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Abstract

This paper presents the first version of SYSMO, the analytical framework employed by the Financial Stability Department at the Banco de la República (the Central Bank of Colombia) to perform its biannual, top-down, stress testing exercise. The framework comprises: (i) a module to produce internally consistent macroeconomic scenarios; (ii) a set of satellite risk models that capture the materialization of credit and market risks in times of stress, and (iii) a bank model that simulates the endogenous response of banks to an adverse scenario. The framework also incorporates endogenous contagion and funding risks, key regulatory constraints (solvency and liquidity), and the feedback effects between the endogenous response of banks and the macroeconomic scenario. The use of SYSMO is illustrated with the example of the stress testing exercise published in the Banco de la República's Financial Stability Report of the second semester of 2017.

Keywords: Stress Testing, DSGE Models, VAR models, Credit Risk, Market Risk, Liquidity Risk, Funding Risk, Contagion Risk.
JEL Codes: E44, E58, G01, G17, G20.

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This paper presents the first version of SYSMO, the analytical framework employed by the Financial Stability Department at the Banco de la República (BR, the Central Bank of Colombia) to perform its biannual, top-down, stress testing exercise. The results of this exercise are published in Chapter III of the BR's Financial Stability Report (FSR). Although SYSMO produces results at the individual bank level, only aggregate results are published in the FSR due to confidentiality reasons.

A stress test is a routine exercise performed by central banks and supervisors that seeks to provide a quantitative assessment of the resilience of financial institutions to a hypothetical, adverse, deliberately extreme macroeconomic scenario lasting a predetermined time horizon. Specifically, the stress testing exercise seeks to calculate the total losses that would be incurred by financial institutions due to the joint, simultaneous materialization of a several risks. From a prudential perspective, the hypothetical macroeconomic scenario should reflect the most salient vulnerabilities of the financial system at a given point in time.

As such, a stress testing exercise is not a forecast or a probabilistic assessment of the economy for a given time horizon. In the case of SYSMO, the hypothetical scenario is constructed without any consideration for the likelihood of any particular course for the economy. Instead, the scenario is designed using the judgement of the staff of the Financial Stability Department with regard to the specific vulnerabilities of the Colombian financial system at a specific point in time. These vulnerabilities are usually identified outside the model, using the analytical toolkit of the FSR and with the help of participants in the Colombian financial industry.

The stress testing is thus useful for quantitatively estimating the potential losses that financial institutions would suffer if the "perfect storm" crystallizes, and as a consequence credit, market, liquidity, funding and contagion risks are materialized. In addition, the stress test sheds light on the specific transmission mechanisms through which macroeconomic stress (or a particular vulnerability) may negatively affect the performance of individual financial institutions, thus translating itself into financial instability.

Although SYSMO is designed to be internally consistent, and to take into account the different feedback effects that may arise between the real and the financial sectors of the economy in times of stress, the model makes use of several extreme assumptions that are intended to capture the response of the financial system to a given macroeconomic scenario under controlled circumstances. Two important assumptions are worth emphasizing. On the one hand, policymakers are assumed to be passive with regard to the financial stress: although monetary policy responds systematically to the macroeconomic scenario (through a standard Taylor rule), economic authorities do not react to the deterioration of balance sheets of individual institutions and do not take preventive or palliative regulatory measures to preserve financial stability. On the other hand, financial institutions themselves are also presumed excessively passive, so long as they neither raise fresh capital (over and above retained earnings) nor take any other strategic initiative (such as mergers or increases in efficiency) to maintain profitability during the stress scenario. These assumptions are intended to assess the resilience of the financial system to the materialization of risks without any sort of external or internal help.

The model comprises three complementary components that interact dynamically with one another. First, a module to generate internally consistent scenarios. At this stage, this module consists of a dynamic, stochastic, general equilibrium (DSGE) model of the Colombian economy and a set of auxiliary specifications that produce internally consistent trajectories for key macroeconomic variables during the time horizon of the stress testing exercise. Second, a set of satellite risk models that capture the materialization of credit and market risks in times of stress. Third, a bank model that simulates the endogenous response of banks to an adverse scenario under the assumptions described above. SYSMO incorporates endogenous contagion and funding risks, key regulatory constraints (solvency and liquidity), and the feedback effects between the endogenous response of banks and the macroeconomic scenario from the DSGE model.

SYSMO resembles other analytical frameworks used by other central banks to perform their respective stress testing assessments. In its aim to include all potential sources of risk that financial institutions face, SYSMO resembles the MacroFinancial Risk Assessment Framework (MFRAF) of the Bank of Canada (Anand et al., 2014). In its approach to modeling banks as having an implicit target for the capital ratio, or in general, a target for the structure of the balance sheet, SYSMO is closest to the Bank of England's RAMSI Model (Burrows et al., 2012).

Earlier work on stress testing in Colombia include Amaya (2005) and Cabrera et al. (2012) (both of which used Vector Error Correction Models to predict credit quality under stress). These papers served as the conceptual basis for some of the components of SYSMO. This first version of it is thus the result of incremental work on stress testing techniques by the BR's Financial Stability Department. SYSMO is a flexible analytical tool that easily allows the incorporation of changes within the Colombian financial system or the introduction of novel techniques in any of its three main components. In this sense, SYSMO is a constantly evolving framework that will be updated in the future to include frontier techniques in stress testing analyses or to capture the natural evolution of the Colombian financial system.

This paper unfolds as follows. Section 1 presents an overview of the literature on stress testing models used by central banks. Section 2 describes the SYSMO framework, whereas Section 3 illustrates its use through the example of the stress testing exercise published in the BR's FSR of September 2017. Finally, Section 4 provides some reflections as concluding comments.

1 An Overview of the Literature on Stress Testing Models

Stress testing of financial institutions has been used since the early 1990s, when individual banks began to develop models for internal risk management purposes. These early stress tests tried to evaluate the effects on trading portfolios of a set of scenarios that resembled extreme past market events such as, for example, Black Monday. Eventually, the practice of using stress tests to evaluate trading portfolios was formalized and incorporated in the Basel Capital Accords. Credit risk stress tests, on the other hand, started to be performed only after the introduction of Basel II in 2004, despite credit risk being the most prominent vulnerability for banks at the time[1].

The use of stress tests within the Financial Sector Assessment Program (FSAP), established in 1999 by the International Monetary Fund (IMF) and the World Bank, stimulated central banks and supervisory authorities to develop their own stress testing techniques. The purpose of these tests is to capture the impact of severe, but plausible, shocks on the entire financial system. Prior to the global financial crisis, however, the results of the stress testing exercises were rarely used for regulatory or policy purposes, and there appear to be a consensus that they were not informative enough (Sorge, 2004). Based on the lessons from the crisis, a new generation of stress testing frameworks was developed.

Since then, stress tests have become an increasingly important tool for supervisors to assess the ability of the financial system to deal with adverse macroeconomic scenarios. Furthermore, the results of this type of exercises may have a strong policy impact. Examples of this include the US Supervisory Capital Assessment Program (SCAP) conducted by the Federal Reserve in 2009. The SCAP stress test assessed whether the largest US

[1] For a detailed historical review on stress testing see Dent et al. (2016).

banks had sufficient capital to absorb losses under a common stress scenario. SCAP results were publicly disclosed on a bank-by-bank basis, and those banks judged to need additional capital resources were given six months to raise that capital[2]. Other examples include the EU-wide stress tests performed by the European Banking Authority (EBA) since 2011.

Despite all the recent developments in stress testing methodologies, the basic conceptual paradigm remains the same. In general, a stress testing exercise may be bottom-up or top-down, or a combination of both[3]. In a bottom-up exercise, the central authority provides individual institutions with a common stress scenario, who use their own models to estimate the impact of this scenario on their financial statements. Finally, the central authority aggregates the results. In a top-down exercise such as SYSMO, the central authority does not involve individual institutions directly, instead estimating the impact of the stress scenario using its own internal models. These type of exercises provide an overall picture of the resilience of the financial system or banking sector, help to identify specific vulnerabilities, allow for transparent comparisons across financial institutions and are able to capture the impact that the actions by one institution may have on others within the financial system (Borio et al., 2014).

Foglia (2009) performs an overview of the core elements of the top-down stress testing frameworks in thirteen central banks. The starting point of a stress testing process is the design of a macroeconomic stress scenario. For this, the effects of the exogenous shocks on the economy are measured using a structural macroeconomic model, a vector autoregressive model or a pure statistical or historical approach.

In a structural macroeconomic model[4], the response to the exogenous shocks of various macroeconomic variables (e.g., GDP, interest rates, exchange rate) is projected over a stress test time horizon, which is typically two or three years[5]. This type of models allow

[2] This program led to a substantial recapitalization by forcing 10 bank holding companies to raise their capital (Acharya et al., 2014).

[3] The joint use of both bottom-up and top-down tests provides a better understanding of the vulnerabilities of the financial system. The results on one test can be used as a cross-check on the results of the other. It may also help financial institutions to improve their own stress testing practices (Borio et al., 2014).

[4] As in SYSMO, macroeconomic models developed for monetary policy purposes are often used in the design of macroeconomic stress scenarios.

[5] There is a trade-off when choosing the length of this time horizon. On the one hand, it must be long enough to allow for the consequences of the shocks to fully crystallize. On the other hand, a longer time horizon may involve an excessive loss of reliability in the estimates, especially in models that do not allow financial institutions to restructure their portfolios in response to shocks (Perez and Trucharte, 2013).

for endogenous policy reactions to the initial shock and imposes consistency across predicted values for key macroeconomic variables. However, they do not take into account the possible nonlinear relationships between macroeconomic variables that may appear during stress times, and it is difficult to determine the likelihood of a specific stress scenario (Foglia, 2009). Structural models have been used in the FSAP exercises as well as in several central bank exercises (e.g., Canada, Italy, the Netherlands, Sweden, Norway).

In a vector autoregressive model a set of macroeconomic variables is jointly affected by the initial shock and the vector process is used to project the impact on the variables over the scenario horizon. The attractiveness of these models relies on their flexibility and relatively simple way of producing consistent paths of macroeconomic variables. Also, as shocks sizes are typically specified in terms of standard deviations of the innovations, the likelihood of a stress scenario can be determined. These models have been used in stress test scenarios developed at the Bank of England, the Bank of Japan, and at the European Central Bank, among others.

Purely statistical approaches provide a statistically consistent path of macroeconomic variables. These methods, however, are less frequently used for policy analysis because they are not considered suitable for storytelling and make the proper communication of results extremely difficult (Drehmann, 2008). The Austrian Central Bank provides an example of the use of a purely statistical approach for scenario design. They develop a multivariate T-copula to model macroeconomic and financial variables. An advantage of using a copula is that macro-financial variables exhibit tail dependence, that is, their correlation increases under stress scenarios.

The second step in a stress testing process usually involves the estimation of satellite models that link the macroeconomic stress scenario to the balance sheets and income statements of financial institutions. Credit risk typically represents the core of the analysis. The approaches that relate macroeconomic variables to credit risk vary in terms of the measure of loan performance (e.g., non-performing loans, loan loss provisions, default probabilities), the level of aggregation (e.g., banking system, bank level, industry level, borrower level) and the estimation methodology[6]. Other satellite models may include models for market, income, liquidity, solvency and funding risks. The most sophisticated stress tests incorporate feedback effects within the financial sector as well as between the financial and real sectors of the economy (Borio et al., 2014).

[6]Non-linear specifications are often used as they capture possible nonlinearities between macroeconomic variables and the credit quality that are likely in stress situations (van den End et al., 2006).

The final step is evaluating whether the financial system is able to withstand the losses generated under the stress scenario. The metrics most frequently used are portfolio losses or capital adequacy ratios. Other metrics include liquidity adequacy, number of defaults or the size of capital injections required to recapitalize the system. At this point, assumptions on the behaviour of financial institutions with respect to their financial statements are usually made (Cihak, 2007). For instance, the assumption of the Bank of England (BoE) is that banks will use their profits to reach their target capital. Once they have reached their target, they will use the remaining profits to increase their risk-weighted assets, keeping the initial distribution of assets constant.

To date, the BoE's Risk Assessment Model of Systemic Institutions (RAMSI) is one of the most comprehensive top-down stress-testing models developed[7]. It allows for feedback channels within a bank and contagion across banks. The mechanism is that as bank fundamentals (i.e., profitability and solvency) worsen with the stress scenario, banks experience higher costs of funding, and if the fundamentals pass certain thresholds, banks can be shut out of certain funding markets. This feedback effect can in turn affect other banks that are perceived to be similar. However, the most direct forms of contagion occur when a bank suffers substantial losses on its portfolios and its capital ratio falls below the failure threshold. In this case, other banks can experience losses due to counterparty credit risk or asset fire sales. The feedback loop goes on until there are no more bank failures. The BoE runs two types of scenarios: an annual cyclical scenario[8], which is associated with the state of the financial cycle (the severity of the scenario increases as risks build up and decreases after those risks crystallize or abate, meaning the size of the shock is greater in good times)[9], and a biannual exploratory scenario, which is related to emerging threats perceived by policy makers to financial stability and individual banks (Burrows et al., 2012).

The Bank of Canada has also developed a rich stress test model - the MacroFinancial Risk Assessment Framework (MFRAF) (Anand et al., 2014). The MFRAF integrates funding liquidity risk as an endogenous outcome of the interactions between solvency risk and the liquidity profiles of banks. In contrast to RAMSI, it does not rely on exogenous thresholds

[7]The macrofinancial projections are generated in the RAMSI using a medium-scale Bayesian vector autoregression model (BVAR) that relies on a set of priors. The BVAR ensures a consistent forecast of macroeconomic variables, and allows the user to set the paths for certain variables.

[8]This stress test scenario is applied to all banks and building societies with total retail deposits greater than GBP 50b.

[9]This approach addresses the frequent criticism that stress test methodologies seem to ignore the dynamics of the financial cycle. According to Borio et al. (2013) financial crises are usually not the results of exogenous shocks, but of financial imbalances.

to impose funding constraints. Gauthier and Souissi (2012) argue that liquidity and solvency should not be treated as two separate dimensions, instead their interconnections should be modelled. This is something that became more evident after the global financial crisis. In MFRAF funding liquidity risk is modelled following the Basel III new Liquidity Coverage Ratio (LCR) framework. However, in contrast to the LCR, runs are not assumed exogenous. Instead, the likelihood of a run on each bank is endogenously determined by the market perceptions of the health of each bank. MFRAF also incorporates network externalities caused by counterparty risk. The size of interbank exposures, as well as factors that reinforce insolvency and funding liquidity risk, increase the likelihood of these spillover effects.

According to a recent overview on stress testing methodologies performed by Breuer and Summer (2017), the Federal Reserve (FED) is the most advanced institution in terms of modelling methodology and model validation. Following on from the 2009 SCAP, the FED launched two related stress testing programs with different purposes. The Dodd-Frank Act Stress Tests (DFAST) is applied to bank holding companies (BHCs) with assets greater than 10 billion USD. It has a sophisticated modelling of losses in which bank reactions and repricing strategies are taken into account. The evaluation of losses uses information from bottom-up exercises performed by banks as well as from top-down evaluations performed by authorities. The results contain information on capital adequacy for public disclosure and internal use of banks. On the other hand, the Comprehensive Capital Analysis and Review (CCAR) uses information from DFAST stress tests to evaluate the capital adequacy, the capital planning process, and the planned capital distributions (i.e., dividend payments and common stock repurchases) of BHCs with total assets of 50 billion USD or more (Federal Reserve, 2016).

Stress tests are likely to develop further over time to increase their usefulness for informing micro and macro prudential policy. Following Dent et al. (2016), some of the directions in which authorities are likely to move forward include: modelling the interconnectivity that exists across different types of risk (e.g., liquidity, funding and solvency risk), exploring a greater set of stress scenarios (to make this feasible, authorities are likely to make stress test more automated), integrating amplification and feedback mechanisms, incorporating realistic bank behavioural responses and extending the scope of the stress tests by including the interconnections between the banking sector and the wider financial system. This last point is important, as these interconnections have the potential to transmit and amplify shocks.

2 The Model

2.1 Design of the Macroeconomic Scenario

The starting point of any stress test is a statement with regard to the behavior of the macroeconomy during the time horizon considered in the exercise. In the case of SYSMO, trajectories for the main macroeconomic variables under stress are obtained using an estimated New Keynesian DSGE model of the Colombian economy and a set of auxiliary econometric specifications. The DSGE model was extended to incorporate a "credit channel" and the stress scenarios are produced given assumptions on total credit growth during the time horizon of the exercise. Specifically, this model produces internally consistent trajectories for GDP growth, inflation, and the short-term interest rate conditional on an assumed trajectory for credit growth.

The original framework used to obtain macroeconomic scenarios is the New Keynesian model of Smets and Wouters (2007). Although this model is the workhorse analytical tool for monetary policy in several central banks, it does not explicitly include a financial module (thus abstracting from depositors, banks, risk, financial distress or a credit channel). Therefore, to make the model suitable for the needs of a financial stress testing, SYSMO extended the original Smets and Wouters (2007) system of equilibrium equations to include a "credit channel" using the methodology proposed by Caldara et al. (2014). As this methodology is crucial to the scenario design module of SYSMO, it deserves some careful consideration.

The baseline Smets and Wouters (2007) model comprises 21 equilibrium equations with 7 exogenous variables described in Table 1. Let $\overline{\Theta}$ denote the parameter vector of the baseline model (which includes the standard deviation of each of the 7 orthogonal shocks). SYSMO estimates $\overline{\Theta}$ for the Colombian economy using standard Bayesian techniques and quarterly data on 7 observables: GDP growth, consumption growth, investment growth, policy rate, real wage growth, inflation and the labour force sourced from the BR and DANE (the Colombian National Statistics Office) for the period 2003:I-2017:II.

In order to extend this model to incorporate the *missing* "credit channel", SYSMO uses the method by Caldara et al. (2014). The method proposes to extend the baseline model by including the missing channel -in this case, credit growth- as a simple autoregressive specification. Letting ΔL_t denote total credit growth at time t, SYSMO proposes the following:

Table 1: Smets and Wouters (2007)

Variable	Written as:	Process
Expenditure	ε_t^g	AR(1)
Euler Equation Spread	ε_t^b	AR(1)
Invesment-Specific Technology	ε_t^i	AR(1)
Total Factor Productivity	ε_t^a	AR(1)
Price Markup	ε_t^p	ARMA(1,1)
Wage Markup	ε_t^w	ARMA(1,1)
Monetary Policy	ε_t^r	AR(1)

$$\Delta L_t = \rho_L \Delta L_{t-1} + \eta_t^L \tag{1}$$

where ρ_L captures the persistence of credit growth and η_t^L is a "structural" credit growth shock (to be specified below). Adding equation (1) to the equilibrium system of equations of the baseline model, Caldara et al. (2014) propose to rewrite these equations substituting the original exogenous variables for the following "extended" variables:

$$\tilde{\varepsilon}_t^j = \varepsilon_t^j + F_t^j \tag{2}$$

for $j \in \{g, b, a, i, p, r, w\}$, with:

$$F_t^j = \lambda^j \times \Delta L_t \tag{3}$$

In this fashion, the method creates correlated disturbances to the baseline model sourced from an observable variable outside of the original system of equations[10]. In this sense, the set of exogenous variables of the model is rewritten as a function of credit growth. As such, a "structural" credit growth shock is interpreted, within the context of the extended model, as a combination of the other structural shocks of the model[11].

[10] One disadvantage of the methodology is that there are excessive degrees of freedom with regard to the choice of F_t^j. In the specification of SYSMO, F_t^j includes only one observable, but in principle it could include more observable or unobservable variables.

[11] In a way, it is fair to argue that structural shocks coming from the missing channel included in the model do not exist as such; instead, they only exist as re-interpretations of combinations of the other, baseline structural shocks. Although this facilitates the introduction of credit shocks in a simple fashion as an alternative to constructing an otherwise extremely complicated and time consuming DSGE model, the extended model used here only captures and interprets credit shocks within the language of baseline shocks and not as independent economic forces. SYSMO operates under the belief that this is a valid way to interpret missing shocks in a DSGE model that, by construction, is not able to include all of the possible

From this extended model, standard Bayesian methods are used again to estimate ρ_L, λ^j for $j \in \{g, b, a, i, p, r, w\}$ and to reestimate the standard deviations of all orthogonal shocks (including η_t^L). As an additional exogenous variable is included in the extended model, an additional observable is required to estimate the model; in this case, quarterly data for the same period as above on total credit growth for all credit institutions (in what follows, CI) is used. All other parameter values in $\overline{\Theta}$ are kept from the first estimation of the baseline model. In this fashion, the data help indicate the dynamic dependence between the exogenous variables and credit growth while respecting the dynamic effects of all orthogonal shocks from the baseline estimation.

To construct trajectories for the key macroeconomic variables (GDP growth, inflation, short-term interest rate) under the hypothetically adverse scenario, an assumption is made regarding average quarterly credit growth during the time horizon of the exercise. Trajectories for all other observables are calculated from the extended DSGE model conditional on the assumed trajectory for credit growth. In this sense, the definition of the stress scenario corresponds to a specific stance on the evolution of credit during the horizon of the exercise.

Besides those trajectories produced by the extended DSGE model, SYSMO employs an additional set of auxiliary specifications to generate trajectories for other key macroeconomic variables. Specifically, additional trajectories are obtained for the unemployment rate, housing market prices, the industrial production index (IPI), the retail sales index and the spread between the intervention rate and rate of interest for every credit category. For these macroeconomic variables, SYSMO aims to guarantee the consistency between the DSGE model and this auxiliary specifications. To achieve this, SYSMO mostly uses vector autorregressions (VAR) containing these additional variables as well as those produced by the DSGE model as indicated in Table 2. Once these VARs are estimated, the trajectories of the DSGE model are used as inputs to construct the trajectories for the additional variables. The only exception corresponds to the Retail Sales Index, in which the assumed trajectory is exogenous and corresponds to a historical, stressed, observed path.

SYSMO incorporates feedback effects between the endogenous response of banks and the macroeconomic scenario from the DSGE model in the following way. The bank model that simulates the response of banks to the adverse scenario (as will be explained in Section 2.3) predicts some trajectory for total credit growth given a path for the growth of bank liabilities. To incorporate feedback effects, the aforementioned assumption on

missing channels.

credit growth is translated into an endogeneity condition for the growth of bank liabilities. More specifically, liabilities are assumed to grow at a rate that implies a trajectory of credit growth from the bank model that is consistent *on average* with the assumed credit growth from the extended DSGE model. In this sense, SYSMO incorporates feedback effects between the bank model and the DSGE model because the trajectories of the DSGE are consistent with the trajectories from the bank model and viceversa simultaneously.

Table 2: Other Variables Included in the Design of Macroeconomic Scenarios

Macroeconomic Variable	Methodology	Explanatory Variables
IPI	VAR	GDP, IPI
Loan Interest Rates Spread	VAR	GDP, TIB, Spread
Retail Sales Index	Stressed Historic Path	Retail Sales Index
Unemployment Rate	VAR	GDP, Unemployment Rate
Housing Price Index	VAR	GDP, Housing Price Index

2.2 Satellite Risk Models

2.2.1 Credit Risk

The purpose of the credit risk model is to evaluate how the macroeconomic scenario affects the loan portfolio composition by credit rating[12]. This is done in order to quantify the loan loss provisions that CI would face as a consequence of the adverse scenario. To perform this analysis, two dimensions are considered. In the first one, a statistical model is used to calculate the aggregate effects of the macroeconomic scenario over the proportion of loans rated different from A for each type of credit. The second dimension consists in the assessment of specific economic sectors or firms that may be vulnerable in the specific scenario that is being evaluated. In this case, an idiosyncratic shock that only affects the credit rating of those firms or sectors is considered.

i. *Aggregate Analysis*

To calculate the aggregate effects of the macroeconomic scenario on the loan portfolio composition by credit rating, a VAR model is estimated. A specific model for each category of loans (i.e. mortgages, consumer, business and micro loans) is utilized. Credit risk

[12] In Colombia, loans are rated as A, B, C, D and E, where A is the best rating and E is the worst.

Table 3: Variables Included in the VAR Models

Business loans	Change of the proportion of loans rated different from A GDP growth Business loans' interest rate IPI	Data since March 2003
Consumer loans	Change of the proportion of loans rated different from A GDP growth Consumer loans' interest rate Unemployment rate	Data since January 2001
Mortgages loans	Change of the proportion of loans rated different from A GDP growth Mortgages' interest rate Housing price index Unemployment rate	Data since January 2004
Micro loans	Change of the proportion of loans rated different from A Retail sales index Micro loans' interest rate Unemployment rate	Data since January 2002

Note: An economic activity indicator calculated by DANE is used as proxy of GDP.

is measured using the proportion of loans that are rated different from A. Additionally, the group of variables included in the estimation comes from the scenario design presented in Section 2.1, and changes depending on the specific loan category that is being analyzed. Likewise, the number of lags used in each model also varies depending on the category. In terms of the data used to estimate the models, monthly time series are used for each variable and the starting date depend on the credit category (see Table 3).

Once the models are estimated, the paths of the different variables that are obtained using the scenario design framework are included to generate trajectories for the proportion of the loan portfolio rated different from A. Additionally, as the purpose is to estimate the change of the loan portfolio distribution by credit rating, the change of the proportion of loans rated as B, C, D or E is calculated as one quarter of the change obtained for the proportion of loans rated different from A.

ii. *Idiosyncratic Analysis*

The second component of the credit risk analysis consists in the assessment of economic sectors or particular firms that are considered to be in a vulnerable financial situation at a given point in time, or are especially sensitive to the economic scenario evaluated in the stress exercise. Below are two examples to illustrate this type of exercise.

In a stress scenario that considers a sudden stop and a complete closure of the external financing sources for the country, it could be assumed that firms that usually find funding abroad, together with the government, increase their demand for loanable funds in the domestic market, potentially causing a crowding-out effect of other domestic borrowers. In this situation, banks usually do not have enough resources to lend and there will be firms unable to get loans. In this case, using the financial dependence indicator proposed by Rajan and Zingales (1998) helps to determine those firms that need external finance to operate. It is assumed that these firms that are thus sensitive to this adverse scenario default on their outstanding obligations with CI.

Another type of exercise consists in evaluating which firms or economic sectors are in a relatively vulnerable financial situation and, therefore, would be more affected in an adverse economic scenario. To perform this analysis we evaluate the behaviour of different credit risk indicators[13]. For those sectors with worse performance, it is assumed that they suffer a downgrade in its credit rating.

In summary, in both exercises the output is the trajectory for the distribution of CI loan portfolio by credit rating for each type of credit. This is used to calculate the loan loss provision expenses using the regulatory models proposed by the Colombian financial supervisor (SFC for its Spanish acronyms) and the CI interest income as will be described in Section 2.3 below.

2.2.2 Market Risk

Fixed income securities take the largest share of Colombian capital markets, accounting for more than 80% of total investments of market intermediaries. These securities are mainly composed of government securities (TES) and corporate bonds, and both of them represent an important portion of the CI investment portfolio. For this reason, SYSMO includes a quantitative mechanism to determine how external shocks affect the valuation of these securities and thus have an impact on the financial statements of CI.

There are several types of shocks that can affect the valuation of securities. SYSMO focuses only on the potential effects of a massive sale of securities held by a group of investors (hereafter referred to as "sellers"). In SYSMO, this methodology is used to model the loss of value produced by both sellers (exogenous risk) and by CI themselves (contagion risk). This section focuses on the former, whereas the latter is explained below in

[13]For instance, non-performing loans ratio or the loan portfolio composition by credit rating.

Section 2.3. Assume that a shock leads sellers to liquidate their positions in the securities they hold. This decision generates a loss in value of all of these securities as their supply increases, thus causing an upward shift of zero coupon yield curves.

To model this shock, there are two steps that have to be accomplished. The first is to determine the position of the sellers in the curve. Naturally, a massive sale of bonds would have a greater impact on the points of the curve where the sellers are positioned relative to those of the rest of the curve. The approach used by SYSMO consists in mapping cash flows onto vertices as proposed by RiskMetrics. Specifically, the cash flows of a given security are divided in different vertices. The criterion to define the place where the cash flow of the security will be assigned is their proximity to each vertex, according to (4):

$$\begin{aligned} CF &= CF_i + CF_{i+1} \\ CF &= \gamma CF + (1-\gamma) CF \end{aligned} \tag{4}$$

where i and $i+1$ are the closest vertices to the maturity of the cash flow, and γ is a proportion of the proximity to each vertex. In this model the vertices are measured in years to maturity as in (5):

$$V = \left\{ \frac{1}{12}, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 25 \right\} \tag{5}$$

The second step is to estimate the variation in the price of each vertex given a certain amount of sales of the securities of the vertex. To measure this impact, the quantity of securities sold is the main parameter in a downward-sloping inverse demand curve function as defined in Gauthier et al. (2012), that relates the price of the security to the amount traded:

$$p_i = \exp\left\{ -\alpha_i \left(\sum_j q_{ij} \right) \right\} \tag{6}$$

where p_i is the price in vertex i, $\sum_j q_{ij}$ is the demand for securities, and α_i is the inverse semi-elasticity of the demand curve. The shock, in this case, will be the sale of securities that will shift the supply curve to the right, changing the price of the securities and their implicit yield rate[14]:

[14] The implicit yield rate of bonds is calculated using the Taylor expansion of first order for a standard equation of price for zero coupon bonds.

$$\Delta y_i = \frac{-\alpha_i \sum_j \Delta q_{ij}}{t_i} \tag{7}$$

Using this expression, it is possible to reestimate the valuation of the position of all those other investors affected by the shock, i.e. those who did not sell. In order to calibrate α_i we use (8):

$$\alpha_i = y^i_{min} \frac{\zeta}{y^p_{min}} \tag{8}$$

where y^i_{min} is the minimum yield observed for vertex i, ζ is the maximum haircut that can be applied to a security and y^p_{min} is the minimum observed yield for the whole portfolio. In this case it is assumed $\zeta = 16\%$ as it is the maximum historical haircut applied to liquid assets by BR for its liquidity operations. This equation allows to estimate the impact of a certain amount of sales of a given group of investors on each vertex.

2.3 The Bank Model: Simulating Financial Statements

The macroeconomic scenario and the materialization of the aforementioned risks would impact the financial statements of CI through different transmission mechanisms. In order to capture these mechanisms, SYSMO uses a model to simulate the behaviour of banks under stress. This model seeks to determine how banks allocate available resources among different asset classes, as well as to capture the endogenous materialization of funding, liquidity, interest rate and solvency risks.

The bank model employs a wide set of simplifying assumptions about investment choices and the dynamics of deposits, interest rates and maturities. Financial institutions themselves are presumed neither to raise fresh capital (over and above retained earnings) nor to take any other strategic initiative to maintain profitability during the stress scenario. This assumption is intended to assess the resilience of CI to the joint materialization of risks without any sort of external or internal help. In addition, capital and liquidity requirements imply that noncompliance by any given CI may lead it to default on their financial obligations. The need to avoid falling below any regulatory requirement will give rise to contagion risk, as CI under stress will attempt to unload positions in order to comply with minimum capital and liquidity, potentially affecting the price for those positions.

2.3.1 Balance Sheet Structure and Regulatory Requirements

This model uses the following balance sheet structure simplification:

$$A_{it} = \sum_{j=1}^{J} A_{it}^{j} - P_{it} \tag{9}$$

$$L_{it} = \sum_{k=1}^{K} L_{it}^{k} \tag{10}$$

$$E_{it} = A_{it} - L_{it} \tag{11}$$

where A_{it} are total assets, L_{it} are total liabilities and E_{it} is total equity for institution i at time period t. On the one hand, total assets are discounted by loan loss provisions (P_{it}) and are divided in J asset classes (A_{it}^{j}): cash and cash equivalent (C_{it}, $j = 1$), loan portfolio (LP_{it}, $j = 2$), trading securities (T_{it}, $j = 3$), held-to-maturity securities (M_{it}, $j = 4$) and other assets (OA_{it}, $j = 5$). The order of the j-indexation is crucial for the reinvestment rule explained below, because it defines a prioritization among asset classes[15]:

$$\left\{A_{it}^{j}\right\}_{j=1}^{J} = \{C_{it}, LP_{it}, T_{it}, M_{it}, OA_{it}\} \tag{12}$$

On the other hand, total liabilities comprise K classes (L_{it}^{k}): deposits (D_{it}, $k = 1$) and other liabilities (OL_{it}, $k = 2$):

$$\left\{L_{it}^{k}\right\}_{k=1}^{K} = \{D_{it}, OL_{it}\} \tag{13}$$

Following Colombian financial regulation, SYSMO proposes that financial institutions must keep their common equity tier I ratio ($CET1_{it}$), capital adequacy ratio (CAR_{it}) and liquidity risk indicator (LRI_{it}) above regulatory limits[16]:

$$CET1_{it} = \frac{C1_{it}}{RWA_{it} + \frac{100}{9} VaR_{it}} \tag{14}$$

$$CAR_{it} = \frac{C2_{it}}{RWA_{it} + \frac{100}{9} VaR_{it}} \tag{15}$$

$$LRI_{it} = \frac{AL_{it}}{NCO_{it}} \tag{16}$$

[15] In a stress scenario, a preference for cash or cash equivalent is assumed, while the prioritization among other asset classes is in accordance to their current share on total assets as will be described below.

[16] In Colombia, regulatory limits for CAR, CET1 and LRI are 9%, 4.5% and 100%, respectively.

where $C1_{it}$ and $C2_{it}$ are the common equity tier 1 and the total regulatory capital, RWA_{it} corresponds to total risk weighted assets, VaR_{it} is the value at risk of the investment portfolio, LA_{it} are the liquid assets and NCO_{it} are the net cash outflows.

The main components that comprise the financial statements simulation are described in the following subsections.

2.3.2 Reinvestment Rule

In each period, CI allocate their available resources among asset classes with the objective of preserving their initial structure, considering solvency and liquidity requirements constraints. The starting point of the simulation is the amount of available resources at the beginning of each period (ar^0_{it}). They consist of loans amortizations (am_{it}) and the change in deposits and other liabilities:

$$ar^0_{it} = am_{it} + \Delta D_{it} + \Delta OL_{it} \tag{17}$$

Loan amortizations reduce the value of loan portfolio prior to the investment decisions. Each financial institution allocates available resources among asset classes following the sequence specified in Equation (12): first cash is determined, then loans, then trading securities, then non-trading securities and finally other assets.

The change in the asset class A^j will approach the amount required to reach the last observed share on total assets as much as permitted by available resources and regulatory target ratios (Equation (18)). In addition, SYSMO assumes a lower bound (lb^j_{it}) of zero for the change in loan portfolio, non-trading securities and other assets (i.e. this asset classes cannot be sold or written off), while for cash and trading assets this bound equals the current amount of the asset. Available resources can increase or decrease in each stage of the investment decision sequence (Equation (19)), highlighting the relevance of the order of asset classes in it. For example, in a stress scenario with scarce resources and massive losses it would be more likely to observe loans disbursements over non-trading assets purchases.

$$\Delta A^j_{it} = \max\left\{lb^j_{it}, \min\left[ar^{j-1}_{it}, \overline{S}^j_{it}, \overline{CET1}^j_{it}, \overline{CAR}^j_{it}, \overline{LRI}^j_{it}\right]\right\} \tag{18}$$

$$ar^j_{it} = ar^{j-1}_{it} - \Delta A^j_{it} \tag{19}$$

where $\bar{S}^j_{it}$ is the change in the asset class that would meet the last observed share of A^j

on total assets, and $\overline{CET1}^{j}_{it}$, $\overline{CAR}^{j}_{it}$ and $\overline{LRI}^{j}_{it}$ corresponds to the maximum changes in A^j that common equity tier I, total capital and liquidity target ratios would allow. These are determined in expressions (20) to (23):

$$
\begin{aligned}
\overline{S}^{j}_{it} &= \frac{A^{j}_{i0}}{A_{i0}} A_{it} - A^{j}_{it} && (20)\\
\overline{CET1}^{j}_{it} &= \frac{1}{\rho^{j}} \left[\frac{C1_{it}}{\beta} - \left(\frac{100}{9} VaR_{it} + RWA_{it} \right) \right] && (21)\\
\overline{CAR}^{j}_{it} &= \frac{1}{\rho^{j}} \left[\frac{C2_{it}}{\gamma} - \left(\frac{100}{9} VaR_{it} + RWA_{it} \right) \right] && (22)\\
\overline{LRI}^{j}_{it} &= \frac{1}{h^{j}_{i}} \left(LA_{it} - \lambda NCO_{it} \right) && (23)
\end{aligned}
$$

where ρ^j is the risk weight associated with asset class A^j, h^j_i is the corresponding average haircut[17] and β, γ and λ are target levels for common equity tier 1, total regulatory capital and liquidity ratios, respectively[18].

Given the new balance sheet structure in each period, the computation of profits before taxes (PBT_{it}) comprises the net interest income (NII_{it}), changes of market valuation (MV_{it}), changes in market valuation due to contagion (MV^c_{it}), loan loss provisions (ΔP_{it}), loan loss provisions due to contagion (ΔP^c_{it}) and net non-interest income (NNI_{it}). Depending on whether profits are positive or negative, CI will pay taxes and capitalize a specific portion of their benefits:

$$
\begin{aligned}
PBT_{it} &= NII_{it} + MV_{it} + MV^{c}_{it} - \Delta P_{it} - \Delta P^{c}_{it} + NNI_{it} && (24)\\
RP_{it} &= PBT_{it} \left(1 - \tau \cdot \mathbb{1}\left(PBT_{it} > 0\right)\right) \left(1 - \delta_i \cdot \mathbb{1}\left(PBT_{it} > 0\right)\right) && (25)
\end{aligned}
$$

where RP_{it} are retained profits (i.e. profits after taxes affecting regulatory capital), τ is the average tax rate and δ_i is the share of profits after taxes paid to shareholders as dividends[19]. According to Colombian regulation, retained profits increase tier 2 capital, while

[17] Colombian regulation establishes that haircuts for cash and public debt securities is equal to zero, while the haircut for loans is equal to one. In addition, the model assumes that other assets' haircut is also equal to one.

[18] In SYSMO, β, γ and λ corresponds to regulatory limits plus prudential margins of 50 basis points, 25 basis points and 10 percentage points, respectively

[19] The indicator function $\mathbb{1}\left(PBT_{it} > 0\right)$ takes the value of 1 if profits are positive and 0 otherwise.

negative profits reduce common equity tier 1:

$$\Delta C1_{it} = PBT_{it}\left(1 - \mathbb{1}\left(PBT_{it} > 0\right)\right) \quad (26)$$
$$\Delta C2_{it} = \Delta C1_{it} + RP_{it}\mathbb{1}\left(PBT_{it} > 0\right) \quad (27)$$

At the end of every fiscal year, institutions may transfer retained profits from tier 2 capital to common equity tier 1.

2.3.3 Exogenous Risks

The exogenous materialization of credit and market risk would impact different accounts of CI financial statements.

Regarding credit risk, downgrades in loan portfolio increase loan loss provisions and non-performing loans; the latter also implies a drop in amortizations and interest income. On the one hand, credit risk models generate paths of loan portfolio dynamics (LP_{rlit}) by credit type l and credit rating r[20]. Provisions are computed following the standard credit risk model of the SFC[21], considering a procyclical (PC) and a countercyclical (CC) component:

$$P_{it} = \sum_{l\in\mathcal{L}}\sum_{r\in\mathcal{R}} PC\left(LP_{rlit}\right) + CC\left(LP_{rlit}\right) \quad (28)$$

On the other hand, the impact of credit risk on amortizations at the beginning of next period depends on the performing loans for each credit type (PL_{lit}) and the proportion of loans that will be amortized each period (μ_{lit}):

$$am_{it} = \sum_{l\in\mathcal{L}} PL_{lit}\cdot\mu_{lit} \quad (29)$$
$$PL_{lit} = \sum_{r\in\mathcal{R}} \pi_{rli}\cdot LP_{rlit-1} \quad (30)$$

where π_{rli} is the share of performing loans for each credit rating r and credit type l, that is assumed to be constant during the time horizon of the stress testing exercise.

[20]Credit type l belongs to $\mathcal{L} = \{business, consumer, mortgage, micro\}$, while credit rating r belongs to $\mathcal{R} = \{A, B, C, D, E\}$.

[21]In Colombia, since the standard model was released in 2007, no credit institution has implemented an internal model.

Concerning market risk, gains or losses due to market valuation directly affect profits before taxes (Equation (24)) and the trading securities accounts according to the results of the market risk model:

$$T_{it} = T_{it-1} + \Delta T_{it} + MV_{it} + \Delta T^{c}_{it} + MV^{c}_{it} \tag{31}$$

Note that MV_{it} correspond to the portfolio valuation given the exogenous sales that stress scenario considers. Moreover, profits and the trading securities could also be endogenously affected by the materialization of contagion risk (ΔT^{c}_{it} and MV^{c}_{it}), interest rate risk and funding risk, explained in the following section.

2.3.4 Endogenous Risks

Along with the impact of exogenous risks materialization, CI response to the adverse scenario would generate the endogenous materialization of funding, banking book interest rate, liquidity, solvency and contagion risks.

i. *Funding Risk*

The model assumes that aggregate liabilities follow a specific path consistent with the macroeconomic scenario. The allocation of aggregate liabilities among CI depends on the type of institution and certain performance indicators (I^{m}_{it}). In particular, as returns on assets (ROA_{it}), total capital ratio (CAR_{it}) and performing loans ratios (PLR_{it}) decrease, institutions would exhibit lower funding growth rates.

$$\{I^{m}_{it}\}_{m=1}^{M} = \{ROA_{it}, CAR_{it}, PLR_{it}\} \tag{32}$$

SYSMO uses a standard panel data model to estimate the relationships between funding and the indicators employed (parameters $\hat{\phi}^{k}_{m}$ below). In addition, the model captures non-linearities in these relations, as well as heterogeneous effects by institution size ($Size_{it}$)[22]. In particular, when indicators exceed a vulnerability thresholds ($\bar{I}^{m}$), the relationships strengthen.

$$\begin{aligned} \Delta L^{k}_{it} &= \hat{\phi}^{0}_{i} k + \sum_{m=1}^{M} \hat{\phi}^{mk}_{1} I^{m}_{it} + \hat{\phi}^{mk}_{2} I^{m}_{it} \cdot \mathbb{1}\left(I^{m}_{it} < \bar{I}^{m}\right) \\ &\quad + \hat{\phi}^{mk}_{3} I^{m}_{it} Size_{it} + \hat{\phi}^{mk}_{4} I^{m}_{it} Size_{it} \cdot \mathbb{1}\left(I^{m}_{it} < \bar{I}^{m}\right) \end{aligned} \tag{33}$$

[22] $Size_{it}$ takes the value of 1 when the assets of institution i are above the 75^{th} percentile of CI assets in each period, 0 otherwise.

Naturally, trajectories from the panel data model are conditional to the path for aggregate liabilities, determined in the macroeconomic scenario.

ii. *Interest Rate Risk in the Banking Book*

The path of the policy interest rate associated with the macroeconomic scenario generates changes in the interest income and expenses. Consistent with the maturity transformation role of CI, liabilities often exhibit shorter maturities than assets. Therefore, an increase of the same magnitude in the interest rates of assets and liabilities would lead to an larger increment of interest expenses than interest income, reducing the intermediation margin.

Interest rates dynamics of different assets and liabilities (X_{it}^n) mainly depend on whether these are instruments of fixed or variable rate. Net interest income (NII_{it}) comprise fixed and variable net interest income (FII_{it} and VII_{it}):

$$NII_{it} = FII_{it} + VII_{it} \tag{34}$$

$$\{X_{it}^n\}_{n=1}^N = \{C_{it}, PL_{it}, T_{it}, M_{it}, OA_{it}, -D_{it}, -OL_{it}\} \tag{35}$$

$$FII_{it} = \sum_{n=1}^{N} \omega^n X_{it} \left[\sigma_{it}^n r_{it}^n + (1 - \sigma_{it}^n)\, r_{i0}^n\right] \tag{36}$$

$$VII_{it} = \sum_{n=1}^{N} (1 - \omega^n)\, X_{it} r_{it}^n \tag{37}$$

where ω^n is the share of assets or liabilities attached to a fixed interest rate and r_{i0}^n and r_{it}^n are the fixed and variable interest rate, respectively. In addition, σ_{it}^n is the share of assets or liabilities renewed in period t and follows an arbitrary path consistent with the assumptions about the dynamics of assets and liabilities' maturity composition in the stress scenario. For example, in an adverse context an increasing path of σ_{it}^n for liabilities would reflect greater difficulties for obtaining long-term funding.

The evolution of all interest rates follows the policy rate (r_t) and a risk premium (s_t^n). The paths for these variables come from the scenario design models as discussed above. The difference between fixed and variable instruments is that fixed interest rates are assumed to be repriced only at maturity[23].

$$r_{it}^n = r_{it-1}^n + \Delta R_t + \Delta s_t^n \tag{38}$$

[23]For the case of deposit interest rates, the model incorporates heterogeneous dynamics across depositor types (i.e. institutional depositors, official depositors and household depositors)

For fixed rate positions, it is assumed that they would be renewed at maturity with an interest rate that follows the dynamics of the policy rate (R_t) and its corresponding spread (s_t^n). For those attached to reference rates or other variables, the model follows the same rule for interest rate adjustment every period.

iii. *Liquidity Risk*

Based on Colombian regulation regarding LRI_{it}, SYSMO captures several mechanisms through which CI may not have enough liquid assets to cover their net cash outflows (i.e. LRI_{it} below the regulatory limit).

On the one hand, liquid assets (cash, trading securities and held-to-maturity securities with their corresponding haircut) can be reduced by endogenous investment decisions (Equation (18)), as well as by losses in market valuation in the case of trading securities (Equation (31)). On the other hand, net cash outflows are affected by changes in deposits and performing loan portfolio.

iii. *Solvency Risk*

In SYSMO, the change in the asset structure and the joint materialization of risks may have an impact on capital ratios through a set of transmission mechanisms, which may lead institutions to a situation of insolvency.

The first channel consists of the reduction of profits as a result of the following factors: i) greater expenditure on provisions, as a consequence of the default of other CI and credit risk materialization, ii) valuation losses generated by market risk materialization and iii) the evolution of net interest income according to the banking book interest rate risk materialization (Equation (21)). This drop in profits implies a reduction in total regulatory capital growth, so long as profits remain positive; if they are negative, a decrease in common equity tier 1 would be observed (equations (26) and (27)).

The second channel is the change in the RWA_{it} attributed to the dynamics of assets allocation. Moreover, in each period loan loss provisions reduce the amount of RWA_{it}. This may potentially have a positive impact on capital ratios, so long as their denominator would fall in these circumstances. The last channel corresponds to the value at risk component that is proportional to the amount of trading securities. Therefore, any change in the size of the portfolio of securities will have an impact on solvency ratios as well.

iii. *Contagion Risk*

As mentioned above, contagion risk emerges in SYSMO from the endogenous response to regulatory requirements by vulnerable institutions. The contagion risk module is a sequence of iterations where each iteration h consists of the following process. First, vulnerable institutions (VI_{ith}) are identified as those that exhibit performance indicators (solvency and liquidity) below the target ratios:

$$VI_{ith} = \mathbb{1}(CET1_{ith} < \beta) \vee \mathbb{1}(CAR_{ith} < \gamma) \vee \mathbb{1}(LRI_{ith} < \lambda) \tag{39}$$

Second, these institutions sell part of their trading securities in order to enhance their performance indicators. The sale amount is determined following the same logic of the aforementioned rule of investment:

$$\Delta T_{ith} = \max\left\{-T_{ith-1}, \min\left[\overline{CET1}^{3}_{it}, \overline{CAR}^{3}_{it}, \overline{LRI}^{3}_{it}\right]\right\} VI_{ith} \tag{40}$$

Third, the simultaneous sale of trading securities by vulnerable institutions produces asset fire sale losses for the whole system. These losses are considered as the indirect component of contagion and are computed using a function f that summarizes the market risk model explained in Section 2.2.2:

$$MV_{ith} = f\left(\sum_{i=1}^{E} \Delta T_{ith}\right) \tag{41}$$

where E is the total number of CI. Both sales and market valuation losses configure positive or negative changes in performance indicators. Those CI that exhibit indicators below regulatory limits, default on their financial obligations. These defaulting institutions (DI_{ith}) generate additional loan loss provisions (P_{ith}) to their counterparties, which constitute the direct component of contagion.

$$DI_{ith} = \mathbb{1}(CET1_{ith} < 4.5\%) \vee \mathbb{1}(CAR_{ith} < 9\%) \vee \mathbb{1}(LRI_{ith} < 100\%) \tag{42}$$

$$P_{ith} = \sum_{e=1}^{E} \Theta^{i}_{et} DI_{eth} \tag{43}$$

where Θ^{i}_{et} is the total debt of institution e to institution i. Finally, at the end of each iteration, losses due to direct and indirect contagion may result in more vulnerable institutions. The last iteration H is reached once the following conditions are satisfied: i) vulnerable institutions exhaust their trading securities so they cannot improve their performance indicators and ii) there are not new vulnerable institutions.

At the end of the contagion module, sales (ΔT_{it}^{c}), market valuation losses (MV_{it}^{c}) and loan loss provisions (P_{it}^{c}) due to spillovers at period t converge to:

$$\Delta T_{it}^{c} = \Delta T_{ith} \quad (44)$$

$$MV_{it}^{c} = MV_{ith} \quad (45)$$

$$P_{it}^{c} = P_{ith} \quad (46)$$

The result of the simulation process is the trajectories of several balance sheet accounts for all CI in the Colombian financial system. Given this set of variables, individual and aggregate financial indicators are computed in order to assess the performance and the resilience of CI under the stress scenario.

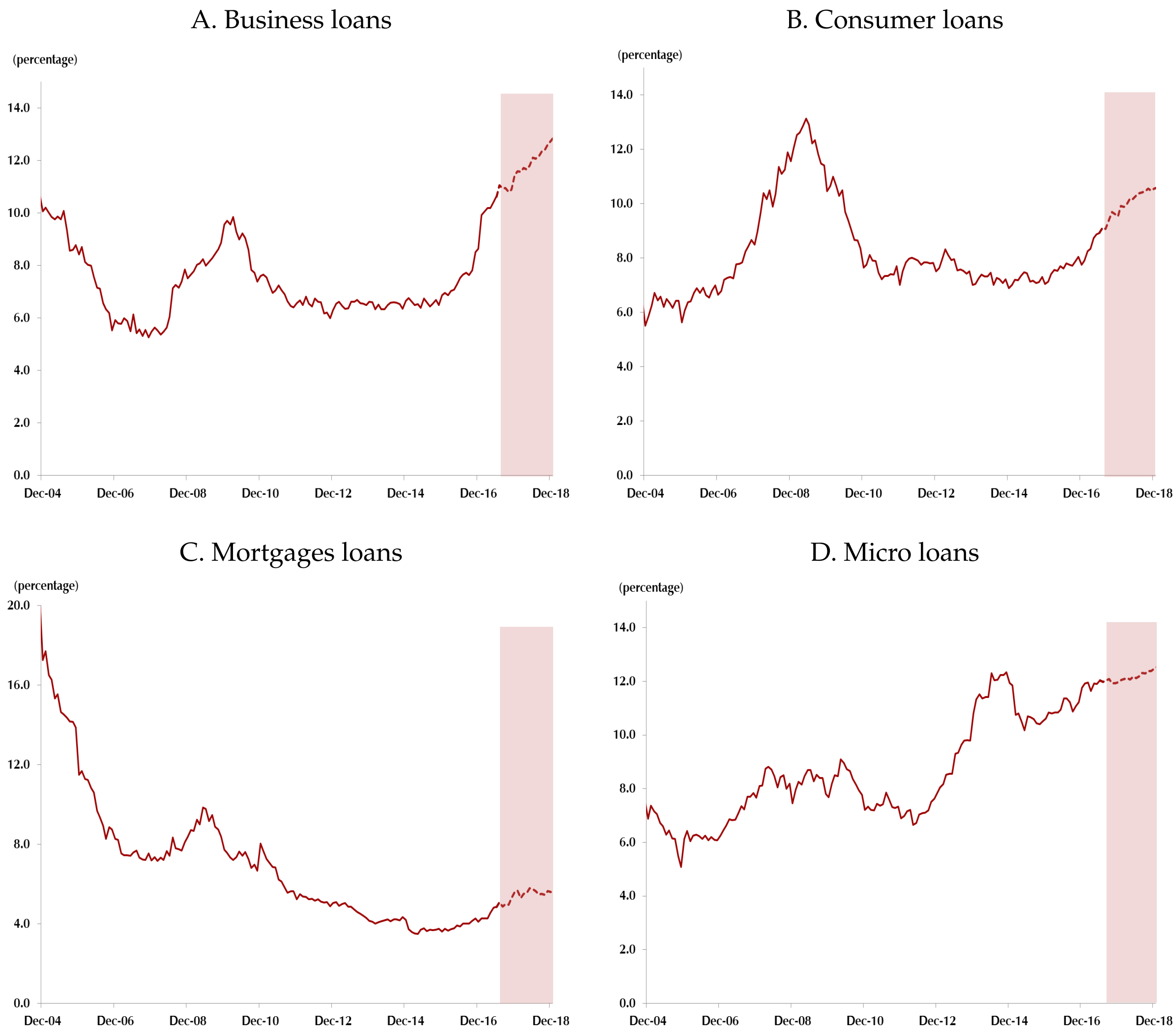
3 SYSMO at Work: An Example

3.1 Scenario

With the aim of illustrating the use of SYSMO, this section presents the results of the stress testing exercise published in the BR's FSR of the second semester of 2017. This exercise covers the time horizon between 2017:II and 2018:IV. Following Section 2.1, the macroeconomic scenario is constructed assuming zero average quarterly credit growth (and the associated path for aggregate liabilities) during the time horizon. This adverse scenario was motivated from the analysis of vulnerabilities of the FSR, which identified macroeconomic weakness as the main potential aggregate risk to CI at the time. The hypothetical aggregate effects of this macroeconomic scenario on the composition of the loan portfolio by credit rating, estimated using the VAR models of Section 2.2.1.i, are presented in Figure 1. In the figure, shaded areas report the trajectories from the VAR models for the time horizon of the stress test exercise, whereas non-shaded areas present observed data up until 2017:II.

As discussed in Section 2.2.1.ii, this scenario also included the materialization of idiosyncratic credit risk. Specifically, in this case all firms from the construction and health sectors were downgraded two credit ratings. These firms were identified to be in a *high risk* situation using a set of indicators within the FSR. In addition, specific firms from the electricity and transportation sectors were downgraded to rating D during the time horizon.

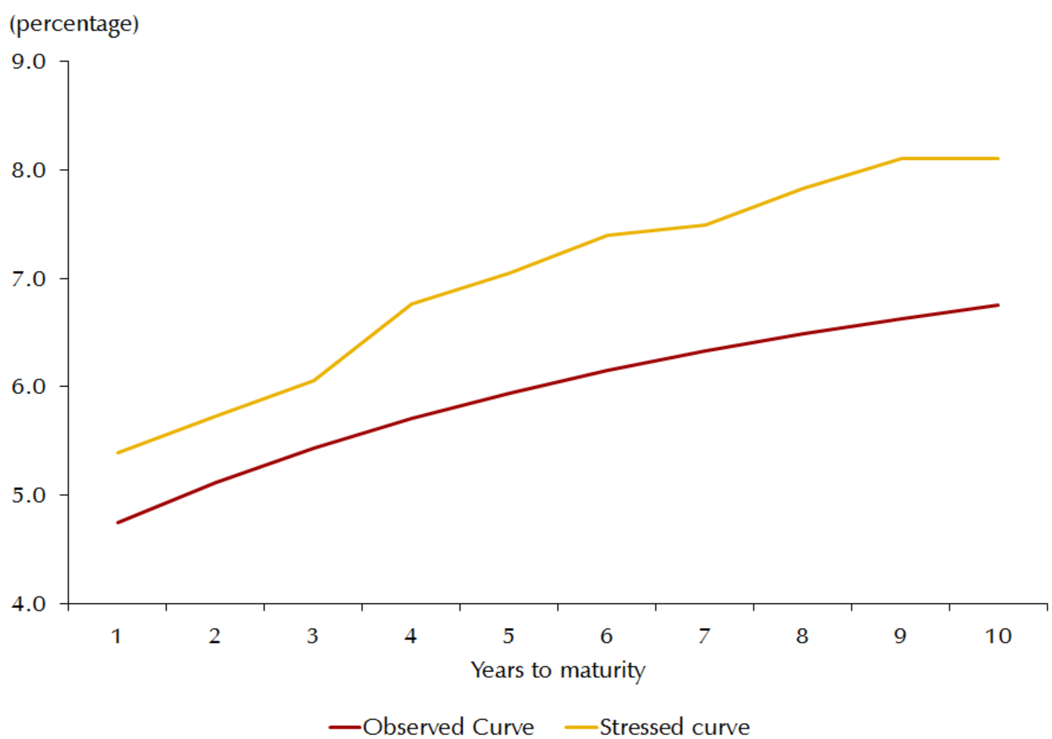
Figure 1: Proportion of Loans Rated Different From A - VAR Models



Source: Superintendencia Financiera de Colombia (until June 2017); authors' calculations.

The latter firms were identified to be in a *vulnerable* situation using additional indicators and expert judgment from the industry and from within the Financial Stability Department. Regarding exogenous market risk (see Section 2.2.2), the adverse scenario of this example assumes that all foreign investors liquidate their whole portfolio of fixed rate securities. Although this hypothetical liquidation is not necessarily consistent with the macroeconomic scenario, it is included in order to assess the resilience of CI to the joint materialization of market and credit risks. The hypothetical effect of this liquidation on the zero-coupon yield curve of Colombian government securities denominated in pesos (COP) is presented in Figure 2. The observed curve corresponds to June 30th, 2017. The massive sales by foreign investors cause a steeper yield curve given that there is a larger increment in long-term yields than in short-term ones. This difference is caused by the different participation of foreign investors in each vertex. In particular, in the short-term segment their average participation is 23.8% of total debt flows, while in the long-term part the average participation rises to 44.8%.

Figure 2: Zero Coupon Yield Curve

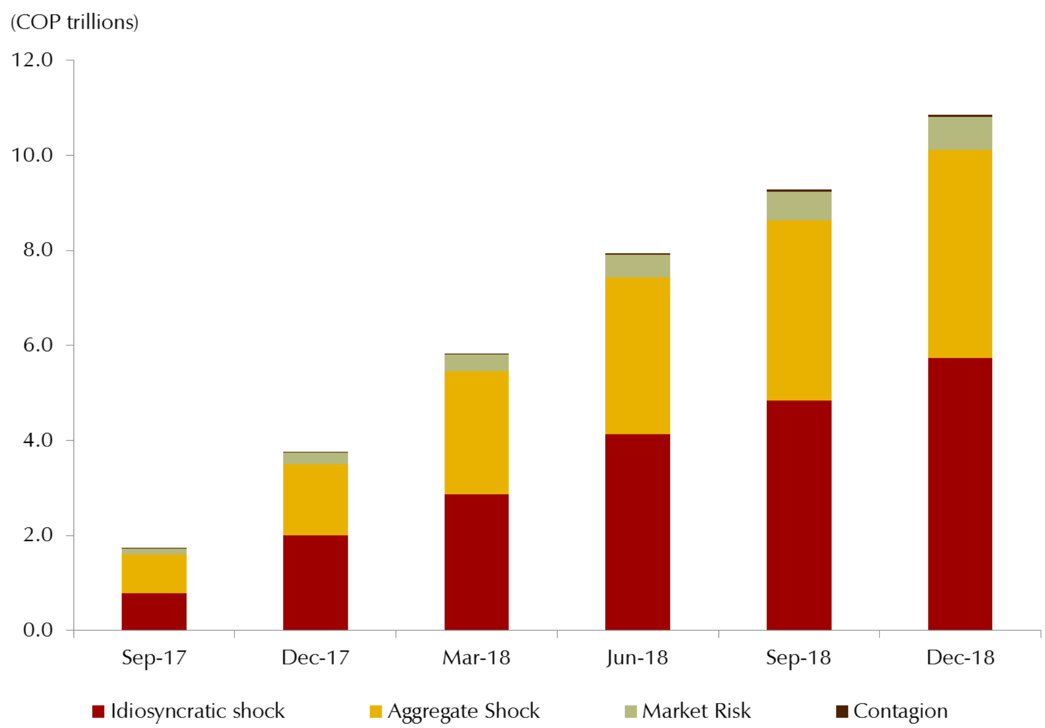


Source: *Infovalmer;* authors' calculations.

3.2 Results

The total cumulative losses of CI for the time horizon of the exercise that result from the macroeconomic stress scenario would be close COP 10.9 trillion(t), representing 13.2% of total regulatory capital of CI as of June 2017. Of this total, COP 5.7t correspond to loan loss provisions associated to *high risk* and *vulnerable* firms, COP 4.4t to loan loss provisions due to the general loan portfolio deterioration of Figure 1, COP 0.7t to market risk losses from the shift shown in Figure 2 and COP 0.1t to endogenous contagion risk losses (see Figure 3).

Figure 3: Credit Institutions Cumulative Losses



Source: Authors' calculations.

The aggregate dynamics of CI main financial variables under the stress scenario are presented in figures 4 to 7. As before, non-shaded areas correspond to observed data, while shaded areas represent the stress test time horizon. The real annual growth of the loan portfolio would be negative since the beginning of the exercise. The latter is a direct result of the low growth of aggregate liabilities that is assumed in the macroeconomic scenario (Figure 4). Figure 5 describes the dynamics of credit risk materialization, which is manifest in a sharp increase in non-performing loans.

The behaviour of CI profitability is presented in Figure 6. Aggregate losses caused by the joint materialization of risks would decrease profitability, causing ROA to fall from 1.5% to -0.1% between June 2017 and December 2018 under the stress scenario. Negative aggregate profitability would persist for 3 quarters, illustrating the severity of the effects from the stress scenario on the profits of financial institutions.

Figure 4: Real Annual Growth of Loan Portfolio

Source: Superintendencia Financiera de Colombia (until June 2017); authors' calculations.

Figure 5: Non-Performing Loans Ratio

Source: Superintendencia Financiera de Colombia (until June 2017); authors' calculations.

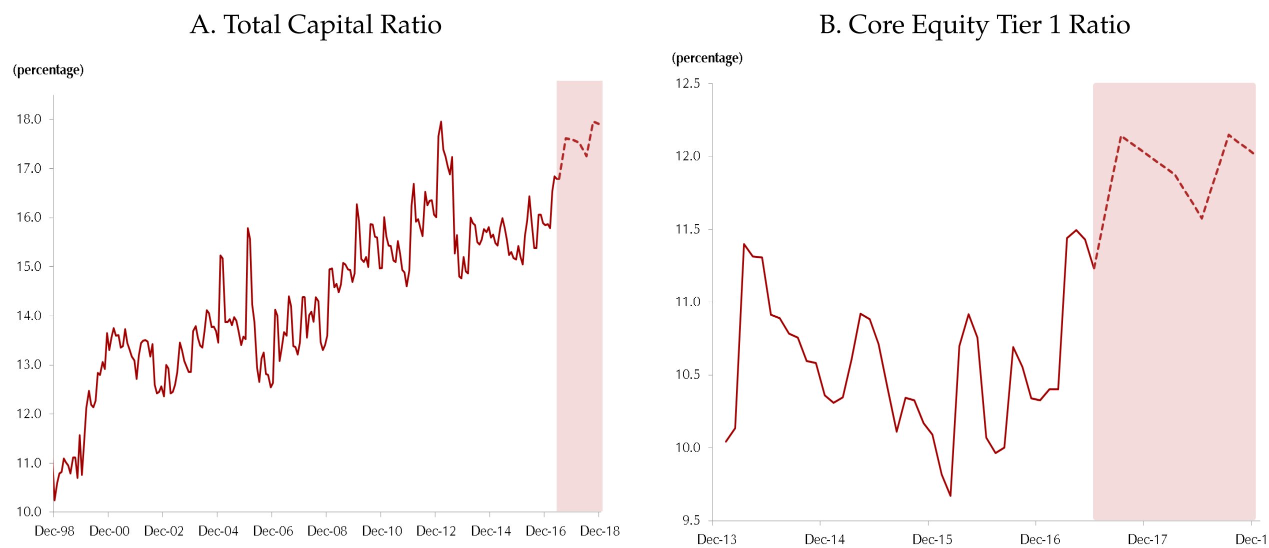
Figure 6: Return on Assets (ROA)

Source: Superintendencia Financiera de Colombia (until June 2017); authors' calculations.

Figure 7 presents the dynamics of aggregate total regulatory capital and common equity tier 1 ratios. Under the stress scenario, these ratios would increase thanks to the combination of two elements. First, the real negative growth of assets during the entire period of analysis would reduce the denominator of both capital ratios. Second, since aggregate profitability is negative only in the last quarters of the exercise, these are the only periods where there would be a negative effect over the numerator of the capital ratios.

Even though aggregate financial indicators would not exhibit high levels of deterioration under the stress scenario, nor would they be below regulatory minima, results for individual institutions show a remarkable heterogeneity. Figure 8 presents the distribution of individual profitability and total capital ratios among CI at the end of the time horizon of the exercise (2018:IV). Regarding the distribution of ROA, as the scenario unfolds an ever greater number of institutions would suffer financial losses: the percentage of CI with negative ROA would increase from 25% to roughly 50% between 2017:II and 2018:IV. With regard to individual total capital ratios, the majority of institutions show a similar dynamics to that of the aggregate indicator, although starting on the second quarter of the exercise some CI would register capital ratios below the regulatory minimum (these CI represent less than 10% of total CI assets). SYSMO operates in such a way that those institutions whose capital ratios fall below the regulatory minimum usually correspond to those that start the exercise with relatively low capital ratios. In this sense, SYSMO "pushes over the edge" those CI that have a weak financial position to begin with.

Figure 7: Capital Ratios



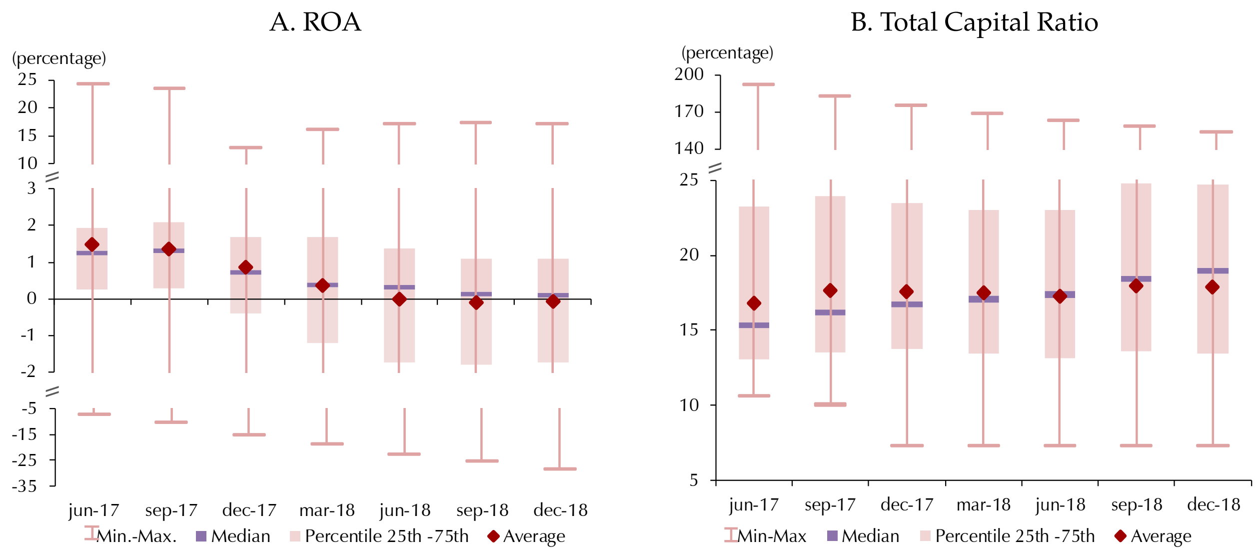
Source: Superintendencia Financiera de Colombia (until June 2017); authors' calculations.

4 Concluding Comments

As illustrated in the example above, a relatively adverse macroeconomic scenario would not present major disruption to the capital ratios of CI, and would manifest itself mostly in the form of reduced loan growth and profits. As is usually the case with stress testing models, there is a number of caveats that are necessary to bear in mind in order to give a correct interpretation of these results.

On the one hand, the exercise has been built along a number of key assumptions. Were some of this assumptions to be relaxed, the impact of the stress scenario would conceivably be reduced. Policymakers are assumed to be passive with regard to the financial stress: economic authorities do not react to the deterioration of balance sheets of individual institutions and do not take preventive or palliative regulatory measures to preserve financial stability. Similarly, financial institutions themselves are also presumed exces-

Figure 8: Results at the individual level - Distributions



Source: Superintendencia Financiera de Colombia (June 2017); authors' calculations.

sively passive, so long as they neither raise fresh capital nor take any other strategic initiative to keep afloat during the stress scenario. In addition, SYSMO produces the joint materialization of several risks, an event that is not necessarily consistent and therefore whose likelihood of occurrence may be extremely small. As said before, SYSMO does not present a forecast or a probabilistic assessment of the economy for a given time horizon.

On the other hand, some assumptions have the effect of reducing the impact of the stress scenario, and were they to be relaxed, its impact would be presumably stronger. Firstly, SYSMO only assesses the effects of a limited number of risks, thus limiting the possibility that risks may materialize or interact in unexpected ways. An example of this is that SYSMO does not analyze, at this stage, the interactions and risks attached to the participation of CI in payments systems' networks and infrastructures. Any shock to a given CI may impair its ability to abide by its compromises in payments systems, thus potentially affecting other institutions. Additionally, the starting point of SYSMO (and a key focus of the results) is the individual capital ratios of CI, leaving aside the analysis of solvency at a consolidated level. Given that solvency ratios tend to fall when computed at a consolidated basis, the effects of the stress scenario in "pushing over the edge" CI would be stronger.

Given these observations, what is the usefulness of a stress testing exercise in general, and of SYSMO in particular? A stress test exercise aims to assess the ability of financial institutions to sail through a hypothetical, adverse and deliberately extreme scenario in which several risks materialize simultaneously. From this point of view, the results of the exercise may serve as the basis to assess the policy response to a predetermined potential scenario. In addition, as any other theoretical model used to simulate the behaviour of the economy, the stress test allows to shed light on how do specific mechanisms operate in translating a given macroeconomic scenario into financial instability. In this case, the exercise allows the policymaker to learn which risks are more relevant for which class or group of CI, and also to disentangle how any particular vulnerability may cristallize after a given macroeconomic shock, something which is not immediately apparent from the analysis of observed data.

Future work on SYSMO will focus simultaneously on several research avenues. In the first place, it is clear that the bank model described above does not represent the solution to an explicit optimization problem for any given bank. In this sense, there is progress to be made in the refinement of the bank model as the solution to a structural bank problem along the lines of Corbae et al. (2017). In addition, there is a challenge to incorporate the materialization of credit and market risks within the macroeconomic model of scenario design. This would allow SYSMO to produce internally consistent risk scenarios for the two most important risks that are included at this stage. Work will focus on introducing new risks that are not being assessed at this stage. Among those, interbank exposures in payment systems and the role of big financial institutions other than CI (e.g. pension funds) in producing systemic risk come to mind. Finally, even though the model includes feedback effects between the macroeconomic scenario and the endogenous response of banks, work will be undertaken to include a more general form of feedback that includes not only the macroeconomic scenario and the response of banks but also the endogenous response of other sectors of the economy and the interaction between different risks and the scenario design module.

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