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Revista ESPE, No. 30, Art. 02, Diciembre de 1996
Páginas 41-112

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his paper studies the performance of two exchange rate regimes by using a dynamic general equilibrium model of the global economy in which nominal prices are temporarily rigid, and producers are monopolistically competitive. The performance of the regimes is evaluated in terms of the welfare effects that each regime supports when the economy is subjected to monetary, fiscal, and productivity shocks. The effort to provide the macroeconomic analysis of the global economy with microfoundations, renders a new view of the mechanisms and incentives operating in the positive reation of the economy to a variety of shocks. Also, it helps in the design of policy recomendations addressed to affect welfare.

* This paper is based on Chapter 1 of the author's Ph.D. dissertation at Columbia University. I would like to thank my advisor, Philip Lane, for extremely valuable guidance. Also, I would like to thank Phillip Cagan, Eduardo Fernández-Arias, Duncan Foley, Roberto Perotti, and Carlos Vegh for very useful discussions on various aspects of this paper. Martin Loser provided excellent editorial assistance.

I. INTRODUCTION

This paper presents an intertemporal two-country model with short-run price rigidities to study the welfare implications of alternative exchange rate regimes. Specifically, the paper evaluates two types of exchange rate regimes (henceforth *EERs*): a perfectly flexible *ERR*, and a completely fixed *ERR*.

The choice of an exchange rate regime is one of the most studied areas in international monetary economics, and one of the most pressing issues debated by policy makers; however, as Krugman (1993) indicates: "I would suggest that the issue of optimum currency areas, or, more broadly, that of choosing an exchange rate regime, should be regarded as the central intelectual question of international monetary economics". The goal of this paper is to contribute to that research agenda by, formally and explicitly, placing the evaluation of alternative ERRs in terms of the analysis of their welfare properties.

Until recently, the debates regarding the choice of the *ERR* were based on one version or another of the so called Mundell-Fleming model. Such a model, essentially, attempts to capture the general features of the way each *ERR* allowed the economy, as a whole, to adjust to a variety of shocks, of domestic or foreign origin. This type of aggregative analysis leaves aside a description of the mechanisms by which different shocks, under different *ERRs*, affect economic incentives, and assumes, implicitly, and for policy purposes, a one to one relationship between welfare and the so called 'intermediate policy targets'[1]. To a large extent such limitations were determined by the need of empirical relevance, but most essentially it reflected the fact that this model lacks the microfoundations required to analyze changes in economic incentives, and to undertake systematic welfare analyses. The need for microfoundations, therefore, is at the core of the analysis of alternative *ERRs* since it allows to explore what amounts to be the essence of such a problem: a trading off between macroeconomic flexibility and microeconomic efficiency. In that respect, the evaluation of alternative *ERRs* in terms of their welfare implications captures both the macroeconomic and the microeconomic dimensions of the effects that shocks of different nature may have on the economy.

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1 For example, output, the terms of trade, and the current account.

The need for a formal welfare evaluation of alternative *ERRs* is not a superfluous technical improvement upon the Mundell-Fleming (MF for short) model; on the contrary, it is essential for a sound policy formulation, otherwise the policy prescriptions could be profoundly misleading[2].

One feature of the *MF* model that is relevant for the analysis of alternative *ERRs* is that it allows the existence of nominal rigidities. The consideration of price rigidities is essential to study the behavior of the exchange rate, and it can be justified for two reasons. First, the empirical evidence suggests that the short-run volatility of the real exchange rate is similar to the short-run volatility of the nominal exchange rate, and that the short-run volatility of the real exchange rate is higher under a flexible *ERR* than under a fixed *ERR*. This evidence contradicts the assumption of perfectly flexible prices[3]. Second, since the evidence just mentioned can not be completely explained on the grounds of dominant real shocks, it gives the possibility that monetary shocks have real output effects. Fortunately it is possible to introduce such nominal rigidities in the context of an intertemporal optimization environment.

In the same line of thought, economists have found that imperfect competition is a salient feature of the global economic landmark, and that the features of the economies' business cycles can not be analyticaly captured unless a non-competitive market structure it is formally introduced in the models[4].

The model used in this paper is similar to the one developed by Obstfeld-Rogoff (1995a). It is a dynamic, two-country model with short-run nominal price rigidities and explicit microfoundations of the aggregate supply side which allows for formal welfare evaluation of international macroeconomic policies and institutions. The evaluation of the two *ERRs* is made by contrasting the effects that monetary, fiscal and productivity shocks have on the level of welfare of both economies operating under different regimes[5]. For the case of the fixed *ERR*, a monetary policy reaction function, specifying the money supply change required to prevent the exchange rate from changing after a particular shock, will be constructed. One limitation of the analysis in this paper is that it does not consider

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2 See Obstfeld-Rogoff (1995a) and Obstfeld-Rogoff (1996).

3 See Musa (1986), and Baxter and Stockman (1989).

4 See Hall (1986).

5 In Chapter II, the evaluation of the two *ERRs* will be based on the effects that the shocks induce on the *variability* of welfare.

issues related with the credibility of a fixed *ERR*, and therefore leaves aside the possibility of speculative attacks on fixed exchange rates. Also, it is assumed that in both countries there is an optimizing government that is concerned with short-run stabilization rather than with the effect of anti-inflation credibility.

The paper is organized as follows. Section II. sets up the structure of the model by spelling out the assumptions, and formulating clearly the decision problem confronted by the agents, as well as the rules by which they interact. In this section, also, a well defined steady-state equilibrium is derived, and the long-run and short-run equilibria of a log-linearized version of the model will be analyzed. Sections III., IV. and V. analyze the performance of each *ERR* under three general types of shocks: monetary, fiscal, and productivity. Finally, Section VI. concludes.

II. STRUCTURE OF THE MODEL

This section sets up the structure of the model and derives its properties under a particular steady-state when prices are completely flexible. It also analyzes the behavior of the model in the short run, when nominal prices are rigid.

A. ASSUMPTIONS REGARDING PREFERENCES, TECHNOLOGY, AND MARKET STRUCTURE

Assumption 1:

The world is populated by a continuum of individual monopolistically competitive producers indexed by $z \in [0,1]$, each of whom produces a single, differentiated, and perishable good[6]. Te world is divided in two countries: home and foreign. The home country is inhabited by producers in the interval $[0,n]$, and the foreign country by producers in the interval $(n,1]$.

Assumption 2:

All individuals in the world exhibit the same preferences, defined over a consumption index, real money balances, and effort spent in production.

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6 Since the goods are perishable, they can not be stored or accumulated. Therefore this model represents an economy in which there is no investment. However, this is not an endowment economy; output is endogenously determined.

The consumption index[7] on which utility is defined, is given by

$$(1)\qquad C=\left[\int_0^1 c(z)^{\frac{\theta-1}{\theta}}\,dz\right]^{\frac{\theta}{\theta-1}}$$

where $\theta>1$[8], and $c(z)$ is the home individual's consumption of good z or the typical individual demand faced by each monopolist. The consumption index for the foreign country, C^*, is defined in a similar way[9]. Note that each commodity enters symmetrically in the definition of the consumption index.

Assumption 3:

Residents of a particular country derive utility from the money of that country only, and not from the money of the other country. Let M_t be the stock of money held by domestic residents entering date $t+1$.

Assumption 4:

The preferences, as defined in assumption 2, are intertemporaly additive, and are represented by the following infinite-horizon time separable utility function

$$(2)\qquad U_t=\sum_{s=t}^{\infty}\beta^{s-t}u\left[\log C_s+\frac{\chi}{1-\varepsilon}\left(\frac{M_s}{P_s}\right)^{1-\varepsilon}-\frac{k}{2}y_s(z)^2\right]$$

where $0<\beta<1$ and $\varepsilon>0$. A foreign individual's utility function is defined similarly.

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7 By this artifice the model is placed in a macroeconomic context, since it looks like if there is a single aggregate commodity over which preferences are defined. Optimizing decisions over C are implicitly defined by optimizing decisions over $c(z)$.

8 θ is defined as the real price elasticity of demand faced by each monopolist. In consequence, for optimal output to be strictly positive, the producer should operate in the elastic portion of its demand curve; hence $\theta>1$.

9 Foreign variables are denoted with asterisks.

Assumption 5:

Each producer, in both countries, has access to the same production technology given by $y = Al^{\alpha}$, where y is the level of output, A is a productivity factor[10], l is the effort level, and $\alpha < 1$[11].

Assumption 6:

There are no barriers to trade between countries. In other words, the goods markets are perfectly integrated internationally.

Let $p(z)$ be the domestic currency price of good z, $p^*(z)$ the foreign currency price of the same good, and E the nominal exchange rate, defined as the home-currency price of the foreign currency. Given Assumption 6 and the previous definitions, by a simple arbitrage argument, the law of one price holds for every good, then

(3) $$p(z) = Ep^*(z) \quad \forall z \in [0,1].$$

Given the definition of the consumption index, by solving the following problem: choose $c(z)$ such that $Z = \int_0^1 p(z)c(z)dz$ is minimized subject to $C = \left[\int_0^1 c(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} = 1$, the consumption-based money price index[12], P, is obtained for the home country. P is therefore the minimum expenditure required to buy one unit of the consumption commodity index C, and it is given by

(4)
$$P = \left[\int_0^1 p(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}},$$
$$P = \left\{\int_0^n p(z)^{1-\theta} dz + \int_n^1 \left[Ep^*(z)\right]^{1-\theta} dz\right\}^{\frac{1}{1-\theta}}.$$

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10 Note that this productivity factor does not, strictly, follows an stochastic process. However, it might be subject to a once and for all unanticipated change.

11 Assuming $\alpha = \frac{1}{2}$ and $k = \frac{2\phi}{A^{\frac{1}{\alpha}}}$, the expression appearing in equation 1, representing the disutility of effort, is obtained. It is clear then that productivity shocks could be modelled as changes in k.

12 P is the price deflator for nominal balances.

Now, expanding this expression and using assumptions 2 and 5, the relationship between P and P^* can be obtained. It can be shown that

$$(5) \qquad P = EP^*.$$

Which means that P and P^* are related by purchasing power parity, *PPP*.

Assumption 7:

There is a perfectly integrated world capital market in which individuals from both countries can borrow and lend. The only asset in which they trade is a riskless real bond denominated in the composite consumption good. Let r_t be the real rate of interest earned on bonds between dates $t-1$ and t, and let F_t be the stock of bonds held by a domestic resident entering date t.

Assumption 8:

Home and foreign government purchases of consumption goods do not affect the private utility or productivity directly[13]. It is also assumed that governments take prices as given when they allocate their spending among goods. Per capita real home government consumption of individual government expenditure is given by

$$(6) \qquad G = \left[\int_0^1 g(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

where $g(z)$ is the home government consumption of individual good z. The same specification holds for G^*.

Assumption 9:

Ricardian Equivalence holds in this model. In consequence, it is assumed the all government spending is financed by taxes and seigniorage. Formally,

[13] The way in which the government is introduced in this version of the model implies that the focus of this paper is on the dynamic implications of fiscal policy.

(7)

$$G_t = T_t + \frac{M_t - M_{t-1}}{P_t},$$

$$G_t^* = T_t^* + \frac{M_t^* - M_{t-1}^*}{P_t^*}.$$

$T(T^*)$ denotes real taxes paid to the domestic (foreign) government.

Given the utility function (2), the home individual's demand for good z in period t is[14]

(8) $$c_t(z) = \left[\frac{p_t(z)}{P_t}\right]^{-\theta} C_t,$$

and similarly for each foreign individual.

Adding up private and government demands generates the world demand curve faced by each monopolist in period t :

(9) $$y_t^d(z) = \left[\frac{p_t(z)}{P_t}\right]^{-\theta} \left(C_t^W + G_t^W\right),$$

where

(10) $$C_t^W = nC_t + (1-n)C_t^*$$

is the world private consumption demand, and

(11) $$C_t^W = nC_t + (1-n)C_t^*$$

is the world government demand. Producers take $\left(C_t^W + G_t^W\right)$ as given.

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14 This demand curve is obtained as a solution of the following problem:

$$\max_{\{c(z)\}} C = \left[\int_0^1 c(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} \quad subject \ to \quad \int_0^1 p(z)c(z)dz = Z.$$

Assumption 10:

There is not uncertainty except for one-time unanticipated shocks.

Assumption 11:

The transversality condition[15]

$$(12)\qquad \lim_{T\to\infty}\left(\frac{1}{1+r_T}\right)^T\left(F_{t+T+1}+\frac{M_{t+T}}{P_{t+T}}\right)=0$$

holds.

Assumption 12:

In this model, all markets clear.

B. THE INDIVIDUAL 'S MAXIMIZATION PROBLEM

The period $t's$ budget constraint confronted by the typical domestic resident is given by

$$(13)\qquad M_t+P_tF_{t+1}=P_t\left(1+r_t\right)F_t+M_{t-1}+P_t.y_t(z)^{\frac{\theta-1}{\theta}}\left(C_t+G_t\right)^{\frac{1}{\theta}}-P_tC_t-P_tT_t$$

which is derived from the period $t's$ current account.

Given the symmetry of the model, the previously defined budget constraint, and the fact that the individuals take global demand, $C_t^W+G_t^W$, as given, the problem confronted by the typical home resident could be characterized as follows:

$$(14)\qquad max\ U_t=\sum_{s=t}^{\infty}\beta^{s-t}\left\{\begin{array}{l}\log\left[\left(1+r_s\right)F_s-F_{s+1}+y_s(z)^{\frac{\theta-1}{\theta}}\left(C_s^W+G_s^W\right)^{\frac{1}{\theta}}-G_s\right]\\ \qquad\qquad+\frac{\chi}{1-\varepsilon}\left(\frac{M_s}{P_s}\right)^{1-\varepsilon}-\frac{k}{2}y_s(z)^2\end{array}\right\}$$

15 This transversality condition includes the *No-Ponzi-Game* condition. In addition, it also includes the optimizing condition that requires the individual not to lend infinitely, in present value terms.

where use was made of the fact that $p_t(z)y_t(z) = P_t y_t(z)^{\frac{\theta-1}{\theta}}(C_t+G_t)^{\frac{1}{\theta}}$. A similar problem is confronted by the typical foreign individual.

The home-currency nominal interest rate at date t, i_t, can be obtained by a simple arbitrage argument as

(15) $$1+i_t = \frac{P_t}{P_{t-1}}(1+r_t),$$

with a similar definition for the foreign-currency nominal interest rate. Now, since purchasing power parity holds, and the real rate of interest is equal for both countries, uncovered interest parity (*UIP)* follows, and is given by

(16) $$1+i_t = \frac{E_t}{E_{t-1}}(1+i_t^*)$$

The first order conditions of the maximization problem with respect to F_{t+1}, M_t, and $y_t(z)$, which hold for any period t, are:

(17) $$C_{t+1} = \beta(1+r_{t+1})C_t$$

(18) $$C_{t+1}^* = \beta(1+r_{t+1})C_t^*,$$

that represent the typical Euler equations for the optimal consumption paths,

(19) $$\frac{M_t}{P_t} = \left[\chi C_t\left(\frac{1+i_{t+1}}{i_{t+1}}\right)\right]^{\frac{1}{\varepsilon}}$$

(20) $$\frac{M_t^*}{P_t^*} = \left[\chi C_t^*\left(\frac{1+i_{t+1}^*}{i_{t+1}^*}\right)\right]^{\frac{1}{\varepsilon}},$$

Tha give the efficiency conditions for hoding real balances. They indicate that the marginal rate of substitution between the services of real balances and the services of composite consumption must equal the opportunity cost of holding real balances. Expressions (19) and (20) indicate that the demand for real balances implied by this model depends on composite consumption and not on aggregate real income. This is due to the fact that the decision to hold money involves an opportunity cost that depends on the marginal utility of consumption.

Finally,

$$(21) \qquad y_t(z)^{\frac{\theta+1}{\theta}} = \left(\frac{\theta-1}{k\theta}\right)\left(C_t^W + G_s^W\right)^{\frac{1}{\theta}} \frac{1}{C_s}$$

$$(22) \qquad y_s^*(z)^{\frac{\theta+1}{\theta}} = \left(\frac{\theta-1}{k\theta}\right)\left(C_t^W + G_t^W\right)^{\frac{1}{\theta}} \frac{1}{C_t^*}$$

confirm that in this model, and under perfect price flexibility, output is endogenously determined. Conditions (21) and (22) indicate that in the output optimal path, the marginal revenue of the additional unit of output, in utility terms, must equal the marginal disutility of the additional effort required to produce it.

C. THE GLOBAL EQUILIBRIUM

Assumption 12 stated that in this model, all markets clear. This formally specifies the rules by which individuals interact in this model. More specifically, this assumption has the following implications:

(i) In the aggregate, the domestic money supply must equal the domestic money demand in each country.

(ii) The global net foreign assets are zero:

$$(23) \qquad nF_{t+1} + (1+n)F_{t+1}^* = 0$$

(iii) The goods markets clear; in other words global demand equals global real income. More precisely[16],

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[16] This condition is obtained from both countries' population-weighted budget constraints, by imposing the restriction in equation (22) and the governments' budget constraints (7).

$$C_t^W + G_t^W \equiv n(C_t + G_t) + (1-n)(C_t^* + G_t^*)$$

(24)

$$= n\frac{p_t(h)}{P_t}y_t(h) + (1-n)\frac{p_t^*(f)}{P_t^*}y_t^*(f) \equiv Y_t^W$$

where $y_t(h)$ and $p_t(h)$ are the typical home output and price, and $y_t^*(f)$ and $p_t^*(f)$ are the typical foreign home output and price[17].

D. A SYMMETRIC STEADY STATE

To analyze the effects that exogenous shocks have on the endogenous variables, a log linearized version of this model will be studied. To implement such linearization, a well defined flexible-price steady state should be first obtained. The steady state that will be analyzed here corresponds to the situation where all exogenous variables are constant[18]. Since in the steady state the consumption and output are constant, the Euler equations (17) and (18) determine the steady state real interest rate

(25) $$\bar{r} = \delta \equiv \frac{1-\beta}{\beta}$$

where δ is the rate of time preference. For notational purposes, all steady sate state variables are denoted with overbars.

In steady state the countries' current accounts are zero; therefore, the intertemporal budget constraints jointly with the global asset market condition (23) imply the following expressions for per capita steady state consumption:

(26) $$\bar{C} = \bar{r}\bar{F} + \frac{\bar{p}(h)}{\bar{P}}\bar{y}(h) - \bar{G}$$

(27) $$\bar{C}^* = -\bar{r}\left(\frac{n}{1-n}\right)\bar{F} + \frac{\bar{p}^*(f)}{\bar{P}^*}\bar{y}^*(f) - \bar{G}^*$$

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17 These expressions for home and foreign prices and output are obtained by exploiting the symmetry of the model.

18 In fact, they are zero.

The particular steady state that will be considered is one in which there are zero net foreign assets, and equal per capita government spending. Specifically, this particular steady state is symmetric, in the sense that both countries will have identical per capita output levels, and real money balances. This symmetric steady state will be denoted by zero subscripts with both $\overline{F}_0 = \overline{F}_0^* = 0$ and $\overline{G}_0 = \overline{G}_0^* = 0$.

From the first order conditions (21) and (22), which give the restrictions on the optimal choice of output, the equilibrium output is given by

$$(28) \qquad \overline{y}_0 = \overline{y}_0^* = \left(\frac{\theta - 1}{\theta_k}\right)^{\frac{1}{2}}.$$

Two specific features of this expression are worth noting. First, with flexible prices the classical dichotomy holds: output is independent of monetary factors. Second, this equilibrium output is typical of models with monopolistic competition as in Blanchard and Kiyotaki (1987) and Mankiw (1988). It reflects the fact that in a decentralized equilibrium the marginal utility of the additional revenue exceeds the marginal disutility from the additional effort[19]. This is basically due to the fact that each individual producer does not have an incentive to reduce his price since he can not appropiate completely the benefit from this action, which is a reduction in the price level.

From the first order conditions (19) and (20), and the steady state real rate of interest condition (25), the steady state real balances are

$$(29) \qquad \frac{\overline{M}_0}{\overline{P}_0} = \frac{\overline{M}_0^*}{\overline{P}_0^*} = \left(\frac{1-\beta}{\chi}\right)^{-\frac{1}{\varepsilon}} \overline{y}_0^{\frac{1}{\varepsilon}}.$$

The main feature of this equation is that, since the steady state inflation is zero, the steady state real money balances depend on the real interest rate.

It is important to note that in this *symmetric* steady state

19 See Obsteld and Rogoff (1996), p. 668, for a detailed exposition of this feature. Note, however, that as $\theta \to \infty$ the steady state output approaches its competitive level.

(30) $\bar{y}_0 = \bar{y}_0^* = \overline{C}_0 = \overline{C}_0^* = \overline{C}_0^W$,

and

(31) $\bar{p}_0(h) = \overline{P}_0, \bar{p}_0^*(f) = \overline{P}_0^*$.

As a consequence of the last two equalities

(32) $\bar{p}_0(h) = \overline{E}_0 \bar{p}_0^*(f)$,

which means that the terms of trade are equal to 1. In general, producers in *each* country are symmetric; for the particular case where $\overline{F}_0 = \overline{F}_0^* = 0$ and $\overline{G}_0 = \overline{G}_0^* = 0$, producers in *both* countries are symmetric.

E. LOG-LINEARIZATION OF THE MODEL AROUND THE SYMMETRIC STEADY STATE

To implement the log-linearization, the model will be expressed in terms of deviations from the symmetric steady state path. Essentially the procedure consists in obtaining the expression $log\, \overline{X}_t = log\, \overline{X}_0 + \frac{dX_t}{\overline{X}_0}$ for any variable X, and its initial steady state value $\overline{X}_0$. Let $\hat{X}_t \equiv \frac{dX_t}{\overline{X}_0}$ be the percentage change from the symmetric steady state at date t. In what follows, a description of the linearized expressions is presented, where use of assumption 10 was made in their derivation.

(33) $\hat{E}_t = \hat{P}_t - \hat{P}_t^*$,

from the purchasing power parity relationship (5). This expression indicates that deviation of the exchange rate from its steady state is explained by inflation differentials.

The inflation rate in each country is the percentage deviation of their consumer price levels from their respective steady states:

(34) $$\hat{P}_t = n\left[\hat{p}_t(h)\right] + (1-n)\left[\hat{E}_t + \hat{p}_t^*(f)\right]$$

and

(35) $$\hat{P}_t^* = n\left[\hat{p}_t(h) - \hat{E}_t\right] + (1-n)\left[\hat{p}_t^*(f)\right],$$

where use was made of the symmetry across each country's producers, and of expression (32).

Linearization of the global goods market equilibrium condition (24) gives the following expression for the change in world private demand:

(36) $$\hat{C}_t^W = n\left[\hat{p}_t(h) + \hat{y}_0 - \hat{P}_t\right] + (1-n)\left[\hat{p}_t^*(f) + \hat{y}_t^* - \hat{P}_t^*\right] - \frac{dG_t^W}{\overline{C}_0^W}.$$

The linearized expressions of the world demand for the typical home product (9), and its foreign version are:

(37) $$\hat{y}_t = \theta\left[\hat{P}_t - \hat{p}_t(h)\right] + \hat{C}_t^W + \frac{dG_t^W}{\overline{C}_0^W}$$

and

(38) $$\hat{y}_t^* = \theta\left[\hat{P}_t^* - \hat{p}_t^*(f)\right] + \hat{C}_t^W + \frac{dG_t^W}{\overline{C}_0^W}.$$

The optimal flexible-price equations (21) and (22), which give the labor-leisure trade-offs, take the following form:

(39) $$(\theta+1)\hat{y}_t = -\theta\hat{C}_t + \hat{C}_t^W + \frac{dG_t^W}{\overline{C}_0^W}$$

and

(40) $$\left(\theta+1\right)\hat{y}_t^* = -\theta\hat{C}_t^* + \hat{C}_t^W + \frac{dG_t^W}{\overline{C}_0^W}.$$

The consumption Euler equations (17) and (22) are aproximated by

(41) $$\hat{C}_{t+1} = \hat{C}_t + (1-\beta)\hat{r}_{t+1}$$

and

(42) $$\hat{C}_{t+1}^* = \hat{C}_t^* + (1-\beta)\hat{r}_{t+1}.$$

Finally, the real balances (19) and (20) take the following expressions:

(43) $$\hat{M}_t - \hat{P}_t = \frac{1}{\varepsilon}\hat{C}_t - \frac{\beta}{\varepsilon}\left(\hat{r}_t + \frac{\hat{P}_{t+1} - \hat{P}_t}{1-\beta}\right)$$

and

(44) $$\hat{M}_t^* - \hat{P}_t^* = \frac{1}{\varepsilon}\hat{C}_t^* - \frac{\beta}{\varepsilon}\left(\hat{r}_t + \frac{\hat{P}_{t+1}^* - \hat{P}_t^*}{1-\beta}\right).$$

Substracting equation (43) from equation (44), and using the *PPP* condition (5) gives the following expression for the change in the exchange rate:

(45) $$\hat{M}_t - \hat{M}_t^* - \hat{E}_t = \frac{1}{\varepsilon}\left(\hat{C}_t - \hat{C}_t^*\right) - \frac{\beta}{\varepsilon(1-\beta)}\left(\hat{E}_{t+1} - \hat{E}_t\right).$$

Having implemented all required linearizations, it is now possible to solve the model. This will be done in two parts: first for the new steady state under flexible prices, and second for the short-run dynamics under temporary sticky prices.

F. SOLVING FOR THE NEW STEADY STATE

Essentially the following log-linearization will be implemented: $\log \overline{X} = \log \overline{X}_0 + \frac{d\overline{X}}{\overline{X}_0}$, for any variable X. Let $\hat{\overline{X}} = \frac{d\overline{X}}{\overline{X}_0}$ denote the percentage change in a steady-state value.

To solve for the new steady state, as a function of exogenous shocks, it is necessary to linearize equations (26) and (27), which yield

$$(46) \qquad \hat{\overline{C}} = \bar{r}\frac{d\overline{F}_0}{\overline{C}_0^W} + \hat{\overline{p}}(h) + \hat{\overline{y}} - \hat{\overline{P}} - \frac{d\overline{G}^W}{\overline{C}_0^W}$$

and

$$(47) \qquad \hat{\overline{C}}^* = -\bar{r}\left(\frac{n}{1-n}\right)\frac{d\overline{F}_0}{\overline{C}_0^W} + \hat{\overline{p}}^*(h) + \hat{\overline{y}}^* - \hat{\overline{P}}^* - \frac{d\overline{G}^W}{\overline{C}_0^W}.$$

Note that in the solution for the new steady state, the change in the net value of foreign assets, the current account, is considered an exogenous variable. As will be seen in the next section, the current account is an endogenous variable which is determined by the whole system. In the steady state its value is zero, as implied by equations (26) and (27); however, in the short run it is determined by the temporary disequilibriums between income and spending induced by shocks in the presence of nominal rigidities.

Finally, observe that equations (33)-(40) hold across steady states, and together with equations (46) and (47), they provide seven equations in the seven unknowns $\hat{\overline{C}}$, $\hat{\overline{C}}^*$, $\hat{\overline{y}}$, $\hat{\overline{y}}^*$, $\hat{\overline{p}}(h)-\hat{\overline{P}}$, $\hat{\overline{p}}^*(h)-\hat{\overline{P}}^*$ and $\hat{\overline{C}}^W$, and are used to obtain the new real steady state.

The method used here to obtain closed form solutions for the level of individual variables exploits the model's symmetry. Specifically, it solves for the differences between the per capita home and foreign variables first, and then for the population-weighted world aggregates[20].

20 This approach was developed by Aoki (1981); it was the one used by Obstfeld and Rogoff (1995a), and it is fully explained in Obstfeld and Rogoff (1996).

G. THE SHORT-RUN EQUILIBRIUM CONDITIONS

The short run is defined as a period of time during which producer nominal prices are fixed. Such rigidity will not be rationalized, it will be stated as an assumption[21].

Assumption 13:

The domestic-currency price of domestic goods $p(h)$, and the foreign-currency price of the foreign good $p^*(f)$, are set one period in advance, but they will completely adjust to flexible-price levels after a period, provided there are no new shocks.

Assumption 13 jointly with the existence of monopolisticaly competitive producers, stated in Assumption 1, imply that in the short run output is demand-determined. This means that in the short run, equations (39) and (40) are not binding. Consequently, output will be determined by equations (37) and (38).

It is important to note, however, that $p(f)$ and $p^*(h)$ change if the exchange rate changes, otherwise the law of one price does not hold.

Under Assumption 13, equations (34) and (35) become

$$(48) \qquad \hat{P} = (1-n)\hat{E}$$

and

$$(49) \qquad \hat{P}^* = -n\hat{E}.$$

For notational purposes, hatted variables without time subscripts denote short-run deviations from the symmetric steady-state path. Using these last two equations together with equations (37) and (38), generate the following short-run aggregate demand expressions:

$$(50) \qquad \hat{y} = \theta(1-n)\hat{E} + \hat{C}^W + \frac{dG^W}{\overline{C}_0^W}$$

[21] A rationalization of this assumption, that is consistent with the methodology of the approach used in this paper, is the small menu cost approach used by Akerlof and Yellen (1985) and Mankiw (1985).

and

$$(51) \qquad \hat{y} = -\theta n\hat{E} + \hat{C}^W + \frac{dG^W}{\overline{C}_0^W},$$

where $\hat{C}^W$ is given by (36), and differentials without time subscripts represent short-run changes.

As was mentioned in the previous section, the current account is determined simultaneously with the whole model's intertemporal equilibrium; in the short run, the current account equations are:

$$(52) \qquad \frac{d\overline{F}}{\overline{C}_0^W} = \hat{y} - \hat{C} - (1-n)\hat{E} - \frac{dG}{\overline{C}_0^W}$$

and

$$(53) \qquad \frac{d\overline{F}^*}{\overline{C}_0^W} = \hat{y}^* - \hat{C}^* + n\hat{E} - \frac{dG^*}{\overline{C}_0^W} = -\left(\frac{n}{1-n}\right)\frac{d\overline{F}_0}{\overline{C}_0^W}.$$

where use was made of (13), (7), (48), (49), and the fact that $\overline{F}_0 = \overline{F}_0^* = 0$.

The remaining equations that characterize the short-run equilibrium are (41)-(44), which always hold. Note that in (41)-(44), all t-subscripted variables now represent short-run variables, while all $t+1$-subscripted variables represent steady-state changes.

In the short run, ten variables are to be determined: $\hat{C}^*, \hat{C}, \hat{y}, \hat{y}^*, \hat{P}, \hat{P}^*, \hat{E}, \hat{C}^W, \hat{r},$ and $d\overline{F}$. The ten equations that jointly determine them are (36), (41)-(44), and (48)-(52). As was indicated before, the approach that is taken in this paper exploits the symmetry of the model.

The stage is now set for the analysis of the effects of shocks to the system under different *ERRs*.

III. MONETARY SHOCKS

This section assumes that the foreign country[22] experiences a permanent unanticipated change in its money supply[23], formally $\hat{M}^* = \hat{\overline{M}}^* \neq 0$ [24], and looks at the effects that these changes have on the welfare of both economies under the two *ERRs*. It is also assumed, for simplification, that $dG = d\overline{G} = dG^* = d\overline{G}^* = 0$. This last assumption does not impinge upon the results of this section since the effects of monetary and fiscal shocks are additive.

One word about timing is important in clarifying the analysis of this section. The shock to the foreign money supply occurs at date 1. During period one the nominal prices remain at their initial symmetric steady-state level, and from period two on, the economies operate on their new steady state.

The new steady state, in the presence of monetary shocks, is governed by following equations:

(54) $$\hat{\overline{y}}^W = \hat{\overline{C}}^W = 0,$$

(55) $$\hat{\overline{C}} = \frac{1+\theta}{2\theta}\left(\frac{\bar{r}d\overline{F}}{\overline{C}_0^W}\right),$$

(56) $$\hat{\overline{C}}^* = -\frac{n}{1-n}\left(\frac{1+\theta}{2\theta}\right)\frac{\bar{r}d\overline{F}}{\overline{C}_0^W},$$

(57) $$\hat{\overline{y}} = -\frac{1}{2}\left(\frac{\bar{r}d\overline{F}}{\overline{C}_0^W}\right),$$

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22 The analysis of the different shocks in this paper is undertaken from the perspective of the home country. That is, it assumes that it is the home country the one that follows the policy of keeping the exchange rate fixed.

23 The difference between a permanent and a transitory shock is that a transitory shock lasts only during the first period, while the permanent shock lasts forever.

24 $\hat{M}\left(\hat{M}^*\right)$ is defined as the percentage deviation of the date 1 money supply from the initial steady state. This section proceeds by analyzing an increase in the foreign money supply only for convenience. The mechanism operating for a decrease in the foreign money supply is identical to that of an increase, although qualified by the sign.

(58) $$\hat{y}^* = \left(\frac{n}{1-n}\right)\frac{1}{2}\left(\frac{\bar{r}d\bar{F}}{\bar{C}_0^W}\right),$$

(59) $$\hat{p}(h) - \hat{p}^*(f) - \hat{\bar{E}} = \left(\frac{1}{1-n}\right)\left(\frac{1}{2\theta}\right)\frac{\bar{r}d\bar{F}}{\bar{C}_0^W},$$

(60) $$\hat{\bar{P}} = \hat{\bar{M}} - \frac{1}{\varepsilon}\hat{\bar{C}} = \hat{\bar{M}} - \frac{1}{\varepsilon}\left(\frac{1+\theta}{2\theta}\right)\left(\frac{\bar{r}d\bar{F}}{\bar{C}_0^W}\right),$$

(61) $$\hat{\bar{P}}^* = \hat{\bar{M}}^* - \frac{1}{\varepsilon}\hat{\bar{C}}^* = \hat{\bar{M}}^* + \frac{1}{\varepsilon}\left(\frac{n}{1-n}\right)\left(\frac{1+\theta}{2\theta}\right)\frac{\bar{r}d\bar{F}}{\bar{C}_0^W},$$

and by substracting (61) from (60), and using (33),

(62) $$\hat{\bar{E}} = \hat{\bar{M}} - \hat{\bar{M}}^* - \frac{1}{\varepsilon}\left(\hat{\bar{C}} - \hat{\bar{C}}^*\right) = \hat{\bar{M}} - \hat{\bar{M}}^* + \left(\frac{1}{1-n}\right)\left(\frac{1+\theta}{2\theta}\right)\frac{\bar{r}d\bar{F}}{\bar{C}_0^W}.$$

Two interesting features charaterize these expressions: first, that monetary shocks have no *long-run* effect on *global* output and consumption[25], and second, that the new steady state depends on the effects that the monetary shock has on the countries' current accounts, *whose dynamics are determined during period 1.* Therefore, to see the implications of the two *ERRs* for the two economies, this section proceeds by first investigating the dynamics of the current accounts under the two *ERRs*, for the given foreign money supply shock, and then by evaluating the effects that such changes have on the welfare of both countries.

This section proceeds in the following way: (1) the flexible *ERR* case will be analyzed, (2) the fixed *ERR* will be analyzed, and (3) a summary of the performance of the two *ERRs* will be presented[26].

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[25] However, their impact will be felt asymmetrically on the respective countries' output and consumption. The assymetric effects are generated by the capital flows induced by the monetary shocks in the presence of nominal price rigidities.

[26] This structure will be used for all the shocks analyzed in this paper.

A. THE FLEXIBLE ERR CASE

1. Positive analysis

The analysis proceeds by solving for the differences between Home and Foreign variables. Starting by substracting (42) from (41) gives

$$(63) \qquad \hat{\bar{C}} - \hat{\bar{C}}^* = \hat{C} - \hat{C}^*,$$

which indicates that the relative changes between home and foreign per capita consumption are permanent[27]. Note, however, that for each country, per-capita consumption may be tilted if the real interest rate deviates from its steady state.

The next relevant equation was obtained before as equation (45):

$$\hat{M} - \hat{M}^* - \hat{E} = \frac{1}{\varepsilon}\left(\hat{C} - \hat{C}^*\right) - \frac{\beta}{\varepsilon(1-\beta)}\left(\hat{\bar{E}} - \hat{E}\right).$$

The implications of this equation for the exchange rate could be appreciated by combining it with equation (62), which is basically equation (45) led by one period. The result, making use of (63), is

$$(64) \qquad \hat{E} = \left(\hat{M} - \hat{M}^*\right) - \frac{1}{\varepsilon}\left(\hat{C} - \hat{C}^*\right).$$

Then $\hat{E} = \hat{\bar{E}}$, which means that, in spite of the short-run price rigidities, the exchange rate moves immediately to the new flexible-price steady-state equilibrium after the monetary shock[28].

It is important to note here that, in the context of nominal price rigidities, the exchange rate change and the relative consumption changes are jointly determined. To obtain a solution for $\hat{E} = \hat{\bar{E}}$, and $\hat{C} - \hat{C}^*$, a second equation, in addition to (64), relating them must be found. By using equations (52), (53), (55),

27 The explanation or this result is that with equal preferences and perfectly integrated bond markets, individuals in both countries face the same real interest rate.

28 This equation gives the relation between $\hat{E}$ and $\hat{C} - \hat{C}^*$ that guarantees equilibrium in the money market. Note that they are inversely related.

and (56), the following expression for steady-state consumption differentials is found:

$$\hat{C}-\hat{C}^{*}=\left(\frac{1+\theta}{2\theta}\right)\bar{r}\left[\left(\hat{y}-\hat{y}^{*}\right)-\left(\hat{C}-\hat{C}^{*}\right)-\hat{E}\right].$$

Now, noting, from (50) and (51), that in the short run $\hat{y}-\hat{y}^{*}=\theta\hat{E}$, the following relationship between $\hat{E}$ and $\hat{C}-\hat{C}^{*}$ is obtained[29]

$$\text{(65)} \qquad \hat{E}=\frac{2\theta+\bar{r}(\theta+1)}{\bar{r}\left(\theta^{2}-1\right)}\left(\hat{C}-\hat{C}^{*}\right).$$

The interaction between equations (64) and (65) gives the short-run equilibrium of the system.

The particular experiment with which this section deals consists in an unanticipated, permanent, foreign country money supply increase $\hat{\bar{M}}^{*}=\hat{M}^{*}>0$. In the new short-run equilibrium, the domestic currency appreciates, although by a smaller amount than the contraction in the relative home money supply, and the relative domestic consumption decreases. The domestic-currency appreciation is induced by the money market disequilibrium produced by the monetary shock[30]; and the relative decline in domestic consumption is the result of the expenditure-switching effects of the exchange rate appreciation. The following equations show the changes in the exchange rate and in the relative consumption induced by the shock[31]:

$$\text{(66)} \qquad \hat{E}=\frac{\varepsilon\left[\bar{r}(\theta+1)+2\theta\right]}{\bar{r}\left(\theta^{2}-1\right)+\varepsilon\left[\bar{r}(\theta+1)+2\theta\right]}\left(-\hat{M}^{*}\right)<0,$$

---

29 This relationship between $\hat{E}$ and $\left(\hat{C}-\hat{C}^{*}\right)$ gives the equilibrium condition in the goods market. It indicates the required depreciation of the home currency required to sustain a given increase in relative domestic consumption. Note that $\hat{E}$ and $\left(\hat{C}-\hat{C}^{*}\right)$ are positively related.

30 With short-run nominal-price rigidities the exchange rate must change to adjust the real relative money supply to the relative money demand. However, the required change in the exchange rate is tempered by the reduction in domestic relative consumption.

31 Note that $\hat{E}\geq-\hat{M}^{*}$.

and

$$(67) \qquad \hat{C}-\hat{C}^* = \frac{\varepsilon\bar{r}\left(\theta^2-1\right)}{\bar{r}\left(\theta^2-1\right)+\varepsilon\left[\bar{r}(\theta+1)+2\theta\right]}\left(-\hat{M}^*\right)<0.$$

These equations confirm the aforementioned intuition regarding the short-run equilibrium[32].

The equilibrium current account, which is found by using equations (55), (56), and (63), is given by

$$(68) \qquad \frac{d\bar{F}}{\overline{C}_0^W} = \frac{2\theta\varepsilon(1-n)(\theta-1)}{\bar{r}\left(\theta^2-1\right)+\varepsilon\left[\bar{r}(\theta+1)+2\theta\right]}\left(-\hat{M}^*\right)<0.$$

This expression indicates that the foreign money supply shock induced a deficit in the domestic current account. The intuition behind this increase in domestic debt is that the reduction in domestic relative consumption does not match the reduction in real income in the home country[33]. Domestic residents borrow abroad to smooth-out their consumption, spreading their reduction in consumption over the future.

Now that $\frac{d\bar{F}}{\overline{C}_0^W}$ was obtained, it is possible to sove for the steady-state variables. From equation (59), the long-run terms of trade are obtained as

$$(69) \qquad \hat{\bar{p}}(h)-\hat{\bar{p}}^*(f)-\hat{\bar{E}} = \frac{\varepsilon\bar{r}(\theta-1)}{\bar{r}\left(\theta^2-1\right)+\varepsilon\left[\bar{r}(\theta+1)+2\theta\right]}\left(-\hat{M}^*\right)<0.$$

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32 It is important to note that, since $\hat{E}=\hat{\bar{E}}$ and $\hat{C}-\hat{C}^*=\hat{\bar{C}}-\hat{\bar{C}}^*$, this occurs also in the long run.

33 The reduction in real domestic relative income is given by $\hat{y}-\hat{y}^*-\hat{E}=(\theta-1)\hat{E}<0$, where the last inequality is explained by de fact that $\theta>1$. To see that $\left|\hat{y}-\hat{y}^*-\hat{E}\right|>\left|\hat{C}-\hat{C}^*\right|$, note that $\hat{C}-\hat{C}^*=(\theta-1)\hat{E}+\frac{2\theta(\theta-1)}{\bar{r}\left(\theta^2-1\right)+\varepsilon\left[\bar{r}(\theta+1)+2\theta\right]}$, where the second term in the right hand side is greater that zero.

Then, a positive shock to the foreign money supply produces a long-run deterioration in the domestic terms of trade, due to the induced reduction in wealth. As equation (57) shows, a lower long-run wealth makes domestic residents to substitute leisure for effort, and consequently, domestic prices get reduced[34]. This result contrasts with the improvement in the short-run domestic terms of trade $\left(-\hat{E}\right)$. Note, however, that even though the long-run terms of trade are of opposite sign of the short-run terms of trade, in absolute value the short run terms of trade are larger.

From the previous discussion, it can be appreciated that the long-run non-neutralities of the foreign money shock are due to its effects on the accumulation of wealth of both countries. Nominal price rigidities induce international capital flows as a result of monetary shocks.

From the equations stated at the begining of section III, the following steady-state changes were obtained:

(i) $\hat{\bar{C}}^W = \hat{\bar{y}}^W = 0,$

(ii) $\hat{\bar{C}} < 0, \hat{\bar{y}} > 0, \hat{\bar{p}}(h) - \hat{\bar{p}}^*(f) - \hat{\bar{E}} < 0,$ and $\hat{\bar{P}} > 0,$ and

(iii) $\hat{\bar{C}}^* > 0, \hat{\bar{y}}^* < 0, \hat{\bar{p}}^*(f) - \hat{\bar{p}}(h) + \hat{\bar{E}} > 0$ and $\hat{\bar{P}}^* > 0.$

That is, in the new steady state, world output and consumption reamain unchanged, residents of the domestic country consume less, work harder, have their terms of trade deteriorated, and have a higher level of inflation; and foreign consume more, enjoy more leisure, have their terms of trade improved, and have a higher inflation level.

These long-run effects of the foreign money shock, are complemented by the short-run effects. To obtain such effects for each country it is necessary to see the short-run effect of the monetary shock on the real interst rate. Using equations (48), (49), (60), and (61), the following expressions for the money market equilibrium conditions (43) and (44) are obtained:

$$\hat{C} + \frac{\beta}{\varepsilon(1-\beta)}\hat{\bar{C}} - \left(\varepsilon + \frac{\beta}{(1-\beta)}\right)\left[\hat{M} - (1-n)\hat{E}\right] = \beta\hat{r},$$

34 Obviously, the opposite happens in the foreign country.

and

$$\hat{C}^* + \frac{\beta}{\varepsilon(1-\beta)}\hat{\bar{C}}^* - \left(\varepsilon + \frac{\beta}{(1-\beta)}\right)\left[\hat{M}^* + n\hat{E}\right] = \beta\hat{r}.$$

Using a population-weighted average of these expressions, jointly with condition (54), equations (41) and (42) imply

$$\text{(70)} \qquad \hat{C}^W = n\hat{C} + (1-n)\hat{C}^* = -(1-\beta)\hat{r}.$$

Therefore,

$$\text{(71)} \qquad \hat{r} = -\left(\varepsilon + \frac{\beta}{(1-\beta)}\right)\hat{M}^W,$$

where

$\hat{M}^W \equiv n\hat{M} + (1-n)\hat{M}^*.$[35]

Equation (71) captures the liquidity effect produced by the monetary shock, and it shows that, no matter where the money supply shock originated, the reduction in the real rate of interest is proportional to the increase in the world money supply $(\hat{M}^W)$.[36] The strenght of this liquidity effect depends on the value of ε, which is the inverse of the interest elasticity of the money demand.

The immediate consequence of the reduction in the real rate of interest is the increase in world consumption in the short run, as indicated in equation (70), which leads to an expansion of world's output.

The main trust of the previous analysis could be summarized as follows. An expansion in the foreign money supply decreases the world real interest rate, increasing global demand along the way. However, due to the nominal price

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35 In the present case $\hat{M}^W = (1-n)\hat{M}^*$, since $\hat{M} = 0$.

36 Another way of explaining $\hat{r} < 0$ for a given $\hat{M}^* > 0$, is by asking what happens to global savings. In order for the rate of interest to decrease, world savings have to increase. Noting that $\hat{Y}^W > 0$, and that $\hat{\bar{Y}}^W = 0$, it is possible to conclude that global savings have increased.

rigidities, the exchange rate appreciates, switching the demand towards foreign goods.

In addition to the current account, other short-run effects of a foreign money shock under a flexible *ERR* are:

(i) $\hat{C}^W = \hat{y}^W > 0,$

(ii) $\hat{P} < 0.$ The results for $\hat{c}$ and $\hat{y}$ depend on the parameter ε[37] although $\hat{C} > \hat{y}.$

(iii) $\hat{C}^* > 0, \ \hat{y}^* > 0, \ \hat{P}^* > 0.$

That is, in the short run, world output and world consumption increase, domestic residents enjoy a consumption higher than income[38], and have a lower inflation. On the other hand, foreign residents enjoy a higher level of consumption, although work harder and have a higher level of inflation.

All the above analysis gives a characterization of the positive impact of a foreign monetary shock under a flexible *ERR*; however, these results do not give a clear cut evaluation of how the welfare of residents in both countries was affected. This issue will be studied in the following section.

2. Normative analysis

As was mentioned in section I, one of the main virtues of the intertemporal approach is that it allows to make a systematic evaluation of the welfare consequences of different shocks, policies, and institutions. In this section, such evaluation, for the residents of both countries, will be undertaken for the foreign monetary shock under a flexible *ERR*.

The analysis will proceed by studying the welfare changes in two parts: one called the *real* welfare, is associated with consumption and output; and the other called the *monetary* welfare, is associated with real money balances. Formally, the

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37 For $\varepsilon = 1$, $\hat{y} < 0$; however, as ε gets larger, $\hat{y} > 0$. The reason being that for larger values of ε the demand for money becomes more inelastic, and the effect of $\hat{M}^* > 0$ on $\hat{r}$ is bigger. Consequently the positive effect on $\hat{y}$ induced by $\hat{C}^W$ dominates the negative effect induced by $\hat{E}$.

38 Although income may in fact decrease.

utility function (2) is written as $U = U^R + U^M$, where U^R represents the utility depending on consumption and output, and U^M represents the utility depending on real money balances[39].

Starting with the *real* component, in general,

$$(72) \qquad dU^R = \left[\hat{C} - \left(\frac{\theta - 1}{\theta}\right)\hat{y}\right] + \frac{\beta}{1-\beta}\left[\hat{\bar{C}} - \left(\frac{\theta - 1}{\theta}\right)\hat{\bar{y}}\right]$$

and

$$(73) \qquad dU^{R^*} = \left[\hat{C}^* - \left(\frac{\theta - 1}{\theta}\right)\hat{y}^*\right] + \frac{\beta}{1-\beta}\left[\hat{\bar{C}}^* - \left(\frac{\theta - 1}{\theta}\right)\hat{\bar{y}}^*\right]$$

represent the domestic and foreign change in welfare associated with short-run and long-run changes in consumption and output.

For the particular foreign monetary shock under study, equations (72) and (73), become

$$(74) \qquad dU^R = \frac{\hat{C}^W}{\theta} = \frac{\beta + \varepsilon(1-\beta)}{\theta}\hat{M}^W = \frac{\beta + \varepsilon(1-\beta)}{\theta}(1-n)\hat{M}^*,$$

and

$$(75) \qquad dU^{R^*} = \frac{\hat{C}^W}{\theta} = \frac{\beta + \varepsilon(1-\beta)}{\theta}\hat{M}^W = \frac{\beta + \varepsilon(1-\beta)}{\theta}(1-n)\hat{M}^*.$$

These results indicate that the *real* welfare effects of a foreign monetary shock are symmetric across the residents of both countries. More precisely, equations (74) and (75) show that only the spillover effects induced by the expansion of foreign money supply, through global demand, effectively affect *real* welfare.

The mechanism operating here works by alleviating the inefficiency produced by the basic distortion of this economy, namely monopolistic competition, which has

39 Note, according to the positive analysis of the previous section, that each component is affected by short-run and long-run variables. The short run lasts one period, after which the economy reaches the steady state.

kept it operating at a level inferior to the maximum possible. The increase in the money supply, irrespective of its origin[40], by expanding global aggregate demand is able to coordinate a higher level of effort[41], inducing producers in both countries to increase output[42].

Another feature of equations (74) and (75) is that neither the expenditure-switching effects nor the intertemporal reallocation of consumption, both induced by changes in the exchange rate, have any bearing on *real* welfare. They simply cancel out. The intuition behind this result is that, since the analysis deals with small shocks, all expenditure-switching and intertemporal reallocation of consumption effects are of strictly second-order[43].

Turning now to the changes in welfare induced by changes in real balances, called *monetary* for convenience, the domestic and foreign residents experience the following effects:

$$dU^{M} = \chi\left(\frac{\overline{M}_0}{\overline{P}_0}\right)^{1-\varepsilon}\left[\left(\hat{M}-(1-n)\hat{E}\right)+\frac{\beta}{1-\beta}\left(\hat{\overline{M}}-\hat{\overline{P}}\right)\right]$$

(76)

$$= \chi\left(\frac{\overline{M}_0}{\overline{P}_0}\right)^{1-\varepsilon}\left[-(1-n)\hat{E}-\frac{\beta}{1-\beta}\hat{\overline{P}}\right],$$

and

(77) $$dU^{M*} = \chi\left(\frac{\overline{M}_0^*}{\overline{P}_0^*}\right)^{1-\varepsilon}\left[\left(\hat{M}^*+n\hat{E}\right)+\frac{\beta}{1-\beta}\left(\hat{\overline{M}}^*-\hat{\overline{P}}^*\right)\right].$$

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40 The size of a country, however, plays an important role in determining the global impact of a particular shock. In the present case, the higher n, the lower the welfare impact of the foreign money supply shock.

41 The decisions of producers in both countries are strategically complementary. See Cooper and John (1988).

42 This result is typical of the literature emphasizing the role of monopolistic competition, or, more generally, stressing the existence of coordination problems in a market economy. See Cooper and John *op cit*.

43 This is a clear application of the envelope theorem. It should be noted that linearization around a steady state implies that only small shocks can be studied.

For the domestic residents, since $\hat{E} < 0$, their real balances increase in the short run. However, equations (60) and (68) show that $\hat{\bar{P}} > 0$, indicating that in the long run, domestic residents see their real balances reduced. As a result, the net effect on the domestic *monetary* welfare derived from the change in real balances is ambiguous. More precisely, its net effect on total welfare depends on the value assumed by the parameters of the model.

As for the foreign residents, from equation (66) it could be seen that $\hat{M}^* > n\hat{E}$, and from equations (56), (61), and (68), it could be concluded that $\hat{\bar{M}}^* - \hat{\bar{P}}^* = \frac{1}{\varepsilon}\hat{\bar{C}}^* > 0.$ Therefore, it is clear that real balances for the foreign residents increase in both periods, and consequently *monetary* welfare has unequivocally increased.

3. Summary

Considering the symmetry of the model, the results obtained in this section are generalized in the following proposition:

Proposition 1:

Under a flexible ERR, a positive (negative) shock to the money supply in a particular country will, unequivocally, increase (decrease) the total welfare of the residents of the expanding (contracting) country. The residents of the non-expanding (non-contracting) country will experience a change in their welfare that depends on their parametric structure. Specifically, if χ is small their welfare will increase (decrease), with the opposite results if χ is large.

B. THE FIXED ERR CASE

Under a fixed *ERR*, the home country's central bank is assumed to have a money-supply reaction function, denoted by $\hat{M}^r$ [44], that specifies the change in the money supply that is required to keep the exchange rate from changing when the economy is hit by a shock. As in the flexible *ERR* case, the focus will be on a positive shock to the foreign money supply.

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[44] Recall that this paper deals only with permanent monetary shocks, therefore $\hat{M}^r = \hat{\bar{M}}$.

1. Positive analysis

It is clear that in order to keep the exchange rate from changing, the domestic-country money reaction function must be $\hat{M}^r = \hat{M}^*$. Intuitively, the disturbance to the money market, which initially created an imbalance between the relative money demand and the realtive money supply, is completely balanced out by the monetary injection undertaken by the domestic central bank. As a consequence, the appreciation of the domestic currency becomes unnecessary, and any expenditure-switching effect is completely muted.

The previous remarks are formally stated by observing that since $\hat{M}^r - \hat{M}^* = 0$, equations (66), (67), (68), and (69) become $\hat{E} = \hat{\bar{E}} = 0$, $\hat{\bar{C}} - \hat{\bar{C}}^* = \hat{C} - \hat{C}^* = 0$, $\frac{d\bar{F}}{C_0^W} = 0$, and $\hat{\bar{p}}(h) - \hat{\bar{p}}^*(f) - \hat{\bar{E}} = 0$, respectively. As a result, and using the equations (55)-(58), (60), and (61), the following consequences for the steady state apply:

(i) $\hat{\bar{C}}^W = \hat{\bar{y}}^W = 0$,

(ii) $\hat{\bar{C}} = 0,\ \hat{\bar{y}} = 0,\ \hat{\bar{p}}(h) - \hat{\bar{p}}^*(f) - \hat{\bar{E}} = 0$, and $\hat{\bar{P}} > 0$, and

(iii) $\hat{\bar{C}}^* = 0,\ \hat{\bar{y}}^* = 0,\ \hat{\bar{p}}(h) - \hat{\bar{p}}^*(f) - \hat{\bar{E}} = 0$ and $\hat{\bar{P}}^* > 0$.

That is, long-run world consumption and output remain constant, domestic and foreign residents have the same level of long-run per-capita consumption, effort per-capita, and terms of trade as in the initial steady state, but also have a higher inflation level[45]. These results underline the role played by the current account in linking the short and the long run; once its potential effects are nullified by the domestic central bank's reaction, the only long-run impact of the monetary shock will be felt on absolute prices.

Essentially, what has happened is that the domestic monetary increase in the money supply has, with the exception of the price levels, returned the whole system to the initial steady-state equlibrium that existed before the foreign money supply shock had occured.

It is in the short run, however, where the action is concentrated. Since the domestic and the foreign money supplies have both increased, the global money supply, $\hat{M}^W$, has increased too. As a consequence, and as shown by equation (71), the short-run reduction in the real rate of interest is larger than in the

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[45] Note that since $\hat{\bar{P}} - \hat{\bar{P}}^* = \hat{\bar{M}} - \hat{\bar{M}}^* = 0$, the long-run inflation is identical in both countries.

flexible *ERR* case. Also, the subsequent expansion in the global demand, $\hat{C}^W$ is larger under the fixed *ERR*.

As a result, and in addition to the current account being zero, the short-run effects of a foreign money shock under a fixed *ERR* are:

(i) $\hat{C}^W = \hat{y}^W > 0$

(ii) $\hat{C} > 0,\ \hat{y} > 0,$ and $\hat{P} = 0,$ and

(iii) $\hat{C}^* > 0,\ \hat{y}^* > 0,$ and $\hat{P}^* = 0.$

That is, world consumption and world output increase, domestic and foreign residents consume and produce more, and pay the same prices. It is important to note that since $\hat{E} = 0,$ the effects induced by $\hat{M}^W$ on $\hat{C}^W$ are distributed symmetrically between the two countries. That is, $\hat{C} = \hat{C}^* = \hat{C}^W = \hat{Y}^W = \hat{y} = \hat{y}^* > 0.$

Now that all the positive effects have been identified, it is possible to make the welfare evaluation of the fixed *ERR* in the presence of a foreign monetary shock.

2. Normative analysis

The welfare evaluation of the foreign money supply shock under a fixed *ERR* follows the same line of analysis used for the flexible *ERR* case. Using equations (72), and (73) and the findings from the positive analysis, the following effects on *real* welfare induced by changes in consumption and effort were found:

(78)
$$dU^R = \frac{\hat{C}^W}{\theta} = \frac{\beta + \varepsilon(1-\beta)}{\theta}\hat{M}^W = \frac{\beta + \varepsilon(1-\beta)}{\theta}\left[n\hat{M}^r + (1-n)\hat{M}^*\right]$$
$$= \frac{\beta + \varepsilon(1-\beta)}{\theta}\hat{M}^*,$$

and

(79)
$$dU^{R*} = \frac{\hat{C}^W}{\theta} = \frac{\beta + \varepsilon(1-\beta)}{\theta}\hat{M}^W = \frac{\beta + \varepsilon(1-\beta)}{\theta}\left[n\hat{M}^r + (1-n)\hat{M}^*\right]$$
$$= \frac{\beta + \varepsilon(1-\beta)}{\theta}\hat{M}^*.$$

Again, but now under a fixed *ERR*, the *real* effects of a positive money supply shock are symmetric across the residents of both countries, with the distinction that now the global money supply includes $\hat{M}^r$. Then, under a fixed *ERR*, the *real* welfare of the residents of both countries increases by the same amount when there is a positive shock to foreign money supply, and such increase is higher than under a flexible *ERR*.

The turn is now for the *monetary* welfare effects that the foreign money shock induced, under a fixed *ERR*. From the positive analysis of this section it is clear that the residents of both countries experience an increase in their short-run and long-run real balances; this fact, plus equations (76) and (77) allow to conclude that their changes in *monetary* welfare are:

(80) $$dU^M = \chi\left(\frac{\overline{M}_0}{\overline{P}_0}\right)^{1-\varepsilon}\hat{M}^r,$$

and

(81) $$dU^{M*} = \chi\left(\frac{\overline{M}_0^*}{\overline{P}_0^*}\right)^{1-\varepsilon}\hat{M}^*.$$

These two equations indicate that both countries' residents have their *monetary* welfare *equally* increased, in an unequivocal way. The source of this increase is the higher level of real balances that the residents of both countries experience in the short run[46]. As was the case for the *real* welfare, the increase in the *monetary* welfare is higher under a fixed *ERR* tan under a flexible *ERR*.

3. Summary

The results of this section are generalized in the following proposition:

Proposition 2:

Suppose a particular country follows the policy of having a fixed ERR. Then, a positive (negative) shock to the money supply in the other country will,

[46] In the long run, inflation eats up all the increase in nominal balances.

unequivocally, increase (decrease) the total welfare of both countries by the same amount.

C. EVALUATION OF THE TWO ERRs UNDER MONETARY SHOCKS

Given the results stated in sections III. A and III. B, the following proposition summarizes the performance of the two *ERRs* under permanent monetary shocks.

Proposition 3:
Suppose that the only shocks that a particular country confronts are 'foreign' monetary shocks. If those shocks are positive, a policy of following a fixed ERR strictly dominates a policy of following a flexible ERR. On the other hand, if the shocks are negative, a policy of following a flexible ERR strictly dominates a policy of following a fixed ERR.

It is important to remark that the optimal policy for a given 'foreign' monetary shock will maximize world welfare, and not only the welfare of the country that is supposedly choosing the *ERR*.

Another point regarding the results is that the basic mechanism operating behind the performance of both *ERRs* is the alleviation and/or the exacerbation of the basic distortion assumed for this economy. In the presence of positive 'foreign' monetary shocks, a fixed *ERR*, *vis-a-vis* a flexible *ERR*, implies a further increase in the global money supply, and consequently a higher reduction in the short-run real interest rate, and a bigger increase in global demand. If on the other hand the 'foreign' monetary shock is negative, a fixed exchange *ERR*, *vis-a-vis* a flexible *ERR*, will force a further contraction in the global money supply, with opposite consequences to the ones described above.

IV. FISCAL POLICY SHOCKS

In this section, the performance of the two *ERRs* will be evaluated for transitory and permanent shocks to domestic and foreign per-capita government expenditure. It will be assumed that, with the exception of those monetary changes required to manage the fixed *ERR*, $\hat{M} = \hat{M}^* = \hat{\bar{M}} = \hat{\bar{M}}^* = 0$, since as was indicated in section III both, fiscal and monetary shocks, have additive effects.

The way government is introduced in the model was specified in section II. A.[47]. It could be said, though, that this particular specification focuses mainly on the dynamic consequences of fiscal policies. The relevant equations that describe the system, and which will be used in the analysis, are equations (33)-(53).

Using the approach developed in section III., the following equations describe the new steady state in the presence of government shocks:

(82) $$\hat{y}^W = \frac{1}{2}\frac{d\overline{G}^W}{\overline{C}_0^W}, \hat{C}^W = -\frac{1}{2}\frac{d\overline{G}^W}{\overline{C}_0^W},$$

(83) $$\hat{C} = \left(\frac{\theta+1}{2\theta}\right)\bar{r}\frac{d\overline{F}}{\overline{C}_0^W} - \left(\frac{\theta+1-n}{2\theta}\right)\frac{d\overline{G}}{\overline{C}_0^W} + \left(\frac{1-n}{2\theta}\right)\frac{d\overline{G}^*}{\overline{C}_0^W},$$

(84) $$\hat{C}^* = -\left(\frac{n}{1-n}\right)\left(\frac{\theta+1}{2\theta}\right)\bar{r}\frac{d\overline{F}}{\overline{C}_0^W} + \left(\frac{n}{2\theta}\right)\frac{d\overline{G}}{\overline{C}_0^W} - \left(\frac{n+\theta}{2\theta}\right)\frac{d\overline{G}^*}{\overline{C}_0^W},$$

(85) $$\hat{y} = -\frac{1}{2}\bar{r}\frac{d\overline{F}}{\overline{C}_0^W} + \frac{1}{2}\frac{d\overline{G}}{\overline{C}_0^W},$$

(86) $$\hat{y}^* = \frac{1}{2}\left(\frac{n}{1-n}\right)\bar{r}\frac{d\overline{F}}{\overline{C}_0^W} + \frac{1}{2}\frac{d\overline{G}^*}{\overline{C}_0^W},$$

(87) $$\hat{p}(h) - \hat{p}^*(f) - \hat{E} = \left(\frac{1}{1-n}\right)\left(\frac{1}{2\theta}\right)\bar{r}\frac{d\overline{F}}{\overline{C}_0^W} - \frac{1}{2\theta}\left(\frac{d\overline{G} - d\overline{G}^*}{\overline{C}_0^W}\right),$$

(88) $$\begin{aligned}\hat{P} &= \hat{M} - \frac{1}{\varepsilon}\hat{C} \\ &= -\frac{1}{\varepsilon}\left[\left(\frac{1+\theta}{2\theta}\right)\bar{r}\frac{d\overline{F}}{\overline{C}_0^W} - \left(\frac{\theta+1-n}{2\theta}\right)\frac{d\overline{G}}{\overline{C}_0^W} + \left(\frac{1-n}{2\theta}\right)\frac{d\overline{G}^*}{\overline{C}_0^W}\right],\end{aligned}$$

and

[47] Specifically, in assumptions 8 and 9. Note also that government expenditures do not affect private utility directly.

(89)
$$\hat{\bar{P}}^* = \hat{\bar{M}}^* - \frac{1}{\varepsilon}\hat{\bar{C}}^*$$
$$= \frac{1}{\varepsilon}\left[\left(\frac{n}{1-n}\right)\left(\frac{\theta+1}{2\theta}\right)\bar{r}\frac{d\bar{F}}{\bar{C}_0^W} - \left(\frac{n}{2\theta}\right)\frac{d\bar{G}}{\bar{C}_0^W} + \left(\frac{n+\theta}{2\theta}\right)\frac{d\bar{G}^*}{\bar{C}_0^W}\right].$$

These equations indicate: first, that only permanent shocks to government spending affect *directly* the steady-state equilibrium, and second, that fiscal policy shocks may also influence the steady-state equilibrium *indirectly*, through the current account.

Another important feature reflected in the previous equations is the fact that an expansion in government spending by any country is financed by the residents of that country, although, as indicated in expression (6), part of the spending falls on goods produced in the other country. This feature of the model explains the signs of the terms involving changes in government spending.

In what follows, the dynamics of fiscal policy shocks that ultimately result in moving the system to a new stady state will be studied, jointly with their welfare implications for the residents of both countries.

Before going into the analysis of each of the fiscal shocks, a generic characterization of the short-run equilibria, containing all the possible fiscal shocks, will be obtained under both, flexible and fixed, *ERRs*[48].

For the flexible *ERR*, the equation representing the money market equilibrium is again given by equation (64), with the proviso that the monetary changes are now zero. The equation representing the equilibrium in the goods market is now given by[49]

(90)
$$\hat{E} = \frac{2\theta+\bar{r}(\theta+1)}{\bar{r}(\theta^2-1)}\left(\hat{C}-\hat{C}^*\right) + \frac{1}{\theta-1}\left[\frac{dG-dG^*}{\bar{C}_0^W} + \frac{1}{\bar{r}}\left(\frac{d\bar{G}-d\bar{G}^*}{\bar{C}_0^W}\right)\right].$$

48 The analysis of this section follows the same approach used for the short-run dynamics of the monetary shocks.

49 Consistent with the notation used in the paper, dG and dG^* represent transitory changes in domestic and foreign government expenditure, while $d\bar{G}$ and $d\bar{G}^*$ represent permanent ones.

Note that the slope of this schedule is the same as the one in equation 65, but its intercept is now the present discounted value of current and future differences in per-capita government spending changes.

From equations (64) and (90) the short-run, and long-run, equilibrium exchange rate and relative consumption differential are obtained as

$$(91) \qquad \hat{E} = \frac{\bar{r}(\theta+1)}{\bar{r}(\theta^2-1)+\varepsilon[\bar{r}(\theta+1)+2\theta]}\left[\frac{dG-dG^*}{\overline{C}_0^W}+\frac{1}{\bar{r}}\left(\frac{d\overline{G}-d\overline{G}^*}{\overline{C}_0^W}\right)\right],$$

and

$$(92) \qquad \hat{C}-\hat{C}^* = \frac{-\varepsilon\bar{r}(\theta+1)}{\bar{r}(\theta^2-1)+\varepsilon[\bar{r}(\theta+1)+2\theta]}\left[\frac{dG-dG^*}{\overline{C}_0^W}+\frac{1}{\bar{r}}\left(\frac{d\overline{G}-d\overline{G}^*}{\overline{C}_0^W}\right)\right].$$

These equations indicate that the changes in the exchange rate are positively related to changes in domestic government spending, and negatively related to the changes in foreign government spending, while the change in domestic per-capita relative consumption has the opposite relationship with the same type of changes in government spending. This result is related to the fact that domestic residents finance, entirely, the changes in their government spending, but part of that spending falls on foreign goods[50].

Finally, the current account is given by

$$(93) \qquad \frac{d\overline{F}}{\overline{C}_0^W} = \frac{\bar{r}(\theta+1)(1-n)(\varepsilon+\theta-1)}{\bar{r}(\theta^2-1)+\varepsilon[\bar{r}(\theta+1)+2\theta]}\left[\frac{dG-dG^*}{\overline{C}_0^W}+\frac{1}{\bar{r}}\left(\frac{d\overline{G}-d\overline{G}^*}{\overline{C}_0^W}\right)\right] - (1-n)\left(\frac{dG-dG^*}{\overline{C}_0^W}\right).$$

An important result regarding this last equation is that both, transitory and permanent, shocks to government spending affect the current account. This is

[50] Obviously, this basic relationship also holds for the fixed *ERR*.

different from the flexible-price model in which only transitory shocks to the net private income are able to affect the current account[51].

Finally, the short-run real rate of interest is given by

$$(94) \qquad \hat{r} = -\left[\frac{(1-\beta)\varepsilon+\beta}{(1-\beta)\varepsilon}\right]\frac{1}{2}\frac{d\overline{G}^W}{\overline{C}_0^W}.$$

As this equation indicates, only permanent shocks to the *world* government spending affect the short-run real rate of interest[52]. The reason for this result is the following: first, $\hat{y}^W = \frac{dG^W}{\overline{C}_0^W}$, and second, $\hat{\overline{y}}^W = 0$. These two characteristics of the model, in the presence of transitory government spending shocks, do not allow the smoothing of consumption, and as a result, the short-run real rate of interest will not change.

Another important feature of equation (94) is that the reaction of the short-run real rate of interes to permanent changes government spending is negative[53].

For the fixed *ERR* case, equations (64) and (90) imply that the generic money-supply reaction function for fiscal policy shoks is given by

$$(95) \qquad \hat{M}^r = \frac{-\bar{r}(\theta+1)}{\varepsilon[\bar{r}(\theta+1)+2\theta]}\left[\frac{dG-dG^*}{\overline{C}_0^W}+\frac{1}{\bar{r}}\left(\frac{d\overline{G}-d\overline{G}^*}{\overline{C}_0^W}\right)\right].$$

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51 The reason being that, under flexible prices, permanent shocks are not able to tilt the path of net private real income.

52 This amounts to say that temporay fiscal policy shocks do not affect *directly* world savings. The reason is that the global net private output in the short-run, $\hat{y}^W - n\frac{dG}{\overline{C}_0^W}$, and the global net private output in the long run, $\hat{\overline{y}}^W$, are both equal to zero.

53 The rationale for this negative relation is that since $\hat{y}^W = \hat{\overline{y}}^W + \hat{\overline{y}}^W\left(\frac{\beta+\varepsilon(1-\beta)}{\varepsilon}\right) > \hat{\overline{y}}^W$, for a given change in permanent spending, then $|\hat{y}^W| > |\hat{\overline{y}}^W|$. Consequently, the real rate of interest will move in a direction opposite to the change in government spending.

Consistent with the remarks made for equations (91) and (92), shocks to domestic goverment spending in a particular direction induce an change in the opposite direction in the domestic money supply, with the reverse results for shocks to foreign government spending.

Given equations (64), (90), and (95) the short-run, and long-run, equilibrium exchange rate change and relative consumption differential are given by

(96) $\hat{E} = 0,$

and

(97) $$\hat{C} - \hat{C}^{*} = \frac{-\bar{r}(\theta+1)}{\left[\bar{r}(\theta+1)+2\theta\right]}\left[\frac{dG - dG^{*}}{\bar{C}_{0}^{W}} + \frac{1}{\bar{r}}\left(\frac{d\bar{G} - d\bar{G}^{*}}{\bar{C}_{0}^{W}}\right)\right].$$

A comparison of equations (92) and (97) shows that by preventing fluctuations in the exchange ratc, a fixed *ERR* produces higher fluctuations in per-capita relative consumption than those produced by the flexible *ERR*[54].

The generic current account for fiscal policy shocks under a fixed *ERR* is given by

(98) $$\frac{d\bar{F}}{\bar{C}_{0}^{W}} = \frac{\bar{r}(\theta+1)(1-n)}{\bar{r}\left(\theta^{2}-1\right)+\varepsilon\left[\bar{r}(\theta+1)+2\theta\right]}\left(\varepsilon + \frac{\bar{r}\left(\theta^{2}-1\right)}{\bar{r}(\theta+1)+2\theta}\right)$$
$$\left[\frac{dG - dG^{*}}{\bar{C}_{0}^{W}} + \frac{1}{\bar{r}}\left(\frac{d\bar{G} - d\bar{G}^{*}}{\bar{C}_{0}^{W}}\right)\right] - (1-n)\left(\frac{dG - dG^{*}}{\bar{C}_{0}^{W}}\right).$$

Finally, the short-run real rate of interest is given by

(99) $$\hat{r} = -\left[\frac{(1-\beta)\varepsilon+\beta}{(1-\beta)\varepsilon}\right]\frac{1}{2}\frac{d\bar{G}^{W}}{\bar{C}_{0}^{W}} - \left[\frac{(1-\beta)\varepsilon+\beta}{(1-\beta)\varepsilon}\right]n\hat{M}^{r}.$$

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54 This feature of the *ERRs* plays an important role in their welfare properties for the different shocks studied in this paper.

Note that, under a fixed *ERR*, the short-run real rate of interest may be affected by transitory fiscal policy shocks inasmuch as they trigger a change in the world money supply.

The stage is now set for analyzing the effects of fiscal policy shocks.

This section proceeds in the following way: (1) the transitory shocks will be analyzed under the Flexible and Fixed *ERRs*, (2) the permanent shocks will be analyzed under the Flexible and Fixed *ERR*, and (3) an evaluation of the performance of the two *ERRs* will be made.

A. TRANSITORY SHOCKS

The analysis of transitory fiscal policy shocks will focus on the effects of an increase in per-capita domestic government spending during the first period, $\frac{dG}{\overline{C}_0^W} > 0$. The results obtained for this particular shock can be generalized, for other transitory shocks, by recurring to the symmetry of the model.

1. The flexible ERR case

Positive analysis: The short-run equilibrium of the model for the shock under study, in the context of a flexible *ERR*, can be obtained by applying the restriction implied by the shock to equations (91) and (92). The following expressions describe such equilibrium:

$$(100) \quad \hat{E} = \frac{\bar{r}(\theta+1)}{\bar{r}(\theta^2-1)+\varepsilon[\bar{r}(\theta+1)+2\theta]}\frac{dG}{\overline{C}_0^W} > 0,$$

and

$$(101) \quad \hat{C}-\hat{C}^* = \frac{-\varepsilon\bar{r}(\theta+1)}{\bar{r}(\theta^2-1)+\varepsilon[\bar{r}(\theta+1)+2\theta]}\frac{dG}{\overline{C}_0^W} < 0.$$

As these equations show, $\frac{dG}{\overline{C}_0^W} > 0$ produces a depreciation in the domestic currency, and a reduction in relative domestic per-capita consumption. The intuition behind this result is that a higher domestic government spending, by increasing domestic taxes, reduces domestic consumption, but increases the foreign one. This change in relative consumption creates a disequilibrium in the monetary market that is solved by a depreciation in the domestic currency. In fact, and given that the real rate of interest is not affected by $\frac{dG}{\overline{C}_0^W} > 0$, the level of domestic consumption decreases, while the level of foreign consumption increases. Also, given that global short-run output increases by $n\frac{dG}{\overline{C}_0^W}$, both countries experience an increase in per-capita output; however, since the exchange rate depreciates, domestic per-capita output increases relatively more than foreign per-capita output, as is reflected in the fact that $\hat{y} - \hat{y}^*$ increases.

Another interesting result concerns global consumption, $\hat{C}^W$, which in the present situation is unaffected. This implies that $(1-n)\hat{C}^* = -n\hat{C}$, which means that the increase in foreign per-capita consumption is essentially a transfer from the domestic residents.

The previous short-run results can be summarized as follows:

(i) $\hat{C}^W = 0 \quad \hat{y}^W = n\frac{dG}{\hat{C}_0^w} > 0$

(ii) $\hat{C} < 0, \ \hat{y} > 0, \ \hat{P} > 0$, and

(iii) $\hat{C}^* > 0, \ \hat{y}^* > 0, \ \hat{P}^* < 0.$

That is, in the short-run, and as a result of a temporary increase in domestic per-capita government spending, world *private* output and world consumption remain unchanged; domestic residents consume less, work more, and tolerate a higher inflation level; and foreign residents consume more, work more, though relatively less than domestic residents, and have a lower inflation level.

In addition to having short-run effects, $\frac{dG}{\overline{C}_0^W} > 0$ will affect the steady-state equilibrium through its impact on the current account. Using equation (93), the current account is obtained as:

$$(102) \quad \frac{d\overline{F}}{\overline{C}_0^W} = (1-n)\left[\frac{\bar{r}(\theta+1)(\varepsilon+\theta-1)}{\bar{r}(\theta^2-1)+\varepsilon[\bar{r}(\theta+1)+2\theta]} - 1\right]\frac{dG}{\overline{C}_0^W} < 0.$$

Which means that a transitory shock to domestic spending will generate a deficit in the domestic current account. The logic behind this result is the same as in the flexible-price model; since the tax increase is temporary, the path of net private real income is tilted upwards, and as a result domestic residents will reduce consumption by less than the increase in the tax, with the difference being financed by external borrowing. This borrowing, however, is mitigated by the increase in domestic output caused by the exchange rate depreciation.

Given the effect on the current account and equations (82)-(89), it is now possible to identify the long-run effects of a transitory shock to domestic spending under a flexible *ERR*. The main point to note from these equations is that, since equation (82) shows that there are no global effects on steady-state output and consumption, all the effects caused by the transitory shock on domestic and foreign steady-variables are the result of the induced redistribution of wealth occured in period 1, and reflected in the current account. Keepping this remark in mind, the positive consequences for the steady state could be described as follows:

(i) $\hat{\bar{C}}^W = \hat{\bar{y}}^W = 0,$

(ii) $\hat{\bar{C}} < 0,\ \hat{\bar{y}} > 0,\ \hat{\bar{p}}(h) - \hat{\bar{p}}^*(f) - \hat{\bar{E}} < 0,$ and $\hat{\bar{P}} > 0,$ and

(iii) $\hat{\bar{C}}^* > 0,\ \hat{\bar{y}}^* < 0,\ \hat{\bar{p}}^*(f) + \hat{\bar{E}} - \hat{\bar{p}}(h) > 0,$ and $\hat{\bar{P}} < 0.$

That is, in the long-run, in addition to world output and consumption being unaffected, domestic residents have a lower consumption, work harder, have their terms of trade deteriorated, and have a higher inflation level. On the other hand, foreign residents enjoy a higher level of consumption, more leisure, improved terms of trade, and lower inflation.

Normative analysis: To evaluate the *real* and *monetary* welfare effects of $\frac{dG}{\bar{C}_0^W} > 0$ under a flexible *ERR*, equations (72), (73), (76), and (77) will be used. Focusing on the *real* effects first, the following changes are obtained:

(103) $$dU^R = -\left(\frac{\theta - n}{\theta}\right)\frac{dG}{\bar{C}_0^W} < 0,$$

and

(104) $dU^{*R} = \frac{n}{\theta} \frac{dG}{\overline{C}_0^W} > 0.$

These expressions indicate that, under a flexible *ERR*, a transitory increase in domestic government spending deteriorates the *real* welfare of domestic residents, and improves the *real* welfare of foreign residents. The driving force behind this result is the fact that domestic residents bear the burden of the taxes required to finance the government spending, while the foreign residents benefit from the induced increase in demand.

Moving now to the *monetary* effects, the following changes are obtained:

(105) $dU^M = \chi\left(\frac{\overline{M}_0}{\overline{P}_0}\right)^{1-\varepsilon}\left[-(1-n)\hat{E} - \frac{\beta}{1-\beta}\hat{\overline{P}}\right] < 0,$

and

(106) $dU^{*M} = \chi\left(\frac{\overline{M}_0^*}{\overline{P}_0^*}\right)^{1-\varepsilon}\left[n\hat{E} - \frac{\beta}{1-\beta}\hat{\overline{P}}^*\right] > 0.$

These results show that the *monetary* welfare of domestic residents deteriorates, while the *monetary* welfare of foreign residents improve. The intuition behind these results is that, since money supplies in both countries have not changed, changes in the real balances will depend on short-run and long-run inflation in both economies. In consequence, given the results from the positive analysis, domestic residents see their real balances decrease, both in the short and long run, while foreign residents see them increase in both periods.

Summary:

The results of this section are generalized in the following proposition:

Proposition 4:

Under a flexible ERR, a positive (negative) transitory shock to government spending in a particular country will, unequivocally, decrease (increase) the total welfare of the residents of the expanding (contracting) country. The *residents of the non-expanding (non-contracting) country will, unequivocally, experience an increase (decrease) in their total welfare.*

2. *The fixed ERR case*

Positive analysis:

Given equation (95), the money-supply reaction function of the home country's central bank is given by

$$(107)\qquad \hat{M}^r = \frac{-\bar{r}(\theta+1)}{\varepsilon\left[\bar{r}(\theta+1)+2\theta\right]}\left[\frac{dG}{\overline{C}_0^W}\right] < 0.$$

This equation indicates that a temporary shock to domestic government spending is met with a *temporary* reduction in the domestic money supply. Such a reduction will prevent the exchange rate from depreciating since it will match the decrease in the relative money demand that the government spending had originally induced.

Given the restriction imposed by $\hat{M}^r$ in the presence of $\frac{dG}{\overline{C}_0^W} > 0$, the short-run equilibrium under a fixed *ERR* is given by the following equations:

$$\hat{E} = 0,$$

and

$$(108)\qquad \hat{C} - \hat{C}^* = \frac{-\bar{r}(\theta+1)}{\left[\bar{r}(\theta+1)+2\theta\right]}\left(\frac{dG}{\overline{C}_0^W}\right) < 0.$$

This reduction in the relative consumption changes is higher than under the flexible *ERR*, and has two reasons: on one hand is the increase in taxes that domestic residents bear, on the other hand is the reduction in the domestic money supply that keeps the exchange rate from changing[55].

The short-run real rate of interest, obtained from equation (99), is given by

[55] These two effects reinforce each other, and in consequence the decrease in relative domestic consumption under a fixed *ERR* is larger than under a flexible *ERR*.

$$\hat{r} = \left[\frac{(1-\beta)\varepsilon+\beta}{(1-\beta)\varepsilon^2}\right]\frac{n\bar{r}(\theta+1)}{\left[\bar{r}(\theta+1)+2\theta\right]}\left[\frac{dG}{\overline{C}_0^W}\right] > 0.$$

This result implies that a transitory increase in domestic spending will, under a fixed *ERR*, increase the short-run real rate of interest, and it is clear that this change is purely a liquidify effect[56]. Since short-run global consumption, $\hat{C}^W$, is affected by the short-run real rate of interest, a subsequent reduction in short-run global consumption follows, and is given by

$$\hat{C}^W = -\left[\frac{(1-\beta)\varepsilon+\beta}{\varepsilon^2}\right]\frac{n\bar{r}(\theta+1)}{\left[\bar{r}(\theta+1)+2\theta\right]}\left[\frac{dG}{\overline{C}_0^W}\right] < 0.$$

This result implies that short-run per-capita consumption will decrease in the home country and will increase in the foreign country.

Another interesting result is that, since global demand is given by $\hat{C}^W + n\frac{dG}{\overline{C}_0^W}$, and the exchange rate remains the same, the short-run output per-capita will increase in both countries by the same amount. That is, $\hat{y} - \hat{y}^* = 0$.

The previous short-run results can be summarized as follows:

(i) $\hat{C}^W < 0$, $\hat{y}^W > 0$, $\hat{y}^W - n\frac{dG}{\overline{C}_0^W} < 0$,

(ii) $\hat{C} < 0$, $\hat{y} > 0$, and $\hat{P} = 0$, and

(iii) $\hat{C}^* > 0$, $\hat{y}^* > 0$, and $\hat{P}^* = 0$.

That is, in the short-run, and as a result of a transitory increase in per-capita domestic government spending, under a fixed *ERR*, world output, increases, met world private output decreases and world consumption decreases; domestic residents will consume less, work more, and have stable prices; and foreign residents will consume more, work more, and similarly to the domestic residents, have also stable prices.

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[56] One way of rationalizing the increase in the short-run real rate of interest is by observing that a reduction in global savings should have occured. Since $\hat{y}^W - n\frac{dG}{\overline{C}_0^W} < 0$, and $\hat{y}^W = 0$, it is clear that world savings have decreased.

It is important to note here that, as in the flexible *ERR* case, the domestic reduction in consumption is less than the increase in the government spending; which means that the domestic economy experiences a deficit in its current account. However, since under a fixed *ERR* the reduction in domestic consumption is larger than under a flexible *ERR*, and since under a fixed *ERR* the increase domestic income is lower than under a flexible *ERR*, the domestic deficit in current account is of a larger magnitude[57]. Therefore, capital flows will be higher under a fixed *ERR* than under a flexible *ERR* for a transitory shock to domestic government spending.

Formally, and using (98), the domestic current account is given by[58]

$$(109) \quad \frac{d\overline{F}}{\overline{C}_0^W} = \left[\frac{\bar{r}(\theta+1)}{\bar{r}(\theta+1)+2\theta} - 1\right](1-n)\frac{dG}{\overline{C}_0^W} < 0.$$

Given this current account effect, it is now possible to obtain the positive long-run implications of a transitory shock to domestic government spending. As was mentioned in section IV. A. 1, any changes in the steady state induced by the transitory government spending shock are driven by transfers of wealth occurred in period 1; therefore, the long-run effects under a fixed *ERR* are of the same direction, but of a higher magnitude than those under a flexible *ERR*. Using equations (83)-(89) the following steady-state changes were obtained:

(i) $\hat{\overline{C}}^W = \hat{\bar{y}}^W = 0,$

(ii) $\hat{\overline{C}} < 0,\ \hat{\bar{y}} > 0,\ \hat{\bar{p}}(h) - \hat{\bar{p}}^*(f) - \hat{\overline{E}} < 0,$ and $\hat{\overline{P}} > 0,$ and

(iii) $\hat{\overline{C}}^* > 0,\ \hat{\bar{y}}^* < 0,\ \hat{\bar{p}}^*(f) + \hat{\overline{E}} - \hat{\bar{p}}(h) > 0,$ and $\hat{\overline{P}} < 0.$

The first characteristic of the new steady state, which is typical for any transitory shock, is that world output and world consumption remain the same. Second, as in the flexible *ERR* case, in the new steady state, domestic residents consume less, work more, have their terms of trade deteriorated, and have a higher inflation level. Third, foreign residents enjoy a higher level of consumption, more leisure, improved terms of trade, and lower inflation.

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57 The basic force behind this result is the contraction in the domestic money supply imposed by the fixed *ERR*.

58 A comparison of (102) and (109) confirms the result stated in the previous paragraph.

Normative analysis:

The *real* welfare effects of $\frac{dG}{\overline{C}_0^W}>0$ under a fixed *ERR* for both countries, are given by the following equations:

$$dU^R = \frac{\hat{C}^W}{\theta} - \left(\frac{\theta-n}{\theta}\right)\frac{dG}{\overline{C}_0^W}$$

(110)

$$= -\left\{\left[\frac{(1-\beta)\varepsilon+\beta}{\theta\varepsilon^2}\right]\frac{n\bar{r}(\theta+1)}{\left[\bar{r}(\theta+1)+2\theta\right]} + \left(\frac{\theta-n}{\theta}\right)\right\}\frac{dG}{\overline{C}_0^W} < 0,$$

and

$$dU^{*R} = \frac{\hat{C}^W}{\theta} + \frac{n}{\theta}\frac{dG}{\overline{C}_0^W}$$

(111)

$$dU^{*R} = \left\{-\left[\frac{(1-\beta)\varepsilon+\beta}{\varepsilon^2}\right]\frac{\bar{r}(\theta+1)}{\left[\bar{r}(\theta+1)+2\theta\right]} + 1\right\}\frac{n}{\theta}\frac{dG}{\overline{C}_0^W} > 0.$$

These results are qualitatively similar to the flexible *ERR*; however, the quantitative changes are different. Specifically, under the fixed *ERR*, the reduction in *real* welfare of domestic residents is bigger, while the increase in welfare of the foreign residents is lower. At issue here is the fact that the required reduction in the money supply has induced a reduction in global aggregate demand, exacerbating the basic distortion of the economy, and causing a first order reduction in *real* welfare.

Moving now to the *monetary* welfare effects, the following changes were derived:

(112) $$dU^M = \chi\left(\frac{\overline{M}_0}{\overline{P}_0}\right)^{1-\varepsilon}\left[\hat{M}^r - \frac{\beta}{1-\beta}\hat{\bar{P}}\right] < 0,$$

and

(113) $$dU^{*M} = \chi\left(\frac{\overline{M}_0^*}{\overline{P}_0^*}\right)^{1-\varepsilon}\left[-\frac{\beta}{1-\beta}\hat{\bar{P}}^*\right] > 0.$$

Similarly to the *real* case studied above, the *monetary* welfare effects under a fixed *ERR* exhibit the same qualitative response to the transitory fiscal shock as the ones exhibited under the flexible *ERR*, but different quantitative effects. In fact, under the fixed *ERR*, domestic residents experience a higher reduction in real balances both in the short and the long run than in the flexible *ERR*. As for the foreign residents, under a fixed *ERR*, they experience a reduction in real balances relative to the flexible *ERR* in the short run, but an increase of their real balances in the long run.

The mechanism operating in the *monetary* effect is, again, the required temporary contraction in the domestic money supply. Such a contraction reduces the real balances of domestic residents in the short run, and, by its wealth-induced effects on steady-state domestic consumption, in the long run. These latter effects explain the increase in the steady-state real balances for the foreign residents[59].

Summary:

The results of this section are generalized in the following proposition:

Proposition 5:

Suppose that a particular country follows the policy of having a fixed ERR. Then, a positive (negative) transitory shock to its government spending will, unequivocally, decrease (increase) the total welfare of the residents of that country, and increase (decrease) the welfare of the residents of the other country.

3. Evaluation of the two ERRs under transitory fiscal policy shocks

Given the results obtained in the previous sections regarding the temporary increase in domestic government spending, it is possible now to make an evaluation of the performance of both *ERRs*. The following proposition makes such evaluation in terms of their welfare consequences:

Proposition 6:

Suppose that the only shocks that a country confronts are 'transitory' shocks to domestic or foreign government spending. If the shocks are domestic and

59 Obviously, these wealth-induced effects are of a higher magnitude under the fixed *ERR* than under the flexible *ERR*.

positive, or foreign and *negative, a policy of following a flexible ERR strictly dominates a policy of following a fixed ERR. On the other hand, if the shocks are domestic* and *negative, or foreign* and *positive, a policy of following a fixed ERR strictly dominates a policy of following a flexible ERR*[60].

An implication of Proposition 6 is that, in the context of transitory fiscal policy shocks, a fixed *ERR* is optimal whenever it forces a country to avoid an appreciation of its currency, while a flexible *ERR* is optimal whenever a country experiences a depreciation of its currency. There is a basic reason for this implication: the implied changes in money supply required in each case. In the first case, an increase in the money supply is required; while in the second case, it means that a contraction in the money supply is avoided[61].

Finally, similarly to the monetary shock, the optimal policy for a given fiscal policy shock will maximize world welfare, and not only the welfare of the country that is supposedly choosing the *ERR*[62].

B. PERMANENT SHOCKS

Parallel to the analysis of section IV. A., the discussion of permanent fiscal policy shocks will focus on the effects of a permanent increase in domestic government spending, $\frac{d\bar{G}}{\bar{C}_0^W} > 0$, announced and implemented in period 1. Also, use will be made of the symmetry of the model in order to generalize the results.

1. Flexible ERR case

Positive analysis:

Using equations (91) and (92), the short-run, and log-run, equilibrium exchange rate and domestic relative consumption differential are given by:

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60 It is interesting to note that, for transitory fiscal policy shocks, the optimal choice of *ERR* by a particular country is not the optimal one for the other country.

61 An additional rationale for the aforementioned circumstances, is that the optimal *ERR* maximizes and/or minimizes the transfers of wealth that the shock has induced through the current account. However, by the envelope theorem, these effects have secondary welfare implications.

62 The reason is, again, the monetary changes that the optimally chosen *ERR* requires.

(114) $$\hat{E} = \frac{(\theta+1)(1+\bar{r})}{\bar{r}(\theta^2-1)+\varepsilon[\bar{r}(\theta+1)+2\theta]}\left(\frac{d\overline{G}}{\overline{C}_0^W}\right) > 0,$$

and

(115) $$\hat{C}-\hat{C}^* = \frac{-\varepsilon(\theta+1)(1+\bar{r})}{\bar{r}(\theta^2-1)+\varepsilon[\bar{r}(\theta+1)+2\theta]}\left(\frac{d\overline{G}}{\overline{C}_0^W}\right) < 0.$$

These results are similar to the transitory case, but the magnitude of the effects for a permanent shock are amplified[63]. The depreciation of the domestic currency, the reduction in domestic relative consumption, and the increase in relative domestic output, $\hat{y}-\hat{y}^*$, are produced by the same mechanism described in section IV. A. 1. However, there is an important difference in the short-run effects of a permanent shock with respect to the transitory one: the real rate of interest will now be affected temporarily. More precisely, by using equation (94), the short-run real rate of interest change for a permanent shock to domestic government spending is obtained as

(116) $$\hat{r} = -\left[\frac{(1-\beta)\varepsilon+\beta}{(1-\beta)\varepsilon}\right]\frac{1}{2}n\frac{d\overline{G}}{\overline{C}_0^W} < 0.$$

The rationale behind the temporary reduction in the real rate of interest is that the permanent shock will cause world short-term private output to increase by more than its long-run counterpart; that is, world private output's path is tilted downwards, which is consistent with a reduction in real interest rate[64]. This asymmetry between short-run and long-run output is due to the fact that, because of the temporary nominal rigidities, output in the short-run is demand determined, and consequently its changes are amplified.

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[63] Note from equation (90), that the vertical intercept of the goods-market equilibrium schedule is proportional to the present discounted value of the permanent changes in domestic spending, $\frac{(1+\bar{r})}{\bar{r}(\theta-1)}\frac{d\overline{G}}{\overline{C}_0^W}$, while in the transitory case this factor is $\frac{1}{(\theta-1)}\frac{dG}{\overline{C}_0^W}$.

[64] This amounts to say that savings have decreased in the short-run. To see this, note that $\hat{y}^W = \hat{\bar{y}}^W + \left[\frac{(1-\beta)\varepsilon+\beta}{\varepsilon}\right]\hat{\bar{y}}^W > \hat{\bar{y}}^W$, which implies an increase in world savings in the short-run.

The immediate effect induced by the reduction in the real interest rate is an increase in short-term global consumption relative to the long-term global consumption. In fact the change in short-term in global consumption is given by

$$
(117)\quad \begin{aligned} \hat{C}^W &= \hat{\bar{C}}^W - (1-\beta)\hat{r} \\ &= \left[\frac{\beta(1-\varepsilon)}{\varepsilon}\right]\frac{1}{2}n\frac{d\bar{G}}{\bar{C}_0^W} \leq 0, \ \ since \ \ \varepsilon \geq 1. \end{aligned}
$$

Obviously, $\hat{C}^W > \hat{\bar{C}}^W$.

The aforementioned nominal rigidities also explain why the current account can be in deficit or surplus for a permanent shock to government spending. It is clear that sticky prices, in this case, will allow the path of net private output to be tilted, which, by consumption smoothing, will produce a discrepancy between current income and spending[65]. From equation (93) the domestic current account for a permanent shock to domestic spending, under a flexible *ERR*, is given by[66]

$$
(118)\quad \frac{d\bar{F}}{\bar{C}_0^W} = \frac{(\theta-1)(\theta+1-\varepsilon)(1-n)}{\bar{r}(\theta^2-1)+\varepsilon[\bar{r}(\theta+1)+2\theta]}\left(\frac{d\bar{G}}{\bar{C}_0^W}\right) > 0, \text{ for } \theta+1 > \varepsilon.
$$

Equation (118) indicates that, in the short run, and following the permanent shock, domestic real output exceeds domestic spending. This means that with sticky prices, the increase in real domestic out put, that the domestic currency devaluation has induced is higher than the net increase in short-run domestic spending[67].

With the results described above, it is now possible to summarize the short-run results as follows:

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65 The flexible-price model predicts a balanced current account for a permanent shock government spending. The reason being, that the permanent shock is not able to tilt the path of output net of government expenditure. See Obstfeld and Rogoff (1996) ch.1,2.

66 Another way of looking at the intuition of this result is the following:

let $\Delta \equiv (\hat{y}-\hat{y}^*) - (\hat{\bar{y}}-\hat{\bar{y}}^*) = \left\{\frac{\theta[\varepsilon-(\theta+1)]}{\varepsilon(\theta+1)}\right\}(\hat{c}-\hat{c}^*)$. Since for $\frac{d\bar{G}}{\bar{C}_0^W} > 0, (\hat{c}-\hat{c}^*) < 0,$ then:

$\theta+1 > \varepsilon \Rightarrow \Delta > 0 \Rightarrow \frac{d\bar{F}}{\bar{C}_0^W} > 0,$ or $\theta+1 < \varepsilon \Rightarrow \Delta < 0 \Rightarrow \frac{d\bar{F}}{\bar{C}_0^W} < 0.$

67 The opposite is happening in the foreign contry.

(i) $\hat{C}^W < 0,\ \hat{y}^W > 0,$

(ii) $\hat{C} < 0,\ \hat{y} > 0,$ and $\hat{P} > 0,$ and

(iii) $\hat{C}^* > 0,\ \hat{y}^* < 0,$ and $\hat{P}^* < 0.$

That is, in the short run, and as a result of a permanent increase in domestic per capita government spending, under a flexible *ERR*, world consumption decreases, and world effort (output) increases. Domestic residents consume less, work harder, and pay higher prices. On the other hand, foreign residents consume more, work less, and pay lower prices.

Additionaly, making use of (118), and equations (82)-(89), the following steady-state changes were obtained:

(i) $\hat{\bar{C}}^W < 0,\ \hat{\bar{y}}^W > 0,$

(ii) $\hat{\bar{C}} < 0,\ \hat{\bar{y}} > 0,\ \hat{\bar{p}}(h) - \hat{\bar{p}}^*(f) - \hat{\bar{E}} < 0,$ and $\hat{\bar{P}} > 0,$ and

(iii) $\hat{\bar{C}}^* > 0,\ \hat{\bar{y}}^* > 0,\ \hat{\bar{p}}^*(f) + \hat{\bar{E}} - \hat{\bar{p}}(h) > 0,$ and $\hat{\bar{P}}^* < 0.$

That is, under a flexible *ERR*, in the new steady state, world consumption decreases, and world output increases. Domestic residents consume less, work more, have their terms of trade deteriorated, and pay higher prices. On the other hand, foreign residents consume more, work more[68], have their terms of trade improved, and pay lower prices.

It is interesting to note that due to the domestic residents' higher level of wealth, the magnitude of the adverse effects induced by the shock on their consumption, effort, terms of trade, and prices, has been reduced. At the same time, the favourable effects for the foreign residents are muted by their wealth reduction.

Normative analysis:

The *real* welfare effects of $\frac{d\bar{G}}{\bar{C}_0^W} > 0,$ under a flexible *ERR*, for both countries are given by the following equations:

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[68] Although less than the domestic residents. This could be seen from the following result:

$$\hat{\bar{y}} - \hat{\bar{y}}^* = \left[1 - \frac{\bar{r}(\theta-1)(\theta-1+\varepsilon)}{\bar{r}\left(\theta^2-1\right)+\varepsilon\left[\bar{r}(\theta+1)+2\theta\right]}\right]\frac{1}{2}\frac{d\bar{G}}{\bar{C}_0^W} > 0.$$

(119) $$dU^{R} = \frac{\hat{C}^{W}}{\theta} - \left[\frac{(1-n)(1+\bar{r})}{\bar{r}} + \frac{n(\theta-1)}{\theta} + n\left(\frac{2\theta-1}{2\theta\bar{r}}\right)\right]\frac{d\overline{G}}{\overline{C}_0^{W}} < 0,$$

and

(120) $$dU^{R*} = \frac{\hat{C}^{W}}{\theta} + \frac{n}{\theta}\left(1 + \frac{1}{2\bar{r}}\right)\frac{d\overline{G}}{\overline{C}_0^{W}} > 0.$$

Similarly to the transitory case, the *real* welfare effects are negative for the domestic residents, and positive for the foreign residents.

The *monetary* welfare effects of $\frac{d\overline{G}}{\overline{C}_0^{W}} > 0$, under a flexible *ERR*, are:

(121) $$dU^{M} = \chi\left(\frac{\overline{M}_0}{\overline{P}_0}\right)^{1-\varepsilon}\left[-(1-n)\hat{E} - \frac{\beta}{1-\beta}\hat{\overline{P}}\right] < 0,$$

and

(122) $$dU^{M*} = \chi\left(\frac{\overline{M}_0^{*}}{\overline{P}_0^{*}}\right)^{1-\varepsilon}\left[n\hat{E} - \frac{\beta}{1-\beta}\hat{\overline{P}}^{*}\right] > 0.$$

Again, these equations reflect that domestic residents decrease their holding of real balances in both periods, while foreign residents experience an increase, also in both periods. These changes in real balances are responsible for the stated changes in *monetary* welfare.

Summary:

The results of this section can be generalized in the following proposition:

Proposition 7:

Under a flexible ERR, a positive (negative) permanent shock to government spending in a particular country will, unequivocally, decrease (increase) the total welfare of the residents of the expanding (contracting) country. The *residents of the non-expanding (non-contracting) country will, unequivocally, experience an increase (decrease) in their welfare.*

2. The fixed ERR case

Positive analysis:

If the domestic central bank, in the presence of $\frac{d\overline{G}^*}{\overline{C}_0^W} > 0$, decides to keep the exchange rate from moving, the required contraction in the money supply would be given by[69]

$$(123) \quad \hat{M}^r = \frac{-(\theta+1)(1+\bar{r})}{\varepsilon[\bar{r}(\theta+1)+2\theta]}\left[\frac{d\overline{G}}{\overline{C}_0^W}\right] < 0.$$

As a consequence of the fiscal shock and the monetary contraction, the short-run equilibrium is characterized by $\hat{E} = 0$, and by

$$(124) \quad \hat{C} - \hat{C}^* = \frac{-(\theta+1)(1+\bar{r})}{[\bar{r}(\theta+1)+2\theta]}\left[\frac{d\overline{G}}{\overline{C}_0^W}\right] < 0.$$

This decrease in domestic relative consumption is higher than in the flexible *ERR* case, where the difference is caused by the contractionary forces created by the reduction in the money supply. Such forces operate through a short-run increase in the real interest rate, as given by the following equation[70]:

$$(125) \quad \hat{r} = \frac{\beta+(1-\beta)\varepsilon}{(1-\beta)\varepsilon}\left[\frac{\bar{r}(\theta+1)+2}{\bar{r}(\theta+1)+2\theta}\right]\frac{n}{2}\frac{d\overline{G}}{\overline{C}_0^W} > 0.$$

This temporary increase in the real rate of interest produces a contraction in short-run world consumption, as given by

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69 Derived from equation (95).

70 Derived from equation (99). Again, note that this increase in the real interest rate obeys to a reduction in savings. Note that $\hat{y}^W - \hat{\bar{y}}^W = \hat{C}^W + \frac{n}{2}\frac{d\overline{G}}{\overline{C}_0^W} < 0$. Note also the different response of $\hat{r}$ under a fixed *ERR*. The differrence obeys to the forced contraction in the money supply that the management of a fixed *ERR* requires.

(126) $$\hat{C}^W = -\left[1+\frac{\beta+(1-\beta)\varepsilon}{\varepsilon}\left(\frac{\bar{r}(\theta+1)+2}{\bar{r}(\theta+1)+2\theta}\right)\right]\frac{n}{2}\frac{d\overline{G}}{\overline{C}_0^W}<0,$$

which generates an absolute reduction in domestic consumption, and moderates the increase in foreign consumption. Since $\hat{E}=0$, domestic and foreign output increase by the same amount, and therefore, the world output increase in the short-run.

The reduction in short-run consumption of the domestic residents is, however, tempered by a deficit in their current account, which is given by

(127) $$\frac{d\overline{F}}{\overline{C}_0^W} = -\frac{(\theta-1)(1-n)}{\bar{r}(\theta+1)+2\theta}\frac{d\overline{G}}{\overline{C}_0^W}<0.$$

Note that the sign of this equation does not depend on the parameters of the model, which means that it is the contraction in the domestic money supply that is driving the result.

The previous short-run results can be summarized as follows:

(i) $\hat{C}^W<0,\ \hat{y}^W>0,$

(ii) $\hat{C}<0,\ \hat{y}>0,\ \hat{P}=0,$ and

(iii) $\hat{C}^*>0,\ \hat{y}^*>0,\ \hat{P}^*=0.$

That is, in the short run, and as a result of a permanent increase in domestic per-capita government spending, under a fixed *ERR*, global consumption decreases, and global output increases. Domestic residents consume less, work more, an pay the same prices. On the other hand, foreign residents consume more, work more, and pay the same prices.

Moving to the long-run effects of the shock, using equations (127), and (82)-(89), the following steady-state changes were obtained:

(i) $\hat{\overline{C}}^W<0,\ \hat{\bar{y}}^W>0,$

(ii) $\hat{\overline{C}}<0,\ \hat{\bar{y}}>0,\ \hat{\bar{p}}(h)-\hat{\bar{p}}^*(f)-\hat{\overline{E}}<0,$ and $\hat{\overline{P}}>0,$ and

(iii) $\hat{\overline{C}}^*>0,\ \hat{\bar{y}}^*>0,\ \hat{\bar{p}}^*(f)+\hat{\overline{E}}-\hat{\bar{p}}(h)>0,$ and $\hat{\overline{P}}<0.$

That is, under a fixed *ERR*, in the new steady state, world consumption decreases, and world output increases, domestic residents consume less, work more, have their terms of trade deteriorated, and pay higher prices. On the other hand, foreign residents consume more, work more, have their terms of trade improved, and pay lower prices.

Normative analysis:

The *real* welfare effects of $\frac{d\overline{G}}{\overline{C}_0^W} > 0$, under a fixed *ERR*, for both countries are given by the following equations:

$$(128) \quad dU^R = \frac{\hat{C}^W}{\theta} - \left[\frac{(1-n)(1+\bar{r})}{\bar{r}} + \frac{n(\theta-1)}{\theta} + n\left(\frac{2\theta-1}{2\theta\bar{r}}\right)\right]\frac{d\overline{G}}{\overline{C}_0^W} < 0,$$

and

$$(129) \quad dU^{R*} = \frac{\hat{C}^W}{\theta} + \frac{n}{\theta}\left(1 + \frac{1}{2\bar{r}}\right)\frac{d\overline{G}}{\overline{C}_0^W} > 0.$$

It is clear that *real* welfare effects are negative for the domestic residents and positive for the foreign residents. It is interesting to note that the *real* welfare effects, under both *ERRs*, as given by equations (119), and (128), and equations (120), and (129) look simililar. However, the difference between each pair obeys to the change in short-run global consumption. Looking at equations (117), and (126) it is clear that the reduction of short-run world consumption under the fixed *ERR* is higher than under the flexible *ERR*. Therefore, for the permanent domestic fiscal shock, the reduction in domestic *real* welfare is larger under the fixed *ERR* than under the flexible *ERR*. By the same token, the increase in foreign real welfare is lower under the fixer *ERR* than under the flexible *ERR*.

The monetary effects of $\frac{d\overline{G}}{\overline{C}_0^W} > 0$, under a Fixed ERR, are:

$$(130) \quad dU^M = \chi\left(\frac{\overline{M}_0}{\overline{P}_0}\right)^{1-\varepsilon}\left[\hat{M}^r - \frac{\beta}{1-\beta}\hat{\overline{P}}\right] < 0,$$

and

$$(131) \quad dU^{M*} = \chi\left(\frac{\overline{M}_0^*}{\overline{P}_0^*}\right)^{1-\varepsilon}\left[-\frac{\beta}{1-\beta}\hat{\overline{P}}^*\right] > 0.$$

Equation (130) indicates that domestic residents see their *monetary* welfare reduced due to a reduction in their real balances in both periods. On the other hand, from equation (131), foreign residents, due to their long-run increase in real balances, experience an increase in their *monetary* welfare. Again, comparing equations (121), and (128) on one hand, and equations (122), and (129) on the other hand, it is possible to conclude that the reduction in monetary welfare, for domestic residents, is higher under a fixed *ERR* than under a flexible *ERR*; also, that the increase in monetary welfare, for foreign residents, is higher under a fixed *ERR* than under a flexible *ERR*.

Summary:

The results of this section can be generalized in the following proposition:

Proposition 8:

Suppose that a particular country follows the policy of having a fixed ERR. Then, a positive (negative) permanent shock to its government spending will, unequivocally, decrease (increase) the total welfare of the residents of that country, and increase (decrease) the welfare of the residents of the other country.

C. EVALUATION OF THE TWO ERRs UNDER PERMANENT FISCAL POLICY SHOCKS

Given the results obtained in sections IV. B. 1. and IV. B. 2., it is now possible to make an evaluation of the performance of both *ERRs*. The following proposition summarizes the performance of both *ERRs* for fiscal permanent shocks:

Proposition 9:

Suppose that the only shocks that a country confronts are 'permanent' shocks to domestic or foreign government spending. If the shocks are domestic and *positive, or foreign* and *negative, a policy of following a flexible ERR strictly dominates a policy of following a fixed ERR. If, on the other hand, the shocks* *are domestic* and *negative, or foreign* and *positive, a policy of following a fixed ERR strictly dominates a policy of following a flexible ERR.*

Again, as in the transitory fiscal shocks, what makes the difference for both *ERRs* is the changes in the money supply that the fixed *ERR* requires. These changes, depending on their direction, either exacerbate or mitigate the basic distortion of the economy.

Also, the optimal choice of an *ERR*, for the given permanent fiscal shock, will maximize world welfare, and not only the welfare of the country choosing the regime.

V. PRODUCTIVITY SHOCKS

This section focuses on the way this economy responds to productivity shocks under the two *ERRs*. Since the only perturbation this economy is subjected to is the shocks to productivity, changes in government expenditure, and changes in the money supply, different from those required to manage the *ERR*, will not be considered.

Assumption 5 specifies that productivity shocks are represented by changes in the parameter A of each producer's production function. Therefore, as indicated in note 9, shocks to A, are captured by shocks to k in equation (2)[71].

As in sections III. And IV., the log-linearized equations (33)-(53) describe the system on which the analysis will be based[72].

[71] There is an inverse relationship between A and k. This implies that a positive shock to A means that less effort is required to produce a unit of output, and viceversa for a negative shock.

[72] It is important to note that the first-order conditions, and the symmetric steady state of the model remain unaffected. In fact all the log-linearized equations remain the same, with exception of equations (39) and (40) which now include the deviations of the parameter $k\left(k^*\right)$ from its steady state $\bar{k}_0\left(\bar{k}_0^*\right)$. These equations become: $(\theta+1)\hat{y}_t = -\theta\hat{C}_t + \hat{C}_t^W + \frac{dG_t^W}{\bar{C}_0^W} - \theta\hat{k}_t$ and $(\theta+1)\hat{y}_t^* = -\theta\hat{C}_t^* - \hat{C}_t^W + \frac{dG_t^W}{\bar{C}_0^W} - \theta\hat{k}_t^*$. As mentioned in the text $\frac{dG_t^W}{\bar{C}_0^W} = 0$, for the analysis of this section.

The following equations describe the new steady state in the presence of productivity shocks:

$$(132)\qquad \hat{C}^W = \hat{y}^W = -\frac{1}{2}\hat{k}^W,$$

$$(133)\qquad \hat{C} = \left(\frac{\theta+1}{2\theta}\right)\bar{r}\frac{d\bar{F}}{\bar{C}_0^W} - \left(\frac{n+\theta-1}{2\theta}\right)\hat{k} - \left(\frac{1-n}{2\theta}\right)\hat{k}^*,$$

$$(134)\qquad \hat{C}^* = -\left(\frac{n}{1-n}\right)\left(\frac{\theta+1}{2\theta}\right)\bar{r}\frac{d\bar{F}}{\bar{C}_0^W} - \left(\frac{n}{2\theta}\right)\hat{k} - \left(\frac{\theta-n}{2\theta}\right)\hat{k}^*,$$

$$(135)\qquad \hat{y} = -\frac{1}{2}\bar{r}\frac{d\bar{F}}{\bar{C}_0^W} - \frac{1}{2}\hat{k},$$

$$(136)\qquad \hat{y}^* = \frac{1}{2}\left(\frac{n}{1-n}\right)\bar{r}\frac{d\bar{F}}{\bar{C}_0^W} - \frac{1}{2}\hat{k}^*,$$

$$(137)\qquad \hat{p}(h) - \hat{p}^*(f) - \hat{E} = \left(\frac{1}{1-n}\right)\left(\frac{1}{2\theta}\right)\bar{r}\frac{d\bar{F}}{\bar{C}_0^W} + \frac{1}{2\theta}\left(\hat{k} - \hat{k}^*\right),$$

$$(138)\qquad \begin{aligned}\hat{P} &= \hat{M} - \frac{1}{\varepsilon}\hat{C} \\ &= -\frac{1}{\varepsilon}\left[\left(\frac{1+\theta}{2\theta}\right)\bar{r}\frac{d\bar{F}}{\bar{C}_0^W} - \left(\frac{n+\theta-1}{2\theta}\right)\hat{k} - \left(\frac{1-n}{2\theta}\right)\hat{k}^*\right],\end{aligned}$$

and

$$(139)\qquad \begin{aligned}\hat{P}^* &= \hat{M}^* - \frac{1}{\varepsilon}\hat{C}^* \\ &= \frac{1}{\varepsilon}\left[\left(\frac{n}{1-n}\right)\left(\frac{\theta+1}{2\theta}\right)\bar{r}\frac{d\bar{F}}{\bar{C}_0^W} - \left(\frac{n}{2\theta}\right)\hat{k} - \left(\frac{\theta-n}{2\theta}\right)\hat{k}^*\right].\end{aligned}$$

Expression (132) states that a permanent increase in global productivity raises steady-state global output and consumption by half of the increase in productivity. The other half goes to an increase in global leisure. Equations (133)-(139) indicate, first, that only permanent productivity shocks may affect the steady state of the economy, and second, that shocks to productivity affect steady-state variables directly, and indirectly through their effect on the current account.

The analysis of this section will focus on the effects of a positive shock to the domestic country's productivity. As usual, a generalization is then obtained by resorting to the symmetry of the model.

A. TRANSITORY SHOCKS

It is clear from equations (132)-(139) that a transitory shock to domestic productivity, $\hat{k} < 0$, does not affect the long-run equilibrium of either economy. In addition, since, in the short run, output in each country is demand determined, productivity shocks, which are supply shocks, do not affect the short-run equilibrium either[73]. Consequently, domestic producers will continue generating the same level of output, but will enjoy more leisure.

The previous analysis implies that, since a temporary productivity shock does not create any pressure on the money market that forces a change in the exchange rate, the choice of an *ERR* is not relevant to manage the effects of such a shock.

The general effect of a transitory shock can be summarized, for the short and the long run, as follows:

(i) $\hat{C}^W = 0, \ \hat{y}^W = 0,$

(ii) $\hat{C} = 0, \ \hat{y} + \hat{k} < 0, \ \hat{P} = 0,$

(iii) $\hat{C}^* = 0, \ \hat{y}^* = 0, \ \hat{P} = 0,$

(iv) $\hat{\bar{C}}^W = 0, \ \hat{\bar{y}}^W = 0,$

(v) $\hat{\bar{C}} = 0, \ \hat{\bar{y}} = 0, \ \hat{\bar{P}} = 0,$ and,

(vi) $\hat{\bar{C}}^* = 0, \ \hat{\bar{y}}^* = 0, \ \hat{\bar{P}}^* = 0.$

[73] Note that equations (39) and (40) are not binding in the short run. The relevant equations in the short run are (37) and (38), and neither of them is affected by $\hat{k} < 0$. As a result, there is no tilting in the paths of net private income, and neither the current account nor the real interest rate will be affected. Of course, neither the exchange rate nor relative domestic consumption will change.

That is, the only effect of a transitory shock to domestic productivity is a reduction in the effort level of the domestic residents in the short run. This change in domestic effort will also be the only source of changes in welfare. Formaly,

$$dU = dU^R = -\left(\frac{\theta - 1}{2\theta}\right)\hat{k} > 0, \text{ and } dU^* = 0.$$

The following proposition summarizes and generalizes the effects of transitory productivity shocks:

Proposition 10:

If a country experiences a transitory and positive (negative) productivity shock, the welfare of that country will increase (decrease), while the welfare of the other country will remain unchanged. Since these effects hold under either ERR, the choice of an ERR is irrelevant in managing the effects of the aforementioned shock.

B. PERMANENT SHOCKS

In studying the effects of a permanent and positive shock to domestic productivity, $\hat{k} < 0$, it is important to understand how its long-run effects may have short-run consequences. It is clear from the previous analysis of the transitory case that, given price stickiness, productivity shocks, either transitory or permanent, will not affect *directly* the short run equilibrium of the system[74]. However, since permanent productivity shocks have long-run effects, future changes in output might influence the short-run equilibrium of the system through the current account and the real rate of interest. In what follows, such mechanism will be studied for both *ERRs*.

1. Flexible ERR

Positive analysis:

To characterize the short-run effects of $\hat{k} < 0$, under a flexible *ERR*, the equations describing the money market and the goods market equilibrium will be used. Equation (64) continues describing the equilibrium in the money market, although

[74] As mentioned earlier, the only short-run effect is the change in domestic leisure.

without money supplies' changes. The equation describing the equlibrium in the goods market is now given by

$$(140)\quad \hat{E} = \frac{\bar{r}(\theta+1)+2\theta}{\bar{r}(\theta^2-1)}\left(\hat{C}-\hat{C}^*\right)+\frac{1}{\bar{r}(\theta+1)}\left(\hat{\bar{k}}-\hat{\bar{k}}^*\right).$$

Using equations (64) and (140) the short-run, and long-run, equlibrium changes in the exchange rate and relative consumption for $\hat{\bar{k}}<0$ are obtained as:

$$(141)\quad \hat{E} = \frac{(\theta-1)}{\bar{r}(\theta^2-1)+\varepsilon[\bar{r}(\theta+1)+2\theta]}\hat{\bar{k}}<0,$$

and

$$(142)\quad \hat{C}-\hat{C}^* = -\frac{\varepsilon(\theta-1)}{\bar{r}(\theta^2-1)+\varepsilon[\bar{r}(\theta+1)+2\theta]}\hat{\bar{k}}>0.$$

Then, a positive domestic productivity shock will appreciate the domestic currency, and increase domestic relative consumption. The mechanism operating here is the following: since, domestic short-run output is not initially affected by the shock, and, as equation (135) shows, its long-run counterpart will be[75], the domestic residents borrow against such future income to finance current consumption[76]. The increase in domestic relative consumption will raise the demand for money, causing an appreciation of the domestic currency.

It is clear from the previous paragraph that the domestic current account will be in deficit, as the following equation confirms:

$$(143)\quad \frac{d\bar{F}}{\bar{C}_0^W} = \frac{(1-n)(\theta-1)[\theta-1+\varepsilon]}{\bar{r}(\theta+1)+2\theta}\hat{\bar{k}}<0.$$

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75 In other words, the domestic path of net private income is tilted upwards.

76 Note that the foreign residents experience a downward tilt in their path of net private income.

Furthermore, the need for financing the short-run consumption is increased by the reduction in relative domestic output, produced by the domestic currency appreciation[77].

Another effect induced by the productivity shock is on the real rate of interest, which is given by

$$(144) \qquad \hat{r} = -\left[\frac{\beta+(1-\beta)\varepsilon}{(1-\beta)\varepsilon}\right]\frac{1}{2}n\hat{\bar{k}} > 0.$$

This effect is also explained by the global decrease in savings produced by the shock[78].

As a consequence of such an increase in the interest rate, short-run global consumption will increase by less than its long-run counterpart. That is,

$$(145) \qquad \hat{\bar{C}}^W > \hat{C}^W = \left[\frac{\beta(1-\varepsilon)}{2\varepsilon}\right]n\hat{\bar{k}} \geq 0.$$

This increase in world consumption is supported by the increase in foreign output, by an amount that more than compensate the reduction in the output of the domestic country.

The previous short-run results can be summarized as follows:

(i) $\hat{y}^W = \hat{C}^W > 0,$

(ii) $\hat{C} > 0, \ \hat{y} < 0, \ \hat{P} < 0,$ and,

(iii) $\hat{C}^* > 0, \ \hat{y}^* > 0, \ \hat{P}^* > 0.$

That is, world output and consumption increase, domestic residents consume more, work less, and pay lower prices, and foreign residents consume more[79], work more, and pay higher prices.

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77 In fact, absolute domestic output also decreases. Consequently, the domestic path of net private output becomes steeper.

78 Note that $\hat{y}^W < \hat{C}^W$, and $\hat{\bar{y}}^W = \hat{\bar{C}}^W$, therefore $\hat{y}^W < \hat{\bar{y}}^W$. This is consistent with a reduction in global savings, and, consequently, with an increase in the real rate of interest.

79 Although relatively less than domestic residents as equation 142 shows.

Two comments about these results are worth mentioning. First, it is interesting to note that a permanent supply shock has short-run effects. The reason is that, because of the presence of sticky prices, a permanent productivity shock is able to induce domestic and foreign residents to smooth out their consumption through borrowing and lending in the international bond market. Second, short-run reduction in domestic residents' effort is higher than in the transitory case, as evidenced by the fact that domestic output declined.

The long-run effects of $\hat{\bar{k}} < 0$ are obtained form equations (132)-(139), and (143), as follows:

(i) $\hat{\bar{C}}^W = \hat{\bar{y}}^W > 0,$

(ii) $\hat{\bar{C}} > 0,\ \hat{\bar{y}} > 0,\ \hat{\bar{p}}(h) - \hat{\bar{p}}^*(f) - \hat{\bar{E}} < 0,$ and $\hat{\bar{P}} < 0,$ and

(iii) $\hat{\bar{C}}^* > 0,\ \hat{\bar{y}}^* < 0,\ \hat{\bar{p}}^*(f) + \hat{\bar{E}} - \hat{\bar{p}}(h) > 0,$ and $\hat{\bar{P}} < 0.$

That is, under a flexible *ERR*, in the new steady state, world consumption and output increase; domestic residents consume more, produce more[80], have their terms of trade deteriorated, and pay lower prices; and foreign residents consume more, produce less, have their terms of trade improved, and pay lower prices[81].

Normative analysis:

Under a flexible *ERR*, the effects of a permanent shock to domestic productivity on *real* welfare for both countries are given by

(146) $$dU^R = \left[\frac{\beta r(1-\varepsilon) - \varepsilon}{\theta r \varepsilon}\right]\frac{n}{2}\hat{\bar{k}} - \left(\frac{\theta - 1}{\theta}\right)\left(\frac{1}{1-\beta}\right)\frac{1}{2}\hat{\bar{k}} > 0,$$

and

(147) $$dU^{*R} = \left[\frac{\beta r(1-\varepsilon) - \varepsilon}{\theta r \varepsilon}\right]\frac{n}{2}\hat{\bar{k}} > 0.$$

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80 Although with less effort per unit of output.

81 Note that one of the effects of $\hat{\bar{k}} < 0$ is to reduce the steady-state world inflation. That is, $\hat{\bar{P}}^W < 0.$

That is, *real* welfare increases for residents of both countries, although the increase is higher for the domestic residents. Equations (146) and (147) share a common term, $\left[\frac{\beta\theta(1-\varepsilon)-\varepsilon}{\theta\theta\varepsilon}\right]\frac{n}{2}\hat{k}$, which explains why foreign residents have their *real* welfare increased. This term is associated with the spillover effects that the domestic productivity shock has on global short-run and long-run consumption, and that, in the context of the model, reduces the basic distortion that this economy experiences[82]. The idiosyncratic effect of $\hat{k}<0$ on domestic real welfare is explained by the fact that now they can enjoy a higher level of leisure.

The *monetary* welfare effects of a permanent shock to domestic productivity are given by

$$(148) \quad dU^{M} = \chi\left(\frac{\overline{M}_0}{\overline{P}_0}\right)^{1-\varepsilon}\left[-(1-n)\hat{E}-\frac{\beta}{1-\beta}\hat{\overline{P}}\right]>0,$$

and

$$(149) \quad dU^{*M} = \chi\left(\frac{\overline{M}_0^*}{\overline{P}_0^*}\right)^{1-\varepsilon}\left[n\hat{E}-\frac{\beta}{1-\beta}\hat{\overline{P}}^*\right] \gtrless 0.$$

Since domestic residents experience an increase in their real balances in both periods, their monetary welfare unequivocally increases. On the other hand, foreign residents see their real balances decrease in the short run, but increase in the long run. The net effect of these changes on foreign residents' monetary welfare is ambiguous, and the way this fare in their total welfare depends on the value of χ.

Summary:

The results of this section are generalized in the following proposition:

Proposition 11:

Under a Flexible ERR, a positive (negative) permanent productivity shock in a particular country will, unequivocally, increase (decrease) the total welfare of

[82] That is, $\hat{k}<0$ coordinates a higher work effort, and moves the world economy closer to efficient production.

the residents of the expanding (contracting) country. The residents of the non-expanding (non-contracting) country will experience a change in their welfare that depends on their parametric structure. Specifically, if χ is small, their welfare will increase (decrease), with the opposite results if χ is large.

2. *Fixed ERR*

Positive analysis:

Under a fixed *ERR*, a positive and permanent shock to domestic productivity forces the domestic central bank to expand, permanently, the money supply, as described by the money supply reaction function

$$(150) \quad \hat{M}^r = -\frac{(\theta-1)}{\varepsilon\left[\bar{r}(\theta+1)+2\theta\right]}\hat{\bar{k}} > 0.$$

The rationale for this expansion of the domestic money supply lies in the fact that, as mentioned in section V. B. 1., the permanent shock causes an increase in the relative money demand that pressures the money market towards an appreciation of the exchange rate. Consequently, a higher relative money supply prevents the exchange rate from appreciating by providing the higher amount of money that is now being demanded.

As a result of keeping the exchange rate constant, the increase in domestic relative consumption is higher than in the flexible *ERR* case, as indicated by

$$(151) \quad \hat{C}-\hat{C}^* = -\frac{(\theta-1)}{\bar{r}(\theta+1)+2\theta}\hat{\bar{k}} > 0.$$

Also, given the expansion in the domestic money supply, the short-run real rate of interest increases by a smaller amount than in the flexible ERR case, as shown by the following expression:

$$(152) \quad \hat{r} = -\left[\frac{\beta+(1-\beta)\varepsilon}{(1-\beta)\varepsilon}\right]\left[\frac{\bar{r}(\theta+1)+\theta}{\bar{r}(\theta+1)+2\theta}\right]\frac{1}{2}n\hat{\bar{k}} > 0.$$

The effect of this smaller increase in the rate of interest is that world consumption will expand by an amount larger than the one obtained in the flexible *ERR* case. That is,

$$(153)\quad \hat{C}^W = \left[\frac{\beta(1-\varepsilon)}{\varepsilon}\right]\frac{n}{2}\hat{k} - \left[\frac{\beta+(1-\beta)\varepsilon}{\varepsilon}\right]\left[\frac{(\theta-1)}{\bar{r}(\theta+1)+2\theta}\right]n\hat{k} > 0.$$

This increase in short-run aggregate consumption, is sustained by an equal increase of output in both countries[83].

The fact that domestic output increases, indicates that the amount that domestic residents borrow from the foreign residents is less than in the flexible *ERR* case[84]. The deficit in the domestic current account is given by the following equation:

$$(154)\quad \frac{d\bar{F}}{\bar{C}_0^W} = \frac{(1-n)(\theta-1)}{\bar{r}(\theta+1)+2\theta}\hat{k} < 0.$$

The mechanism operating behind this result is essentially the same described for the flexible *ERR* case; however, in the fixed *ERR* the increase in the money supply counterbalances the forces generating the deficit. Essentially, what occurs in the fixed *ERR* is the simultaneous operation of two separate shocks with additive effects on both economies. On one hand are the effects of the positive productivity shock, and on the other are the effects of a positive money supply shock.

The effects that these shocks have on the short-run variables are summarized as follows:

(i) $\hat{y}^W = \hat{C}^W > 0,$

(ii) $\hat{C} > 0,\ \hat{y} > 0,\ \hat{P} = 0,$ and,

(iii) $\hat{C}^* > 0,\ \hat{y}^* > 0,\ \hat{P}^* = 0$.

That is, world output and consumption increase; domestic residents consume more, have a higher level of output, although enjoy relatively more leisure, and

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[83] That is, given that $\hat{E} = 0$, $\hat{y} = \hat{y}^* = \hat{y}^W = \hat{C}^W$.

[84] The paths of net private output are now less steeper than in the flexible *ERR* case.

have their prices constant; and foreign residents consume more, have a higher level of output[85], and have their prices constant.

The long-run effects of $\hat{k} < 0$ are obtained from equations (132)-(139) and (154), as follows:

(i) $\hat{C}^W = \hat{y}^W > 0,$

(ii) $\hat{C} > 0,\ \hat{y} > 0,\ \hat{p}(h) - \hat{p}^*(f) - \hat{E} < 0,$ and $\hat{P} < 0,$ and

(iii) $\hat{C}^* > 0,\ \hat{y}^* < 0,\ \hat{p}^*(f) + \hat{E} - \hat{p}(h) > 0,$ and $\hat{P} < 0.$

That is, under a fixed *ERR*, in the new steady state, world consumption and output increase; domestic residents consume more, produce more[86], have their terms of trade deteriorated, and pay lower prices; and foreign residents consume more, produce less, have their terms of trade improved, and pay lower prices.

Normative analysis:

Under a fixed *ERR*, the real effects of a permanent shock to domestic productivity for both countries are given by

$$dU^R = \left[\frac{\beta\bar{r}(1-\varepsilon)-\varepsilon}{\theta\bar{r}\varepsilon}\right]\frac{n}{2}\hat{k} - \left(\frac{\theta-1}{\theta}\right)\left(\frac{1}{1-\beta}\right)\frac{1}{2}\hat{k} - \frac{(\theta-1)[\beta+(1-\beta)\varepsilon]}{\theta\varepsilon[\bar{r}(\theta+1)+2\theta]}n\hat{k} > 0, \tag{155}$$

and

$$dU^{*R} = \left[\frac{\beta\bar{r}(1-\varepsilon)-\varepsilon}{\theta\bar{r}\varepsilon}\right]\frac{n}{2}\hat{k} - \frac{(\theta-1)[\beta+(1-\beta)\varepsilon]}{\theta\varepsilon[\bar{r}(\theta+1)+2\theta]}n\hat{k} > 0. \tag{156}$$

85 In fact, foreign residents work more.

86 Again, with less effort per unit of output.

Again, as in the flexible ERR case, *real* welfare increases for the residents of both countries, and the increase is relatively higher for the domestic residents[87]. However, there is an important difference with respect to the flexible *ERR* case: under the fixed *ERR*, the increase in *real* welfare is higher for both countries. The source of the higher *real* welfare is captured by the last, common term of equations (155), and (156), $-\frac{(\theta-1)[\beta+(1-\beta)\varepsilon]}{\theta\varepsilon[\bar{r}(\theta+1)+2\theta]}\hat{nk}$; which reflects the increase in short-run world consumption that the expansion of the money supply induced.

The *monetary* welfare effects of a permanent shock to domestic productivity are given by

$$(157) \quad dU^{M} = \chi\left(\frac{\overline{M}_0}{\overline{P}_0}\right)^{1-\varepsilon}\left[\hat{M}^{r} + \frac{\beta}{\varepsilon(1-\beta)}\hat{\bar{C}}\right] > 0,$$

and

$$(158) \quad dU^{*M} = \chi\left(\frac{\overline{M}_0^{*}}{\overline{P}_0^{*}}\right)^{1-\varepsilon}\left[\frac{\beta}{\varepsilon(1-\beta)}\hat{\bar{C}}^{*}\right] > 0.$$

That is, *monetary* welfare increases for the residents of both countries. This increase reflects the fact that, for the domestic residents, real balances increase in both periods; while for foreign residents, there is an increase in steady-state real balances caused by a reduction in its steady-state inflation.

A comparison between equations (148) and (157), and between equations (149) and (157), show that under the fixed *ERR*, the increase in *monetary* welfare for the residents of both countries is higher than under the flexible *ERR*.

Summary:

The results of this section are generalized in the following proposition:

Proposition 12:

Suppose that a particular country follows a policy of having a fixed ERR. Then, a positive (negative) permanent shock to its productivity will, unequivocally,

[87] The source of this higher increase is, again, the idiosyncratic positive shock to domestic productivity.

increase (decrease) the total welfare of the residents of both countries. Also, the increase in the welfare of the country experiencing the shock is relatively higher than the increase in the welfare of the other country.

C. EVALUATION OF THE TWO ERRs UNDER PERMANENT PRODUCTIVITY SHOCKS

The following proposition summarizes the results obtained in sections V. B. 1. and V. B. 2. with respect to the performance of both *ERRs*:

Proposition 13:

Suppose that the only shocks that a country confronts are permanent shocks to domestic and foreign productivity. If a shocks is positive, a policy of following a fixed ERR strictly dominates a policy of following a flexible ERR, independent of where *the shock has originated. If, on the other hand, the shock is negative, a policy of following a flexible ERR strictly dominates a policy of following a fixed ERR, independent of* where *the shock has originated.*

As was mentioned above, the reason for this result lies in the fact that a fixed *ERR* forces a change in the world's money supply, adding to the expansionary or to the contractionary forces induced by the productivity shock. In other words, under a fixed *ERR*, the two forces affecting the system either reduce or exacerbate the basic distortion assumed for this economy.

VI. CONCLUSSIONS

The first observation that can be made about the role played by an *ERR*, is that it does not by itself reverse the basic, positive and normative, effects of particular shocks. Its role is to manage them. In this sense, a flexible *ERR* smooths down the impact of the shocks, while a fixed *ERR* amplifies them. For this reason, a flexible *ERR* works well when the effects of the shocks are negative, but it does not allow the system to take full advantadge of the benefits when the effects of the shocks are positive[88]. A second observation is that an *ERR* operates through the, direct and/or indirect, effects that a particular shock has on the money market; either by its effects on the money supply or on the money demand, but ultimately

88 This idea clearly illustrates the point made in the introduction regarding the trade off between macroeconomic flexibility and microeconomic efficiency.

on the exchange rate. However, even if, in general, this last statement is valid, the actual operation of an *ERR* depends on the structure of the whole economy.

From the analysis of the previous three sections it could be concluded that the performance of the two *ERRs* depends greatly on the two basic features of the economy, namely the existence of short-run price rigidities, and the inefficiency created by the presence of monopolistic competition. In the case of the nominal rigidities, the shocks affecting the system would not have had any dynamics if the prices would have been completely flexible[89]. Consider, for example, the foreign monetary shock. Without nominal rigidities, there would not have been any real effects, and only the long-run price levels would have been affected. For fiscal and productivity shocks, the changes in welfare would also have been different.

The relevance of the assumption of monopolistic competition for the performance of the *ERRs* is even more startling. The basic effect of the shocks on the system is to mitigate or magnify the implicit distortion of the system. Furthermore, the *ERRs* effect is determined by how they lessen or reinforce such a distortion.

The basic question emerges: in the context of the model used in this paper, which *ERR* is better? The answer is that it depends on the stochastic properties of the shocks affecting the economy, and on the assumed parametric structure. For example if the majority of shocks confronting the system tend to have negative effects, the economy would be better off with a flexible *ERR*, and vice-versa for the opposite effects. In this paper the goal has been to evaluate the performance of each regime under different shocks, and to point out that the mechanisms behind such performance are better understood in the context of a model with imperfect competition and nominal price rigidities.

Finally, as was mentioned in the introduction, the analytical framework used in the paper allowed a clear cut evaluation of the relevant effects of the shocks under the two *ERRs*; that is, their effects on the welfare of the residents of both countries. Without such a framework, it would have been necessary to rely on some ad-hoc criteria to evaluate the positive implications of the shocks.

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89 And this short-run dynamics does have significant welfare effects.

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