ENSAYOS

sobre política económica

A Market Risk Approach to Liquidity Risk and Financial Contagion

Dairo Estrada
Daniel Osorio

Revista ESPE, núm. 50, junio 2006
Páginas 242-271

La interacción entre el riesgo de liquidez y el riesgo de mercado como fuente de crisis financieras

Dairo Estrada[*]
Daniel Osorio[**]

[*] Director, Departamento de Estabilidad Financiera, Banco de la República. Correo electrónico: destrada@banrep.gov.co
[**] Profesional en Análisis y Estabilidad Financiera, Banco de la República. Correo electrónico: dosoriro@banrep.gov.co

Documento recibido el 13 de febrero 2006; versión final aceptada el 1 de julio de 2006.

Resumen

De acuerdo con la literatura tradicional, el riesgo de liquidez individual puede generar problemas sistémicos únicamente en presencia de exposiciones crediticias entre bancos o de corridas bancarias. Este artículo muestra que este fenómeno también ocurre cuando el riesgo de liquidez individual se convierte en riesgo de mercado para el conjunto del sistema financiero (aún en ausencia de las características mencionadas anteriormente). Ello sucede cuando, en presencia de una crisis de liquidez, los bancos resuelven liquidar parte de su portafolio de inversiones en el mercado. Si la demanda por estas inversiones no es perfectamente elástica, este procedimiento conduce a una caída en el precio de mercado de las inversiones. Dado que los bancos valoran su portafolio de inversiones a precios de mercado, la caída del precio reduce el valor de los activos de todos los bancos del sistema, dejándolos en condiciones menos favorables para enfrentar futuros choques de liquidez y, por lo tanto, más expuestos a la bancarrota. El artículo presenta esta idea por intermedio de la simulación de un modelo microeconómico que intenta capturar el comportamiento del administrador de liquidez de un banco que actúa en un ambiente de incertidumbre en torno a su hoja de balance. Los resultados sugieren que este fenómeno es más crítico en tanto menos profundo sea el mercado de las inversiones.

Clasificación JEL: G21, G33, L14.

Palabras claves: *administrador de liquidez, riesgo de liquidez, riesgo de mercado, riesgo sistémico, contagio,* mark-to-market.

A Market Risk Approach to Liquidity Risk and Financial Contagion

Dairo Estrada [*]
Daniel Osorio [**]

According to traditional literature, liquidity risk in individual banks can turn into a wide-system financial crisis when either interbank credit exposures or bank runs are present. This paper shows that this phenomenon can also arise when individual liquidity risk transforms into wide-system market risk (even in the absence of bank runs and interbank credit networks). This happens when banks try to sell some portion of its assets in order to overcome a liquidity shortage (individual liquidity risk). These sales depress the market price of assets if demand is not perfectly elastic. Given the fact that banks mark to market the asset book, the fall of market price reduces the value of assets of every bank in the system (wide-system market risk), leaving

---

The opinions contained here are those of the authors and do not necessarily represent those of the Banco de la República or its Board of Directors.

[*] Director of Financial Stability Department, Banco de la República. E-mail: destrada@banrep.gov.co

[**] Profesional of Analysis and Financial Stability, Banco de la República. E-mail: dosoriro@banrep.gov.co

Document received 13 february 2006, final version accepted 1 July 2006.

them less suited for future liquidity shortages and therefore more prone to bankruptcies. The paper rationalizes this idea through the simulation of a model that tries to capture the behavior of a liquidity manager that faces shocks on bank deposits and loans. The main results suggest that the extent of financial contagion depends crucially on the size of the market for assets.

JEL Classification: G21; G33; L14.

Keywords*:* liquidity manager, liquidity risk, market risk, systemic risk, financial contagion, mark-to-market.

I. INTRODUCTION

There is a wide agreement, especially after the recent experience of a number of countries, that financial crises entail huge costs. As a consequence, financial stability has become an important concern for economic authorities. Central banks, responsible for the maintenance of a well-functioning payments system, now undertake several activities in order to promote financial stability.

According to Large (2005a), one of those activities should be the "assessment of threats to financial stability", which comprises particularly the monitoring of the risks faced by financial institutions. A correct monitoring depends crucially on the "stock of knowledge" about how risks arise and operate in individual financial institutions. And perhaps more relevant, about how risks faced by individual institutions turn into systemic risk, and eventually become material in the form of a wide-system financial crisis.[1]

This paper is an attempt to contribute to the understanding of the mechanics of liquidity risk and, particularly, of the underlying forces that allow liquidity crises in individual financial institutions to spread throughout the financial system in a contagion-fashion.[2] Specifically, the objective of this paper is to explore the possibility

[1] For a concrete definition of systemic risk, see De Bandt and Hartmann (2000).

[2] Liquidity risk is related to the possibility of a bank being unable to pay its liabilities as they fall due, independently of its solvency situation. This «institutional liquidity risk» (Large, 2005b) –as shows this paper– relates itself to the inability to liquidate positions «in a timely manner and at a reasonable prices» (Muranaga and Ohsawa, 1997).

that liquidity risk faced by individual banks turn into systemic risk through its relationship with market risk: as will be shown, the sales of assets that banks conduct in order to overcome a liquidity shortage (individual liquidity risk) disturb the market for assets of every bank in the system (wide-system market risk), possibly leading to a financial crisis.[3] This source of systemic risk has been relatively left aside by recent literature.

The paper rationalizes this idea through the simulation of a model that tries to capture the behavior of a liquidity manager that faces shocks on bank deposits and loans. The main results suggest that the extent of financial contagion depends crucially on the size of the market for assets, which is related particularly to the aggregate demand of credit in the economy.

The paper unfolds in three sections. The II section reviews some of the relevant literature, in order to locate the contribution of this paper. The III section focuses on the above-mentioned model, while IV and V sections will present, respectively, the results of the simulations of the model and some reflections that serve as concluding comments.

II. A TALE OF THE LITERATURE

The appearance of systemic risk in a financial system, arising from liquidity risk in individual financial institutions, is probably a phenomenon as old as banks themselves.[4]

Following Gorton (1988) and Furfine (1999), the recent literature on the topic can be classified in three groups. It is worth noting, however, that this classification is in some sense arbitrary and that these three groups are not necessarily mutually excluding. Its only usefulness is to locate in a straightforward manner the contribution of this paper.

The first group considers that systemic risk is a natural result of the possibility of bank runs. The pioneering work by Diamond and Dybvig (1983) emphasizes the

[3] Market risk is associated to the losses suffered by financial institutions in the possible event of a disruption in the general conditions of the markets in which banks concur.

[4] The materialization of systemic risk into financial crises is also an old issue. See Kindleberger (1978) for an interpretation of the history of the most important financial crises.

role of the liquidity structure of banks in inducing runs.[5] In particular, the *raison d'être* of banks is characterized by the transformation of liquid liabilities into illiquid assets. The model exhibits multiple equilibria: besides showing how the presence of banks can improve on private (non-bank) market allocation, one of these equilibria is characterized by a bank run, in which depositors simultaneously rush to withdraw their deposits.[6] The common thread of the story is that banks face shocks on deposits that can be backed only by a portfolio of illiquid assets, redeemable only with discount (i. e.: banks face liquidity risk).[7]

Gorton (1988) denies the idea that sunspots are behind bank runs. Instead, he shows that bank runs can be understood as the rational response of depositors whose perceptions of aggregate risk are altered. One source of alteration of these perceptions is the business cycle. His theoretical model, however, lacks an explicit modelling of the liquidity structure of the balance sheet of banks, which is so crucial in the model of Diamond and Dybvig.[8]

An ample majority of recent researchers can be grouped into the second strand of the literature. According to them, individual liquidity risk becomes systemic risk whenever "the failure of one or a small number of institutions [is] transmitted to others due to explicit financial linkages across institutions" (Furfine, 1999, p. 1), even if bank runs are not present. These financial linkages are commonly associated to interbank credit exposures. In most papers, banks are subject to a common source of uncertainty (e. g. demand for liquidity from depositors). Shocks then hit one or some banks.

Eventually, these banks will default on their payments in interbank credit market, leaving creditors (other banks) in a hard financial situation. Again eventually, these creditors will default on its liabilities to other banks, and so on. Financial trouble spreads from one bank to another in a domino-fashion.

---

5 Only one bank is modelled in the framework of Diamond and Dybvig (1983).

6 The selection between the bank run equilibrium and other equilibria is not a clear issue. According to Diamond and Dybvig, the bank run may appear as a result of the release of some sort of negative information, or «even sunspots» (Diamond and Dybvig, 1983). This is a very criticized feature of the model.

7 The discount is exogenous.

8 The empirical record of bank runs has been analyzed, among others, by Gorton (1988), and Hasan and Dwyer (1994).

The work by Allen and Gale (2000) is representative of this group. In their model, banks are uncertain about their individual demand for liquidity from depositors (and therefore subject to liquidity risk). According to these authors, a complete interbank market allows optimal risk sharing between depositors and banks and between banks themselves. An incomplete interbank market, instead, is prone –under certain states of nature– to contagion of bankruptcy from banks with unusually high demand of liquidity to other banks, through credit expositions in the interbank market. In much as the same way as Diamond and Dybvig, Allen and Gale give a crucial role to the liquidity structure of the banks balance sheet.[9]

To Rochet and Tirole (1996), the definition of systemic risk is limited to only the "propagation of an agent's economic distress to other agents linked to that agent through financial transactions". These authors build a model of interbank lending, where under certain conditions contagion is an issue. Furthermore, they explore how the presence of contagion-prone interbank exposures may benefit from interbank monitoring, enhancing the results obtained with a centralized scheme of credit insurance or liquidity management.

The coordinating role of a centralized authority (central bank) in presence of interbank linkages is also analyzed by Freixas, Parigi and Rochet (2000). These authors coincide with Allen and Gale in stating that the absence of an interbank credit market reduces the capability of the banking system to face an uncertain demand for liquidity from depositors. Interbank credit networks, however, bring the possibility of contagion and rationalize an orderly (in terms of liquidating insolvent banks) intervention by the central bank.

The contribution of an interbank credit market to liquidity risk management of a financial system is analyzed, in the context of the simulation of microeconomic models, by Iori and Jafarey (2000), Iori, Jafarey and Padilla (2003) and Estrada (2001). Liquidity risk arises in these models because banks are subject to deposit as well as investment shocks, that make liquidity shortages likely. Banks are allowed to overcome shortages by borrowing in the interbank market. As the abovementioned literature suggests, the interbank market is subject to a trade-off: it improves the resilience of the system in comparison with a situation in which there is no interbank

9 Reformulations of the original model by Castiglionesi (2004), in order to allow an active role by the central bank, reach similar conclusions.

transactions, but makes the system prone to contagion of financial trouble from one bank to another. Importantly, these authors suggest that rather than empirical exercises, simulation of microeconomic models is a natural strategy to analyze these issues, given data limitations on bilateral interbank credit exposures.[10]

Finally, the third group of literature remains relatively underdeveloped. This strand emphasizes the disruption of financial markets that can be provoked by a troubled bank. Insofar as this disruption alters the value of the positions of every bank in the system, it is possible to state that the distress of the former bank spreads to other banks through the disruption of the market. As individual risks end up disturbing markets, it is possible to state that, according to this group, individual risks turn into market risk. The work by Cifuentes, Ferrucci and Shin (2005), Schnabel and Shin (2004) and Plantin, Sapra and Shin (2005) can be grouped into this strand.[11]

In the models by Cifuentes, Ferrucci and Shin (2005) and Schnabel and Shin (2004), a distressed bank resorts to the sale of illiquid assets in order to avoid default on its interbank liabilities. This unambiguosly depresses the price of the assets (disturbs the market), and consequently the value of the portfolio of every bank in the system (in presence of mark-to-market rules). If price falls enough, the bank will default, leaving other banks in a distress similar to the one suffered by the initial bank.

This strand of literature, however, imposes several restrictions on the mechanisms that allow individual risks turning into market risk. These models, for example, do not include explicitly shocks on the demand for liquidity, and hence neglect the role of the source of liquidity risk in previous literature. Instead, the initial distress comes from the exogenous default of any bank in the system. Furthermore, in the case of the model by Cifuentes, Ferrucci and Shin (2005), banks are obliged to comply with an exogenous capital adecuacy ratio. By this mechanism, further sales of illiquid assets are feeded by previous sales, exacerbating the disruption of the market.[12]

---

10 An obliged reference on the empirical relevance of contagion from interbank credit exposures is Furfine (1999). For an application of physics network theory to an interbank market, see Boss et al. (2004).

11 These studies follow previous ideas by Allen and Gale (2003) and Diamond and Rajan (2003).

12 In a sense, the source of perturbation is exogenous.

This paper belongs to the third strand of literature. The model to be presented later tries to emphasize the idea that liquidity risk can distress banks, and distressed banks can perturb the markets in which all banks concur. This perturbation affects the capability of other banks to withstand liquidity shocks.[13] In doing so, however, the model overcomes the flaws mentioned in the last paragraph by relying in some of the insights provided by the first and second strands. In particular, the model will explicitly include an uncertain demand for liquidity from depositors, as most models of the first and second group do.

Furthermore, the model is intented to show that the appearance of market risk and financial contagion does not depend upon either the presence of interbank credit exposures, bank runs or regulatory capital requirements. A contribution of the model is to show that the perturbation of markets by distressed banks is an issue even if those elements are not present.

III. THE FRAMEWORK

A. A NOTE ON MICROFOUNDATIONS

In this section, the intraday problem of the liquidity manager of a representative bank is modelled. The liquidity manager is not only in charge of the handling of the liquid resources of the bank, but also of the transactions that banks conduct in the market. The handling of liquidity and the participation in the market lie at the heart of the model.

The problem that faces a bank as a whole is associated, traditionally, to the maximization of an objective function (e. g. profits, market share) in presence of resource as well as technological constraints (Freixas and Rochet, 1997).

In this model, however, the intraday problem of the liquidity manager has nothing to do with optimization. The absence of an optimization problem is justified on two grounds.

[13] To the extent that the actions of a distressed bank end up altering the financial resilience of other banks, the model captures financial contagion (in the sense of De Bandt and Hartmann, 2000).

First, the problem that faces the liquidity manager is not necessarily the same problem that faces the bank as a whole. Rather than to maximize, the only objective of the liquidity manager is to pay his/her intraday liabilities, subject to aggregate liquidity, and the sizes of the income statement and the balance sheet of the bank; independent of the effect that his/her actions have on the non-modelled objective function of the bank as a whole. When the manager is not able to accomplish his/her objective, the bank goes bankrupt and disappears from the financial system.

Second, the model considers the effect of the disappearance of bankrupt banks. General equilibrium models such as those by Goodhart, Tsunirand and Tsomocos (2004, 2005a, 2005b) and Tsomocos (2003), do not take into account the possibility of forced liquidation of banks as the response to financial trouble. Rather, banks exhibit in equilibrium a probability of default, and they either only consider to be liquidated as a consequence of the end of the world (in the two-period version of the model) or simply do not consider it at all (in the infinite-horizon version).

This model is instead based on the belief that the disappearance of bankrupt banks affects other banks through several mechanisms, in particular through its effect on competition and aggregate liquidity. So, as will be shown, the liquidation of banks is crucial in the understanding of how systemic risk arise.

B. THE MODEL

The model presented in this subsection follows previous work by Iori and Jafarey (2000) and Estrada (2001).

This exercise seeks to model the day-to-day problem faced by the liquidity manager of a bank, in a context where the bank faces deposit as well as loans shocks, that make liquidity shortages likely. Under a shortage, the liquidity manager is obliged to seek for liquid funds in order to pay the liabilities of the bank. For Iori and Jafarey (2000) and Estrada (2001), banks can obtain these funds by borrowing in an interbank market for loans.

As said before, the objective of this paper is to show the possible existence of financial contagion in the absence of exposures in the interbank market for loans. Therefore, in sharp contrast with these papers, the interbank market for loans is not present in this model, so liquidity managers cannot solve liquidity problems

borrowing in it. Instead, this model assumes that banks are endowed with a stock of assets (as government debt securities) that can be traded with other banks in an interbank market for assets, and whose book value (as in Cifuentes, Ferrucci and Shin, 2005) is marked-to-market.

When a liquidity manager founds itself in a liquidity shortage, he/she can solve the problem by selling to other banks a portion of its tradeable asset. As asset demand is not perfectly elastic, the sales will lead to a fall in asset prices. By the mark-to-market valuation procedure, the assets of other banks will loss value correspondingly, which leaves other banks in a weaker position to face future liquidity shocks. In this model, then, financial contagion is the result of the depression of asset prices triggered by liquidity shocks in an individual bank.

1. The appearance of liquidity risk

Suppose that, at any point in time, the financial system is composed by a finite number of operating banks, N_t. Each bank is labelled by the superscript k, where $k \in \{1, 2, \cdots, N_t\}$. This model does not allow for replacement of bankrupt banks.

At the start of period t, the liquidity manager of bank k inherits an amount of cash holdings from period t - 1 given by:

$$(1) \qquad M^k_{t-1} = D^k_{t-1}(1 - b) - L^k_{t-1} - p_{t-1}A^k_{t-1}$$

where:

M^k_{t-1}: Cash holdings inherited from period t - 1.
D^k_{t-1}: Deposits held by the public in bank k at the end of t - 1.
b: Reserve requirement on deposits set by the central bank.[14]
L^k_{t-1}: Stock of loans extended by bank k until the end of t - 1.
A^k_{t-1}: Stock of tradeable asset of bank k at the end of t - 1.

[14] The central bank is modelled as a strategic dummy whose tasks are to set the reserve requirement and to oversee the liquidation of bankrupt banks. This model does not take into account any scheme of financial regulation in terms of capital requirements. As was mentioned, the objective of this exercise is to show the possibility of financial contagion in the absence of capital requirements forcing the balance sheet of banks.

p_{t-1}: Unitary market price of tradeable asset at the end of t - 1.[15]

According to expression (1), the cash holdings of bank k at the beginning of t corresponds to after-b deposits minus the total operations in which bank k has engaged. Note that the only balance sheet item that is valued at market prices is the stock of tradeable asset. This fact implicitly assumes that the set of remaining items have a constant price of 1.

At the beginning of t, the bank simultaneously receives payments from loans, must pay interest on deposits, and faces shocks in its stock of deposits. The model also assumes that the tradeable asset produces income to its holder.

Loans must be repaid after two periods. Therefore, the payment from loans received by bank k at the beginning of period t is:

$$(2) \qquad (1 + r_l)^2 L^k_{t-2}$$

where r_l is the one-period interest rate that banks charge on loans. Note that this expression implies that borrowers do not pay interest income one period after the loan was extended.

The interest rate payed on deposits is determined endogenously according to the model of Salop (1979) in the following way (for a detailed analysis see Freixas and Rochet, 1998):

$$(3) \qquad r_{dt} = r_l - (a / N_t)$$

This specification attempts to capture the effect of competition on deposits rate. When the financial system is large (N_t is large), the deposit rate will tend to r_l and the margin will tend to zero. The reduction in the number of banks caused by bankruptcies will move the deposits rate downwards and will consequently increase the margin. Finally, interests are paid on the stock of deposits observed at the end of t - 1.

The interest proceeds from the holding of the tradeable asset are exogenous. To simplify, this model assumes that the tradeable asset has no maturity, so the proceeds

[15] Because of mark-to-market rules, market price equals book price.

from holding it correspond only to the stock of the asset multiplied by an exogenous and constant interest rate, r_a. For reasons that will become clear later, the only restriction on the value of r_a is $r_a \leq r_l$.

The gap in the timing between the payments from loans and the payments to depositors is precisely a source of liquidity risk in this framework. If the effect of other balance sheet items is isolated, banks are prone to liquidity shortages because they have not still received income from loans at the moment when the must pay to depositors.[16]

The other source of liquidity risk in this model is the stochastic pattern of withdrawals and creation of deposits. Given that the behavior of deposits is unpredictable, a liquidity manager can find itself unable to pay depositors due to the illiquidity of its investments.

This model assumes that aggregate deposits are exogenously deterministic, and are divided in two portions that are distributed among banks. The first portion is the deterministic component of aggregate deposits that is distributed prorrata among banks. The second portion is the stochastic component of aggregate deposits, and is distributed randomly among banks. The decomposition of aggregate deposits into a deterministic and a stochastic component is exogenous. The behavior of deposits in an individual bank follows then:

$$(4) \qquad D_t^k = pD_{t-1}^k + (1 - p)[(1 - s_d)\,(D_{t-1} / N_{t-1}) + s_d\, e_t^k\, D_{t-1}]$$

where:

D_t = Aggregate deposits at t.

p : The autorregressive component of deposits of bank k.

s_d: The stochastic share of aggregate deposits. As can be seen, $s_d \in [0, 1]$. This means that the stochastic share is an exogenous percentage of aggregate deposits.

e_t^k: The share of the stochastic component of aggregate deposits that goes to bank k at the start of period t. Bank deposits are random because e_t^k is random.

16 The gap between the maturity of assets and the maturity of liabilities, that traces back to Diamond and Dybvig, is precisely used as a measure of liquidity risk by, for example, the Financial Sector Assessment Programs (FSAPs) of the International Monetary Fund.

e_t^k is uniformly distributed across banks and satisfies $\sum_k^{Nt} e_t^k = 1$.

The uniformity of e_t^k implies that any individual bank face the same conditions than its competitors, unless the initial size of banks differs. This is indicative of the competitive environment in which banks act.[17]

Before proceeding, it is worth noting that equation (4) does not allow the possibility of bank runs. In accordance with (4), depositors shift deposits from one bank to another in a random way, not as a rational response to their forecast about the financial soundness of the bank. Moreover, aggregate deposits are not affected by the behavior of depositors. This fact implicitly assumes that depositors do not withdraw resources from the financial system as a whole, as is the case with a bank run.

2. Liquidity shortages

After receiving interest income from assets, and pay interest and face shocks on deposits, the liquidity manager of an individual bank finds it self with an intraperiod -*t* position in cash given by the following expression:

$$\widehat{M}_t^k = M_{t-1}^k + (D_t^k - D_{t-1}^k)(1 - b) - r_{dt} D_{t-1}^k + (1 + r_l)^2 L_{t-2}^k + r_a A_{t-1}^k \quad (5)$$

Substituting M_{t-1}^k from equation (1), it is possible to get:

$$\widehat{M}_t^k + bD_t^k = D_t^k - (p_{t-1} A_{t-1}^k) - L_{t-1}^k - r_{dt} D_{t-1}^k + (1 + r_l)^2 L_{t-2}^k + r_a A_{t-1}^k \quad (6)$$

Equation (6) corresponds to the liquid resources of the bank during period *t*: the sum of intraperiod position in cash ($\widehat{M}_t^k$) and deposits in the central bank in the form of reserve requirements (bD_t^k).

Liquidity risk materializes in the form of a liquidity shortage when the liquid resources during period *t* are negative, i. e.: when $\widehat{M}_t^k + bD_t^k < 0$. It is easy to show that, if liquid resources are negative:

17 The behavior of individual bank deposits may be justified on the grounds of depositors that move randomly across geographic regions that are associated with a specific bank.

$$(7) \qquad D_t^k - (p_{t-1} A_{t-1}^k) - L_{t-1}^k + (1 + r_l)^2 L_{t-2}^k + r_a A_{t-1}^k \leq r_{dt} D_{t-1}^k$$

Equation (7) states that the bank is unable to pay his/her obligations to depositors, because of the illiquidity of its assets. On the contrary, when a bank has positive liquid resources, it has resources in excess of its obligations to depositors, which may be referred to as "liquidity excess". The liquidity excess is available for investment in loans or tradable assets.

3. The Interbank Market for Tradeable Assets

In the absence of an interbank market, a liquidity shortage would imply the liquidation of the bank.[18] Notwithstanding, before liquidation, the liquidity manager can try to solve his/her liquidity problem by selling a portion of his/her stock of the tradeable asset. The stock of the tradeable asset that needs to be sold by bank *k* during period *t*, s_t^k, is therefore equivalent to its liquidity shortage (of course, s_t^k will be zero if the bank has a liquidity excess):

$$(8) \qquad s_t^k = -min\ [0, ((\widehat{M}_t^k + bD_t^k)/p_{t-1})]$$

Note that the bank calculates the necessary sales with the market price of assets observed at the end of *t* - 1, i. e.: before the market for tradeable assets open.

Tradeable assets can be sold only to other banks (and as will be showed later, to the central bank only in case of liquidation of the bank). This means that the potential purchasers of tradeable assets are banks with liquidity excesses. However, banks with liquidity excesses give priority to extend loans, which is granted by making the interest rate on loans greater than the interest rate on tradeable assets. The loans a bank with liquidity excess can extend behave stochastically in a similar way as individual bank deposits. In particular, the opportunity for bank *k* in period *t* to extend loans is:

$$(9) \qquad o_t^k = [(1 - s_o)(W / N_t)] + v_t^k\ s_o\ W_t$$

where:

[18] In this model, the central bank is not allowed to offer liquidity assistance to solvent banks. Therefore, the «Lender of last resort» role for the central bank is not present.

W : Aggregate demand for loans, assumed constant.

s_o :The stochastic component of aggregate demand for loans. As can be seen, $s_o \in [0, 1]$. This means that the stochastic portion is an exogenous percentage of aggregate demand.

v_t^k: The share of the stochastic component of demand that goes to bank k during period t.

The actual amount of loans extended in period t is constrained, of course, by the liquidity excess (net of the reserve requirement) of the bank. In other words, if o_t^k is greater than the liquidity excess (net), the amount of loans extended must be equivalent to the liquidity excess (net). That amount therefore corresponds to:

(10) $\quad w_t^k = min\{max[0, \widehat{M}_t^k], o_t^k\}$

Note, that according to (10), banks with liquidity shortages do not extend loans.

If after extending loans, the bank is still left with liquid resources, this excess is spent by the bank purchasing tradeable assets to banks with a liquidity shortage. The stock of assets purchased by bank k during t is given by:

(11) $\quad b_t^k = (\widehat{M}_t^k - w_t^k) / p_{t-1}$

From (11) note that the greater w_t^k (i. e.: the greater the opportunity to extend loans), the smaller b_t^k and hence the smaller the demand for tradeable assets (the smaller the market of the tradeable asset). Finally, the working of the market for the tradeable asset is given by the following definition.

Definition 1. Market for tradeable asset. The market for tradeable asset is a double (X_t, p_t) consisting of a matrix of transactions of tradeable assets, X_t, whose dimensions are $N_0 \; X \; N_0$ and a new price for the tradeable asset at which all transactions are conducted, p_t such that:

(a) $\quad x_{ij} \leq min[s_t^i, b_t^j] \; \forall \; i, j \in [1, \ldots, N_0]$

(b) $\quad \sum_{j=1}^{Nt} x_{ij} \leq s_t^i$

(c) $x_{ij} \leq \sum_{j=1}^{Nt} b_t^j \ \forall \ i, j \in [1, \ldots, N_0]$

(d) $p_t = p_{t-1} e^{-l(\sum_{j=1}^{Nt} s_t^j - \sum_{j=1}^{N_t} b_t^j)} \leq 1$

Item (a) defines the elements of X. The element $(i; j)$ of X correspondsto a transaction conducted in the market by a seller bank (i) and a purchaser bank (j). According to item (b), the sum of each of the columns of X (which corresponds to the total purchases by bank j) must be equal to the liquid resources that bank j had destined to it. Item (c) states that total sales by bank i may be smaller than its liquidity shortage (never greater, bank i cannot short sell the tradeable asset). In this sense, the market will not necessarily clear. Item (d) determines the evolution of the market price of the tradeable asset as a negative function of the excess supply in the market, where l , is a positive constant that represents the sensitivity of p_t to the excess supply. This price is not intended to clear the market, but to capture the idea that even if the market does not clear, a greater excess supply depress unambiguously the price. Finally, p_t is the price at which all transactions in the interbank market for the tradeable asset are conducted.

As far as the simulation is concerned, the code starts by taking a bank facing a liquidity shortage, making sure that the shortage satisfies two conditions: first, it can be satisfied with the liquidity excesses of the other banks. Second, the sale of its total stock of tradeable assets is at least enough to cover the shortage.[19] If both conditions are satisfied, the market is open: this bank contacts any other bank in a random order. If the latter faces a liquidity excess, a transaction between the two banks is agreed according to item (a). The transaction is agreed at price p_t. If the former bank does not cover the shortage with this transaction, then it contacts any other bank, until the shortage is covered. The code takes then another shortage bank, and so on. The code stops when there are no more liquidity excesses to buy tradeable assets (the market closes). From that moment, it is clear that some banks with liquidity shortage will not be able to cover it.

4. End of Period *t* and the Channel of Contagion

At the end of period t, after the closure of the market for tradeable asset, all banks left with liquidity shortages are liquidated(i. e. are bankrupt). As was mentioned, a

[19] If the shortage do not satisfy any of these conditions, the bank is liquidated according to the procedure that will be presented later.

bank is bankrupt if the market closed before it could agree any transaction, or if its stock of assets was not enough to cover its shortage.

The central bank oversees the liquidation of every bankruptcy. In particular, the central bank distributes the liquidation value of the bankrupt bank among its depositors who, in turn, redeposit these resources in the remaining banks.[20] The liquidation value is the sum of the intraperiod position and a fraction g of its assets (loans and tradeable assets), minus total liabilities (deposits). Whenever the liquidation value falls short of the obligations of the bank, aggregate deposits are reduced by the difference.

The evolution of total deposits will then be given by:

$$(12) \qquad D_t = D_{t-1} + \sum_{j \in B} \{ \widehat{M}_t^j + g\,[L_t^j + p_t A_t^j] - D_t^j \}$$

Note that the sum correspond to those banks that fall bankrupt (B). According to (a), the cash holdings at the end of period t for surviving banks are:

$$(13) \qquad M_t^k = D_t^k\,(1 - b) - L_t^k - p_t A_t^k$$

Note that A_t^k is valued at price p_t. So, even if the bank has not engaged in transactions of tradeable asset, the new stock of the tradeable asset is valued at the new market price according to the mark-to-market valuation procedure. This is precisely the channel of financial contagion emphasized in this model: the fall in the value of assets caused by the shortages of some banks (as the supply of tradeable assets exceeds demand) has an important effect on other banks: it leaves them less well-suited for future liquidity needs, insofar as the buffer to face liquidity shortages lose value. The model shows, then, how liquidity risk can turn into market risk for banks, and how the interaction between market and liquidity risk can spread between banks causing financial crises, which is understood here as a large number of almost simultaneous bankruptcies.

The next section will try to understand in more detail, by means of simulations of the previous model, this «mechanics» of the interaction between liquidity and market risk.

20 These resources add up to aggregate deposits, that will be distributed among banks according to (4).

IV. SIMULATIONS

This section focuses on the results of several simulations of the above model. All simulations share two features in common. First, to the end of comparability, all of them used 10 starting banks ($N_0 = 10$) and 150 iterations (periods of time), with the exception of the simulation of the Colombian banking system. Second, to the end of exacerbating liquidity risk, starting banks are divided arbitrarily in two groups: the first group does receive interest proceeds from loans in $t = 0$ and does not in $t = 1$ (i. e.: they did extended loans in $t = -2$, and did not in $t = -1$), while the second group does not receive interest proceeds from loans in $t = 0$, and does in $t = 1$ (i. e.: they did not extended loans in $t = -2$, and did in $t = -1$).

Additionally, because of the random features of the simulations and the consequent caveats about robustness, each one of them (i. e.: 150 iterations for 10 starting banks under a given set of parameters) was run 1,000 times. The results showed below correspond therefore to an average of those 1,000 exercises.

The first set of simulations were performed under the assumption of homogeneous starting banks.[21] This assumption is removed in the second and third set of simulations. Second set imposes randomly heterogeneous starting banks, as in Iori, Jafarey and Padilla (2003), while third set explores the effect that the model dynamics has on a system whose balance sheet structure mirrors that of the Colombian financial system.

From here on, the number of surviving banks in each moment of time will be the main measure of financial resilience.[22]

A. HOMOGENEOUS BANKS

Starting 10 banks are endowed with 1,000 units of deposits and tradeable assets. 1,000 units of starting loans are given to the first group of starting banks in $t = -2$ and to the second group in $t = -1$.[23]

---

21 Besides the properties of e_t^k, homogeneity refers to an identical starting balance sheet.

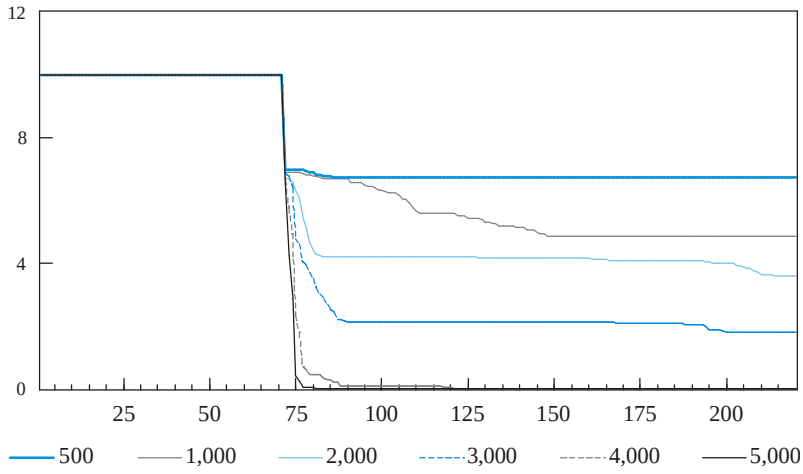
22 The model is inherently prone to instability. Only under certain sets of parameter values, a positive number of banks survive the horizon of 150 periods of time. Therefore, financial resilience can also be captured by the number of periods that are needed to go bankrupt the whole financial system.

Graph 1 shows the effect of W (aggregate demand for credit) on the resilience of the system (and therefore on the extent of financial contagion). On average, the smaller the demand for credit, the sounder the financial system. When aggregate demand equals 500 (a twentieth of aggregate starting deposits), an average of 6.4 banks survive the 150 periods of time, after a huge financial crisis in early iterations. The number of surviving banks falls to 4.8, 3.6 and 1.7 when W is 1,000, 2,000 and 3,000 respectively.

For levels of W of 4,000 or 5,000, no bank survives the horizon of 150 iterations. In the former case, however, all banks go bankrupt in an average of 51 iterations, while in the latter it requires only 11 iterations.

Graph 1

Surviving Banks and Aggregate Demand for Credit
(homogeneous banks)



Source: authors' calculations.

23 The set of parameters employed in these simulations were: $a = 0.1$, $b = 0.2$, s_d (when not variable) $= s_o = g = p = 0.5$, $r_l = 0.1$, $r_a = 0.05$, $w = 2000$ (when not variable), $l = 0.01$.

These results are interpreted in terms of the effect of W on the size of the market for tradeable assets. When the aggregate demand for credit is low (high), the opportunity of banks to extend loans falls (rises) in concordance with (9), which at least decreases (increases) the actual amount of loans extended according to (10). If this amount falls (rises), the demand for tradeable asset (11) rises (falls) for banks with liquidity excesses, which in turn widens (narrows) the market.

Finally, if demand is high(low) the fall of market price will be less (more) pronounced (see item (e) in definition 1), which inhibits (exacerbates) the perturbation of the market emphasized in the model. In summary, the negative effect of W on the extent of contagion can be understood in the light of its negative effect on the size of the market for tradeable assets, and therefore, on market price.

Graphs 2 and 3 show the effect of s_d (random share of deposits) and b (reserve requirement) on the resilience of the system. According to Graph 2, a more random pattern of individual deposits (in terms of a greater s_d) has not a clear effect on resilience. Except in the case with s_d = 0.1, no value for s_d has a similar effect. For values of s_d ranging from 0.3 to 0.9, a number of banks ranging from 2.3 and 3.1 survives after a financial crisis that leaves half of the financial system in bankruptcy. If the dispersion of simulations is included, these results are not «statistically» different. The extent of contagion via market perturbation does not depend therefore clearly on the volatility of deposits.

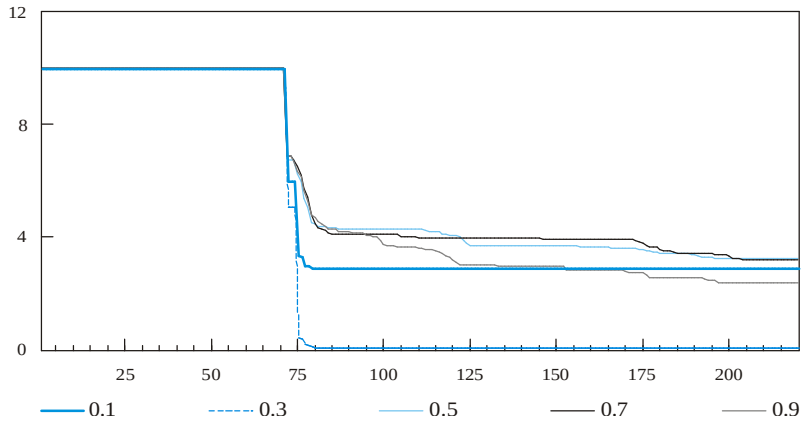
The reserve requirement has a negative effect on financial resilience in much as the same way as the aggregate demand for credit. As far as this model assumes that marginal reserve requirement equals total reserve requirement, a higher b makes liquidity shortages more likely according to (1).

B. RANDOMLY HETEROGENEOUS BANKS

To induce heterogeneity of banks, each one of the 10 starting banks is endowed with an equal stock of deposits, investment and loans (in t = -1 or t = -2), randomly chosen in the interval [0,1000]. To enhance robustness in face of an additional source of randomness, these simulations were performed 5,000 times.

Graph 2

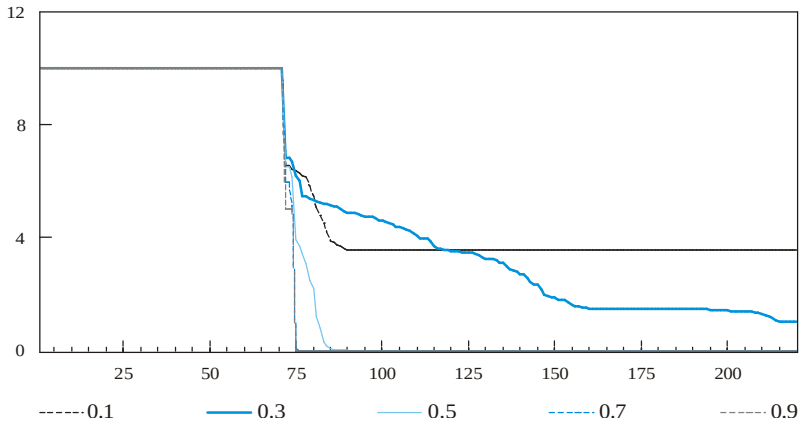
Surviving Banks and Deposit Randomness (Homogeneous Banks)



Source: authors' calculations.

Graph 3

Surviving Banks and Reserve Requirements (Homogeneous Banks)



Source: authors' calculations.

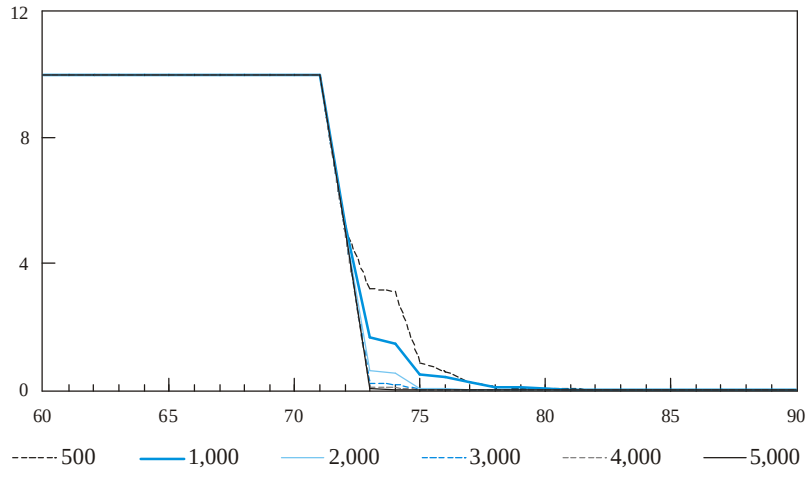
In the case of heterogeneous banks, the dispersion of results is smaller.[24] The negative effect of *W* on the resilience of the system remains (see Graph 4), although in all simulations all banks went bankrupt in a range of 4 to 16 iterations.

Moreover, as seen in Graph 5, the effect of the volatility of deposits remains unclear, although in this case the abnormal value of s_d is 0.9 instead of 0.1.

Similar results are obtained with the reserve requirement (Graph 6): rather than having a clear one-directional effect on contagion, the case of heterogeneous

Graph 4

Surviving Banks and Aggregate Demand for Credit
(Heterogeneous Banks)



Source: authors' calculations.

24 The set of parameters employed in these simulations were: $a = 0.1$, $b = 0.2$ (when not variable), s_d (when not variable) = $s_o = g = p = 0.5$, $r_l = 0.1$, $r_a = 0.05$, $w = 2000$ (when not variable), $l = 0.01$.

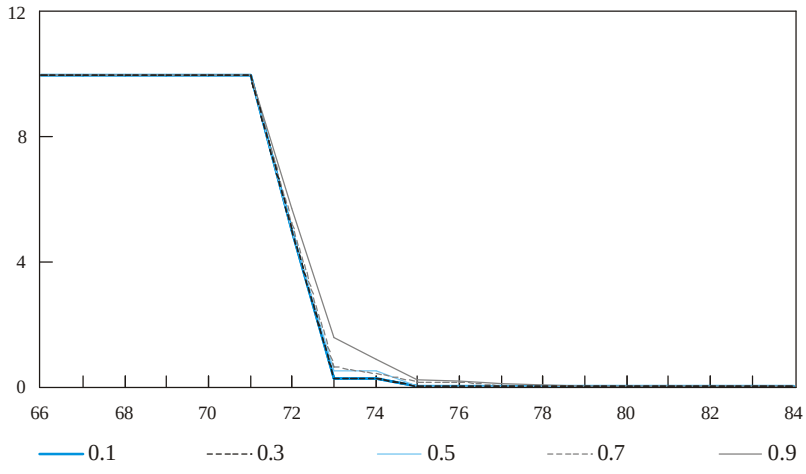
banks do not exhibit a clear effect of reserve requirement on resilience. Differences are not, moreover, «statistically» significant.

C. *HETEROGENEOUS BANKS: A SIMULATION OF COLOMBIAN FINANCIAL SYSTEM*

On November 2005, 16 banks took place in the Colombian banking system. This is the only set of simulations with a different number of starting banks. To mirror the structure of this system, the 1,000 units of the homogeneous case were distributed among the 16 banks according to the participation that each bank had on November 2005 on the total stock of the respective balance sheet item. For example, if Bancolombia (the biggest bank in the system) had 25% of deposits on November 2005, the simulations give to Bancolombia 250 units as starting deposits.

Graph 5

Surviving Banks and Deposit Randomness
(Heterogeneous Banks)



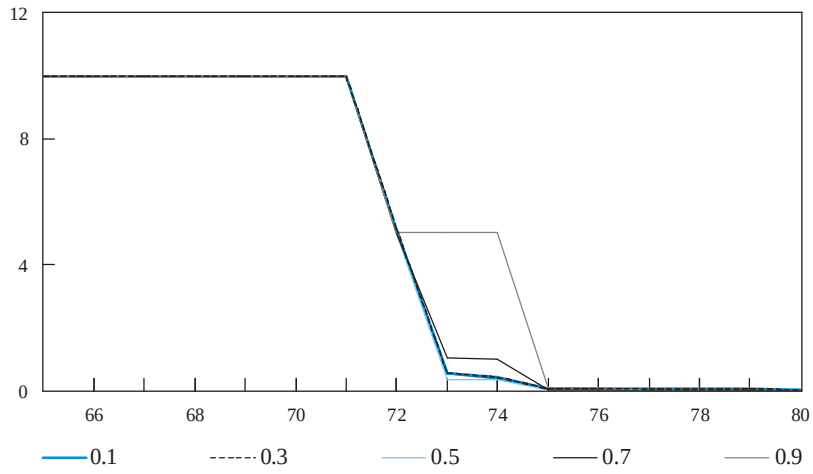
Source: authors' calculations.

It is important to note that, in this case, the distribution of banks with loans in t = -1 and t = -2 alters not only the results of the simulations but the concept of the incidence of liquidity risk in the financial system. This occurs because the distribution imposes a misleading conjecture on the real exposure to liquidity risk of Colombian banks. This obliges to present the results under two extreme distributions of banks: first, assuming that the 8 biggest banks extend loans in t = -1; second, assuming that the 8 smallest banks extend loans in t = -1.[25]

Graph 7 shows the effect of the aggregate demand for credit (W) on the extent of contagion under the first distribution of banks.[26] Yet again, on average, the demand

Graph 6

Surviving Banks and Reserve Requirements
(Heterogeneous Banks)



Source: authors' calculations.

25 Banks are ordered according to its share of total stock of loans.

26 The set of parameters employed in the simulation of the colombian financial system were: $a = 0.1$, $b = 0.06$ (not variable in this case), $s_d = 0.9$ (estimated, not variable), $s_o = g = 0.5$, $p = 1$ (estimated), $r_l = 0.152$ (calculated), $r_a = 0.00132$ (calculated), $l = 0.01$.

for credit exerts a negative impact on the resilience of the simulated Colombian financial system. In this case, if demand for credit equals a half of the observed aggregate stock of loans (W = 500), an average of 15.94 banks survive the 150 iterations. This number reduces slightly to 15.90 if aggregate demand rises to W = 1,000. All banks fall bankrupt only when the demand of credit is more than ten times the observed stock of loans.

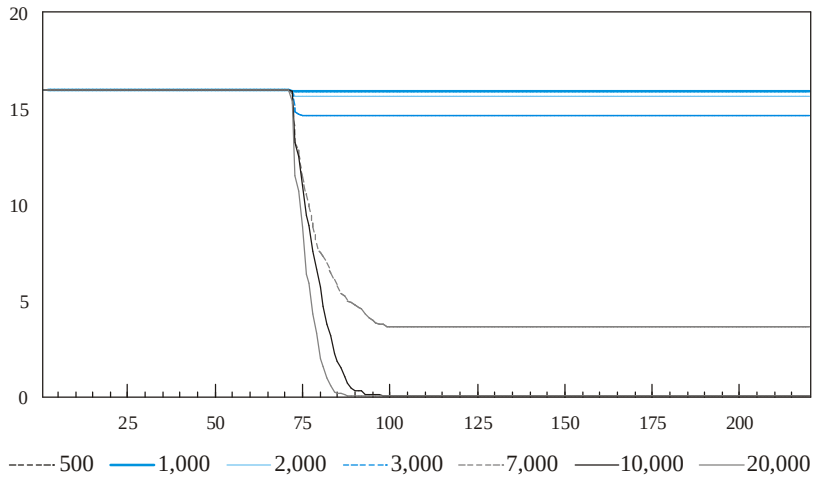
Graph 8 demonstrates the robustness of this result in face of a different distribution of starting banks. Results are qualitatively similar under the second distribution of banks.

V. CONCLUDING COMMENTS

The materialization of individual liquidity risk into systemic risk and, eventually, into a wide-system financial crisis, is a widely studied issue. It has been associated,

Graph 7

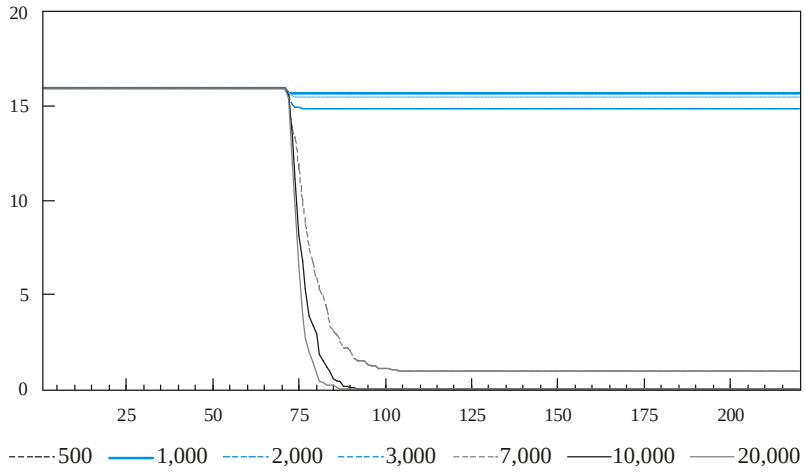
Surviving Banks and Aggregate Demand for Credit in Colombia



Source: authors' calculations.

Graph 8

Surviving Banks and Aggregate Demand for Credit in Colombia



Source: authors' calculations.

traditionally, to the presence of bank runs and/or financial interlinkages in banking systems. The purpose of this paper was to show that liquidity risk may also end up in systemic risk when troubled banks disturb financial markets. In this sense, individual liquidity risk turns into market risk, before hitting other banks in a contagion fashion.

This paper showed by means of the simulation of a microeconomic model not only that the aforementioned mechanism works, but that it depends crucially on the depth of the financial markets in which banks concur. By this way, this paper contributes to the understanding of the mechanics of liquidity risk, and does so by relying on some of the most important insights of current literature. Indeed, the mechanisms emphasized in this paper have been relatively left aside in recent works, and when taken into account, severely restricted. This paper lifted several of those restrictions.

In order to complement these contributions, some additional reflections are noteworthy.

The practical usefulness of these results may be questioned on the grounds that they arise from a very limited theoretical specification.[27]

Despite the controlled environment of the microeconomic model, it is possible to state that these results leave some practical lessons: firstly, the very simple fact of liquidity risk turning into market risk. In the case of Colombia, for example, recent statements from the economic authorities point to the growing relevance of market risk for the stability of the financial system.[28] Market risk is actually considered the most important risk facing the financial system, and perhaps under a situation of liquidity stress, fears about market risk may exacerbate.

Moreover, the results point to several variables that on a pure environmentneed to be monitored closely in order to avoid the exacerbation of market risk arising from liquidity risk. Among them, the results suggest the need to monitor the depth of the markets in which banks concur.

Also in practical grounds, fears can arise that the mechanisms outlined above are a by-product of the increasing complexity of current financial systems and instruments. The work by Schnabel and Shin (2004) (about the European financial crisis of 1763) is there to remember that it is not necessary a huge and complex system of institutions and instruments to see how liquidity risks transforms into market risk. This is a caveat in favor of the simplicity of the above framework.

Finally, it is worth to mention a subtle practical lesson emerging from the mechanics of the model. According to Plantin, Sapra and Shin (2005), fair accounting in terms of marking the bank book to market (despite its transparency benefits) poses a threat to financial stability by accentuating financial cycles. In the context of the presented model, mark-to-market rules place the same restrictions on financial stability.

---

27 The story told in this paper makes no sense, for example, if the model includes a central bank ready to inject any needed amount of liquidity at any point in time.

28 See, for example the *Financial Stability Reports* of the Banco de la República de Colombia. Around a third of the assets of the colombian banking system is placed in tradeable assets, particularly government debt securities.

REFERENCES

Allen, F.; Gale, D. (2000) "Financial Contagion", in *Journal of Political Economy,* v. 108, num. 1, Feb., pp. 1-33.

_____; _____ (2003) "Financial Fragility, Liquidity and Asset Prices", in *Wharton Financial Institutions Center,* num. 01-37-B.

Boss, M.; Elsinger, H.; "Summer, M.; Thurner, S. (2004) "An Empirical Analysis of the Network Structure of the Austrian Interbank Market", *Oesterreichische Nationalbank Financial Stability Report*, num.7, pp. 77-87.

Castiglionesi, F. (1994) "Financial Contagion and the Role of the Central Bank", *Universitat Autónoma de Barcelona.*

Cifuentes, R., Ferrucci, G., Shin, H. (2005) "Liquidity Risk and Contagion", *Bank of England, Working Paper Series,* num. 264.

De Bandt, O.; Hartmann, P. (2000) "Systemic Risk: a Survey", *European Central Bank Working, Paper Series,* num. 35, November.

Diamond, D.; Dybvig, P. (1983) "Bank Runs, Deposit Insurance, and Liquidity", *Journal of Political Economy,* 91 num. 3, June, pp. 401-419.

_____; R. Rajan (2003) "Liquidity Shortages and Banking Crises", *NBER, Working Paper Series,* num. 10071, October.

Dwyer, G.; Hasan, I. (1994) "Bank Runs in the Free Banking Period", *Journal of Money, Credit and Banking,* v. 26, num. 2, May, pp. 271-288.

Estrada, D. (2001) "Reserve Requirements, Systemic Risk and Interbank Market",Thesis, Universitat Autónoma de Barcelona, International Doctoral Program in Economic Analysis.

Freixas, X.; Parigi, B.; Rochet, J. (2000) "Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank", *Journal of Money, Credit and Banking* 32, num. 3, Part 2, August, pp. 611-638.

_____; J. Rochet (1997) *Microeconomics of Banking*. MIT Press, Cambridge.

Furfine, C. (1999) Interbank exposures: Quantifying the Risk of Contagion. *BIS, Working Papers*, num. 70, June.

Goodhart, C.; P.; Tsomocos, D.; Sunirand (2005) "A Model to Analyse Financial Fragility", *Economic Theory*, num. 70.

_____; _____; _____ (2004a) "A Model to Analyse Financial Fragility: Applications", *Journal of Financial Stability*, v. 1, num. 1, September, pp. 1-30.

_____; _____; _____ (2004b) "A Time Series Analysis of Financial Fragility in the UK Banking System, Oxford Financial Research Center, OFRC working Papers Series, num. 2004fe18.

Gorton, G. (1988) "Banking Panics and Business Cycles", *Oxford Economic Papers*, num. 40, pp.751-781

Iori, G.; Jafarey, S. (2000) "Interbank Lending, Reserve Requirements and Systemic Risk", *University of Wales at Swansea Discussion Paper Series*, num. 00-08.

_____; _____; Padilla, F. (2003) "Interbank Lending and Systemic Risk", *Computing Economic and Finance*, num. 63.

Kindleberger, C. (1978) *Manias, Panics and Crashes: a History of Financial Crises*, Basic Books, New York.

Large, A. (2005a) "A Framework for Financial Stability", *Bank of England Financial Stability Review*, num. 18, June, pp. 135-141.

_____ (2005b) Financial Stability: Managing Liquidity Risk in a Global System", *Bank of England Financial Stability Review*, num. 19, December, pp. 78-84.

Muranaga, J.; Ohsawa, M. (1997) "Measurement of Liquidity Risk in the Context of Market Risk Calculation", Bank of Japan, Institute for Monetary and Economic Studies.

Plantin, G., Sapra, H. Shin, H. (2005). "Marking to Market, Liquidity and Financial Stability", *Monetary and Economic Studies*, October, pp.133-155.

Rochet, J.; Tirole, J. (1996). "Interbank Lending and Systemic Risk", *Journal of Money, Credit and Banking*, v. 28, num. 4, Part 2, November, pp. 733-762.

Salop, S. (1979). "Monopolistic Competition with outside Goods", *Bell Journal of Economics*, v. 10, num. 1, pp 141-156.

Schnabel, I.; Shin, H. (2004). *Liquidity and Contagion: the Crisis of 1763*.

Tsomocos, D. (2003). Equilibrium Analysis, Banking and Financial Stability", *Journal of Mathematical Economics*, num. 30, pp. 619-655.