Box 2

FORMATION OF INFLATION EXPECTATIONS IN COLOMBIA

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For central banks that have adopted an inflation targeting strategy, measuring and understanding inflation expectations are important tasks when designing monetary policy, because they influence the dynamics of price increases in a number of ways. Therefore, if expectations are anchored to the target, it is less costly to keep inflation low, future developments in prices and wages are more predictable, and it is easier to implement monetary policy.

Several econometric exercises are presented in this section to evaluate whether or not expectations in Colombia are rational in a linear sense. How much the inflation target contributes to the formation of expectations is estimated as well, and we assess if agents who form their expectations adaptively, based on the inflation target, follow a learning process.

Five sources of information on inflation expectations are used in the exercises; namely, the monthly survey of economic analysts, the quarterly survey of economic expectations, [1] break-even inflation one year forward (BEI), [2] forward break-even inflation at two years,[3] (FBEI two years), and the inflation target.[4]

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1 (Note 1, Box 2) The survey is a quarterly. The expectations one year forward were obtained for each month, through linear interpolation.

2 (Note 2, Box 2) These expectations are constructed on the basis of government bonds (TES). The break-even point is the difference between the spot rates on nominal and real bonds. In other words, it is the compensation required for the yield on both types of bonds to be the same for a given maturity.

3 (Note 3, Box 2) Constructed based on government bonds (TES). It indicates the one-year forward inflation expectations agents have during a particular a year.

4 (Note 4, Box 2) The mid-point of the target range was used. The monthly path of the inflation target one year forward was obtained through linear interpolation.

A. Are inflation expectations in Colombia formed rationally?

An initial step in understanding the development of inflation expectations is to know if they are formed rationally; that is, if agents use all the information at hand to predict how much consumer prices (CPI) will increase in the future. In this context, inflation expectations formed at time t for s periods in advance ($\pi^e_{t+s/t}$) can be defined as the expected value (E_t) of inflation (π_{t+s}) subject to all available data (Ωt) (Equation 1). If that expected value were obtained with regression models, the expectation would be rational in a linear sense.[5]

$$\pi e_{t+s/t} = E_t \left[\pi_{t+s} / \Omega_t \right] (1)$$

A first step in analyzing if there are inflation expectations in Colombia that match definition (1) is to evaluate whether or not a long-term relationship exists between the projections ($\pi^e_{t+s/t}$) and actual inflation ($\pi\pi_{t+s}$). As illustrated in Table B2.1, all inflation expectations are cointegrated with registered inflation, a fact that allows for accepting the hypothesis of a long-term relationship.[6]

Table B2.1
Unit Root and Cointegration Tests

Inflation Expectation	Unit root	Cointegration with π
Analysts 1 year forward	Yes	Yes
Quarterly 1 year forward	Yes	Yes
BEI 1 year forward	Yes	Yes
FBEI 2 years forward	Yes	Yes
Target 1 year forward	Yes	Yes

Source: Authors' calculations.

Another desirable property to meet the definition in equation (1) is when the inflation expectation gener-

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ates unbiased forecasts of the increase in the CPI. A linear regression model is estimated between expected and actual inflation to evaluate this property (Equation 2). In addition, the null hypothesis Ho: $\alpha = 0$ y $\beta = 1$ was verified to assess if inflation expectations by themselves generate unbiased projections of actual inflation. As indicated in Table B2.2, the hypothesis is not rejected in any of the measurements, with the exception of expectations derived from government bonds

$$\pi_t = \alpha + \beta\pi^e_{t/t-j} + \mu_t \quad (2)$$

Table B2.2
Unbiasedness Tests
Inflation Expectations

Inflation Expectations	Ho: $\alpha = 0$ y $\beta = 1$
Analysts 1 year forward	Unbiased
Quarterly 1 year forward	Unbiased
BEI 1 year forward	Not unbiased
FBEI 2 years forward	Not unbiased
Target 1 year forward	Unbiased

Source: Authors' calculations.

If an expectation is rational, it should not be possible to explain the forecast errors $(\pi_t - \pi^e_{t/t-j})$ by means of any other variable (z_t). However, if is, it would indicate the agents were not efficient because not all the data was used to form their expectation. To assess this, multiple models were estimated to explain the errors (Equation 3) and the set of explicative variables zt was used, as described in Table B2.3. Then, hypothesis Ho: $c=\lambda=0$ was tested. If accepted, it would indicate there was efficiency. In other words, variable zt does not explain the error and it was included in formation of the expecta-

tion. As illustrated in the same table, the one-year forward BEI was the only "inefficient" expectation, since real activity (measured by a dynamic factor) can explain the forecast errors.

$$\pi_t - \pi^e_{t/t-j} = c + \lambda z_t + \eta_{t+s} \quad (3)$$

In short, the inflation expectations of analysts and those from the quarterly survey, as well as the expectations based on the inflation target have enough desirable properties to be considered rational expectations. Those derived from government bonds (the one-year forward BEI and the two-year forward FBEI), by themselves, do not generate unbiased forecasts of actual inflation. Furthermore, the one-year forward BEI has efficiency problems.

B. What is the importance of the inflation target when forming expectations?

As mentioned earlier, using the inflation target to predict inflation in the future can be considered a rational expectation. However, agents can correct that expectation adaptively by observing previous forecast errors. This is modeled in equation 4, where the coefficient c_1 represents the importance agents would give to the inflation target (π_{t+s}). The complement $(1-c_1)$ would be the weight of the projection made p periods earlier $(\pi^e_{t/t-p})$, corrected by the error against actual inflation $c_2 (\pi_t - \pi^e_{t/t-p})$.

$$\pi^e_{t+s/t} = c_1\pi_{t+s} + (1-c_1)[\pi^e_{t/t-p} + c_2 (\pi_t - \pi^e_{t/t-p})] \quad (4)$$

Table B2.3
Efficiency Tests

Inflation Expectations	Analysts 1 year forward	Quarterly 1 year forward	BEI 1 year forward	FBEI 2 years forward	Target 1 year forward
Inflation in t	Not rejected	Not rejected	Not rejected	Not rejected	Not rejected
Target I + h	Not rejected	Not rejected	Not rejected	Not rejected	n. a.
GDP GAP	Not rejected	Not rejected	Not rejected	Not rejected	Not rejected
IIR	Not rejected	Not rejected	Not rejected	Not rejected	Not rejected
F_prices	Not rejected	Not rejected	Not rejected	Not rejected	Not rejected
F-monetary	Not rejected	Not rejected	Not rejected	Not rejected	Not rejected
F-real activity	Not rejected	Not rejected	Rejected	Not rejected	Not rejected
F_nat.accts	Not rejected	Not rejected	Not rejected	Not rejected	Not rejected
F_external	Not rejected	Not rejected	Not rejected	Not rejected	Not rejected

n.a. Not applicable.
Source: Authors' calculations.

Graph B2.1
Evolution of the C1 Coefficient
Proxy for the Rational Expectation Variable:
Inflation Target t + 12
Correction with Forecast Error in t -1
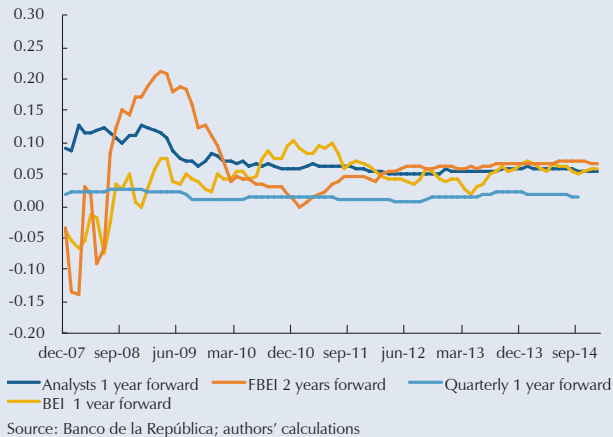
Source: Banco de la República; authors' calculations

Graph B2.2
C1 Coefficient
Proxy for the Rational Expectation Variable:
Inflation Target t + 12
Correction with Forecast Error in t -p
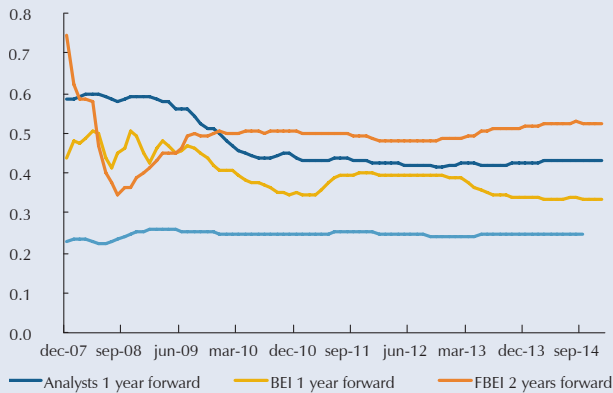
Source: Banco de la República; authors' calculations

Graphs B2.1 and B2.2 show the results in terms of the importance agents give to the inflation target (c_1) when forming expectations about inflation one year forward ($s = 12$) and regularly correct their forecasts made one month ($p = 1$) or one year ($p = 12$) before. The first conclusion is that, when agents continually correct forecasts made one month before ($p = 1$), the weight of the target is low (for example, in 2014 $0\% \leq c_1 \leq 10\%$); the importance of the target increases when the correction is made using the forecast error from the projections made one year earlier (for example, in 2014 and ($p = 12$), $25\% \leq c_1 \leq 55\%$). Also, generally speaking, it was found that agents with more access to information (eco-

nomic analysts and stock brokers) apparently give more importance to the inflation target than other agents (quarterly survey).[7]

C.   Is there adaptive learning with respect to the inflation target?

The adaptive learning hypothesis assumes agents do not know the structure of the economy and behave as econometricians to forecast the annual increase in the CPI (π^e). To do so, they create a prediction rule (Equation 5) and update its parameters ($\Phi_{0,t-1}, \Phi_{1,t-1}$) when they know the estimates error and use the newly available information. As agents learn about the structure of the economy, their expectations converge towards rational ones. Using econometric methods,[8] the parameters that best explain inflation expectations are measured at each point in time, and it is determined if learning has occurred.

$$\pi^e_{t/t-j} = \Phi_{0,t-1} + \Phi_{1,t-1}\pi_t + \varepsilon_t \quad (5)$$

Graphs B2.3 through B2.6 show the estimated values for $\Phi_{0,t}$ and $\Phi_{1,t}$ for each of the expectations variables analyzed. In the case of analysts' expectations and the quarterly survey, these values indicate the adaptive law of motion that best explained the forecasts prior to 2008 pointed to inflation expectations above target ($\Phi_{0,t} = 0$ y $\Phi_{1,t} > 1$). Since then, parameter $\Phi_{1,t}$ has declined and currently, in statistical terms, it is not possible to reject that its value is ($\Phi_{1,t} = 1$). The latter is indicative, to some extent, of inflation expectations anchored to the target. In the case of inflation expectations derived from government bonds, parameter $\Phi_{1,t}$ has grown in the last five years and, considering the confidence intervals, it is statistically the same as 1.

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7   (Note 7, Box 2) This survey is applied to commerce, industry, academics and consultants, transport and communications, the financial sector and labor unions.

8   (Note 8, Box 2) The parameters are updated as follows when using least squares.
$\widehat{\Phi}_t = \widehat{\Phi}_{t-1} + vR_{t-1}^{-1} X'_{t-(2j+1)} (\pi_{t-j} - X'_{t-(2j-1)}\widehat{\Phi}_{t-(j+1)})$, donde $X_t = [1, \bar{\pi}_t]$ y $\Phi_t = [\Phi_{0,t-1}, \Phi_{1,t-1}]'$ $R_t = R_{t-1} + v(X_{t-(2j-1)} X'_{t-(2j-1)} - R_{t-1})$. R_t is the matrix of second moments and v is the gain or learning. The method consists of simulating several series of πe with different combinations of v and Φ. The objective is to find a combination of initial values and gain parameters to replicate each agent's inflation expectations as closely as possible.

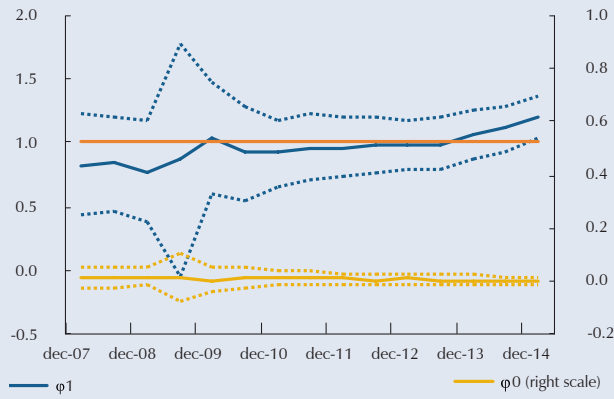Graph B2.3
Analysts Expectations 1 Year Forward
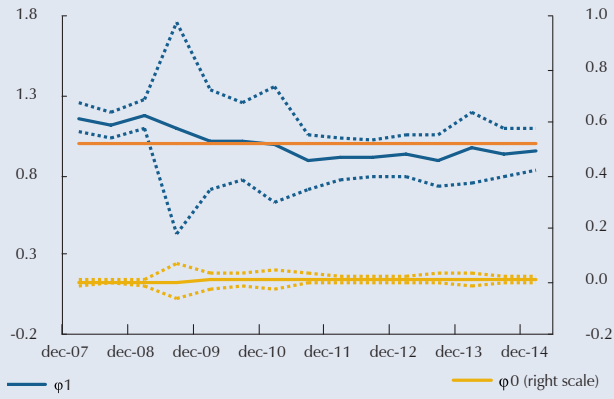
Source: Banco de la República; authors' calculations

Graph B2.4
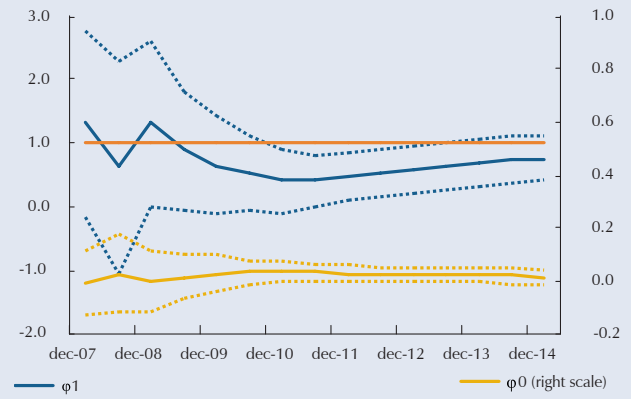Inflation Expectations in the Quarterly Survey 1 Year Forward
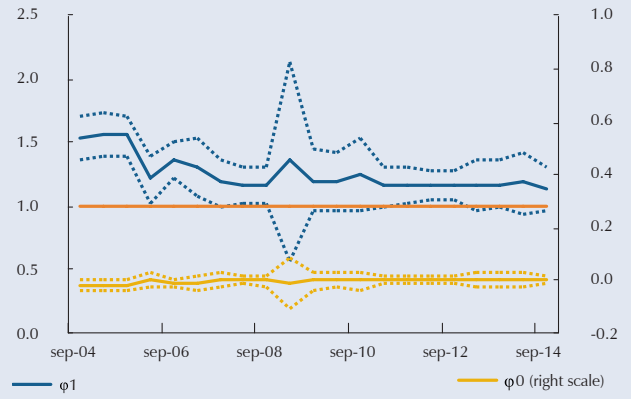
Source: Banco de la República; authors' calculations

Graph B2.5
Break Even Inflation – 1 Year Forward

Source: Banco de la República; authors' calculations

Graph B2.6
Forward Break Even Inflation 2 Years Forward

Source: Banco de la República; authors' calculations