MICROECONOMIC APPROACHES IN THE CHRISTALLER'S CENTRAL PLACES THEORY

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Introduction

- Along past century, it has been clear an increase in the growth rate of the cities and a higher population concentration in urban areas. According to The Economist, with the trade liberalization, companies are looking for being located in cities more than countries. So, the cities power has risen compared to the governments.
- To explain this type of phenomenons, the spatial economy has made significant developments, as the one made by the Christaller's CPT.
- Although the different studies about the Christaller's CPT, there is no consensus about the best theory or more suitable micro-fundamentals that can pick up the most important aspects of this theory. One famous work is the one made by Eaton and Lipsey (1982) and maybe the most outstanding is the one made by Fujita, Krugman and Mori (1999).
- This document will use some tools borrowed from the game theory and from the industrial organization, to model some aspects of the Christaller's CPT. More than determine the model that capture the most important aspects of the theory, the goal is to present an alternative aproximation.

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Theories in the Spatial Economics(I)

- The importance of the space in the economy came from the beginnings of the economic theory. Cantillon and even Smith, considered the relationship between labor and space, as well as the ramifications in productive subjects and relationship between urban and rural areas.
- Von Thünen (1783-1850) (considered "the father of localization theories ") was the precursor of this theories. He found that of according to the distance and cost of transport, activities find their best location, forming areas economic or concentric rings around a city with homogeneous space; around this is created different yield in terms of the distance and transportation costs. That generate differential prices from the center to the periphery.
- Subsequently, Weber made a similar analysis for industries saying that they decide the location taking into account minimize their transport costs, given the proximity to market and production factors. Weber added the concept of agglomeration that considers the advantages of transportation costs, access to skilled labor, access to roads and resources, giving great importance to technological innovations.

Theories in the Spatial Economics(II)

- Hotelling (1929) uses Cournot duopoly models for competition between firms to deal with transport costs, so they compete in prices to be located along a straight line and get the largest portion of consumers.
- Then Christaller (1933) with his work " Die Zentralen Orte in Süddeutschland " and Losch (1940) would become the precursors of the central place theory. They said that exists some regularity in production and market areas and their location depends on transportation costs and production of each central place. Similarly, this theory incorporates the idea of threshold and range creating a hexagonal framework of central places as well as a system of hierarchical central places.
- According to Fujita y Thisse, the economic agglomeration has to deal with regional specialization and trade and additionally it has to consider:
- **Heterogeneity of the space** (it came from the Neoclassic Theory). Models of comparative advantages, under which the heterogeneity introduces an uneven distribution of resources.

Theories in the Spatial Economics(III)

- **Externalities in production and consumption** (From the urban economics). Models of externalities in terms of agglomeration forces due to the spillovers of knowledge, communications and social interactions.
- **Imperfect Competition** (From the new economic geography). Firms take their decisions on prices base on the spatial distribution of consumers and firms.
- Other models like **Center-Periphery**, such as exogenous development models and autochthon development. (Poles of development, Perroux (1964), accumulative causation, Boudeville (1966), with channels of diffusion , Berry (1972) and the stages of creation and diffusion, Friedmann (1972))
- **Nueva Economic Geography.** Krugman, among others, argues the existence of centripetal forces (or sizes of the market) as causes of geographical agglomeration. Exist too some centrifugal forces lead by competition among firms, the lower wages on the periphery that disperse the agglomeration. In this process, *the forward linkage effect* and *the backward linkage effect* are important.[2]

[2] Forward linkage effect: the more producers concentrated the more consumers are attracted, increasing the size of the market. Backward linkage effect: the size of the market attracts more producers.

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Approximations of the Christaller's theory

- Eaton and Lipsey (1982) propose an economic model of central places. For them, their model is primitive and should be consider as first attempt to model the CPT.
- Weyl(1935), Beckmann-McPherson (1970) and Bogataj y Bogataj (2001), Michael Sonis (2005) made attempts of modeling from the viewpoint of mathematics and geometry.
- Garrocho(2003) is another one who from a microeconomic approach put together the consumer theory with the Classic Geography (CPT) with its ideas of centrality, threshold and range. Finally, he included the Land Auction Theory in his model (David Ricardo and von Thunen).
- Fujita, Krugman and Mori (1999) develop a general equilibrium model with a dynamic adjustment in the labor market, under an environment of monopolistic competition. The most interesting is that they are able to create the hierarchical framework using the dynamic adjustment and the growth of population.

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Christaller´s vision. Basic Assumptions of the CPT

- An isotropic surface.
- Population evenly distributed.
- A similar power of purchase among all consumers, which are rationals and they head to the nearest market.
- Equal costs of transport and proportional to the distance from the consumer to the market.
- No economic benefits (Perfect Competition). **This assumption is relaxed in the document.**
- Centrality. Christaller mentioned the central place as a centralizer of goods and services more than its influence in location terms.

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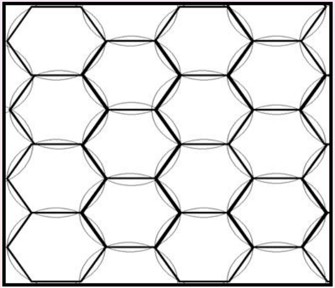
The idea of Threshold and Range

- Threshold of demand. It is defined as the distance or minimum demand a central place needs to supply the consumers and remain in the market, until reaching a point of balance between income and expenditure.
- Range. It is the maximum distance as a consumer you are willing to go in order to buy a good or service.
- Both concepts interact with prices, costs of production and distances.
- In a situation with threshold larger than the range, some consumers are not able to get the goods, so the emergence of new central places is required for the market to reach an equilibrium.

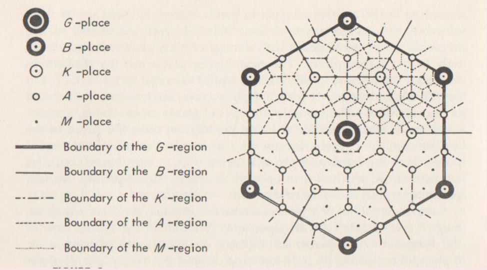
From Central Places to the hexagonal framework

- With the entrance of new central places, the most efficient way to share the market area is the framework of circles, which lead to a hexagonal framework as appear in the next figure:



Hexagonal framework of Christaller

- The Christaller´s idea is clearly seen in the next figure:



- Therefore, the Christaller´s CPT rise to a configuration of hierarchies for different level of central places.

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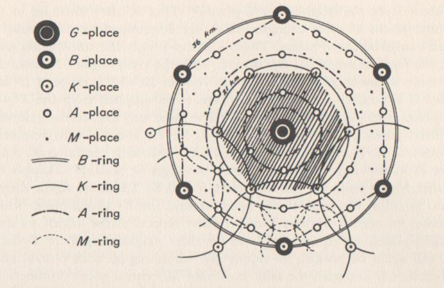
The Market Principle(system $k = 3$)

Although exists the Traffic Principle (that consider basically routes or traffic connections) and Administrative Principle (taking into account the territorial limits), the analysis is more related with the Market (connections between central places of different hierarchical level).



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Assumptions

- Oligopolistic competition that leads to an equilibrium with equality on prices.
- The refinement of hexagonal configuration is given.
- The location of the firms is given.
- The problem faced competition between firms in the same hierarchical level (symmetry)
- The analysis focuses on a representative central location and then generalizes. Market Principle
- The benchmark is the model of competition in terms of location of Gabszewicz and Thisse (1992), assuming the location as given (at the end this assumption is mildly relaxed).

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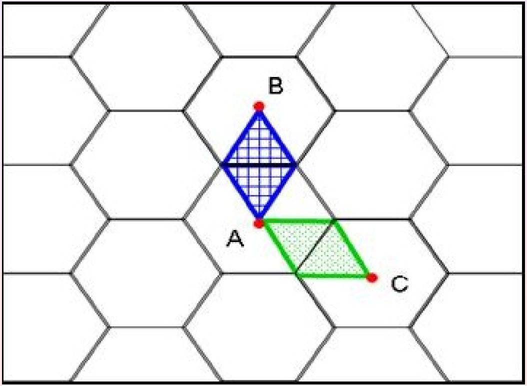
Proposed Model

- Isotropic space
- Firms compete for space around them.
- Individuals head to the nearest central place.
- Continuum of consumers.
- Exists central place A and B with homogenous product and production costs of 0, installed at distances a and b such that satisfy:
  $a \geq 0, b \geq 0, a + b \geq 1$
- It is assumed that:
  $a = 0$ and $1 - b = 1$
- Consumers are distributed along a rhomboidal area (see figure below) according to a density function f uniformly positive and continuously differentiable. A and B compete for that area.

Hexagonal Framework of Christaller and Proposal of modelation



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Model

- Each consumer buys the good to the firm that offers the lowest full price (price + transportation cost) and transportation costs are assumed by the consumer to move to A or B.
- Exist a transportation cost function $C(x)$ continuous, increasing and convex in x, with $C(0) = 0$.
- P_1 and P_2 are the full prices of A and B and $m(P_1, P_2)$, margin consumer with $y \epsilon [0,1]$; then, it is possible to define 3 situations:
  1. $P_1 + c(|y|)\frac{Dd}{2} = P_2 + c(|1-y|)\frac{Dd}{2}$, where $m(P_1, P_2)$ if exists is unique. If it does not exist, then,
  2. $P_1 + c(|y|)\frac{Dd}{2} < P_2 + c(|1-y|)\frac{Dd}{2}$, for all $y \epsilon [0,1]$
  3. $P_1 + c(|y|)\frac{Dd}{2} > P_2 + c(|1-y|)\frac{Dd}{2}$, for all $y \epsilon [0,1]$

Model

- $\frac{Dd}{2}$ corresponds to the rhomboidal area in competition by A and B.
- In case 1, A obtaines consumers of the area $[0, m(P_1, P_2)\frac{Dd}{2}]$ and B the ones from $[m(P_1, P_2)\frac{Dd}{2}, 0]$, so that would be assuming that $\frac{Dd}{2} = 1$.
- In case 2, A supplies all the area $\frac{Dd}{2}$ to prices (P_1, P_2), since the full price (production cost + cost of transportation) provides by A is lower than that offered by B it will capture all the market. The opposite happens in case 3.

Model

The strategies for the firms are $P_1 \epsilon [0, \infty]$ and $P_2 \epsilon [0, \infty]$ and the payment function for A is:

$$\pi_1(P_1, P_2) = \begin{cases} P_1 \int_0^{m(P_1,P_2)} f(z)\,dz & \text{if} \quad m(P_1,P_2) \text{ exists} \\ P_1 & \text{if for all } y \in [0,1] \quad P_1 + c(|y|)\frac{Dd}{2} < P_2 + c(|1-y|)\frac{Dd}{2} \\ 0 & \text{if for all } y \in [0,1] \quad P_1 + c(|y|)\frac{Dd}{2} > P_2 + c(|1-y|)\frac{Dd}{2} \end{cases}$$

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Nash-Cournot Equilibrium

- The Non Cooperative Nash-Cournot Equilibrium in prices under pure strategies would be a pair of prices $({P_1}^*, {P_2}^*)$ such that $\pi_i({P_i}^*, {P_j}^*) \geq \pi_i(P_i, {P_j}^*)$ for all $P_{\geq}0$ with $i = 1, 2$ and $i \neq j$. Namely, the pair of points $({P_1}^*, {P_2}^*)$, such that ${P_1}^*$ is the best response, given that the best response of the firm B is ${P_2}^*$ and vice versa.
- As in Hotelling(1929), it is assumed that $C(x) = cx$ with $c > 0$, where c is the transportation rate. If $m(P_1, P_2)$ exists, it holds that $a = 0 \leq m(P_1, P_2) \leq 1 = 1 - b$ and $m(P_1, P_2)$ is obtained by solving for y in $P_1 + c(|y|)\frac{Dd}{2} = P_2 + c(|1-y|)\frac{Dd}{2}$:

$$y = m(P_1, P_2) = \frac{P_2 - P_1}{cDd} + \frac{1}{2} \qquad (1)$$

Nash-Cournot Equilibrium

- In case 1, the payment function for the central place A is:

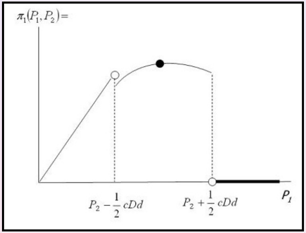
$$\pi_1 = \left(\frac{P_2 - P_1}{cDd} + \frac{1}{2}\right)P_1 \qquad \textit{with} \qquad P_1 + c(y)\frac{Dd}{2} = P_2 + c(1-y)\frac{Dd}{2} \tag{2}$$

- The payment function for B is obtained similarly. Solving this for A and B, it is found that:

$$P_1 = P_2 = c\left[\frac{Dd}{2}\right] \qquad \textit{and} \qquad \pi_1 = \pi_2 = c\left[\frac{Dd}{4}\right] \tag{3}$$

- Therefore, we get equality in prices and profits in the equilibrium, where the firms share the rhomboidal area in halves.

Payment function for A



The other two situations encourage the competition on prices, trying to capture all the market. If *A* offers a lower price than *B*, takes all the market and *B* nothing. But this leads that *B* offers a lower price. The opposite also happens, and so on. In general, it could be said that the equilibrium in this model is Cournot-Nash due to the firms are looking for the bigger market areas, but it is Bertrand in terms of the price competition.

Outline

Consistency and Existence of equilibrium

- To check the consistency of the results, the model was revised by changing the function of transportation cost like $C(x) = cx^2$, with $c > 0$, obtaining the same results and the same equilibrium as before.
- Using the proposition given by Gabszewicz and Thisse (1992) in their model and made by d'Aspremont et al (1979), (necessary and sufficient conditions for the existence of equilibrium in pure strategies):
  1. ***Proposición 1***: For $a + b = 1$, the unique equilibrium price is given by $P_1{}^* = P_2{}^* = 0$. For $a + b < 1$, there is an equilibrium if and only if:

$$(1 + \frac{a-b}{3})^2 \geq \frac{4}{3}(a + 2b) \tag{4}$$

$$(1 + \frac{b-a}{3})^2 \geq \frac{4}{3}(b + 2a) \tag{5}$$

- Whenever this is accomplished, the equilibrium in prices is unique.

Consistency and Existence of equilibrium

- In our case it was assumed that $a = 0$ and $1 - b = 1$, so it holds that $a + b < 1$ and the result is:

$$(1)^2 \geq 0, \quad and \quad that \quad (1)^2 \geq 0, \quad which \ always \ is \ fulfilled. \quad (6)$$

- To Gabszewicz and Thisse (1992), there is an equilibrium when firms are located far enough apart and in our case that $a = b$ (symmetric), the above inequalities make firms are located outside the first and third quartile. item If this result is overlapped to the competition between A and C, we found that they gather the rhomboidal area in halves as before. The union of those triangular areas is consistent with the hexagonal framework of Christaller.

Outline

Relaxing the equality in cost of production

- Basically, the payments functions were modified including different costs of production:

$$\pi_1(P_1,P_2)=\begin{cases}(P_1-C_1)\int_0^{m(P_1,P_2)} f(z)\,dz & \text{if} \quad m(P_1,P_2) \text{ exists.}\\ P_1-C_1 & \text{if for all } y\in[0,1] \quad P_1+c(|y|)\frac{Dd}{2}<P_2+c(|1-y|)\frac{Dd}{2}\\ 0 & \text{if for all } y\in[0,1] \quad P_1+c(|y|)\frac{Dd}{2}>P_2+c(|1-y|)\frac{Dd}{2}\end{cases}$$

- The payment function for *B* is obtained in a similar way:

$$\pi_2(P_1,P_2)=\begin{cases}(P_2-C_2)\int_{m(P_1,P_2)}^{1} f(z)\,dz & \text{if} \quad m(P_1,P_2) \text{ exists}\\ P_2-C_2 & \text{if for all } y\in[0,1] \quad P_1+c(|y|)\frac{Dd}{2}<P_2+c(|1-y|)\frac{Dd}{2}\\ 0 & \text{if for all } y\in[0,1] \quad P_1+c(|y|)\frac{Dd}{2}>P_2+c(|1-y|)\frac{Dd}{2}\end{cases}$$

Relaxing the equality in costs of production (II)

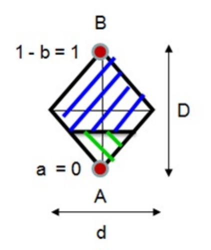
- When $m(P_1, P_2)$ exist the result will depend on the size of C_1 and C_2. When $C_1 = C_2$ the result is similar as before. If $C_1 \neq C_2$, particularly, $C_1 = C_2 + \varepsilon$ with ε as a number very small, so the central place B has a marginal cost lower than A. In this case, the prices and payment functions are:

$$P_1 = \frac{cDd}{2} + \frac{3C_2 + 2\varepsilon}{3} \qquad \text{and} \qquad P_2 = \frac{cDd}{2} + \frac{3C_2 + \varepsilon}{3} \tag{7}$$

$$\pi_1 = \left(\frac{3cDd - 2\varepsilon}{6}\right)^2\left(\frac{1}{cDd}\right) \qquad \text{and} \qquad \pi_2 = \left(\frac{3cDd + 2\varepsilon}{6}\right)^2\left(\frac{1}{cDd}\right) \tag{8}$$

- Under this difference in costs of production, in equilibrium, the central place B offers a price lower than A ($P_1 > P_2$). The central place B has a margin to do that, so it will receive a higher benefit than A ($\pi_1 < \pi_2$). The higher the difference in costs of production, the higher the difference in prices and benefits.

Relaxing the equality in costs of production (III)



- The last figure shows that central place B captured a higher portion of the rhomboidal area (blue area). This result makes us think that differences in costs of production break the hexagonal configuration of Christaller. The result when $m(P_1, P_2)$ does not exist is the same as in the benchmark model, but now, the benefits for A and B are modified according to the difference in costs of production.

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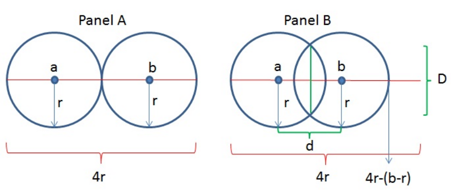
Alternative Modelation

- In this modelation, the location of the central place A is given and the best option for the central place B is a location according to the Christaller model. Firstly, A is located in a point a on the isotropic space and exist a ratio r determined by the threshold and range of the central place (even more it will be assumed that $a = r$). A central place B with the same ratio of A is trying to get into the market and is trying to choose a location b along a line $4r$ determined by the initial location of A. It is assumed $b \geq r$ to guarantee that all the circumference of B will be inside of the line $4r$. The maximum distance in competition between A and B is $4r - (b - r)$.



Alternative Modelation II

- Then, the payment function for A is:

$$\pi_1(P_1,P_2,b)=\begin{cases} P_1\int_0^{m(P_1,P_2)} f(z)\,dz & \text{if} \quad m(P_1,P_2) \text{ exists.} \\ P_1\int_0^{4r-(b-r)} f(z)\,dz & \text{if for all } y\in[r,4r-(b-r)] \\ & P_1+c(|T_A(y,b,r)|)<P_2+c(|T_B(y,b,r)|) \\ 0 & \text{if for all } y\in[r,4r-(b-r)] \\ & P_1+c(|T_A(y,b,r)|)<P_2+c(|T_B(y,b,r)|) \end{cases}$$

- Where $T_A(y,b,r)$ and $T_B(y,b,r)$ correspond to functions that measure the area between location a and $m(P_1,P_2)$, and between $m(P_1,P_2)$ and b, respectively.

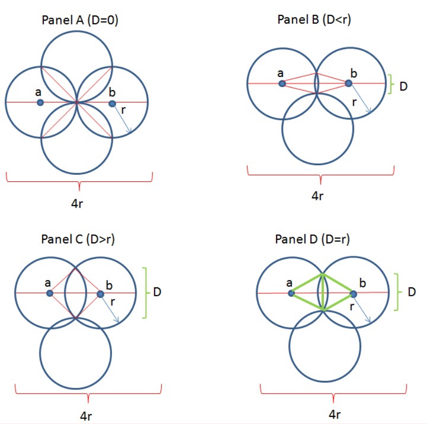
Alternative Modelation III

- The payment function for B is similar:

$$\pi_2(P_1,P_2,b)=\begin{cases} P_2\displaystyle\int_{m(P_1,P_2)}^{4r-(b-r)} f(z)\,dz & \text{if} \quad m(P_1,P_2) \text{ exists} \\ P_1\displaystyle\int_{0}^{4r-(b-r)} f(z)\,dz & \text{if for all } y\in[r,4r-(b-r)] \\ & P_1+c(|T_A(y,b,r)|)<P_2+c(|T_B(y,b,r)|) \\ 0 & \text{if for all } y\in[r,4r-(b-r)] \\ & P_1+c(|T_A(y,b,r)|)<P_2+c(|T_B(y,b,r)|) \end{cases}$$

- D is the vertical distance as appeared in the figure before, then, $0 \leq D \leq 2r$.
- When $D = 2r$ B is located in the same position like A so, each firm obtain $\frac{\Pi r^2}{2}$.

Alternative Modelation IV



Alternative Modelation V

- If $D = 0$ (see next figure, Panel A), $b = r$ and the solution is a quadratic geometric configuration, as soon as new firms enter to the market(this solution is discarded because the hexagonal configuration permits to get a bigger area)
- When $D < r$ (see Panel B) the areas are not symmetrical and are smaller than the ones found with the hexagonal form due to the entrance of new firms in the future.
- If $D > r$ (see Panel C) the analysis is quite similar as before, and this is not a solution.
- Finally, when $D = r$ (Panel D), the areas are bigger than in the other cases. The two triangles in green correspond to the areas in competition between A and B (each edge is equal to r). To determine the distance between the two central places d, we could break each triangle as $90 - 60 - 30$ triangles, with the opposite edge to each angle being $2x - x\sqrt{3} - x$ (in this case $r - \frac{r\sqrt{3}}{2} - \frac{r}{2}$). Then, $d = r\sqrt{3} = 4r - r - b$, namely, $b = r(3 - \sqrt{3})$

Alternative Modelation VI

- According to that, the restriction for the consumer $m(P_1, P_2)$ will be:

$$P_1 + c(\frac{\Pi r^2}{3} + \frac{r(y-r)}{2}) = P_2 + c(\frac{\Pi r^2}{3} + \frac{r(4r-b-y)}{2}) \quad (9)$$

- Solving this for y and replacing in the payment functions for A and B, will give us as solution a symmetric equilibrium again:

$$P_1 = P_2 = \frac{(5r-b)cr}{2} \quad and \quad \pi_1 = \pi_2 = \frac{cr(5r-b)^2}{4} \quad (10)$$

- The central places compete one by one against each other for the area they have in common(rhomboidal area). The same happens between A and C gathering the rhomboidal area on equal parts, and so on. The union of this areas is consistent with the hexagonal area proclaim by Christaller.

Conclusions

- The modeling gives overtones of how oligopolistic competition between firms is within the hexagonal structure of Christaller. Firms compete in prices taking into account transportation costs and such competition leads to deal out the rhomboidal areas in halves.
- The balance is achieved in a decentralized manner without cooperation due to the interaction of firms.
- Firms compete one by one for the consumer areas closest to them and assigned these areas equally.
- As the location for the central places or firms is given, the distance of consumers to these determines transportation costs they incur.
- Competition in prices makes that the firms offer similar prices and the difference in terms of capturing all the market depends on the cost of transportation and the distance. As was showed, the hexagonal framework is sensible to the equality in costs of production.
- Other points to explore will be the interaction in terms of location and prices under new hierarchical levels. An additional point is the spatialization of the model including contiguities or neighborhoods as well as territorial borders.