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## ***Efectos del capital financiero sobre la eficiencia bancaria en Colombia***

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### **Resumen**

*Este trabajo analiza los determinantes de la eficiencia en costos y beneficios del sistema financiero colombiano durante el periodo 1989-2003. Usando un enfoque de frontera estocástica, durante el periodo, la eficiencia en costos se deteriora mientras que la eficiencia en beneficios es relativamente estable. Del análisis empírico deducimos que existen grandes diferencias cuando analizamos las medidas de eficiencia entre costos y beneficios. Adicionalmente, nuestro análisis muestra que las medidas de eficiencia en beneficios y costos tienen distribuciones diferentes y existen grandes diferencias entre los diferentes tipos de intermediarios. Este resultado favorece la creencia de que puede existir poder de mercado para los intermediarios del sector financiero al fijar precios de sus productos reflejando comportamiento colusivo para obtener rentas monopolísticas.*

**Clasificación JEL:** C23, D24, G21, L11.

**Palabras clave:** *frontera, eficiencia, costes, beneficios, capital financiero.*

## *Effects of Financial Capital on Colombian Banking Efficiency*

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*In this paper we discuss cost and profit efficiency on the Colombian financial market in the period 1989-2003, using stochastic frontier efficiency analysis. During the period, the cost efficient frontier deteriorates, but profit efficient frontier is relatively stable. We found significant difference when we compare the efficiency scores among different types of financial intermediaries. Additionally, our analysis shows that the scores for profit and cost efficiency have different distributions. Also, we found big differences between profit and cost efficiency among the different type of banks. This is evidence in favor of the existence of collusive behavior of some banks, which allows them to capture oligopoly rents.*

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**JEL Code:** C23, D24, G21, L11.

**Key Words:** frontier, efficiency, cost, profit, financial capital.

## **I. INTRODUCTION**

In the last years, there have been a series of changes related to global trends in the supply of financial services. These trends include economic integration, technological change, increased competition, disintermediation, deregulation and financial crises.<sup>1</sup> Colombia has not been apart from this phenomenon in its financial market where it is assumed that this phenomenon have led to an increase in competition during the last years. In this environment, both for banks and non-bank financial institutions, cost efficiency becomes a prerequisite for survival. Efficiency analysis thus becomes a leading indicator of how the financial firms adopt some strategies to face the consolidation process.

With efficiency analysis we can study the effects of the liberalization, and distinguish between cost and profit analysis. We use stochastic frontier analysis (SFA) to identify the different levels of inefficiency for each financial firm. It yields a best practice frontier as well as individual firm performance measures benchmarks against this frontier. We apply the analysis to a sample of firms of the Colombian financial system in the period 1989-2003. During this period changes occurred in the Colombian financial system originated by a financial crises, deregulation and a consolidation processes.

We analyze the efficiency with cost and profit functions using variables such as financial capital and linear and quadratic trend term to determine whether shifts in the efficient cost and profit frontiers occur. In addition, we check for trend changes in average cost efficiency and average profit efficiency between 1989–2003. We conclude that the cost and profit efficiency differ among firms and during the studied period. Profit efficiency was nearly constant while the cost efficiency showed a significant change during the period. To find differences among financial institutions, we test equality between cost and profit efficiency for the different types of financial institutions, following to the traditional division of the Colombian

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<sup>1</sup> For an overview see Berger, A. N. (1999).

financial system. The results show different levels of cost and profit efficiency, suggesting that some banks benefit from sheer size and market power in the financial system. This is evidence in favor of some banks are behaving collusively and capturing oligopoly rents.

Additionally, in this paper we find that the incorporation of the financial capital as a control variable is relevant to measure the cost and profit efficiency.

The remainder of this paper is organized as follows. In section II, a general review of the literature to model inputs and outputs for the financial firms and the role of the financial capital is presented. Next, in section III we present the *translog* function to estimate the efficiency measures. Section IV presents the empirical evidence for the Colombian case. Section V presents estimation results. We conclude in section VI.

## II. PRODUCTION FUNCTION

### A. INPUTS AND OUTPUTS FOR THE FINANCIAL INSTITUTION

There is a wide debate on the accurate specification of the production function. We can distinguish two alternatives approaches, on the one hand, the *production* approach distinguishes labor and physical capital as inputs to be combined to obtain outputs measured as credit and deposit transactions.<sup>2</sup> On the other hand, the *intermediation* approach starts from the traditional core function of financial institutions and takes deposits as inputs and defines loans and investments as outputs. This approach has been widely used in the literature: Benston *et al.* (1982), Murray, J. D.; White, R. W. (1983), and Mester, L. J. (1993).<sup>3</sup> Some authors such as Hancock D. (1991) and Hughes *et al.* (2000) do not establish a priori if the deposits transactions are inputs or outputs in the production function. They use a regression for the profit function, using different variables that must be checked as input and output. In their empirical exploration for US financial institutions, they found that credit and deposit transactions are outputs in the estimated profit function.

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<sup>2</sup> See Ferrer, G. D. *et al.* (1996). This approach has been used recently for analyzing branch bank behavior, in which there is no total dependency between the intermediation strategies.

<sup>3</sup> There are another approaches, that pretend to compute specifically another roles of financial institutions, such as risk administration, information management and/or agency problems.

*B. THE ROLE OF THE FINANCIAL CAPITAL*

In this paper, we incorporate financial capital to measure the effects related to risk and information management on the efficiency of the financial institutions. Note that the approaches mentioned previously fail to incorporate all the aspects of risk, information processing and the solution of agency problems arising from the differences between loans and deposits and from the separation between management and ownership. Potential solutions to the shortcomings could be a different formulation of the constraints under which banks solve their minimization and maximization problems respectively. Berger and Mester (1997) argue that a bank's insolvency risk depends on its financial capital available to absorb portfolio losses, as well as on the portfolio risks themselves. Insolvency risk affects the cost and profit structure via the risk premium on uninsured debt, and through the intensity of risk management activities the bank undertakes.

Apart from risk, a bank's capital level directly affects cost by providing an alternative funding resource for assets. Interest paid on deposits represents a cost, but dividend payments do not. On the other hand, raising equity typically involves higher costs than raising deposits. In this way, banks with different relation equity/deposits can see modified their cost and profit structure. In some cases, large banks depend more on deposits funding to finance their portfolios than small banks do, so a failure to control for equity could yield a scale bias.<sup>4</sup>

Additionally, if we consider the size of the assets, banks with less risky positions can choose to set higher capitalization levels to send good signals. While banks with low capital level and higher risk position cannot imitate those actions given the opportunity cost incurred by having additional capital position. These kind of banks need to have riskier assets that are compensated with higher interest rate to alleviate higher variance and risk level. The specification of the capital in the cost and profit function also discriminates the different risk preferences of banks. If the banks are more risk averse than others, they may hold a higher level of financial capital to maximize profits or minimize cost. If financial capital is ignored, the efficiency of these banks would be mismeasured, even though they behave optimally given their risk preferences.

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<sup>4</sup> For a brief summary describing the role of financial capital within the financial technology see Lucas, D. and McDonald, R. J. (1992), and Berger, A. N. and Humphrey, D. (1997).

The financial firms combine inputs such as labor, physical capital and financial capital (equity and/or debt) to offer certain outputs: loans, investments and off-balance-sheet operations. The production process for these assets and products involves collecting relevant information, taking credit risk positions, monitoring activities, and relationships between managers, owners and borrowers. Banks that are more efficient at accomplishing these tasks expect a higher return and a lower variance of return on individual loans. Hence, banks that are more efficient producers can reduce both the systematic and idiosyncratic components of the individual risk's total variance through better credit assessment, contract writing, and monitoring. Unlike individual investors, banks can influence the magnitude of the systematic risk of an individual asset. When loans are combined in banks portfolios, more efficient banks can expect a lower variance for any given return on their portfolios. Thus, capital markets price this efficiency.

Most of the literature about financial efficiency has ignored the role of financial capital to estimate bank efficiency. The financial capital is a source of resources to finance loans and other assets and it serves as instrument to protect banks against financial crises and as we mentioned before, it serves as a signal to the agents about bank's credit and management risk position. Banks that finance their assets with a lower proportion capital-deposits, need more liabilities and then a higher insolvency risk, *ceteris paribus*.

Hughes *et al.* (2000) try to solve the following question: how is the cost of equity capital taken into account in computing efficiency? They formulate the answer by conditioning the minimum cost on the level of equity capital and computing equity capital's shadow price from this conditional optimum. In the same way, we compute the optimization problem of the banks taking in account the cost and profit functions both conditioned by the financial capital.

Now, we will do a brief description of the two optimization problems, considering a financial technology that is represented according with the function  $F(x, y, z) \leq 0$  where  $y$  denotes different assets such as information-intensive loans, financial interbank services, and other investments;  $x = x_d + x_p$ , denotes the level of inputs;  $x_d$  representing deposits,  $x_p$  denoting labor and physical capital and  $z$ , denotes equity capital. The prices for each inputs are denoted by  $w_i$ . The economic cost of producing the output vector  $y$  is given by  $w_d x_d + w_p x_p + w_z z$ , omitting the cost of

equity capital, the function cost is denominated *cash-flow* cost and is represented by  $w_d x_d + w_p x_p$ .

### C. BANK PRODUCTION, COST AND PROFIT

Here, we summarize the main aspects related to the intermediation approach, widely used in the literature. Based on the minimization cost and profit maximization methods, we evaluate efficiency with respect to certain objective function. In the first case, the inefficiency is caused by suboptimal choices of used inputs, given input prices, output quantities and available financial capital. In the second case, the profit-inefficiency measures foregone profits due to a suboptimal choice of output quantities given output prices (or suboptimal output prices given quantities). In perfectly competitive markets, the two approaches could yield identical results fixing the output quantity. However, in the case of imperfect competition, market power might lead to a profit efficient bank that is inefficient in terms of cost or viceversa. The combined use of cost minimization problem and profit maximization problem will therefore shed light on the character of inefficiencies. Using the same notation of the previous subsection, we present the two mentioned approaches.

### D. COST MINIMIZATION

The minimization problem is set up as follows. Based in Hughes *et al.*, (2000), we consider a function  $C(\cdot)$  consisting of the cost incurred due to buy input quantities  $x$  at price  $w$ . We distinguish three alternative cost functions: operating cost function, cash flow cost function and economic cost function.

Given a deposits level  $x_d$  and a financial capital level  $z$ , the *operating cost function*  $C_p(y, w_p, x_p, x_d^0, z^0)$  is defined by:

$$(1) \quad \text{Min}_{x_p} = w'_p x_p \quad s.t. \quad F(x, y, z) \leq 0; \quad x_d = x_d^0; \quad z = z^0$$

The operating cost function considers capital structure by conditioning cost on levels of financial capital while excludes their expenses from the cost function. Deposits and financial capital are taken as given.



A cash-flow measure of cost  $C_{cf}(y, w_p, w_d, z^0)$  includes the cost of deposits but excludes the cost of equity capital. The minimum cash-flow cost function is defined by

$$(2) \quad \text{Min}_{x_p, x_d} = w'_p x_p + w'_d x_d \quad s.t. \quad F(x, y, z) \leq 0; \quad z = z^0$$

The level of deposits minimizes cost while cost is conditioned on the level of financial capital. Hence, the level of equity capital does not have to minimize cost. This formulation accounts for capitalization but does not require a price for financial capital.

In contrast, the minimum economic cost function  $C_e(y, w_i)$  is conditioned on the price of financial capital rather than on the quantity and, hence, the level of equity capital minimizes cost:

$$(3) \quad \text{Min}_{x_p, x_d, z} = w'_p x_p + w'_d x_d + w'_z z \quad s.t. \quad F(x, y, z) \leq 0$$

While these three formulations of cost incorporate financial capital's influence on production, many studies on bank costs omit any role of financial capital in defining cash-flow cost in the following way:

$$(4) \quad \text{Min}_{x_p, x_d} = w'_p x_p + w'_d x_d \quad s.t. \quad F(x, y, z) \leq 0$$

The differences among these four formulations cost are important. The last expression is very similar to (3) but does not consider  $z$ . The differences between (2)–(4) are important, given that in the last equation we don't consider financial capital, so when this variable changes, the equation (4) does not capture those variations in the cost function.

If there are two banks with different capital-deposits ratio. Given (4), the bank with less capital appears with a higher cash flow cost compared with the other bank. As we mentioned before, the level of financial capital effects the risk position of banks and the incurred costs in managing risk. A specification comparable to (4) does not take in account these kind of decisions of the banks and then, it can generate wrong conclusions when we evaluate efficiency in the cost function for the banks with different capitalization level.

The corresponding Lagrangian function can be formulated as:

$$\mathcal{L} = \sum_i (w'_i x_i) - \lambda F(\cdot)$$

Taking first derivatives and solving the result yields the conditional factor demand equations, or, in terms of Hughes and Mester (1993), the restricted input requirement set:

$$x_i^* = x_i^*(y, w, z)$$

The minimum cost level is obtained by substituting into the cost function:

$$(5) \quad TC^* = w' x_i^*(y, w, z) = \tilde{c}(y, w, z)$$

The conditional demand for inputs depends on the amount of output sold at prevailing prices, the given factor prices in input markets and the level of capital in production period.

#### *E. PROFIT MAXIMIZATION*

Like the minimization cost problem, we can deduce the maximization problem. When we assume that the market is perfectly competitive in inputs and outputs, in which banks choose optimal quantities of inputs and outputs, given prices, we use the standard approach, expressed in the following form:

$$(6) \quad \text{Max}_{y,x} = p'y - w'x \quad \text{s.t.} \quad F(y, x) = 0$$

With  $F(y, x)$  is the transformation function of the factors vector  $x$  to outputs vector  $y$ . The Lagrangian system can be written as:

$$\mathcal{L} = p'y - w'x - \lambda F(\cdot)$$

The simultaneous solution for  $x$  and  $y$ , produces the optimal output and input vectors:

$$y^* = y^*(p, w)$$

$$x^* = x^*(p, w)$$

Substituting into the profit function, we obtain a optimal profit level:

$$(7) \quad \pi = py^*(p, w) - w'x^*(p, w) = \pi^*(p, w)$$

The problem related with this approach is associated with the assumption of perfect competition among banks. It could be an unrealistic assumption. Following Humphrey, D. and Pulley, L. (1997) and Bos, J. and Kool, C. (2001), we modify the profit function and allow banks to exercise a form of market power in choosing output prices. This market power is limited to output markets, banks remain being competitive purchasers of inputs.<sup>5</sup>

We assume that banks maximize profits for a given output quantities,  $y$ , and input prices  $w$ , by choosing output prices  $p$ , along with input quantities,  $x$ . The associated indirect profit function is derived as the solution to the problem:

$$(8) \quad \text{Max}_{p,x} = p'y - w'x \quad \text{s.t.} \quad F(y, x) = 0 \text{ and } G(y, p, w, z) = 0$$

Where  $G(y, p, w, z)$  represents a bank's pricing opportunity set for transforming given values of  $y$ ,  $w$  and  $z$  into output prices. This reflects the bank's assessment of the willingness of customers to pay the prices the bank wishes to charge. The function  $G(\cdot)$  also reflects any conjetural variations incorporated in pricing rules that the bank may follow, such as differentiability marking up the cost of funds; hence the inclusion of input prices.

The Lagrangian system can be written as:

$$\mathcal{L} = p'y - w'x - \lambda F(\cdot) - \theta G(\cdot)$$

And the solution give us the optimal choice for output prices  $p^* = p^*(y, w, z)$  and input quantities  $x^* = x^*(y, w)$ :

$$p^* = p^*(y, w, z)$$

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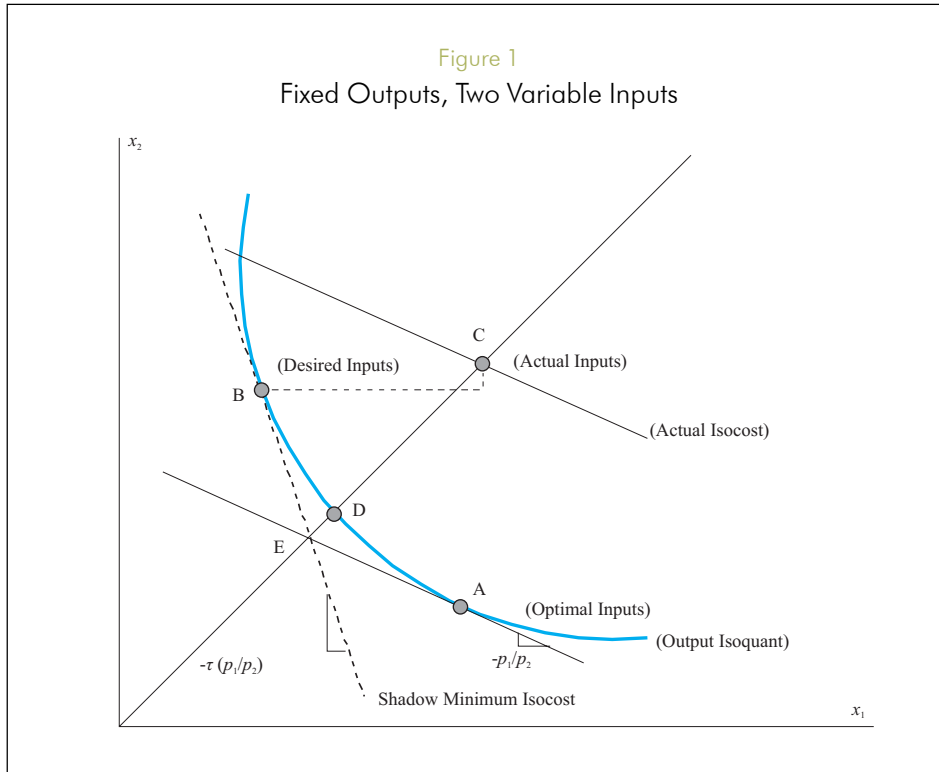
<sup>5</sup> In practice, banks exploit local market power for certain deposit and loan services and have the ability to differentiate output prices among customer groups, across geographic areas, and over time.

$$x^* = x^*(y, w, z):$$

Substituting in the *alternative* profit function (8), the optimal profit level will be:

$$(9) \quad \pi = p^*(y, w, z)'y - w' x^*(y, w, z) = \tilde{\pi}(y, w, z)$$

The appealing feature of this profit function is that it allows for market imperfections on the output side. Additionally, output prices, which are required for the traditional profit function estimation are not required for the empirical analysis of the alternative profit function.<sup>6</sup>



<sup>6</sup> Berger, A. N. and L. J. Mester (1997) argue that alternative profit function may provide useful information when one or more of the following conditions affect the bank behavior: i. there are a substantial unmeasured differences in the quality of banking services; ii. output is not completely

To illustrate the efficiency concept, we could think in the simple case in which all outputs are fixed and there are two variable inputs,  $x_1$  and  $x_2$ . The figure (1) explain us the case with fixed outputs, the variable profit function is equivalent to the negative of the variable cost function. The technically efficient set of input requirements for the fixed outputs, i.e., the efficient frontier is given by the isoquant which passes through points  $A$ ,  $D$  and  $B$ . Any  $(x_1, x_2)$  combination along this curve could be used by a technically efficient firm to produce the fixed output bundle. Actual observed input usage is at point  $C$ . A firm that is allocatively as well as technically efficient would use the input combination at point  $A$ , the tangency between the lowest isocost line and the efficient frontier, where the slope of the isoquant and the isocost curves both equal the actual relative price ratio  $-(p_1/p_2)$ . Total inefficiency is measured by the difference in cost between the isocost line passing through points  $C$  and  $A$  in Figure (1). Total efficiency is often measured as the ratio of minimum cost to actual cost. In the figure is the ratio  $OE/OC$ , since point  $E$  has the same cost as point  $A$ .<sup>7</sup>

The next section presents the functional form used to estimate the different cost and profit systems within the financial system.

### III. SPECIFICATION

For the estimation of cost and alternative profit frontier functions a *translog* functional form is chosen with three inputs and three outputs. This form has been employed widely and has proven to allow for the necessary flexibility when estimating the frontier function.<sup>8</sup>

Berger and Master (1997) compared the *translog* and the Alternative Fourier Flexible Form. Despite the latter's added flexibility, the difference in results

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variable, so that a bank cannot achieve every output scale and product mix; iii. output markets are not completely competitive, then, there is market power; iv. output prices are not accurately measured, a very common problem for empirical analysis.

<sup>7</sup> See Berger, A. Hancock, D. and Humphrey, D. (1993b).

<sup>8</sup> Fuss, M., McFadden, D. and Mundlak, Y. (1978) describe the different characteristics that must be considered to choose a functional form and summarize the main functional forms used in the literature, such as Cobb-Douglas, CES, Leontief/Lineal, Translog, Generalized Cobb-Douglas, Quadratic and Concave generalized.

between both methods appears to be negligible. Additionally, given the larger number of parameters in the second functional form, we avoid its implementation, since we don't have enough data. For this reason we adopt the *translog* functional form in our analysis. The frontier cost function for a  $k$  bank in the period  $t$  is represented by:

$$(10) \quad \tilde{c}_{kt}(y, w, z) = \beta_0 + \sum_{i=1}^3 \beta_i \ln y_{ikt} + (1/2) \sum_{i=1}^3 \sum_{j=1}^3 \beta_{ij} \ln y_{ikt} \ln y_{jkt} + \sum_{i=1}^3 b_i \ln w_{ikt} \\ + (1/2) \sum_{i=1}^3 \sum_{j=1}^3 b_{ij} \ln w_{ikt} \ln w_{jkt} + \sum_{i=1}^3 \sum_{j=1}^3 d_{ij} \ln w_{ikt} \ln y_{jkt} + U_{kt} + V_{kt}$$

We denoted this 3-input/3-output model as model 1 (M1). Here,  $U_{kt}$  and  $V_{kt}$  are the inefficiency and random error terms, respectively. For the profit function, the left-hand side is replaced with net profits and the inefficiency term is  $-U_{kt}$ .

In the model 2 (M2), we incorporate variables related to financial capital and its interactions with the explanatory variables to analyze the effect of financial capital on cost and profit functions of the financial intermediaries. The new cost function will be:

$$(11) \quad \tilde{c}_{kt}(y, w, z) = M1 + d_0 \ln z_{kt} + (1/2) d_1 (\ln z_{kt})^2 + \sum_{i=1}^3 e_i \ln w_{ikt} \ln z_{kt} \\ + \sum_{i=1}^3 f_i \ln y_{ikt} \ln z_{kt}$$

To allow the impact of consolidation and deregulation on the efficient frontier, we alternatively include a linear and quadratic trend term as well as a trend. These will be referred to model 3 (M3):<sup>9</sup>

$$(12) \quad \tilde{c}_{kt}(y, w, z) = (M2) + g_0 t + 1/2 g_1 t^2$$

The alternative profit function for each model is similar except for the before-mentioned modifications:

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<sup>9</sup> A negative number and statistical significance of the parameter  $t$  is indicative of multi-factor productivity growth. Obviously, these trend terms may capture pure technological change as well as effects of consolidation and deregulation jointly. We are not able to determine the relative contribution of each factor separately.

$$(13) \quad \tilde{\pi}_{kt}(y, w, z) = \beta_0 + \sum_{i=1}^3 \beta_i \ln y_{ikt} + (1/2) \sum_{i=1}^3 \sum_{j=1}^3 \beta_{ij} \ln y_{ikt} \ln y_{jkt} + \sum_{i=1}^3 b_i \ln w_{ikt} \\ + (1/2) \sum_{i=1}^3 \sum_{j=1}^3 b_{ij} \ln w_{ikt} \ln w_{jkt} + \sum_{i=1}^3 \sum_{j=1}^3 d_{ij} \ln w_{ikt} \ln y_{jkt} - U_{kt} + V_{kt}$$

Following Lang, G. and Welzel, P. (1999), to ensure symmetry and linear homogeneity in input prices, we impose the usual restrictions:

$$(14) \quad \beta_{ij} = \beta_{ji} \quad \forall_{ij}, \quad b_{ij} = b_{ji} \quad \forall_{ij}, \\ \sum_{i=1}^3 b_i = 1, \quad \sum_{i=1}^3 b_{ij} = 0 \quad \forall_i, \quad \sum_{i=1}^3 b_{ij} = 0 \quad \forall_j, \\ \sum_{i=1}^3 e_i = 0, \quad \sum_{i=1}^3 f_i = 0$$

In the empirical estimation, linear homogeneity in input prices is imposed by normalizing the dependent variable (total cost or profit) and all factor price variables ( $w_i$ ) before taking logarithms.<sup>10</sup> Each one of the variables is included as a ratio relative to one of the factor price variables. Note that this imposes homogeneity of degree one in factor prices only.<sup>11</sup> Therefore, this implies that only two coefficients ( $b_i$ ) for the input factor price variables are obtained, while the third can be inferred from the imposed restriction. The random error term  $V_{kt}$  is assumed i.i.d. with  $V_{kt} \sim N(0, \sigma_v^2)$  and represents those shocks that are not directly controlled by the financial intermediaries and it is assumed to be independently of the explanatory variables.<sup>12</sup>

The inefficiency term  $U_{kt}$  is i.i.d. with  $U_k \sim N(\mu, \sigma_u^2)$  and is independent of  $V_{kt}$ . It is drawn from a non-negative distribution truncated in  $\mu$  instead than in zero.<sup>13</sup>

<sup>10</sup> See Coelli, T.; Rao, D.P. and Battese, G. E. (1998).

<sup>11</sup> To impose constant returns to scale, normalization of the output variables would be required too.

<sup>12</sup> See Aigner, D. J.; Lovell, C. A. K. and Schmidt, P. (1977), and Coelli, T. (1996).

<sup>13</sup> Coelli, T., Rao, D. P. and Battese, G. E. (1998) argue that the truncated distribution is a generalization of the half-normal distribution. It is obtained by the truncation at zero of the normal distribution with mean,  $\mu$ , and variance  $\sigma^2$ . If  $\mu$  is pre-assigned to be zero, then the distribution is the half-normal. The distribution may take a variety of shapes, depending on the size and sign of  $\mu$ . The

For the cost model, let  $E_{kt} = V_{kt} + U_{kt}$ . The specific cost efficiency estimation of a bank  $k$  at time  $t$  is given by the mean of the conditional distribution of  $U_{kt}$  given  $E_{kt}$ , defined as:

$$EFF_{kt}(\tilde{c}) = E [\exp (U_{kt}) | E_{kt}]$$

This measure takes values in the interval  $(1, \infty)$ . Values equal to one mean fully efficient. Values close to one, indicate that efficiency on bank's cost, conditional on its outputs, input prices and capital level, is above of the cost that fully efficient bank could incur under the same conditions. For the profit function,  $E_{kt} = V_{kt} - U_{kt}$ , firm specific profit efficiency is again the mean of the conditional distribution of  $U_{kt}$  given  $E_{kt}$ , and is defined as:

$$EFF_{kt}(\tilde{\pi}) = E [\exp (-U_{kt}) | E_{kt}]$$

which takes values on the interval  $(0, 1)$ , where 1 indicates a fully efficient financial intermediary.

The frontier functions are estimated through ML methods. For this purpose we used the computer program Frontier 4.1.<sup>14</sup> Following Coelli (1996), the terms  $\sigma_U^2$  and  $\sigma_V^2$  are replaced by  $\sigma^2 = \sigma_U^2 + \sigma_V^2$  and  $\gamma = \sigma_U^2 / (\sigma_U^2 + \sigma_V^2)$ .<sup>15</sup> The parameter  $\gamma$  represents the share of inefficiency in the overall residual variance with values in the interval  $(0, 1)$ . A value of 1 suggests the existence of a deterministic frontier, whereas a value of 0 can be seen as evidence in favor of a standard OLS estimation. In the latter case, no structural inefficiency exists.

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estimation of the truncated-normal stochastic frontier involves the estimation of the parameter,  $\mu$ , together with the other parameters of the model. The log-likelihood function required for the Maximum-Likelihood (ML) estimation of the parameters of the model was first given by Stevenson, R. E. (1980). Expressions for appropriate predictors of the technical efficiencies of firms were given in Battese, G. E. and Coelli, T. (1988).

<sup>14</sup> The computer program Frontier 4.1 has been written to provide ML estimates of a wide variety of stochastic frontier production and cost functions. See Coelli (1996).

<sup>15</sup> The log-likelihood function for this stochastic frontier and inefficiency model is presented in the appendix in Battese, G. E. and Coelli, T. J. (1993), together with the first partial derivatives of the log-likelihood function with respect to the different parameters of the model.



#### IV. EMPIRICAL EVIDENCE

##### A. COLOMBIAN BANKING SECTOR

Table (1) illustrates the different papers related with the Colombian banking sector efficiency. We can say that the empirical evidence is not enough given the lag in the studied period and the methodology used in the estimations. Another limitation is that the results just apply for one type of financial intermediary. Under intermediation approach, we supposed a common stochastic frontier for all financial institutions, because of freedom degrees problems. In this paper, we consider a period of time between 1989 and 2003 and the majority of financial institutions of the Colombian banking sector.

##### B. THE DATA

We extend the data set of the previous papers considering a wider period. Our analyzed period runs from the first quarter of 1989 to the third quarter of 2003. Additionally, we incorporate the different types of Colombian financial institutions

Table 1  
Colombian Bank Efficiency  
Literature

Date	Author	Period	Method 1/	Institution Type 2/
1996	Misas y Suescún	1989-1995	TFA	CB
2000	Mendoza	1996-1999	DEA	CB
2001	Castro	1994-1999	DFA	CB
2002	Badel	1998-2000	DFA	CB
2003	Janna	1992-2002	SFA	CB

1/ DEA: Data Envelopment Analysis, DFA: Distribution Free Approach, SFA: Stochastic Frontier Analysis, TFA: Thick Frontier Analysis.

2/ CB: Comercial Bank.

Source: Calculations by the author.

jointly: commercial banks, specialized mortgage loan banks,<sup>16</sup> financial corporations (investment banks) and specialized commercial loan banks. This gives us a general perspective of the bank efficiency of the sector.

During this period, the Colombian banking system has been affected by a process of deregulation and consolidation. For this reason, Colombian financial institutions have reacted to the new market conditions. They were forced to reconsider their strategic options and to restructure.<sup>17</sup> Between 1989 and 2003, the financial sector had 46 mergers, take overs and transformations. Additionally, at the beginning of the 90's, the Colombian financial system was affected by an internationalization process with the incorporation of foreign banks, principally Spanish institutions like BBVA and Santander. In 1998, 14 foreign banks existed, but now, there are only 9.

After the consolidation process in the 90's, the financial sector had 74 financial intermediaries divided in 22 commercial banks, 6 specialized mortgage loan banks, 5 financial corporation (investment banks), 14 specialized commercial loan banks, 7 financial cooperative institutions, 11 leasing financial firms and 9 public specialized financial institutions. Table (2) shows the composition by sectors of the financial system for 2003.

We use the data set provided by the Colombian superintendency of banks. For each year, we include only those banks with data available for all variables. This leaves us with a non-balanced panel, of 57 periods and 5326 observations.<sup>18</sup> We don't have special treatment of mergers or absorptions, the main bank continues (with jump in its balance) and the other disappears. This is because the effects of financial system composition on efficiency it is not the purpose of this paper.

### *C. SELECTION OF VARIABLES*

We identify three outputs: loans ( $y_1$ ) as the total stock of all loans supplied, investments ( $y_2$ ), as the sum of total securities, equity investments, bond (private

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<sup>16</sup> Since 1997, these institutions are transformed from saving and loan banks to specialized mortgage loan banks.

<sup>17</sup> In 1989 the data set includes 84 financial institutions (33 commercial banks and specialized mortgage loan banks, 22 financial corporations and 30 specialized commercial loan banks).

<sup>18</sup> The included financial intermediaries in the sample represent more than 96% of the total assets of the Colombian financial sector during the period 1989-1 to 2003-1.

Table 2  
Colombian Financial Institutions: 2003

	<b>Public Banks</b>	<b>Private Banks</b>	<b>Foreign Banks</b>	<b>Total</b>
Commercial Banks	3	10	9	22
Mortgage Loan Banks	1	5	0	6
Financial Corporations	1	4	0	5
Commercial Loan Banks	0	10	4	14
Leasing Banks	1	8	2	11
Financial Cooperative Institutions	0	7	0	6
Public Specialized Banks	9	0	0	9
<b>Total</b>	<b>15</b>	<b>44</b>	<b>15</b>	<b>74</b>

Source: Superintendency of Banks.

and public) investments and other investments. The third output is deposits held with other banks ( $y_3$ ). As explained before and in line with Hughes and Mester (1993), we include ( $z$ ) as a control variable, so this variable affect directly the production function and not the efficiency level.<sup>19</sup>

Finally, we identify three input prices. The price of financial capital ( $w_1$ ), expressed in percentage and computed as: (interest expense/customer and short-term funding + other funding)\*100. Next, we compute the price of labor ( $w_2$ ). Unfortunately, the information about the number of employees of banks is not complete. Therefore, we approximate the number of employees as follows: we assume a constant relationship between number of employees and fixed assets. For all banks in the Colombian sector, that we have information on the number of employees, we regress the logarithm of the number of employees on the logarithm of fixed assets.

The main inconvenience that arises is that almost certainly the relationship is far from being lineal and on the other hand, contemporary. The linearity imposed on

<sup>19</sup> This variable includes social capital, earnings, reserves and banks's funds with specific destination.

the data may very well be shading the real effects of efficiency related to the efficiency of the labor factor. Additionally, it imposes a rigid structure of substitution between labor and capital factors, something which methodologically one would want the data to reveal. This naturally enforces rigidities on the allocative efficiency (as must be known, x-efficiency is comprised of both technical and allocative efficiency) but does not affect technical efficiency. On the other hand, it is clear that the adjustment costs of the fixed asset to new market conditions are more rigid than those of employees, hence, there may exist a different time structure to the contemporaneous one between the two variables (employees and fixed assets).<sup>20</sup>

The result of this estimation is used to estimate the number of employees for all banks. Our proxy for the price of labor is then composed as follows: Personnel Expenses / Estimated number of Employees.

The price of physical capital ( $w_3$ )<sup>21</sup> is: Administrative fees / Fixed assets. Before the estimations, we divide profit before tax  $PBT$ , total cost  $TC$ ,  $w_1$  and  $w_2$  by  $w_3$ , the physical-capital price, to impose input-price lineal homogeneity.

In Table (3), we present a brief summary of statistics for the variables involved. All quantity variables are expressed in millions of Pesos and deflated for inflation.<sup>22</sup> The explanatory variables are (PBT) and (TC). Both are taken from the banks' profit and loss account, where the latter is the sum of interest expenses, personnel expenses and other operating expenses.

In the period 1989-2003, commercial banks present a higher level of dispersion in the analyzed variables.<sup>23</sup> Based on Table (3), the banks have, in real terms, the highest levels of cost and profit, but with higher dispersion than the rest.

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<sup>20</sup> The rest of the number of employees data was estimated using a regression between the number of employees and fixed assets:

$$\ln(\text{employees}) = -1.983 + 0.945 * \ln(\text{fixed assets}) - 0.0478 * t$$

(0.23)    (0.024)    (0.002)

with  $R^2 = 0.787$ . Standard Error in parenthesis.

<sup>21</sup> Administrative fees includes those fees different from personnel fees: indirect operating cost, depreciation and amortizations. Fixed assets include own used goods and another assets.

<sup>22</sup> We used the CPI, 100=dec/98.

<sup>23</sup> From now on, we denote banks as the sum of commercial banks and mortgage loan banks.

Table 3  
Summary Statistics: Millions of Pesos (\*)

Variable	Max.	Min.	Mean	Median	SD	Asymmetry	Kurtosis
<b>Total System (161)</b>							
<i>TC</i>	13208.6	0.3820	344.1	98.6	582.4	4.4	52.3
<i>PBT</i>	10530.2	1.0000	8217.2	8209.5	219.9	-14.3	437.7
<i>y1</i>	40169.4	0.0407	4054.1	1055.4	6579.3	2.3	5.4
<i>y2</i>	22270.7	0.0001	950.4	187.7	2098.4	4.7	29.0
<i>y3</i>	5187.8	0.0031	135.1	25.0	320.5	5.6	47.9
<i>z</i>	12207.5	3.4102	859.4	256.4	1501.6	3.1	11.7
<i>w1</i>	1634.1	0.0087	5.6	5.3	22.7	69.2	4943.7
<i>w2</i>	51940.0	0.9580	535.0	277.3	1161.3	21.0	781.6
<i>w3</i>	10202.2	0.2093	27.5	17.0	156.5	55.1	3433.2
<b>Commercial Banks (49)</b>							
<i>TC</i>	5063.8	10.82	755.1	498.2	716.8	1.6	3.3
<i>PBT</i>	9866.2	1.00	8232.5	8235.2	333.1	-11.1	217.6
<i>y1</i>	40169.4	1.32	8882.6	5740.7	8263.3	1.2	0.8
<i>y2</i>	22270.7	1.27	2018.7	1033.4	2909.2	3.4	13.9
<i>y3</i>	5187.8	0.04	302.0	132.7	462.7	3.8	21.8
<i>z</i>	12207.5	68.58	1669.7	870.7	1944.2	2.2	5.3
<i>w1</i>	1634.1	0.59	4.6	3.6	36.4	44.6	1996.1
<i>w2</i>	14060.4	0.96	523.5	348.7	679.4	7.4	110.2
<i>w3</i>	10202.2	0.61	29.5	19.4	229.1	43.8	1942.8
<b>Investment Banks (29)</b>							
<i>TC</i>	1986.3	1.57	162.9	83.8	209.4	2.5	8.9
<i>PBT</i>	10530.2	5906.34	8208.5	8207.9	168.8	-2.0	102.5
<i>y1</i>	16258.8	0.14	2551.0	1074.6	3679.9	2.2	3.8
<i>y2</i>	9158.2	0.93	830.0	230.7	1523.4	2.8	8.0
<i>y3</i>	1182.9	0.02	68.6	19.2	146.9	4.1	18.9
<i>z</i>	5430.2	16.62	881.6	289.2	1336.8	2.0	2.7
<i>w1</i>	81.4	0.34	5.6	5.6	3.2	14.9	319.6
<i>w2</i>	7790.3	2.02	327.4	185.1	503.3	5.8	57.6
<i>w3</i>	387.6	0.21	18.4	11.6	24.8	5.6	56.4
<b>Specialized Commercial Loan Banks (83)</b>							
<i>TC</i>	13208.6	0.3820	66.4	39.5	281.7	44.3	2063.5
<i>PBT</i>	10343.7	7989.6200	8207.8	8206.3	48.0	38.3	1712.1
<i>y1</i>	29925.8	0.0407	511.5	213.7	925.2	15.0	446.1
<i>y2</i>	4371.9	0.0001	71.7	38.5	125.6	18.3	599.9
<i>y3</i>	1721.6	0.0031	19.1	8.2	45.6	23.6	846.3
<i>z</i>	8217.4	3.4102	142.2	92.2	212.0	24.5	917.5
<i>w1</i>	197.2	0.0087	6.4	6.3	5.8	24.8	738.7
<i>w2</i>	51940.0	8.3497	637.7	300.3	1607.0	17.7	486.0
<i>w3</i>	3941.1	0.4348	29.7	16.4	103.5	28.0	962.9

(\*) *TC*: Total Cost, *PBT*: Profits Before Taxes, *y1*: Credit, *y2*: Investments, *y3*: Deposit in other banks, *z*: Capital, *w1*: Financial-Capital Price, *w2*: Labor-Price, *w3*: Physical-Capital Price.  
Source: Bank's Profit and Loss Account and Balance Sheet of Banks. Superintendency of Banks. Period 1989-2003.

The analysis of the outputs data, illustrates us significant differences between the different types of financial intermediaries. Commercial banks (CB) had superior output levels compared to the rest of the banks. Loans for commercial banks were higher in 3.5% and 17% compared to investment banks (IB) and specialized commercial loan banks (SCB), respectively. Investment was higher for commercial banks 2.4% and 28.2% respectively. For the deposits in other banks, commercial banks had superior levels (4.4% and 15.8%) with respect to the other type of intermediaries. However, it is important to highlight that the level of outputs for commercial banks present the highest dispersion among them.

During the period, the financial capital has increased, in real terms, for all type of intermediaries. This variable increased 332% for the investment banks, while commercial banks and specialized commercial banks raised their financial capital 164% and 28% respectively. See Table (4).

The pertinent variables have varied differently among type of financial institutions.<sup>24</sup> Regarding the cost variable, the commercial banks presented higher cost levels compared to the another type of institutions, this behavior is accentuated in the crisis period. For the profit variable, commercial banks had profit level below the levels of IB and SCB. However, it is important to emphasize the significant difference between the mean and median values for the analyzed variables for each type of intermediary. This explains the high dispersion among the different type of banks, where the case the SCB had the lowest one.

## **V. ESTIMATION RESULTS**

We now turn to the empirical analysis. In the next subsection, we show the estimation of the different models for both, cost and profit translog functions. We test the three models and select one as the preferred model and interpret it. Also we investigate how the consolidation process may change the estimation results in both the efficient frontier and the estimated mean cost and profit efficiency relative to the frontier. In sub-section V.B we use the preferred

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<sup>24</sup> To evaluate the evolution of variables, we have divided the period into three sub-periods: 1989-I to 1998-III; 1998-IV to 2000-IV (crises period); and 2001-I to 2003-III.

Table 4  
Summary Statistics: Millions of Pesos (\*)

	CB		IB		SCB	
	Mean	SD	Mean	SD	Mean	SD
<b>1989-1998</b>						
<i>TC</i>	742.3	712.2	140.4	171.6	70.6	320.6
<i>PBT</i>	8270.8	154.6	8222.5	139.6	8209.0	53.6
<i>y1</i>	8073.8	7401.6	2132.6	3059.1	538.2	1010.9
<i>y2</i>	1357.7	1473.1	534.9	1008.0	73.0	137.2
<i>y3</i>	256.6	372.2	52.4	112.1	17.8	49.0
<i>z</i>	1396.1	1633.7	718.1	1235.2	131.6	230.1
<i>w1</i>	4.1	1.4	5.7	2.2	6.8	5.3
<i>w2</i>	311.6	313.8	263.9	313.6	423.9	1035.6
<i>w3</i>	24.3	30.5	18.5	24.9	27.4	105.7
<b>1998-2000</b>						
<i>TC</i>	929.2	856.1	294.7	365.1	59.5	65.8
<i>PBT</i>	7997.2	755.7	8114.2	272.3	8198.5	22.0
<i>y1</i>	11444.8	10506.4	4468.3	5717.2	434.7	598.6
<i>y2</i>	2777.8	3391.0	1840.0	2058.9	66.9	81.7
<i>y3</i>	439.7	632.7	148.7	267.3	25.2	35.2
<i>z</i>	2572.5	2697.7	1623.6	1598.8	178.8	139.2
<i>w1</i>	3.8	2.2	4.9	1.8	5.9	4.1
<i>w2</i>	952.0	631.1	598.1	1032.7	996.8	3068.7
<i>w3</i>	23.2	23.6	18.9	29.0	34.6	117.7
<b>2001-2003</b>						
<i>TC</i>	626.6	495.2	318.7	241.5	42.0	37.9
<i>PBT</i>	8276.4	164.0	8157.1	280.8	8212.5	13.9
<i>y1</i>	10847.0	9334.1	6389.5	5024.0	416.3	475.0
<i>y2</i>	5222.7	5523.0	3948.4	2759.1	70.8	67.0
<i>y3</i>	420.0	644.7	198.7	222.6	19.5	24.8
<i>z</i>	2290.7	2176.9	2385.2	1157.6	168.7	117.7
<i>w1</i>	8.8	105.8	2.8	0.6	3.8	10.7
<i>w2</i>	1328.9	1287.5	932.1	875.8	1724.2	1567.7
<i>w3</i>	69.2	659.9	19.3	17.2	39.4	41.3

(\*) **CB:** Commercial Banks, **IB:** Investment Banks, **SCB:** Specialized Comercial Banks.  
Source: Bank's Profit and Loss Account and Balance Sheet of Banks. Superintendency of Banks. Period 1989-2003.

models to compute individual efficiency scores. We use the efficiency scores for individual financial institutions to analyze differences between different type of banks.

#### *A. ESTIMATED COST AND PROFIT FRONTIERS*

The detailed estimation results for the different versions of cost and profit models respectively are presented in tables 5 and 6. We also present both LR test and LR test (one side) of the standard response function (OLS) versus full frontier model.<sup>25</sup> The LR test results show that we can reject the restrictions imposed by OLS. Consequently, we use the specification including a stochastic inefficiency term for all models.

Regarding the estimated cost function in Table (5), we have that  $\gamma$ , the proportion of inefficiency in the global residual variance, is significantly different from 1, which indicates a stochastic frontier. Also, for our cost model  $\mu$  is significantly positive with a value of 1.10 in model 3. This means that the top of the half normal distribution of our inefficiency term  $U$  lies close to 3, as we can verify in Graph (1). Hence, most of our financial institutions are relatively cost inefficient and the average cost inefficiency is high.

The profit efficiency results in Table (6) show again that  $\gamma$  is significantly different from 1 so that efficient frontier is stochastic. The estimated value of  $\mu$  changes significantly between the different models. The impact of a different value of  $\mu$  can be easily observed comparing the distribution of cost and profit efficiency scores in Graph (1). In the case of cost efficiency, the relatively large value of  $\mu$  indicates that the peak of the density function of inefficiency term  $U$  is not close to zero. As a result, most individual efficiency scores are not close to the full efficiency value of 1. This is reflected in the specially flat path of the efficiency scores. The large negative  $\mu$  for profit function in model 3 implies that the peak of the density function on inefficiency terms is far away from zero. Consequently, most individual banks are in the tail of the density, leading to wider dispersion in profit efficiency than in cost efficiency.

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<sup>25</sup> Kodde and Palm (1986). The null hypothesis in this test is  $\gamma = 0$  versus the alternative  $\gamma > 0$ .



Table 5  
 Estimation Results under Cost Minization (\*)

	Model 1		Model 2		Model 3	
	Coefficient	t-ratio	Coefficient	t-ratio	Coefficient	t-ratio
Constant	-2.3375	-20.70	-3.362	-24.14	-3.405	-23.89
lny <sub>1</sub>	-0.0348	-1.90	-0.037	-1.54	-0.046	-1.91
lny <sub>2</sub>	0.2104	11.96	0.261	8.90	0.269	9.23
lny <sub>3</sub>	0.1525	9.69	0.100	5.25	0.092	4.89
lnw <sub>13</sub>	0.7755	34.38	0.866	32.98	0.828	30.24
lnw <sub>23</sub>	0.2568	8.09	0.128	3.73	0.176	4.95
0.5lny <sub>1</sub> lny <sub>1</sub>	0.1444	40.25	0.128	32.22	0.128	32.11
0.5lny <sub>1</sub> lny <sub>2</sub>	-0.0648	-11.38	-0.021	-3.21	-0.019	-2.95
0.5lny <sub>1</sub> lny <sub>3</sub>	-0.0502	-9.89	-0.041	-8.20	-0.039	-7.78
0.5lny <sub>2</sub> lny <sub>2</sub>	0.0373	15.36	0.035	12.76	0.035	12.77
0.5lny <sub>2</sub> lny <sub>3</sub>	0.0265	5.41	0.057	10.02	0.056	9.71
0.5lny <sub>3</sub> lny <sub>3</sub>	0.0009	0.27	-0.004	-1.30	-0.004	-1.37
0.5lnw <sub>13</sub> lnw <sub>13</sub>	0.0138	2.48	0.014	2.57	0.008	1.36
0.5lnw <sub>23</sub> lnw <sub>23</sub>	0.0114	1.64	0.031	4.15	0.016	2.00
0.5lnw <sub>13</sub> lnw <sub>23</sub>	0.0152	1.64	0.011	1.08	0.034	3.07
lnw <sub>13</sub> lny <sub>1</sub>	0.0067	1.92	0.008	2.29	0.009	2.66
lnw <sub>13</sub> lny <sub>2</sub>	0.0120	3.34	0.017	4.17	0.018	4.54
lnw <sub>13</sub> lny <sub>3</sub>	-0.0114	-3.72	-0.009	-2.92	-0.010	-3.09
lnw <sub>23</sub> lny <sub>1</sub>	-0.0127	-2.51	-0.003	-0.63	-0.004	-0.71
lnw <sub>23</sub> lny <sub>2</sub>	-0.0195	-4.04	0.004	0.67	0.003	0.61
lnw <sub>23</sub> lny <sub>3</sub>	-0.0123	-3.53	0.001	0.28	0.002	0.54
lnz			0.461	10.78	0.477	10.70
lnz lnz			0.018	2.57	0.017	2.37
lnw <sub>13</sub> lnz			-0.021	-3.14	-0.022	-3.37
lnw <sub>23</sub> lnz			-0.033	-4.15	-0.033	-4.13
lny <sub>1</sub> lnz			-0.018	-3.08	-0.016	-2.78
lny <sub>2</sub> lnz			-0.054	-8.15	-0.055	-8.41
lny <sub>3</sub> lnz			-0.016	-3.26	-0.015	-3.18
t					-0.005	-3.39
0.5t <sup>2</sup>					0.000	4.49
$\sigma^2 = \sigma_V^2 + \sigma_U^2$	0.479	11.26	0.396	11.92	0.388	11.25
$\gamma = \sigma_U^2 / \sigma^2$	0.816	64.45	0.796	55.17	0.792	50.29
$\mu$	1.251	10.52	1.124	10.96	1.109	10.47
LR Test	-1495.1		-1244.4		-1232.5	
LR Test (1 side)	5097.9		4398.3		4380.8	
Iterations	31		38		41	

(\*) Frontier 4.1 program was used for the estimations.  
 Source: Calculations by the author.

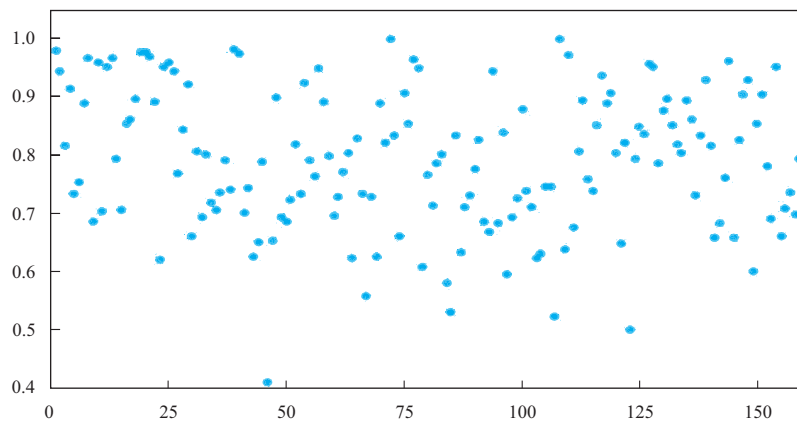
Table 6  
Estimation Results under Profit Maximization (\*)

	Model 1		Model 2		Model 3	
	Coefficient	t-ratio	Coefficient	t-ratio	Coefficient	t-ratio
Constant	7.848	55.82	7.928	53.03	6.601	54.64
$\ln y_1$	0.044	2.20	-0.105	-3.87	-0.138	-5.23
$\ln y_2$	-0.029	-1.51	-0.045	-1.34	-0.039	-1.20
$\ln y_3$	0.027	1.64	0.017	0.76	-0.026	-1.25
$\ln w_{13}$	1.000	37.99	1.019	34.84	0.858	29.04
$\ln w_{23}$	-0.032	-0.94	-0.047	-1.25	0.164	4.30
$0.5 \ln y_1 \ln y_1$	-0.010	-2.68	-0.013	-2.99	-0.009	-2.02
$0.5 \ln y_1 \ln y_2$	0.005	0.78	-0.013	-1.86	-0.002	-0.29
$0.5 \ln y_1 \ln y_3$	-0.007	-1.33	-0.010	-1.72	-0.006	-1.17
$0.5 \ln y_2 \ln y_2$	-0.002	-0.94	0.003	1.05	0.005	1.60
$0.5 \ln y_2 \ln y_3$	0.032	6.03	0.030	4.63	0.028	4.46
$0.5 \ln y_3 \ln y_3$	-0.013	-3.97	-0.013	-3.69	-0.011	-3.26
$0.5 \ln w_{13} \ln w_{13}$	0.047	7.89	0.047	7.92	0.020	3.36
$0.5 \ln w_{23} \ln w_{23}$	0.047	6.38	0.047	5.60	0.011	1.27
$0.5 \ln w_{13} \ln w_{23}$	-0.132	-12.56	-0.124	-11.31	-0.034	-2.89
$\ln w_{13} \ln y_1$	0.005	1.30	0.004	1.05	0.005	1.22
$\ln w_{13} \ln y_2$	0.000	-0.13	0.000	0.03	0.007	1.53
$\ln w_{13} \ln y_3$	0.000	-0.04	-0.001	-0.20	-0.006	-1.86
$\ln w_{23} \ln y_1$	-0.003	-0.54	-0.003	-0.52	0.010	1.72
$\ln w_{23} \ln y_2$	0.002	0.47	0.007	1.18	0.005	0.91
$\ln w_{23} \ln y_3$	-0.015	-4.14	-0.016	-3.89	-0.012	-3.13
$\ln z$			0.238	4.94	0.410	8.51
$\ln z \ln z$			-0.060	-7.60	-0.054	-7.08
$\ln w_{13} \ln z$			-0.005	-0.73	-0.009	-1.29
$\ln w_{23} \ln z$			0.002	0.20	-0.019	-2.18
$\ln y_1 \ln z$			0.042	6.19	0.033	5.08
$\ln y_2 \ln z$			0.010	1.35	0.006	0.79
$\ln y_3 \ln z$			0.004	0.82	0.007	1.35
$t$					-0.031	-19.30
$0.5 t^2$					0.001	18.26
$\sigma^2 = \sigma_V^2 + \sigma_U^2$	0.255	20.60	0.256	14.33	0.523	6.43
$\gamma = \sigma_U^2 / \sigma^2$	0.577	24.20	0.598	22.84	0.806	26.45
$\mu$	0.767	6.78	0.783	8.57	-1.298	-4.57
LR Test	-1869.8		-1806.9		-1656.0	
LR Test (1 side)	1022.1		1084.6		1030.0	
Iterations	34		36		50	

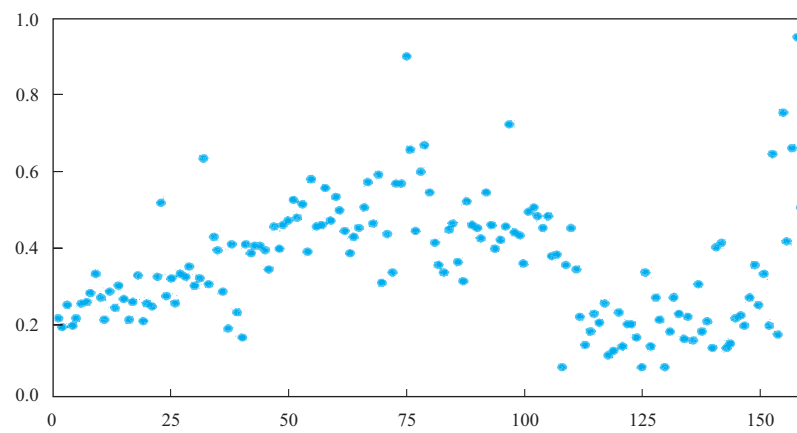
(\*) Frontier 4.1 program was used for the estimations.  
Source: Calculations by the author.

Graph 1  
Distribution Efficiency

Profit



Cost



Source: Calculations by the author.

Table (7) reports likelihood tests for all considered models. For both, cost and profit frontiers, all restrictions are rejected. Therefore, model 3 is choiced for cost and profit functions.<sup>26</sup>

Interpretation of the regression coefficients requires more attention, given that there are many interrelations between the different explanatory variables in the *translog* function. The marginal effect of an increase in the loan variable  $\ln(y_1)$  on the respective dependent variables total cost (TC) and before tax profits (BTP) must include not only the magnitude of the coefficient on  $\ln(y_1)$ , but also the combination of all coefficients on explanatory variables that include  $\ln(y_1)$ . With these caveats in mind, the following holds for the direct effects, excluding the no less important interaction terms.

For cost function, model 3 coefficients on the output variables have significant t-value and the coefficient of  $\ln(y_1)$  has a negative value, representing scale economies. High financial capital are significantly positively correlated with total

Table 7  
Likelihood Ratio Test

	Restrictions	Test Statistic	$\chi^2_{0.95}$ - value	Decision
<b>Cost Function</b>				
Model <sub>1,3</sub>	9	525.20	16.92	Reject H0
Model <sub>2,3</sub>	2	23.81	5.99	Reject H0
Model <sub>1,2</sub>	7	501.39	14.07	Reject H0
<b>Profit Function</b>				
Model <sub>1,3</sub>	9	427.61	16.92	Reject H0
Model <sub>2,3</sub>	2	301.87	5.99	Reject H0
Model <sub>1,2</sub>	7	125.74	14.07	Reject H0

Source: Calculations by the author.

<sup>26</sup> Remember that the model 1 corresponds to the estimations of the functions without taking into account the role of financial capital and technological change, while model 2 introduce financial capital, but does not includes trend variables

cost. The direct effect of input prices is diverse; it is high and significantly positive (0.828) for the price of financial capital ( $w_1$ ); low and significantly positive (0.176) for the price of labor ( $w_2$ ); and negative ( $1-0.828-0.176 = -0.004$ ) for the price of physical capital ( $w_3$ ). The negative coefficient on ( $w_3$ ) suggests that total cost decrease with higher physical capital price.

The negative coefficient on the linear trend term ( $t$ ) suggest a shifting cost curve with lower cost (on the frontier) through time. The positive square trend coefficient offsets the linear trend effect throughout time. From the estimated results on the linear and quadratic trend term we derive an improvement of the cost function between 1989 and 2003.

For the profit frontier, the model M3 has been chosen. The coefficients on the outputs are negative and significant in the first variable. Overall, increasing the size of production leads to lower profits, implying diseconomies of scale (again excluding the interaction effects). The coefficient on financial capital is positive and significant. The coefficient on the price of financial resource ( $w_1$ ) is significantly positive (0.858). The coefficient on the prices of personnel ( $w_2$ ) was 0.164 and the one on physical capital ( $w_3$ ) was  $-0.022 (1 - 0.858 - 0.164 = -0.022)$ .

## B. EFFICIENCY SCORES

Now, we turn to the mean efficiency scores that result from the M3 to the cost and profit frontiers. Remember that profit efficiency scores are in a range from 0 to 1, where 1 indicates a bank is efficient and operates on the frontier. For cost efficiency, scores lie range from 1 to  $\infty$ , where an efficient bank again has a score of 1. In Table (8), we report a few summary statistics on cost and profit efficiency scores.

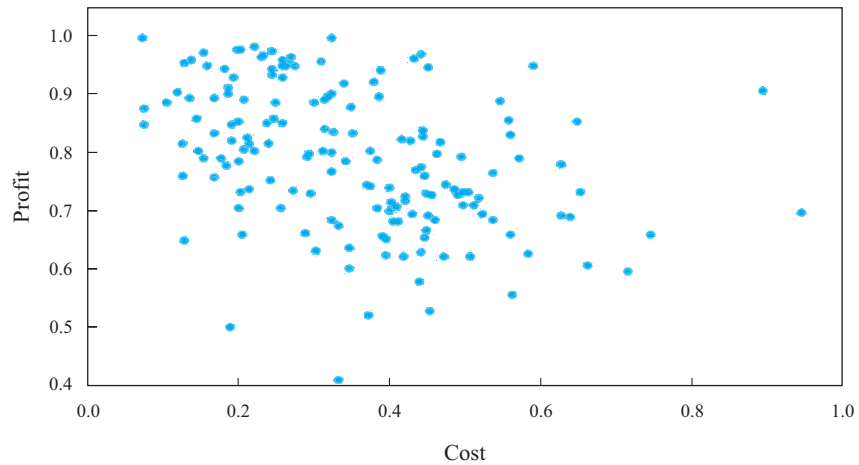
Table (8) shows that individual cost efficiencies vary from 1.05 to 13.64. Moreover, the mean of cost efficiency of 3.62 suggest that most of financial institutions have an efficiency score not close to 1. While to the profit function, the individual scores vary from 0.41 to 1, suggesting that the most of banks have scores close to 1. This is consistent with the graphical evidence in Graph (1). The distribution of individual profit efficiency score is more uniform and less concentrated than in the case of cost function. In addition, Graph (2) provides graphical evidence on the relation between cost and profit efficiency scores for individual bank firms. The scatter plot suggests a weak

Table 8  
Summary Efficiency Statistics (\*)

	N	Max.	Mean	Min.	SD
<b>Cost Function</b>					
Total System	161	0.95	0.35	0.07	0.16
Commercial Banks	49	0.63	0.30	0.15	0.10
Investment Banks	29	0.90	0.50	0.30	0.11
Specialized Commercial Banks	83	0.95	0.33	0.07	0.18
<b>Profit Function</b>					
Total System	161	1.00	0.79	0.41	0.12
Commercial Banks	49	0.98	0.81	0.41	0.13
Investment Banks	29	1.00	0.79	0.55	0.11
Specialized Commercial Banks	83	1.00	0.77	0.50	0.11

(\*) Profit efficiency scores are in a range from 0 to 1, where 1 indicates that a bank is efficient and operates on the frontier. In the same line, for cost efficiency, score lie range from 0 to 1, where an efficient bank, yet again, has a score of 1. The selected model to compute efficiency scores was the model 3.  
Source: Calculations by the author.

Graph 2  
Cost vs. Profit Efficiency Scores



Source: Calculations by the author.

correlation between both scores. This is confirmed by the bilateral correlation coefficient between two scores.<sup>27</sup> However, these results are related with the differences in efficiency scores among types of institutions, which could be also reflecting heterogeneity in the products and objective market.<sup>28</sup> It provides evidence in support of our claim that both cost and profit efficiency need to be investigated.

*C. SPECIALIZATION EFFECTS ON EFFICIENCY SCORES*

In this section we analyze differences in cost and profit efficiency across individual banks in more detail. To this purpose, we first distinguish among different type of financial institutions. In Table (9) we report independently the efficiency scores for each type of financial institution. In the case of cost efficiency scores, the specialized

Table 9  
Independent Samples Type of Banks <sup>1/</sup>

	Cost Efficiency			Profit Efficiency		
	CB	IB	SCB	CB	IB	SCB
N	49	29	83	49	29	83
Min.	1.59	1.11	1.05	0.41	0.55	0.50
Max.	6.48	3.33	13.64	0.98	1.00	10.00
Mean	3.62	2.10	4.15	0.81	0.79	0.77
S.D	1.07	0.44	2.69	0.13	0.11	0.11
t-Statistic		7.28	-1.05		0.69	1.89
t-Value <sup>2/</sup>		2.29	-2.27		2.29	2.27

<sup>1/</sup> Profit efficiency scores are in a range from 0 to 1, where 1 indicates a bank is efficient and operates on the frontier. For cost efficiency, score lie range from 1 to  $\infty$ , where an efficient bank, yet again, has a score of 1. The selected model to compute efficiency scores was the M3.  
<sup>2/</sup> 5% level of significance.  
 Source: Calculations by the author.

<sup>27</sup> Both, the Spearman rank correlation test and Pearson correlation coefficient were 0.44 and 0.36 respectively.

<sup>28</sup> Greene (2003) exposes that familiar approaches to inefficiency estimation mistakenly measure heterogeneity as inefficiency.

commercial banks presented the highest mean level of inefficiency scores (4.15), while commercial banks and investments banks had mean inefficiency scores levels of 3.62 and 2.10 respectively.

The results in Table (9) show that cost efficiency is marginally higher for both (IB) and (SCB) with respect to (CB). The t-test shows that this difference is statistically significant. In the case of profit efficiency, the conclusions are quite different. The difference in mean profit efficiency was not significant when comparing commercial banks to the other type of intermediaries. Overall, our results suggest that making the differentiation among type of bank firms does not allow them to exploit their specialization on profit side. A possible explanation is the presence of more opportunities to achieve scale economies on the input size than on the output size.

Some comparative results between the dispersion found in efficiency both inter and intra intermediaries are deeply influenced by the heterogeneity in the product offered by such agents and not differences in efficiency.<sup>29</sup> For example, this factor may explain why more dispersion exists between the scores related to cost efficiency than to income efficiency, which would in turn be reflected on the low correlation between both indicators and which is mentioned (in the paper) to be caused mainly by market power exercised by some institutions. The latter statement may be correct, however, it should also be mentioned that heterogeneity both between different types of firms and across firms of the same type, has an effect on this differential effect. The results of the distributions (variance and mean analysis) for each of the different types of firms strongly suggest this heterogeneity problem. For such reason, it is not strange that commercial banks and commercial financing companies are less cost-efficient than financial corporations (in fact, it would be interesting to break-up the average scores of commercial banks, between traditional commercial banks and savings-and-loans corporations; one would tend to think that the latter, having a restricted portfolio aimed towards mortgage loans for most of the analyzed time-period, would thus present a lower dispersion than the traditional commercial banks group).

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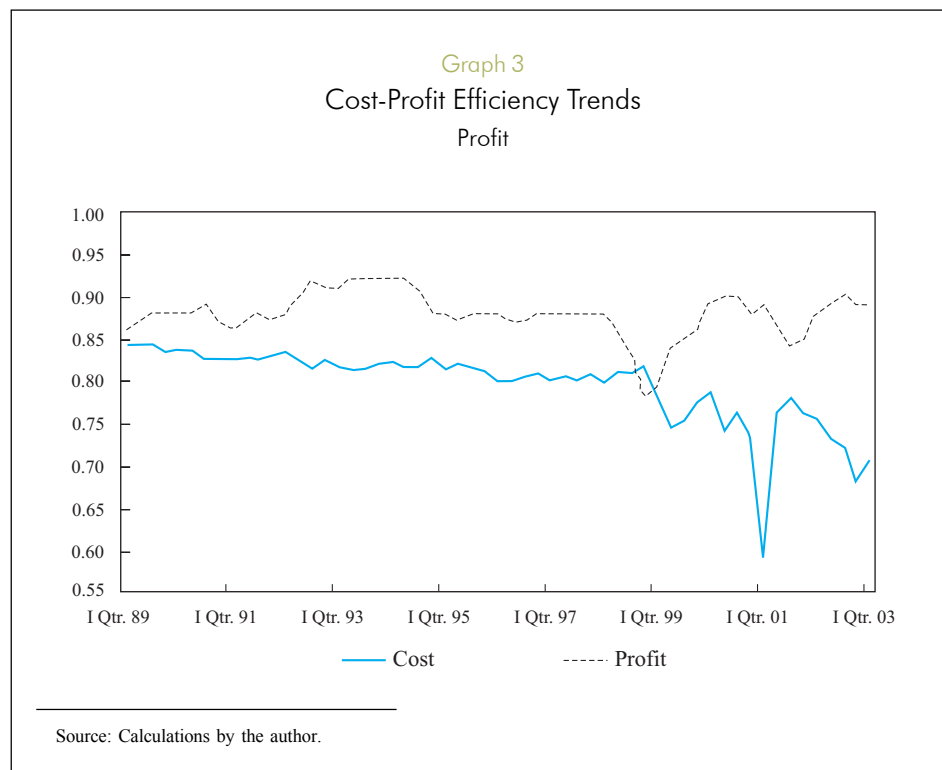
<sup>29</sup> Considering it is hard to compare the costs of an intermediary dedicated exclusively to consumption credit, which is much more costly to administer, to those costs of an intermediary dedicated to commercial loans.



*D. TEMPORAL ANALYSIS*

During the studied period, the trend variables were significant for the frontier estimations for cost and profit functions. Consequently, it is important to analyze if the mean efficiency scores has changed, specially, when three different subperiods have been identified in our sample.<sup>30</sup>

Graph (3) reports the time path of mean cost and profit efficiency for the years 1989-2003, both weighted by total assets. The graph shows that weighted mean cost efficiency is relatively more variable than profit efficiency over time. In the crisis period we found a impairment in the mean profit efficiency. The evolution



<sup>30</sup> The estimated coefficients for the time varying frontier analysis have been left out. They are available from the authors upon request.

after the crises period suggests us that the impact of the consolidation process affected the financial intermediaries differently adjusting their cost and profit functions. The mean cost efficiency for the period was 79%, while that for alternative profit efficiency was 88%. Comparing the standard deviations, we found that the mean cost efficiency (4.6%) was more irregular than mean profit efficiency (2.8%).

## **VI. CONCLUSIONS**

In this paper, we analyzed the cost and profit efficiency scores to the Colombian financial system during the period 1989-2003, in which the banking system has been affected by different consolidation, liberation and crises process. We used the parametric method of stochastic frontier to estimate cost and alternative profit functions, using a translog specification, that includes financial capital and trend time terms.

Our results show that there is significant difference between cost and profit estimations. Both Cost and profit functions, must be estimated using stochastic frontier method. The incorporation of financial capital was determinant to the frontier estimation in both cases. Furthermore, the inclusion of trend terms was important to determine the best frontier. The efficiency scores presented a higher variance in cost efficiency than profit efficiency, However, profit efficiency had more uniform distribution among financial intermediaries.

We have offered evidence showing the importance of incorporating capital structure in the bank production to consider banks' risk-taking behavior. In this way, incorporating financial capital plays an important role in the determination of the production efficiency to the financial firms and if we ignore this variable, we can generate bias in the efficiency estimation.

Analyzing microeconomic duality between minimization cost and maximization profits, the results suggest, with the empirical data in the analyzed period, that there is not perfect competition in the Colombian banking system. The correlation between cost and profit efficiency scores wasn't high. For this reason, when we want to analyze efficiency, we need to use both cost and profit functions. To our knowledge, this is the first paper of the efficiencies considering a long period, the first to compare cost and profit efficiency of Colombian financial intermediaries and the first using an important control variable such as financial capital.

Finally, distinguishing by type of financial intermediaries, we found significant differences between commercial banks and the rest of bank firms in the case of the cost function. We find that whereas all banks appear to perform rather similarly in terms of profit efficiency, in terms of cost efficiency there are differences when we consider efficiency mean.

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