Measuring the systemic importance of a financial institution

Carlos Castro, Juan Sebastián Ordoñez and Stijn Ferrari

Universidad del Rosario and National Bank of Belgium

Universidad del Rosario, October 6, 2011

Introduction.

1. Define the concepts of systemic risk and systemic importance, discusses the challenges of measuring them and mentions the different approaches that have been proposed.

2. We develop a series of testing procedures, based on a particular Market information based approach to identify and rank the systemically important institutions. We stress the importance of statistical testing (to complement economic significance) in interpreting the measure of systemic importance.

3. Discuss preliminary results on use of balance-sheet counterparty information from the Colombian financial system, to map the interdependencies between the institutions.

Systemic Risk and Systemic Importance.

$\diamond$ Financial Crisis (2007,2008).

$\diamond$ Basel 2 (Individual Resilience) $\rightarrow$ Basel 3 (systemic approach) wrt Financial Institutions.

$\diamond$ Δ Macro-prudential supervision and Regulation.

$\diamond$ Identify Systemically Important Financial Institutions (SIFI's) Tax or Capital surcharge.

We argue that Systemic risk and SIFI's are distinct and have different drivers, hence any measurement must take this issue into account.

Systemic Risk and Systemic Importance.

Definition: Systemic Risk (Acharya et al. (2009, p.283) and IMF/BIS/FSB (2009, p.2))
"the risk of a crisis in the financial sector and its spillovers to the economy at large" or "a risk of disruption to financial services that is (i) caused by an impairment of all or parts of the financial system and (ii) has the potential to have serious negative consequences for the real economy".
Definition: Systemic Importance of a Financial Institution (IMF/BIS/FSB (2009, p.8)) SI of financial institutions depends on *"their potential to have a large negative impact on the financial system and the real economy."*

The impact results from: **Spillover and contagion effects**.

Systemic Risk and Systemic Importance: Drivers

Level of Systemic Risk:

1. Individual Default Probabilities.
2. Dependence across defaults.

   a) Common exposures: portfolios vulnerable similar risk factors.

   b) Spillover Channels: Direct (interbank mkt, counterparty relations) and Indirect (asset fire sales, imperfect and asymmetric information, negative feedback loops).

Systemic Importance of a Financial Institution:

1. Default Probabilities Institution in question: Not really, sound banks may be SI.
2. Default Probabilities of other Institution in system: Strengthen the effect, but no driver.
3. Dependence across defaults.

   a) Common exposures: Strengthen the effect, complicates measure in particular identification.

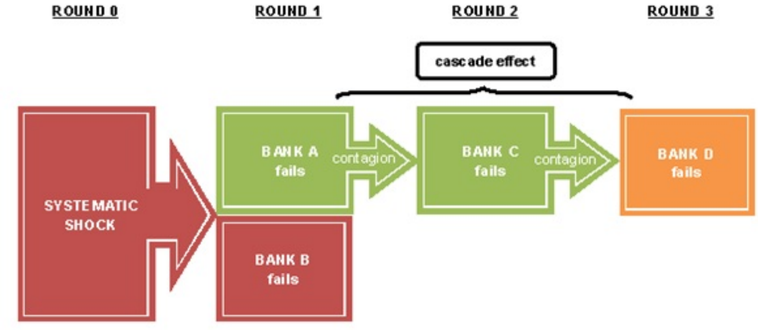
   b) **Spillover Channels:** Main driver, interconnectedness through direct and indirect channels.
4. Others: Size, substitutability.

Systemic Importance: Identification

Spillover effects vs common exposures: Need to separate both and keep only the former for systemic importance. Ideally, identification of systemic importance, needs to look at failures cause by an idiosyncratic shock and its propagation throughout the system. Systematic shocks may overestimate systemic importance.

Cascade or Domino Effects: First round and second+ round effects, account total impact of first failure. Should all round effect be taken into account in order to levy a tax or a capital surcharge on a systemically important institution?

Systemic Importance: Identification



Measuring Systemic Importance: Approaches

1. Indicator based approach: Syntectic indicators based on: total assets, interbank liabilities, share non-traditional banking activities, etc..
2. Network approach: map interconnections between institutions.
3. Market information based approach:

   a) Co-Risk Approach: infer impact of failure or distress of financial institution thought market data.
   b) Portfolio Approach: First, quantify total risk in the system and second, determine contribution of each institution to system-wide risk.

Assess existing market-based measures of Systemic Importance

$\diamond$ Measuring impact rather than fragility.

$\diamond$ Identification, need to disentangle common exposure from spillover channels, in order to properly measure systemic importance.

$\diamond$ Market based measures may never capture cascade or domino effects $\rightarrow$ Network based approach.

Measuring the systemic importance (SI) of financial institutions (FIs): $\Delta CoVaR$.

$\diamond$ Co-risk measures have attracted considerable attention in both academic and policy research.

$\diamond$ Adrian and Brunnermeier (2009,2010): compare VaR of the financial system conditional on FI in distress (CoVaR) to VaR of the financial system in normal times $< 2009 >$ or the CoVaR of the financial system in normal times $< 2010 >$ (both versions extensively applied).

$\diamond$ However, statistical testing procedures to assess the significance of the findings and interpretations based on this co-risk measure "have not yet been developed".

$\diamond$ Emerging literature, Chuang, Kuan and Lin (2009), Billio, Getmansky, Lo and Pelizzon (2010), White, Kim, and Manganelli(2010).

Quantile-based Risk Measures.

- $\diamond$ $VaR_X(\tau) := inf\,\{x \in \mathbb{R} : F_X(x) \geq \ \tau\} .,\ \tau \in (0,1).$
- $\diamond$ $ES_X(\tau)$ (Expected Shortfall).

Add $CoVaR_{X^{index|i(\tau_X)}}(\tau)$ to this family of measures. Where X^{index} returns on index of financial institutions (representing the system)and X^i stock return of the financial institution i (possibly the root of distress).

$$P(X^{index} \leq CoVaR_{X^{index|i(\tau_X)}}(\tau) \mid X^i = VaR_{X^i}(\tau_X)) = \tau,$$

$$\Delta CoVaR^{index|i}(\tau) = CoVaR_{X^{index|i}}(\tau) - VaR_{X^{index}}(\tau).$$

Then $\Delta CoVaR^{index|i}(\tau)$ is the marginal risk contribution (incremental VaR) of institution i; determines the SI.

CoVaR estimation.

Linear Location/Scale Model

$$X_t^{index} = K_t\delta + (\gamma K_t)\varepsilon_t,$$

Quantile (response) Function Representation

$$\begin{aligned} Q_{X^{index}|K}(\tau) &= K_t\delta + (\gamma K_t)Q_\varepsilon(\tau) \\ &= K_t\beta(\tau) \end{aligned}$$

where $\beta(\tau) = \delta + \gamma Q_\varepsilon(\tau)$.
Most applications of Adrian and Brunnermeier's methodology (Linear location-shift model, $\gamma K_t = 1$).

$$X_t^{index} = \mathbf{K}_t\delta + \varepsilon_t,$$

where $\mathbf{K}_t = [\mathbf{Z}_t, X_t^i]$.

Might be extremely restrictive model(s), more on that at the end!

Measuring the SI of FIs: application of $\Delta CoVaR$

◇ Data: daily stock returns (1986-2010) for individual FIs and index of FIs.

◇ CoVaR: conditional quantile function (CQF) (also: quantile response function).

Table: Size and $\Delta CoVaR$ of three European banks

Bank	Assets (millions)	Quantile Regression Results	$\Delta CoVaR$
A	$1,571,768$	$X^{index\|A}(0.99) = 0.026 + 0.526X^{A}(0.99)$	1.38
B	$102,185$	$X^{index\|B}(0.99) = 0.042 + 0.231X^{B}(0.99)$	1.18
C	$10,047$	$X^{index\|C}(0.99) = 0.037 + 0.028X^{C}(0.99)$	0.03

Our contribution: Testing for the SI of FIs.

- ⋄ Conclusion: A is more SI than B and C, and B is more SI than C?
- ⋄ Testing for the strength of the results.

Significance

$$H_0 : \Delta CoVaR^{index|i}(\tau) = 0,$$

test whether CQF differs from un-CQF for FI i

Dominance

$$H_0 : CoVaR_{X^{index|i}}(\tau) > CoVaR_{X^{index|j}}(\tau),$$

test whether CQF conditional on FI i differs from CQF conditional on FI j

Universidad del Rosario NationalBank OF BELGIUM Eurosystem

Quantile treatment effects and $\Delta CoVaR$.

Two-sample treatment effects

- $\diamond$ Treatment group (CQF), with distribution G.
- $\diamond$ Control group (un-CQF), with distribution F.

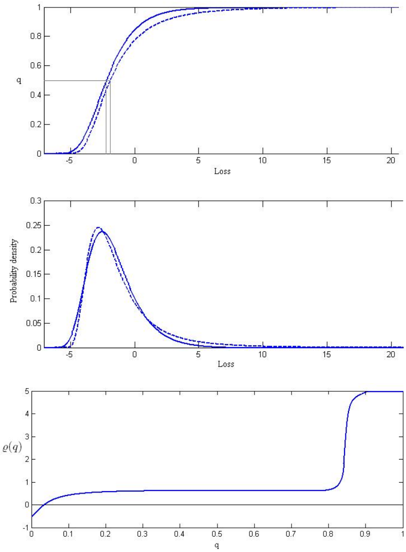
(Non-parametric) estimator of quantile treatment effects

$$\hat{\varrho}(\tau) = \hat{G}_T^{-1}(\tau) - \hat{F}_S^{-1}(\tau),$$

$\Delta CoVaR$ as a quantile treatment effect:

$$\begin{aligned} \Delta\widehat{CoVaR}^{index|i}(\tau) &= \widehat{Q}_{X^{index}|X^i}(\tau) - \widehat{Q}_{X^{index}}(\tau) \\ &= \widehat{F}^{-1}_{X^{index}|X^i}(\tau) - \widehat{F}^{-1}_{X^{index}}(\tau), \end{aligned}$$

Graphical depiction of $\Delta CoVaR$



Inference for Quantile Regression.

H_0 in both significance and dominance test involves CQF. Since CQF is linear, both tests fit in: general linear hypothesis framework:

$$H_0 : R\beta(\tau) = r(\tau), \tau \in \mathcal{T}$$

where $\beta(\tau)$ is p dimensional and q is the rank of matrix R, $(q \leq p)$. Wald (process, indexed by τ) statistic under the null, is:

$$W_T(\tau) = T\frac{(R\widehat{\beta}(\tau) - r(\tau))'(R\hat{\Omega}(\tau)R')^{-1}(R\widehat{\beta}(\tau) - r(\tau))}{(\tau(1-\tau))}$$

where $\hat{\Omega}(\tau)$ is a consistent estimator of $\Omega(\tau)$.

Universidad del Rosario

NationalBank of Belgium Eurosystem

Inference for Quantile Regression.

The Kolmogorov-Smirnov (KS) type statistic:

$$K_T = \sup_{\tau \in \mathcal{T}} \| \hat{W}_T(\tau) \| .$$

$$K_T' = \sup_{\tau \in [\tau_0, \tau_1]} \frac{\hat{W}_T(\tau) - \hat{W}_T(\tau_0)}{\sqrt{\tau_1 - \tau_0}}.$$

Test statistic is distribution free. Critical values: DeLong (1981) and Andrews (1993, 2003) by simulation methods, and more recently by exact methods by Estrella (2003) and Anatolyev and Kosenok (2011).

Simple Test of Significance for $\Delta CoVaR$.

$$Q_{X^{index}|X^i}(\tau) = \beta_0(\tau) + X^i\beta_1(\tau),$$

Theorem

Testing the hypothesis $H_0 := \beta_1(\tau) = 0$ *is equivalent to testing the hypothesis* $H_0 := \Delta CoVaR_{X^{index}|i}(\tau) = 0$, *for a given* τ.

For such simple (two-sided) test $H_0 := \beta_1(\tau) = 0$ we use Wald statistic $W_T(\tau)$.

Define R as a selection matrix $R = [0 : 1]$ and the restriction $r(\tau) = 0$.

Test of significance and dominance using quantile response function.

Theorem

From Theorem 4.1 *and let us define some continuous mapping* $g(\beta(\tau)) = \mathbf{X}\beta(\tau)$, *where this mapping defines the quantile response function, evaluated at some point in the design space.*

$$\sqrt{n}(\hat{Q}_{\mathbf{Y}|\mathbf{X}}(\tau) - Q_{\mathbf{Y}|\mathbf{X}}(\tau)) \rightarrow_d N(0, \tau(1-\tau)\mathbf{X}\Omega(\tau)\mathbf{X}')$$

Test of significance and dominance using quantile response function.

Two different (at least one column is different) design matrices $\mathbf{X}$ and $\mathbf{Z}$ (two different continuous treatment effects applied to the same population Y. The respective empirical quantile response functions are a follows:

$$\hat{Q}_{\mathbf{Y}|\mathbf{X}}(\tau) = \mathbf{X}\hat{\beta}^{x}_{T}(\tau)$$

and

$$\hat{Q}_{\mathbf{Y}|\mathbf{Z}}(\tau) = \mathbf{Z}\hat{\beta}^{z}_{T}(\tau)$$

Test of significance and dominance using quantile response function.

Without loss of generality, we consider equal amount of observations T through out the design space. Therefore, we have the following parametric empirical process:

$$
\begin{aligned}
W_T(\tau) &= \sqrt{T}(\hat{Q}_{\mathbf{Y}|\mathbf{X}}(\tau) - \hat{Q}_{\mathbf{Y}|\mathbf{Z}}(\tau)) \\
&= \sqrt{T}(\tilde{\mathbf{X}}\hat{\beta}^x_T(\tau) - \tilde{\mathbf{Z}}\hat{\beta}^z_T(\tau))
\end{aligned}
$$

Where $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{Z}}$ implies the quantile response function is evaluated at any point of the design space (centroid $(\bar{\mathbf{X}}, \bar{\mathbf{Z}})$ or an extreme quantile of interest).

Recall hypothesis test and statistic

Significance: Two-sided.

$$H_0 : \Delta CoVaR^{index|i}(\tau) = 0,$$

Dominance: One-sided.

$$H_0 : CoVaR_{X^{index|i}}(\tau) > CoVaR_{X^{index|j}}(\tau),$$

Recall hypothesis test and statistic

Statistic

$$W_T(\tau) = T\frac{(R\widehat{\beta}(\tau) - r(\tau))'(R\hat{\Omega}(\tau)R')^{-1}(R\widehat{\beta}(\tau) - r(\tau))}{(\tau(1-\tau))}$$

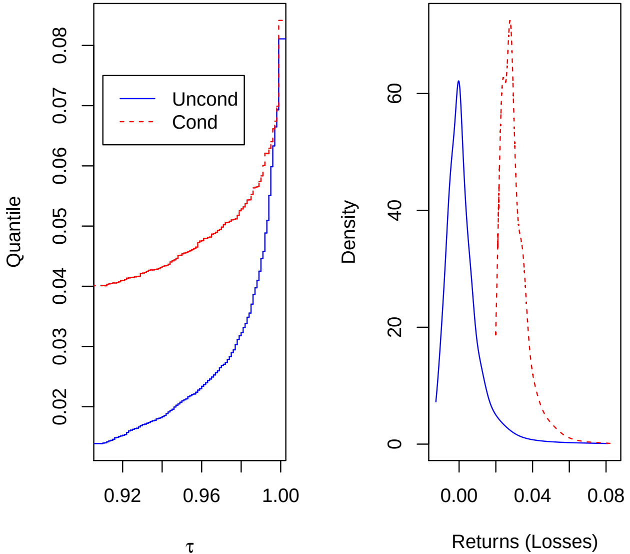
Hypothesis	Significance	Dominance
R	$[\tilde{\mathbf{X}}^i, -1]$	$[\tilde{\mathbf{X}}, -\tilde{\mathbf{Z}}]$
$\hat{\beta}(\tau)$	$[\hat{\beta}^i(\tau), Q_{X^{index}}(\tau)]$	$[\hat{\beta}^i(\tau), \hat{\beta}^j(\tau)]$
r	0	0

Testing for the SI of FIs: significance

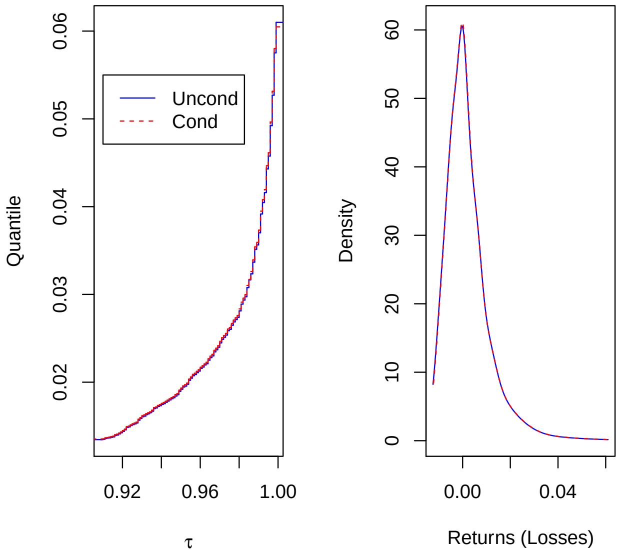
Table: Testing for Significance (p-values)

FI	$\Delta CoVaR$	$H_0 : \beta(0.99) = 0$	$H_0 : \Delta CoVaR(0.99) = 0$
A	1.38	0.000	0.000
B	1.18	0.039	0.000
C	0.03	0.782	0.424

Testing for the SI of FI A: significance



Testing for the SI of FI C: significance

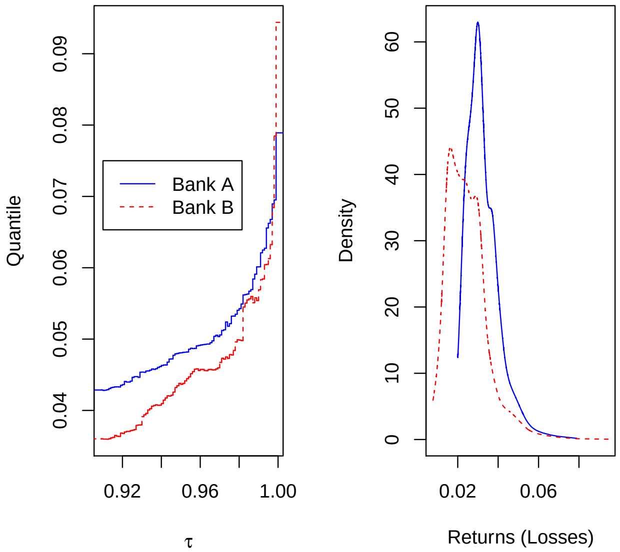


Testing for the SI of FIs: dominance

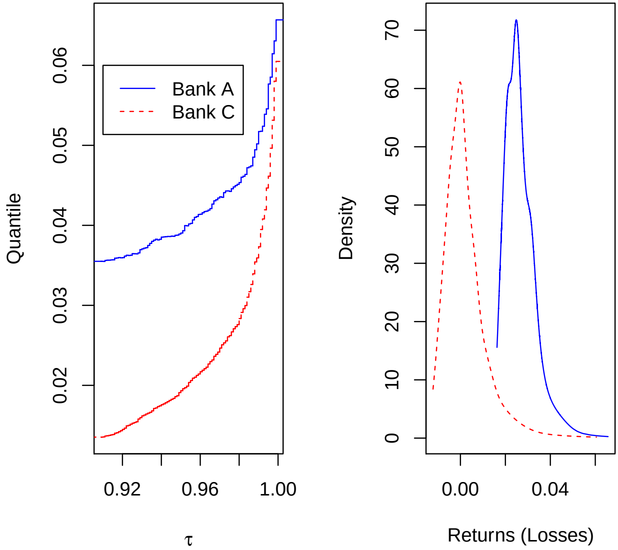
Table: Testing for Dominance (p-values)

FI	$\Delta CoVaR$	$[\tau_0, \tau_1] = [0.90, 0.99]$	$[\tau_0, \tau_1] = [0.10, 0.99]$
AB	1.38	0.000	0.913
AC	1.18	0.000	0.874
BC	0.03	0.000	0.482

Testing for the SI of FI A and B: dominance



Testing for the SI of FI A and C: dominance



Concluding remarks: Measuring and Testing for systemically important financial institutions.

$\diamond$ $\Delta CoVaR$ is interesting tool for measuring SI, but statistical testing is required before interpreting results.

$\diamond$ We develop such tests in linear quantile regression framework. This linear framework (location-shift model and location/scale model) is restrictive.

$\diamond$ work in progress.

- $\diamond$ Power of the test.
- $\diamond$ At some point when $\tau \to 1$, the convergence of the statistic breaks down, Chernozhukov (2000).
- $\diamond$ Test for stochastic dominance at the extremum for a general class of (models) conditional and unconditional quantile functions.

Network/ Graph Theory approach.

Definition

A network (graph) G consist of a non-empty set of elements V(G) called vertices, and a list of unordered pairs of these elements called edges E(G). The set of vertices (nodes) of the network is called a vertex set and the list of edges is called edge list.

If i and j are vertices of G, then an edge of the form (i,j) is said to joint or connect i and j.

In Financial networks:

> Vertices = (financial) institutions.
>
> Edges = counterparty relationships on the asset or liability side of the balance sheet.

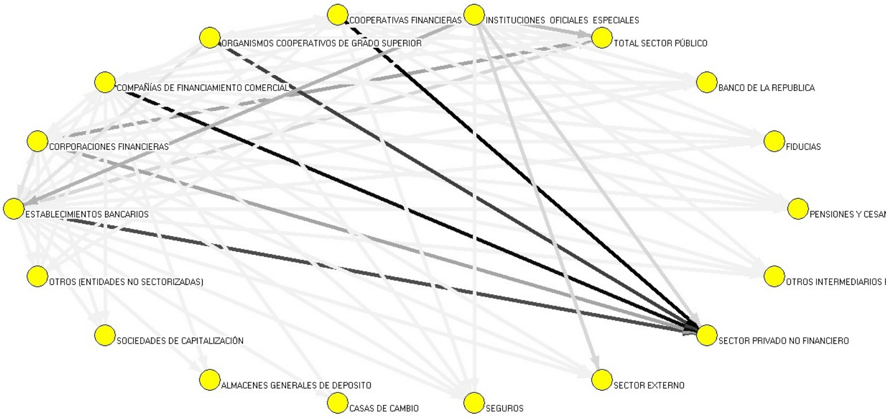
We take counterparty balance sheet annual data (2010) to determine the interconnections of the type of institutions within Colombia's financial sector. The data also allows us to link the financial sector to the public and real sectors of the economy.

Simple and descriptive measures of network topology.

Density: quotient between the number of edges observed in a network over the potential number of edges that could exist. For **complete networks**, density is equal to 1. Overview of strength of interconnections between all vertices in the network. With respect to systemic risk denser networks can be both a *blessing and a curse*.

Degree: measures the number of edges observed for each vertex. **Indegree** of a vertex is the number of edges that it receives, **Outdegree** is the number of edges that it sends, and the **Netdegree** is the sum of the last two.

Core: A k-core is a maximal subnetwork in which each vertex has at least degree k within the subnetwork. This measure allows us to identify similarities in terms of degree between vertices.


COOPERATIVAS FINANCIERAS
INSTITUCIONES OFICIALES ESPECIALES
ORGANISMOS COOPERATIVOS DE GRADO SUPERIOR
TOTAL SECTOR PÚBLICO
COMPAÑÍAS DE FINANCIAMIENTO COMERCIAL
BANCO DE LA REPUBLICA
CORPORACIONES FINANCIERAS
FIDUCIAS
ESTABLECIMIENTOS BANCARIOS
PENSIONES Y CESAN
OTROS (ENTIDADES NO SECTORIZADAS)
OTROS INTERMEDIARIOS F
SECTOR PRIVADO NO FINANCIERO
SOCIEDADES DE CAPITALIZACIÓN
SECTOR EXTERNO
ALMACENES GENERALES DE DEPOSITO
CASAS DE CAMBIO
SEGUROS

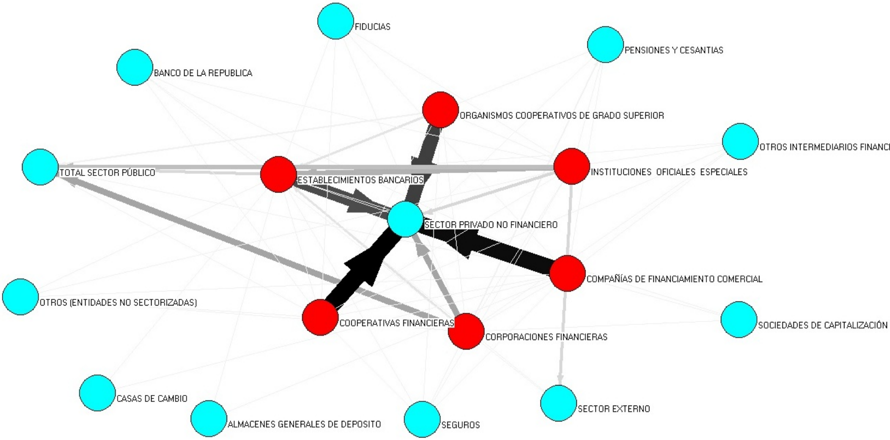
Colombia's financial, public and real sector: Financial Network.

◇ 18 institutions (asset side).

◇ Edges are weighted wrt total asset of individual institution.

◇ Direction of the arrow indicates a position on the asset side of the balance sheet for the originating (sending) institutions.

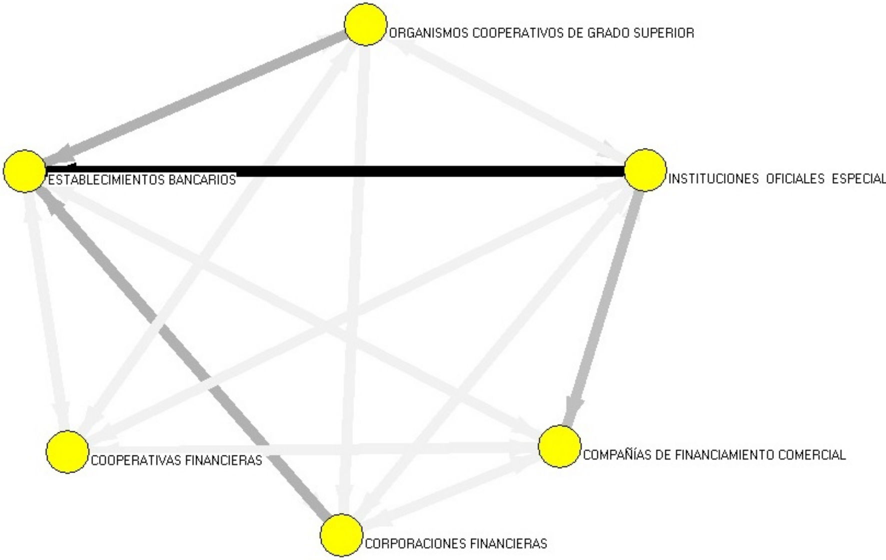
◇ density of this network is 0.27.


FIDUCIAS
PENSIONES Y CESANTIAS
BANCO DE LA REPUBLICA
ORGANISMOS COOPERATIVOS DE GRADO SUPERIOR
OTROS INTERMEDIARIOS FINANCI
TOTAL SECTOR PÚBLICO
ESTABLECIMIENTOS BANCARIOS
INSTITUCIONES OFICIALES ESPECIALES
SECTOR PRIVADO NO FINANCIERO
COMPAÑÍAS DE FINANCIAMIENTO COMERCIAL
OTROS (ENTIDADES NO SECTORIZADAS)
COOPERATIVAS FINANCIERAS
CORPORACIONES FINANCIERAS
SOCIEDADES DE CAPITALIZACIÓN
CASAS DE CAMBIO
ALMACENES GENERALES DE DEPOSITO
SEGUROS
SECTOR EXTERNO

Colombia's financial Network: Core

◇ Only 6 of 18 institutions are on the sending-end of the counterparty relationship.

◇ Relevance of out-degree for only these 6 institutions, performing financial intermediation.


ORGANISMOS COOPERATIVOS DE GRADO SUPERIOR
ESTABLECIMIENTOS BANCARIOS
INSTITUCIONES OFICIALES ESPECIAL
COOPERATIVAS FINANCIERAS
COMPAÑÍAS DE FINANCIAMIENTO COMERCIAL
CORPORACIONES FINANCIERAS

Colombia's financial Subnetwork.

- ◇ Sub-network of 6 institutions.
- ◇ density of this network is 0.83.