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A Proposal on Macro-prudential Regulation

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Uma proposta em regulação macroprudencial

Carolina Osorio*

Este artigo avalia a decisão de implementar diferentes instrumentos de política regulatória para a prevenção e gestão de crises. Com tal objetivo, constrói um modelo monetário de equilíbrio geral em dois períodos, com bancos comerciais, colateral, titularização e descumprimento, a fim de explicar a crise financeira de 2007-2009 nos Estados Unidos. O equilíbrio do modelo está caracterizado por um fenômeno de contágio que começa com um incremento nas taxas de descumprimento do setor hipotecário que posteriormente se propaga ao resto dos sectores nominais da economia. Os resultados mostram que em períodos de crise, uma política monetária suave mitiga a crise do mercado imobiliário, mas somente produz uma melhora parcial na estabilidade financeira do sistema. As medidas de regulação são as ferramentas primárias para combater a instabilidade financeira; os requerimentos de capital reduzem a alavancagem no setor bancário e induzem os bancos a internalizar as perdas por conceito de descumprimento sem gerar custos para o contribuinte; os requerimentos de margem prevêem uma alavancagem excessiva nos mercados imobiliários e derivados e contêm, portanto, os efeitos adversos da crise imobiliária; os requerimentos de liquidez reduzem a exposição dos bancos a ativos de risco, estimulando a oferta de crédito em épocas de crise y amenizando a deflação no preço dos imóveis.

Classificação JEL: D5, E43, E44, E5, G01, G2.

Palavras chave: equilíbrio geral, titularização, colateral, descumprimento, política monetária, regulação.

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Una propuesta en regulación macroprudencial

Carolina Osorio*

Este artículo evalúa la decisión de implementar diferentes instrumentos de política regulatoria para la prevención y manejo de crisis. Con este fin, se construye un modelo monetario de equilibrio general en dos períodos, con bancos comerciales, colateral, titularización, e incumplimiento con el fin de explicar la crisis financiera de 2007-2009 en Estados Unidos. El equilibrio del modelo está caracterizado por un fenómeno de contagio que comienza con un incremento en las tasas de incumplimiento del sector hipotecario y luego se propaga al resto de los sectores nominales de la economía. Los resultados muestran que en tiempos de crisis, una política monetaria laxa mitiga la crisis del mercado de vivienda, pero produce solo una mejora parcial en la estabilidad financiera del sistema. Las medidas de regulación son las herramientas primarias para lograr combatir la inestabilidad financiera; los requerimientos de capital reducen el apalancamiento en el sector bancario e induce a los bancos a internalizar las pérdidas por concepto de incumplimiento sin generar costos para el contribuyente; los requerimientos de margen previenen un apalancamiento excesivo en los mercados de vivienda y derivados, por tanto conteniendo los efectos adversos de la crisis de vivienda; los requerimientos de liquidez reducen la exposición de los bancos a activos riesgosos, lo cual estimula la oferta de crédito en épocas de crisis y suaviza la deflación en el precio de la vivienda.

Clasificación JEL: D5, E43, E44, E5, G01, G2.

Palabras clave: equilibrio general, titularización, colateral, incumplimiento, política monetaria, regulación.

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A Proposal on Macro-prudential Regulation

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This paper assesses the choice of different regulatory policy instruments for crisis management and prevention. To this end a two-period, rational expectations, monetary general equilibrium model with commercial banks, collateral, securitization and default is contructed in order to explain the 2007-2009 U.S. financial crisis. The equilibrium outcome is characterized by a contagion phenomenon that commences with increased default in the mortgage sector, and then spreads to the rest of the nominal sector of the economy. The resuslts show that in times of financial distress accommodative monetary policy mitigates housing crises, but it achieves only a partial improvement on financial stability. Regulatory measures are the primary tools to achieve financial stability; capital requirements reduce leverage in the banking sector, and induce banks to internalize (default) losses without taking a toll on the taxpayer; margin requirements prevent excess leverage in the housing and derivatives markets, thus containing the adverse effects of the housing crisis; and, liquidity requirements reduce banks´ exposure to risky assets, thereby promoting lending in times of financial distress and stemming house price deflation.

JEL classification: D5, E43, E44, E5, G01, G2.

Keywords: general equilibrium, securitization, collateral, default, monetary policy, regulation.

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I. INTRODUCTION

Even though the relative importance of some of the identified causes of the recent crisis[1] is still being debated, there seems to be consensus that the inadequacy of the financial regulatory framework was essential to the development of the worldwide financial meltdown. Academics and policy makers seem to agree that the foundations of financial regulation need to be reviewed, and most importantly, that regulatory reform needs to move in the direction of (or be complemented by) macro-prudential rules.

The former regulatory system is based on a micro-prudential tenet, whereby the financial system is expected to be safe by properly supervising and auditing financial institutions in isolation (partial equilibrium approach). However, the 2007-2009 crisis proved that making each agent and product safe, so as to prevent their individual failure, does not safeguard against the collapse of the financial system as a whole.

This regulatory framework failed to prevent the recent boom-bust cycle in the financial system and its devastating effects on the real economy; it provided agents with incentives to leverage and erode credit standards during the booming stage (procyclical leverage and risk taking); it allowed excessive build-up of systemic risk on the balance sheets of unregulated intermediaries and off the balance sheets of regulated ones (regulatory arbitrage); and, it failed to mitigate the adverse effects when the boom turned to bust —and financial institutions engaged in fire asset sales— and reduced the amount of lending to the real economy.

1 Such as large global imbalances and extremely accommodative monetary policy in the developed world.

Macro-prudential regulation is meant to provide the means to improve upon the former regulatory framework by taking into account the general equilibrium effects brought about by the insolvency and illiquidity of a single individual; in other words, it is concerned with the interconnectedness among different agents, or systemic risk.

Agents' decision to default can be due to balance sheet insolvency (liabilities are greater than assets) or illiquidity (borrowers cannot meet their due payments in the short- term), therefore in a model of default agents should make choices about the structure of their portfolios, their level of indebtedness and whether they are willing, or able, to honour their obligations.

Furthermore, to assess the scope of *macro-prudential* regulation, a framework of interconnected heterogeneous agents is required. Representative agent models are inconsistent with the existence of default (idiosyncratic risk) and contagion (systemic risk), because either no one defaults ever, or everyone defaults simultaneously, at some point in time.

This paper proposes a group of necessary macro-prudential tools that address different types of inefficiencies. To this end, a monetary general equilibrium model with commercial banks, collateral, securitization, and endogenous default is built. Under the right choice of parameters, the framework yields an equilibrium outcome which is consistent with the features of the recent financial crisis: the economy is subject to a shock that initially increases default in the mortgage market, and then spreads to the rest of the nominal sector of the economy, as several other agents default. This hurts the financial system, creates a credit crunch, and reduces households' welfare.

Such a contagion phenomenon reveals the systemic risk inherent in the system. Each agent makes an optimal choice upon its level of leverage, risk taking and default (idiosyncratic risk), but due to the interconnectedness among individuals, the recoiling from risk, or excessive risk taking, from one agent materializes into greater risk by others (systemic risk).

The results show that just as higher capital requirements on banks protect the economy against 'structural' default, higher liquidity requirements reduce the overall risk of banks' portfolios and to extend more credit in times of financial distress and the application of higher margin requirements to all borrowers prevents against excess leverage.

Capital requirements mitigate the externality banks impose on the economy by taking (excessive) credit risk. Although it induces banks to internalize default loses, it also renders the economy financially unstable. Prior to a crisis, capital requirements provide banks with incentives to wardoff capital charges; hence, they reduce their assets by securitizing and selling them to unregulated (and, generally, highly leveraged) non-bank financial institutions (NBFIs).

During a crisis, in an effort to meet capital requirements, banks seek to protect their own positions by liquidating assets (fire sales), refusing loans to, or requiring higher margins or collateral from, other borrowers (credit crunch) in such a way that can also cause adverse externalities[2]. Such externalities provide a rationale for implementing liquidity and margin requirements, in addition to capital adequacy measures.

Liquidity requirements contain the contagious effects of small shocks by stemming the collapse of collateral and asset prices (fire sales); and, margin requirements protect against ex-ante excessive leverage and securitization (a means of implementing regulatory arbitrage). Both of these tools mitigate the real effects of a credit crunch on the economy.

The paper now proceeds as follows. Section 2 outlines a literature review. Section 3 presents the formal model. The optimization procedures, the equilibrium of the model, and its properties, are set out in the Appendix. A benchmark model is described in Section 4. To examine the effects of different regulatory frameworks, Section 5 extends the basic model to allow for the implementation of Capital Adequacy Ratios (CAR), Margin Requirement (MR), and Liquidity Ratios (LR) policies. Finally, Section 6 concludes.

II. LITERATURE REVIEW

Models which abstract from the essential role played by default imply a series of unrealistic assumptions. First, an economy without default requires complete markets and no uncertainty; but, in practice, lenders and investors do not know future state probabilities, which is why market participants cannot make a market for the unknown. Second, a representative agent model is inconsistent with the existence of

2 Hanson, Kashyap and Stein (2010) provide evidence of this. This phenomenon is referred to as the pro-cyclical property of capital requirements.

default. Either no one - the representative agent - defaults ever, or everyone defaults, simultaneously, at some point in time, and the model then comes to an immediate end. Moreover, representative agent models cannot be used to explore contagion between agents, most importantly between banks. Finally, in a model without default, there is no essential role for money, banking or finance. Thus, any agent could simply write a risk-free IOU note on herself which, under these circumstances (no risk premia), will be perfectly acceptable in payment. Also most kinds of financial intermediation would be unnecessary because everyone could borrow or lend at the same single risk-free interest rate.

Yet, most models in mainstream macroeconomic theory have been specified in a way that either abstracts from, or largely ignores, default and the financial frictions implied by its existence. This is due to the fact that default is very difficult to incorporate into a general model. Furthermore, in normal circumstances, such as the years prior to the 2007 crisis, default-free models, namely Dynamic Stochastic General Equilibrium (DSGE) models, gained a large comparative advantage on the grounds of analytical and forecasting power.

But, representative agent models, without financial frictions, yield equilibrium outcomes that are inconsistent with empirical realities in financial markets. This is why they have not been useful for understanding the origins and implications of financial crises. Any attempt to incorporate financial frictions into models that are otherwise essentially in a real business cycle format, as are most DSGEs, implicitly assumes the existence of default. Some examples include the financial accelerator model of Bernanke, Gertler and Gilchrist (1999), and the Kiyotaki-Moore (1997) framework; both introduce credit restrictions in the form of collateral constraints[3]. However, in these models there is no default, and thus no need for collateral to support borrowing. Similarly, incorporating (exogenous) risk-spread adjusted interest rates into expenditure functions and Taylor rules of an otherwise standard DSGE model, as in Curdia and Woodford (2009, 2010), must logically imply the existence of default.

This does not entail that the economics literature on default is non-existent. Several academics have developed models of bankruptcy which provide important insights into optimal debt contracts, optimal corporate debt-to-equity ratios, and the dynamics of credit cycles. Key seminal pieces include the papers by Townsend (1979), Sti-

3 In Bernanke et al.(1999) model firms' ability to borrow depends on the market value of their net worth, and in Kiyotaki-Moore (1997) framework land is used as both productive input and collateral.

glitz and Weiss (1981), Aghion and Hermalin (1990), Hart and Moore (1994), White (1998), and Wang and White (2000). However, some of these are partial equilibrium models which fail to capture the feedback effects that arise naturally in a general equilibrium framework, and are essential for the analysis of systemic risk. Furthermore, some of these models do not allow agents to be buyers of some assets and sellers of others, and have little heterogeneity among the economic agents involved.

General equilibrium financial models have become the workhorse models in the fields of macroeconomics and finance. What makes them so applicable is that they are characterized by two main features: heterogeneous agents and financial frictions.

The first paper to include bankruptcy in a general equilibrium model with complete markets appears to be that by Shubik (1972). In a more extensive model of general equilibrium with complete markets, the seminal papers by Kehoe and Levine (1993), Kocherlakota (1996), Zhang (1997), and Alvarez and Jermann (2000) build on the literature of dynamic consistency and introduce individual rationality constraints as endogenous debt limits. These constraints imply that, in equilibrium, agents do not default in any state. Although the models provide important insights in understanding consumption and risk sharing, by their very construction, they cannot explain the existence of default as an equilibrium phenomenon. In these papers, households want to default in the states where earnings are high, but the binding individual rationality constraint prevents them from doing so.

To overcome this limitation, some macroeconomists developed models where there is limited dependence of loan contracts on future shocks, which is closer to the literature of default with incomplete markets (discussed in detail in the next sub-section). The general equilibrium models typified by the work of Lehnert and Maki (2000), Athreya (2002, 2004), Li and Sarte (2002), Livshits, MacGee and Tertilt (2003), and Chatterjee, Corbae, Nakajima and Rios-Rull (2007) propose a setting where households optimize, and equilibrium conditions implied by competition, market clearing, and resource feasibility are imposed.

Lehnert and Maki (2000) propose a model where competitive risk-neutral financial intermediaries can pre-commit to long-term credit contracts, and, thus, charge the same interest rate on the loans extended to different borrowers. Consequently, small borrowers end up subsidizing large borrowers because default probabilities increase with loan size. However, this form of cross-subsidization is not sustainable with free-entry of intermediaries. The work by Athreya (2002) is innovative because equilibrium default emerges in the (heterogeneous) household sector subject to stochastic punishment spells, but the aforementioned cross-subsidization-free-entry inconsistency also arises.

In contrast, Livshits et al. (2003, 2004) and Mateos-Planas and Seccia (2004), and Chatterjee et al. (2007) recognize that creditors' willingness to lend is related to the observable characteristics of the borrower. Livshits et al. (2004) use a life-cycle model where various debtor characteristics are observed by the creditors. Mateos-Planas and Seccia (2004) assume lack of observability beyond population averages, and derive borrowing limits consistent with zero profits under the restriction of observability. Furthermore, they use an institutional structure, whereby banks issue securities backed by repayments on the unsecured loans they make, in order to finance lending.

Chatterjee et al. (2007) use an infinite-horizon setting where loan contracts specify the household's next-period obligation, independent of any future shock, but gives the household the option to default. The interest rate depends on the household's current total debt, credit rating, and demographic characteristics, and a zero profit condition is imposed on loans of varying size. This implies that in equilibrium there is a schedule of loan prices, rather than a single lending interest rate. Similarly, Chatterjee, Corbae and Ríos-Rull (2008) model intermediaries that use a household´s bankruptcy history to update their beliefs about a household's private information.

The problem with all these models is that they assume a representative bank which absorbs all the risks in the economy and is not allowed to default. Although the representative agent bank has many uses and advantages, it cannot be used to analyze the banking system because it cannot capture the relationships between agents (especially between banks) that lead to systemic risk externalities. Moreover, in a model with a single bank there is no interbank market, which poses several limitations: first, it cannot properly capture the transmission mechanism of shocks, especially in times of financial distress; second, the whole banking system (as represented by the one agent) fails, or the whole banking system survives, in the face of some assumed shock; third, in reality, banks have different levels of risk-aversion and capital, which is why failures tend to occur in financial institutions with riskier portfolios and evolve into a contagion phenomenon that may threaten the survival of other banks and deteriorate households' welfare.

The process of contagion has many channels: through interbank relationships, lending flows, and changes in asset flows and prices (e.g. collateral and fire sales). These phenomena are most likely impossible to study in a representative bank model.

A. THE GEI FRAMEWORK

The model presented in this paper belongs to the class of economic models of general equilibrium with incomplete markets, or GEI-models. These models are based on Radner's extenstion of the Arrow-Debreu model (see Radner 1968, 1972, and Magill and Quinzii, 1996). This stream of the literature includes the seminal papers by Zame (1993), Geanakoplos and Zame (2002), and Dubey, Geanakoplos and Zame (DGZ, 2005) who have, thus far, proposed the best approach to modelling default and contagion. While Geanakoplos and Zame (2002) focus on secured credit markets, the other two papers focus on unsecured loans (the model proposed in this paper comprises both types of loans).

In these models, outcomes are studied in stylized two-period models in which all uncertainty is resolved at the terminal date. The interaction of the different states of the world with agents' individual characteristics, such as risk- aversion, wealth and honesty/duplicity, determines the probabilities of default. Agents take commodity and asset prices as given, but they also take as given delivery rates, and hence, effective returns on assets. As penalty for default, an agent incurs a loss in utility which increases proportionately on the value of default. Finally, insurance markets against second-period risk are incomplete.

Although these models normally focus on endowment economies, extending the framework to a productive economy setting is straightforward. For a given set of claims of financial contracts, market prices are endogenously determined so as to satisfy aggregate consistency conditions. Generally, financial contracts are in zero net supply, in such a way that aggregate consistency requires the total number of sold and purchased contracts to be equal..

In the case of a productive economy, financial contracts would include claims to the production bundle of a public firm (a "stock"), so these would be in unit net supply. Moreover, in a productive economy, companies would be allowed to default and borrow against collateral to finance capital investments. Since such a model would involve too large a dimension, which is not necessary to examine financial regulation, I abstract from including firms in the present model.

DGS show that a unique equilibrium always exists in this environment, while Zame shows that with incomplete markets it is efficient to allow for a bankruptcy option because default is uniquely useful in overcoming insurance-market incompleteness. Intuitively, there is a nontrivial amount of uninsurable risk, in the face of which borrowers do risk-management by saving in good times and borrowing in bad times. Thus, as default allows for partial repayment according to debtors' immediate needs, it can provide a "state-contingency" to smooth consumption. Moreover, since insurance against risk requires promising to occasionally pay very large amounts to the counter-party, borrowers will have to promise very large payments in some states in order to obtain insurance against other states. In such a setting, the addition of more assets with similar structure does not help.

When a friction is introduced, straightforward analysis requires a comparison between the resulting equilibrium and the "frictionless" equilibrium allocations, as well as an assessment of whether optimal welfare properties obtain.

A competitive complete-markets´ equilibrium for this type of economy is Pareto efficient because there is perfect risk sharing between agents. In contrast, with incomplete markets, even as more assets are added, there is no welfare improvement so long as all liabilities are required to be satisfied with certainty (Zame's argument).

On the other hand, while the complete markets case of GEI models obtains Pareto efficient equilibrium allocations because households are able to transfer wealth across time and states using a complete set of assets, Hart (1975) argues that if the set of assets is incomplete, then for a generic subset of endowments the resulting equilibrium allocations are Pareto suboptimal.

Consequently, with incomplete markets there is scope for welfare improving economic policy. If an economy requires an omniscient planner to implement transfers that can potentially leave all agents at least as well-off as prior to the intervention, then the resulting allocation is Pareto suboptimal. However, in the presence of incomplete markets, like agents, the planner can only make transfers that respect the fixed asset structure. Thus, the allocation resulting from the planner's intervention is "constrained suboptimal". The constrained suboptimality result was introduced by Stiglitz (1982),and proved formally by Geanakoplos and Polemarchakis (1986).

This paper examines whether a planner, subject to the constraints arising from the incomplete asset structure, can generate a Pareto improvement. The planner can intervene in one of three ways: by imposing tighter capital, liquidity, or margin requirements.

III. THE BASELINE MODEL

Consider a Monetary General Equilibrium model with Incomplete Markets, Commercial Banks and Default (MGEICD) in which time extends over two periods $t = \{0,1\}^t$. The first period consists of a single initial state and the second period consists of S possible states. Suppose there are 2 possible states of the world in the second period $s = \{1,2\}$, and let $s^* \in S^* = 0 \cup \{S\} = \{0,1,2\}$.

This (endowment) economy has two goods, a basket of perishable consumption goods and housing, which are denoted by subscripts 1 and 2, respectively. Housing is a durable good, which provides utility in every period, and for tractability purposes, it is assumed to be infinitely divisible.

There are two households $h \in H = \{\alpha, \theta\}$, two commercial banks $j \in J = \{\gamma, \delta\}$, a non-bank financial institution (NFBI), which I denote by (ψ), and a Central Bank/Government/Financial Supervisory Agency (FSA), which operates as a strategic dummy. Households are risk-averse[4], and heterogeneous in their endowments of goods and money. Commercial banks are also risk-averse agents that face a portfolio allocation problem whereby they try to diversify idiosyncratic risks[5], and are heterogeneous in their endowments of capital and riskaversion. Finally, in contrast to commercial banks, the NBFI is assumed to be risk neutral.

There are 9 active markets: the goods and housing markets in the real sector; two short-term default-free credit markets (repo and consumer loans); three long-term credit markets (mortgage, deposit and interbank), of which the deposit market is assumed to be default-free; and two securities markets (Mortgage Backed Securities or (MBSs), and Collateralized Debt Obligations (CDOs)).

4 I assume households have a Constant Relative Risk Aversion (CRRA) utility function over the stream of goods and housing consumption; this functional form allows us to capture wealth effects of prices and interest rates movements.

5 i.e. I suppose commercial banks have quadratic preferences over their expected second period profits.

Following Goodhart, Sunirand and Tsomocos (2006), an important friction is introduced in short- term consumer credit markets. Individual borrowers are assigned, by history or by informational constraints, to a single bank over t = {0,1}[6]. Thus, without loss of generality, let household $\alpha(\theta)$ borrow from bank $\gamma(\delta)$ in the short- term credit market. In contrast, I assume that households make transactions with the bank offering the best rate when they engage in long-term (inter-period) loan or deposit contracts.

Furthermore, I introduce four financial frictions: money, default, collateral and securitisation. Money and (continuous) default are modelled as in the canonical MGEICDs. Money is introduced via a cash-in-advance technology, whereby all commodities and assets can be traded only for money, and all asset deliveries are paid in money. Following Dubey and Geanakoplos (1992), money is fiat and is the stipulated medium of exchange; it does not render utility to agents, it cannot be privately produced, and it is perfectly durable. Moreover, money enters the system as outside or inside money.

Outside money enters the system free and clear of any offsetting obligations (private monetary endowments), whereas inside money enters the system accompanied by an offsetting obligation (money supplied by the Central Bank to commercial banks in the repo market). Since money is fiat, it must exit the system at the final period. Hence, inside and outside money exit the economy via loan repayments by households/investors to commercial banks, loan repayments by commercial banks to the Central Bank, or by the Central Bank's liquidation of commercial banks.

The model incorporates two types of (endogenous) default. On the one hand, following Shubik and Wilson (1977) and Dubey et al. (2005), default arises as a *continuous* phenomenon in unsecured credit markets (the interbank market). Subject to a default penalty, borrowers are allowed to choose the fraction they are willing to repay on their outstanding obligations; this penalty reduces borrowers' utility/profits by an amount $\overline{\tau}_s^k$ for $s \in S$ and $k = \{H, J, \psi\}$, per unpaid monetary unit of account. Thus,

6 Restricted participation can also arise as an outcome of banks aiming to outperform each other by introducing a relative performance criterion into their objective functions (see Bhattacharya et al., 2007)

in equilibrium, agents will equalize the marginal utility of defaulting (additional consumption/profits) to its marginal cost (the bankruptcy penalty)[7].

On the other hand, default is highly *discontinuous* in markets where collateral is required. Collateral is introduced in the mortgage market, where banks seize housing from mortgage borrowers in the event of default. Thus, home buyers default whenever the value of their collateral falls below their outstanding mortgage obligation[8].

The scarcity of collateral induces agents to create innovations to economize it, e.g. via securitization and derivatives markets (see Geanakoplos and Zame, 2002). In the framework presented here, mortgage issuers create and sell mortgage backed assets (MBSs) to risk neutral NBFIs; these institutions structure CDOs by attaching Credit Default Swaps (CDS) to MBSs, and sell them to commercial banks for a premium, which is the cost of insurance against mortgage default.

Let $\tilde{q}^{\alpha}$ denote the price of the CDO, and $\overline{r}^{\gamma\alpha}$ the interest rate on the mortgage contract. Further, assume household α honours his its mortgage if s = 1, and defaults if s = 2. Thus, the mortgage can be regarded as an asset with the following vector of payoffs across states:

$$R^{\alpha} = \begin{bmatrix} 1+\overline{r}^{\gamma\alpha} \\ 1+\overline{r}_{s}^{\gamma\alpha} \end{bmatrix}$$

where $\left(1+\overline{r}_{s}^{\gamma\alpha}\right)$ is the *effective* mortgage rate in case of default (the ratio of collateral to mortgage outstanding debt). Consequently, the CDO security has the following payoffs:

$$R^{CDO} = \begin{bmatrix} \left(1+\overline{r}^{\gamma\alpha}\right) / \tilde{q}^{\alpha} \\ 1 \end{bmatrix}$$

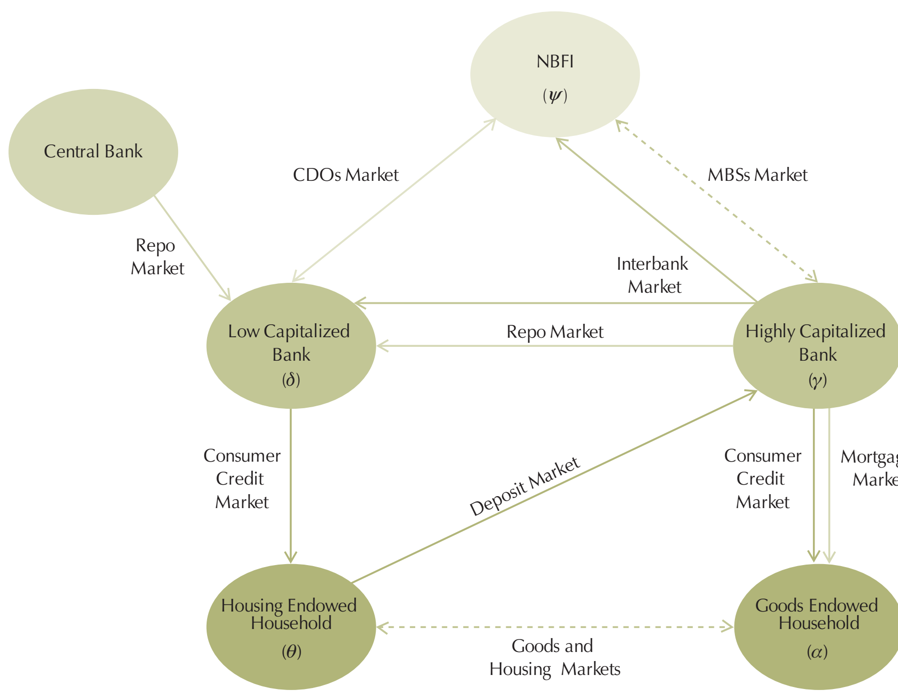
7 In the literature this requirement is known as the "on-the-verge" condition; see Dubey et. al. (2005)

8 I model the "walk-away" option of U.S. mortgage markets, and I abstract from recourse mortgage loans which are common in Europe and the U.K. Nevertheless, it can be argued that allowing for mortgage contracts with recourse would induce lower default rates, because the default penalty on these types of contracts is implicitly higher.

In the good states, the CDO investor earns the monetary payoff of the mortgage asset net of the premium paid to the NBFI; and in the bad states of nature, it has a zero rate of return on its investment but with no capital loss. Either.

The structure of the economy is summarized in Figure 1.

Figure 1
Structure of the Economy



Straight lines and their direction represent lending flows.
Dashed lines indicate trade.

Source: Author calculations.

The time structure of the model is equivalent to that of Monetary General Equilibrium models with Incomplete Markets, Commercial Banks and Default (see Goodhart et al. 2006). Initially $\forall\, t \in T$, commercial banks $\left(j \in J\right)$ organize a short term credit market with the Central Bank, which provides liquidity through open market operations $\left(M_s^{CB}\right)$ for $s \in S$ or by entering into (reversal) repurchase agreements with commercial banks. Since bank γ is assumed to be highly capitalized, it enters into a reverse repurchase agreement with the Central Bank (makes a deposit), while the poor bank (δ) enters into a repurchase agreement (borrows).

Households $h \in H$ take out short- term loans at $s^* \in S^*$ because cash-in-advance is needed for all market transactions. In the initial period, long- term credit markets meet after short- term consumer credit and repo markets close; hence, household α takes out a mortgage and household θ makes a long- term deposit with bank γ.

Furthermore, at t = 0 the NBFI buys the mortgage asset from bank γ in the MBS market, and securitizes it into a CDO containing the mortgage backed security and a CDS. Since the NBFI has a small endowment of capital, and bank δ is poorly capitalized, both borrow from bank γ in the interbank market before making their respective investments in the derivatives markets. At the end of the first period, consumption and settlement of short-term loans take place.

In the second period, the repo and consumer short- term credit markets meet before settlement and default takes place in the mortgage, MBS, CDO, and interbank markets. At the end of this period, consumption and settlement of oneperiod loans take place, and the Central Bank liquidates commercial banks by taking over their profits[9].

Figure 2 describes the time line of the economy. The full mathematical characterization of the baseline model is in the Appendix.

IV. BENCHMARK EQUILIBRIUM

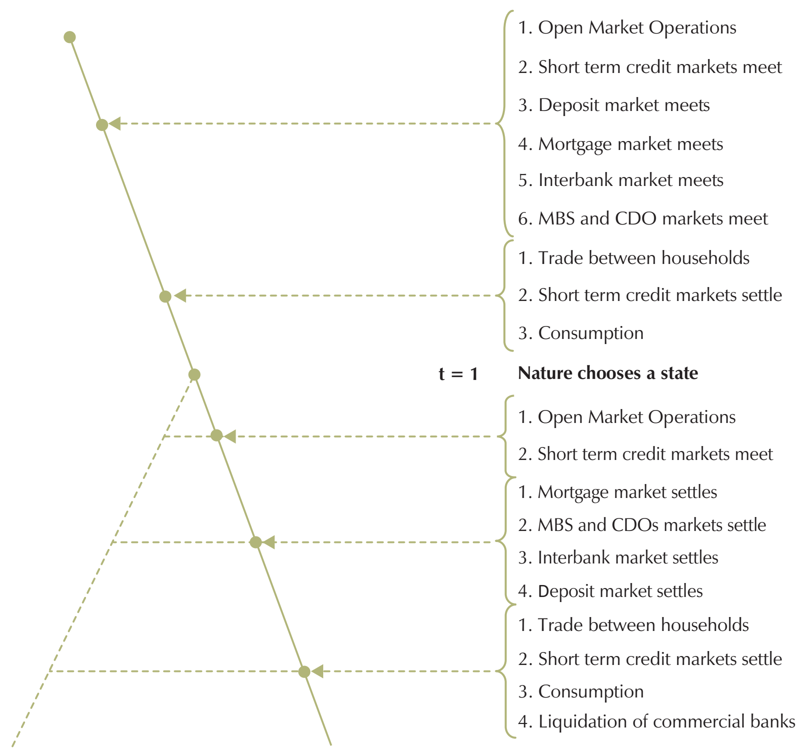
Hereafter, a parameterized version of the model is analyzed. The chosen vector of parameter values allows for an illustration of how default in the mortgage market spreads through the rest of the nominal sector of the economy.

A. EXOGENOUS VARIABLES

The chosen set of exogenous variables has a twofold purpose. On the one hand, it determines the structure of the economy, the degree of heterogeneity between households and financial institutions, and hence, the relationships and interconnectedness between the different agents, markets and sectors of the economy.

9 For simplicity, but without loss of generality, I abstract from modelling an equity market for banks (where financial institutions raise capital, and according to which they distribute second period profits to shareholders). Nevertheless, this model can easily be adapted to allow for such an extension (see Tsomocos, 2003, and Goodhart et al., 2006).

Figure 2
Time Structure



Source: Author calculations.

On the other hand, the parameters of the benchmark model describe the set of actions of the authorities. The economy has three official players: a Central Bank, which can inject (withdraw) money into (from) the system; the Government, which can increase or decrease private monetary or commodity endowments; and a Financial Supervisory Agency (FSA), which imposes penalties on defaults and violations of the financial regulatory framework. Since I do not, at this stage, model the *optimal response* of these agents, they operate as strategic dummies and their actions are described by a set of exogenous variables.

Assume two possible states of nature in the second period, and that a (good) state 1 realisation (ϖ_1) is more likely than a (bad) state 2 realisation (ϖ_2) (see column 6 in Table 1, below). Therefore, in the first period agents make decisions under a set of beliefs, where the probability of a housing and financial crisis is relatively small.

Households are assumed to be homogenous in their risk-aversion (hence their risk-aversion coefficients, c^{α} and c^{θ} are equal; see Table 1 below, first column), but they differ in their endowments of goods and money. While household α is endowed with consumption goods (e^{α}_{01} being its endowment of goods at time 0, and e^{α}_{11} and e^{α}_{21} being its endowment in the good and bad state respectively[10]), consumer θ is endowed with a large amount of housing at t = 0 (e^{θ}_{02}). Moreover, consumer θ is relatively richer than α in monetary endowments in the initial period (hence $e^{\theta}_{m,0}$ is greater than $e^{\theta}_{m,0}$, see Table 1 below, fourth column). I have chosen this parameterization to induce household α to borrow in the mortgage market and household θ to make long-term deposits.

In the first period, bank γ is better capitalised than bank δ and the NBFI (ψ), and in the second period, only commercial banks are endowed with capital (hence e^{γ}_{0} is larger than e^{δ}_{0} and e^{ψ}_{0}, see Table 1, fourth column). With this assumption, bank γ is provided with incentives to lend in the interbank market, while financial institutions δ and ψ borrow to finance their investments in CDOs and MBSs. Households engage in long-term loan or deposit contracts with bank γ because, due to its high capitalization, this institution offers the best mortgage and deposit rates.

Financial institutions also differ in their attitude towards risk. While commercial banks are risk-averse (therefore, $c^{\gamma}$and c^{δ}denoting the risk- aversion coefficients of bank γ and δ respectively, are positive; see Table 1, first column), the NBFI is assumed to be risk-neutral (which is why its risk- aversion coefficient is zero). I assume that bank γ is more risk-tolerant than bank δ. Under this parameterization, I motivate the NBFI to sell CDOs, or to bear the risks of mortgage default and a short CDS position. Similarly, I induce bank δ to buy CDOs, because due to its high risk aversion, it only invests in credit enhanced derivatives. I also provide bank γ with incentives to reduce the overall risk of its portfolio, which is why it sells its mortgage assets in the MBS market; however, since bank γ's risk- aversion is relatively low, it also makes credit extensions to investors in MBSs and CDOs in order to (re)gain some exposure to mortgage risk.

10 I denote the endowments of goods, prices and traded quantities with subscripts *sl*, where *s* is the state and *l* represents a specific good; recall that *l* = 1 and *l* = 2 denote consumption goods and housing respectively.

The economy experiences an adverse availability[11] shock in the goods sector that is moderate in the first state and severe in the second state (therefore $e^{\alpha}_{01} > e^{\alpha}_{11} > e^{\alpha}_{21} >$, see Table 1 second column). This type of shock increases inflation and deters trade in the goods market (thus goods prices p_{01} rise in the good state p_{11}, and much more in the bad state p_{12}, whereas the amount of goods sold q^{α}_{01} decreases, especially in the bad state, see Table 2, first and fifth columns). Hence, by assuming that the Central Bank has a price stability objective whereby it tries to stabilize goods, but not asset price inflation (i.e. changes in house prices), I set the parameters of the model so as to reflect a tightening of the Central Bank's monetary policy stance across time. Given the relative magnitude of this shock, the policy change is more drastic in the second than in the first state (with M_0^{CB} denoting the quantity of money supplied by the Central Bank in the first period, and M_1^{CB} and M_2^{CB} the supply of money in the good and bad states, respectively; column 6 in Table 1 shows that these parameters satisfy $M_0^{CB} > M_1^{CB} > M_2^{CB}$).

As mentioned earlier, commercial banks are endowed with capital in both periods. At $t = 0$ these endowments represent equity raised by banks in the stock market or through an initial public offering, whereas in the second period they correspond to capital injections by the fiscal authority. Upon this assumption, I capture the perception that in hazardous times ($t = 1$) commercial banks may not be able to raise capital in the equity market, thereby calling for Government intervention (see Catarineu-Rabell et al., 2005). Moreover, this Government initiative mitigates the adverse effects of the negative availability shock in the goods markets: by provisioning banks with additional capital in the second period, their capacity to absorb losses improves, thus allowing them to offer better credit conditions to the household sector.

I also assume that capital injections into bank δ are higher than into bank γ (therefore, $e^{\gamma}_1 < e^{\delta}_1$ and $e^{\gamma}_2 < e^{\delta}_2$ in the good and bad states, respectively). This is due to the fact that financial institution δ, having to repay short-term obligations in the repo market, is more vulnerable than γ, which is a net lender in the repo market. However, bank δ does not require a significantly higher provision of equity, since it has fewer assets and less exposure to creditrisk than bank γ.

Finally, I choose a set of bankruptcy penalties that reflects the actions of an FSA interested in protecting the average consumer. I set very high default penalties in the

11 In versions of this model which include a production sector – but not a housing sector would be a productivity shock.

deposit market $\overline{\tau}_s^{\gamma}$ (see fifth column of Table 1 below), which is why default does not even arise as an equilibrium outcome in this market. However, I allow financial institutions to default partially in the interbank market by imposing lower bankruptcy penalties for its borrowers.

Furthermore, I assume that the punishment for NBFIs is higher than for commercial banks (hence, bank δ's default penalties in the second period $\overline{\tau}_s^{\delta}$ are smaller than NBFI default penalties $\overline{\tau}_s^{\Psi}$, where s denotes state 1 or 2 (see Table 1 below, column 5), because when the former file for bankruptcy they pose higher costs on society than the latter. This is due to the assumption that firstly, commercial banks are not as leveraged as NBFIs, and secondly, liquidation tends to be more organized for banks than non-banking financial institutions (e.g. because governments provide partial consumer deposit insurance). I could, of course, easily revise such assumptions.

Table 1
Exogenous Variables

Risk Aversion Coecients		Goods Endowments		Housing Endowments		Monetary Endowments		Default Penalties		Others	
c^{α}	3	e_{01}^{α}	13	e_{02}^{α}	20	$e_{m,0}^{\alpha}$	3	$\overline{\tau}_1^{\gamma}$	1000	M_0^{CB}	30
c^{θ}	3	e_{11}^{α}	11			$e_{m,1}^{\alpha}$	1	$\overline{\tau}_2^{\gamma}$	1000	M_1^{CB}	20
c^{γ}	0,001	e_{21}^{α}	9			$e_{m,2}^{\alpha}$	1	$\overline{\tau}_1^{\delta}$	0,05	M_2^{CB}	0,1
c^{δ}	0,025					$e_{m,0}^{\alpha}$	60	$\overline{\tau}_2^{\delta}$	0,05	ω_1	0,85
						$e_{m,1}^{\alpha}$	1	$\overline{\tau}_1^{\Psi}$	1	ω_2	0,15
						$e_{m,2}^{\alpha}$	1	$\overline{\tau}_2^{\delta}$	1		
						e_0^{γ}	50				
						e_1^{γ}	10				
						e_2^{γ}	10				
						e_0^{γ}	1				
						e_1^{γ}	12				
						e_2^{γ}	12				
						e_0^{Ψ}	1				

Source: Authors calculations.

B. ENDOGENOUS VARIABLES

Given the parameterization described in the previous section, in the benchmark equilibrium house and goods prices move in opposite directions; the relative price of houses drops from the first to the second period, and is lower in state 2 than in state 1 (thus, $p_{01} < p_{11} < p_{21}$ and house prices p_{02} fall in the good state p_{12} and *plummet* in the bad state p_{22}, see Table 2, first two columns). This is a consequence of the negative supply shock in the goods market. Intuitively, agent α defaults on his mortgage when the value of his house is low, and house prices fall when goods endowments are scarce. Stated differently, as household α's sales revenues decrease, he is forced to demand fewer houses; consequently, lower demand in the housing market reduces house prices, while lower supply in the goods market raises the price of goods.

Household α is rich in his endowment of goods e_{01}^{α} in the first period. Therefore, he can finance a rather large percentage of his desired housing expenditure with short-term credit μ_0^{α}, and he is not required to take out a very large loan-to-value mortgage $\bar{\mu}^{\alpha}$ (see Table 2, second column). Nevertheless, the severe depreciation of housing in state 2 induces household α to default on his mortgage, thereby lowering repayment rates in the interbank market (hence, state 2 mortgage repayment rates $\bar{v}_2^{\alpha}$ as well as interbank repayment rates by bank δ and the NBFI, $\bar{v}_2^{\delta}$ and $\bar{v}_2^{\psi}$ respectively, fall below 100%; see Table 2, column 4).

In state 1, the average default rate in the interbank market $\bar{R}_1$ is very low. Since the home buyer honours his mortgage (hence, $\bar{v}_1^{\alpha}$ equals 100%; see column 4 of Table 2), and the latter is the underlying asset of MBSs and CDOs, bank δ and NBFI ψ are able to repay most of their interbank obligations with the proceeds from the CDO return and the securitisation premium, respectively.

In contrast, in state 2 default in the mortgage market creates significant losses in the NBFI sector. The CDS leg of the CDO contract is executed, thereby forcing bank δ to deliver the mortgage's collateral to the NBFI in return for its initial capital investment. The NBFI ψ assumes an additional write-off loss because it is obliged to sell the collateral when the housing market meets, which pushes house prices further down and induces ψ to default more.

Likewise, bank δ's repayment rate in state 2 ($\bar{v}_2^{\delta}$) decreases for two reasons. Firstly, because bank δ's CDS insurance only provides protection against capital losses, so it remains unable to pay for the service on its interbank debt. And secondly, because in equilibrium bank δ uses its short- term revenues to pay for its repo market obligations at t = 0[12]; this prevents the bank from accumulating profits (thus $\pi_0^\delta = 0$, see column 3, in Table 2, below), thereby leaving it with insufficient capital to pay for its long- term liabilities.

This is not the case with bank γ, because its balance sheet consists of short and long-term assets and of long- term liabilities (*only*). Hence, bank γ accumulates profits by carrying forward the proceeds from its firstperiod, short-term loan repayments (hence $\pi_0^\gamma > 0$), which allows it to hoard enough capital to pay its depositors back in both states of the second period.

In state 2, lower repayment rates in the mortgage and interbank markets prevent commercial banks from making credit extensions to households efficiently (relative to the first period, bank γ and δextend fewer consumer loans, thus $m^\gamma{}_0 > m^\gamma{}_2$ and $m^\delta{}_0 > m^\delta{}_2$, respectively; see Table 2, third column). Consequently, households' welfare and profits in the banking sector fall relative to the first period and state 1. Following the criteria proposed by Tsomocos (2003) and Goodhart et al. (2006)[13], this implies that the economy becomes *financially unstable* in the second state.

Repo rates (denoted by ρ_0^{CB} in the first period and by ρ_1^{CB} and ρ_2^{CB} in the good and bad states) equal short- term interest rates in all states ($r^\gamma{}_s$ representing the rates on short- term consumer loans taken out with bank γ, and $r^\delta{}_s$ with bank δ). Furthermore, in the first period, the deposit rate $\overline{r}_d^\gamma$ equals the repo rate ρ_0^{CB} as the no arbitrage conditions for default-free loans hold (see Table 2, first column). By the same argument, the price of MBS p^α is equal to the gross return on first-period, risk- free assets $\left(1+\rho_0^{CB}\right)$. This is due to the fact that the former is a riskless security from the perspective of the issuer and seller of the mortgage asset. In other words, bank γ sells its mortgage prior to its settlement, and just before the second period state is revealed; therefore, it substitutes the cash flows of a risky (mortgage) asset for that

12 For bank δ, it is optimal not to accumulate profits in the first period. This is due to the fact that banks are not punished for being *undercapitalized* at $t = 0$ in the benchmark model (or in its extended versions), and because at the beginning of the second period, financial institutions (*rationally*) expect the Government to intervene. Moreover, when the model is extended to allow for regulatory policy, I assume the FSA examines banks at the beginning of $t = 1$.

13 According to these authors, an economy is financially unstable whenever substantial default of a 'number' of households and banks occurs, and the aggregate profitability of the banking sector decreases significantly.

of a constant pay-off security. No arbitrage requires these pay-offs to match those of a risk-less loan.

Furthermore, the interbank rate $\bar{\rho}$ is higher than the deposit rate $\bar{r}_d^{\gamma}$, because long-term interest rates are ex-ante and pricein a default premium; therefore, no arbitrage for long-term defaultable loans requires lending rates to be at least as high as borrowing rates. Similarly, the expectation of high default rates in the mortgage market in state 2 induce bank γ to offer a very high mortgage rate $\bar{r}^{\gamma\alpha}$ (see column 1, in Table 2 below).

The choice of parameter values illustrates the trade-off between price and financial stability, which is common to Central Bank practice at certain points of the business

Table 2
Initial Equilibrium

		Housholds		**Financial Sector**			
Prices		Lending Borrowing		Profits, Lending and Borrowing		Repayment Rates	
p_{01}	2,6	μ_0^{α}	51,33	$d_0^{G\gamma}$	6,67	$\bar{v}_1^{\alpha}$	100%
p_{11}	6,41	μ_1^{α}	101	$d_1^{G\gamma}$	0,61	$\bar{v}_2^{\alpha}$	62,5%
p_{21}	75,22	μ_2^{α}	85,35	$d_2^{G\gamma}$	7,67	$\bar{v}_1^{\delta}$	95,35%
p_{02}	12,64	$\bar{\mu}^{\alpha}$	40,08	m_0^{γ}	36,06	$\bar{v}_2^{\delta}$	64,9%
p_{12}	11,13	μ_0^{θ}	51,43	m_1^{γ}	83,25	$\bar{v}_1^{\psi}$	99,5%
p_{22}	6,16	μ_1^{θ}	44,18	m_2^{γ}	55,82	$\bar{v}_2^{\psi}$	62,97%
r_0^{γ}	0,40	μ_2^{θ}	31,75	$\bar{m}^{\alpha}$	11,82	$\bar{R}_1$	96,95%
r_1^{γ}	0,21	$\bar{d}^{\theta}$	45,34	$\bar{\mu}_d^{\gamma}$	63,58	$\bar{R}_2$	64,12%
r_2^{γ}	0,53			$\bar{d}^{\gamma}$	40,24		
r_0^{δ}	0,40			$\mu_0^{G\delta}$	51,43		
r_1^{δ}	0,21			$\mu_1^{G\delta}$	25		
r_2^{δ}	0,53			$\mu_2^{G\delta}$	11,88		
$\bar{r}_d^{\gamma}$	0,40			m_0^{δ}	36,67		
$\bar{r}^{\gamma\alpha}$	2,39			m_1^{δ}	36,42		
ρ_0^{CB}	0,40			m_2^{δ}	20,77		
ρ_1^{CB}	0,21			$\bar{\mu}^{\delta}$	38,04		
ρ_2^{CB}	0,53			$\bar{\mu}^{\psi}$	24,03		
$\bar{\rho}$	0,54			π_0^{γ}	81,34		
p^{α}	1,40			π_0^{δ}	0,00		
$\tilde{q}^{\alpha}$	2,17						

Source: Authors calculations.

cycle. The benchmark equilibrium shows the consequences of disregarding financial imbalances in the pursuit of low and stable inflation; since monetary policy is non-neutral and a non-trivial quantity theory of money holds, tighter monetary policy in the last period exacerbates the adverse effects of the availability of goods shock.

V. MODELLING FINANCIAL REGULATION

In this section I analyze the effects of three different types of financial regulation: capital, margin, and liquidity requirements. Moreover, since the Central Bank has a monopoly over the supply of money, in order to make a comprehensive analysis of shocks to aggregate liquidity, I also assess the impact of Central Bank interventions in times of financial distress.

	Trade and Spending					
	Goods		Housing		Derivatives	
	q^{α}_{01}	23,72	q^{θ}_{02}	4,07	$\tilde{m}^{\alpha}$	16,58
	q^{α}_{11}	15,75	q^{θ}_{12}	3,97	$\hat{m}^{\alpha}$	25,66
	q^{α}_{21}	1,14	q^{θ}_{22}	5,15		
	b^{θ}_{01}	51,33	b^{α}_{02}	51,42		
	b^{θ}_{11}	101	b^{α}_{12}	44,78		
	b^{θ}_{21}	85,35	b^{α}_{22}	56,81		

A. CAPITAL ADEQUACY RATIOS

In this section I examine the effects of changing the regulatory framework, whereby commercial banks are punished if their capital adequacy ratio lies below some minimum requirement. For a mathematical description of how this policy changes the behaviour of commercial banks, see Appendix.

This policy induces commercial banks to finance their portfolios with less debt. Bank δ borrows less from the interbank market, and bank γ takes fewer deposits from households, thus causing interbank and deposit rates to fall.

Looser credit conditions in the interbank market allow the NBFI to make larger investments in the MBS market. Higher demand for this security raises its price, thereby lowering the securitization premium.

On the other hand, a lower deposit rate induces rich households (θ) to save less and consume more in the first period. Since the wealthy consumer is also the supplier of houses, to smooth consumption efficiently, he spends more in the goods market and offers fewer houses for sale. This increases the price of both goods and houses in the initial period.

Consequently, home buyers expect future housing prices to decline more than in the initial equilibrium. Hence, household α is motivated to take out a smaller mortgage in order to postpone consumption. This pushes house prices further down in the second period, and provides the home buyer with stronger incentives to default in the bad states (thus, mortgage repayment rates $\overline{v}_2^{\alpha}$ decrease; see third column of Table 3 below).

As repayment rates in the mortgage market decrease, aggregate default in the bad states of nature increases due to a contagion phenomenon that spreads to the NBFI sector. Since the investment bank ψ is highly leveraged and exposed to mortgage credit risk, its capacity to meet its interbank obligation falls (hence, the NBFI repayment rate $\overline{v}_2^{\psi}$ decreases, causing the *aggregate* expected delivery rate on interbank loans $\overline{R}_2$ to fall; see Table 3 below, column 3).

This is not the case for commercial banks. By lowering its debt-to-equity ratio in the first period, bank δ has the ability to repay a larger fraction on its interbank loan (thus $\overline{v}_2^{\delta}$ rises), whereas bank γ uses the additional equity buffer to absorb the losses generated by lower *aggregate* interbank repayments.

This policy reduces leverage in the banking sector and promotes lending to the household sector in the second period, so consumers' welfare improves. In particular, as home buyers expect higher rates of depreciation in the value of houses, they are betteroff by defaulting and purchasing more houses in the bad states (which is why the utility of consumer α in state 2 ($U^{\alpha}{}_{2}$) increases; see Table 3, first column). Put differently, banks internalize (default) losses without taking a toll on the taxpayer (bank γ and δ's expected second period profits - π^{γ} and π^{δ}- decrease, as well as their profits in the bad states, see Table 3 below, second column; however, $U^{\alpha}{}_{2}$ increases, and household θ's welfare across time (U^{θ}) improves, see first column of Table 3 below).

Table 3
Eects of imposing Capital Adequacy Ratios

Households Welfare		Banking Sector Profits		Repayment Rates	
U^{α}	–	π^{γ}	–	$\overline{v}_2^{\alpha}$	–
U^{θ}	+	π^{δ}	–	$\overline{v}_2^{\delta}$	+
U_2^{α}	+	π_2^{γ}	–	$\overline{v}_2^{\psi}$	–
U_2^{θ}	–	π_2^{γ}	–	$\overline{v}_2$	–

$U^{\alpha} \equiv$ household α's utility stream

$U^{\theta} \equiv$ household θ's utility stream

$U_2^{\alpha} \equiv$ household α's utility if $s \notin S_1^{\alpha}$

$U_2^{\theta} \equiv$ household θ's utility if $s \notin S_1^{\alpha}$

$\pi^{\gamma} \equiv$ bank γ's stream of profits

$\pi_2^{\gamma} \equiv$ bank γ's profits if $s \notin S_1^{\alpha}$

$\pi^{\delta} \equiv$ bank δ's stream of profits

$\pi_2^{\delta} \equiv$ bank δ's profits if $s \notin S_1^{\alpha}$

$\overline{v}_2^{\alpha} \equiv$ mortgage repayment rate if $s \notin S_1^{\alpha}$

$\overline{v}_2^{\delta} \equiv$ interbank repayment rate if $s \notin S_1^{\alpha}$

$\overline{v}_2^{\psi} \equiv$ investment bank's repayment rate if $s \notin S_1^{\alpha}$

Source: Authors calculations.

B. MARGIN REQUIREMENTS

Suppose the FSA issues a regulatory policy that imposes maximum Loan-to-value (LTV) ratio constraints to mortgage borrowers, and minimum margin requirements to interbank borrowers of the NBFI sector. For a mathematical description of how this policy changes the behaviour of households and investors, see Appendix.

These constraints induce home buyers and NBFIs to leverage less. As demand for mortgages decreases, its interest rate $\bar{r}^{\gamma\alpha}$ falls (see Table 4 below, column 4). Therefore, the expected return on CDOs decreases, thereby reducing demand for these assets as well as for their underlying security (MBSs). This provides commercial banks with incentives to lower their debt-to-equity ratio, which is why repayment rates in the interbank market ($\bar{v}_2^{\delta}$ and $\bar{v}_2^{\psi}$) increase (see Table 4, third column).

Higher repayment rates in the bad states of nature allow commercial banks to make larger credit extensions to the household sector, thus allowing consumers to spend more in the goods and housing markets. In particular, higher demand bids housing prices up, hence raising the value of collateral (which is depicted by a higher *effective* mortgage rate $\bar{r}_2^{\gamma\alpha}$; see Table 4 below, fourth column) and lowering mortgage default rates (or increasing mortgage repayment rates $\bar{v}_2^{\alpha}$; see column 3, Table 4). Consequently, solvency of the NBFI sector improves, which pushes interbank default rates further down.

Binding loan-to-value and margin constraints improve home buyers′ welfare by allowing them to smooth consumption efficiently across time (which is why the overall utility of household α- U^{α}-, as well as his welfare in state 2 -U_2^{α}- improve; see Table 4 below, first column). Moreover, these measures promote financial stability, because they induce households and financial institutions to default less and allow the banking sector to improve profitability (therefore, bank γ and δ's expected second period profits - π^{γ} and π^{δ} - increase; see Table 4, second column).

Table 4
Eects of imposing tighter LTV and Margin Requirements

Households Welfare		Banking Sector Profits		Repayment Rates		Housing Prices		Lending at $s = 2$	
U^{α}	+	π^{γ}	+	$\bar{v}_2^{\alpha}$	+	p_{02}	+	m_2^{γ}	+
U^{θ}	–	π^{δ}	+	$\bar{v}_2^{\delta}$	+	p_{22}	+	m_2^{δ}	+
U_2^{α}	+	π_2^{γ}	+	$\bar{v}_2^{\psi}$	+	(p_{22}/p_{02})	+		
U_2^{θ}	–	π_2^{δ}	–						

$U^{\alpha} \equiv$ household α 's utility stream
$U^{\theta} \equiv$ household θ 's utility stream
$U_2^{\alpha} \equiv$ household α 's utility if $s \notin S_1^{\alpha}$
$U_2^{\theta} \equiv$ household θ 's utility if $s \notin S_1^{\alpha}$
$\pi^{\gamma} \equiv$ bank γ 's stream of profits

Table 4 (continued)
Eects of imposing tighter LTV and Margin Requirements

Households Welfare	Banking Sector Profits	Repayment Rates	Housing Prices	Lending at $s = 2$
$\pi_2^{\gamma} \equiv$ bank γ 's profits if $s \notin S_1^{\alpha}$				
$\pi^{\delta} \equiv$ bank δ 's stream of profits				
$\pi_2^{\delta} \equiv$ bank δ's profits if $s \notin S_1^{\alpha}$				
$\bar{v}_2^{\alpha} \equiv$ mortgage repayment rate if $s \notin S_1^{\alpha}$				
$\bar{v}_2^{\delta} \equiv$ interbank repayment rate if $s \notin S_1^{\alpha}$				
$\bar{v}_2^{\psi} \equiv$ investment bank's repayment rate if $s \notin S_1^{\alpha}$				
$(p_{22}/p_{02}) \equiv$ housing inflation if $s \notin S_1^{\alpha}$				
$\bar{r}^{\gamma\alpha} \equiv$ contract mortgage rate				
$\bar{r}_2^{\gamma\alpha} \equiv$ effective mortgage rate if $s \notin S_1^{\alpha}$				
$m_2^{\gamma} \equiv$ lending to household α if $s \notin S_1^{\alpha}$				
$m_2^{\theta} \equiv$ lending to household θ if $s \notin S_1^{\alpha}$				

Source: Authors calculations.

C. LIQUIDITY REQUIREMENTS

Assume the FSA implements a regulatory framework whereby commercial banks are required to maintain a minimum liquidity ratio. If banks violate this requirement, then they are punished with a reputational penalty. For a mathematical description of how this policy changes the behaviour of commercial banks, see Appendix.

Higher liquidity requirements induce banks to make fewer credit extensions in the interbank and mortgage markets in the first period. This provides the NBFI with incentives to reduce its leverage and make smaller investments in MBSs. Moreover, as the volume of MBSs available to structure CDOs falls, the securitization premium increases and induces bank δ to make a smaller investment in the CDO market.

Since banks reduce their exposure to MBSs and CDOs, their revenues at the beginning of the last period improve, which allows them to make larger credit extensions to the household sector (thus, consumer loans in state 2, m_2^{γ} and m_2^{δ}, increase; see Table 5 below, fifth column). Looser credit conditions in the second period provide consumers with incentives to spend more in the bad states of nature. Specifically, as demand by home buyers becomes stronger in the bad states, property values rise, thereby stemming the collapse of housing prices (hence, house price *deflation* (p_{22}/p_{02}) decreases; see column 4 on Table 5). This increases the worth of collateral (represented by a higher effective mortgage rate $\bar{r}_2^{\gamma\alpha}$, see column 4 on Table 5) and reduces default rates in the mortgage market $\bar{v}_2^{\alpha}$. Therefore, the NBFI sector derives higher income in the bad states, which lowers the probability of its defaulting on its interbank obligation (thus, the NBFI repayment rate $\bar{v}_2^{\psi}$ rises; see Table 5 below, third column).

A minimum liquidity ratio policy reduces the exposure of commercial banks to risky assets, thereby promoting consumer lending in the second period and in the bad states of nature. Moreover, this measure stimulates financial stability because it reduces default rates in the mortgage and interbank markets, and boosts profits in the banking sector (therefore, bank γ and δ's expected second period profits - π^{γ} and π^{δ}- increase; see Table 5, second column).

Table 5
Effects of imposing Liquidity Requirements

Households Welfare		Banking Sector Profits		Repayment Rates		Housing Prices		Lending at $s = 2$	
U^{α}	+	π^{γ}	+	$\bar{v}_2^{\alpha}$	+	(p_{22}/p_{02})	+	m_2^{γ}	+
U^{θ}	–	π^{δ}	+	$\bar{v}_2^{\delta}$	+	$\bar{r}_2^{\gamma\alpha}$	+	m_2^{δ}	+
U_2^{α}	+	π_2^{γ}	+	$\bar{v}_2^{\psi}$	+	$\bar{r}^{\gamma\alpha}$	+		
U_2^{θ}	–	π_2^{δ}	–						

$U^{\alpha} \equiv$ household α 's utility stream
$U^{\theta} \equiv$ household θ 's utility stream
$U_2^{\alpha} \equiv$ household α 's utility if $s \notin S_1^{\alpha}$
$U_2^{\theta} \equiv$ household θ 's utility if $s \notin S_1^{\alpha}$
$\pi^{\gamma} \equiv$ bank γ 's stream of profits
$\pi_2^{\gamma} \equiv$ bank γ 's profits if $s \notin S_1^{\alpha}$
$\pi^{\delta} \equiv$ bank δ 's stream of profits
$\pi_2^{\delta} \equiv$ bank δ's profits if $s \notin S_1^{\alpha}$
$\bar{v}_2^{\alpha} \equiv$ mortgage repayment rate if $s \notin S_1^{\alpha}$
$\bar{v}_2^{\delta} \equiv$ interbank repayment rate if $s \notin S_1^{\alpha}$
$\bar{v}_2^{\psi} \equiv$ investment bank's repayment rate if $s \notin S_1^{\alpha}$
$(p_{22}/p_{02}) \equiv$ housing inflation if $s \notin S_1^{\alpha}$
$\bar{r}^{\gamma\alpha} \equiv$ contract mortgage rate

Table 5 (continued)
Effects of imposing Liquidity Requirements

Households Welfare	Banking Sector Profits	Repayment Rates	Housing Prices	Lending at $s = 2$
$\bar{r}_2^{\gamma\alpha} \equiv$ effective mortgage rate if $s \notin S_1^\alpha$				
$m_2^\gamma \equiv$ lending to household α if $s \notin S_1^\alpha$				
$m_2^\theta \equiv$ lending to household θ if $s \notin S_1^\alpha$				

Source: Authors calculations.

D. MONETARY POLICY

Let the Central Bank engage in expansionary monetary policy in state 2 by increasing the monetary base and letting the repo rate clear the market. This policy reduces the repo rate in $s = 2$, thus inducing commercial banks to make larger credit extensions in short- term markets, which lowers short- term interest rates. Better credit conditions provide households with incentives to raise their levels of spending and lead to an increase of both prices and trade.

Since consumers purchase a larger quantity of goods and houses in the bad states of nature, they smooth consumption across time by buying more goods and houses in the first period. Household θ borrows more short-term and makes fewer deposits, thereby raising the deposit rate; while household α substitutes mortgage with short-term borrowing, which lowers the mortgage rate. Due to the fact that consumers shift to short- term financing in the first period, interest rates on these loans rise.

A lower mortgage rate and a smaller rate of deflation in the housing market, enhance the value of collateral in the bad states. This reduces default rates in the mortgage market $\bar{v}_2^\alpha$ (see column 3 on Table 6 below), thereby inducing the NBFI to repay more on its interbank obligation (thus, the investment bank ψ's repayment rate $\bar{v}_2^\psi$ increases; see Table 6, third column). Consequently, the interbank rate decreases in response to a lower default premium.

As the credit spread between interbank and deposit rates narrows, bank γ shifts its portfolio from long-term (mortgage and interbank) to short-term loans. Moreover, since mortgage loans become scarcer, the price of MBSs increases, and induces the NBFI to securitize fewer of these assets.

Similarly, lower expected CDO returns (via a mortgage rate decline) induce bank δ to re-allocate its portfolio from derivatives to short- term loans. Overall, this strategy reduces the liquidity available to bank δ at the beginning of the second period, thus providing it with incentives to default more on its interbank loan (hence, the repayment rate of financial institution δ in state 2 - $\overline{v}_2^{\delta}$ - decreases; see Table 6, third column).

In the bad states of nature, tighter credit conditions (narrower credit spreads) and lower returns on long- term investments, prevent commercial banks from lending efficiently. Hence, banks' profits decrease in $s = 2$ (bank γ and δ's expected second period profits - π^{γ} and π^{δ}- fall, as well as their profits in the bad states, see Table 6 below, second column).

Note that in times of financial distress, accommodative monetary policy mitigates the housing crisis, but it achieves only a *partial* improvement on financial stability: although aggregate default decreases, profits in the banking sector fall. This result suggests that the Central Bank cannot manage the trade-off between price and financial stability with only one instrument at its disposal (the repo rate), and that financial stability should be primarily achieved through regulatory measures.

Table 6
Eects of Expansionary Monetary Policy at $s = 2$

Households Welfare		Banking Sector Profits		Repayment Rates	
U^{α}	–	π^{γ}	+	$\overline{v}_2^{\alpha}$	+
U^{θ}	–	π^{δ}	≈	$\overline{v}_2^{\delta}$	+
U_2^{α}	+	π_2^{γ}	–	$\overline{v}_2^{\psi}$	+
U_2^{θ}	–	π_2^{δ}	–		

$U^{\alpha} \equiv$ household α 's utility stream
$U^{\theta} \equiv$ household θ 's utility stream
$U_2^{\alpha} \equiv$ household α 's utility if $s \notin S_1^{\alpha}$
$U_2^{\theta} \equiv$ household θ 's utility if $s \notin S_1^{\alpha}$
$\pi^{\gamma} \equiv$ bank γ 's stream of profits
$\pi_2^{\gamma} \equiv$ bank γ 's profits if $s \notin S_1^{\alpha}$
$\pi^{\delta} \equiv$ bank δ 's stream of profits
$\pi_2^{\delta} \equiv$ bank δ's profits if $s \notin S_1^{\alpha}$
$\overline{v}_2^{\alpha} \equiv$ mortgage repayment rate if $s \notin S_1^{\alpha}$
$\overline{v}_2^{\delta} \equiv$ interbank repayment rate if $s \notin S_1^{\alpha}$
$\overline{v}_2^{\psi} \equiv$ investment bank's repayment rate if $s \notin S_1^{\alpha}$

Source: Authors calculations.

VI. CONCLUDING REMARKS

The framework presented in this paper incorporates heterogeneous agents, endogenous default, an essential role for money, and incomplete financial markets. Moreover, to understand the 2007-2009 financial crisis, I introduced collateral and securitization frictions to capture financial market innovations over the recent past. In stark contrast with mainstream macroeconomic models, these elements ensure that default (idiosyncratic risk) and contagion (systemic risk) arise as equilibrium outcomes, thereby justifying a role for welfare improving economic policy.

The simulation results shed light on the efficiency of different macro-prudential policies for crisis prevention and management. I analyze the effects of three different types of financial regulation: capital, margin, and liquidity requirements. Furthermore, since the Central Bank has a monopoly over the supply of money, I also assess the effects of Central Bank interventions in times of financial distress in order to make a comprehensive analysis of shocks to aggregate liquidity.

I find that in times of financial distress, accommodative monetary policy mitigates the housing crisis, but it achieves only a *partial* improvement on financial stability: although aggregate default decreases, profits in the banking sector fall. This result suggests that the Central Bank cannot manage the trade-off between price and financial stability with only one instrument at its disposal (the repo rate), and that financial stability should be primarily achieved through regulatory measures.

Capital requirements reduce leverage in the banking sector, and induce banks to internalize (default) losses without taking a toll on the taxpayer; margin requirements prevent excess leverage in the housing and derivatives markets, thus reducing/containing the adverse effects of the housing crisis via lower housing deflation rates; and liquidity requirements reduce banks´ exposure to risky assets, thereby promoting lending in times of financial distress and stemming house price deflation.

This model is a stepping-stone of my ongoing research. Agents' heterogeneity leads to greater complexity, with which greater reality is achieved. However, since the initial equilibrium has been chosen, the simulation outputs depend on the assumed inputs. Moreover, since the model has only two periods, it cannot track dynamic effects of shocks and policies.

This framework could be extended to an infinite horizon setting, and enriched with a stochastic structure for the different types of shocks, which would allow for an exercise in which the parameters of the model are calibrated to match actual data.

I have not modelled the production sector. By using a representation of an endowment economy, policy welfare implications are never clear-cut because some agents gain and others lose. Hence, in future work the model will be adjusted to a production economy setting, thereby allowing shocks to affect the redistribution and level of aggregate real income.

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APPENDIX

This appendix sets out the optimization procedures, the equilibrium of the baseline model and its properties.

AGENTS' BEHAVIOUR

Household α's Optimization Problem

Consumer α maximizes the utility derived from his consumption of goods and housing. He is endowed with goods and money in every state. At $t = 0$, α takes out a short- term loan and a mortgage to buy housing, and pledges the latter as collateral to the mortgage.

If a good state is realized in the second period, he takes out a short- term loan to buy more housing and pay back his mortgage. In contrast, in the bad states of the world, household α defaults on his mortgage, and his house is seized by bank γ. Nevertheless, he takes out yet another short- term loan to purchase housing because he still needs housing services.

Denote by $S_1^\alpha \subset S$ the set of states in which α honours his mortgage. Then

$$S_1^\alpha = \left\{ s \in S : \frac{b_{02}}{p_{02}} p_{s2} \geq \bar{\mu}^\alpha \right\}$$

where (b_{02}/p_{02}) is the amount of housing purchased at $t = 0$, p_{s2} is the price of housing at $s \in S_1^\alpha$ in the second period, and $\bar{\mu}^\alpha$ is the value of outstanding mortgage debt.

Finally, α repays his short- term obligations with the proceeds of goods sales at the end of each period. His maximization problem is as follows.

$$\max_{q_{s*1}, b_{s*2}, \mu_{s*}, \bar{\mu}} U^\alpha = u\left(\frac{b_{02}^\alpha}{p_{02}}\right) + u\left(e_{s1}^\alpha - q_{s1}^\alpha\right) + \sum_{s \in S} \varpi_s u\left(e_{s1}^\alpha - q_{s1}^\alpha\right)$$
$$+ \sum_{s \in S_1^\alpha} \varpi_s u\left(\frac{b_{02}^\alpha}{p_{02}} + \frac{b_{s2}^\alpha}{p_{s2}}\right) + \sum_{s \notin S_1^\alpha} \varpi_s u\left(\frac{b_{s2}^\alpha}{p_{s2}}\right) \tag{A.1}$$

s.t.

$$b_{02}^{\alpha} \leq \frac{\overline{\mu}^{\alpha}}{1+\overline{r}^{\gamma\alpha}} + \frac{\mu_0^{\alpha}}{1+r_0^{\gamma}} + e_{m,0}^{\alpha} \tag{A.2}$$

(i.e. housing expenditure at $t = 0 \leq$ mortgage loan + short-term borrowing + private monetary endowments at $t = 0$)

$$\mu_0^{\alpha} \leq p_{01}\, q_{01}^{\alpha} \tag{A.3}$$

(i.e. short- term loan repayment at $t = 0 \leq$ goods sales revenues at $t = 0$)

$$b_{s2}^{\alpha} + \overline{\mu}^{\alpha} \leq \frac{\mu_s^{\alpha}}{1+r_s^{\gamma}} + e_{m,s}^{\alpha}, \qquad for \quad s \in S_1^{\alpha} \tag{A.4}$$

(i.e. housing expenditure at $s \in S_1^{\alpha}$ + mortgage repayment $\leq$short-term borrowing + private monetary endowments at $s \in S_1^{\alpha}$)

$$b_{s2}^{\alpha} \leq \frac{\mu_s^{\alpha}}{1+r_s} + e_{m,s}^{\alpha}, \qquad for \quad s \notin S_1^{\alpha} \tag{A.5}$$

(i.e. housing expenditure at $s \notin S_1^{\alpha} \leq$ short-term borrowing + private monetary endowments at $s \notin S_1^{\alpha}$)

$$\mu_s^{\alpha} \leq p_{s1} q_{s1}^{\alpha}, \qquad for \quad s \in S \tag{A.6}$$

(i.e. short- term loan repayment $\leq$ goods sales revenues at t = 0)

$$q_{s^*1}^{\alpha} \leq e_{s^*1}^{\alpha} \tag{A.7}$$

(i.e. quantity of goods sold at $s^* \in S^* \leq$ goods endowments at $s^* \in S^*$)

where

$b_{s^*2}^{\alpha} \equiv$ fiat money spent by α to trade in the housing market in s^*
$q_{s^*1}^{\alpha} \equiv$ goods offered for sale by α in s^*
$\overline{\mu}^{\alpha} \equiv$ repayment value of the mortgage loan that γ extends to α
$\overline{r}^{\gamma\alpha} \equiv$ mortgage rate offered to α by bank γ
$r_{s^*}^{\gamma} \equiv$ short- term rate offered to α by bank γ in s^*

μ_{s*}^{α} ≡ short- term loan γ extends to α in s^*
p_{s*2} ≡ price of housing in state s^*
p_{s*1} ≡ price of goods in state s^*
e_{s*1}^{α} ≡ α's endowment of goods in state s^*
$e_{m,s*}^{h}$ ≡ monetary endowments of $h \in (\alpha,\theta)$ in state s^*
ϖ_s ≡ probability of state s

$u\ (x) = x^{1-c^h} \big/ \left(1-c^h\right) \equiv$ households have a CRRA utility function, where c^h is the risk aversion coefficient of $h \in H = \{\alpha,\theta\}$.

Household θ's Optimization Problem

Household θ is endowed with money in every state and with a large amount of housing at $t = 0$. He sells houses and buys goods in both periods. At the beginning of the first period, θ uses his cash inflows to buy goods and to make a long- term deposit with bank γ. In the second period, he uses the gross return on his deposits, his monetary endowment, and a short- term loan to purchase consumption goods.

Finally, θ repays his short- term obligations with the proceeds from housing sales at the end of each period. His maximization problem is as follows.

$$\max_{q_{s*2}, b_{s*1}, \mu_{s*}, \bar{d}} U^{\theta} = u\left(\frac{b_{01}^{\theta}}{p_{01}}\right) + u\left(e_{02}^{\theta} - q_{02}^{\theta}\right) + \sum_{s \in S} \varpi_s u\left(\frac{b_{s1}^{\theta}}{p_{s1}}\right) + \sum_{s \in S} \varpi_s u\left(e_{02}^{\theta} - q_{02}^{\theta} - q_{s2}^{\theta}\right) \tag{A.8}$$

s.t.

$$b_{01}^{\theta} + \bar{d}^{\theta} \leq \frac{\mu_0^{\theta}}{1 + r_0^{\delta}} + e_{m,0}^{\theta} \tag{A.9}$$

(i.e. goods expenditure at t = 0 + inter-period deposits ≤ short-term borrowing + private monetary endowments at t = 0)

$$\mu_0^{\theta} \leq p_{02} q_{02}^{\theta} \tag{A.10}$$

(i.e. short- term loan repayment at t = 0 ≤ housing sales revenues at t = 0)

$$b_{s1}^{\theta} \leq e_{m,s}^{\theta} + \frac{\mu_s^{\theta}}{1 + r_s^{\delta}} + \bar{d}^{\theta}\left(1 + \bar{r}_d^{\gamma}\right) \tag{A.11}$$

(i.e. goods expenditure at $s \in S \leq$ short-term borrowing + deposits and interest payment + private monetary endowments at $s \in S$)

$$\mu_s^\theta \leq p_{s2} q_{s2}^\theta \tag{A.12}$$

(i.e. short- term loan repayment at $s \in S \leq$ housing sales revenues at $s \in S$)

$$q_{s2}^\theta \leq e_{02}^\theta - q_{02}^\theta \tag{A.13}$$

(i.e. number of housing units sold at $s \in S \leq$ endowment of housing at t = 0 - units of housing sold at $s \in S$)

where

$b_{s^*1}^\theta \equiv$ fiat money spent by θ to trade in the goods market in s^*
$\mathrm{q}_{s^*2}^\theta \equiv$ amount of housing offered for sale by θ in s^*
$\overline{d}^\theta \equiv$ amount deposited deposits by θ in bank γ
$\mu_{s^*}^\theta \equiv$ short- term loan that δ extends to θ in s^*
$r_{s^*}^\delta \equiv$ short- term interest rate offered by bank δ to θ in s^*
$\overline{r}_d^\gamma \equiv$ deposit rate offered by bank γ on $\overline{d}^\theta$
$e_{s^*2} \equiv \theta$'s endowment of housing

Bank γ's Optimization Problem

Bank γ is a risk- averse agent that maximizes the utility provided by its expected second period profits. It has quadratic preferences over its expected profits, and a high level of capital endowments in the first period. Initially, bank γ interacts with the Central Bank in the repo market by entering into a reverse repurchase agreement, and makes short- term credit and mortgage extensions to household α. Then, it sells its mortgage assets to NBFI ψ, takes deposits from household θ, and makes credit extensions in the (long-term) interbank market.

In the second period, bank γ uses its firstperiod earnings and repayments on its interbank loans, to make short-term credit extensions to household α and bank δ in the consumer and repo markets respectively, and repay its depositors (θ). Bank γ's second period profits are the sum of the gross returns on its reverse repurchase agreement and short- term loans; these profits are taken over by the Central Bank at the end of t = 1.

$$\max_{m_{s*},\bar{m},d_{s*}^{G\gamma},\bar{d},d_d,\pi_{s*1}} \Pi^\gamma = \sum_{s\in S} \varpi_s \left(\pi_s^\gamma - c^\gamma \left(\pi_s^\gamma\right)^2\right) \tag{A.14}$$

s.t.

$$d_0^{G\gamma} + m_0^\gamma + \bar{m}^\alpha + \bar{d}^\gamma \le e_0^\gamma + \frac{\bar{\mu}_d^\gamma}{1+\bar{r}_d^\gamma} \tag{A.15}$$

(i.e. deposits in the repo market + short-term lending + mortgage extension + interbank lending ≤ capital endowment at t = 0 + consumer deposits)

$$m_s^\gamma + d_s^{G\gamma} + \bar{\mu}_d^\gamma \le K_s^\gamma + \bar{R}_s \bar{d}^\gamma \left(1+\bar{\rho}\ \right) + p^\alpha \bar{m}^\alpha \tag{A.16}$$

(i.e. short-term lending + deposits in the repo market at $s \in S$ + consumer deposits repayment ≤ bank's capital + interbank loan repayments $s \in S$ + MBSs sales revenues)

$$\pi_0^\gamma = m_0^\gamma \left(1+r_0^\gamma\right) + d_0^{G\gamma}\left(1+\rho_0^{CB}\right) \tag{A.17}$$

(i.e. profits at t = 0 = short- term loan repayment + repo deposits and interest payment at t = 0)

$$K_s^\gamma = e_s^\gamma + \pi_0^\gamma \tag{A.18}$$

(i.e. bank's capital at $s \in S$ = capital endowment at $s \in S$ + accumulated profits from previous period)

$$\pi_s^\gamma = m_s^\gamma \left(1+r_s^\gamma\right) + d_s^\gamma \left(1+\rho_s^{CB}\right) \tag{A.19}$$

(i.e. profits at $s \in S$ = short- term loan repayment + repo deposits and interest payment at $s \in S$)

where

$\pi_{s*}^\gamma \equiv$ bank γ's profits at state s^*
$\bar{m}^\alpha \equiv$ mortgage extension to α
$m_s^\gamma \equiv$ short- term credit extension to α in state s^*
$\bar{d}^\gamma \equiv$ long- term deposits in the interbank market by bank γ in s^*
$d_{s*}^{Gj} \equiv$ cash sent by bank $j \in J$ to enter a reverse repurchase agreement in state s^*

$\bar{\mu}_d^{\gamma} \equiv$ long- term borrowing by γ from household θ
$\bar{\rho}$ $\equiv$ long- term interbank market rate
ρ_{s*}^{CB} $\equiv$ short- term interest rate on government bonds in state s^*
e_{s*}^{j} $\equiv$ capital endowment of bank $j \in J$ in state s^*
p^{α} $\equiv$ price of MBSs sold to ψ
$K_s^j \equiv$ capital of bank $j \in J$ in state s
$\bar{R}_s$ $\equiv$ expected delivery rate on the interbank loans

Bank δ's Optimization Problem

Bank δ is also a risk- averse financial institution with quadratic preferences, which maximizes the utility provided by its expected second period profits.

At every $s \in S^*$, bank δ enters into a repurchase agreement with the Central Bank and uses household θ's short- term credit repayment to meet its repo market obligation. In the first period, bank δ borrows money in the interbank market, and buys CDOs from the NBFI (ψ); therefore, in the second period it uses the repayment on its CDO investment to meet its interbank market obligation. Bank δ's second period profits are equal to the repayment on its short- term loans, less its outstanding debt in the repo market. These earnings are liquidated by the Central Bank at the end of t = 1.

$$\max_{m_{s*}\mu_{s*}^{G\delta}\hat{m}^{\alpha},\bar{\mu}^{\delta},\mu_s^{\delta},\bar{v}_s^{\delta},\pi_s} \prod^{\delta} = \sum_{s\in S}\varpi_s\left(\pi_s^{\delta} - c^{\delta}\left(\pi_s^{\delta}\right)^2\right) - \sum_{s\in S}\varpi_s\bar{\tau}_s^{\delta}\left[\bar{D}_s^{\delta}\right]^+ \tag{A.20}$$

s.t.

$$m_0^{\delta} + \hat{m}^{\alpha} \le e_0^{\delta} + \frac{\mu_0^{G\delta}}{(1+\rho_0^{CB})} + \frac{\bar{\mu}^{\delta}}{(1+\bar{\rho})} \tag{A.21}$$

(i.e. short-term lending at t = 0 + CDOs investment ≤ capital endowment capital endowment + short-term borrowing in the repo market at t = 0 + interbank borrowing)

$$\mu_0^{G\delta} \le m_0^{\delta}\left(1 + r_0^{\delta}\right) \tag{A.22}$$

(i.e. repo loan repayment at t = 0 ≤ short-term loan repayment at t = 0)

$$m_S^{\delta} + \bar{v}_S^{\delta}\,\bar{\mu}^{\delta} \leq e_S^{\delta} + \hat{m}^{\alpha}\;\frac{(1+\bar{r}^{\gamma\alpha})}{\tilde{q}^{\alpha}} + \frac{\mu_S^{G\delta}}{(1+\rho_S^{CB})} \text{ for } s \in S_1^{\alpha} \tag{A.23}$$

(i.e. short-term lending + interbank loan repayment at $s \in S_1^{\alpha} \leq$ bank's capital endowment + CDO repayment + short-term borrowing at $s \in S_1^{\alpha}$)

$$m_s^{\delta} + \bar{v}_s^{\delta}\,\bar{\mu}^{\delta} \leq e_s^{\delta} + \hat{m}^{\alpha} + \frac{\mu_s^{G\delta}}{(1+\rho_s^{CB})} \text{ for } s \notin S_1^{\alpha} \tag{A:24}$$

(i.e. short-term lending + interbank loan repayment at $s \notin S_1^{\alpha} \leq$ bank's capital endowment + CDO repayment + short-term borrowing at $s \notin S_1^{\alpha}$)

$$\pi_s^{\delta} = m_s^{\delta}\left(1 + r_s^{\delta}\right) - \mu_s^{G\delta} \tag{A.25}$$

(i.e. profits at $s \in S$ = short- term loan repayment - repo loan repayment at $s \in S$)

where

$\pi_s^{\delta} \equiv$ bank δ's profits in state s
$m_{s*}^{\delta} \equiv$ mshort- term credit extended by δ to θ in state s^*
$\hat{m}^{\alpha} \equiv$ amount of money sent by δ to purchase CDOs from ψ
μ_{s*}^{Gj} amount due by bank $j \in J$ in the repo-market in state s^*
$\bar{\mu}^{\delta} \equiv$ long- term borrowing by δ in the interbank market
$\bar{v}_s^{\delta} \equiv$ repayment rate to γ on long- term interbank loans
$\bar{\tau}_s^{\delta} \equiv$ marginal disutility to δ for defaulting on its interbank loan in state s
$\bar{D}_s^{\delta} = \left(1 - \bar{v}_s^{\delta}\right)\bar{\mu}^{\delta} \equiv \delta$'s nominal value of long- term interbank debt due to default in state s.

NBFI ψ's Optimization Problem

The NBFI has risk- neutral preferences over its second period expected profits. It buys mortgage assets from bank γ, securitizes them into CDOs as explained in section 3, and sells these to bank δ. In t = 0, the NBFI finances the purchase of mortgage assets by taking out a loan in the interbank market, which is payable in the next period.

In the second period, if a bad state of the world is realized, ψ becomes the final owner of the mortgage's collateral and sells it in the housing market. Also, as the CDS leg of the CDO contract is executed, ψ returns to bank δ an amount equivalent to its initial investment[1]. However, if nature picks a good state in the second period, then ψ earns the premium for buying and securitizing mortgage assets $\left(\tilde{q}^{\alpha}/p^{\alpha}\right)$.

$$\max_{\tilde{m},\bar{\mu}^{\Psi},\bar{v}_s^{\Psi}} \Pi^{\Psi} = \sum_{s\in S} \varpi_s \pi_s^{\Psi} - \sum_{s\in S} \varpi_s \bar{\tau}_s^{\Psi}\left[\bar{D}_s^{\Psi}\right]^{+} \tag{A.26}$$

s.t.

$$\tilde{m}^{\alpha} \leq e_0^{\Psi} + \frac{\bar{\mu}^{\Psi}}{(1+\bar{\rho})} \tag{A.27}$$

(i.e. expenditure in the MBS's market ≤ capital endowments at t = 0 + interbank borrowing)

$$\pi_s^{\Psi} = e_s^{\Psi} + \frac{\tilde{m}^{\alpha}}{p^{\alpha}}\tilde{q}^{\alpha} - \bar{v}_s^{\Psi}\bar{m}^{\Psi} \leq \quad for \quad s \in S_1^{\alpha} \tag{A.28}$$

(i.e. profits at $s \in S_1^{\alpha}$ = capital endowment + CDO's sales revenues - interbank market loan repayment at $s \in S_1^{\alpha}$)

$$\pi_s^{\Psi} = e_s^{\Psi} + \tilde{m}^{\alpha}\frac{\tilde{q}}{p^{\alpha}}^{\alpha} + \bar{R}_s^{\alpha}\left(1+\bar{r}^{g\alpha}\right)\frac{\tilde{m}^{\alpha}}{p^{\alpha}} - \tilde{m}^{\alpha}\frac{\tilde{q}}{p^{\alpha}}^{\alpha} - -\bar{v}_s^{\Psi}\bar{\mu}^{\Psi} \quad \Leftrightarrow$$

$$\pi_s^{\Psi} = e_s^{\Psi} + \bar{R}_s^{\alpha}\left(1+\bar{r}^{\gamma\alpha}\right)\frac{\tilde{m}^{\alpha}}{p^{\alpha}} - \bar{v}_s^{\Psi}\bar{\mu}^{\Psi} \qquad \text{for } s \in S_1^{\alpha} \tag{A.29}$$

(i.e. profits at $s \notin S_1^{\alpha}$ = capital endowment + CDO's sales revenues + mortgage collateral sales - CDS settlement - interbank market loan repayment $s \in S_1^{\alpha}$)

where:

$\pi_s^{\Psi} \equiv$ bank ψ's profits at state s
$\tilde{m}^{\alpha} \equiv$ amount of money sent by ψ to purchase mortgage assets from γ
$\bar{\mu}^{\Psi} \equiv$ inter-period borrowing from γ
$\bar{v}_s^{\Psi} \equiv$ ψ's repayment rate on the loan extended by δ in state s

1 For simplicity I have abstracted from allowing the investment bank to default on its CDS obligation, which would capture counterparty risk in the derivatives markets.

$\bar{R}_s^{\alpha} = \dfrac{p_{22} b_{02}^{\alpha}}{\bar{\mu}^{\alpha} p_{02}} \equiv$ expected effective mortgage repayment rate in state $s \notin S_1^{\alpha}$

$\tilde{q}^{\alpha} \equiv$ price of CDO

$\bar{\tau}_s^{\Psi} \equiv$ marginal disutility to ψ for defaulting on the long- term loan with δ in state s

$\bar{D}_s^{\Psi} = \left(1 - \bar{v}_s^{\Psi}\right) \bar{\mu}^{\Psi} \equiv$ ψ's nominal value of interbank debt due to default in state s

Market Clearing Conditions

There are 9 markets in the economy: the goods, housing, mortgage, short- term loans, consumer deposit, repo, interbank, MBS and CDO markets. In each of these markets, the price equating supply and demand is determined.

Goods Market

In every state-period, the goods market clears when the amount of money offered for goods is exchanged for the quantity of goods offered for sale.

$$p_{01} = \frac{b_{01}^{\theta}}{q_{01}^{\alpha}} \tag{A.30}$$

$$p_{s1} = \frac{b_{s1}^{\theta}}{q_{s1}^{\alpha}} \quad for \quad s \in S \tag{A.31}$$

Housing Market

In every state-period, the housing market clears when the amount of money offered for housing is exchanged for the quantity of housing offered for sale. In every $s \notin S^{\alpha}{}_1$, since agent α defaults on his mortgage, the amount of housing he pledged as collateral in the previous period is also offered for sale by the investment bank.

$$p_{02} = \frac{b_{02}^{\alpha}}{q_{02}^{\theta}} \tag{A.32}$$

$$p_{s2} = \frac{b_{s2}^{\alpha}}{q_{s2}^{\theta}} \quad for \quad s \in S_1^{\alpha} \tag{A.33}$$

$$p_{s2} = \frac{b_{s2}^{\alpha}}{q_{s2}^{\theta} + b_{02}^{\alpha} / p_{02}} \quad for \quad s \notin S_1^{\alpha} \tag{A.34}$$

Mortgage Market

The mortgage market clears when the amount offered to be repaid in the second period is exchanged for the mortgage extension offered in the first period.

$$\left(1+\bar{r}^{\gamma\alpha}\right) = \frac{\bar{\mu}^{\alpha}}{\bar{m}^{\alpha}} \tag{A.35}$$

The effective return on the mortgage at any state in the second period is defined as:

$$\left(1+\bar{r}_s^{\gamma\alpha}\right) = \frac{\min\left\{p_{22}\left(b_{02}^{\alpha}/p_{02}\right), \bar{\mu}^{\alpha}\right\}}{\bar{m}^{\alpha}}$$

therefore, the clearing conditions for effective returns on mortgages is given by:

$$\left(1+\bar{r}_s^{\gamma\alpha}\right) = \begin{cases} \left(1+\bar{r}^{\gamma\alpha}\right) & for \quad s \in S_1^{\alpha} \\ \\ \dfrac{p_{22} b_{02}^{\alpha}}{p_{02}} \left(\dfrac{\bar{\mu}^{\alpha}}{1+\bar{r}^{\gamma\alpha}}\right)^{-1} & for \quad s \notin S_1^{\alpha} \end{cases} \tag{A.36}$$

Short-term consumer credit markets

For any state-period, short-term consumer credit markets clear when the amount offered to be repaid at the end of the period is exchanged for the short- term credit extension offered at the beginning of that period.

$$\left(1+r_{s*}^{\gamma}\right) = \frac{\mu_{s*}^{\alpha}}{m_{s*}^{\gamma}} \tag{A.37}$$

$$\left(1+r_{s*}^{\delta}\right) = \frac{\mu_{s*}^{\theta}}{m_{s*}^{\delta}} \tag{A.38}$$

Deposit market

The consumer deposit market clears when the amount commercial banks offer to repay to households in the second period is exchanged for the amount of savings offered to deposit in the first period.

$$\left(1+\bar{r}_d^{\gamma}\right)=\frac{\bar{\mu}_d^{\gamma}}{\bar{d}^{\theta}} \tag{A.39}$$

Repo market

In every state-period, the repo market clears when the amount offered to be repaid at the end of the period is exchanged for the short- term credit extension and the liquidity provided by the Central Bank (through OMOs) at the beginning of the period.

$$\left(1+\rho_{s*}{}^{CB}\right)=\frac{\mu_{s*}^{G\delta}}{M_{s*}^{CB}+d_{s*}^{G\gamma}} \tag{A.40}$$

Interbank market

The interbank market clears when the amount offered to be repaid in the second period is exchanged for the long- term credit extension in the first period.

$$\left(1+\bar{\rho}\right)=\frac{\bar{\mu}^{\delta}+\bar{\mu}^{\psi}}{\bar{d}^{\gamma}} \tag{A.41}$$

MBSs market

The MBS market clears when the amount of money offered for these securities is exchanged for the quantity of MBSs offered for sale.

$$p^{\alpha}=\frac{\tilde{m}^{\alpha}}{\bar{m}^{\alpha}} \tag{A.42}$$

CDO market

The CDO market clears when the amount of money offered for these securities is exchanged for the quantity of CDOs offered for sale.

$$\tilde{q}^{\alpha} = \frac{\hat{m}^{\alpha}}{\tilde{m}^{\alpha} / p^{\alpha}} \tag{A.43}$$

Conditions on Expected Delivery Rates (Rational Expectations)

Rational expectations conditions imply that commercial banks are correct in their expectations about the fraction of loans that will be repaid to them. The expected rate of interbank market loan delivery for bank γ is given by:

$$\bar{R}_s = \begin{cases} \dfrac{\bar{v}_s^{\Psi}\bar{\mu}^{\Psi} + \bar{v}_s^{\delta}\bar{\mu}^{\delta}}{\bar{\mu}^{\Psi} + \bar{\mu}^{\delta}} & if \quad \bar{\mu}^{\Psi} + \bar{\mu}^{\delta} > 0 \\ \\ arbitrary & if \quad \bar{\mu}^{\Psi} + \bar{\mu}^{\varphi} = 0 \qquad \forall\, s \in S \end{cases} \tag{A.44}$$

Equilibrium Definition

Let

$$\sigma^{\alpha} = \left(q_{s1}^{\alpha}, b_{s2}^{\alpha}, \mu_s^{\alpha}, \bar{\mu}^{\alpha}\right) \in \Re^{s+1} \times \Re^{s+1} \times \Re^{s+1} \times \Re\,;$$
$$\sigma^{\theta} = \left(q_{s2}^{\theta}, b_{s1}^{\theta}, \mu_s^{\theta}, \bar{d}^{\theta}\right) \in \Re^{s+1} \times \Re^{s+1} \times \Re^{s+1} \times \Re\ ;$$
$$\sigma^{\gamma} = \left(\pi_s^{\gamma}, m_s^{\gamma}, d_s^{G\gamma}, \bar{m}^{\alpha}, \bar{\mu}_d^{\gamma}, \bar{d}^{\gamma}\right) \in \Re^{s} \times \Re^{s+1} \times \Re^{s+1} \times \Re \times \Re \times \Re\,;$$
$$\sigma^{\delta} = \left(\pi_s^{\delta}, m_s^{\delta}, \mu_s^{G\gamma}, \bar{v}_s^{\delta}, \hat{\mu}^{\alpha}, \bar{\mu}^{\delta}\right) \in \Re^{s} \times \Re^{s+1} \times \Re^{s+1} \times \Re^{s} \times \Re \times \Re;$$
$$\sigma^{\Psi} = \left(\bar{v}_s^{\Psi}, \bar{\mu}^{\Psi}, \tilde{m}^{\alpha}\right) \in \Re^{s} \times \Re \times \Re\,;$$

Also, let the vector of macroeconomic variables be represented by:

$$\eta = \left(p_{s1}, p_{s2}, \rho_s^{CB} r_s^{\gamma}, r_s^{\delta}, \bar{r}^{\gamma\alpha}, \bar{r}_d^{\gamma}, \bar{\rho}, p^{\alpha}, \tilde{q}^{\alpha}\right)$$
$$\in \Re^{s+1} \times \Re^{s+1} \times \Re^{s+1} \times \Re^{s+1} \times \Re^{s+1} \times \Re \times \Re \times \Re \times \Re \times \Re$$

and the budget set of all agents be denoted by:

$B^{\alpha}(\eta) = \{\sigma^{\alpha}: (2) - (7) hold\}$
$B^{\theta}(\eta) = \{\sigma^{\theta}: (9) - (13) hold\}$

$B^{\gamma}(\eta) = \{\sigma^{\gamma}: (15) – (19) hold\}$
$B^{\delta}(\eta) = \{\sigma^{\delta}: (21) – (25) hold\}$
$B^{\psi}(\eta) = \{\sigma^{\psi}: (27) – (29) hold\}$

Then $(\sigma^{\alpha}, \sigma^{\theta}, \sigma^{\gamma}, \sigma^{\delta}, \sigma^{\psi}, \eta)$ is a monetary equilibrium with commercial banks, collateral, securitisation, and default (MEBCSD) if:

(i) All agents optimise given their budget sets:

(a) $\sigma^{h} \in \arg\max_{\sigma^{h} \in B^{h}(h)} U^{h}\left(c_{s*}{}^{h}\right), h \in H = \{\alpha, \theta\}, s^{*} \in S^{*}$
(b) $\sigma^{j} \in \arg\max_{\sigma^{j} \in B^{j}(\eta)} \Pi^{j}\left(\pi_{s}{}^{j}\right), j \in J = \{\gamma, \delta\}, s \in S$
(c) $\sigma^{k} \in \arg\max_{\sigma^{k} \in B^{k}(\eta)} \Pi^{k}\left(\pi_{s}{}^{k}\right), k = \{\Psi\}, s \in S$

Where $\chi_{s*}{}^{h}$ is the vector of quantities of housing and goods consumed by agent h at state $s^{*} \in S^{*}$, $U^{h}(.)$ is households' utility function over consumption streams of goods and houses, and $\Pi(.)$ is the commercial banks and investors' utility function over their second period profits.

(ii) All markets clear. Hence, equations (30) – (43) hold.
(iii) Expectations are rational. Thus, conditions (44) are satisfied.

Properties of the MEBCSD

At each market meeting, money is exchanged for another commodity or security. Hence, the traditional transaction motive for holding money and the standard Hicksian IS/LM determinants of money demand, namely interest rates and income, are at work in this model.

Credit spreads, and the term structure of interest rates proposition

Banks' portfolios and default determine the money multiplier in the economy, as well as credit spreads. Since ex-ante interest rates are considered, in the presence of default, borrowing rates have to be at least as high as lending rates to preclude arbitrage opportunities.

Similarly, the term- structure of interest rates is affected by aggregate liquidity and default, because interest rates pricein anticipated default rates (default premium).

Put formally, $\forall\, s \in S$ aggregate ex-post interest rate payments to commercial banks adjusted by default equal the economy's total amount of outside money, plus interest payments of commercial banks' accumulated profits. This is not the case in the first period, where uncertainty induces commercial banks to accumulate profits and/or make investments in the derivatives markets; hence, aggregate interest payments will be less than, or equal to. aggregate initial monetary endowments.

Monetary policy non-neutrality proposition

I have introduced two nominal frictions to the model: private monetary endowments and default on credit markets, which ensure positive nominal interest rates by pinning down the price of money (Dubey and Geanakoplos, 1992; Shubik and Wilson, 1977; Shubik and Tsomocos, 1992; Espinoza, Goodhart and Tsomocos, 2008). Consequently, there is a wedge between selling and purchasing prices; hence, monetary policy is non-neutral.

The Quantity Theory of Money proposition

In this model, a non-trivial quantity theory of money holds. An agent will not hold idle cash he does not want to spend; instead, he will lend it out to someone who is willing to use it. It follows that if all the interest rates are positive, then in equilibrium the quantity theory of money holds with money velocity equal to one. Moreover, since quantities supplied in the markets are chosen by agents (unlike the representative agent model's sell-all assumption), the real velocity of money is endogenous. Consequently, nominal changes affect both prices and quantities.

At each state in the second period, nominal income equals the stock of money because all the liquidity available in the economy is channelled to commodity markets. However, at $t = 0$ uncertainty and the inability of agents to complete the asset span will induce commercial banks to accumulate profits and/or make investments in the derivatives markets.

The Fisher Effect proposition

The model has an integral monetary sector where equilibrium interest rates are determined in nominal terms. Therefore, long- term nominal interest rates equal their corresponding real interest rate plus the expected rate of inflation and a risk premium.

See Tsomocos (2003) or Goodhart etal. (2006) for a sketch of the proofs of these propositions.

EXTENSIONS

In this section I describe how the implementations of different regulatory frameworks affect the behaviour of agents.

Capital Adequacy Ratios

Let the FSA set the minimum capital requirement $\bar{k}$, as well as the penalties for violating it $\left(\lambda_s^j\right)$, for $s \in S$ and $j \in J$. This policy changes the behaviour of commercial banks as their objective functions now take into account the reputational cost they bear if they violate the FSA's capital requirement. Thus, bank utility γ function (equation 14) becomes:

$$\max_{m_{s*},\bar{m},d_{s*}^{G\gamma},\bar{d},d_d,\pi_{s*1}} \Pi^\gamma = \sum_{s\in S} \varpi_s \left(\pi_s^\gamma - c^\gamma \left(\pi_s^\gamma\right)^2 - \lambda_s^\gamma \max\left\{0,\left(\bar{k} - k_s^\gamma\right)\right\}\right)$$

s.t.

$$k_s^\gamma = \frac{K_s^\gamma}{\xi_1 \bar{R}_s \left(1+\bar{\rho}\right)\bar{d}^\gamma + \xi_2 p^\alpha \bar{m}^\alpha}, \xi_1 \geq \xi_2$$

where

$k_s^j \equiv$ capital adequacy ratio of bank $j \in J$ in state $s \in S$
$\xi_1^s \equiv$ risk weight on interbank loans
$\xi_2 \equiv$ risk weight on mortgage loans
$\lambda_s^j \equiv$ capital requirement violation penalty for bank $j \in J$ in state $s \in S$
$\bar{k} \equiv$ minimum capital requirement

Similarly, bank δ' s utility function (equation 20) changes as follows:

$$\max_{m_{s*}\mu_{s*}^{G\delta} \hat{m}^\alpha,\bar{\mu}^\delta,\mu_s^\delta,\bar{v}_s^\delta,\pi_s} \Pi^\delta = \sum_{s\in S} \varpi_s \left(\pi_s^\delta - c^\delta\left(\pi_s^\delta\right)^2 - \bar{\tau}_s^\delta\left[\bar{D}_s^\delta\right]^+ - \lambda_s^\delta \max\left\{0,\left(\bar{k} - k_s^\delta\right)\right\}\right)$$

s.t.

$$k_s^{\delta} = \frac{e_s^{\delta}}{\xi_3 \hat{m}^{\alpha}\left(1+\overline{r}^{\gamma\alpha}\right)/\widetilde{q}^{\alpha}} \text{ for } s \in S_1^{\alpha}$$

$$k_s^{\delta} = \frac{e_s^{\delta}}{\xi_3 \hat{m}^{\alpha}} \text{ for } s \notin S_1^{\alpha}$$

where:

$\xi_3 \equiv$ risk weight on CDOs

Margin Requirements

Let the FSA impose the following constraints, a maximum Loan-to-value (LTV) ratio on mortgage borrowing (χ^{LTV}), and a minimum margin requirement for interbank lending to the NBFI sector (χ^{MG}).

As the FSA sets the maximum LTV ratio, mortgage borrowers can only obtain credit extensions for a fraction of *at most* χ^{LTV} of the value of the house they intend to buy. Consequently, in addition to constraints (2-7), household α is now subject to the following restriction:

$$\frac{\overline{\mu}^{\alpha}}{1+\overline{r}^{\gamma\alpha}} \leq \chi^{LTV} b_{02}^{\alpha}, 0 \leq \chi^{LTV} \leq 1$$

(i.e. mortgage borrowing ≤ Loan-to-value × value of housing at $t = 0$).

where:

$\chi^{LTV} \equiv$ Loan-to-value ratio.

The FSA also imposes a minimum margin requirement on NBFI interbank borrowing. Under this measure, non-banking investors are obliged to put up capital for *at least* fraction χ^{MG} of the value of the asset they intend to purchase. Therefore, NBFI ψ is subject to the following additional constraint:

$$\frac{\overline{\mu}^{\Psi}}{1+\overline{\rho}} \leq \left(1-\chi^{MG}\right)\tilde{m}^{\alpha}, 0 \leq \chi^{MG} \leq 1$$

(i.e. interbank borrowing ≤ (1-margin requirement) × MBSs investment value).

where:

$\chi^{MG} \equiv$ Margin or haircut requirement on interbank lending to the NBFI sector.

Liquidity Ratios

Suppose the FSA issues a regulation that requires commercial banks to maintain a minimum liquidity ratio $\left(\bar{L}\right)$. If banks violate this requirement, then they will assume a reputational cost $\bar{\lambda}^j$ for $j \in J$, which is determined by the Regulator. As with capital requirements, this policy changes the objective function of commercial banks.
The utility of bank γ becomes:

$$\max_{m_{s*},\bar{m},d_{s*}^{G\gamma},\bar{d},d_d,\pi_{s*1}} \Pi^{\gamma} = \sum_{s\in S} \varpi_s \left(\pi_s^{\gamma} - c^{\gamma}\left(\pi_s^{\gamma}\right)^2 - \bar{\lambda}^{\gamma} \max\left\{0,\left(\bar{L} - L^{\gamma}\right)\right\}\right)$$

s.t.

$$L^{\gamma} = \frac{m_0^{\gamma}\left(1+r_0^{\gamma}\right) + d_0^{G\gamma}\left(1+\rho_0^{CB}\right)}{m_0^{\gamma}\left(1+r_0^{\gamma}\right) + d_0^{G\gamma}\left(1+\rho_0^{CB}\right) + \left(1+\bar{\rho}\right)\bar{d}^{\gamma}\sum_{s\in S}\omega_s \bar{R}_s + p^{\alpha}\bar{m}^{\alpha}}$$

where:

$L^j \equiv$ liquidity ratio of bank $j \in J$

Likewise, bank δ now maximizes:

$$\max_{m_{s*}\mu_{s*}^{G\delta}\hat{m}^{\alpha},\bar{\mu}^{\delta},\mu_s^{\delta},\bar{v}_s^{\delta},\pi_s} \Pi^{\delta} = \sum_{s\in S}\varpi_s\left(\pi_s^{\delta} - c^{\delta}\left(\pi_s^{\delta}\right)^2 - \bar{\tau}_s^{\delta}\left[\bar{D}_s^{\delta}\right]^{+} - \bar{\lambda}^{\delta}\max\left\{0,\left(\bar{L}-L^{\delta}\right)\right\}\right)$$

s.t.

$$L^{\delta} = \frac{m_0^{\delta}\left(1+r_0^{\delta}\right)}{m_0^{\gamma}\left(1+r_0^{\gamma}\right) + \hat{m}^{\alpha}\left(\varpi_1\left(1+\bar{r}^{\gamma\alpha}\right)/\tilde{q}^{\alpha} + \varpi_2\right)}$$