation rate.

Similarly, by aggregating across all households budget constraints we obtain the aggregate resource constraint:[8]

$$
\begin{aligned}
c_t^F + \frac{p_t^{xF}}{p_t^{cF}} x_t^F &+ \frac{s_t p_t^{c\star}}{p_t^{cF}} \frac{b_{t-1}^{\star}}{\left(1+\overline{n}\right)\left(1+g_t\right)} \left(\frac{1+i_{t-1}^{\star}}{1+\pi_t^{c\star}}\right) + \Psi^X\left(x_t^F, x_{t-1}^F\right) = && (4.9)\\
r_t^k \frac{u_t k_{t-1}}{\left(1+\overline{n}\right)\left(1+g_t\right)} &+ w_t \left(1-TD_t\right) TBP_t h_t^F + \xi_t + \frac{s_t p_t^{c\star}}{p_t^{cF}} tr_t^{\star} + \frac{s_t p_t^{c\star}}{p_t^{cF}} b_t^{\star}
\end{aligned}
$$

Note that, equation (4.4) describes the trade-off in adjusting the degree of capacity utilization; equations (4.5) and (4.6) describe the investment and capital stock decisions respectively, which are often used as the basis for partial equilibrium estimations of investment; equations (4.3) and (4.7) together produce the standard Euler equation for intertemporal consumption. Finally combining equations (4.7) and (4.8) we obtain the uncovered interest rate parity.

4.2 Domestic and imported consumption choice

In a separate hypothetical second stage, the households choose consumption of domestically produced goods and imported goods by minimizing cost. The aggregate consumption bundle includes domestically produced

[7]We also assume the standard transversality conditions

$$\lim_{t\to\infty} \left(\beta\left(1+\overline{n}\right)\right)^t \tilde{\lambda}_t^c \tilde{f}_t = 0 \qquad \lim_{t\to\infty} \left(\beta\left(1+\overline{n}\right)\right)^t \tilde{\lambda}_t^c \tilde{b}_t = 0$$

$$\lim_{t\to\infty} \left(\beta\left(1+\overline{n}\right)\right)^t \tilde{\lambda}_t^x \tilde{k}_t = 0$$

where $\tilde{\lambda}_t^c$ is the Lagrange multiplier associated with the budget constraint and $\tilde{\lambda}_t^x$ is the Lagrange multiplier associated with the capital accumulation constraint. They are then the shadow prices of consumption and capital respectively in terms of utility.

[8]Where we use the relations $\int\int p_{t+1,t}^a(j)\, a_{t+1}(j)\, d\omega_{t+1,t}(j)\, dj = \int a_t(j)\, dj$, $\int b_t(j)\, dj = 0$ and $\int w_t(j)\, h_t(j)\, dj = w_t h_t^F$

goods, $c_t^{dF}(j)$, and imported goods adapted for local consumption, $c_t^{mF}(j)$. These are aggregated in utility as:

$$c_t^F(j) = \left[(\gamma^c)^{\frac{1}{\omega^c}} \left(c_t^{dF}(j) \right)^{\frac{\omega^c - 1}{\omega^c}} + (1-\gamma^c)^{\frac{1}{\omega^c}} \left(c_t^{mF}(j) \right)^{\frac{\omega^c - 1}{\omega^c}} \right]^{\frac{\omega^c}{\omega^c - 1}} \tag{4.10}$$

where ω^c is elasticity of substitution between domestic consumption and imported consumption. γ^c controls the participation of domestic consumption on total consumption.

The minimization problem is:

$$\min_{\left\{ \tilde{c}_t^{dF}(j), \tilde{c}_t^{mF}(j) \right\}} \quad p_t^{cdF} \tilde{c}_t^{dF}(j) + p_t^{mF} \tilde{c}_t^{mF}(j)$$

$$\tilde{c}_t^F(j) = \left[(\gamma^c)^{\frac{1}{\omega^c}} \left(\tilde{c}_t^{dF}(j) \right)^{\frac{\omega^c - 1}{\omega^c}} + (1-\gamma^c)^{\frac{1}{\omega^c}} \left(\tilde{c}_t^{mF}(j) \right)^{\frac{\omega^c - 1}{\omega^c}} \right]^{\frac{\omega^c}{\omega^c - 1}}$$

and the first-order conditions of this problem are

$$c_t^{dF}(j) = \gamma^c \left(\frac{p_t^{cdF}}{p_t^{cF}} \right)^{-\omega^c} c_t^F(j) \tag{4.11}$$

and

$$c_t^{mF}(j) = (1-\gamma^c) \left(\frac{p_t^{mF}}{p_t^{cF}} \right)^{-\omega^c} c_t^F(j) \tag{4.12}$$

where p_t^{cdF} is the price of domestically produced consumption and p_t^{mF} is the price of imported goods. By substituting equations (4.11) and (4.12) into the definition of total cost of consumption we obtain the expression for the aggregate consumption deflator:

$$p_t^{cF} = \left[\gamma^c \left(p_t^{cdF} \right)^{1-\omega^c} + (1-\gamma^c) \left(p_t^{mF} \right)^{1-\omega^c} \right]^{\frac{1}{1-\omega^c}}$$

and, by an algebraic manipulation of the later equation, also an expression for the consumer price inflation

$$\left(1+\pi_t^{cF}\right) = \left[\gamma^c \left(\frac{p_{t-1}^{cdF}}{p_{t-1}^{cF}} \left(1+\pi_t^{cdF}\right) \right)^{1-\omega^c} + (1-\gamma^c) \left(\frac{p_{t-1}^{mF}}{p_{t-1}^{cF}} \left(1+\pi_t^{mF}\right) \right)^{1-\omega^c} \right]^{\frac{1}{1-\omega^c}}.$$

4.3 Wage setting problem

The households offer differentiated labour in a monopolistically competitive labour market. But wages are rigid in nominal terms; it is assumed that each household must wait for a stochastic signal before adjusting the nominal wage rate.

Representative labour aggregator

As in Erceg, Henderson, and Levin (2000), workers are hired by an intermediary firm which operates under perfect competition. This firm combines the work effort of individual workers and supply a joint labour input. The problem of the intermediary firm is to minimize its costs given technological restrictions. The problem is described as:

$$\begin{aligned} \min_{\{h_t(j)\}} \quad & \int_0^1 \tilde{w}_t(j)\,\tilde{h}_t(j)\,dj \\ \text{s.t} \quad & \tilde{h}_t^F \leq \int_0^1 \left[\tilde{h}_t(j)^{\frac{\theta^w-1}{\theta^w}}\,dj\right]^{\frac{\theta^w}{\theta^w-1}} \end{aligned}$$

where θ^w represents the elasticity of substitution among differentiated labour from households.

The first order conditions for this problem are the demand for labour of the household j

$$\tilde{h}_t(j) = \left(\frac{\tilde{w}_t(j)}{\tilde{w}_t}\right)^{-\theta^w} \tilde{h}_t^F$$

and the wage index

$$\tilde{w}_t \equiv \left[\int_0^1 \tilde{w}_t(j)^{1-\theta^w}\,dj\right]^{\frac{1}{1-\theta^w}}. \tag{4.13}$$

Household's decisions over nominal wages

As in xxx, nominal wages are sticky. Given the demand for their differentiated labour, a household adjust wages according to a rule, but it is only free to optimally set a salary when it receives a random signal which arrives every quarter with probability $1-\varepsilon^w$. This probability is independent of its own history and of other shocks in the model. Thus the probability that the wage is set by a rule for the next i quarters is $(\varepsilon^w)^i$. This rule implies that nominal wages increase in line with previous period's inflation and the rate of increase of productivity:

$$\tilde{w}_t^{Rule}(j) = \tilde{w}_{t-1}(j)\,(1+g_t)\left(\frac{1+\pi_{t-1}^{cF}}{1+\pi_t^{cF}}\right).$$

If, on the other hand, the household j receives the signal to adjust its wage at period t, it will face the following problem:

$$\begin{aligned}
\max_{\tilde{w}_t(j)} \quad & E_t \sum_{i=0}^{\infty} \left(\beta\varepsilon^w \left(1+\overline{n}\right)\right)^i u\left(\tilde{c}_{t+i}^F, \bar{l} - \left(1 - TD_{t+i}\right) TBP_{t+i}\tilde{h}_{t+i}\left(j\right)\right) \\
\text{s.t} \quad & \tilde{h}_{t+i}\left(j\right) = \left(\frac{\tilde{w}_{t+i}\left(j\right)}{\tilde{w}_{t+i}}\right)^{-\theta^w} \tilde{h}_{t+i}^F \\
& \tilde{w}_{t+i}\left(j\right) = \tilde{w}_t\left(j\right) \prod_{k=1}^{i} \left(\frac{\left(1+g_{t+k}\right)\left(1+\pi_{t+k-1}^{cF}\right)}{1+\pi_{t+k}^{cF}}\right)
\end{aligned}$$

and subject to the budget constraint (4.1).

It follows that the optimal salary of household j, if renegotiated at time t, will have to obey:

$$w_t^{opt}\left(j\right) = \frac{\theta^w}{\theta^w - 1} \frac{num_t^w\left(j\right)}{den_t^w\left(j\right)} \tag{4.14}$$

where we define the terms $num_t^w\left(j\right)$ and $den_t^w\left(j\right)$ as:

$$\begin{aligned}
num_t^w\left(j\right) \quad \equiv \quad & E_t \sum_{i=0}^{\infty} \left(\beta\varepsilon^w \left(1+\overline{n}\right)\right)^i \prod_{k=1}^{i} \left[\left(1+g_{t+k}\right)^{1-\sigma}\right] \\
& z_{t+i}^h \left(\left(1 - TD_{t+i}\right) TBP_{t+i}\right)^{1+\eta} \left(h_{t+i}^F \left(\frac{w_t^{opt}\left(j\right)}{w_{t+i}}\right)^{-\theta^w} \left(\frac{1+\pi_t^{cF}}{1+\pi_{t+i}^{cF}}\right)^{-\theta^w}\right)^{1+\eta}
\end{aligned} \tag{4.15}$$

$$\begin{aligned}
den_t^w\left(j\right) \quad \equiv \quad & E_t \sum_{i=0}^{\infty} \left(\beta\varepsilon^w \left(1+\overline{n}\right)\right)^i \prod_{k=1}^{i} \left[\left(1+g_{t+k}\right)^{1-\sigma}\right] \\
& \lambda_{t+i}^c \left(1 - TD_{t+i}\right) TBP_{t+i} \left(h_{t+i}^F \left(\frac{w_t^{opt}\left(j\right)}{w_{t+i}}\right)^{-\theta^w} \left(\frac{1+\pi_t^{cF}}{1+\pi_{t+i}^{cF}}\right)^{1-\theta^w}\right).
\end{aligned} \tag{4.16}$$

Notice that all households able to choose optimally their wage, will choose the same wage because the market for assets allows them to eliminate the idiosyncratic risk associated with not being able to adjust optimally in the future. We can therefore omit the subscript j from equations (4.15) and (4.16). Additionally, it can be shown that aggregated effective real wage follows:

$$w_t = \left[\varepsilon^w \left(w_{t-1}\left(\frac{1+\pi_{t-1}^{cF}}{1+\pi_t^{cF}}\right)\right)^{1-\theta^w} + \left(1-\varepsilon^w\right)\left(w_t^{opt}\right)^{1-\theta^w}\right]^{\frac{1}{1-\theta^w}} \tag{4.17}$$

Aggregation of the labour supply for the households implies:

$$\int_0^1 \tilde{h}_t\left(j\right) dj = \int_0^1 \left(\frac{\tilde{w}_t\left(j\right)}{\tilde{w}_t}\right)^{-\theta^w} dj \tilde{h}_t^F$$

$$h_t^S = aw_t h_t^F$$

where $aw_t = \int_0^1 \left(\tilde{w}_t\left(j\right)/\tilde{w}_t\right)^{-\theta^w} dj$ is the wage distortion that appears from the fact that there are two fractions of households adjusting at different wages[9], h_t^S is the aggregate supply of labor and h_t^F is the aggregate demand from the representative labour aggregator.

5 Firms

This section contains a detailed description of the production process in the model. The process starts with the production of a raw domestic good that is combined with imported goods and later commercialized.

The production structure of our model differs from the production structure found in other DSGE models. There are several reasons why we build a more complex production sector. First, we found that distribution of goods is an important component of the total cost of the final good (see Parra (2010)). Second, final goods are produced using both imported goods and domestically produced goods. Similarly, imported goods have to be distributed and the distribution costs are not negligible.

The complete production process comprises eight stages. First, a set of firms produce a domestic generic good (gross output). These firms combine domestic factors, such as labor and capital, with imported raw materials. At a second stage, output of the intermediate domestic producers is bought by imaginary warehousing firms as inputs of production of a homogeneous good. In the third stage, the homogeneous good is transformed into four different intermediate goods: domestic consumption goods, domestic investment goods, export goods, and distribution services. We call firms involved in this stage transforming firms. They are imaginary firms because they do not produce any value added. However, having this transforming stage in the model allow us to create a set of relative prices that are useful in the calibration process. In the fourth stage of the production process, we generate distinct distribution services. That is, there are a number of firms buying distribution services from the transforming firms to produce differentiated distribution services at no cost. In the fifth stage a final producer of either domestic consumption good, domestic investment good or exports, combines the differentiated distribution services with intermediate goods from the transforming firm.

The last three stages in the production process are carried out by the following firms. First, there are importers of consumption and investment goods. These importers combine raw imports with the distribution

[9] This expression can be expressed in a recursive way:

$$aw_t = \varepsilon^w \left((1+g_t) \frac{\left(1+\pi_{t-1}^{cF}\right)}{\left(1+\pi_t^{cF}\right)} \frac{\tilde{w}_{t-1}}{\tilde{w}_t} \right)^{-\theta^w} aw_{t-1} + (1-\varepsilon^w) \left(\frac{\tilde{w}_t^{opt}}{\tilde{w}_t} \right)^{-\theta^w}$$

sector output to produce an imported good that is useful for domestic consumption or investment. The second type of firms are the importers of raw materials who sell their product to producers of the domestic output. Finally, we have the producers of investment goods, they combine the final domestic investment good with the imported investment good and produce an aggregate investment good.

5.1 Gross output producers

There is a continuum of firms indexed by $z \in (0,1)$. Each produces a differentiated product z given a production function. In model units, the technological restriction is:

$$\begin{aligned} q_t^C(z) &= z_t^q \left[\alpha^{\frac{1}{\rho}} \left(va_t(z) \right)^{\frac{\rho-1}{\rho}} + (1-\alpha)^{\frac{1}{\rho}} \left(rm_t^F(z) \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \\ va_t(z) &= \left[\alpha_v^{\frac{1}{\rho_v}} \left(k_t^s(z) \right)^{\frac{\rho_v-1}{\rho_v}} + (1-\alpha_v)^{\frac{1}{\rho_v}} \left((1-TD_t) TBP_t h_t(z) \right)^{\frac{\rho_v-1}{\rho_v}} \right]^{\frac{\rho_v}{\rho_v-1}} \end{aligned} \tag{5.1}$$

where $k_t^s(z)$ is the demand for capital from firm z, $h_t(z)$ is the demand for labour in hours, $rm_t^F(z)$ is the demand for imported raw materials and z_t^q is an aggregate temporary technology shock that follows

$$\ln z_t^q = \rho_{z^q} \ln z_{t-1}^q + (1-\rho_{z^q}) \ln \overline{z}^q + \epsilon_t^{z^q}$$

$\overline{z}^q$ is the mean of the exogenous process , $0 < \rho_{z^q} < 1$, and $\epsilon_t^{z^q} \sim n\left(0, \sigma^{z^q}\right)$.

Notice that the production function in Equation (5.1) implies different degrees of substitutability between value added and raw materials, and between capital and labor. This feature allows us to control the degree of substitutability between domestic and imported factors, see Bruno and Sachs (1985, p. 64).

The elasticities of substitution between $va_t(z)$ and $rm_t^F(z)$ and between $k_t^s(z)$ and $h_t(z)$ are $(\rho)^{-1}$ and $(\rho_v)^{-1}$. The participation of $k_t^s(z)$ in the value added is controlled by α_v and the participation of $va_t(z)$ in the production is controlled by α.

These firms find themselves in a state of monopolistic competition and each period have a constant probability of adjusting their price. They sell the total production $Q_t^C(z)$ to an intermediary at a price $p_t^{qF}(z)$. They must solve two problems: first they decide on their demand for factors of production and then they choose its output price upon receiving a stochastic signal that allows them to change the price.

Cost minimization problem

Intermediate firms solve the following minimization problem[10]:

$$\begin{aligned} \min_{K_t^s(z),h_t(z),RM_t(z)} \quad & \frac{W_t}{p_t^{qF}(z)} L_t h_t(z) + \frac{R_t^k}{p_t^{qF}(z)} K_t^s(z) + \frac{p_t^{rmF}}{p_t^{qF}(z)} RM_t^F(z) \\ \text{s.t} \quad & Q_t^C(z) \leq z_t^q \left[\alpha^{\frac{1}{\rho}} \left(VA_t(z)\right)^{\frac{\rho-1}{\rho}} + (1-\alpha)^{\frac{1}{\rho}} \left(RM_t^F(z)\right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \\ & VA_t(z) = \left[\alpha_v^{\frac{1}{\rho_v}} \left(K_t^s(z)\right)^{\frac{\rho_v-1}{\rho_v}} + (1-\alpha_v)^{\frac{1}{\rho_v}} \left(A_t L_t h_t(z)\right)^{\frac{\rho_v-1}{\rho_v}} \right]^{\frac{\rho_v}{\rho_v-1}} \end{aligned} \tag{5.2}$$

to determine their demand for factors. In Equation (5.2), W_t is the nominal wage, R_t^K is the nominal rent of capital and p_t^{rmF} is the domestic price of the raw materials used for the production. The first-order conditions for the problem above are:

$$w_t = \lambda_t^q(z) z_t^q \left(\frac{\alpha q_t^C(z)}{z_t^q va_t(z)} \right)^{\frac{1}{\rho}} \left(\frac{(1-\alpha_v) va_t(z)}{(1-TD_t) TBP_t h_t(z)} \right)^{\frac{1}{\rho_v}} \tag{5.3}$$

$$r_t^k = \lambda_t^q(z) z_t^q \left(\frac{\alpha q_t^C(z)}{z_t^q va_t(z)} \right)^{\frac{1}{\rho}} \left(\frac{\alpha_v va_t(z)}{k_t^s(z)} \right)^{\frac{1}{\rho_v}} \tag{5.4}$$

$$\frac{p_t^{rmF}}{p_t^{cF}} = \lambda_t^q(z) z_t^q \left(\frac{(1-\alpha) q_t^C(z)}{z_t^q rm_t^F(z)} \right)^{\frac{1}{\rho}} \tag{5.5}$$

w_t is the real wage, $r_t^k = \frac{R_t^k}{p_t^{cF}}$ is the real rent of capital and $\lambda_t^q(z)$ is the real marginal cost. In model units and measured at the consumption price index $\lambda_t^q(z)$ is defined as:

$$\lambda_t^q(z) = \frac{1}{(z_t^q)} \left[\alpha \left(\left[\alpha_v \left(r_t^k\right)^{1-\rho_v} + (1-\alpha_v)(w_t)^{1-\rho_v} \right]^{\frac{1}{1-\rho_v}} \right)^{1-\rho} + (1-\alpha) \left(\frac{p_t^{rmF}}{p_t^{cF}} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}.$$

Note that $\lambda_t^q(z)$ is the same for every firm z, because all firms face the same marginal cost and therefore will set the same market prices.

[10]Note that the minimization problem is solved in levels and the first order conditions are presented in model units.

Profit maximization problem

The problem of setting prices is motivated by assuming that each period, firms face a constant probability $(1-\varepsilon^{q})$ of receiving a signal which tells them that they can adjust their price optimally. This probability is independent of the firm and also time. This set up is as in Calvo (1983). The other ε^{q} firms set their price according with the following backward looking indexation rule:

$$p_{t}^{rule}(z)=p_{t-1}^{qF}(z)\left(1+\pi_{t-1}^{qF}\right)^{\iota^{q}}(1+\overline{\pi})^{1-\iota^{q}} \tag{5.6}$$

where $\overline{\pi}$ is the central bank's inflation target, $1\geq\iota^{q}\geq 0$ is the weight assigned to past inflation as opposed to this target.

The problem of the representative firm z is to choose a price $p_{t}^{q}(z)$ so that its expected stream of profits will be highest, given the constraint that it will only be allowed to change its price optimally on receipt of a random signal. The problem of the adjusting firm is to maximize

$$E_{t}\sum_{i=0}^{\infty}(\varepsilon^{q})^{i}\Delta_{t+i,t}\left[\left(\frac{p_{t+i}^{qF}(z)}{p_{t+i}^{cF}}\right)\tilde{q}_{t+i}^{C}(z)-\tilde{\lambda}_{t+i}^{q}(z)\,\tilde{q}_{t+i}^{C}(z)\right]$$

subject to the demand curve for its product

$$\tilde{q}_{t}^{C}(z)=\left(\frac{p_{t}^{qF}(z)}{p_{t}^{qF}}\right)^{-\theta^{q}}\tilde{q}_{t}^{F} \tag{5.7}$$

and the price-setting rule

$$p_{t+i}^{qF}(z)=p_{t}^{qF}(z)\prod_{l=1}^{i}\left\{\left(1+\pi_{t-1+l}^{qF}\right)^{\iota^{q}}\right\}(1+\overline{\pi})^{i(1-\iota^{q})}$$

where $\Delta_{t+i,t}=(\beta(1+\overline{n}))^{i}\frac{\tilde{\lambda}_{t+i}^{c}}{\tilde{\lambda}_{t}^{c}}=(\beta(1+\overline{n}))^{i}\prod_{k=1}^{i}\left[(1+g_{t+k})^{-\sigma}\right]\frac{\lambda_{t+i}^{c}}{\lambda_{t}^{c}}$ is the discount factor.

The first order condition for the optimal price, $p_{t}^{qopt}(z)$, is:

$$\frac{p_{t}^{qopt}(z)}{p_{t}^{qF}}=\frac{\theta^{q}}{\theta^{q}-1}\frac{E_{t}\sum_{i=0}^{\infty}(\varepsilon^{q})^{i}\Delta_{t+i,t}\frac{A_{t+i}}{A_{t}}\left[\frac{\lambda_{t+i}^{q}(z)\left(\frac{p_{t+i}^{qF}}{p_{t}^{qF}}\right)^{\theta^{q}}q_{t+i}^{F}}{\left(\prod_{l=1}^{i}\left\{\left(1+\pi_{t-1+l}^{qF}\right)^{\iota^{q}}\right\}(1+\overline{\pi})^{i(1-\iota^{q})}\right)^{\theta^{q}}}\right]}{E_{t}\sum_{i=0}^{\infty}(\varepsilon^{q})^{i}\Delta_{t+i,t}\frac{A_{t+i}}{A_{t}}\left[\frac{\left(\frac{p_{t+i}^{qF}}{p_{t}^{qF}}\right)^{\theta^{q}-1}\frac{p_{t+i}^{qF}}{p_{t+i}^{cF}}q_{t+i}^{F}}{\left(\prod_{l=1}^{i}\left\{\left(1+\pi_{t-1+l}^{qF}\right)^{\iota^{q}}\right\}(1+\overline{\pi})^{i(1-\iota^{q})}\right)^{\theta^{q}-1}}\right]} \tag{5.8}$$

Since all firms are identical and $\lambda_t^q(z) = \lambda_t^q$ then $p_t^{qopt}(z) = p_t^{qopt}$. That is, all adjusting firms choose the same price. Given Calvo's pricing arrangement at each moment, $1-\varepsilon^q$ firms will choose the price p_t^{qopt} and the remaining ε^q firms will adjust following the rule. Consequently the output price aggregator is given by

$$p_t^{qF} = \left[(1-\varepsilon^q) \left(p_t^{qopt}\right)^{1-\theta^q} + \varepsilon^q \left[p_{t-1}^{qF} \left(1+\pi_{t-1}^{qF}\right)^{\iota^q} (1+\overline{\pi})^{i(1-\iota^q)} \right]^{1-\theta^q} \right]^{\frac{1}{1-\theta^q}} \tag{5.9}$$

and from Equation (5.9), it is easy to obtain an expression for the producer's inflation, which is :

$$\left(1+\pi_t^{qF}\right) = \left[(1-\varepsilon^q) \left(\frac{p_t^{qopt}}{p_t^{qF}}\right)^{1-\theta^q} \left(1+\pi_t^{qF}\right)^{1-\theta^q} + \varepsilon^q \left[\left(1+\pi_{t-1}^{qF}\right)^{\iota^q} (1+\overline{\pi})^{i(1-\iota^q)} \right]^{1-\theta^q} \right]^{\frac{1}{1-\theta^q}} \tag{5.10}$$

Profits and aggregation

Profits to each firm are:

$$\xi_t^q(z) = \left(\frac{p_t^q(z)}{p_t^{cF}} - \lambda_t^q(z)\right) q_t^C(z).$$

Integrating across firms we obtain the aggregate profits,

$$\xi_t^q = \frac{p_t^{qF} q_t^F}{p_t^{cF}} - \lambda_t^q z_t^q \left[\alpha^{\frac{1}{\rho}} (va_t)^{\frac{\rho-1}{\rho}} + (1-\alpha)^{\frac{1}{\rho}} \left(rm_t^F\right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}.$$

Following, Yun (1996) we have the output equilibrium condition

$$q_t^C = ap_t^q q_t^F$$

where $ap_t^q = \int \left(p_t^q(z)/p_t^{qF}\right)^{-\theta^q} dz$ is the price distortion that appears from the fact that there are two fractions of firms adjusting at different prices[11], q_t^F is the aggregate demand for the raw good, that must satisfy:

$$q_t^F = \frac{p_t^{cdC}}{p_t^q} c_t^{dC} + \frac{p_t^{xdC}}{p_t^q} x_t^{dC} + \frac{p_t^{disC}}{p_t^q} dis_t^C + \frac{p_t^{eC}}{p_t^q} e_t^C$$

[11]This expression can be expressed in a recursive way:

$$ap_t^q = \varepsilon^q \left(\frac{\left(1+\pi_{t-1}^{qF}\right)^{\gamma_p} (1+\overline{\pi})^{1-\gamma_p}}{\left(1+\pi_t^{qF}\right)} \right)^{-\theta^q} ap_{t-1}^q + (1-\varepsilon^q) \left(\frac{p_t^{opt}}{p_t^{qF}}\right)^{-\theta^q}$$

and q_t^C is the total supply from all z goods, defined as[12]:

$$q_t^C \equiv \int q_t^C(z)\,dz \quad = \quad z_t^q \left[\alpha^{\frac{1}{\rho}} (va_t)^{\frac{\rho-1}{\rho}} + (1-\alpha)^{\frac{1}{\rho}} \left(rm_t^F\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}},$$

where

$$va_t \quad = \quad \left[\alpha_v^{\frac{1}{\rho_v}} \left(k_t^s\right)^{\frac{\rho_v-1}{\rho_v}} + (1-\alpha_v)^{\frac{1}{\rho_v}} \left((1-TD_t)\,TBP_t h_t^F\right)^{\frac{\rho_v-1}{\rho_v}}\right]^{\frac{\rho_v}{\rho_v-1}},$$

with

$$h_t^F \quad = \quad \int h_t(z)\,dz,$$

and

$$k_t^s \quad = \quad \int k_t^s(z)\,dz = u_t k_{t-1} / \left((1+\overline{n})(1+g_t)\right).$$

5.2 Warehousing firm

In order to obtain the demand for the intermediate raw domestic product we assumed a warehousing firm that aggregates the intermediate output to produce an homogeneous good and, for simplicity we also assume that they do not use any other input. Their transformation happens according to the following function:

$$q_t^F = \left[\int_0^1 \left(q_t^C(z)\right)^{\frac{\theta^q-1}{\theta^q}} dz\right]^{\frac{\theta^q}{\theta^q-1}}.$$

The elasticity of substitution among the z firms is given by θ^q. The problem of warehousing firms is then to minimize cost subject to this constraint, that is:

$$\min_{q_t^C(z),\, z\in[0,1]} \quad \int_0^1 p_t^{qF}(z)\, q_t^C(z)\,dz$$

$$\text{s.t} \quad \left[\int_0^1 \left(q_t^C(z)\right)^{\frac{\theta^q-1}{\theta^q}} dz\right]^{\frac{\theta^q}{\theta^q-1}} \geq q_t^F$$

[12]To obtain this expression we use the fact that we are in a symmetric equilibrium, and that the ratios $\frac{k_t^s(z)}{v_t(z)}$, $\frac{h_t(z)}{v_t(z)}$, $\frac{v_t(z)}{q_t(z)}$ and $\frac{rm_t^F(z)}{q_t(z)}$ are the same for all firms.

The optimality condition is the demand function

$$q_t^C(z) = \left(\frac{p_t^q(z)}{p_t^{qF}} \right)^{-\theta^q} q_t^F$$

This demand was used in the minimization problem of the transforming firms (see Equation (5.7)). The aggregate producer price is given by

$$p_t^{qF} = \left[\int_0^1 \left(p_t^q(z) \right)^{1-\theta^q} dz \right]^{\frac{1}{1-\theta^q}}$$

5.3 Transforming firms

At a next stage, the generic raw base product is taken from the warehouse and transformed into four different types of intermediate goods: domestic consumption goods, C_t^{dC}, domestic investment goods, X_t^{dC}, exports, E_t^C, and distribution services, DIS_t^C. An example of domestically produced consumption would be services. Construction is an example of a domestically produced investment good. In Colombia, oil, coal, nickel, coffee and industrial products are good examples of domestically produced exports. And for a domestically produced distribution services would simply be transport and commerce.

These transforming firms take one input, and produce four types of output using the following technology

$$\begin{aligned} Q_t^F &= \left[\nu_{nt}^{\omega_q - 1} (NT_t)^{\omega_q} + \nu_e^{\omega_q - 1} \left(E_t^C \right)^{\omega_q} \right]^{\frac{1}{\omega_q}} \qquad (5.11) \\ NT_t &= \left[\nu_c^{\omega_{nt} - 1} \left(C_t^{dC} \right)^{\omega_{nt}} + \nu_x^{\omega_{nt} - 1} \left(X_t^{dC} \right)^{\omega_{nt}} + \nu_{dis}^{\omega_{nt} - 1} \left(DIS_t^C \right)^{\omega_{nt}} \right]^{\frac{1}{\omega_{nt}}} \end{aligned}$$

which represents the minimum quantity of real resources (in terms of the final good of the economy) that are needed to produce these many outputs as in Edwards and Végh (1997) model of banking production. In Equation (5.11), ω_q governs the elasticity of substitution between domestic uses of output and exports, and ω_{nt} governs the elasticity of substitution among domestic uses of output. The parameters ν_{nt}, ν_e, ν_c, ν_x, ν_{dis} define the shares.

The functional form in Equation (5.11) assumes that substitution between export production and any other domestic use of output is no necessarily the same. In Colombia nearly half export production is in five commodities, and these are often called traditional exports. These goods (oil, minerals and coffee) cannot easily be transformed for domestic use. That is the why we would want to allow for a different elasticity to capture the special rigidity in transforming between domestic production and traditional exports.

The maximization problem of this firm is[13]:

$$\max_{\left\{C_t^{dC}, X_t^{dC}, DIS_t^C, E_t^C\right\}} \quad p_t^{cdC}C_t^{dC} + p_t^{xdC}X_t^{dC} + p_t^{disC}DIS_t^C + p_t^{eC}E_t^C - p_t^{qF}Q_t^F$$

$$Q_t^F = \left[\nu_{nt}^{\omega_q - 1}\left(NT_t\right)^{\omega_q} + \nu_e^{\omega_q - 1}\left(E_t^C\right)^{\omega_q}\right]^{\frac{1}{\omega_q}}$$

$$NT_t = \left[\nu_c^{\omega_{nt}-1}\left(C_t^{dC}\right)^{\omega_{nt}} + \nu_x^{\omega_{nt}-1}\left(X_t^{dC}\right)^{\omega_{nt}} + \nu_{dis}^{\omega_{nt}-1}\left(DIS_t^C\right)^{\omega_{nt}}\right]^{\frac{1}{\omega_{nt}}}$$

and the first order conditions are given by

$$\frac{p_t^{cdC}}{p_t^{qF}} = \left(\frac{\nu_{nt}nt_t}{q_t^F}\right)^{\omega_q-1}\left(\frac{\nu_c c_t^{dC}}{nt_t}\right)^{\omega_{nt}-1} \tag{5.12}$$

$$\frac{p_t^{xdC}}{p_t^{qF}} = \left(\frac{\nu_{nt}nt_t}{q_t^F}\right)^{\omega_q-1}\left(\frac{\nu_x x_t^{dC}}{nt_t}\right)^{\omega_{nt}-1} \tag{5.13}$$

$$\frac{p_t^{disC}}{p_t^{qF}} = \left(\frac{\nu_{nt}nt_t}{q_t^F}\right)^{\omega_q-1}\left(\frac{\nu_{dis} dis_t^{C}}{nt_t}\right)^{\omega_{nt}-1} \tag{5.14}$$

$$\frac{p_t^{eC}}{p_t^{qF}} = \nu_e^{\omega_q-1}\left(\frac{e_t^C}{q_t^F}\right)^{\omega_q-1} \tag{5.15}$$

equation (5.12) to (5.15) are supply functions.

5.4 Distribution of manufactured goods

In every economy, there is a sector which takes finished manufactured goods and brings them to the consumer. This sector combines retailing, marketing and transport. The role of this sector is becoming ever important as goods acquire the essential characteristic of services, which is to be designed for each consumer. The economics of this sector has a very important effect on final consumer prices, which is often referred to as distributor's margins. For example, they play a role in shaping the pass-through of exchange rate changes into the economy (see Campa and Goldberg (2008)). In this section we describe how the output of that sector is produced, and then used to transform the raw outputs of other sectors so that they are ready for final or intermediate uses.

5.4.1 Distributing firms

There is a continuum of distributing firms indexed by z who take the product from the transforming firm and shape it into an intermediate input that can be used to take other goods to their respective markets. They buy this good at a price p_t^{disC} and sell it at a price $p_t^{dis}(z)$. Their final output is $DIS_t(z)$.

[13]Note that the maximization problem is solved in levels and the first order conditions are presented in model units.

Profit maximization problem

These firms operate under monopolistic competition. Each firm z, as a producer of the differentiated intermediate good faces an downward sloping demand curve of the form

$$dis_t(z) = \left(\frac{p_t^{dis}(z)}{p_t^{disF}}\right)^{-\theta^{dis}} dis_t^F$$

where $dis_t^F = \left[\int_0^1 (dis_t(z))^{\frac{\theta^{dis}-1}{\theta^{dis}}} dz\right]^{\frac{\theta^{dis}}{\theta^{dis}-1}}$ is the final output of distribution, $p_t^{disF} = \left[\int_0^1 \left(p_t^{dis}(z)\right)^{1-\theta^{dis}} dz\right]^{\frac{1}{1-\theta^{dis}}}$ is the aggregate price of distribution services and θ^{dis} is the elasticity of substitution among the z firms.

In addition, we assume that nominal prices are determined as in the previous sections, with a probability $\left(1-\varepsilon^{dis}\right)$ firms receive a stochastic signal that tells them whether they can set their price optimally. If not the firm follows a rule of the form

$$p_t^{disrule}(z) = p_{t-1}^{dis}(z)\left(1+\pi_{t-1}^{disF}\right)^{\iota^{dis}}(1+\overline{\pi})^{1-\iota^{dis}} \tag{5.16}$$

as before ι^{dis} is the weight assigned to past inflation as opposed to the central bank target and π_t^{disF} is the inflation of the final output of distribution.

The problem of the z^{th} firm is to choose the output price $p_t^{dis}(z)$ to maximize the discounted sum of profits subject to its demand curve and to the price rule that has to follow if the firm is not allow to reset its price. The optimum price on receiving the signal obeys

$$\frac{p_t^{disopt}(z)}{p_t^{disF}} = \frac{\theta^{dis}}{\theta^{dis}-1}\frac{E_t\sum_{i=0}^{\infty}\left(\varepsilon^{dis}\right)^i \Delta_{t+i,t}\frac{A_{t+i}}{A_t}\left[\frac{\frac{p_{t+i}^{disC}}{p_{t+i}^{cF}}\left(\frac{p_{t+i}^{disF}}{p_t^{disF}}\right)^{\theta^{dis}} dis_{t+i}^F}{\left(\prod_{l=1}^{i}\left\{\left(1+\pi_{t-1+l}^{disF}\right)^{\iota^{dis}}\right\}(1+\overline{\pi})^{i\left(1-\iota^{dis}\right)}\right)^{\theta^{dis}}}\right]}{E_t\sum_{i=0}^{\infty}\left(\varepsilon^{dis}\right)^i \Delta_{t+i,t}\frac{A_{t+i}}{A_t}\left[\frac{\frac{p_{t+i}^{disF}}{p_{t+i}^{cF}}\left(\frac{p_{t+i}^{disF}}{p_t^{disF}}\right)^{\theta^{dis}-1} dis_{t+i}^F}{\left(\prod_{l=1}^{i}\left\{\left(1+\pi_{t-1+l}^{disF}\right)^{\iota^{dis}}\right\}(1+\overline{\pi})^{i\left(1-\iota^{dis}\right)}\right)^{\theta^{dis}-1}}\right]}. \tag{5.17}$$

From the price aggregation and the fact that a fraction ε^{dis} of firms follow an indexation rule and a fraction $\left(1-\varepsilon^{dis}\right)$ firms adjust the price optimally, we arrive at the following equation for the inflation of the distribution sector final output:

$$\left(1+\pi_t^{disF}\right) = \left[\left(1-\varepsilon^{dis}\right)\left(\frac{p_t^{disopt}}{p_t^{disF}}\right)^{1-\theta^{dis}}\left(1+\pi_t^{disF}\right)^{1-\theta^{dis}} + \varepsilon^{dis}\left[\left(1+\pi_{t-1}^{disF}\right)^{\iota^{dis}}(1+\overline{\pi})^{1-\iota^{dis}}\right]^{1-\theta^{dis}}\right]^{\frac{1}{1-\theta^{dis}}} \tag{5.18}$$

Profits and aggregation

The profits of each firm are:

$$\xi_t^{dis}(z) = \left(\frac{p_t^{dis}(z)}{p_t^{cF}}\right) dis_t(z) - \frac{p_t^{disC}}{p_t^{cF}} dis_t(z)$$

and after aggregating across all firms, the aggregate profit for the distribution sector is

$$\xi_t^{dis} = \frac{p_t^{disF} dis_t^F}{p_t^{cF}} - \frac{p_t^{disC}}{p_t^{cF}} dis_t^C.$$

The market equilibrium condition for the distribution sector is:

$$dis_t^C = ap_t^{dis} dis_t^F \tag{5.19}$$

where $ap_t^{dis} = \int \left(p_t^{dis}(z) / p_t^{disF}\right)^{-\theta^{dis}} dz$ is the price distortion, $dis_t^C = \int dis_t(z)\, dz$ is the total supply from the z firms and dis_t^F is the aggregate demand for distribution services, that must satisfy:

$$dis_t^F = dis_t^{cd} + dis_t^{xd} + dis_t^e + dis_t^m$$

where dis_t^{cd}, dis_t^{xd}, dis_t^e, dis_t^m are the distribution output for final domestic consumption, final domestic investment, final exports and final imports.

5.4.2 Final good producers

In this section we describe how the distribution sector transforms the domestically produced items into goods ready for final use. In what follows, J_t is a dummy variable which could refer to domestic consumption, C_t^d, domestic investment, X_t^d or exports, E_t. The superscript j represents cd, xd or e.

There is a continuum of firms, indexed by z, which produce $J_t(z)$ in monopolistic competition. They buy the raw good $J_t^C(z)$ from the transformation sector at a price p_t^{jC}, and combine it with the distribution sector's output to transform this good into a good ready for final use according to the production function

$$J_t(z) = \left[\left(\gamma^j\right)^{\frac{1}{\omega^j}} \left(J_t^C(z)\right)^{\frac{\omega^j - 1}{\omega^j}} + \left(1-\gamma^j\right)^{\frac{1}{\omega^j}} \left(DIS_t^j(z)\right)^{\frac{\omega^j - 1}{\omega^j}}\right]^{\frac{\omega^j}{\omega^j - 1}}$$

here ω^j represents the elasticity of substitution between distribution services output and other domestically produce good. γ^j is a participation coefficient.

These firms are in monopolistic competition and each period can be allowed to change their prices with a constant probability. They need to chose the demand for factors, and also the optimal price in the event that they are allowed to change.

Cost minimization problem

Given the output, the demand for factors are the first order conditions of the cost minimization problem:

$$\min_{J_t^C(z),\, DIS_t^j(z)} \quad \frac{p_t^{jC}}{p_t^j(z)} J_t^C + \frac{p_t^{disF}}{p_t^j(z)} DIS_t^j$$

$$\text{s.t} \quad J_t(z) \leq \left[\left(\gamma^j\right)^{\frac{1}{\omega^j}} \left(J_t^C(z)\right)^{\frac{\omega^j-1}{\omega^j}} + \left(1-\gamma^j\right)^{\frac{1}{\omega^j}} \left(DIS_t^j(z)\right)^{\frac{\omega^j-1}{\omega^j}} \right]^{\frac{\omega^j}{\omega^j-1}}$$

The first order conditions are:

$$\frac{p_t^{jC}}{p_t^{cF}} = \lambda_t^j(z) \left(\frac{\gamma^j j_t(z)}{j_t^C} \right)^{\frac{1}{\omega^j}}$$

$$\frac{p_t^{disF}}{p_t^{cF}} = \lambda_t^j(z) \left(\frac{\left(1-\gamma^j\right) j_t(z)}{dis_t^j} \right)^{\frac{1}{\omega^j}}$$

where $\lambda_t^j(z)$ is the marginal cost associated with producing the good $j_t(z)$. The marginal cost is a weighted average of the relative prices of inputs as shown in the following equation:

$$\lambda_t^j(z) = \lambda_t^j = \left[\gamma^j \left(\frac{p_t^{jC}}{p_t^{cF}} \right)^{1-\omega^j} + \left(1-\gamma^j\right) \left(\frac{p_t^{disF}}{p_t^{cF}} \right)^{1-\omega^j} \right]^{\frac{1}{1-\omega^j}}.$$

Profit maximization problem

We assume that each firm z faces a downward sloping demand curve of the form

$$j_t(z) = \left(\frac{p_t^j(z)}{p_t^{jF}} \right)^{-\theta^j} j_t^F$$

where θ^j represents the elasticity of substitution among the z varieties, $j_t^F = \left[\int_0^1 \left(j_t(z)\right)^{\frac{\theta^j-1}{\theta^j}} dz \right]^{\frac{\theta^j}{\theta^j-1}}$ and $p_t^{jF} = \left[\int_0^1 \left(p_t^j(z)\right)^{1-\theta^j} dz \right]^{\frac{1}{1-\theta^j}}$.

As in the rest of the paper, we assume a price setting structure as in Calvo (1983). With probability $\left(1-\varepsilon^j\right)$ firms receive a stochastic signal which lets them know if they can choose the price in an optimal way. If not, prices follow a rule of the form

$$p_t^{jrule}(z) = p_{t-1}^j(z)\left(1+\pi_{t-1}^{jF}\right)^{\iota^j}(1+\overline{\pi})^{1-\iota^j} \tag{5.20}$$

As before, ι^j represents the indexation to past inflation for each firm that produces the $j_t(z)$ good, and π_t^{jF} represents the final inflation of good j_t.

The firm z has the problem of choosing $p_t^j(z)$ to maximize his discounted future stream of profits given a demand function for variety z and Equation (5.20). The solution to this problem implies that

$$\frac{p_t^{jopt}(z)}{p_t^{jF}} = \frac{\theta^j}{\theta^j-1}\frac{E_t\sum_{i=0}^{\infty}\left(\varepsilon^j\right)^i\Delta_{t+i,t}\frac{A_{t+i}}{A_t}\left[\frac{\lambda_{t+i}^j\left(\frac{p_{t+i}^{jF}}{p_t^{jF}}\right)^{\theta^j}j_{t+i}^F}{\left(\prod_{l=1}^{i}\left\{\left(1+\pi_{t-1+l}^{jF}\right)^{\iota^j}\right\}(1+\overline{\pi})^{i\left(1-\iota^j\right)}\right)^{\theta^j}}\right]}{E_t\sum_{i=0}^{\infty}\left(\varepsilon^j\right)^i\Delta_{t+i,t}\frac{A_{t+i}}{A_t}\left[\frac{\frac{p_{t+i}^{jF}}{p_{t+i}^{cF}}\left(\frac{p_{t+i}^{jF}}{p_t^{jF}}\right)^{\theta^j-1}j_{t+i}^F}{\left(\prod_{l=1}^{i}\left\{\left(1+\pi_{t-1+l}^{jF}\right)^{\iota^j}\right\}(1+\overline{\pi})^{i\left(1-\iota^j\right)}\right)^{\theta^j-1}}\right]} \tag{5.21}$$

As in previous sections, the inflation rate for each j_t is defined through the following equation:

$$\left(1+\pi_t^{jF}\right) = \left[\left(1-\varepsilon^j\right)\left(\frac{p_t^{jopt}}{p_t^{jF}}\right)^{1-\theta^j}\left(1+\pi_t^{jF}\right)^{1-\theta^j} + \varepsilon^j\left[\left(1+\pi_{t-1}^{jF}\right)^{\iota^j}(1+\overline{\pi})^{1-\iota^j}\right]^{1-\theta^j}\right]^{\frac{1}{1-\theta^j}}. \tag{5.22}$$

Profits of each firm z are given by

$$\xi_t^j(z) = \frac{p_t^j(z)\,j_t(z)}{p_t^{cF}} - \lambda_t^j j_t(z)$$

and the aggregate profit of sector j_t is

$$\xi_t^j = \frac{p_t^{jF}}{p_t^{cF}}j_t^F - \lambda_t^j j_t^{CF}.$$

The market equilibrium condition for the j_t good is:

$$j_t^{CF} = ap_t^j j_t^F$$

where $ap_t^j = \int\left(p_t^j(z)/p_t^{jF}\right)^{-\theta^j}dz$ is the price distortion, $j_t^{CF} = \int j_t(z)\,dz$ is the total supply from all z firms in each j_t sector and j_t^F is the total demand for each j_t sector output . Equation (4.11) is the demand for final domestic consumption c_t^{dF}, Equation (5.36) is the demand for final domestic investment x_t^{dF}, and Equation (6.1) is the demand for final exports.

5.4.3 Importers of consumption and investment goods

Imports are transformed and made ready for final and intermediate consumption by a continuum of intermediary firms. There is a sector that buys raw imports on the world market, combines it with the distribution output to produce an imported good that is fit for local use. We assume that these firms operate in monopolistic competition and consequently they solve two problems: A cost minimization problem to obtain the demand for inputs and a profit maximization problem to choose its optimal prices, the probability of setting prices is as in Calvo (1983).

Cost minimization problem

The z^{th} firm produces imports $M_t(z)$ using the raw import $M_t^{\star}(z)$ and the distribution sector's output $DIS_t^m(z)$. The technology for producing $M_t(z)$ is

$$M_t(z) = \left[(\gamma^m)^{\frac{1}{\omega^m}} \left(DIS_t^m(z)\right)^{\frac{\omega^m-1}{\omega^m}} + (1-\gamma^m)^{\frac{1}{\omega^m}} \left(M_t^{\star}(z)\right)^{\frac{\omega^m-1}{\omega^m}}\right]^{\frac{\omega^m}{\omega^m-1}}$$

where ω^m represents the elasticity of substitution and γ^m defines the share of raw imports.

The intermediate firms determine their demand for inputs by solving the following problem:

$$\min_{DIS_t^m, M_t^E} \quad \frac{p_t^{disF}}{p_t^{mF}} DIS_t^m(z) + \frac{p_t^{mC}}{p_t^{mF}} M_t^{\star}(z)$$

$$\text{s.t} \quad M_t(z) \leq \left[(\gamma^m)^{\frac{1}{\omega^m}} \left(DIS_t^m(z)\right)^{\frac{\omega^m-1}{\omega^m}} + (1-\gamma^m)^{\frac{1}{\omega^m}} \left(M_t^{\star}(z)\right)^{\frac{\omega^m-1}{\omega^m}}\right]^{\frac{\omega^m}{\omega^m-1}}$$

The optimality conditions are:

$$\frac{p_t^{disF}}{p_t^{cF}} = \lambda_t^m(z) (\gamma^m)^{\frac{1}{\omega^m}} \left(\frac{m_t(z)}{dis_t^m(z)}\right)^{\frac{1}{\omega^m}} \tag{5.23}$$

and

$$\frac{p_t^{mC}}{p_t^{cF}} = \lambda_t^m(z) (1-\gamma^m)^{\frac{1}{\omega^m}} \left(\frac{m_t(z)}{m_t^{\star}(z)}\right)^{\frac{1}{\omega^m}} \tag{5.24}$$

where $\lambda_t^m(z)$ is the real marginal cost. As before $\lambda_t^m(z) = \lambda_t^m$ for all firms.

Profit maximization problem

In keeping with the literature on new open economy models, we assume that there is a degree of stickiness in local currency pricing. As ever, the stickiness is model of as in Calvo (1983). With probability $(1-\varepsilon^m)$ each firm receives a stochastic signal that it can choose its optimal price. If not it should follow the rule

$$p_t^{mrule}(z) = p_{t-1}^m(z)\left(1+\pi_{t-1}^{mF}\right)^{\iota^m}(1+\overline{\pi})^{1-\iota^m} \tag{5.25}$$

with ι^m defining the degree of indexation and π_t^{mF} the imports final inflation.

As before the z^{th} firm's problem when it receives the signal is to choose $p_t^m(z)$ to maximize its discounted profit stream:

$$\max_{p_t^m(z)} \qquad E_t\sum_{i=0}^{\infty}(\varepsilon^m)^i\,\Delta_{t+i,t}\left[\left(\frac{p_{t+i}^m(z)}{p_{t+i}^{mF}}\right)\frac{p_{t+i}^{mF}}{p_{t+i}^{cF}}\tilde{m}_{t+i}(z)-\tilde{\lambda}_{t+i}^m\tilde{m}_{t+i}(z)\right]$$

subject to the demand function, $m_t(z)=\left(\frac{p_t^m(z)}{p_t^{mF}}\right)^{-\theta^m}m_t^F$ and the price rule in Equation (5.25). In this case $m_t^F=\left[\int_0^1(m_t(z))^{\frac{\theta^m-1}{\theta^m}}dz\right]^{\frac{\theta^m}{\theta^m-1}}$ is the aggregate amount of imports adapted for domestic use, $p_t^{mF}=\left[\int_0^1(p_t^m(z))^{1-\theta^m}dz\right]^{\frac{1}{1-\theta^m}}$ is the aggregate price of imported goods and θ^m is the elasticity of substitution among z varieties.

The optimal price is determined by:

$$\frac{p_t^{mopt}(z)}{p_t^{mF}}=\frac{\theta^m}{\theta^m-1}\frac{E_t\sum\limits_{i=0}^{\infty}(\varepsilon^m)^i\,\Delta_{t+i,t}\frac{A_{t+i}}{A_t}\left[\frac{\lambda_{t+i}^m(z)\left(\frac{p_{t+i}^{mF}}{p_t^{mF}}\right)^{\theta^m}m_{t+i}^F}{\left(\prod\limits_{l=1}^{i}\left\{\left(1+\pi_{t-1+l}^{mF}\right)^{\iota^m}\right\}(1+\overline{\pi})^{i(1-\iota^m)}\right)^{\theta^m}}\right]}{E_t\sum\limits_{i=0}^{\infty}(\varepsilon^m)^i\,\Delta_{t+i,t}\frac{A_{t+i}}{A_t}\left[\frac{\frac{p_{t+i}^{mF}}{p_{t+i}^{cF}}\left(\frac{p_{t+i}^{mF}}{p_t^{mF}}\right)^{\theta^m-1}m_{t+i}^F}{\left(\prod\limits_{l=1}^{i}\left\{\left(1+\pi_{t-1+l}^{mF}\right)^{\iota^m}\right\}(1+\overline{\pi})^{i(1-\iota^m)}\right)^{\theta^m-1}}\right]} \tag{5.26}$$

using the aggregate price for imported goods and the fact that a fraction $1-\varepsilon^m$ of firms choose the price optimally, the aggregate price in the sector is

$$p_t^{mF} = \left[(1-\varepsilon^m)\left(p_t^{mopt}\right)^{1-\theta^m}+\varepsilon^m\left[p_{t-1}^{mF}\left(1+\pi_{t-1}^{mF}\right)^{\iota^m}(1+\overline{\pi})^{1-\iota^m}\right]^{1-\theta^m}\right]^{\frac{1}{1-\theta^m}},$$

and the inflation for the final imported good is

$$\left(1+\pi_t^{mF}\right)=\left[\left(1-\varepsilon^m\right)\left(\frac{p_t^{mopt}}{p_t^{mF}}\right)^{1-\theta^m}\left(1+\pi_t^{mF}\right)^{1-\theta^m}+\varepsilon^m\left[\left(1+\pi_{t-1}^{mF}\right)^{\iota^m}\left(1+\overline{\pi}\right)^{1-\iota^m}\right]^{1-\theta^m}\right]^{\frac{1}{1-\theta^m}} \tag{5.27}$$

Note that the inflation rate in this sector depends directly on the exchange rate through the price of the imported goods. In fact, using the purchasing power parity condition, $p_t^{mC}=s_t p_t^{m\star}$, where $p_t^{m\star}$ is the external price of imports, we have that imported inflation is

$$\left(1+\pi_t^{mC}\right)=\left(1+d_t\right)\left(1+\pi_t^{m\star}\right) \tag{5.28}$$

where $\pi_t^{m\star}$ is the inflation rate of imports in foreign currency that follows the exogenous process

$$\ln\pi_t^{m\star}=\rho_{\pi^{m\star}}\ln\pi_{t-1}^{m\star}+\left(1-\rho_{\pi^{m\star}}\right)\ln\overline{\pi}^{m\star}+\epsilon_t^{\pi^{m\star}}$$

where $\overline{\pi}^{m\star}$ is the mean of the foreign inflation, $0<\rho_{\pi^{m\star}}<1$, and $\epsilon_t^{\pi^{m\star}}\sim n\left(0,\sigma^{\pi^{m\star}}\right)$.

Profits and aggregation

The profits of the z^{th} firm are given by $\xi_t^m(z)=\frac{p_t^m(z)m_t(z)}{p_t^{cF}}-\lambda_t^m m_t(z)$, and the total profits of the sector are

$$\xi_t^m=\frac{p_t^{mF}}{p_t^{cF}}m_t^F-\lambda_t^m m_t^C.$$

As before, the market equilibrium is

$$m_t^C=ap_t^m m_t^F \tag{5.29}$$

where, $ap_t^m=\int\left(p_t^m(z)/p_t^{mF}\right)^{-\theta^m}dz$, is the price distortion, $m_t^C=\int m_t(z)\,dz$ is the total supply of imports and m_t^F is the total demand for imports, this demand satisfy

$$c_t^{mF}+x_t^{mF}=m_t^F. \tag{5.30}$$

5.5 Raw materials importers

There is a continuum of firms indexed by z, these firms operate under monopolistic competition. These firms import raw materials and sell them to domestic producers. They buy a quantity, $RM_t^\star$, of raw materials at the port price, p_t^{rmC}, and transform them into a differentiated raw material, $RM_t(z)$, without cost. These firms face a negative slope demand curve for its product

$$rm_t(z) = \left(\frac{p_t^{rm}(z)}{p_t^{rmF}}\right)^{-\theta^{rm}} rm_t^F$$

where θ^{rm} is the elasticity of substitution among firms, $rm_t^F = \left[\int_0^1 (rm_t(z))^{\frac{\theta^{rm}-1}{\theta^{rm}}} dz\right]^{\frac{\theta^{rm}}{\theta^{rm}-1}}$ is the imported good adapted for aggregate use and $p_t^{rmF} = \left[\int_0^1 (p_t^{rm}(z))^{1-\theta^{rm}} dz\right]^{\frac{1}{1-\theta^{rm}}}$ is the aggregate price of raw materials adapted for production. In this case the raw materials price, p_t^{rmF}, already includes the distribution cost, that is, the distribution of raw materials is not explicitly model[14].

As for the importers of goods for consumption and investment, these firms receive a signal that allows them to adjust optimally their prices, otherwise prices are set by a rule

$$p_t^{rmrule}(z) = p_{t-1}^{rm}(z)\left(1+\pi_{t-1}^{rmF}\right)^{\iota^{rm}}(1+\overline{\pi})^{1-\iota^{rm}} \tag{5.31}$$

where ι^{rm} represents the indexation to past inflation and π_t^{rmF} the raw materials inflation. When the signal is received, the z^{th} firm's problem is to choose the price $p_t^{rmopt}(z)$ to maximize the discounted stream of real profits subject to the demand for its product and subject to the price rule (Equation (5.31)). In this case the first order condition for the optimal price is given by

$$\frac{p_t^{rmopt}(z)}{p_t^{rmF}} = \frac{\theta^{rm}}{\theta^{rm}-1}\frac{E_t\sum_{i=0}^{\infty}(\varepsilon^{rm})^i\Delta_{t+i,t}\frac{A_{t+i}}{A_t}\left[\frac{\frac{p_{t+i}^{rmC}}{p_{t+i}^{cF}}\left(\frac{p_{t+i}^{rmF}}{p_t^{rmF}}\right)^{\theta^{rm}}rm_{t+i}^F}{\left(\prod_{l=1}^{i}\left\{\left(1+\pi_{t-1+l}^{rmF}\right)^{\iota^{rm}}\right\}(1+\overline{\pi})^{i(1-\iota^{rm})}\right)^{\theta^{rm}}}\right]}{E_t\sum_{i=0}^{\infty}(\varepsilon^{rm})^i\Delta_{t+i,t}\frac{A_{t+i}}{A_t}\left[\frac{\frac{p_{t+i}^{rmF}}{p_{t+i}^{cF}}\left(\frac{p_{t+i}^{rmF}}{p_t^{rmF}}\right)^{\theta^{rm}-1}rm_{t+i}^F}{\left(\prod_{l=1}^{i}\left\{\left(1+\pi_{t-1+l}^{rmF}\right)^{\iota^{rm}}\right\}(1+\overline{\pi})^{i(1-\iota^{rm})}\right)^{\theta^{rm}-1}}\right]}. \tag{5.32}$$

The aggregate raw materials inflation is defined as

$$\left(1+\pi_t^{rmF}\right) = \left[(1-\varepsilon^{rm})\left(\frac{p_t^{rmopt}}{p_t^{rmF}}\right)^{1-\theta^{rm}}\left(1+\pi_t^{rmF}\right)^{1-\theta^{rm}} + \varepsilon^{rm}\left[\left(1+\pi_{t-1}^{rmF}\right)^{\iota^{rm}}(1+\overline{\pi})^{1-\iota^{rm}}\right]^{1-\theta^{rm}}\right]^{\frac{1}{1-\theta^{rm}}}. \tag{5.33}$$

Note that the raw materials inflation rate depends on the nominal exchange rate through the price of imported raw materials. This comes from the fact that we assume that purchasing power parity holds, that is, $p_t^{rmC} = s_t p_t^{rm\star}$, where $p_t^{rm\star}$ is the external price of raw materials. Therefore, the inflation rate of raw

[14]We decided not to model the distribution of imported raw materials because there is no statistical information about this in the Colombian national accounts.

materials at the port is

$$\left(1+\pi_t^{rmC}\right) = \left(1+d_t\right)\left(1+\pi_t^{rm\star}\right) \tag{5.34}$$

where $\pi_t^{rm\star}$ is the raw material inflation in foreign currency, which we assume exogenous.

$$\ln \pi_t^{rm\star} = \rho_{\pi^{rm\star}} \ln \pi_{t-1}^{rm\star} + \left(1-\rho_{\pi^{rm\star}}\right) \ln \overline{\pi}^{rm\star} + \epsilon_t^{\pi^{rm\star}}$$

where $\overline{\pi}^{rm\star}$ is the mean of the exogenous process , $0 < \rho_{\pi^{rm\star}} < 1$, and $\epsilon_t^{\pi^{rm\star}} \sim \left(0, \sigma^{\pi^{rm\star}}\right)$.

Profits and aggregation

The profit of each raw materials importer is:

$$\xi_t^{rm}(z) = \left(\frac{p_t^{rm}(z)}{p_t^{cF}}\right) rm_t(z) - \frac{p_t^{rmC}}{p_t^{cF}} rm_t(z)$$

and integrating over firms we have the aggregate profit for the raw materials importers sector

$$\xi_t^{rm} = \frac{p_t^{rmF} rm_t^F}{p_t^{cF}} - \frac{p_t^{rmC}}{p_t^{cF}} rm_t^C.$$

The market equilibrium is

$$rm_t^C = ap_t^{rm} rm_t^F$$

where, as before, the price distortion is $ap_t^{rm} = \int \left(p_t^{rm}(z)/p_t^{rmF}\right)^{-\theta^{rm}} dz$. The total demand for raw materials, rm_t^F, comes from the aggregation of the raw good producer (equation (5.1)) [15]

$$rm_t^F = \left(\lambda_t^q z_t^q\right)^{\rho} \left(\frac{p_t^{rmF}}{p_t^{cF}}\right)^{-\rho} \frac{(1-\alpha)\, q_t^C}{z_t^q}.$$

and the total supply of raw materials, $rm_t^C = \int rm_t(z)\, dz$, comes from the aggregation over the raw materials importers.

5.6 Investment producers

As we have seen investment goods can be produced abroad and domestically. Both are important in explaining Colombian growth and fluctuations. An example of the former would be machinery equipment, an example of

[15] In the aggregation across all the raw good producers it must be that $\frac{q_t}{rm_t}$ is equal for all firms.

the latter would be infrastructure. These two types of capital are aggregated to produce combined investment. This then enables us to work with an aggregate capital stock in production. The technology for combining the two types of capital is well described by:

$$x_t^F = z_t^x \left[(\gamma^x)^{\frac{1}{\omega^x}} \left(x_t^{dF} \right)^{\frac{\omega^x - 1}{\omega^x}} + (1-\gamma^x)^{\frac{1}{\omega^x}} \left(x_t^{mF} \right)^{\frac{\omega^x - 1}{\omega^x}} \right]^{\frac{\omega^x}{\omega^x - 1}} \tag{5.35}$$

where x_t^F is final investment, x_t^{dF} is the input of domestically produced investment, x_t^{mF} is the input of imported and distributed investment, ω^x is the elasticity of substitution between domestically and imported investment, γ^x define the share of domestic investment into total investment and z_t^x is an investment efficiency shock

$$\ln z_t^x = \rho_{z^x} \ln z_{t-1}^x + (1-\rho_{z^x}) \ln \overline{z}^x + \epsilon_t^{z^x}$$

where $\overline{z}^x$ is the mean of the exogenous process , $0 < \rho_{z^x} < 1$, and $\epsilon_t^{z^x} \sim n\left(0, \sigma^{z^x}\right)$.

The firms which carry out this aggregation operate under perfect competition. Hence they solve the following problem to find the final demand for x_t^{dF} and x_t^{mF}.

$$\begin{aligned} \max_{\left\{x_t^{dF},\, x_t^{mF}\right\}} \quad & \left(p_t^{xF} X_t^F - p_t^{xdF} X_t^{dF} - p_t^{mF} X_t^{mF} \right) \\ \text{s.t} \quad & x_t^F \leq z_t^x \left[(\gamma^x)^{\frac{1}{\omega^x}} \left(x_t^{dF} \right)^{\frac{\omega^x - 1}{\omega^x}} + (1-\gamma^x)^{\frac{1}{\omega^x}} \left(x_t^{mF} \right)^{\frac{\omega^x - 1}{\omega^x}} \right]^{\frac{\omega^x}{\omega^x - 1}} \end{aligned}$$

The expressions for these demands are:

$$x_t^{dF} = (\gamma^x) \left(\frac{p_t^{xdF}}{z_t^x p_t^{xF}} \right)^{-\omega^x} \frac{x_t^F}{z_t^x} \tag{5.36}$$

$$x_t^{mF} = (1-\gamma^x) \left(\frac{p_t^{mF}}{z_t^x p_t^{xF}} \right)^{-\omega^x} \frac{x_t^F}{z_t^x} \tag{5.37}$$

Combining Equations (5.36), (5.37) and (5.35), we obtain an expression for the price of the final investment good

$$p_t^{xF} = \frac{1}{z_t^x} \left[(\gamma^x) \left(p_t^{xdF} \right)^{1-\omega^x} + (1-\gamma^x) \left(p_t^{xmF} \right)^{1-\omega^x} \right]^{\frac{1}{1-\omega^x}} . \tag{5.38}$$

6 Foreign variables

It is assumed that exports E_t^F are demanded according to following function

$$e_t^F = \left(\frac{p_t^{e\star}}{p_t^{c\star}}\right)^{-\mu} c_t^\star \tag{6.1}$$

where $c_t^\star$ is the external demand, that follows the autoregressive process

$$\ln c_t^\star = \rho_{c^\star} \ln c_{t-1}^\star + (1-\rho_{c^\star}) \ln \overline{c}^\star + \epsilon_t^{c^\star}$$

where $\overline{c}^\star$ is the mean of the exogenous process , $0 < \rho_{c^\star} < 1$, and $\epsilon_t^{c^\star} \sim n\left(0, \sigma^{c^\star}\right)$.

$p_t^{e\star}$ is the foreign currency price of Colombian exports and is determined in the world market, that is, $p_t^{eF} = s_t p_t^{e\star}$, and consequently, exports inflation rate is define as

$$\left(1+\pi_t^{eF}\right) = (1+d_t)\left(1+\pi_t^{e\star}\right). \tag{6.2}$$

The ratio $-\frac{1}{\mu}$ is the slope of the demand for Colombian exports.

Following Schmitt-Grohe and Uribe (2003) we impose a further condition to ensure that external debt converges to a predetermined ration with GDP. Let us assume that the external sector offers resources at a rate $i_t^\star$ which depends on the deviation of debt from this target ratio. In that case, we define the rate of interest as:

$$i_t^\star = \overline{i}^\star z_t^{i\star} \exp\left(\Omega_u \left(\frac{s_t p_t^{c\star}}{p_t^{cF}} \frac{b_t^\star}{y_t} - \overline{b^\star}\right)\right) \tag{6.3}$$

where $\overline{i}^\star$ is the nominal risk free international interest rate, $\frac{s_t p_t^{c\star}}{p_t^{cF}} \frac{b_t^\star}{y_t}$ is the external debt to GDP ratio in the same currency, $\overline{b^\star}$ is its target value and $\Omega_u > 0$ is a parameter that affects the model's dynamics, $z_t^{i\star}$ is a risk premium shock which follows an exogenous process

$$\ln z_t^{i\star} = \rho_{z^{i\star}} \ln z_{t-1}^{i\star} + (1-\rho_{z^{i\star}}) \ln \overline{z}_t^{i\star} + \epsilon_t^{z^{i\star}}.$$

$\overline{z}_t^{i\star}$ is the mean of the exogenous process , $0 < \rho_{z^{i\star}} < 1$, and $\epsilon_t^{z^{i\star}} \sim n\left(0, \sigma^{z^{i\star}}\right)$.

7 Monetary policy

Monetary policy in the model follows the simple rule

$$i_t = \rho_s i_{t-1} + (1-\rho_s)\left(\bar{i} + \varphi_\pi\left(\pi_t^{cF} - \overline{\pi}\right)\right) + \varphi_y\left(\frac{y_t}{y_t^{flex}} - 1\right) + z_t^i \tag{7.1}$$

where ρ_s is the smoothing coefficient, $\bar{i}$ is the steady state value for the nominal interest rate, φ_π determines the response to deviations in the inflation from its target, y_t^{flex} is the level of output if prices were flexible, and φ_y is the response to deviations of GDP from its flexible prices value. Deviations from the observed nominal interest rate and the nominal interest rate dictated for this rule are define by an exogenous process z_t^i.

$$\ln z_t^i = \rho_{z^i} \ln z_{t-1}^i + (1-\rho_{z^i}) \ln \overline{z}_t^i + \epsilon_t^{z^i}$$

where $\overline{z}_t^i$ is the mean of the exogenous process , $0 < \rho_{z^i} < 1$, and $\epsilon_t^{z^i} \sim n\left(0, \sigma^{z^i}\right)$

8 National Accounts

From the aggregate budget constraint of the households (equation (4.9)), follows the balance of payments identity as[16]:

$$\begin{aligned} &\frac{s_t p_t^{c\star}}{p_t^{cF}} \frac{b_{t-1}^\star}{(1+\overline{n})(1+g_t)} \frac{1+i_{t-1}^\star}{1+\pi_t^{c\star}} - \frac{s_t p_t^{c\star}}{p_t^{cF}} b_t^\star = \\ &\frac{p_t^{eF}}{p_t^{cF}} e_t^F - \frac{p_t^{mC}}{p_t^{cF}} m_t^E - \frac{p_t^{rmC}}{p_t^{cF}} rm_t^C + \frac{s_t p_t^{c\star}}{p_t^{cF}} tr_t^\star - \Psi^X\left(x_t^F, x_{t-1}^F\right) \end{aligned} \tag{8.1}$$

Defining $\tilde{y}_t \equiv \frac{1}{p_t^{cF}} \frac{Y_t}{A_t N_t}$, we can express real GDP as:

$$y_t = c_t^F + \frac{p_t^{xF}}{p_t^{cF}} x_t^F + \frac{p_t^{eF}}{p_t^{cF}} e_t^F - \frac{p_t^{mC}}{p_t^{cF}} m_t^E - \frac{p_t^{rmC}}{p_t^{cF}} rm_t^C. \tag{8.2}$$

9 Conclusions and further research

In this document we described the structure of PATACON, a DSGE model for the Colombian economy. PATACON was designed to be useful for analyzing the Colombian macroeconomic data and to help guide the monetary policy discussion. However, in its design we ignored three characteristics of the Colombian

[16]The derivation is presented in Appendix C.

economy that may be important. Currently, the Department of Macroeconomic Modeling at Banco de la República is continuously working on these and other related issues.

The first omission of PATACON is that it does not incorporate frictions in the labour market and consequently it cannot explain the dynamics of employment and unemployment over the business cycle. A first attempt to incorporate these features in a DSGE model for the Colombian economy can be found in González, Ocampo, Rodríguez, and Rodríguez (2011). The second aspect, in which we are working on, is in the introduction of a financial sector to understand its role in the economic business cycle. Some work has also being done in this respect. See, for example, López and Rodríguez (2008), López, Prada, and Rodríguez (2009) , López and Prada (2010) or Bustamante (2011). The third modification is to include explicitly the government within the model.

Finally, the in-house construction and design of a general equilibrium model creates spill overs that are also valuable. Indeed, the design and implementation of PATACON has created a need to improve the understanding of some macroeconomic phenomena and to develop econometric techniques useful for its implementation and suitable for routine use. Members of the Department of Macroeconomic Modelling have responded to these needs producing several academics works that include a descriptive analysis of the Colombian business cycle, the development of a database consistent with a general equilibrium model, a measure of the pass-through effect of the exchange rate to the consumer price inflation, the implementations of a numerical algorithm useful for calibrating the model steady-state, an estimate of the Frisch elasticity[17], and an estimation of the relative importance of various nominal and real rigidities within a general equilibrium model for the Colombian economy[18] .

References

Adolfson, M., S. Laséen, J. Lindé, and M. Villani (2008): "Evaluating an estimated new Keynesian small open economy model," *Journal of Economic Dynamics and Control*, 32(8), 2690–2721.

Bonaldi, P., A. González, J. D. Prada, D. Rodríguez, and L. E. Rojas (2009): "Método numérico para la calibración de un modelo DSGE," Borradores de Economía 548, Banco de la República.

Bonaldi, P., A. González, and D. Rodríguez (2010): "Importancia de las rigideces nominales y reales

[17] This elasticity measures how much the labour supply changes after a change in the real wage keeping all other factor unchanged

[18] Mahadeva and Parra (2008); Parra (2008); Bonaldi, González, Prada, Rodríguez, and Rojas (2009); González, Rincón, and Rodríguez (2010); Prada and Rojas (2010); Bonaldi, González, and Rodríguez (2010).

en Colombia: un enfoque de equilibrio general dinámico y estocástico," Borradores de Economía 591, Banco de la República.

——— (2011): "Estimating PATACON," Discussion paper, Banco de la República.

Bruno, M., and J. Sachs (1985): *Economics of Worldwide Stagflation*. Harvard University Press.

Burriel, Pablo, F.-V. J., and J. F. Rubio-Ramirez (2009): "MEDEA: A DSGE Model for the Spanish Economy," PIER Working Paper 09-017.

Bustamante, C. (2011): "Política monetaria contracíclica y encaje bancario," Borradores de economía 646, Banco de la República.

Calvo, G. A. (1983): "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, 12(3), 383–398.

Campa, J. M., and L. S. Goldberg (2008): "Pass-Through of Exchange Rates to Consumption Prices: What Has Changed and Why?," in *International Financial Issues in the Pacific Rim: Global Imbalances, Financial Liberalization, and Exchange Rate Policy (NBER-EASE Volume 17)*, NBER Chapters, pp. 139–176. National Bureau of Economic Research, Inc.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

Christiano, L. J., M. Trabandt, and K. Walentin (2007): "Introducing Financial Frictions and Unemployment into a Small Open Economy Model," Working Paper Series 214, Sveriges Riksbank (Central Bank of Sweden).

Edwards, S., and C. Végh (1997): "Banks and macroeconomic disturbances under predetermined exchange rate," *Journal of Monetary Economics*, 40, 239–278.

Erceg, C. J., D. W. Henderson, and A. T. Levin (2000): "Optimal monetary policy with staggered wage and price contracts," *Journal of Monetary Economics*, 46(2), 281–313.

González, A., L. Mahadeva, D. Rodríguez, and L. E. Rojas (2009): "Monetary Policy Forecasting in a DSGE Model with Data that is Uncertain, Unbalanced and About the Future," Borradores de Economía 559, Banco de la República.

González, A., S. Ocampo, D. Rodríguez, and N. Rodríguez (2011): "Asimetrías del empleo y el producto, una aproximación de equilibrio general," Discussion paper, Banco de la República.

González, A., H. Rincón, and N. Rodríguez (2010): *La transmisión de los choques a la tasa de cambio sobre la inflación de los bienes importados en presencia de asimetrías,*chap. 10, Mecanismos de transmisión de la política monetaria en Colombia. Banco de la República y Universidad Externado de Colombia.

Hofstetter, M. (2010): "Sticky prices and moderate inflation," *Economic Modelling*, 27(2), 535–546.

Iregui, A. M., L. A. Melo, and M. T. Ramírez (2009): "Are wages rigid in Colombia?: Empirical evidence based on a sample of wages at the firm level," Borradores de Economia 571, Banco de la República.

Julio, J. M. (2010): "Heterogeneidad observada y no observada en la formación de los precios del IPC Colombiano," Borradores de Economia 597, Banco de la República.

Julio, J. M., H. M. Zárate, and M. D. Hernández (2009): "The Stickiness of Colombian Consumer Prices," Borradores de Economia 578, Banco de la República.

King, R. G., C. I. Plosser, and S. T. Rebelo (1988): "Production, Growth and Business Cycles," *Journal of Monetary Economics*, 21, 195–232.

López, M. R., and J. D. Prada (2010): "Optimal monetary policy and asset prices: the case of Colombia," *Ensayos sobre política económica*, 28(61).

López, M. R., J. D. Prada, and N. Rodríguez (2009): "Evidence for a Financial Accelerator in a Small Open Economy, and implications for monetary policy," *Ensayos sobre política económica*, 27(60).

López, M. R., and N. Rodríguez (2008): "Financial Accelerator Mechanism: Evidence for Colombia," Borradores de Economía 481, Banco de la República.

Mahadeva, L., and J. G. Gómez (2010): *Los factores externos que afectan la política monetaria colombiana,*chap. 1, Mecanismos de transmisión de la política monetaria en Colombia. Banco de la República y Universidad Externado de Colombia.

Mahadeva, L., and J. C. Parra (2008): "Testing a DSGE model and its partner database," Borradores de Economía 478, Banco de la República.

Misas, M., E. López, and J. C. Parra (2009): "La formación de precios en las empresas colombianas: evidencia a partir de una encuesta directa," Borradores de Economia 569, Banco de la República.

Parra, J. (2010): *La sensibilidad de los precios del consumidor a la tasa de cambio en Colombia: una medición de largo plazo,*chap. 9, Mecanismos de transmisión de la política monetaria en Colombia. Banco de la República y Universidad Externado de Colombia.

Parra, J. C. (2008): "Hechos estilizados de la economía colombiana: fundamentos empíricos para la construcción y evaluación de un modelo DSGE," Borradores de Economía 509, Banco de la República.

Prada, J. D., and L. E. Rojas (2010): *La elasticidad de Frisch y la transmisión de la política monetaria en Colombia,*chap. 13. Banco de la República y Universidad Externado de Colombia.

Romer, D. (2005): *Advanced Macroeconomics*. McGraw-Hill/Irwin, 3rd edn.

Schmitt-Grohe, S., and M. Uribe (2003): "Closing small open economy models," *Journal of International Economics*, 61(1), 163–185.

Smets, F., and R. Wouters (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

Yun, T. (1996): "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," *Journal of Monetary Economics*, 37(8), 345–370.

A Variables

Symbol	**Description**
Real quantities	
c^F	Consumption bundle
c^{dF}	Final domestic consumption (demand)
c^{dC}	Intermediate domestic consumption
c^{dCF}	Final domestic consumption (supply)
c^{mF}	Final imported consumption
x^F	Final investment
x^{dF}	Final domestic investment (demand)
x^{dC}	Intermediate domestic investment
x^{dCF}	Final domestic investment (supply)
x^{mF}	Final imported investment
m^F	Final imports (demand)
$m^{\star}$	Imports at dock
continue...	

Symbol	Description
m^C	Final imports (supply)
e^F	Final exports (demand)
e^C	Intermediate exports
e^{CF}	Final exports (supply)
λ^x	Multiplier for capital accumulation equation
λ^c	Multiplier for budget constraint
h^F	Labour
k^s	Capital supply
k	Total capital
u	Variable capital utilization
$\delta(u)$	Endogenous depreciation
rm^F	Final raw materials (demand)
rm^C	Final raw materials (supply)
y	Real gross domestic product
q^F	Gross output (demand)
nt	Domestic uses of output
q^C	Gross output (supply)
va	Value added
dis^F	Final distribution services (supply)
dis^{cd}	Distribution services used for domestic consumption
dis^{xd}	Distribution services used for domestic investment
dis^e	Distribution services used for exports
dis^m	Distribution services used for imports
dis^C	Final distribution services (demand)
$b^\star$	External debt
Interest rates	
i_t	Nominal interest rate
$i_t^\star$	External nominal interest rate
Inflation rates and nominal devaluation	
continue...	

Symbol	Description
π_t^{cF}	Consumption bundle inflation
π_t^{cdF}	Domestic consumption inflation
π_t^{eF}	Exports inflation
π_t^{mF}	Imported goods inflation
π_t^{mC}	Imported goods inflation at dock
π_t^{qF}	Homogeneous good inflation
π_t^{rmF}	Raw materials inflation
π_t^{disF}	Distribution services inflation
π_t^{xF}	Investment inflation
π_t^{xdF}	Domestic investment inflation
d_t	Nominal devaluation
Marginal cost	
λ^q	Marginal cost of producing good q
λ^{cd}	Marginal cost of producing good cd
λ^e	Marginal cost of producing good e
λ^m	Marginal cost of producing good m
λ^{rm}	Marginal cost of producing good rm
λ^{xd}	Marginal cost of producing good xd
Relative prices	
$\frac{p_t^{cdF}}{p_t^{cF}}$	Final domestic consumption / Consumption bundle
$\frac{p_t^{cdF}}{p_t^{qF}}$	Final domestic consumption / Gross product
$\frac{p_t^{cdC}}{p_t^{qF}}$	Intermediate domestic consumption / Gross product
$\frac{p_t^{eF}}{p_t^{cF}}$	Final exports / Consumption bundle
$\frac{p_t^{eF}}{p_t^{qF}}$	Final exports / Gross product
$\frac{p_t^{eC}}{p_t^{qF}}$	Intermediate exports / Gross product
$\frac{p_t^{mF}}{p_t^{cF}}$	Final imports / Consumption bundle
$\frac{p_t^{mF}}{p_t^{xF}}$	Final imports / Final investment
$\frac{p_t^{mC}}{p_t^{cF}}$	Intermediate imports / Consumption bundle
$\frac{p_t^{mC}}{p_t^{mF}}$	Intermediate imports / Final investment
continue...	

Symbol	Description
$\frac{p_t^{rmC}}{p_t^{cF}}$	Intermediate raw materials / Consumption bundle
$\frac{p_t^{rmC}}{p_t^{rmF}}$	Intermediate raw materials / Final raw materials
$\frac{p_t^{rmF}}{p_t^{cF}}$	Final raw materials / Consumption bundle
$\frac{p_t^{qF}}{p_t^{cF}}$	Gross output / Consumption bundle
$\frac{p_t^{disF}}{p_t^{cF}}$	Final distribution services / Consumption bundle
$\frac{p_t^{disF}}{p_t^{mF}}$	Final distribution services / Final imports
$\frac{p_t^{disF}}{p_t^{qF}}$	Final distribution services / Gross product
$\frac{p_t^{disC}}{p_t^{qF}}$	Intermediate distribution services / Gross product
$\frac{p_t^{xF}}{p_t^{cF}}$	Final investment / Consumption bundle
$\frac{p_t^{xdF}}{p_t^{cF}}$	Final domestic investment / Consumption bundle
$\frac{p_t^{xdF}}{p_t^{cdF}}$	Final domestic investment / Final domestic consumption
$\frac{p_t^{xdF}}{p_t^{mF}}$	Final domestic investment / Final imports
$\frac{p_t^{xdF}}{p_t^{qF}}$	Final domestic investment / Gross product
$\frac{p_t^{xdF}}{p_t^{xF}}$	Final domestic investment / Final investment
$\frac{p_t^{xdC}}{p_t^{qF}}$	Intermediate domestic investment / Gross product
$\frac{p_t^{e\star}}{p_t^{c\star}}$	Foreign currency price of exports / Foreign consumer price index
$\frac{s_t p_t^{c\star}}{p_t^{cF}}$	Real exchange rate
$\frac{p_t^{m\star}}{p_t^{c\star}}$	Foreign currency price of imports / Foreign consumer price index
$\frac{p_t^{rm\star}}{p_t^{c\star}}$	Foreign currency price of raw materials / Foreign consumer price index
w	Real wage
r^k	Real rent of capital
Optimal prices and wage	
w^{opt}	Optimal real wage
$\frac{p_t^{qopt}}{p_t^{qF}}$	Optimal price of gross output q
$\frac{p_t^{cdopt}}{p_t^{cdF}}$	Optimal price of domestic consumption goods cd
$\frac{p_t^{eopt}}{p_t^{eF}}$	Optimal price of exports e
$\frac{p_t^{mopt}}{p_t^{mF}}$	Optimal price of imports m
$\frac{p_t^{rmopt}}{p_t^{rmF}}$	Optimal price of raw materials rm
$\frac{p_t^{disopt}}{p_t^{disF}}$	Optimal price of distribution services dis
continue...	

Symbol	Description
$\frac{p_t^{xdopt}}{p_t^{xdF}}$	Optimal price of domestic investment goods xd
Profits	
ξ	Aggregated Profits
ξ^q	q gross output producer's profits
ξ^{cd}	cd domestic consumption goods producer's profits
ξ^e	e exports producer's profits
ξ^m	m imports producer's profits
ξ^{rm}	rm raw materials producer's profits
ξ^{dis}	dis distribution services producer's profits
ξ^{xd}	xd domestic investment goods producer's profits
Auxiliary variables	
f_1	Recursive equation for optimal wage
f_2	Recursive equation for optimal wage
Ψ^q	Recursive equation for optimal price of gross output q
Ψ^{cd}	Recursive equation for optimal price of domestic consumption goods cd
Ψ^e	Recursive equation for optimal price of exports e
Ψ^m	Recursive equation for optimal price of imports m
Ψ^{rm}	Recursive equation for optimal price of raw materials rm
Ψ^{dis}	Recursive equation for optimal price of distribution services dis
Ψ^{xd}	Recursive equation for optimal price of domestic investment goods xd
Θ^q	Recursive equation for optimal price of gross output q
Θ^{cd}	Recursive equation for optimal price of domestic consumption goods cd
Θ^e	Recursive equation for optimal price of exports e
Θ^m	Recursive equation for optimal price of imports m
Θ^{rm}	Recursive equation for optimal price of raw materials rm
Θ^{dis}	Recursive equation for optimal price of distribution services dis
Θ^{xd}	Recursive equation for optimal price of domestic investment goods xd
Price distortions	
ap^q	Price distortion of gross output q
continue...	

Symbol	Description
ap^{cd}	Price distortion of domestic consumption goods cd
ap^{e}	Price distortion of exports e
ap^{m}	Price distortion of imports m
ap^{rm}	Price distortion of raw materials rm
ap^{dis}	Price distortion of distribution services dis
ap^{xd}	Price distortion of domestic investment goods xd
Exogenous variables	
$c^{\star}$	External demand
g	Growth rate of the technological progress
$\pi^{c\star}$	External imported inflation
$\pi^{m\star}$	External imported inflation
$\pi^{rm\star}$	External raw materials inflation
$tr^{\star}$	Remittances
z^{u}	Shock to marginal utility of consumption
z^{h}	Shock to marginal disutility of labour
z^{q}	Temporary technological shock
z^{x}	Investment efficiency shock
z^{i}	Nominal interest rate shocks
$z^{i^{\star}}$	Risk premium shock

B Equations

B.1 Households

Utility maximization

$$\frac{s_t\, p_t^{c\star}}{p_t^{cF}}\, tr_t^{\star} + y_t + \frac{s_t\, p_t^{c\star}}{p_t^{cF}}\, b_t^{\star} - \Psi^X\left(x_t^F, x_{t-1}^F\right) = c_t^F + \frac{p_t^{xF}}{p_t^{cF}}\, x_t^F + \frac{\left(1+i_{t-1}^{\star}\right)}{\left(1+\pi_t^{c\star}\right)}\, \frac{s_t\, p_t^{c\star}}{p_t^{cF}}\, \frac{b_{t-1}^{\star}}{\left(1+\bar{n}\right)\left(1+g_t\right)} \tag{B.1}$$

$$\delta\left(u_t\right) = \bar{\delta} + \frac{b}{1+\Upsilon}\, \left(u_t\right)^{1+\Upsilon} \tag{B.2}$$

$$k_t = x_t^F + \frac{(1-\delta(u_t))\ k_{t-1}}{(1+\bar{n})\ (1+g_t)} \tag{B.3}$$

$$\lambda_t^c = z_t^u \ \left(c_t^F - hab\,\bar{c}_{t-1}^F\right)^{-\sigma} \tag{B.4}$$

$$\begin{aligned} \lambda_t^c \frac{p_t^{xF}}{p_t^{cF}} \quad = \quad & \lambda_t^x - \frac{\lambda_t^c\,\psi^x\ \left(x_t^F - x_{t-1}^F\right)}{x_{t-1}^F} \\ & +\beta\,(1+\bar{n})\,E_t\,(1+g_{t+1})^{1-\sigma}\ \lambda_{t+1}^c\ \left(\frac{\psi^x\ \left(x_{t+1}^F - x_t^F\right) + \frac{\frac{\psi^x}{2}\left(x_{t+1}^F - x_t^F\right)^2}{x_t^F}}{x_t^F}\right) \end{aligned} \tag{B.5}$$

$$\lambda_t^x = \beta E_t\,(1+g_{t+1})^{-\sigma}\ \lambda_{t+1}^c r_{t+1}^k u_{t+1} + \ \beta E_t\,(1+g_{t+1})^{-\sigma}\ \lambda_{t+1}^x\,(1-\delta\,(u_t)) \tag{B.6}$$

$$r_t^k = b\,\frac{\lambda_t^x}{\lambda_t^c}\,u_t^{\Upsilon} \tag{B.7}$$

$$\lambda_t^c = \beta\,E_t\ (1+g_{t+1})^{-\sigma}\ \lambda_{t+1}^c\,\frac{(1+i_t)}{\left(1+\pi_{t+1}^{cF}\right)} \tag{B.8}$$

$$\lambda_t^c = \beta\,E_t\ (1+g_{t+1})^{-\sigma}\ \lambda_{t+1}^c\,\frac{(1+i_t^{\star})\ (1+d_{t+1})}{\left(1+\pi_{t+1}^{cF}\right)} \tag{B.9}$$

Domestic and imported consumption choice

$$c_t^F = \left(\gamma^{c\,\frac{1}{\omega^c}}\ \left(c_t^{dF}\right)^{\frac{\omega^c-1}{\omega^c}} + (1-\gamma^c)^{\frac{1}{\omega^c}}\ \left(c_t^{mF}\right)^{\frac{\omega^c-1}{\omega^c}}\right)^{\frac{\omega^c}{\omega^c-1}} \tag{B.10}$$

$$c_t^{dF} = \gamma^c\ \left(\frac{p_t^{cdF}}{p_t^{cF}}\right)^{-\omega^c}\ c_t^F \tag{B.11}$$

$$c_t^{mF} = (1-\gamma^c)\ \left(\frac{p_t^{mF}}{p_t^{cF}}\right)^{-\omega^c}\ c_t^F \tag{B.12}$$

$$1+\pi_t^{cF} = \left(\gamma^c\ \left(1+\pi_t^{cdF}\right)^{1-\omega^c}\ \left(\frac{p_{t-1}^{cdF}}{p_{t-1}^{cF}}\right)^{1-\omega^c} + (1-\gamma^c)\ \left(1+\pi_t^{mF}\right)^{1-\omega^c}\ \left(\frac{p_{t-1}^{mF}}{p_{t-1}^{cF}}\right)^{1-\omega^c}\right)^{\frac{1}{1-\omega^c}} \tag{B.13}$$

Wage setting problem

$$f_{1t} = f_{2t} \tag{B.14}$$

$$\begin{aligned} f_{1t} &= \lambda_t^c \, w_t^{opt} \, (1 - TD) \; TBP \; (\theta^w - 1) \left(\frac{w_t^{opt}}{w_t} \right)^{-\theta^w} h_t^F \\ &\quad + \beta \, (1 + \bar{n}) \, E_t \, (1 + g_{t+1})^{1-\sigma} \, \varepsilon^w \left(\frac{1 + \pi_t^{cF}}{1 + \pi_{t+1}^{cF}} \frac{w_t^{opt}}{w_{t+1}^{opt}} \right)^{1-\theta^w} f_{1t+1} \end{aligned} \tag{B.15}$$

$$\begin{aligned} f_{2t} &= \theta^w \, z_t^h \; ((1 - TD) \; TBP)^{1+\eta} \left(\left(\frac{w_t^{opt}}{w_t} \right)^{-\theta^w} h_t^F \right)^{1+\eta} \\ &\quad + \beta \, (1 + \bar{n}) \, E_t \, (1 + g_{t+1})^{1-\sigma} \, \varepsilon^w \left(\frac{1 + \pi_t^{cF}}{1 + \pi_{t+1}^{cF}} \frac{w_t^{opt}}{w_{t+1}^{opt}} \right)^{-\theta^w (1+\eta)} f_{2t+1} \end{aligned} \tag{B.16}$$

$$w_t = \left(\varepsilon^w \left(w_{t-1} \frac{1 + \pi_{t-1}^{cF}}{1 + \pi_t^{cF}} \right)^{1-\theta^w} + (1 - \varepsilon^w) \left(w_t^{opt} \right)^{1-\theta^w} \right)^{\frac{1}{1-\theta^w}} \tag{B.17}$$

B.2 Firms

Gross output producer

$$q_t^C = z_t^q \left(\alpha^{\frac{1}{\rho}} \; (va_t)^{\frac{\rho-1}{\rho}} + (1 - \alpha)^{\frac{1}{\rho}} \; \left(rm_t^F \right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \tag{B.18}$$

$$va_t = \left(\alpha_v^{\frac{1}{\rho_v}} \; (k_t^s)^{\frac{\rho_v - 1}{\rho_v}} + (1 - \alpha_v)^{\frac{1}{\rho_v}} \; \left((1 - TD) \; TBP \; \left(h_t^F \right) \right)^{\frac{\rho_v - 1}{\rho_v}} \right)^{\frac{\rho_v}{\rho_v - 1}} \tag{B.19}$$

$$w_t = z_t^q \, \lambda_t^q \left(\frac{\alpha \, q_t^C}{z_t^q \, va_t} \right)^{\frac{1}{\rho}} \left(\frac{(1 - \alpha_v) \; va_t}{(1 - TD) \; TBP \, h_t^F} \right)^{\frac{1}{\rho_v}} \tag{B.20}$$

$$r_t^k = z_t^q \, \lambda_t^q \left(\frac{\alpha \, q_t^C}{z_t^q \, va_t} \right)^{\frac{1}{\rho}} \left(\frac{\alpha_v \, va_t}{k_t^s} \right)^{\frac{1}{\rho_v}} \tag{B.21}$$

$$\frac{p_t^{rmF}}{p_t^{cF}} = z_t^q \, \lambda_t^q \left(\frac{(1 - \alpha) \; q_t^C}{z_t^q \, rm_t^F} \right)^{\frac{1}{\rho}} \tag{B.22}$$

$$k_t^s = \frac{u_t\, k_{t-1}}{(1+\bar{n})\;(1+g_t)} \tag{B.23}$$

$$\frac{p_t^{qopt}}{p_t^{qF}} = \frac{\theta^q}{\theta^q - 1}\,\frac{\Theta_t^q}{\Psi_t^q} \tag{B.24}$$

$$\Theta_t^q = \lambda_t^q\, q_t^F + \varepsilon^q\, \beta\; (1+\bar{n})\; E_t \frac{(1+g_{t+1})^{1-\sigma}\;\left(1+\pi_{t+1}^{qF}\right)^{\theta^q}}{\left(1+\pi_t^{qF}\right)^{\theta^q}}\,\frac{\lambda_{t+1}^c}{\lambda_t^c}\;\Theta_{t+1}^q \tag{B.25}$$

$$\Psi_t^q = \frac{p_t^{qF}}{p_t^{cF}}\, q_t^F + \varepsilon^q\, \beta\; (1+\bar{n})\; E_t \frac{(1+g_{t+1})^{1-\sigma}\;\left(1+\pi_{t+1}^{qF}\right)^{\theta^q-1}}{\left(1+\pi_t^{qF}\right)^{\theta^q-1}}\,\frac{\lambda_{t+1}^c}{\lambda_t^c}\;\Psi_{t+1}^q \tag{B.26}$$

$$1+\pi_t^{qF} = \left((1-\varepsilon^q)\;\left(\frac{p_t^{qopt}}{p_t^{qF}}\right)^{1-\theta^q}\;\left(1+\pi_t^{qF}\right)^{1-\theta^q} + \varepsilon^q\;\left(1+\pi_{t-1}^{qF}\right)^{1-\theta^q}\right)^{\frac{1}{1-\theta^q}} \tag{B.27}$$

$$q_t^C = ap_t^q\, q_t^F \tag{B.28}$$

$$ap_t^q = \varepsilon^q\;\left(\frac{1+\pi_{t-1}^{qF}}{1+\pi_t^{qF}}\right)^{-\theta^q}\; ap_{t-1}^q + (1-\varepsilon^q)\;\left(\frac{p_t^{qopt}}{p_t^{qF}}\right)^{-\theta^q} \tag{B.29}$$

$$\xi_t^q = \frac{p_t^{qF}}{p_t^{cF}}\, q_t^F - \lambda_t^q\, q_t^C \tag{B.30}$$

Transforming firms

$$q_t^F = \left(\nu_{nt}^{\omega_q-1}\;(nt_t)^{\omega_q} + \nu_e^{\omega_q-1}\;\left(e_t^C\right)^{\omega_q}\right)^{\frac{1}{\omega_q}} \tag{B.31}$$

$$nt_t = \left(\nu_c^{\omega_{nt}-1}\;\left(c_t^{dC}\right)^{\omega_{nt}} + \nu_x^{\omega_{nt}-1}\;\left(x_t^{dC}\right)^{\omega_{nt}} + \nu_{dis}^{\omega_{nt}-1}\;\left(dis_t^C\right)^{\omega_{nt}}\right)^{\frac{1}{\omega_{nt}}} \tag{B.32}$$

$$\frac{p_t^{cdC}}{p_t^{qF}} = \nu_c^{\omega_{nt}-1}\,\nu_{nt}^{\omega_q-1}\;\left(\frac{nt_t}{q_t^F}\right)^{\omega_q-1}\;\left(\frac{c_t^{dC}}{nt_t}\right)^{\omega_{nt}-1} \tag{B.33}$$

$$\frac{p_t^{xdC}}{p_t^{qF}} = \nu_x^{\omega_{nt}-1}\,\nu_{nt}^{\omega_q-1}\;\left(\frac{nt_t}{q_t^F}\right)^{\omega_q-1}\;\left(\frac{x_t^{dC}}{nt_t}\right)^{\omega_{nt}-1} \tag{B.34}$$

$$\frac{p_t^{disC}}{p_t^{qF}} = \nu_{dis}^{\omega_{nt}-1}\,\nu_{nt}^{\omega_q-1}\;\left(\frac{nt_t}{q_t^F}\right)^{\omega_q-1}\;\left(\frac{dis_t^C}{nt_t}\right)^{\omega_{nt}-1} \tag{B.35}$$

$$\frac{p_t^{eC}}{p_t^{qF}} = \nu_e^{\omega_q - 1} \left(\frac{e_t^C}{q_t^F}\right)^{\omega_q - 1} \tag{B.36}$$

Distributing firms

$$\frac{p_t^{disopt}}{p_t^{disF}} = \frac{\theta^{dis}}{\theta^{dis} - 1} \frac{\Theta_t^{dis}}{\Psi_t^{dis}} \tag{B.37}$$

$$\Theta_t^{dis} = \frac{p_t^{qF}}{p_t^{cF}} \frac{p_t^{disC}}{p_t^{qF}}\, dis_t^F + \varepsilon^{dis}\, \beta\, (1 + \bar{n})\; E_t\, \frac{(1 + g_{t+1})^{1-\sigma}\, \left(1 + \pi_{t+1}^{disF}\right)^{\theta^{dis}}}{\left(1 + \pi_t^{disF}\right)^{\theta^{dis}}} \frac{\lambda_{t+1}^c}{\lambda_t^c}\, \Theta_{t+1}^{dis} \tag{B.38}$$

$$\Psi_t^{dis} = \frac{p_t^{disF}}{p_t^{cF}}\, dis_t^F + \varepsilon^{dis}\, \beta\, (1 + \bar{n})\; E_t\, \frac{(1 + g_{t+1})^{1-\sigma}\, \left(1 + \pi_{t+1}^{disF}\right)^{\theta^{dis}-1}}{\left(1 + \pi_t^{disF}\right)^{\theta^{dis}-1}} \frac{\lambda_{t+1}^c}{\lambda_t^c}\, \Psi_{t+1}^{dis} \tag{B.39}$$

$$\frac{p_t^{disF}}{p_t^{cF}} = \frac{p_t^{qF}}{p_t^{cF}} \frac{p_t^{disF}}{p_t^{qF}} \tag{B.40}$$

$$1 + \pi_t^{disF} = \left(\left(1 - \varepsilon^{dis}\right) \left(\frac{p_t^{disopt}}{p_t^{disF}}\right)^{1-\theta^{dis}} \left(1 + \pi_t^{disF}\right)^{1-\theta^{dis}} + \varepsilon^{dis}\, \left(1 + \pi_{t-1}^{disF}\right)^{1-\theta^{dis}}\right)^{\frac{1}{1-\theta^{dis}}} \tag{B.41}$$

$$\frac{1 + \pi_t^{disF}}{1 + \pi_t^{qF}} = \frac{p_t^{disF}}{p_t^{qF}} / \frac{p_{t-1}^{disF}}{p_{t-1}^{qF}} \tag{B.42}$$

$$dis_t^C = ap_t^{dis}\, dis_t^F \tag{B.43}$$

$$ap_t^{dis} = \varepsilon^{dis}\, \left(\frac{1 + \pi_{t-1}^{disF}}{1 + \pi_t^{disF}}\right)^{-\theta^{dis}} ap_{t-1}^{dis} + \left(1 - \varepsilon^{dis}\right)\, \left(\frac{p_t^{disopt}}{p_t^{disF}}\right)^{-\theta^{dis}} \tag{B.44}$$

$$\xi_t^{dis} = \frac{p_t^{disF}}{p_t^{cF}}\, dis_t^F - \frac{p_t^{qF}}{p_t^{cF}}\, \frac{p_t^{disC}}{p_t^{qF}}\, dis_t^C \tag{B.45}$$

$$dis_t^F = dis_t^e + dis_t^{cd} + dis_t^{xd} + dis_t^m \tag{B.46}$$

Final good producers

Final domestic consumption

$$c_t^{dCF} = \left(\gamma^{cd\,\frac{1}{\omega^{cd}}} \, \left(c_t^{dC}\right)^{\frac{\omega^{cd}-1}{\omega^{cd}}} + \left(1-\gamma^{cd}\right)^{\frac{1}{\omega^{cd}}} \, \left(dis_t^{cd}\right)^{\frac{\omega^{cd}-1}{\omega^{cd}}} \right)^{\frac{\omega^{cd}}{\omega^{cd}-1}} \tag{B.47}$$

$$\frac{p_t^{qF}}{p_t^{cF}} \frac{p_t^{cdC}}{p_t^{qF}} = \lambda_t^{cd} \left(\frac{\gamma^{cd} \, c_t^{dCF}}{c_t^{dC}} \right)^{\frac{1}{\omega^{cd}}} \tag{B.48}$$

$$\frac{p_t^{disF}}{p_t^{cF}} = \lambda_t^{cd} \left(\frac{\left(1-\gamma^{cd}\right) \, c_t^{dCF}}{dis_t^{cd}} \right)^{\frac{1}{\omega^{cd}}} \tag{B.49}$$

$$\frac{p_t^{cdopt}}{p_t^{cdF}} = \frac{\theta^{cd}}{\theta^{cd}-1} \frac{\Theta_t^{cd}}{\Psi_t^{cd}} \tag{B.50}$$

$$\Theta_t^{cd} = \lambda_t^{cd} \, c_t^{dF} + \varepsilon^{cd} \, \beta \, (1+\bar{n}) \, E_t \, \frac{(1+g_{t+1})^{1-\sigma} \, \left(1+\pi_{t+1}^{cdF}\right)^{\theta^{cd}}}{\left(1+\pi_t^{cdF}\right)^{\theta^{cd}}} \, \frac{\lambda_{t+1}^c}{\lambda_t^c} \, \Theta_{t+1}^{cd} \tag{B.51}$$

$$\Psi_t^{cd} = \frac{p_t^{cdF}}{p_t^{cF}} \, c_t^{dF} + \varepsilon^{cd} \, \beta \, (1+\bar{n}) \, E_t \, \frac{(1+g_{t+1})^{1-\sigma} \, \left(1+\pi_{t+1}^{cdF}\right)^{\theta^{cd}-1}}{\left(1+\pi_t^{cdF}\right)^{\theta^{cd}-1}} \, \frac{\lambda_{t+1}^c}{\lambda_t^c} \, \Psi_{t+1}^{cd} \tag{B.52}$$

$$\frac{p_t^{cdF}}{p_t^{qF}} = \frac{p_t^{cdF}}{p_t^{cF}} \, \left(\frac{p_t^{qF}}{p_t^{cF}} \right)^{-1} \tag{B.53}$$

$$1+\pi_t^{cdF} = \left(\left(1-\varepsilon^{cd}\right) \, \left(\frac{p_t^{cdopt}}{p_t^{cdF}} \right)^{1-\theta^{cd}} \, \left(1+\pi_t^{cdF}\right)^{1-\theta^{cd}} + \varepsilon^{cd} \, \left(1+\pi_{t-1}^{cdF}\right)^{1-\theta^{cd}} \right)^{\frac{1}{1-\theta^{cd}}} \tag{B.54}$$

$$\frac{1+\pi_t^{cdF}}{1+\pi_t^{qF}} = \frac{p_t^{cdF}}{p_t^{qF}} / \frac{p_{t-1}^{cdF}}{p_{t-1}^{qF}} \tag{B.55}$$

$$c_t^{dCF} = ap_t^{cd} \, c_t^{dF} \tag{B.56}$$

$$ap_t^{cd} = \varepsilon^{cd} \, \left(\frac{1+\pi_{t-1}^{cdF}}{1+\pi_t^{cdF}} \right)^{-\theta^{cd}} \, ap_{t-1}^{cd} + \left(1-\varepsilon^{cd}\right) \, \left(\frac{p_t^{cdopt}}{p_t^{cdF}} \right)^{-\theta^{cd}} \tag{B.57}$$

$$\xi_t^{cd} = \frac{p_t^{cdF}}{p_t^{cF}} \, c_t^{dF} - \lambda_t^{cd} \, c_t^{dCF} \tag{B.58}$$

Final domestic investment

$$x_t^{dCF} = \left(\gamma^{xd\,\frac{1}{\omega^{xd}}} \left(x_t^{dC}\right)^{\frac{\omega^{xd}-1}{\omega^{xd}}} + \left(1-\gamma^{xd}\right)^{\frac{1}{\omega^{xd}}} \left(dis_t^{xd}\right)^{\frac{\omega^{xd}-1}{\omega^{xd}}}\right)^{\frac{\omega^{xd}}{\omega^{xd}-1}} \tag{B.59}$$

$$\frac{p_t^{qF}}{p_t^{cF}}\,\frac{p_t^{xdC}}{p_t^{qF}} = \lambda_t^{xd} \left(\frac{\gamma^{xd}\, x_t^{dCF}}{x_t^{dC}}\right)^{\frac{1}{\omega^{xd}}} \tag{B.60}$$

$$\frac{p_t^{disF}}{p_t^{cF}} = \lambda_t^{xd} \left(\frac{\left(1-\gamma^{xd}\right) x_t^{dCF}}{dis_t^{xd}}\right)^{\frac{1}{\omega^{xd}}} \tag{B.61}$$

$$\frac{p_t^{xdopt}}{p_t^{xdF}} = \frac{\theta^{xd}}{\theta^{xd}-1}\,\frac{\Theta_t^{xd}}{\Psi_t^{xd}} \tag{B.62}$$

$$\Theta_t^{xd} = \lambda_t^{xd} x_t^{dF} + \varepsilon^{xd}\,\beta\,(1+\bar{n})\,E_t\,\frac{\left(1+g_{t+1}\right)^{1-\sigma}\left(1+\pi_{t+1}^{xdF}\right)^{\theta^{xd}}}{\left(1+\pi_t^{xdF}\right)^{\theta^{xd}}}\,\frac{\lambda_{t+1}^{c}}{\lambda_t^{c}}\,\Theta_{t+1}^{xd} \tag{B.63}$$

$$\Psi_t^{xd} = \frac{p_t^{xdF}}{p_t^{cF}}\,x_t^{dF} + \varepsilon^{xd}\,\beta\,(1+\bar{n})\,E_t\,\frac{\left(1+g_{t+1}\right)^{1-\sigma}\left(1+\pi_{t+1}^{xdF}\right)^{\theta^{xd}-1}}{\left(1+\pi_t^{xdF}\right)^{\theta^{xd}-1}}\,\frac{\lambda_{t+1}^{c}}{\lambda_t^{c}}\,\Psi_{t+1}^{xd} \tag{B.64}$$

$$\frac{p_t^{xdF}}{p_t^{cF}} = \frac{p_t^{xF}}{p_t^{cF}}\,\frac{p_t^{xdF}}{p_t^{xF}} \tag{B.65}$$

$$1+\pi_t^{xdF} = \left(\left(1-\varepsilon^{xd}\right)\left(\frac{p_t^{xdopt}}{p_t^{xdF}}\right)^{1-\theta^{xd}}\left(1+\pi_t^{xdF}\right)^{1-\theta^{xd}} + \varepsilon^{xd}\left(1+\pi_{t-1}^{xdF}\right)^{1-\theta^{xd}}\right)^{\frac{1}{1-\theta^{xd}}} \tag{B.66}$$

$$\frac{1+\pi_t^{xdF}}{1+\pi_t^{cdF}} = \frac{p_t^{xdF}}{p_t^{cdF}} / \frac{p_{t-1}^{xdF}}{p_{t-1}^{cdF}} \tag{B.67}$$

$$x_t^{dCF} = ap_t^{xd}\,x_t^{dF} \tag{B.68}$$

$$ap_t^{xd} = \varepsilon^{xd}\left(\frac{1+\pi_{t-1}^{xdF}}{1+\pi_t^{xdF}}\right)^{-\theta^{xd}} ap_{t-1}^{xd} + \left(1-\varepsilon^{xd}\right)\left(\frac{p_t^{xdopt}}{p_t^{xdF}}\right)^{-\theta^{xd}} \tag{B.69}$$

$$\xi_t^{xd} = \frac{p_t^{xdF}}{p_t^{cF}}\,x_t^{dF} - \lambda_t^{xd}\,x_t^{dCF} \tag{B.70}$$

Final exports

$$e_t^{CF} = \left(\gamma^{e\,\frac{1}{\omega^e}} \; \left(e_t^C\right)^{\frac{\omega^e - 1}{\omega^e}} + (1-\gamma^e)^{\frac{1}{\omega^e}} \; \left(dis_t^e\right)^{\frac{\omega^e-1}{\omega^e}} \right)^{\frac{\omega^e}{\omega^e - 1}} \tag{B.71}$$

$$\frac{p_t^{qF}}{p_t^{cF}} \frac{p_t^{eC}}{p_t^{qF}} = \lambda_t^e \; \left(\frac{\gamma^e \, e_t^{CF}}{e_t^C} \right)^{\frac{1}{\omega^e}} \tag{B.72}$$

$$\frac{p_t^{disF}}{p_t^{cF}} = \lambda_t^e \; \left(\frac{(1-\gamma^e) \; e_t^{CF}}{dis_t^e} \right)^{\frac{1}{\omega^e}} \tag{B.73}$$

$$\frac{p_t^{eopt}}{p_t^{eF}} = \frac{\theta^e}{\theta^e - 1} \frac{\Theta_t^e}{\Psi_t^e} \tag{B.74}$$

$$\Theta_t^e = \lambda_t^e \, e_t^F + \varepsilon^e \, \beta \; (1+\bar{n}) \; E_t \, \frac{(1+g_{t+1})^{1-\sigma} \; \left(1+\pi_{t+1}^{eF}\right)^{\theta^e}}{\left(1+\pi_t^{eF}\right)^{\theta^e}} \frac{\lambda_{t+1}^c}{\lambda_t^c} \, \Theta_{t+1}^e \tag{B.75}$$

$$\Psi_t^e = \frac{p_t^{eF}}{p_t^{cF}} \, e_t^F + \varepsilon^e \, \beta \; (1+\bar{n}) \; E_t \, \frac{(1+g_{t+1})^{1-\sigma} \; \left(1+\pi_{t+1}^{eF}\right)^{\theta^e - 1}}{\left(1+\pi_t^{eF}\right)^{\theta^e - 1}} \frac{\lambda_{t+1}^c}{\lambda_t^c} \, \Psi_{t+1}^e \tag{B.76}$$

$$1+\pi_t^{eF} = \left((1-\varepsilon^e) \; \left(\frac{p_t^{eopt}}{p_t^{eF}} \right)^{1-\theta^e} \; \left(1+\pi_t^{eF}\right)^{1-\theta^e} + \varepsilon^e \; \left(1+\pi_{t-1}^{eF}\right)^{1-\theta^e} \right)^{\frac{1}{1-\theta^e}} \tag{B.77}$$

$$\frac{1+\pi_t^{eF}}{1+\pi_t^{qF}} = \frac{p_t^{eF}}{p_t^{qF}} / \frac{p_{t-1}^{eF}}{p_{t-1}^{qF}} \tag{B.78}$$

$$e_t^{CF} = ap_t^e \, e_t^F \tag{B.79}$$

$$ap_t^e = \varepsilon^e \; \left(\frac{1+\pi_{t-1}^{eF}}{1+\pi_t^{eF}} \right)^{-\theta^e} \; ap_{t-1}^e + (1-\varepsilon^e) \; \left(\frac{p_t^{eopt}}{p_t^{eF}} \right)^{-\theta^e} \tag{B.80}$$

$$\xi_t^e = \frac{p_t^{eF}}{p_t^{cF}} \, e_t^F - \lambda_t^e \, e_t^{CF} \tag{B.81}$$

Importers of consumption and investment goods

$$m_t^C = z_t^m \; \left((\gamma^m)^{\frac{1}{\omega^m}} \; (m_t^\star)^{\frac{\omega^m - 1}{\omega^m}} + (1-\gamma^m)^{\frac{1}{\omega^m}} \; (dis_t^m)^{\frac{\omega^m - 1}{\omega^m}} \right)^{\frac{\omega^m}{\omega^m - 1}} \tag{B.82}$$

$$\frac{p_t^{mC}}{p_t^{cF}} = \lambda_t^m \; \left(\frac{\gamma^m m_t^C}{m_t^\star} \right)^{\frac{1}{\omega^m}} \tag{B.83}$$

$$\frac{p_t^{disF}}{p_t^{cF}} = \lambda_t^m \left(\frac{(1-\gamma^m)\ m_t^C}{dis_t^m} \right)^{\frac{1}{\omega^m}} \tag{B.84}$$

$$\frac{p_t^{disF}}{p_t^{mF}} = \frac{p_t^{qF}}{p_t^{cF}}\ \frac{p_t^{disF}}{p_t^{qF}}\ \left(\frac{p_t^{mF}}{p_t^{cF}} \right)^{-1} \tag{B.85}$$

$$\frac{p_t^{mC}}{p_t^{mF}} = \left(\frac{p_t^{mF}}{p_t^{cF}} \right)^{-1}\ \frac{s_t\, p_t^{c\star}}{p_t^{cF}}\ \frac{p_t^{m\star}}{p_t^{c\star}} \tag{B.86}$$

$$\frac{p_t^{mC}}{p_t^{cF}} = \frac{p_t^{mF}}{p_t^{cF}} \frac{p_t^{mC}}{p_t^{mF}} \tag{B.87}$$

$$\frac{p_t^{mopt}}{p_t^{mF}} = \frac{\theta^m}{\theta^m - 1} \frac{\Theta_t^m}{\Psi_t^m} \tag{B.88}$$

$$\Theta_t^m = \lambda_t^m m_t^F + \varepsilon^m\, \beta\ (1+\bar{n})\ E_t\, \frac{(1+g_{t+1})^{1-\sigma}\ \left(1+\pi_{t+1}^{mF}\right)^{\theta^m}}{\left(1+\pi_t^{mF}\right)^{\theta^m}} \frac{\lambda_{t+1}^c}{\lambda_t^c} \Theta_{t+1}^m \tag{B.89}$$

$$\Psi_t^m = \frac{p_t^{mF}}{p_t^{cF}}\, m_t^F + \varepsilon^m\, \beta\ (1+\bar{n})\ E_t\, \frac{(1+g_{t+1})^{1-\sigma}\left(1+\pi_{t+1}^{mF}\right)^{\theta^m-1}}{\left(1+\pi_t^{mF}\right)^{\theta^m-1}} \frac{\lambda_{t+1}^c}{\lambda_t^c}\, \Psi_{t+1}^m \tag{B.90}$$

$$1+\pi_t^{mF} = \left((1-\varepsilon^m)\ \left(\frac{p_t^{mopt}}{p_t^{mF}} \right)^{1-\theta^m}\ \left(1+\pi_t^{mF}\right)^{1-\theta^m} + \varepsilon^m\ \left(1+\pi_{t-1}^{mF}\right)^{1-\theta^m} \right)^{\frac{1}{1-\theta^m}} \tag{B.91}$$

$$1+\pi_t^{mC} = (1+d_t)\ (1+\pi_t^{m\star}) \tag{B.92}$$

$$\frac{1+\pi_t^{mF}}{1+\pi_t^{cF}} = \frac{p_t^{mF}}{p_t^{cF}} / \frac{p_{t-1}^{mF}}{p_{t-1}^{cF}} \tag{B.93}$$

$$\frac{1+\pi_t^{m\star}}{1+\pi_t^{c\star}} = \frac{p_t^{m\star}}{p_t^{c\star}} / \frac{p_{t-1}^{m\star}}{p_{t-1}^{c\star}} \tag{B.94}$$

$$m_t^C = ap_t^m\, m_t^F \tag{B.95}$$

$$ap_t^m = \varepsilon^m\ \left(\frac{1+\pi_{t-1}^{mF}}{1+\pi_t^{mF}} \right)^{-\theta^m}\ ap_{t-1}^m + (1-\varepsilon^m)\ \left(\frac{p_t^{mopt}}{p_t^{mF}} \right)^{-\theta^m} \tag{B.96}$$

$$\xi_t^m = \frac{p_t^{mF}}{p_t^{cF}}\, m_t^F - \lambda_t^m\, m_t^C \tag{B.97}$$

$$m_t^F = c_t^{mF} + x_t^{mF} \tag{B.98}$$

Importers of raw materials

$$\lambda_t^{rm} = \frac{p_t^{rmC}}{p_t^{cF}} \tag{B.99}$$

$$\frac{p_t^{rmopt}}{p_t^{rmF}} = \frac{\theta^{rm}}{\theta^{rm}-1}\,\frac{\Theta_t^{rm}}{\Psi_t^{rm}} \tag{B.100}$$

$$\Theta_t^{rm} = \lambda_t^{rm}\, rm_t^F + \varepsilon^{rm}\,\beta\,(1+\bar{n})\;E_t\,\frac{(1+g_{t+1})^{1-\sigma}\left(1+\pi_{t+1}^{rmF}\right)^{\theta^{rm}}}{\left(1+\pi_t^{rmF}\right)^{\theta^{rm}}}\,\frac{\lambda_{t+1}^c}{\lambda_t^c}\,\Theta_{t+1}^{rm} \tag{B.101}$$

$$\Psi_t^{rm} = \frac{p^{rmF_t}}{p_t^{cF}}\, rm_t^F + \varepsilon^{rm}\,\beta\,(1+\bar{n})\;E_t\,\frac{(1+g_{t+1})^{1-\sigma}\;\left(1+\pi_{t+1}^{rmF}\right)^{\theta^{rm}-1}}{\left(1+\pi_t^{rmF}\right)^{\theta^{rm}-1}}\,\frac{\lambda_{t+1}^c}{\lambda_t^c}\,\Psi_{t+1}^{rm} \tag{B.102}$$

$$1+\pi_t^{rmF} = \left((1-\varepsilon^{rm})\,\left(\frac{p_t^{rmopt}}{p_t^{rmF}}\right)^{1-\theta^{rm}}\;\left(1+\pi_t^{rmF}\right)^{1-\theta^{rm}} + \varepsilon^{rm}\,\left(1+\pi_{t-1}^{rmF}\right)^{1-\theta^{rm}}\right)^{\frac{1}{1-\theta^{rm}}} \tag{B.103}$$

$$\frac{1+\pi_t^{rmF}}{1+\pi_t^{cF}} = \frac{p_t^{rmF}}{p_t^{cF}}/\frac{p_{t-1}^{rmF}}{p_{t-1}^{cF}} \tag{B.104}$$

$$\frac{p_t^{rmC}}{p_t^{rmF}} = \frac{p_t^{rmC}}{p_t^{cF}}\,\left(\frac{p_t^{rmF}}{p_t^{cF}}\right)^{-1} \tag{B.105}$$

$$\frac{p_t^{rmC}}{p_t^{cF}} = \frac{s_t\,p_t^{c\star}}{p_t^{cF}}\,\frac{p_t^{rm\star}}{p_t^{c\star}} \tag{B.106}$$

$$\frac{(1+\pi_t^{rm\star})}{(1+\pi_t^{c\star})} = \frac{p_t^{rm\star}}{p_t^{c\star}}/\frac{p_{t-1}^{rm\star}}{p_{t-1}^{c\star}} \tag{B.107}$$

$$rm_t^C = ap_t^{rm}\, rm_t^F \tag{B.108}$$

$$ap_t^{rm} = \varepsilon^{rm}\,\left(\frac{1+\pi_{t-1}^{rmF}}{1+\pi_t^{rmF}}\right)^{(-\theta^{rm})}\,ap_{t-1}^{rm} + (1-\varepsilon^{rm})\,\left(\frac{p_t^{rmopt}}{p_t^{rmF}}\right)^{(-\theta^{rm})} \tag{B.109}$$

$$\xi_t^{rm} = \frac{p_t^{rmF}}{p_t^{cF}}\, rm_t^F - \lambda_t^{rm}\, rm_t^C \tag{B.110}$$

Investment producers

$$x_t^F = z_t^x \left((\gamma^x)^{\frac{1}{\omega^x}} \left(x_t^{dF}\right)^{\frac{\omega^x-1}{\omega^x}} + (1-\gamma^x)^{\frac{1}{\omega^x}} \left(x_t^{mF}\right)^{\frac{\omega^x-1}{\omega^x}} \right)^{\frac{\omega^x}{\omega^x-1}} \tag{B.111}$$

$$x_t^{dF} = \gamma^x \left(\frac{p_t^{xdF}}{z_t^x \, p_t^{xF}} \right)^{-\omega^x} \frac{x_t^F}{z_t^x} \tag{B.112}$$

$$x_t^{mF} = (1-\gamma^x) \left(\frac{p_t^{mF}}{z_t^x \, p_t^{xF}} \right)^{-\omega^x} \frac{x_t^F}{z_t^x} \tag{B.113}$$

$$1+\pi_t^{xF} = \frac{1}{z_t^x} \left(\gamma^x \left(1+\pi_t^{xdF}\right)^{1-\omega^x} \left(\frac{p_{t-1}^{xdF}}{p_{t-1}^{xF}} \right)^{1-\omega^x} + (1-\gamma^x) \left(1+\pi_t^{mF}\right)^{1-\omega^x} \left(\frac{p_{t-1}^{mF}}{p_{t-1}^{xF}} \right)^{1-\omega^x} \right)^{\frac{1}{1-\omega^x}} \tag{B.114}$$

$$\frac{p_t^{xdF}}{p_t^{xF}} = \frac{p_t^{qF}}{p_t^{cF}} \frac{p_t^{xdF}}{p_t^{qF}} \left(\frac{p_t^{xF}}{p_t^{cF}} \right)^{-1} \tag{B.115}$$

$$\frac{p_t^{mF}}{p_t^{xF}} = \frac{p_t^{mF}}{p_t^{cF}} \left(\frac{p_t^{xF}}{p_t^{cF}} \right)^{-1} \tag{B.116}$$

$$\frac{p_t^{xdF}}{p_t^{mF}} = \frac{p_t^{xdF}}{p_t^{xF}} \left(\frac{p_t^{mF}}{p_t^{xF}} \right)^{-1} \tag{B.117}$$

$$\frac{p_t^{xdF}}{p_t^{cdF}} = \frac{p_t^{xdF}}{p_t^{qF}} \left(\frac{p_t^{cdF}}{p_t^{qF}} \right)^{-1} \tag{B.118}$$

B.3 Foreign variables

$$e_t^F = \left(\frac{p_t^{e\star}}{p_t^{c\star}} \right)^{-\mu} c_t^\star \tag{B.119}$$

$$\frac{p_t^{eF}}{p_t^{qF}} = \frac{s_t \, p_t^{c\star}}{p_t^{cF}} \frac{p_t^{e\star}}{p_t^{c\star}} \left(\frac{p_t^{qF}}{p_t^{cF}} \right)^{-1} \tag{B.120}$$

$$\frac{p_t^{eF}}{p_t^{cF}} = \frac{p_t^{qF}}{p_t^{cF}} \frac{p_t^{eF}}{p_t^{qF}} \tag{B.121}$$

$$i_t^\star = \bar{i}^\star \, z_t^{i^\star} \, \exp\left(\Omega_U \left(\frac{s_t p_t^{c\star}}{p_t^{cF}} \frac{b_t^\star}{y_t} - \bar{b^\star} \right) \right) \tag{B.122}$$

$$\frac{(1+\pi_t^{c\star}) \, (1+d_t)}{1+\pi_t^{cF}} = \frac{s_t \, p_t^{c\star}}{p_t^{cF}} / \frac{s_{t-1} \, p_{t-1}^{c\star}}{p_{t-1}^{cF}} \tag{B.123}$$

B.4 Monetary policy

$$i_t = (i_{t-1})^{\rho^i} \left(\bar{i} \left(\frac{\pi_t^{cF}}{\bar{\pi}} \right)^{\varphi_\pi} \left(\frac{y_t}{y^{flex}} \right)^{\varphi_y} \right)^{1-\rho^i} \left(z_t^i \right) \tag{B.124}$$

B.5 National Accounts

$$y_t = \xi_t + \frac{p_t^{qF}}{p_t^{cF}} q_t^F - \frac{p_t^{rmF}}{p_t^{cF}} rm_t^F \tag{B.125}$$

$$\xi_t = \xi_t^q + \xi_t^e + \xi_t^{dis} + \xi_t^{xd} + \xi_t^{cd} + \xi_t^m + \xi_t^{rm} \tag{B.126}$$

B.6 Exogenous Processes

$$\ln z_t^u = \rho_{z^u} \ \ln z_{t-1}^u + (1 - \rho_{z^u}) \ \ln \bar{z}^u + \epsilon_t^{z^u} \tag{B.127}$$

$$\ln z_t^h = \rho_{z^h} \ \ln z_{t-1}^h + (1 - \rho_{z^h}) \ \ln \bar{z}^h + \epsilon_t^{z^h} \tag{B.128}$$

$$\ln z_t^q = \rho_{z^q} \ \ln z_{t-1}^q + (1 - \rho_{z^q}) \ \ln \bar{z}^q + \epsilon_t^{z^q} \tag{B.129}$$

$$\ln z_t^x = \rho_{z^x} \ \ln z_{t-1}^x + (1 - \rho_{z^x}) \ \ln \bar{z}^x + \epsilon_t^{z^x} \tag{B.130}$$

$$\ln z_t^{i^\star} = \rho_{z^{i^\star}} \ \ln z_{t-1}^{i^\star} + (1 - \rho_{z^{i^\star}}) \ \ln \bar{z}^{i^\star} + \epsilon_t^{z^{i\star}} \tag{B.131}$$

$$\ln z_t^i = \rho_{z^i} \ \ln z_{t-1}^i + (1 - \rho_{z^i}) \ \ln \bar{z}^i + \epsilon_t^{z^\mu} \tag{B.132}$$

$$\ln \pi_t^{c\star} = \rho_{\pi^{c\star}} \ \ln \pi_{t-1}^{c\star} + (1 - \rho_{\pi^{c\star}}) \ \ln \bar{\pi}^{c\star} + \epsilon_t^{\pi^{c\star}} \tag{B.133}$$

$$\ln \pi_t^{m\star} = \rho_{\pi^{m\star}} \ \ln \pi_{t-1}^{m\star} + (1 - \rho_{\pi^{m\star}}) \ \ln \bar{\pi}^{m\star} + \epsilon_t^{\pi^{m\star}} \tag{B.134}$$

$$\ln \pi_t^{rm\star} = \rho_{\pi^{rm\star}} \ \ln \pi_{t-1}^{rm\star} + (1 - \rho_{\pi^{rm\star}}) \ \ln \bar{\pi}^{rm\star} + \epsilon_t^{\pi^{rm\star}} \tag{B.135}$$

$$\ln tr_t^\star = \rho_{tr^\star} \ \ln tr_{t-1}^\star + (1 - \rho_{tr^\star}) \ \ln \bar{tr}^\star + \epsilon_t^{z^{tr^\star}} \tag{B.136}$$

$$\ln c_t^{\star} = \ln c_{t-1}^{\star}\, \rho_{c^\star} + (1-\rho_{c^\star})\, \ln \bar{c}^{\star} + \epsilon_t^{c^\star} \tag{B.137}$$

$$\ln g_t = \rho_g\, \ln g_{t-1} + (1-\rho_g)\, \log \bar{g} + \epsilon_t^{g} \tag{B.138}$$

C Balance of payment's identity

Starting from he aggregate household budget constraint we have:

$$\begin{aligned} c_t^F + \frac{p_t^{xF}}{p_t^{cF}} x_t^F \quad &+ \quad \frac{s_t p_t^{c\star}}{p_t^{cF}} \frac{b_{t-1}^{\star}}{(1+\overline{n})(1+g_t)} \left(\frac{1+i_{t-1}^{\star}}{1+\pi_t^{c\star}} \right) + \Psi^X \left(x_t^F, x_{t-1}^F \right) = \\ r_t^k \frac{u_t k_{t-1}}{(1+\overline{n})(1+g_t)} \quad &+ \quad w_t (1-TD_t)\, TBP_t h_t^F + \xi_t + \frac{s_t p_t^{c\star}}{p_t^{cF}} tr_t^{\star} + \frac{s_t p_t^{c\star}}{p_t^{cF}} b_t^{\star} \end{aligned}$$

combining the former with the equilibrium condition of the consumption bundle

$$\begin{aligned} \frac{p_t^{cdF}}{p_t^{cF}} c_t^{dF} + \frac{p_t^{mF}}{p_t^{cF}} c_t^{mF} + \frac{p_t^{xF}}{p_t^{cF}} x_t^F \quad &+ \quad \frac{s_t p_t^{c\star}}{p_t^{cF}} \frac{b_{t-1}^{\star}}{(1+\overline{n})(1+g_t)} \left(\frac{1+i_{t-1}^{\star}}{1+\pi_t^{c\star}} \right) + \Psi^X \left(x_t^F, x_{t-1}^F \right) = \\ r_t^k \frac{u_t k_{t-1}}{(1+\overline{n})(1+g_t)} \quad &+ \quad w_t (1-TD_t)\, TBP_t h_t^F + \xi_t + \frac{s_t p_t^{c\star}}{p_t^{cF}} tr_t^{\star} + \frac{s_t p_t^{c\star}}{p_t^{cF}} b_t^{\star} \end{aligned}$$

again, combining the former with the equilibrium condition of the aggregate investment

$$\begin{aligned} \frac{p_t^{cdF}}{p_t^{cF}} c_t^{dF} + \frac{p_t^{xdF}}{p_t^{cF}} x_t^{dF} + \frac{p_t^{mF}}{p_t^{cF}} \left(c_t^{mF} + x_t^{mF} \right) \quad &+ \quad \frac{s_t p_t^{c\star}}{p_t^{cF}} \frac{b_{t-1}^{\star}}{(1+\overline{n})(1+g_t)} \left(\frac{1+i_{t-1}^{\star}}{1+\pi_t^{c\star}} \right) + \Psi^X \left(x_t^F, x_{t-1}^F \right) = \\ r_t^k \frac{u_t k_{t-1}}{(1+\overline{n})(1+g_t)} \quad &+ \quad w_t (1-TD_t)\, TBP_t h_t^F + \xi_t + \frac{s_t p_t^{c\star}}{p_t^{cF}} tr_t^{\star} + \frac{s_t p_t^{c\star}}{p_t^{cF}} b_t^{\star} \end{aligned}$$

combining with the total imports, we get:

$$\begin{aligned} \frac{p_t^{cdF}}{p_t^{cF}} c_t^{dF} + \frac{p_t^{xdF}}{p_t^{cF}} x_t^{dF} + \frac{p_t^{mF}}{p_t^{cF}} m_t^F \quad &+ \quad \frac{s_t p_t^{c\star}}{p_t^{cF}} \frac{b_{t-1}^{\star}}{(1+\overline{n})(1+g_t)} \left(\frac{1+i_{t-1}^{\star}}{1+\pi_t^{c\star}} \right) + \Psi^X \left(x_t^F, x_{t-1}^F \right) = \\ r_t^k \frac{u_t k_{t-1}}{(1+\overline{n})(1+g_t)} \quad &+ \quad w_t (1-TD_t)\, TBP_t h_t^F + \xi_t + \frac{s_t p_t^{c\star}}{p_t^{cF}} tr_t^{\star} + \frac{s_t p_t^{c\star}}{p_t^{cF}} b_t^{\star} \end{aligned}$$

using the from the profits ξ_t:

$$\begin{aligned}
\frac{p_t^{cdF}}{p_t^{cF}} c_t^{dF} + \frac{p_t^{xdF}}{p_t^{cF}} x_t^{dF} + \frac{p_t^{mF}}{p_t^{cF}} m_t^F \quad &+ \quad \frac{s_t p_t^{c\star}}{p_t^{cF}} \frac{b_{t-1}^{\star}}{(1+\overline{n})(1+g_t)} \left(\frac{1+i_{t-1}^{\star}}{1+\pi_t^{c\star}} \right) + \Psi^X \left(x_t^F, x_{t-1}^F \right) = \\
r_t^k \frac{u_t k_{t-1}}{(1+\overline{n})(1+g_t)} \quad &+ \quad w_t (1 - TD_t) \, TBP_t h_t^F + \frac{s_t p_t^{c\star}}{p_t^{cF}} tr_t^{\star} + \frac{s_t p_t^{c\star}}{p_t^{cF}} b_t^{\star} \\
&+ \quad \xi_t^q + \xi_t^e + \xi_t^{dis} + \xi_t^{xd} + \xi_t^{cd} + \xi_t^m + \xi_t^{rm}
\end{aligned}$$

using the homogeneity of degree one in the production and also using the factor demands

$$\begin{aligned}
\frac{p_t^{cdF}}{p_t^{cF}} c_t^{dF} + \frac{p_t^{xdF}}{p_t^{cF}} x_t^{dF} + \frac{p_t^{mF}}{p_t^{cF}} m_t^F \quad &+ \quad \frac{s_t p_t^{c\star}}{p_t^{cF}} \frac{b_{t-1}^{\star}}{(1+\overline{n})(1+g_t)} \left(\frac{1+i_{t-1}^{\star}}{1+\pi_t^{c\star}} \right) + \Psi^X \left(x_t^F, x_{t-1}^F \right) = \\
\frac{p_t^{qF} q_t^F}{p_t^{cF}} \quad &- \quad \frac{p_t^{rmF}}{p_t^{cF}} rm_t^F + \frac{s_t p_t^{c\star}}{p_t^{cF}} tr_t^{\star} + \frac{s_t p_t^{c\star}}{p_t^{cF}} b_t^{\star} \\
&+ \quad \xi_t^e + \xi_t^{dis} + \xi_t^{xd} + \xi_t^{cd} + \xi_t^m + \xi_t^{rm}
\end{aligned}$$

From the final production for c_t^{dF}, x_t^{dF} y m_t^F

$$\begin{aligned}
\frac{p_t^{cdC}}{p_t^{cF}} c_t^{dC} + \frac{p_t^{disF}}{p_t^{cF}} dis_t^{cd} \quad &+ \quad \frac{p_t^{xdC}}{p_t^{cF}} x_t^{dC} + \frac{p_t^{disF}}{p_t^{cF}} dis_t^{xd} + \frac{p_t^{mC}}{p_t^{cF}} m_t^{\star} + \frac{p_t^{disF}}{p_t^{cF}} dis_t^m \\
+ \frac{s_t p_t^{c\star}}{p_t^{cF}} \frac{b_{t-1}^{\star}}{(1+\overline{n})(1+g_t)} \left(\frac{1+i_{t-1}^{\star}}{1+\pi_t^{c\star}} \right) \quad &+ \quad \Psi^X \left(x_t^F, x_{t-1}^F \right) = \frac{p_t^{qF} q_t^F}{p_t^{cF}} - \frac{p_t^{rmF}}{p_t^{cF}} rm_t^F \\
+ \frac{s_t p_t^{c\star}}{p_t^{cF}} tr_t^{\star} + \frac{s_t p_t^{c\star}}{p_t^{cF}} b_t^{\star} \quad &+ \quad \xi_t^e + \xi_t^{dis} + \xi_t^{rm}
\end{aligned}$$

From the uses of final output q_t^F

$$\begin{aligned}
+ \frac{p_t^{disF}}{p_t^{cF}} dis_t^{cd} + \frac{p_t^{disF}}{p_t^{cF}} dis_t^{xd} \quad &+ \quad \frac{p_t^{mC}}{p_t^{cF}} m_t^{\star} + \frac{p_t^{disF}}{p_t^{cF}} dis_t^m \\
\frac{s_t p_t^{c\star}}{p_t^{cF}} \frac{b_{t-1}^{\star}}{(1+\overline{n})(1+g_t)} \left(\frac{1+i_{t-1}^{\star}}{1+\pi_t^{c\star}} \right) \quad &+ \quad \Psi^X \left(x_t^F, x_{t-1}^F \right) = \frac{p_t^{disC}}{p_t^q} dis_t^C + \frac{p_t^{eC}}{p_t^{cF}} e_t^C \\
- \frac{p_t^{rmF}}{p_t^{cF}} rm_t^F + \frac{s_t p_t^{c\star}}{p_t^{cF}} tr_t^{\star} + \frac{s_t p_t^{c\star}}{p_t^{cF}} b_t^{\star} \quad &+ \quad \xi_t^e + \xi_t^{dis} + \xi_t^{rm}
\end{aligned}$$

Final raw materials also generate profits

$$\begin{aligned}
+ \frac{p_t^{disF}}{p_t^{cF}} dis_t^{cd} + \frac{p_t^{disF}}{p_t^{cF}} dis_t^{xd} \quad &+ \quad \frac{p_t^{mC}}{p_t^{cF}} m_t^{\star} + \frac{p_t^{disF}}{p_t^{cF}} dis_t^m \\
\frac{s_t p_t^{c\star}}{p_t^{cF}} \frac{b_{t-1}^{\star}}{(1+\overline{n})(1+g_t)} \left(\frac{1+i_{t-1}^{\star}}{1+\pi_t^{c\star}} \right) \quad &+ \quad \Psi^X \left(x_t^F, x_{t-1}^F \right) = \frac{p_t^{disC}}{p_t^q} dis_t^C \\
+ \frac{p_t^{eC}}{p_t^{cF}} e_t^C - \frac{p_t^{rmC}}{p_t^{cF}} rm_t^C + \frac{s_t p_t^{c\star}}{p_t^{cF}} tr_t^{\star} \quad &+ \quad \frac{s_t p_t^{c\star}}{p_t^{cF}} b_t^{\star} + \xi_t^e + \xi_t^{dis}
\end{aligned}$$

Distribution profits implies

$$\begin{aligned}
+\frac{p_t^{disF}}{p_t^{cF}}dis_t^{cd}+\frac{p_t^{disF}}{p_t^{cF}}dis_t^{xd} &+ \frac{p_t^{mC}}{p_t^{cF}}m_t^{\star}+\frac{p_t^{disF}}{p_t^{cF}}dis_t^{m} \\
\frac{s_t p_t^{c\star}}{p_t^{cF}}\frac{b_{t-1}^{\star}}{(1+\overline{n})(1+g_t)}\left(\frac{1+i_{t-1}^{\star}}{1+\pi_t^{c\star}}\right) &+ \Psi^X\left(x_t^F,x_{t-1}^F\right)= \\
+\frac{p_t^{disf}}{p_t^{cF}}dis_t^{F}+\frac{p_t^{eC}}{p_t^{cF}}e_t^{C}-\frac{p_t^{rmC}}{p_t^{cF}}rm_t^{C} &+ \frac{s_t p_t^{c\star}}{p_t^{cF}}tr_t^{\star}+\frac{s_t p_t^{c\star}}{p_t^{cF}}b_t^{\star}+\xi_t^{e}
\end{aligned}$$

using finals exports

$$\begin{aligned}
+\frac{p_t^{disF}}{p_t^{cF}}dis_t^{cd}+\frac{p_t^{disF}}{p_t^{cF}}dis_t^{xd} &+ \frac{p_t^{mC}}{p_t^{cF}}m_t^{\star}+\frac{p_t^{disF}}{p_t^{cF}}dis_t^{m} \\
\frac{s_t p_t^{c\star}}{p_t^{cF}}\frac{b_{t-1}^{\star}}{(1+\overline{n})(1+g_t)}\left(\frac{1+i_{t-1}^{\star}}{1+\pi_t^{c\star}}\right) &+ \Psi^X\left(x_t^F,x_{t-1}^F\right)= \\
+\frac{p_t^{disf}}{p_t^{cF}}dis_t^{F}+\frac{p_t^{eF}}{p_t^{cF}}e_t^{F}-\frac{p_t^{disF}}{p_t^{cF}}dis_t^{e} &- \frac{p_t^{rmC}}{p_t^{cF}}rm_t^{C}+\frac{s_t p_t^{c\star}}{p_t^{cF}}tr_t^{\star}+\frac{s_t p_t^{c\star}}{p_t^{cF}}b_t^{\star}
\end{aligned}$$

finally, using the distribution services, we derive the balance of payment identity:

$$\begin{aligned}
\frac{s_t p_t^{c\star}}{p_t^{cF}}\frac{b_{t-1}^{\star}}{(1+\overline{n})(1+g_t)}\left(\frac{1+i_{t-1}^{\star}}{1+\pi_t^{c\star}}\right) &- \frac{s_t p_t^{c\star}}{p_t^{cF}}b_t^{\star}= \\
+\frac{p_t^{eF}}{p_t^{cF}}e_t^{F}-\frac{p_t^{mC}}{p_t^{cF}}m_t^{\star} &- \frac{p_t^{rmC}}{p_t^{cF}}rm_t^{C}-\Psi^X\left(x_t^F,x_{t-1}^F\right)
\end{aligned}$$