Reconciling Views on Banking Competition and Stability: The Role of Leverage

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Abstract

This paper reexamines the classical issue of the possible trade-offs between competition and financial stability by highlighting the key role of leverage. By means of a simple model we show that, even in the cases where competition could contribute to the reduction in the risk of each individual bank's investment, the endogenous increase in leverage will mitigate the reduction of bankruptcy risk and increase the risk of illiquidity and contagion. Thus, the analysis of the relationship between banking competition and financial stability should carefully distinguish between the different types of risks. This allow us to revisit the existing empirical literature from a more rigorous theoretical perspective.

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1 Introduction

Understanding the link between bank competition and financial stability is essential to the design of an efficient banking industry and its appropriate regulation. Because of the relevance of this topic, there is a large body of literature on the issue with path-breaking contributions from both theoretical and empirical perspectives. Yet, in spite of the critical importance of the subject and notwithstanding today's improved understanding of its complexity, brought in by rigorous empirical analysis, there is no clear-cut consensus on the impact of competition on banks' risk taking and the resulting level of stability for the banking industry. Two main contending views rise in the literature: the charter value view that assumes that banks choose their level of risk, e.g. Keeley (1990), and the risk shifting view that risks are chosen instead at the firm level, e.g. Boyd and Nicolo (2005). The charter value approach argues that less competition makes banks more cautious in their investment decisions, as in case of bankruptcy they will loose their future market power rents. The alternative, put forward by the tenants of the risk shifting hypothesis, points out that, as banks inherit the risk that results from the behavior of firms, higher interest rates will lead firms to take more risk and therefore increase the riskiness of the banks' portfolio of loans, leading to the opposite result.

The theoretical debate on banking competition is further obfuscated by the apparently contradictory empirical results. The ambiguous relationship between competition and financial stability is highlighted by Beck, Jonghe, and Schepens (2011) who show the relationship displays considerable cross-country variation. Part of the ambiguity can stem from the difficulty in the choice of a measure for "competition" and for "banks' financial stability". Our aim in this paper is to provide a simple framework that clarifies the concept of financial stability by distinguishing three different types of banking crisis so as to derive a set of testable implications and a better understanding of the empirical literature. This is obtained by means of a theoretical framework where firms' risk choice as a response to the cost of funding is explicitly modeled and banks' leverage is endogenous.

The literature on banking competition has largely ignored the issue of banks' capital structure, taking it as exogenously given. [1] However a classical justification of financial intermediation is precisely the role of banks in security transformation, as in Gurley and Shaw (), and in liquidity insurance, as in Diamond and Dybvig (1983). To fulfill their liquidity provision role, banks have to have the ability to access funds quickly, which also

[1] For example, Boyd and Nicolo (2005) considers banks solely financed by debt; and Martinez-Miera and Repullo (2010) assumes the cost of equity to be independent of banks' risk.

give them the ability to adjust their leverage ratio. The objective of this paper is to recover the role of the leverage ratio in the analysis of the link between competition and financial stability and consider the more complex link of bank competition-leverage-financial stability. Once we acknowledge the key role of leverage as an endogenous variable, the perspective regarding banking competition and its effect on financial stability varies considerably, as the issue of the riskiness of the banks' portfolio of loans is disentangled from the banks' solvency, liquidity and contagion risk. Indeed, banking competition affects both portfolio risk and leverage choices which in turn determine the solvency risk, liquidity risk and systemic risk. in exchange it provides a clarification of the different driving forces in place and their implication on the empirical predictions of the theoretical approach.

The introduction of endogenous leverage implies a higher degree of complexity, but, in exchange, enables us to distinguish the impact of competition on risk at three different levels. First, competition has an impact on the riskiness of a bank's portfolio of loans: fiercer competition can make the loan portfolio safer according to the risk-shifting view. Second, competition and the reduced portfolio risk have an impact on banks' leverage, and, ultimately, on its solvency and liquidity risk: under reasonable assumptions, a safer portfolio leads banks to take on more debt. The high leverage erodes the pro-solvency effects of competition and increases the bank's exposure to funding liquidity risk. Third, contagion from one bank to another resulting in a systemic crisis is more likely when banks are highly leveraged. In short, even if banking competition leads to safer loans, because of the endogeniety of leverage, the solvency risk of banks is not necessarily reduced, their funding liquidity risk is increased and so is systemic risk.

1.1 Overview of the paper

Our framework allow us to explore the impact of competition on the three types of risk and how taking into account the optimal banks' strategy regarding their the structure of their liabilities affects the results.

Because the risk shifting view considers the micro-foundations of risks in the banks' portfolio of loans, we start with a tractable framework where individual loan defaults and portfolio risk diminish with banking competition. The model is in the spirit of Boyd and Nicolo (2005), but allows imperfect correlation in loan default.[2] This is a key realistic assumption that makes leverage relevant in the determination of banks' risk: under per-

[2] Martinez-Miera and Repullo (2010) allows also imperfectly correlated defaults; but the present model is far more tractable and delivers analytical solutions.

fectly correlated loan defaults, unless a bank holds one hundred percent equity, its capital is unable to provide a buffer against the massive loan losses, and the bank's capital level is irrelevant to its solvency. At the other extreme, if correlation is zero, no capital is required because of the law of large numbers.

Using this framework, we show how banks optimally increase their leverage in response to a decrease in their portfolio risk, thus eroding the benefits of a safer portfolio. The implication is that safer loan portfolios are not necessarily associated with safer banks. Indeed, at the bank level, the leverage choice presents a countervailing force to portfolio riskiness. When the asset becomes safer the bank takes higher leverage which reduces solvency; when the asset becomes more risky, banks adapts to lower leverage to avoid high bankruptcy probability. Adjusted by the endogenous leverage, solvency risk does not fluctuate as much as the underlying portfolio risk does. In principle, we may even have a bank's solvency risk independent of its portfolio risk—if the change in portfolio risk is exactly offset by the changing leverage[3]; or, as an extreme case we do not consider in our model, the portfolio risk and solvency risk move in the opposite direction—if the induced large adjustment in leverage is overwhelming.

In addition, a bank's funding liquidity risk—the risk of facing a run—closely links to its leverage. As it is intuitive, and in line with the predictions of a number of models (e.g. **?**), the risk of a bank run depends upon the combination of two factors: high leverage and a fire-sale price for liquidating long-term assets. In particular, the fire-sale penalty, a loss in firm value, will first born by equity holders and then by debt holders. This implies for a bank with high leverage and little equity, the loss is likely to be born by debt holders for the thin equity buffer. The short-term nature of banks' debts allows creditors to be able to withdraw (or, equivalently, not to roll over) their credit on a short notice. As a result, the party who withdraws early won't be hurt by the costly liquidation, while the debt holders who do not run bear the fire-sale loss. As a result, distrust rises among debt holders and the coordination failure can lead to runs on illiquid solvent institutions.

For the financial system as a whole, contagion happens in a similar manner with one more ingredient—greater downward pressure on asset price when more banks fail. We illustrate the point for the two banks case. When both banks sell, the secondary market price becomes even lower; a bank is more vulnerable due to the lower secondary market price. As a result, the chance for a bank to be solvent but illiquid becomes higher when the other one faces a run than when the other does not. Indeed, the systemic risk of contagion

[3] This would be the result of a value-at-risk approach to capital management in line with the Basel internal risk approach.3

can be measured by the difference in the probabilities. We show that such contagion is more likely as leverage increases.

The paper proceeds as follows. Section 2 lays out the model. Section 3 establishes the benchmark case by exploring how banking industry risk levels are determined in equilibrium under the assumption of exogenous leverage. Section 4 is the core of the paper as we analyze the impact of endogenizing banks' leverage on the banks' solvency, liquidity and systemic risk. We devote section 5 to empirical literature. We make testable hypotheses and reinterpret the empirical findings with the refined definition of "financial stability". Relevant policy implications are discussed in section 6. Section 7 concludes.

2 Model Setup

2.1 Portfolio risk and competition

We consider a one good three dates, $t = 0, 1, 2$, economy where all the agents are assumed to be risk neutral and interest rates are normalized to zero in the wholesale competitive market. There are three types of active agents: entrepreneurs, banks and banks' wholesale financiers and one type of purely passive agent: retail depositors. Entrepreneurs are penniless but have access to long-term risky projects. A project costs one unit of investment. It yields a gross return of $x > 1$ if succeeds and 0 if fails. The projects are subject to moral hazard: each entrepreneur chooses the probability of success $P \in [0, 1]$ in order to maximize his expected utility,

$$E(U) = P(x - r) - \frac{P^2}{2B}. \tag{1}$$

Here r is the gross loan rate charged by banks. $B \in [0, \overline{B}]$ represents an entrepreneur's type, with a higher B implying a lower marginal cost of efforts. Entrepreneur types are private information, and in particular, unknown to banks, who hold common prior beliefs that B is uniformly distributed in the interval $[0, \overline{B}]$. Entrepreneurs' reservation utility is normalized to zero.

Because idiosyncratic risk diminishes in a bank's well diversified portfolio of loans we dispense with the modeling of this type of risks and focus, instead, on a bank level risk that affects the whole bank portfolio in the following way: whether the project succeeds or not is jointly affected by entrepreneurs' choice P and a risk factor z. The risk is assumed to be identical for all loans in a bank's portfolio, but can change across banks. It is assumed

z follows a standard normal distributions. Following Vasicek (2002) and Martinez-Miera and Repullo (2010), we assume the failure of a project is represented by a random variable y: when $y < 0$, the project fails. y takes the following specific form

$$y = -\Phi^{-1}(1 - P) + z, \tag{2}$$

where Φ denotes the c.d.f. of standard normal distribution. A project defaults either because of the entrepreneur's moral hazard (a low P) or an unfortunate risk realization affecting the bank's whole portfolio (a low z). For the sake of consistency, note that the probability of success P is given by:

$$Prob(y \geq 0) = 1 - Prob(y < 0) = 1 - Prob(z_i < \Phi^{-1}(1 - P)) = 1 - \Phi(\Phi^{-1}(1 - P)) = P.$$

Banks are assumed to invest in a continuum of projects. We further assume the loan market is fully covered and all types of entrepreneurs are financed. The loan portfolio generates a constant total cash flow, denoted by θ. We denote the maximum possible cash flow by $\overline{\theta}$ and the minimum possible level by $\underline{\theta}$.

In order to focus on bank leverage and risks, we dispense with the specific modeling of loan market competition and consider the loan rate r as an exhaustive information on the degree of competition. On the one hand, low prices (loan rates) are predicted by the mainstream competition model[4] and constitute the driving force in reducing risk in the Boyd and De Nicolo setup. On the other hand, low interest margins are also found associated with less concentrated market, Degryse, Kim, and Ongena (2009). The stylized modeling that financial intermediaries take loan rate as given can also be interpreted in the context of shadow banking. For example, a special investment vehicle (SPV), which buys loans from commercial banks, would take the loan rate as given by some equilibrium of commercial bank competition. The SPV therefore inherits the portfolio risk associated with a given r, chooses its leverage by weighting a variety of risk.

2.2 Funding liquidity risk

Each banks holds a portfolio of loans, and finances it with debt and equity. A bank's debt combines risk-free retail deposits and risky debt issued to wholesale financiers. It is assumed that the insured retail deposits are senior and its supply is fixed. Because retail

[4] The exception are the models based on Broeker (1990) where when the number of banks increases the probability for a bad borrower to get funded in equilibrium increases and so does the equilibrium interest rate.

depositors are insensitive to the banks' risks and play a purely passive role, we assume the amount is inelastically set and equal to F. Each bank's assets consist exclusively of its portfolio of loans and have a size equal to 1.

The debt issued to wholesale financiers is of face value D, risky and uninsured. The debt is jointly financed by a continuum of creditors whose measure is 1. Thus each creditor holds an equal share of the bank's debt, i.e., D. More importantly, the debt is short-term and demandable. The short-term nature of debt also allows a debt holder to withdraw before the risky investment matures. In that case she receives $q \cdot D$, where $1 - q \in (0, 1)$ represents an early withdrawal penalty. Equivalently, the debt contract can simply be viewed as promising an interest rate $q \cdot D$ at time $t = 1$ and D at time $t = 2$. At $t = 0$, a bank finances its total investment of size 1 by raising F and D_0 from creditors and the rest from equity holders ($E_0 + D_0 + F = 1$).

The bank's risky loan portfolio takes two periods to mature. When the bank faces early withdraws, it has to sell part of its portfolio for an illustration. in a secondary market at a discount:[5] for one unit asset with cash flow θ, the bank obtains only for $\theta/(1 + \lambda)$, with $\lambda > 0$.[6] The maturity mismatch and fire-sale discount together expose the bank to the risk of a bank run.

In principle, a bank can fail either at $t = 1$ or $t = 2$. In the former case, the liquidation value of all asset is insufficient to repay early withdrawals. In the latter case, while partial liquidation generates sufficient cash to pay early withdrawals at $t = 1$, the residual portfolio is insufficient to pay creditors who wait until $t = 2$. Once a bank's cash flow is insufficient to repay its debt, either at $t = 1$ or $t = 2$, the bank declares bankruptcy and incurs a bankruptcy cost. For simplicity, we assume the bankruptcy cost is sufficiently high such that once a bankruptcy happens, the wholesale financiers get zero payoffs and only retail deposits are paid.

At $t = 1$ each (wholesale) creditor, observes a noisy signal $x_i = \theta + \epsilon_i$. Based on the information, the creditors play a bank-run game with each others. Each player has two actions: to wait until maturity or to withdraw early. If the bank does not fail at $t = 2$, depositors who wait receive the promised repayment $D \equiv D_0 r_D$. If a creditor withdraws, she receives nothing if the bank fails at $t = 1$ and $D_0 q r_D$ if the bank does not fail at $t = 1$.

[5] The alternative assumption of banks using collateralized borrowing is to generate similar results. See Morris and Shin (2009). The discount can be due to moral hazard, e.g., banks' inalienable human capital in monitoring entrepreneurs, or adverse selection—buyers concerned that banks are selling 'lemon' projects.

[6] The proportional form assumes that buyers of the asset can observe better information than banks creditors. Some justification is provided in Rochet and Vives (2004)

If the bank is only able to pay early withdrawals but goes bankrupt at $t = 2$, the creditors who do not run receive nothing at $t = 2$.

2.3 Contagion

The risk of contagion is illustrated with a two-bank setup: we make a stylized assumption that when both banks need to sell the fire-sale discount increases from λ_1 to λ_2, where the subscripts $1, 2$ denote the number of bank failures. The assumption captures the observation that the secondary market price tends to fall when more banks fail and sell, due to either cash-in-the-market pricing or informational contagion. In the former case, market prices are driven down by the limited supply of cash in the short run. In the latter, investors infer bad macroeconomic fundamental from more bank failures and lower the willingness to pay. The exposure to the same asset price provides a way of financial contagion: when the first bank goes under and sell, the asset price is driven down; this magnifies concerns among debt holders of the other banks', leading to further bank runs.[7]

2.4 Endogenous leverage

Regarding the choice of leverage, we take the simplest standard textbook representation: capital structure is chosen to maximize the leveraged firm values of banks'. In particular, each bank chooses to issue debt of face value D. The debt repayment is exempt from corporate tax, which is assume to be a constant marginal rate τ. Recall that the bank is to incur bankruptcy costs in case of liquidation. A bank therefore trades off between the tax shield and the probability of bankruptcy and the associated (expected) bankruptcy cost. In deciding the their leverage, the bank takes into account both solvency risk and illiquidity risk. To further simplify the model, we further assume that the bankruptcy cost is sufficiently high, such that a bank can only repay its retail deposits F and has zero residual value for wholesale financiers.

2.5 Time line

The timing of the model is summarized in the figure below.

[7] For a full-fledged model that shows asset prices drop with bank runs, see Li and Ma (2012).

t = 0	t = 1	t = 3
1. Banks choose capital structure (D).	1. upon signals, wholesale financiers decide on run or not	1. bank returns realize.
2. Entrepreneurs choose P for given r.	2. possible asset sales	2. creditor who do not run get repaid.

3 Banking risks with exogenous leverage

In this section, we analyze various risk for a fixed level of leverage. We move upward the spectrum of types of risk: from the bottom—individual loan default risk, to the top—risk of the systemic risk of contagion. The full spectrum, including portfolio risk, solvency risk, and funding liquidity risk, is analyzed for its relationship with competition and leverage.

3.1 Loan default and risk shifting

In the spirit of Boyd and Nicolo (2005), we first show that bank competition reduces the default risk of individual loans by curbing entrepreneurs' moral hazard.[8] Note the utility maximization of an entrepreneur of type B yields the following probability of success.[9]

$$P_B^*(r) = \begin{cases} 1 & \text{if } B \in [1/(x-r), \overline{B}] \\ B(x-r) & \text{if } B \in [0, 1/(x-r)) \end{cases}$$

An entrepreneur of type $B \geq 1/(x-r)$ will not default for any finite realization of z. This gives us a natural partition between risk-free and risky loans. For the uniform distribution of B, it implies that α fraction of loans

$$\alpha \equiv 1 - \frac{1}{\overline{B}(x-r)} \tag{3}$$

are risk free, and the complementary fraction $1-\alpha$ of loans

$$1 - \alpha \equiv \frac{1}{\overline{B}(x-r)} \tag{4}$$

[8] We discuss the possible different results by taking a alternative priori of charter values in section 6.
[9] Note $U_E(P_B^*) \geq 0$ such that the participation constraint of entrepreneurs is always satisfied for optimal P.

are risky and have positive probabilities of default. The fraction of risky loans α measures the riskiness of a bank's loan portfolio: a larger α implies greater portfolio risk.

The risk of the portfolio decreases with bank competition. When banks charge lower loan rates under fierce competition, entrepreneurs have more skin in the game': having more stake in their projects, entrepreneurs put in more efforts. Note that as the incentive of risk shifting dampens, the pool of safe loans grows.

$$\partial\alpha/\partial r = \frac{-1}{\overline{B}(x-r)^2} < 0. \tag{5}$$

3.2 Portfolio risk: loan loss and cash flow

We now characterize banks' portfolio risk more precisely: deriving the distribution of loan losses and that of cash flow. Denote the fraction of non-performing loans *in the risky pool* by γ. We show γ follows a uniform distribution on $[0, 1]$.

Lemma 1. *The loan loss γ, defined as the fraction of defaults in the risky pool, follows a uniform distribution on* $[0, 1]$.

Proof. Take a risky type $\hat{B} < 1/(x-r)$; and define the fraction of entrepreneurs below $\hat{B}$ in the risky pool by $\hat{\gamma}$. We have

$$\hat{\gamma} = \frac{\hat{B} - 0}{1/(x-r) - 0} = \hat{B}(x-r).$$

Consider $z = \Phi^{-1}(1 - P^*_{\hat{B}})$ such that $\hat{B}$ does not default but all types $B < \hat{B}$ do. One will have $\gamma = \hat{\gamma}$. To derive the distribution of γ, notice that

$$\begin{aligned} F(\hat{\gamma}) &\equiv Prob(\gamma < \hat{\gamma}) = Prob(z > \Phi^{-1}(1 - P^*_{\hat{B}})) = 1 - Prob(z < \Phi^{-1}(1 - P^*_{\hat{B}})) \\ &= 1 - \Phi(\Phi^{-1}(1 - P^*_{\hat{B}})) = P^*_{\hat{B}} = \hat{B}(x-r). \end{aligned}$$

Note that by $\hat{\gamma} \equiv \hat{B}(x-r)$, one has $\hat{B} = \hat{\gamma}/(x-r)$. Substitution yields

$$F(\hat{\gamma}) = \hat{\gamma}.$$

It is an identity function over the interval $[0, 1]$ and therefore implies $\gamma \sim U(0, 1)$. □

The uniform distribution implies the expected loan loss in the risky pool is always $1/2$. The riskiness of the portfolio depends only on the proportion of the risky pool, which in turn shrinks with competition and lower loan rates r. That is, banking competition reduces the risk of each individual loans, and it also reduces the riskiness of a bank's loan portfolio.

Denote θ the cash flow generated by one unit investment,

$$\theta \equiv \alpha r + (1-\alpha)[0 \cdot \gamma + r \cdot (1-\gamma)] = r - (1-\alpha) r \cdot \gamma. \tag{6}$$

For the loan loss enters the cash flow expression in a linear way, the portfolio cash flow follows a uniform distribution as well. The distribution function of θ can be derived from that of γ:

$$\begin{aligned} G(\hat{\theta}) &\equiv Prob(\theta < \hat{\theta}) = Prob(r - (1-\alpha) r \cdot \gamma < \hat{\theta}) = Prob(\gamma > \frac{r - \hat{\theta}}{(1-\alpha) \cdot r}) \\ &= 1 - Prob(\gamma < \frac{r - \hat{\theta}}{(1-\alpha) \cdot r}) = 1 - \frac{r - \hat{\theta}}{(1-\alpha) \cdot r} = \frac{\hat{\theta} - \alpha r}{r - \alpha r}. \end{aligned}$$

It is a uniform distribution on $[\alpha r, r]$. Denote the length of the support by $w \equiv (1-\alpha) r$, the cash flow can be represented as $\theta = r - w\gamma$.

A higher loan rate presents a trade-off between risk and returns: a bank's expected cash flow is lower but less volatile in a competitive environment.

$$\begin{aligned} E(\theta) &= r - wE(\gamma) \\ Var(\theta) &= w^2 Var(\gamma) \end{aligned}$$

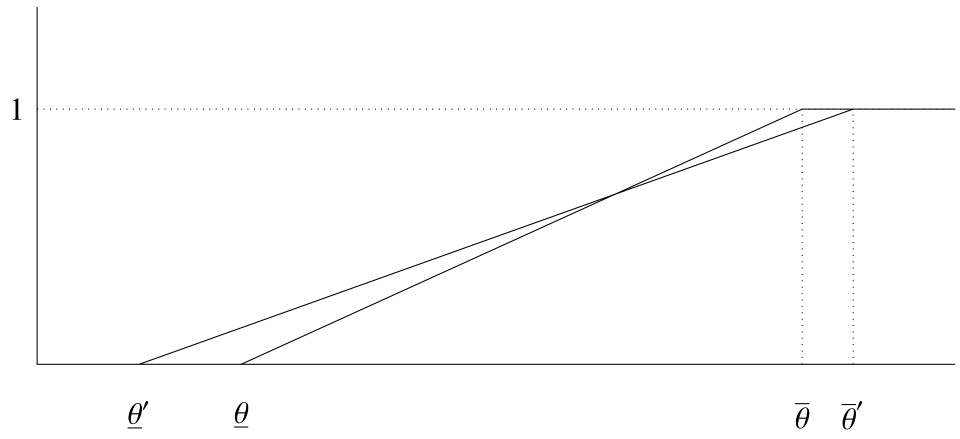
[The comments below are not entirely correct: $E(\theta)$ is not a monotonic function of r.] When competition reduces r, both the expectation and variance of the cash flow decrease, for $dE(\theta)/dr > 0$ and $dVar(\theta)/dr > 0$. Intuitively, as r increases, a larger fraction of loan portfolio becomes risky, but the bank earns a higher interest income for all the loans that do not default.[10] The results are summarized in the lemma below.

Lemma 2. *The random cash flow $\theta \sim U(\alpha r, r)$. When banking competition reduces the loan rate r, both the expectation and the variance of the cash flow drops.*

The figure above depicts two distribution functions of cash flows, associated with different levels of competition. When competition intensifies, the distribution function becomes steeper, implying a smaller variance. The mean (middle point) however can move shift to the right with more competition.

[10]Intuitively the expected cash flow is the r minus the expected loss from the risky pool, whose weight is $\frac{1}{B(x-r)}$. Since the randomness of of the cash flow only comes from $\gamma \sim U[0,1]$, the variance only depends on the size of risky pool.

Figure 1: Cash flow distribution under two different levels of competition



Note a monopolistic bank that maximizes the expected net cash flow would set $r = x - \sqrt{x/2\overline{B}} \equiv \overline{r}$.[11] It is never rational to set loan rate higher than $\overline{r}$, in which case the risk shifting effect on entrepreneurs' side is dominant such that the increase in loan losses erode all the extra repayment from non-default projects. On the other hand, for the portfolio features constant returns to scale, the net expected cash flow must be positive. Denote $\underline{r}$ such that $\theta(\underline{r}) = 1$. We therefore focus on loan rates $r \in [\underline{r}, \overline{r}]$.

3.3 Solvency Risk

In this subsection, we define a bank's solvency risk under exogenous leverage. A bank is solvent if its cash flow meets its liability,

$$\theta = r - w\gamma \geq F + D.$$

The inequality gives a critical level of loan loss

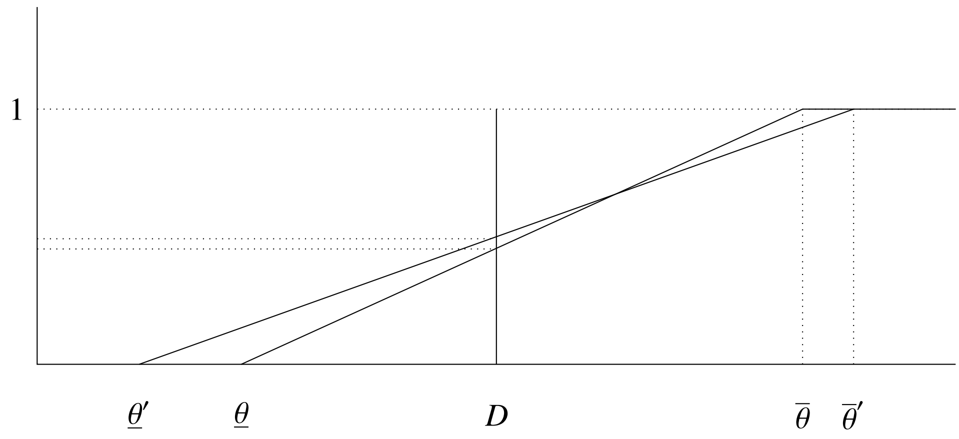
$$g \equiv \frac{r - (F + D)}{(1 - \alpha)r}.$$

A bank with a realized loan loss greater than g is to be insolvent. Given $\gamma \sim U(0, 1)$, it implies the solvency probability is equal to g. The bank's pure solvency risk (SR), i.e., the probability of failure in the absence of liquidity risk is

$$SR \equiv \frac{(F + D) - \alpha r}{r - \alpha r} \tag{7}$$

[11] It is straightforward to check $E(\theta)$ is strictly concave in r.

Figure 2: Change of solvency risk under exogenous leverage



Note that the solvency probability is not monotonic in r. The reason is as in Martinez-Miera and Repullo (2010). While a lower loan rate decreases the portfolio loss, it also makes interest margin thinner and banks less profitable, providing less cushion to absorb loan losses.[12]

Proposition 1. *For given leverage, a bank's solvency risk is reduced by competition, i.e.,* $\partial SD/\partial r > 0$*, if and only if* $-r^2 + (x - r + 1)(F + D) < 0$.

As we want to explore the Boyd and Nicolo (2005) argument in the most favorable context, we will focus on the case where a lower loan rate decreases solvency risk and examine whether leverage and liquidity risk increase as a result. Therefore throughout the paper we assume for fixed leverage the risk-shifting effect always dominates the margin effect,

$$\frac{\partial SR}{\partial r} > 0. \tag{8}$$

Graphically, this requires $\partial\underline{\theta}/\partial r > 0$ so that the distribution function should satisfy a single crossing condition and the fixed face value of debt should lie to the left of the crossing point. The picture below illustrates the increase in bankruptcy probability when the loan rate hikes from r to r', as a result of reduced banking competition.

Expression (7) implies that the solvency risk of a financial institution is determined by both its portfolio risk and its leverage ratio, a causal link that we explore in section 4.

[12]**[To be checked in the new setup]** It is worth mentioning that the result of Martinez-Miera and Repullo (2010) hinges on distributional assumptions. In the current setup, replace the utility function of entrepreneurs' by $U_E = P(x - r) - \frac{B}{2}P^2$. One can derive a new distribution function $\tilde{F}(\gamma) = (x - r)/[\overline{B} - (\overline{B} - \underline{B})\gamma]$, according to which the probability of solvency monotonically decreasing in r.

3.4 Funding liquidity risk and bank run

In this section we examine banks' funding liquidity risk and derive a critical level of cash flow for a bank to be solvent but illiquid: the bank is able to repay in full its $t = 2$ liability if no one runs it at $t = 1$, but will default if sufficient many short-term debt holders withdraw early. In particular, we show the illiquidity threshold is proportional to the face value of debt. The model is based on the global games approach due to Carlsson and Van Damme (1993). In the appendix we show that for 2-by-2 games, the same result holds under the argument of risk dominance of Harsanyi and Selten (1988).[13]

In principle, a bank can fail either at $t = 1$ or $t = 2$. In the former case, the liquidation value of all assets is insufficient to repay early withdrawals. In the latter case, while partial liquidation generates sufficient cash to pay early withdrawals, the residual portfolio is insufficient to pay creditors who wait until $t = 2$. Once a bank's cash flow is insufficient to repay its debt, either at $t = 1$ or $t = 2$, the bank declares bankruptcy and incurs a bankruptcy cost. For simplicity, we assume the bankruptcy cost is sufficiently high such that a bank can only repay F after bankruptcy; $\theta - F$ is dissipated in bankruptcy.

We will focus on the case where runs make it more difficult for a bank to meet its debt obligation. Otherwise, the repayment is more easily met under fire sale price, which would be paradoxical. This implies the following inequality holds:

$$\frac{1}{1+\lambda} < q, \tag{9}$$

which is naturally true as q approaches 1.

Denote by L the fraction of wholesale financiers who run the bank. The bank will fail at $t = 1$ iff $\theta/(1+\lambda) \leq LqD$, or

$$L \geq \frac{\theta}{1+\lambda} \cdot \frac{1}{qD} \equiv L'.$$

The bank is to survive $t = 1$ withdraws but fail at $t = 2$ iff $(1 - f)\theta < F + (1 - L)D$, where $f = (1+\lambda)LqD/\theta$ denotes the fraction of asset sales to meet $t = 1$ debt obligation. In terms of L, the bank fails at $t = 2$ iff

$$L \geq \frac{\theta - F - D}{[(1+\lambda)q - 1)]D} \equiv L''.$$

[13] For an elegant reference on the concept, its connection to the popular approach of global games and its relevance in this context, see Van Damme (2002).

The lack of common knowledge leads to the so-called Laplacian property of global games: no matter what signal a player i observes, he has no information on the rank of his signal as compared to the signals observed by the others. Denote by M the fraction of players that player i believes to observe a higher signal than his. The Laplacian property implies $M \sim [0, 1]$.

It is further assumed that players take a switching strategy: to run the bank if the observed signal is smaller than s^* and to wait otherwise. As the Laplacian property in particular holds for the player who observes the critical signal, the player will hold a belief that $M \sim [0, 1]$ fraction of players will not run the bank and the rest $L \equiv 1 - M$ run. Consequently, from the perspective of the player who observes the the probability for the bank to survive at $t = 1$ is

$$Prob(t = 1 \text{ survival}) = Prob(1 - M \leq L'|s = s^*) = \min\{L', 1\}.$$

For the retail deposits to be risk-free, a bank should be able to repay F in the worst scenario: all wholesale financiers withdraw early and γ's realization turns to be 1. It implies

$$\alpha r - qD(1 + \lambda) > F, \tag{10}$$

which further implies

$$\theta > (1 + \lambda)qD$$

Therefore, $L' > 1$ and $Prob(t = 1 \text{ survival}) = 1$: banks do not default at $t = 1$.

The probability of $t = 2$ bankruptcy is

$$Prob(t = 2 \text{ survival}) = Prob(1 - M \leq L''|s = s^*) = L''.$$

Depending on the outcome of bank run games, the payoffs for "run" and "wait" are tabulated as follows.

	t = 1 failure	t = 2 failure	otherwise
run	0	qD	qD
wait	0	0	D

When running the bank, a creditor receives qD with $Prob(t = 1 \text{ survival}) = L'$. By waiting, she receives D with a lower probability $Prob(t = 2 \text{ survival}) = L''$. Therefore, in playing the bank run game, a creditor trades off between a higher chance of receiving non-zero payoff and the higher payoff from waiting.

To be consistent with the definition of switching strategy, a creditor who observes the critical signal s^* should be indifferent between running the bank or not.

$$Prob(1 - M \leq L'|s = s^*) \cdot q = Prob(1 - M \leq L''|s = s^*) \quad \text{or} \quad q = \frac{\theta - F - D}{[(1+\lambda)q - 1)]D},$$

The indifference condition implies the following critical cash flow $\hat{\theta}$ for a given fire-sale price.

$$\theta^* = F + F + [1 - q(1 - (1+\lambda)q)]D \tag{11}$$

Banks run successfully happens when θ falls below the critical θ^*. Define $\mu \equiv 1 - q[1 - (1+\lambda)q]$ and note that $\mu > 1$ for $(1+\lambda)q > 1$. We know a bank is solvent but illiquid if

$$F + D < \theta \leq F + \mu D. \tag{12}$$

Further notice that the critical cash flow decreasing in λ such that lower asset price leads to a higher chance of illiquidity.

Proposition 2. *There exists a critical level $\theta^* = F + \mu D$, $\mu = 1 - q[1 - (1+\lambda)q] > 1$, a bank whose cash flow $\theta \in [F + D, \theta^*]$ becomes solvent but illiquid: being able to sustain in the absence of bank runs, but going bankrupt as a run happens.*

Proposition 2 states that there is pure liquidity risk for banks in the range $[F + D, F + \mu D]$, as those banks are solvent in the absence of bank runs but insolvent if a run occurs. A bank's funding liquidity risk is reflected by the probability to be solvent but illiquid,

$$\frac{(\mu - 1)D}{(1 - \alpha)r}. \tag{13}$$

Once a bank's debt obligation is exogenous, its funding liquidity risk increases with competition. The result follows directly from the first order derivative. Intuitively, the lower cash flow due to intensified competition provides a thinner buffer against fire-sale losses. Early creditors who early withdraw are more likely to incur a loss to those who wait. The coordination failure is therefore aggravated, and bank run happens more frequently as a results.

Proposition 3. *For a given level of debt obligation, the chance for a bank to be solvent but illiquid increases with competition.*

Two three factors contribute to illiquidity: (1) low fire-sale prices for banks' assets, (2) low capital buffer (or equivalently, high debt obligations) and (3) weak fundamental and low cash flows. For fire-sale price and leverage are held exogenous, competition contributes to illiquidity by reducing the expected cash flows.

3.5 Systemic risk of contagion

The price for banks' assets tends to fall when more banks fail and are forced to liquidate, due to either cash-in-the-market pricing or informational contagion. In the former case, market prices are driven down by the limited supply of cash in the short run. In the latter case, investors infer bad macroeconomic fundamental from more bank failures and lower the willingness to pay. We illustrate such contagion and its relationship with leverage in a two-bank model. In particular, we make a stylized assumption that when both banks need to sell the fire-sale penalty increases from ξ_1 to ξ_2, where the subscripts $1, 2$ denote the number of bank failures.

Following the same procedure of the last section, we will be able to derive a critical cash flow level

$$\theta_2^* = F + \mu_2 D > \theta_1^*, \tag{14}$$

where $\mu_2 = 1 - q[1 - (1 + \lambda_2)q] > \mu_1$. A bank whose cash flow falls between $[\theta_1^*, \theta_2^*]$ will be solvent and liquid if the other bank does not face a run, but will become solvent but illiquid if runs happen to the other bank. Namely, contagion happens to a bank whose cash flow falls between $[F + \mu_1 D, F + \mu_2 D]$, if the other bank's falls between $[F + D, F + \mu_2 D]$. We therefore define the exposure to the risk of contagion by

$$\frac{(\mu_2 - \mu_1)D}{w} \tag{15}$$

The exposure to the same asset market provides a way of financial contagion. When the first bank goes under and sell, the asset price is driven down; this raises concerns among debt holders of the other banks'.

4 Endogenous leverage and its impacts

It is important to notice that the portfolio risk is not identical to the default risk of a bank. Leverage plays a crucial role: a low risk portfolio financed with high leverage can still fail with a significant chance. Furthermore, leverage is endogenously chosen based on the portfolio risk: a bank with safer portfolio will be able to use higher leverage and one with risky portfolio will find it more difficult to issue debts. In this sense, leverage is a countervailing force to the change in asset riskiness. In this section, we endogenize banks' choice of leverage and analyze its impact on different types of risks.

4.1 Endogenous leverage

For the sake of simplicity we choose the text book view of capital structure, where the optimal structure is determined by the marginal cost of bankruptcy equaling the marginal tax shield loss of additional equity. This is obtained for the banks' capital structure that maximize the market value of equity:

We assume banks' capital structure is chosen to maximize their leveraged firm values. For the market value of equity

$$V_E = \int_0^g (1-\tau)[\theta - F - D] d\gamma$$

and the market value of debts

$$V_F = F \qquad \text{and} \qquad V_D = \int_0^g D d\gamma,$$

the optimal capital structure maximizes $V_E + V_F + V_D$, with the positive net present value condition implying $V_E + V_F + V_D > 1$. In choosing the debt face value D, a bank trades off between tax shield of debt and the expected bankruptcy cost. In particular, a bank solves the following maximization program.

$$\max_D \int_0^g \big[(1-\tau)\theta + \tau(F+D)\big] d\gamma + \int_g^1 F d\gamma$$

with $\theta = r - w\gamma$. The first order condition

$$(1-\tau)\Big[r\frac{\partial g}{\partial D} - \frac{w}{2} 2g \frac{\partial g}{\partial D}\Big] + \tau g + \tau(F+D)\frac{\partial g}{\partial D} - F\frac{\partial g}{\partial D} = 0$$

yields

$$D^* = \frac{\tau \cdot (r-F)}{(1-\tau)\mu^2 + 2\tau\mu}. \tag{16}$$

And it is straightforward to check that the second order condition satisfies,

$$-(1-\tau)\frac{w}{2} 2 \big(\frac{\partial g}{\partial D}\big)^2 + 2\tau \frac{\partial g}{\partial D} < 0.$$

Denote the constant coefficient $c \equiv \tau/[(1-\tau)\mu^2 + 2\tau\mu]$. The optimal debt obligation is written for convenience

$$D^* = c \cdot (r - F). \tag{17}$$

The result is summarized in the following theorem.

Proposition 4. *A bank that maximizes its value by trading off tax shields versus bankruptcy cost sets its debt* $D^* = c \cdot (r - F)$.

A few natural observations can be made. The optimal debt obligation increases in the tax shield, $\partial D^*/\partial \tau > 0$, and decreases in the liquidity risk, $\partial D^*/\partial \mu < 0$. Further notice that as $\mu \searrow 1$, the coefficient $c \nearrow \tau/(1+\tau)$. Given the monotonicity of c in μ, it holds that $c < \tau/(1+\tau) < 1$: a bank cannot issue more risky debt claims than its maximum cash flow after paying the risk-free F and is unwilling to issue risky debt more than $\tau/(1-\tau)$ fraction of $(r - F)$.

A bank's leverage ratio, l, can be measured by the face value of its risky debt over the available expected cash flow

$$l \equiv \frac{D^*}{E(\theta - F)} = \frac{2c(r-F)}{(1+\alpha)r - 2F}. \tag{18}$$

The leverage ratio is not monotonically in r. Its comparative statics with respect to competition depends on the relative strength of two countervailing effects: (1) When r increases, a bank generates a higher cash flow and can issue more claims, including debts, which we call margin effects. And (2) a higher r implies stronger risk-shifting by entrepreneurs, leading to higher portfolio risk and curbs leverage via bankruptcy costs, which we call risk effects.

Proposition 5. *The leverage ratio* $D^*/E(\theta - F)$ *decreases with competition (decreases with loan rate r) if and only if* $xF < r^2$. *Otherwise, the result reverses.*

Proof. The result follows directly from the first order condition.

$$\begin{aligned}\frac{\partial l}{\partial r} &= \frac{1}{[(1+\alpha)r - 2F]^2}\left[-(1-\alpha)F - \frac{\partial \alpha}{\partial r} r(r-F)\right] \\ &= \frac{1-\alpha}{[(1+\alpha)r - 2F]^2}\frac{r^2 - xF}{x - r}\end{aligned}$$

The comparative statics depends on the sign of

$$r^2 - xF. \tag{19}$$

When it is positive, competition leads to a lower leverage ratio. □

In the current setup, as F decreases, a bank has more cash flow available to its wholesale financiers. In that case, when r increases, the margin effect dominates the risk effect. And overall the leverage ratio rises.

4.2 Risk under endogenous leverage

Proposition 6. *As competition intensifies, pure solvency risk and funding liquidity risk move in opposite directions. In particular, if* $xF < r^2$*, pure solvency risk decreases with competition and funding liquidity risk increases in competition. Otherwise, the result reverses.*

Proof. The result follows directly the first order derivatives. We start with pure solvency risk.

$$SR \equiv 1 - \frac{r - F - D^*}{w} = 1 - \frac{(1-c)(r-F)}{w}.$$

The comparative statics follows

$$\frac{\partial SR}{\partial r} = -(1-c)\frac{\partial}{\partial r}\left(\frac{r-F}{w}\right)$$

For $c < 1$, the expression shares the same sign as

$$-\frac{\partial}{\partial r}\left(\frac{r-F}{w}\right).$$

Now examine liquidity liquidity risk

$$IL \equiv (\mu - 1)\frac{D^*}{w} = (\mu - 1)c\frac{r-F}{w}.$$

It follows

$$\frac{\partial IL}{\partial r} = (\mu - 1)c\frac{\partial}{\partial r}\left(\frac{r-F}{w}\right),$$

whose sign is the same as

$$\frac{\partial}{\partial r}\left(\frac{r-F}{w}\right).$$

It is now clear that the with the change in competition r, the two types of risks move in the opposite direction. □

Corollary 1. *The total credit risk of individual bank, i.e., pure solvency risk plus funding liquidity risk, reduces with competition, if and only if* $xF < r^2$*. Otherwise, the result reverses.*

Proof. Notice that the total credit risk

$$TCR \equiv 1 - \frac{r - F - \mu D^*}{w} = 1 - (1 - \mu c)\frac{r - F}{w}$$

The result follows directly the first order derivative.

$$\frac{\partial TCR}{\partial r} = -(1 - \mu c)\frac{\partial}{\partial r}\left(\frac{r - F}{w}\right)$$

Since $\mu c < 1$, the total credit risk is determined by

$$-\frac{\partial}{\partial r}\left(\frac{r - F}{w}\right).$$

The result is therefore proved. □

To grasp some intuition on the proposition, consider the two types of banks corresponding to the two possible signs of $xF - r^2$.

A negative sign, with $xF < r^2$, corresponds to less productive firms, with banks financed through market funding and high interest rates on loans. When this is the case, total credit risk is reduced with competition. When this is the case, the above proposition shows that the Boyd and Nicolo (2005) result obtains. This will occur for a low x implying a risky portfolio of loans ($P^*_B(r)$ increases with x for every value of r). Banks have a relatively high average cost of funds because of a low F and a high income as r is high. As a particular case, $F = 0$ in which case x is irrelevant, corresponds to this case and might be interpreted as investment banking. More competition means safer investment banking.

A positive sign, with $xF > r^2$, corresponds to highly productive firms, with banks mainly financed through deposits and low interest rates on loans. When this is the case, a high x implies a safer portfolio of loans ($P^*_B(r)$ is higher for every value of r). Banks have a lower average cost of funds because of the higher F and a lower income as r is low. This might be closer to retail banking. The opposite result occurs, and more competition will increase total risk.

Proposition 7. *The exposure to the risk of contagion,* $(\mu_2 - \mu_1)D^*/(1 - \alpha)r$*, decreases with competition (decreases with loan rate r) if and only if* $xF < r^2$*. Otherwise, the result reverses.*

Proof. The result follows directly the first order derivative

$$\frac{\partial}{\partial r}\left(\frac{(\mu_2 - \mu_1)D^*}{(1-\alpha)r}\right) = (\mu_2 - \mu_1)\frac{\partial}{\partial r}\left(\frac{r - F}{w}\right)$$

□

4.3 Comparison to the exogenous leverage case

To make it clear the impact of endogenous leverage, we tabulate below the results for a side-by-side comparison.

Table 1: As r decreases, banking risk under exogenous and endogenous leverage

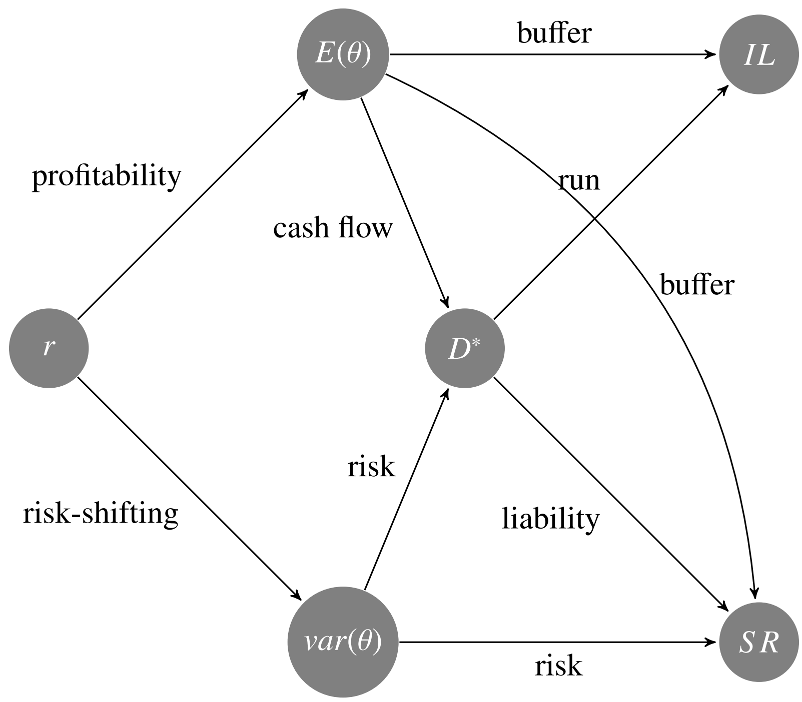
	$xF \leq r^2$		$xF > r^2$	
	exogenous D	endogenous D	exogenous D	endogenous D
Pure solvency risk	-	decreasing	decreasing	increasing
Pure liquidity risk	increasing	increasing	increasing	decreasing
Total credit risk	-	decreasing	decreasing	increasing
Exposure to contagion	increasing	increasing	increasing	decreasing

5 Reinterpreting the empirical literature

The difficulties in analyzing the link between competition and financial stability are exponentially increased when we turn to the empirical studies. The empirical analysis has led to a multiplicity of results that are sometimes difficult to reconcile and susceptible of alternative interpretations. The task at hand is not an easy one. Indeed, there is not a unique measurement of financial stability and there is not a unique measurement of competition. The measurement of competition ranges from franchise value (Tobin's Q), to market structure (e.g., HHI, C-n), to structural measurement (i.e., P-R H-stat., Lener's index, Boone's indicator), and to institutions (contestability of the market, e.g., activity and entry restrictions). There is no unambiguous answer to which measurement reflect competition best.[14]. In our opinion, competition does not only change the cash flow characteristics but also provides an environment where banks optimize and interact. Its impact is present in every aspect of bank behaviors and the resulting risk. The complexity, as mentioned before, is

[14]See Degryse, Kim, and Ongena (2009) for a comprehensive review on measuring banking competition. It should be mentioned, nevertheless, that concentration measures may be poor proxies for bank competition, Claessens and Laeven (2004) and Schaeck, Cihák, and Wolfe (2009)

Figure 3: Linkages between Banking Competition, Leverage and Various Risk



Despite of our efforts to build a comprehensive model, the presented still considerably understates the complexity of the issue, for competition also affects banks' portfolio choice, e.g., the correlation of their portfolios, cash hoarding, and so on. For simplicity, the figure also suppresses the feedback from risk to *endogenous* leverage.

beyond the scope of our framework. Bearing this limitation in mind, we focus on different facets of bank risk.

Financial stability has multiple dimensions. The empirical implication of our model is that banks' risks should be measured at four fundamentally different levels: first, at the level of banks' assets; second, at the level of banks' solvency; third, at the level of banks' liquidity risk and fourth at the level of the overall systemic risk and contagion. Competition can affect solvency risk and liquidity/systemic risk differently. Banks' endogenous leverage presents a central hub that connects all types of risk. As banks optimally set their leverage in light of the resulting risk, the optimal leverage is jointly determined with banks' solvency, liquidity and systemic risk. Banking competition sets the stage for a rich interaction: competition directly affects various risks via changing bank cash flows; and once leverage is endogenous and reacts to competition, the risks are also affected by the endogenous leverage. Depending on the magnitude of the direct and indirect forces, a full diversity of predictions can rise.

As a consequence, our reading of the empirical literature results introduces drastic differences depending on whether the evidence concerns the riskiness of banks' assets, the riskiness of banks themselves, either their solvency or their liquidity, or systemic risk. The long distance that separates the cup from the lips is here simply the banks' leverage choice.

Our theoretical framework suggests a progressive approach to the understanding of the impact of competition on banks' risk taking by refining the questions that are asked as successive layers.

1. Does competition increase the safety of a banks portfolio of assets? In other words is the Boyd & DeNicole's basic assumption true?

   Next, once we take into account the optimal reallocation of assets and the fundamental effect of optimal leverage tuning by the bank, the following issues are to be addressed.

2. Does competition increase the riskiness of banks?

3. Does competition increase the liquidity risk of banks?

4. Does competition increase banks' systemic risk?

Revisiting the empirical literature through this filter lead us to regroup the empirical results in a different way, as some papers jointly address the first three questions without any reference to the endogenous risk reallocation and leverage. In the end of the section, we summarize in table 2 the empirical literature by highlighting the used risk, competition measurements and the results on whether competition leads to financial instability.

5.1 Portfolio risk: non-performing loans

Our model follows the assumptions of Boyd and Nicolo (2005) and, because it postulates competition will reduce the riskiness of the portfolio of loans it makes no empirical predictions regarding the impact of increased competition on the banks' asset risks. Still, knowing whether Boyd & De Nicolo's basic assumption is in line with empirical evidence is a crucial step forward. In order to measure the riskiness of assets, some measures, e.g. stock volatility in Demsetz, Saidenberg, and Strahan (1996); Brewer and Saidenberg (1996), are contaminated by leverage. In our judgment, the non-performing loans (NPLs) ratio is the most accurate measurement for the riskiness of banks' assets, and the huge bulk of literature appears to support this view, as it considers non-performing loans as one of the key variables in the assessment of banks' risks.

Some caveats are here in order regarding the accuracy of this measure. First, banks can manipulate it by rolling over bad loans. Second, a risky loan granted today will only default in the future (e.g. after a two-year lag if we follow Salas and Saurina (2003) and the rate of default will depend upon the business cycle (Shaffer 98). Although the latter might be corrected by the introduction of macroeconomic risk controls, such as the GDP growth rate, the time lag may be more difficult to correct because of the persistence of the non-performing loans ratios. Third, the riskiness of assets could also be altered by changing the portfolio allocation among the different classes of risks. A bank with higher market power may be willing to take more risks on its assets that will result in higher NPLs in order to obtain a higher expected return while its market power on, say, deposits provides a natural buffer that prevents its financial distress.

Restricting the measurement of risk to NPLs implies focusing on a very limited part of the broad link between competition and financial stability where we might hope for some consensus on the empirical results. Unfortunately even with this is drastic reduction the evidence is mixed. So, in spite of the fact the charter value and risk shifting theories have completely opposite predictions regarding the impact of banks' competition on non-performing loans, empirical studies give no definitive answer on which one should be the predominant view.

The initial paper on the charter value Keeley (1990) did not consider NPLs measures but rather estimates of overall bank risk of failure. The prediction on NPLs is backed by more recent work, such as the analysis of Salas and Saurina (2003) and Yeyati and Micco (2007). The authors found an increase in non-performing loans as bank competition increased in Spain and in eight Latin American countries respectively. Support for the risk shifting hypothesis comes from Boyd et al. (2006), Berger et al.(2008) find an interesting set of results based upon both loan risk and overall bank risk. Using cross-sectional data on 29 developed countries for the years 1999 through 2005, they find that banks with a higher degree of market power exhibit significantly more loan portfolio risk.

The impact of the US introduction of Nationwide banking also leads to contradictory results: while Jayaratne and Strahan (1998) report that "Loan losses decrease by about 29 basis points in the short run and about 48 basis points in the longer run after statewide branching is permitted", Dick (2006) finds out that "charged-off losses over loans (...) appears to increase by 0.4 percentage point following deregulation.

5.2 Individual bank risk: insolvency

It should be clear from our theoretical model that portfolio risk is not the same as banks' default risk: leverage plays a key role. Such divergence between the impact of competition on the riskiness of banks' assets (as measured by NPL) and its overall risk is perfectly illustrated in Berger, Klapper, and Turk-Ariss (2009): in spite of finding confirmation of the risk shifting hypothesis in the NPL analysis show that banks with a higher degree of market power have lower overall risk exposure mainly due to their higher equity capital levels.

Since Keeley (1990) the literature has been focusing on the risk of individual bank failure. In his classic paper, Kelley considers the market-value capital-to-asset ratio and the interest cost on large, uninsured CD's. Following his approach, Demsetz, Saidenberg, and Strahan (1996) use seven different measures of BHCs' risks and in each of them franchise value is statistically significant providing support to the charter value theory (annualized standard deviation of weekly stock returns, Systematic risk, Firm-specific risk, Capital-to-Assets Ratio, Loans-to-Assets Ratio, Commercial and Industrial Loans-to-Assets Ratio and Loan Portfolio Concentration). Brewer and Saidenberg (1996) found also corroborating evidence that the standard deviation of B&S stock returns was negatively related to S&L franchise values as measured by the market-to-book asset ratio. Also confirming the charter value perspective, Salas and Saurina (2003) obtain that capital ratio increases with Tobin's Q, thus providing some evidence on the possible behavior of the (endogenous) leverage ratio.

In our opinion, pure solvency risk can be best measured by z-scores. Still, there are important nuances in these results.Beck, Jonghe, and Schepens (2011) show on average, a positive relationship between banks' market power, as measured by the Lerner index, and banks' stability, as proxied by the Z-score that measures the distance from insolvency. Nevertheless, they find large cross-country variation in this relationship. Jiménez, Lopez, and Salas (2010) empirical evidence supports the franchise value paradigm but only if market power is measured by Lerner indexes based on bank-specific interest rates and bank risk.

Opposing this view, Boyd and Jalal (2009) provided cross-country empirical evidence supporting the risk-shifting model using several proxies for the measure of bank risk (a z-score based on bank returns on assets (ROA), its dispersion measured as ¦Ò(ROA), and the ratio of equity to total assets) Using a US sample and a cross-country one they consistently find that banks' probability of failure is negatively and significantly related to measures of competition. Confirming this view, Nicolo and Ariss (2010) analyze the impact of large deposit and loan rents and show that they predict higher probabilities of bank failures and lower bank capitalization.

5.3 Individual bank risk: illiquidity

Funding liquidity risk has largely been overlooked in the empirical study.[15] One might argue that upon observing bank failures, it is difficult, if ever possible, to distinguish pure solvency issue from illiquidity ones, (Goodhart, 1987). However, just as solvency risk can be measured ex ante by z-scores, illiquidity can be measured from an ex ante perspective too. For example, in their study of bond pricing, **?** identify the extra yield due to illiquidity risk: as far as the yield reflects default probability, the liquidity risk can be reflected in bond pricing. While the relationship between funding liquidity risk and competition has not been studied, theoretical models do provide sound guide for estimating the risk: funding liquidity risk is negatively affected by returns (e.g., ROA) and the assets market liquidity (e.g., reserves and cash), and positively by the amount of uninsured short-term funding. Morris and Shin (2009) provide further practical guide and in the context of herding behavior **?** present an attempt to measure the risk by liquidity ratio.

[15] On contrast, even though theoretical models made no prediction on how competition affects leverage, which in turn affects solvency risk, the empirical study has taken into account solvency risk adjusted by leverage by using measurements like z-scores.

5.4 Systemic risk

The analysis of systemic risk is, obviously, the most difficult one as it has to deal with cross-country analysis and the main driving force for changes in market power are related to banking deregulation, market entry, deposit insurance and a number of joint measures of which increased competition is only one of the consequences.[16] The precise definition of a banking crisis itself as well as its timing is subject to different interpretations. Thus, while some authors consider the intervention of exceptional measures by the Treasury, or a 10% of the banking industry being affected, others like Anginer, Demirgüç-Kunt, and Zhu (2012) and De Nicolo et al. (2004) prefer measuring the probability of systemic risk by pairwise distance to default correlation or constructing an indicator of the probability of failure for the five largest banks.

According to Beck, Demirgüç-Kunt, and Levine (2006) on a sample of 69 countries over a 20 year period more concentrated national banking systems are subject to a lower probability of systemic banking crisis. Still, they point out that concentration need not be related to market power, as already mentioned by Claessens and Laeven (2004), and that other measures of competition may lead to the opposite result. Contradicting the result of Beck, Demirgüç-Kunt, and Levine (2006), Schaek et al. (2006) show, using the Panzar and Rosse H-Statistic as a measure for competition in 38 countries during 1980-2003, that more competitive banking systems are less prone to systemic crises and that time to crisis is longer in a competitive environment even if concentration and the regulatory environment is controlled for.

[16] It should be noted that with newly developed measurements on systemic risk such as *CoVaR* in Adrian and Brunnermeier (2010), one can in principle regress an individual bank's systemic risk contribution on its learner index.

Table 2: Does banking *competition* lead to *instability*? Diverse risk/competition measurements and results from the empirical literature.

Paper	Risk	Competition	Results	Data Source	Comments
Keeley (1990)	Interest Cost	Tobin's q	Yes	US	also via capital
Demsetz, Saidenberg, and Strahan (1996)	Stock Volatility	Market-Book Value	Yes	US	via capital/diversification
Brewer and Saidenberg (1996)	Stock Volatility	Market-Book Value	Yes	US S&L banks	
Jayaratne and Strahan (1996, 1998)	NPLs	Deregulation	No	US	
Salas and Saurina (2003)	Loan Loss	Tobin's q	Yes	Spain	also via capital
De Nicolo and Loukoianova (2005)	Z-Score	HHI	No	Non-industrialized	interaction with ownership
Beck, Demirgüç-Kunt, and Levine (2006)	Crisis Dummy	Concentration	Yes	Cross-Country	
Beck, Demirgüç-Kunt, and Levine (2006)	Crisis Dummy	Contestability	No	Cross-Country	
Dick (2006)	Loan Loss	Deregulation	Yes	US	
Yeyati and Micco (2007)	Z-Score & NPLs	P-R H-Stat.	Yes	Latin America	
Schaeck and Cihák (2010a)	Capitalization	P-R H-Stat.	No	Developed	IV
Dell'Ariccia, Igan, and Laeven (2008)	Lending standard	Number of banks	Yes	US	mortgage market
Boyd and Jalal (2009)	Loan Loss	HHI	No	US/Cross-Country	
Boyd and Jalal (2009)	Z-Score	HHI	No	US/Cross-Country	
Boyd, Nicolo, and Loukoianova (2009)	Crisis Dummy	HHI/C3	No	Cross-Country	
Berger, Klapper, and Turk-Ariss (2009)	NPLs	Lerner Index/HHI	Yes	Developed	via capital
Berger, Klapper, and Turk-Ariss (2009)	Z-Score	Lerner Index/HHI	No	Developed	via capital
Schaeck, Cihák, and Wolfe (2009)	Crisis Dummy	P-R H-Stat.	No	Cross-Country	
Schaeck, Cihák, and Wolfe (2009)	Duration until crisis	P-R H-Stat.	No	Cross-Country	
Schaeck and Cihák (2010b)	Z-Score	Boone's Indicator	No	US/EU	via efficiency
Jiménez, Lopez, and Salas (2010)	NPLs	Lerner Index	Yes	Spain	
Jiménez, Lopez, and Salas (2010)	NPLs	HHI/C5		Spain	
Beck, Jonghe, and Schepens (2011)	Z-Score	Lerner Index	Yes	Cross-Country	cross-country heterogeniety
Nicolo and Ariss (2010)	Z-Score	Deposit market rent	Yes	Europe	
Nicolo and Ariss (2010)	Z-Score	Loan market rent	No	Europe	
Dick and Lehnert (2010)	Personal bankruptcy	Deregulation	Yes	US	
Dick and Lehnert (2010)	NPLs/risk management	Deregulation	No	US	
Anginer, Demirgüç-Kunt, and Zhu (2012)	D-to-D Correlation	Lerner Index	No	Cross-Country	supervision/ownership

Our paper's empirical prediction states here that an increase of competition may have different effects depending upon the amount of deposits, the profitability of projects and banks' spreads, thus suggesting new lines for future empirical research based on the differentiation of different types of banking systems. It would be interesting to pursue this research by distinguishing among different types of banks. If we interpret literally our model, this would be to distinguish banks with low deposit to asset ratios from those with a high deposit to asset ratio. Still, more generally, this could be interpreted as dividing the banks according to their different access to short maturity market funds.

6 Conclusion and discussion

Caveats:

- Welfare effects of competition
- Focus only on loan market price competition. Little on deposit market competition and non-price competition.
- Focus only on liquidity risk due to increasing leverage and liability. The incentive to hold liquid asset is not analyzed.
- Focus only on contagion due to asset price/collateral value, says nothing on common risk exposure.

Appendix A The bank run game

In the appendix, we derive the critical cash flow for the bank to be solvent but illiquid. The first subsection assumes specific distribution on private signal, which allows deriving explicit belief updates of creditors. The second subsection shows the application of risk dominance concept, which is potentially a intuitive shortcut for the result.

Appendix A.1 Bank run games with explicit belief updating

Denote the fraction of creditors who run the bank by L. The bank will fail at $t = 1$ if $LD > (D + E)\theta/(1 + \xi)$, or, by denoting leverage $d \equiv D/(D + E)$

$$\theta < (1 + \lambda)L \cdot d \equiv \theta^*. \tag{A.20}$$

Similarly, the bank will sustain $t = 1$ bank run but fail at $t = 2$ if $(1 - L)Dr_D > (1 - f)(D + E)\theta$. Here f denotes the fraction of partial liquidation carried out on date-1. Specifically,

$$f = (1 + \lambda)\frac{L \cdot d}{\theta}.$$

We can derive the critical level cash flow for $t = 2$ failure

$$\theta < (1 - L) \cdot d \cdot r_D + (1 + \lambda)L \cdot d \equiv \theta^{**}. \tag{A.21}$$

Depending on the value of θ, the payoffs for "run" and "wait" are tabulated as follows.

	$\theta < \theta^*$	$\theta^* < \theta < \theta^{**}$	$\theta^{**} < \theta$
run	0	D	D
wait	0	0	Dr_D

When running the bank, a creditor receives D with $Prob(\theta > \theta^*)$. By waiting, she receives Dr_D with a lower probability $Prob(\theta > \theta^{**})$. Therefore, in playing the bank run game, a creditor trades off between a higher chance of receiving non-zero payoff and the higher payoff from waiting.

A creditor chooses her action based on the private noisy signal $x_i = \theta + \epsilon_i$. To facilitate the solution I impose the following distributional assumption: the noise follows a uniform distribution on $[-\sigma, \sigma]$. The resulting ex post distribution of θ conditional on observing x_i is a uniform distribution on $[x_i - \sigma, x_i + \sigma]$. We focus on the case where σ approaches 0: the private signal is as much accurate as desired but is still imperfect and underpins the lack of common knowledge on θ.

It is assumed that players take a switching strategy: to run the bank if the observed signal is smaller than x^* and to wait otherwise. Equilibrium is characterized by a set of three variables $\{x^*, \theta^*, \theta^{**}\}$, such that (1) upon observing the critical signal x^*, a creditor is indifferent between running the bank or not, and (2) conditional on the realization of θ, creditors act according to the equilibrium switching strategy, which lead to outcomes consistent with the definition of θ^* and θ^{**}.

$$\begin{cases} Pr(\theta > \theta^* | x^*) = Pr(\theta > \theta^{**} | x^*) \cdot r_D \\ \theta^* = (1 + \lambda)L(\theta^*, x^*) \cdot d \\ \theta^{**} = [1 - L(\theta^{**}, x^*)] \cdot d \cdot r_d + (1 + \lambda)L(\theta^{**}, x^*) \cdot d \end{cases}$$

The first equation is the indifference condition. And the second and the third equations correspond to the regime change and define θ^* and θ^{**}. The distributional assumption yields a system of linear equations in x^*, θ^* and θ^{**}.

$$\begin{cases} \frac{x^*-\theta^*+\sigma}{2\sigma} = \frac{x^*-\theta^{**}+\sigma}{2\sigma} \cdot r_D \\ \theta^* = (1+\lambda)\frac{x^*-\theta^*+\sigma}{2\sigma} \cdot d \\ \theta^{**} = [1 - \frac{x^*-\theta^{**}+\sigma}{2\sigma}] \cdot d \cdot r_d + (1+\lambda)\frac{x^*-\theta^{**}+\sigma}{2\sigma} \cdot d \end{cases}$$

Solving the system, the 2nd and 3rd equations yield

$$\begin{cases} \theta^* = \frac{x^*+\sigma}{\frac{2\sigma}{(1+\lambda)d}+1} \\ \theta^{**} = \frac{(\sigma-x^*)dr_D+(1+\lambda)(x^*-\sigma)d}{2\sigma-dr_D+(1+\lambda)d} \end{cases}$$

Plug the expression of θ^* and θ^{**} in the the 1st equation. The left hand side becomes

$$\frac{x^*+\sigma}{2\sigma+(1+\lambda)d}.$$

The right hand side becomes

$$\frac{x^*+\sigma-dr_D}{2\sigma-dr_D+(1+\lambda)d}r_D.$$

When the two are equal, x^* obtains. We focus on the limit case where $\sigma \to 0$. It yields solution

$$x^* = \frac{1+\lambda}{1+(1+\lambda)(1-1/r_D)} \cdot d \cdot r_D$$

Also note that

$$\lim_{\sigma\to 0} \theta^* = x^* \quad \text{and} \quad \lim_{\sigma\to 0} \theta^{**} = x^*.$$

We therefore derive a single critical level of $\hat{\theta}$ such that the a bank run successfully happens at $t = 1$ and otherwise the bank survives.

$$\hat{\theta} = \mu d r_D\,,\ \mu = \frac{1+\lambda}{1+(1+\lambda)(1-1/r_D)} \tag{A.22}$$

The critical level increases in d and λ, but decreases in the early withdrawal penalty r_D.

Result 1. *As σ approaches 0 such that private signals tend to be fully accurate, there exists a critical level $\hat{\theta} = \mu D$, $\mu > 1$, a bank whose cash flow $\theta \in (D, \hat{\theta})$ becomes solvent but illiquid: being able to sustain in the absence of bank runs, but going bankrupt as a run happens.*

Appendix A.2 liquidity risk under risk dominance

The two-creditor setup is apparently a stylized assumption, but allows us to illustrate bank-runs as a debt holder coordination failure in its simplistic form. We pick the risk the dominant equilibrium to highlight the possible coordination failure.

In this section, we analyze an individual bank's funding liquidity risk at early withdraw and fire-sales. Recall that we assume a loan portfolio whose cash flow is θ can only be sold for $\xi_1\theta$, $\xi_1 \in (0, 1)$ in the secondary market. This implies when one of the debt holders withdraw early and demand $D/2r_D$, a fraction

$$\lambda_1 = \frac{D}{2r_D} \cdot \frac{1}{\xi_1\theta}$$

of the loan portfolio has to be liquidated. This leaves residual cash flow

$$(1 - \lambda_1) \cdot \theta = \theta - \frac{D}{2r_D} \cdot \frac{1}{\xi_1}$$

within the bank. The expression shows a low secondary market price can raise concerns for the remaining debt holder: the fraction of liquidation λ climbs as the secondary market price drops (a lower ξ_1), resulting in a low residual cash flow for the remaining creditor. In this sense prices lower than fundamental values can ignite panic among a bank's debt holders and lead to a bank-run. To be more precise, the standard debt contract implies the debt holder who does not run receives

$$min\Big\{(1 - \lambda_1) \cdot \theta, \frac{D}{2}\Big\}.$$

It is a best response for the waiting depositor to run the bank as well when

$$\frac{\xi_1\theta}{2} > min\Big\{(1 - \lambda_1) \cdot \theta, \frac{D}{2r_D}\Big\}.$$

When the inequality holds, the 2-by-2 bank run game has two strict equilibria: (1) both debt holders run or (2) both wait. The inequality indicates a bank run is possible under weak fundamentals (low θ), high leverages (high D) or secondary market illiquidity (low P). The reasonings are all the same: weak fundamentals, high leverages, low secondary market prices—all contribute to low residual cash flow for the party who does not run. So the bank run equilibrium can rise as long as

$$\frac{\xi_1\theta}{2} > (1 - \lambda_1) \cdot \theta.$$

We select one of the equilibria according to the argument of risk dominance, Harsanyi and Selten (1988). The deviation loss from the "wait" equilibrium is

$$(1 - \frac{1}{r_D})\frac{D}{2}.$$

And the deviation loss from the "run" equilibrium is

$$\frac{\xi_1 \theta}{2} - \frac{1 - \lambda_1}{2 r_D}\theta.$$

For simplicity, we consider the case where banks have monopolistic power in deposit markets and the outside option for debt holders is zero. Further more, the deposits are insured so that it does not reflect risk. This indicates $r_D \to 1$ and the deviation loss from "wait" approaches zero. "Run" becomes risk dominant as long as

$$\frac{\xi_1 \theta}{2} > \theta - \frac{D}{2\xi_1}.$$

It defines a critical level of cash flow below which a bank run is to take place.

$$\theta_1^* = \frac{D}{\xi_1(2 - \xi_1)}$$

The fire-sale penalty and leverage makes it possible for a bank to be solvent but illiquid.

Proposition 1. *$\exists\ \theta_1^* = D/[\xi_1(2 - \xi_1)] > D$, a bank whose cash flow $\theta \in [D, \theta_1^*]$ becomes solvent but illiquid: being able to sustain in the absence of bank runs, but going bankrupt if a run happens.*

Proof. Simply note for $\xi_1 \in (0, 1)$,

$$\frac{1}{\xi_1(2 - \xi_1)} > 1.$$

This is to give $\theta_1^* > D$ and the existence proves. □

Naturally, when the fire-sale discount approaches zero ($\xi \to 0$), θ_1^* converges to D.

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